Neutrino Oscillations and Theory Biases

Peter B. Denton

BNL Summer Student Lecture

June 17, 2022

Speaking from the Setauket territory

About Me

1. Grew up in Michigan
2. Bachelors in physics and math from Rice, '10
3. PhD from Vanderbilt, '16
4. Year at Fermilab working with Stephen Parke, '15-'16
5. Postdoc at the Niels Bohr International Academy, '16-'18
6. Faculty at Brookhaven, '18-present

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Research interests

- Neutrino oscillations
- New physics in neutrinos
- Astroparticle physics
- Black holes
- Dark matter

Other interests

- Ultimate frisbee
- Hiking
- Piano
- Photography

Stop by 2-16 anytime

Key points

- Measuring neutrinos requires the biggest detectors
- Quantum mechanical neutrino oscillations occur on human scales
- Neutrinos continue to **surprise**

Neutrino masses: only left handed neutrinos?

- Neutrinos: fermions only feel the weak (left) interaction
- Measure right handed fermions through electric charge
- Right handed neutrinos won't scatter off anything
- They don't exist?
- Neutrinos are massless?

This was the standard assumption until 1998!

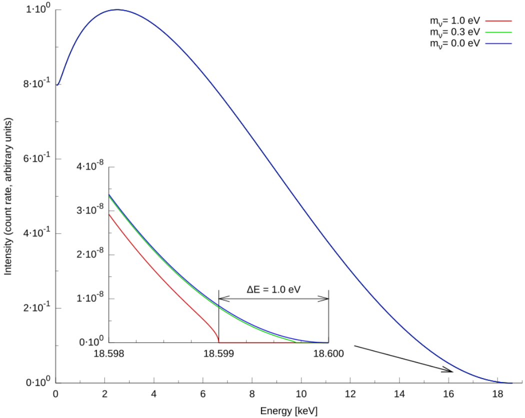
Let's do a direct kinematic search

KATRIN 2006

$$^3_1H \rightarrow ^3_2 He + e^- + \bar{\nu}_e$$

For massless neutrinos, what is the maximum electron energy?

Neutrino masses: kinematic end point is hard



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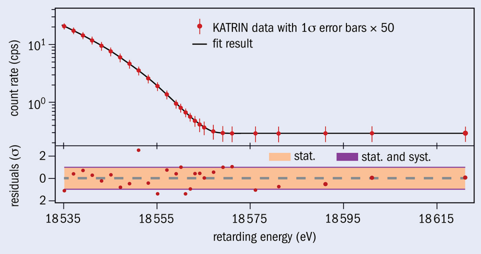


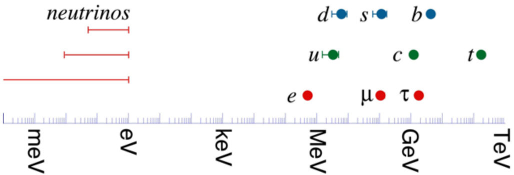
KATRIN 2018

$$m_\nu \lesssim 1 \text{ eV}$$

Neutrino masses: small numbers?

- Other fermions get their mass from the Higgs field

  See H. Davoudiasl's lecture on Tuesday, June 14
- "Expect" Yukawa couplings: $y \sim 1$
- Top quark: $y_t \sim 1$, but electron: $y_e \sim 10^{-6}$
- Neutrinos: $y_\nu < 10^{-12}$ or nothing if no right handed neutrinos
- Weird?



Big surprise of 1998

- Electroweak understood, mediators (γ, W, Z) found
- Strong understood, mediators (gluon) found
- All fermions detected except tau neutrino (2000), but no surprises expected
- Higgs boson still to be found
- Standard Model looks to be in great shape
- Top movie: Titanic

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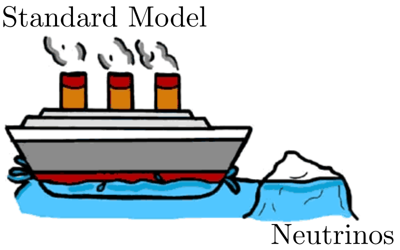
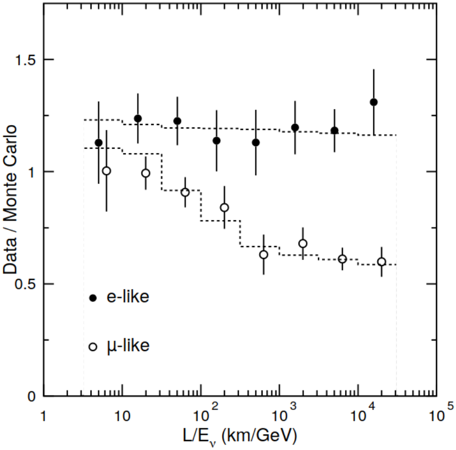


Atmospheric neutrinos disappear

Cosmic rays hit the atmosphere, produce π^+, μ, and ν_μ



SuperKamiokande hep-ex/9807003

Neutrinos really oscillate

1. Neutrinos experience time $\Rightarrow$ must have mass
2. Neutrino oscillate $\Rightarrow$ must mix & masses must be different

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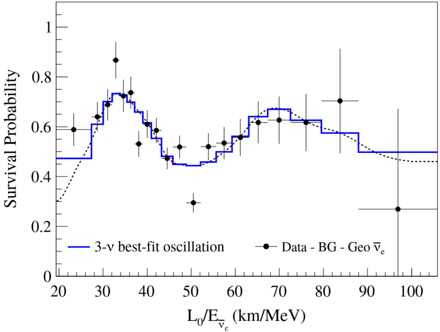
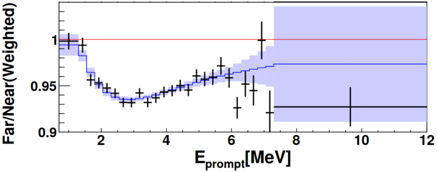


KamLAND 1303.4667



Daya Bay 1809.02261

Two flavor oscillation probability

Only one mixing angle, one $\Delta m^2_{31} \equiv m^2_3 - m^2_1$, no complex phase

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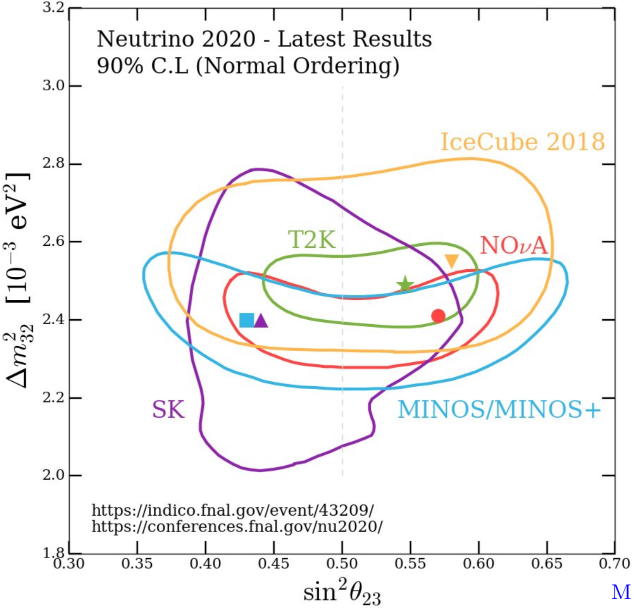
What angle leads to maximal oscillations?

Atmospheric parameters



M. Ross-Lonergan

Maximal mixing: atmospheric neutrinos

Mixing for atmospheric angles seems to be maximal $\theta_{23} \sim 45^\circ$

	θ_{23}	θ_{13}	θ_{12}	δ
Quarks	2.4°	0.20°	13°	69°
Leptons	$\sim 45^\circ$	X	X	unknown

Was an expectation that mixing angles should be small

Other atmospheric experiments had hints for oscillations, didn't frame it since "mixing angles should be small"

Solar neutrinos

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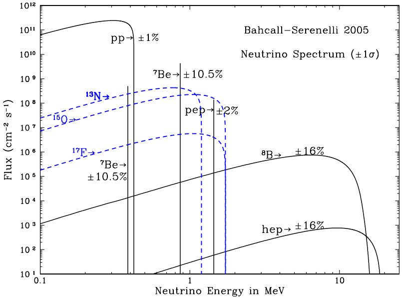
Problem: Too few neutrinos from the sun

Solar neutrinos

Problem: Too few neutrinos from the sun

1. John Bahcall predicted the solar neutrino flux



^{8}B flux $\propto T^{24}$

J. Bahcall et al. nucl-th/9601044

Solar neutrinos

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$E_{\nu,\mathrm{tr}} = 0.8$ MeV

Homestake ApJ. 496 (1998) 505-526

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Perhaps both are wrong

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3. 1990s: SNO used heavy water

Leased the water from a reactor for 1CAD (+ lots of insurance)

SNO nucl-ex/0204008

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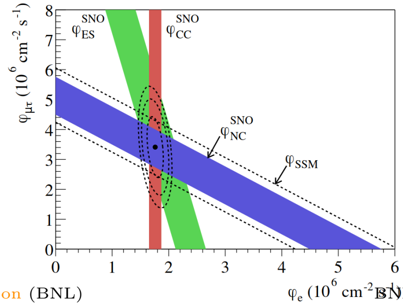

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Solar neutrinos: matter effect

Presence of a dense electron field modifies oscillations

L. Wolfenstein PRD 17 (1978)

S. Mikheev, A. Smirnov Nuovo Cim. C9 (1986) 17-26

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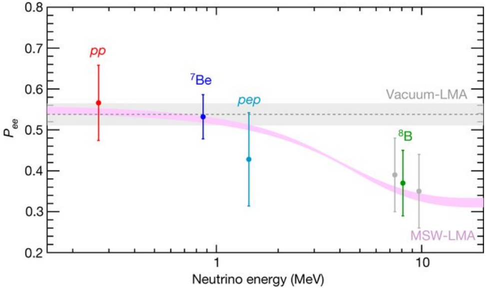
Low energy: no matter effect

$$P_{ee} \simeq 1 - \frac{1}{2}\sin^2 2\theta_{12}$$

High energy: large matter effect

$$P_{ee} \simeq \sin^2 \theta_{12}$$



Borexino

What mixing angle fits this data?

	θ_{23}	θ_{13}	θ_{12}	δ
Quarks	2.4°	0.20°	13°	69°
Leptons	$\sim 45^\circ$	X	33°	unknown

Two large angles

Surely θ_{13} will be small?!

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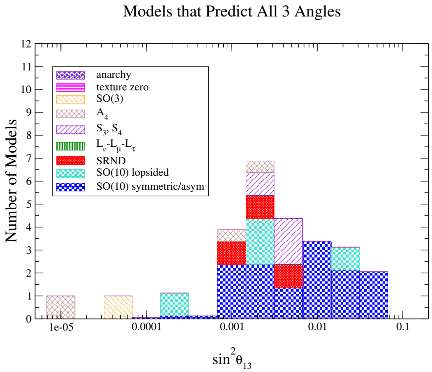

C. Albright, M-C. Chen hep-ph/0608137

	θ_{23}	θ_{13}	θ_{12}	δ
Quarks	2.4°	0.20°	13°	69°
Leptons	$\sim 45^\circ$	8.5°	33°	unknown

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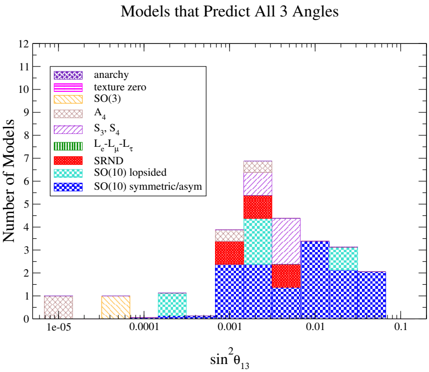

True value:

$\sin^2\theta_{13} = 0.02$, $\theta_{13} = 8.5^\circ$

Quite large!

C. Albright, M-C. Chen hep-ph/0608137

Complex phase: δ

CP violation $\Rightarrow$ particles and antiparticles act differently

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To see CPV in oscillations need:

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In vacuum at first maximum:

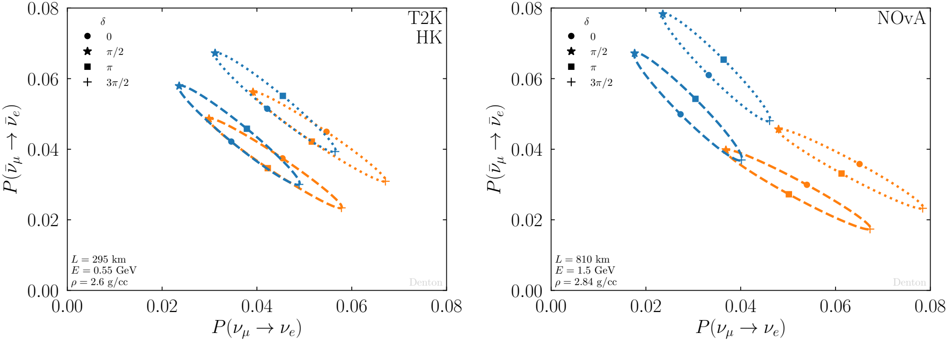
$$P_{\mu e} - \bar{P}_{\mu e} \approx 8\pi J \frac{\Delta m^2_{21}}{\Delta m^2_{32}}$$

$$J \equiv s_{12} c_{12} s_{13} c^2_{13} s_{23} c_{23} \sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

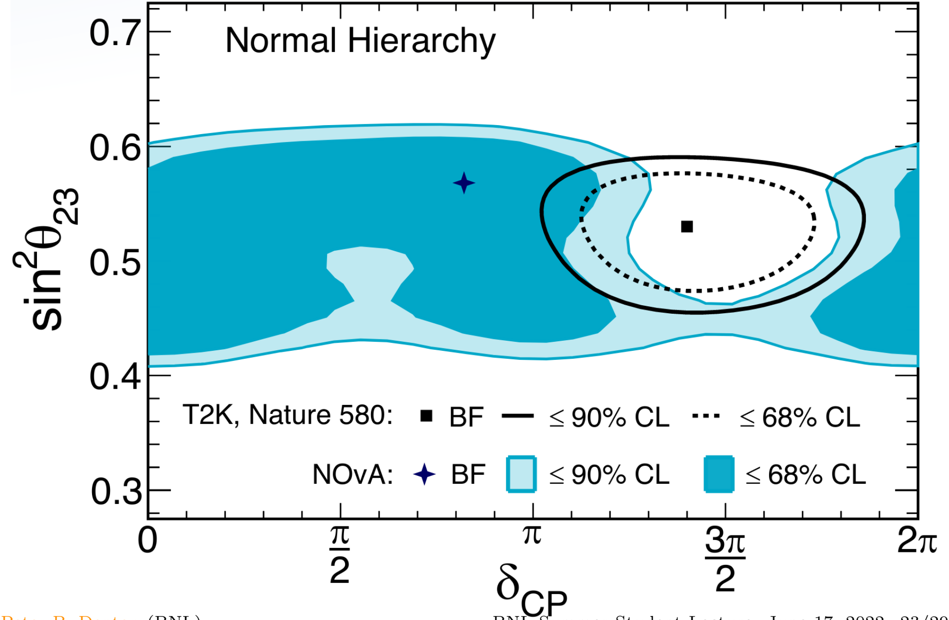
Matter effects are easily accounted for: PBD, S. Parke 1902.07185

Complex phase: δ: how is it measured?



Complex phase: δ: the data

A. Himmel 10.5281/zenodo.3959581



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With three new right handed neutrinos

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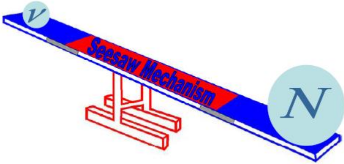

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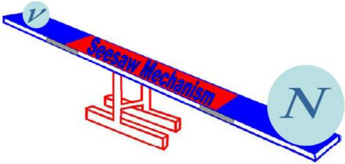
Seesaw!
We don't know if/how
this works though



Neutrino oscillation status: today

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     Only have one good measurement of this
  2. Δm^2_{31}: atmospheric, accelerator, & reactor:
     know the magnitude, not the sign
  3. θ_{12}: solar & reactor: good
  4. θ_{13}: reactor: good
  5. θ_{23}: atmospheric & accelerator:
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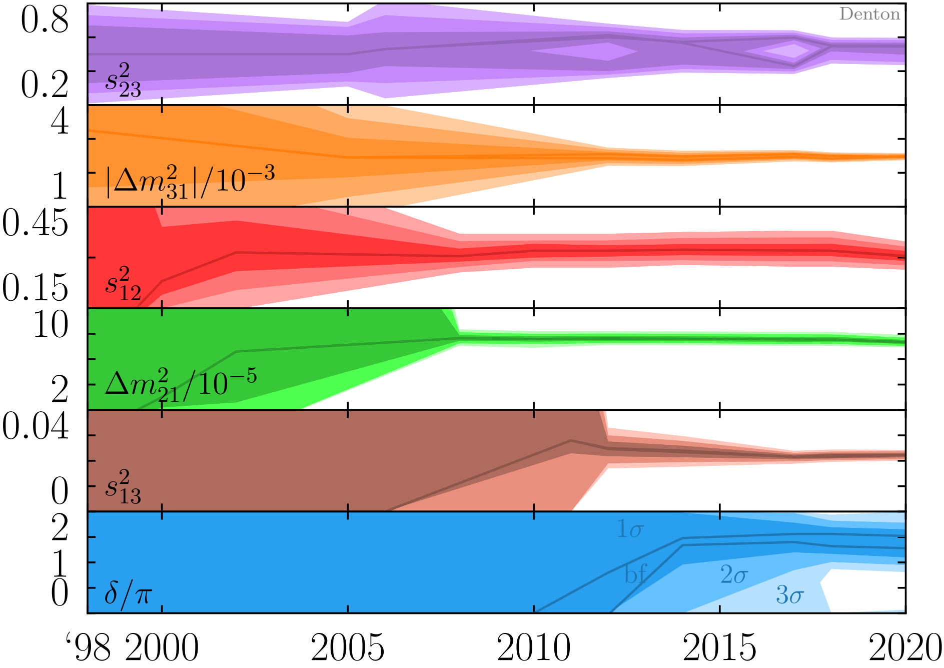
Four remaining known unknowns in particle physics: all neutrinos!


Denton
0.8
0.2
s_{23}^2
4
1
$|\Delta m_{31}^2|/10^{-3}$
0.45
0.15
s_{12}^2
10
2
$\Delta m_{21}^2/10^{-5}$
0.04
0
s_{13}^2
2
1
0
δ/π
1σ
bf
2σ
3σ
'98 2000
2005
2010
2015
2020

Precision is coming to neutrino physics

Discussion time!

Backups

Schrödinger equation

Neutrinos propagate in eigenstates of the Hamiltonian

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$$|\nu_\alpha\rangle = \sum_{i=1}^3 U_{\alpha i}^*|\nu_i\rangle \qquad \alpha\in\{e,\mu,\tau\}$$

U is a unitary 3×3 matrix which has four degrees of freedom

Unitarity $\Rightarrow$ 9 dofs, rephasing $\Rightarrow 9-5=4$

The probability

The physical observable is the probability: $P(\nu_\alpha \to \nu_\beta; L, E)$

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First calculate the transition amplitude:

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3. Project from ν_i to ν_β via U^T
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Discrete symmetries:

$$T : L \to -L\,, \qquad CP : \nu \leftrightarrow \bar{\nu} \;\Leftrightarrow\; U_{\alpha i} \to U_{\alpha i}^* \;\Leftrightarrow\; E \to -E$$

Assume CPT is conserved: $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$

The probability

The physical observable is the probability: $P(\nu_\alpha \to \nu_\beta; L, E)$

First calculate the transition amplitude:

1. Project from ν_α to ν_i via U^*
2. Propagate mass state ν_i
3. Project from ν_i to ν_β via U^T
4. Sum over all intermediate states*

*coherency must apply

$$\mathcal{A}(\nu_\alpha \to \nu_\beta; L, E) = \sum_{i=1}^{3} U_{\alpha i}^* e^{-im_i^2 L/2E} U_{\beta i}$$

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Assume that E and direction don't change during propagation

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  - Solar neutrinos: decohere from sun to Earth
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- Decohered probabilities are easy!

$$P(\nu_\alpha \to \nu_\beta) = \sum_{i=1}^{3} P_{\alpha i} P_{ii} P_{i\beta} = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Everything is at the probability level not the amplitude level

This is the same expression as oscillation averaged probabilities

Three flavor

Three angles, three Δm^2 (two are close), one complex phase

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It is less easy to show that:

$$
\begin{aligned}
P(\nu_\alpha \to \nu_\alpha) = 1 &- 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2\left(\frac{\Delta m^2_{21}L}{4E}\right) \\
&- 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) \\
&- 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta m^2_{32}L}{4E}\right)
\end{aligned}
$$

Many different ways to write these probabilities

Three flavor: appearance

$$
\begin{aligned}
P(\nu_\alpha \to \nu_\beta) = & -4\Re[U_{\alpha1}U^*_{\beta1}U^*_{\alpha2}U_{\beta2}]\sin^2\left(\frac{\Delta m^2_{21}L}{4E}\right)\\
& -4\Re[U_{\alpha1}U^*_{\beta1}U^*_{\alpha3}U_{\beta3}]\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)\\
& -4\Re[U_{\alpha2}U^*_{\beta2}U^*_{\alpha3}U_{\beta3}]\sin^2\left(\frac{\Delta m^2_{32}L}{4E}\right)\\
+8\Im[U_{\alpha1}U^*_{\beta1}U^*_{\alpha2}U_{\beta2}] & \sin\left(\frac{\Delta m^2_{21}L}{4E}\right)\sin\left(\frac{\Delta m^2_{31}L}{4E}\right)\sin\left(\frac{\Delta m^2_{32}L}{4E}\right)
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\begin{aligned}
P(\nu_\alpha \to \nu_\beta) = & -4\Re[U_{\alpha 1}U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2}] \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) \\
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\end{aligned}
$$

Final coefficient:

$$
8\Im[U_{\alpha 1}U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2}] \equiv 8J = 8 s_{12} c_{12} s_{13} c_{13}^2 s_{23} c_{23} \sin\delta
$$

This is the same for all appearance channels (up to sign)

C. Jarlskog PRL 55 (1985)

$s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

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   Care is required because of the matter effect
5. This follows from CPT. CP: $\delta \to -\delta$ and T is $L \to -L$

   Matter effect causes apparent CPT violation

Matter effect: constant

Call Schrödinger equation's eigenvalues m_i^2 and eigenvectors U_i.

$$\mathcal{A}(\nu_\alpha \to \nu_\beta) = \sum_{i=1}^{3} U_{\alpha i}^* \, e^{-im_i^2 L/2E} \, U_{\beta i} \qquad\qquad P = |\mathcal{A}|^2$$

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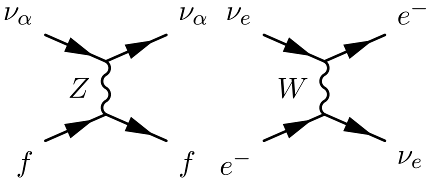
In matter ν's propagate in a new basis that depends on $a \propto N_e E_\nu$.



L. Wolfenstein PRD 17 (1978)

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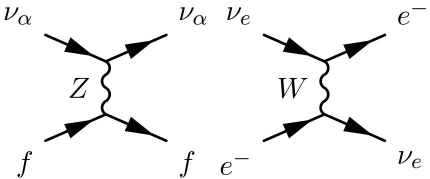

L. Wolfenstein PRD 17 (1978)

Eigenvalues: $m_i^2 \to \widehat{m^2}_i(a)$

Eigenvectors are given by $\theta_{ij} \to \widehat{\theta}_{ij}(a) \qquad \Leftarrow \qquad$ Unitarity

Hamiltonian dynamics

$$H_{\text{flav}} = \frac{1}{2E}\left[U\begin{pmatrix}0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31}\end{pmatrix}U^\dagger + \begin{pmatrix}a & & \\ & 0 & \\ & & 0\end{pmatrix}\right]$$

$$a = 2\sqrt{2}G_F N_e E$$

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$$U = \begin{pmatrix}1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1\end{pmatrix}$$

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Find eigenvalues and eigenvectors:

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H. Zaglauer, K. Schwarzer Z.Phys. C40 (1988) 273

K. Kimura, A. Takamura, H. Yokomakura hep-ph/0205295

PBD, S. Parke, X. Zhang 1907.02534

Matter effect: varying

Solar neutrinos in an adiabatically changing matter potential

Solution = MSW effect

S. Mikheev, A. Smirnov Nuovo Cim. C9 (1986) 17-26

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Bonus question: do we see more solar neutrinos at day or night?

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Neutrinos in SNe experience MSW effect too,
but they also experience neutrino-neutrino interactions

Propagation in SNe is much more involved

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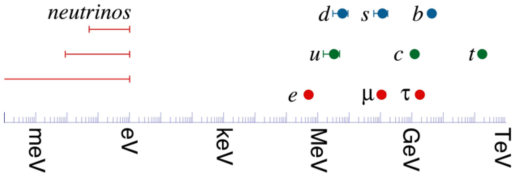
Perfectly valid way to acquire mass, but ...



Neutrino Yukawa couplings $\lesssim 10^{-12}$

But electron Yukawa coupling $\sim 10^{-6}$

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5. Difference is only relevant phenomenologically for $p_\nu \sim m_\nu$

   Cosmic neutrino background

   Internal leg in neutrinoless double beta decay diagram

Seesaw

Majorana mass term does not forbid Dirac mass term
Many different seesaw realizations

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$$\mathcal{L} \supset -m_D \overline{\nu_L} N_R - \frac{1}{2} M_R \overline{(N^c)}_L N_R$$

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5. Diagonalize the mass matrix between bare and mass bases

$$\mathbb{N}^\dagger \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \mathbb{N}^* = \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix}, \quad \begin{pmatrix} \nu \\ N^c \end{pmatrix}_L = \mathbb{N} \begin{pmatrix} \nu_m \\ N^c_m \end{pmatrix}$$

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6. Physical mass terms for $M_R \gg m_D$:

$$m_\nu \approx -\frac{m_D^2}{M_R}, \quad M_N \approx M_R$$

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    Daya Bay, RENO, Double Chooz

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- Accelerator LBL ν_e appearance: $\pm\Delta m^2_{31}$, $\pm\cos 2\theta_{23}$, θ_{13}, δ

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Experiment to Oscillation Parameters

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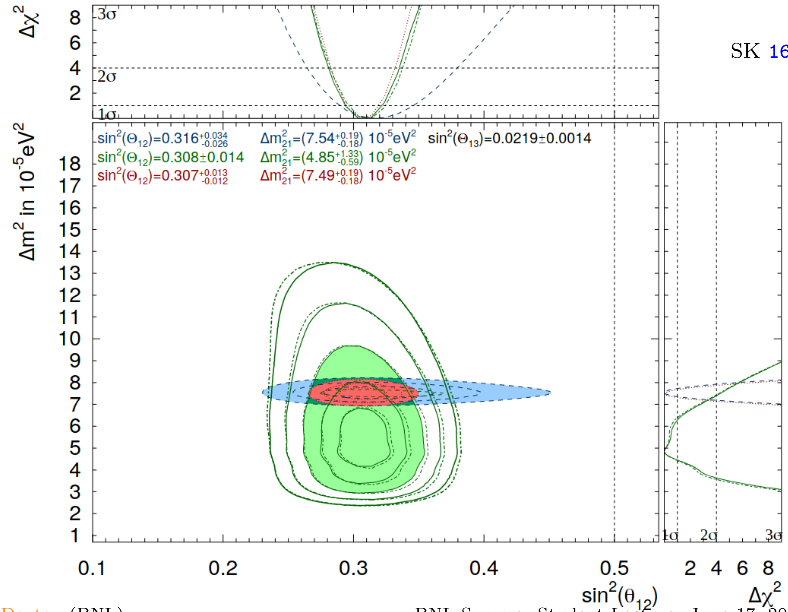
7th parameter: absolute mass scale

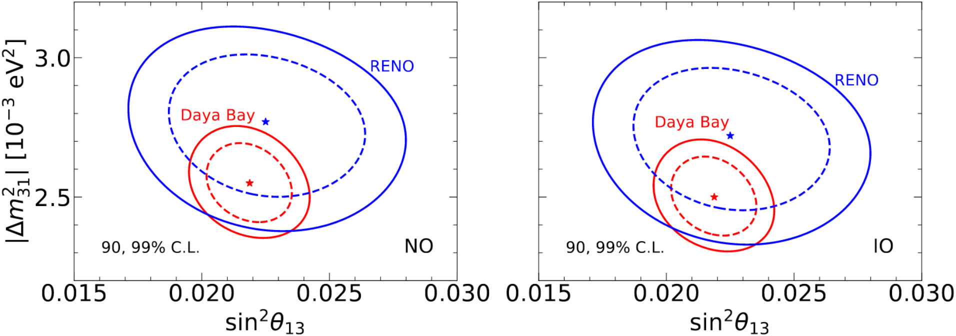
Cosmology, KATRIN, $0\nu\beta\beta$

Solar parameters: SK, SNO, KamLAND

SK 1606.07538



Reactor parameters



P. F. de Salas, et al. 2006.11237

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2. Saw an excess of $\bar{\nu}_e$ events: $87.9 \pm 22.4 \pm 6.0$

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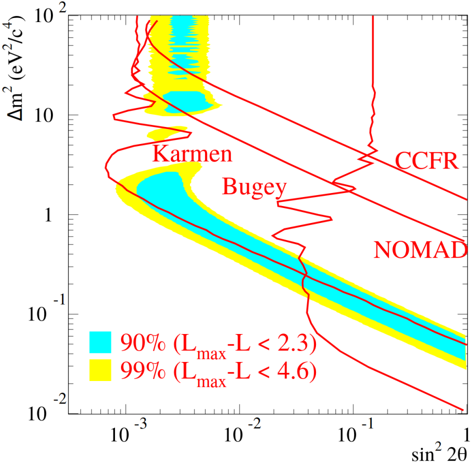

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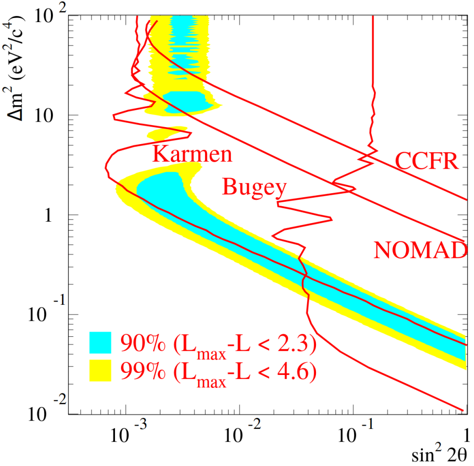

LSND hep-ex/0104049

Could be a cut problem:
J. Hill hep-ex/9504009

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7. Excess is generally consistent with LSND under the oscillation hypothesis

Latest MiniBooNE results

MiniBooNE 1805.12028

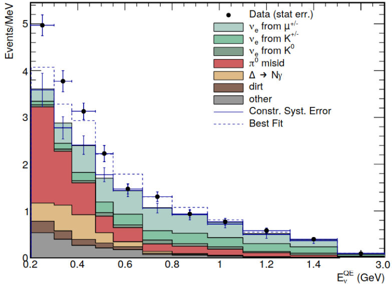

FIG. 1: The MiniBooNE neutrino mode E_ν^{QE} distributions, corresponding to the total 12.84×10^{20} POT data, for ν_e CCQE data (points with statistical errors) and background (histogram with systematic errors). The dashed curve shows the best fit to the neutrino-mode data assuming two-neutrino oscillations. The last bin is for the energy interval from 1500-3000 MeV.

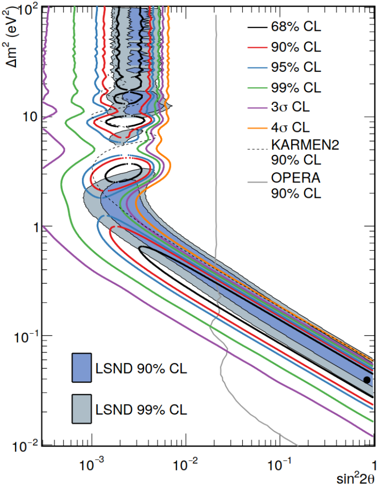

FIG. 3: MiniBooNE allowed regions in neutrino mode (12.84×10^{20} POT) for events with $200 < E_\nu^{QE} < 3000$ MeV within a two-neutrino oscillation model. The shaded areas show the 90% and 99% C.L. LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ allowed regions. The black point shows the MiniBooNE best fit point. Also shown are 90% C.L. limits from the KARMEN [37] and OPERA [38] experiments.

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GALLEX PLB 342 (1995) 440

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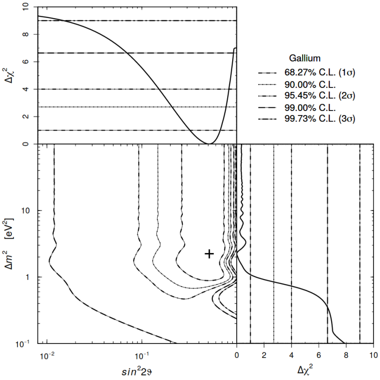

C. Giunti, M. Laveder 1006.3244

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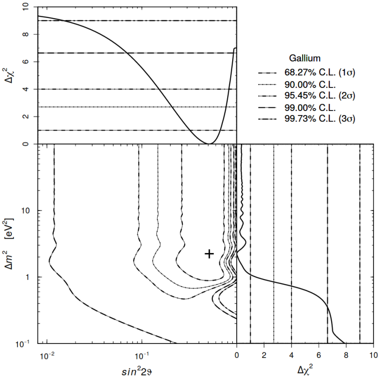

C. Giunti, M. Laveder 1006.3244

4. Using improved nuclear shell models: $3.0\sigma \to 2.3\sigma$

J. Kostensalo, et al. 1906.10980

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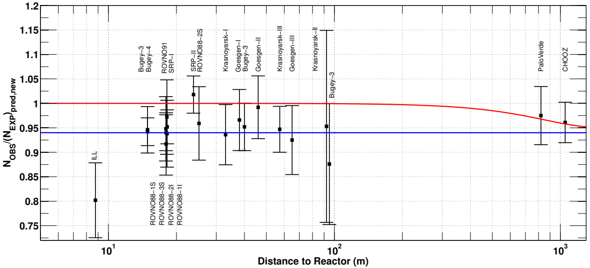

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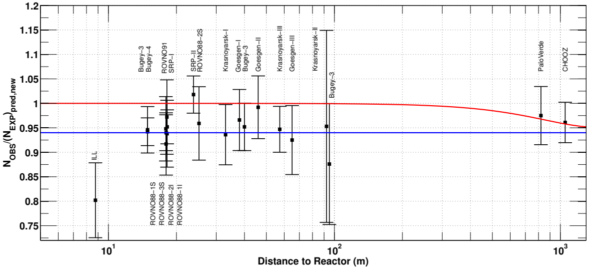

- Deficit compared to theory
  $\Rightarrow \Delta m^2_{41} \gtrsim 1.5\text{ eV}^2\ \sin^2 2\theta_{14} \sim 0.14$

G. Mention, et al. 1101.2755

Reactor anti-neutrino anomaly: fuel evolution

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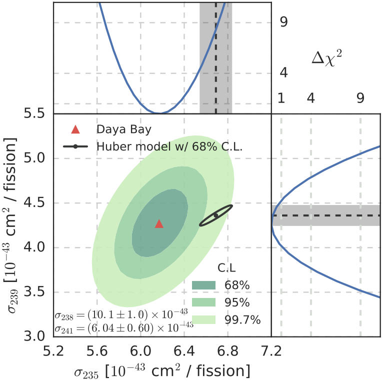

Daya Bay 1704.01082

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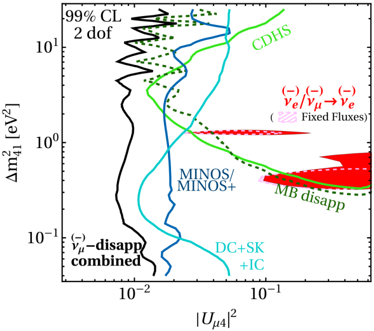

M. Dentler, et al. 1803.10661

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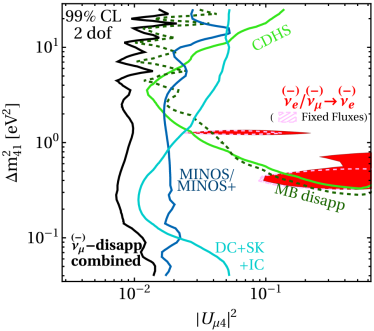

M. Dentler, et al. 1803.10661

- Are also cosmological bounds

Other anomalies

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ANITA 1803.05088

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    PBD, I. Tamborra 1805.05950

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- NOvA and T2K slightly disagree
  - Flavor changing CP violating non-standard interactions
  - Model preference is slight $\sim 2\sigma$
  - Testable at IceCube and COHERENT

    PBD, J. Gehrlein, R. Pestes 2008.01110

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Steriles: 1 eV

For:

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2. MiniBooNE
3. Gallium
4. Reactor anti-neutrino

Against:

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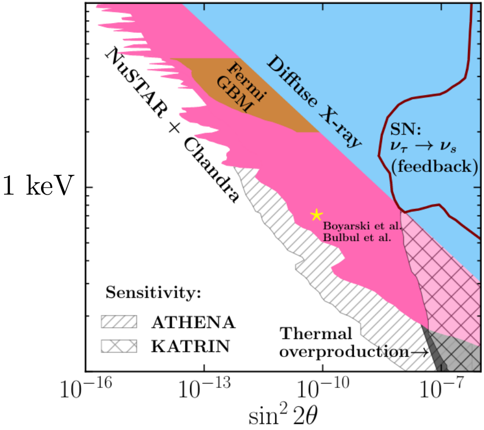
Ongoing/upcoming probes:

1. MicroBooNE → Short baseline neutrino program (three detectors)
2. Short baseline reactor experiments: see wiggles directly!

NEOS, DANSS, PROSPECT

Steriles: keV

- keV sterile neutrinos can be DM
- Would be a bit high in temperature
- A possible hint of their existence at 7 keV
- Would also affect SNe



A. Suliga, I. Tamborra, M. Wu 2004.11389

Steriles: GeV+

If they are heavy they won't affect oscillations, just kinematics

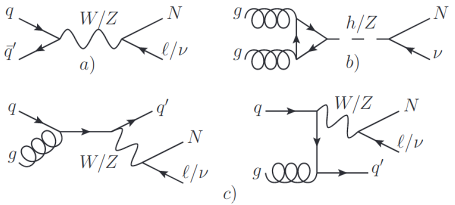
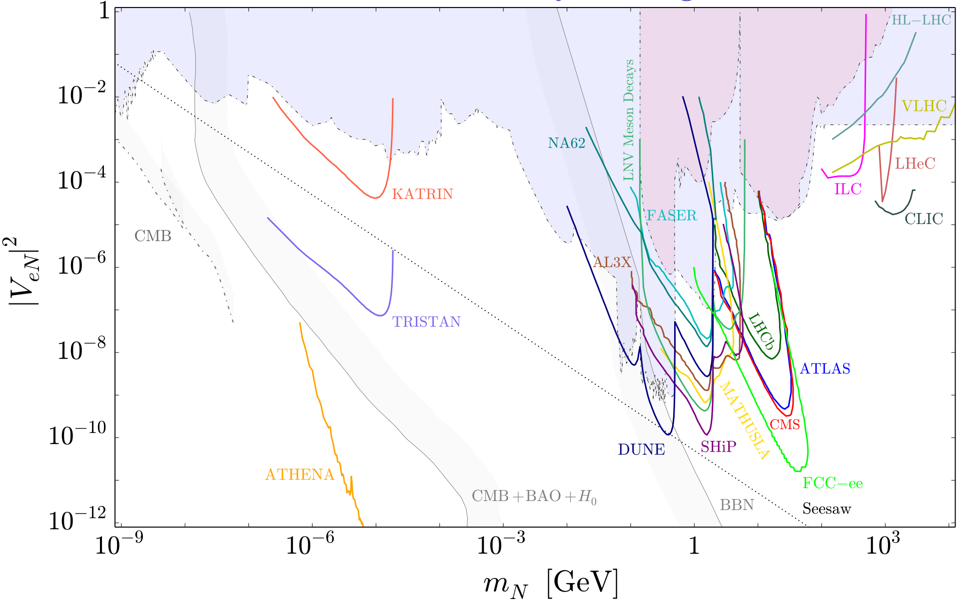


Figure 7. HNL production channels: a) Drell-Yan-type process; b) gluon fusion; c) quark-gluon fusion.

K. Bondarenko, et al. 1805.08567

- Look in colliders, beam dumps
- Battle between energy and intensity

Sterile Neutrinos: Where are they Hiding?



F. Deppisch CERN Neutrino Platform '19

Non-standard neutrino interactions

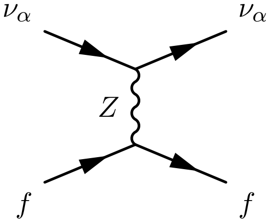
What if there was a new matter-effect like interaction?

L. Wolfenstein PRD 17 (1978)

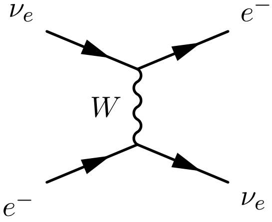
Recent overview: PBD, et al. 1907.00991

- Can affect propagation, production, detection
- Scales like the matter potential
- Can have own non-trivial flavor & CP violating structure
- Testable in scattering experiments, early universe, and SNe
- Leads to a degeneracy: mass ordering can't be determined

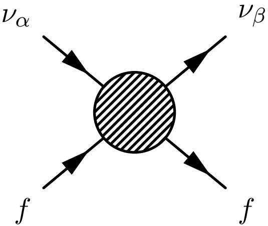
Matter Effects in Feynman Diagrams



$$V_{\mathrm{NC}} = \mp\frac{1}{2}\sqrt{2}G_F n_n$$

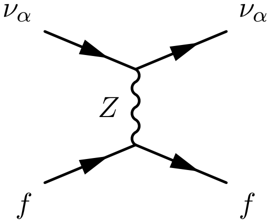


$$V_{\mathrm{CC}} = \pm\sqrt{2}G_F n_e$$

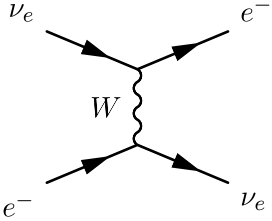


$$V_{\mathrm{NSI}} = \pm\epsilon_{\alpha\beta}^{f,X}\sqrt{2}G_F n_f$$

Matter Effects in Feynman Diagrams



$$V_{\rm NC}=\mp\frac{1}{2}\sqrt{2}G_Fn_n$$



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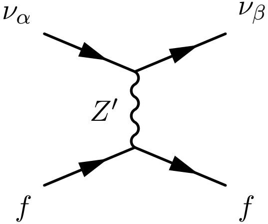

$$V_{\rm NSI}=\frac{n_f}{2}\frac{g_\nu g_f}{q^2+m_{Z'}^2}$$

NSI at the Hamiltonian Level

$$H^{\text{vac}} = \frac{1}{2E} U \begin{pmatrix} 0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31} \end{pmatrix} U^\dagger$$

$$H^{\text{mat,SM}} = \frac{a}{2E} \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

NSI at the Hamiltonian Level

$$H^{\text{vac}} = \frac{1}{2E} U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger$$

$$H^{\text{mat,SM}} = \frac{a}{2E} \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

$$H^{\text{mat,NSI}} = \frac{a}{2E} \begin{pmatrix} \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}$$

$$H = H^{\text{vac}} + H^{\text{mat,SM}} + H^{\text{mat,NSI}}$$

NSI at the Lagrangian Level

EFT Lagrangian:

$$\mathscr{L}_{\rm NSI}=-2\sqrt{2}G_F\sum_{f,P,\alpha,\beta}\epsilon_{\alpha,\beta}^{f,P}(\bar\nu_\alpha\gamma^\mu P_L\nu_\beta)(\bar f\gamma_\mu Pf)$$

with $\Lambda=\frac{1}{\sqrt{2\sqrt{2}\epsilon G_F}}$.

NSI at the Lagrangian Level

EFT Lagrangian:

$$\mathscr{L}_{\rm NSI} = -2\sqrt{2}G_F \sum_{f,P,\alpha,\beta} \epsilon_{\alpha,\beta}^{f,P}(\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(\bar{f}\gamma_\mu P f)$$

with $\Lambda = \frac{1}{\sqrt{2\sqrt{2}\epsilon G_F}}$.

Simplified model Lagrangian:

$$\mathscr{L}_{\rm NSI} = g_\nu Z'_\mu \bar{\nu}\gamma^\mu \nu + g_f Z'_\mu \bar{f}\gamma^\mu f$$

which gives a potential

$$V_{\rm NSI} \propto \frac{g_\nu g_f}{q^2 + m_{Z'}^2}$$

NSI at the Lagrangian Level

EFT Lagrangian:

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which gives a potential

$$V_{\rm NSI} \propto \frac{g_\nu g_f}{q^2 + m_{Z'}^2}$$

Models with large NSIs consistent with CLFV:

Y. Farzan, I. Shoemaker 1512.09147 Y. Farzan, J. Heeck 1607.07616
D. Forero and W. Huang 1608.04719 U. Dey, N. Nath, S. Sadhukhan 1804.05808
K. Babu, A. Friedland, P. Machado, I. Mocioiu 1705.01822 Y. Farzan 1912.09408
PBD, Y. Farzan, I. Shoemaker 1804.03660

Neutrino Decay

Since neutrinos have different masses, they decay

- Loop suppressed
- Long lifetime: $\tau \gtrsim 10^{35}$ years

Test this!

Typical Lagrangian for $\nu_i \to \nu_j + \phi$ with $m_i > m_j$

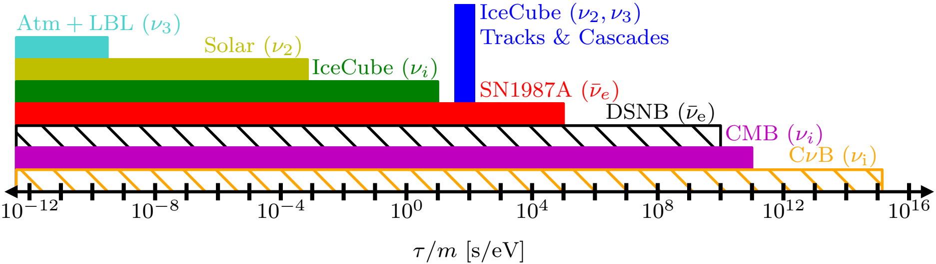
$$\mathcal{L} \supset \frac{g_{ij}}{2}\bar{\nu}_j \nu_i \phi + \frac{g'_{ij}}{2}\bar{\nu}_j i\gamma_5 \nu_i \phi$$

Neutrino Decay Phenomenology

Neutrino decay is phenomenologically classified into:

- **Invisible decay**:
  - The decay products are sterile or too low energy to be detected
  - Results in a *depletion* of the flux below the relevant energy
- **Visible decay**:
  - Decay products are detected
  - In addition to depletion, there is *regeneration*
  - Regeneration happens at a lower energy than depletion

Invisible ν Decay Constraints and Evidence



M. Gonzalez-Garcia and M. Maltoni 0802.3699

J. Berryman, A. de Gouvea, D. Hernandez 1411.0308

G. Pagliaroli, et al. 1506.02624

PBD, I. Tamborra 1805.05950

Kamiokande-II, PRL 58 1490 (1987)

S. Ando hep-ph/0307169

S. Hannestad, G. Raffelt hep-ph/0509278

A. Long, C. Lunardini, E. Sabancila 1405.7654

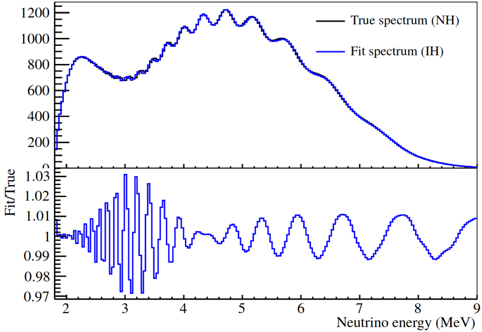
Other new physics searches

1. Unitary violation
2. Decoherence
3. Lorentz invariance violation and CPT violation
4. Dark matter interactions
5. Neutrino magnetic moment
6. Combination of new physics scenarios
7. $\vdots$

Next generation oscillation experiments

JUNO: KamLAND 2.0, coming online in ~ 1 year

1. Improved measurement of solar parameters θ_{12}, Δm^2_{21}
2. Measurements of MBL reactor parameters θ_{13}, Δm^2_{31}
3. Mass ordering measurement by Δm^2_{31} vs. Δm^2_{32} discrimination



JUNO 1508.07166

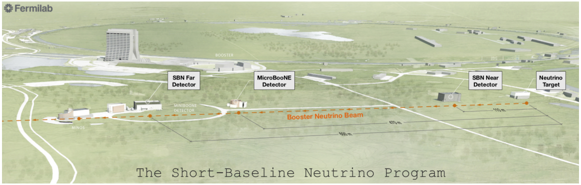
Next generation oscillation experiments

- Accelerator
  - Short baseline neutrino program at Fermilab

MicroBooNE: taking data since 2015

Short baseline neutrino detector (SBND): near detector, coming online nowish

ICARUS: far detector, coming online nowish



P. Machado, O. Palamara, D. Schmitz 1903.04608

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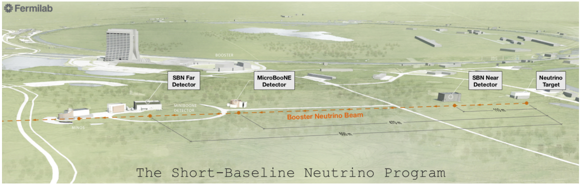

P. Machado, O. Palamara, D. Schmitz 1903.04608

- DUNE from Fermilab to SURF in South Dakota: 6+ years out

1300 km: longest long-baseline accelerator experiment

Broadband beam peaked at ~ 2.5 GeV: highest energy accelerator experiment

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  - Short baseline neutrino program at Fermilab

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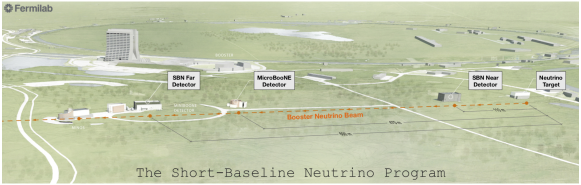


P. Machado, O. Palamara, D. Schmitz 1903.04608

- DUNE from Fermilab to SURF in South Dakota: 6+ years out

1300 km: longest long-baseline accelerator experiment

Broadband beam peaked at ~ 2.5 GeV: highest energy accelerator experiment

- T2HK in Japan: Similar to T2K: 5+ years out

Increasing protons on target (POT)
New far detector, HyperK

Next generation oscillation experiments

Hyper-KamiokaNDE: A new much larger SuperK-like detector under a different mountain

- Long-baseline program is called T2HK
- Will have additional solar neutrino physics

  Less sensitive than SK due to less overburden and more backgrounds
- Atmospheric neutrinos
- Galactic supernova neutrinos
- Diffuse supernova neutrino background (DSNB)

Super-K was loaded with Gadolinium last year to reduce backgrounds to detect the DSNB

Next generation oscillation experiments

Possible future oscillation experiments

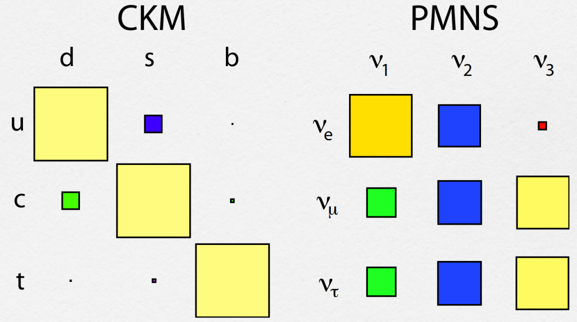
- T2HKK: Put one of the HK detectors in Korea
- ESSnuSB: Long baseline accelerator experiment in Sweden

  The above two are targeting the second oscillation appearance maximum

- INO: Magnetized atmospheric experiment in India
- Neutrino factory: muon storage ring
- $\vdots$

Flavor models

Quark matrix (CKM) is perturbative
Lepton matrix (PMNS) isn't



Review: S. King 1510.02091

Flavor models

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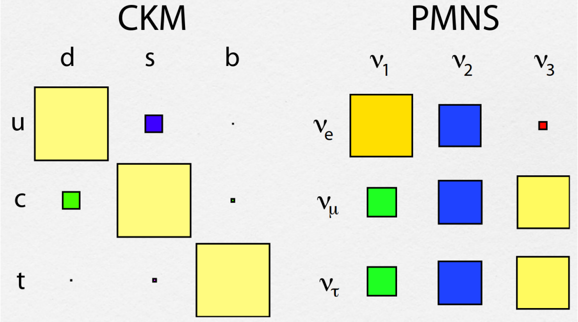

Review: S. King 1510.02091

Is there any structure?

Flavor models

Popular early models: Bimaximal, tri-bimaximal, & golden ratio

All predicted $U_{e3}=0\Rightarrow\theta_{13}=0$

Now know $\theta_{13}=8.5^\circ$

$$U_{TBM}=\begin{pmatrix}\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{pmatrix}$$

Flavor models

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Now know $\theta_{13} = 8.5^\circ$

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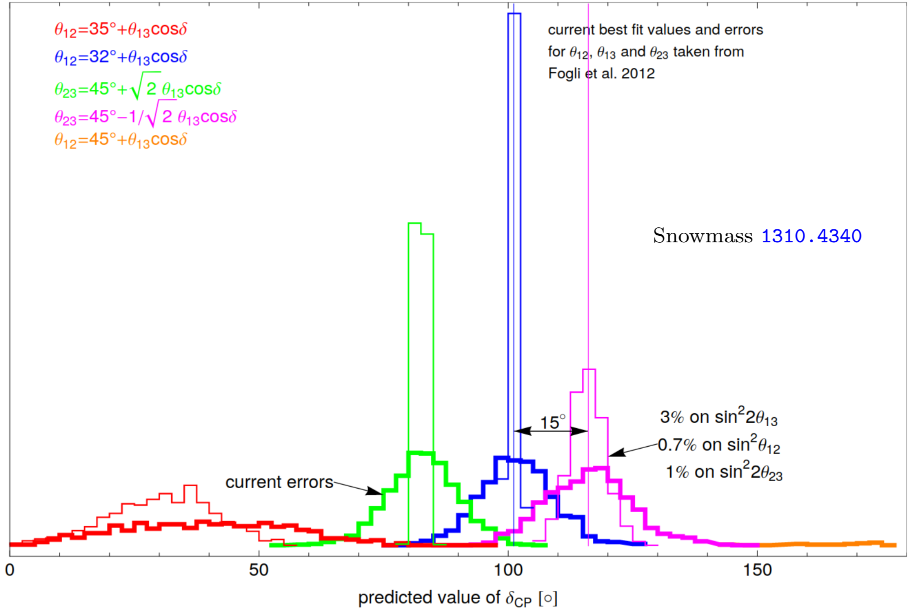
Need more degrees of freedom: sum rules
Perhaps:

$$U = \begin{pmatrix} c_\phi & s_\phi e^{-i\psi} & 0 \\ -s_\phi e^{i\psi} & c_\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} U_{TBM}$$

which predicts:

$$\cos\delta \approx \frac{\theta_{12} - \sin^{-1}\frac{1}{\sqrt{3}}}{\theta_{13}}$$

Flavor models



Related topics that were skipped

- Absolute mass scale measurements
  - Cosmological/astrophysical measurements
  - Neutrino-less double beta decay
  - Tritium end point
- Supernova neutrinos
  - Galactic and diffuse background
  - Physics during propagation and inside SNe
- High energy astrophysical flux
  - IceCube (10 years ago) and its upgrade (soon)
  - KM3NeT/ARCA/ANTARES (construction ongoing)
  - Baikal GVD (construction ongoing)
  - ANITA (has performed several balloon flights)
  - GRAND, POEMMA, P-ONE, ARA, ARIANNA, RNO, PUEO, BEACON, TAROGE (none are funded . . . yet!)
- Many other oscillation BSM scenarios
  - Decoherence
  - Lorentz invariance or CPT violaion
  - Dark matter interactions
  - Unitary violation
- Leptogenesis
- Early universe measurements of neutrino properties
- Neutrino cross sections
  - Coherent elastic ν nucleus scattering (CEvNS) at COHERENT, . . .
- Geoneutrinos

Hamiltonian Dynamics

$$H_{\text{flav}} = \frac{1}{2E}\left[U\begin{pmatrix}0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31}\end{pmatrix}U^\dagger + \begin{pmatrix}a & & \\ & 0 & \\ & & 0\end{pmatrix}\right]$$

$a = 2\sqrt{2}G_F N_e E$

Hamiltonian Dynamics

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$a = 2\sqrt{2}G_F N_e E$

$$U = \begin{pmatrix}1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1\end{pmatrix}$$

$s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

PBD, R. Pestes 2006.09384

Hamiltonian Dynamics

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$s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Find eigenvalues and eigenvectors:

PBD, R. Pestes 2006.09384

$$H_{\text{flav}} = \frac{1}{2E}\widehat{U}\begin{pmatrix}0 & & \\ & \Delta\widehat{m^2}_{21} & \\ & & \Delta\widehat{m^2}_{31}\end{pmatrix}\widehat{U}^\dagger$$

J. Kopp physics/0610206

Computationally works, but we can do better than a $\boxed{\textbf{black box}}$...

Analytic expression?

Analytic Oscillation Probabilities in Matter

- ☑ Solar: $P_{ee} \simeq \sin^2\theta_\odot$

  Approx: S. Mikheev, A. Smirnov Nuovo Cim. C9 (1986) 17-26

  Exact: S. Parke PRL 57 (1986) 2322

- ☑ Long-baseline: All three flavors

  Exact: H. Zaglauer, K. Schwarzer Z.Phys. C40 (1988) 273

  Approx: PBD, H. Minakata, S. Parke, 1604.08167

  Review: G. Barenboim, PBD, S. Parke, C. Ternes 1902.00517

- ☑ ν_e disappearance (neutrino factory):
  $\Delta\widehat{m^2}_{ee} = \widehat{m^2}_3 - (\widehat{m^2}_1 + \widehat{m^2}_2 - \Delta m^2_{21} c^2_{12})$

  PBD, S. Parke, 1808.09453

- ☐ Atmospheric

Get the eigen**values**

Eigenvalues Analytically: The Exact Solution

Solve the cubic characteristic equation: eigen**values**

$$(\widehat{m^2}_i)^3 - A(\widehat{m^2}_i)^2 + B\widehat{m^2}_i - C = 0$$

$$A \equiv \sum_i \widehat{m^2}_i = \Delta m^2_{31} + \Delta m^2_{21} + a$$

$$B \equiv \sum_{i>j} \widehat{m^2}_i \widehat{m^2}_j = \Delta m^2_{31} \Delta m^2_{21} + a(\Delta m^2_{ee} c^2_{13} + \Delta m^2_{21})$$

$$C \equiv \prod_i \widehat{m^2}_i = a \Delta m^2_{31} \Delta m^2_{21} c^2_{13} c^2_{12}$$

G. Cardano *Ars Magna* 1545

V. Barger, et al. PRD 22 (1980) 2718

H. Zaglauer, K. Schwarzer Z.Phys. C40 (1988) 273

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H. Zaglauer, K. Schwarzer Z.Phys. C40 (1988) 273

Then write down eigen**vectors** (mixing angles)

H. Zaglauer, K. Schwarzer Z.Phys. C40 (1988) 273

K. Kimura, A. Takamura, H. Yokomakura `hep-ph/0205295`

PBD, S. Parke, X. Zhang `1907.02534`

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PBD, S. Parke, X. Zhang 1907.02534

"Unfortunately, the algebra is rather impenetrable."

V. Barger, et al.

Eigenvalues Analytically: The Exact Solution

The cubic solution (in neutrino terms)

$$\widehat{m^2}_1 = \frac{A}{3} - \frac{1}{3}\sqrt{A^2 - 3B}S - \frac{\sqrt{3}}{3}\sqrt{A^2 - 3B}\sqrt{1 - S^2}$$

$$\widehat{m^2}_2 = \frac{A}{3} - \frac{1}{3}\sqrt{A^2 - 3B}S + \frac{\sqrt{3}}{3}\sqrt{A^2 - 3B}\sqrt{1 - S^2}$$

$$\widehat{m^2}_3 = \frac{A}{3} + \frac{2}{3}\sqrt{A^2 - 3B}S$$

$$A = \Delta m^2_{21} + \Delta m^2_{31} + a$$

$$B = \Delta m^2_{21}\Delta m^2_{31} + a\left[c^2_{13}\Delta m^2_{31} + (c^2_{12}c^2_{13} + s^2_{13})\Delta m^2_{21}\right]$$

$$C = a\Delta m^2_{21}\Delta m^2_{31}c^2_{12}c^2_{13}$$

$$S = \cos\left\{\frac{1}{3}\cos^{-1}\left[\frac{2A^3 - 9AB + 27C}{2\left(A^2 - 3B\right)^{3/2}}\right]\right\}$$

H. Zaglauer, K. Schwarzer Z.Phys. C40 (1988) 273

Get the eigen**vectors**

Values and Vectors

Probability amplitude:

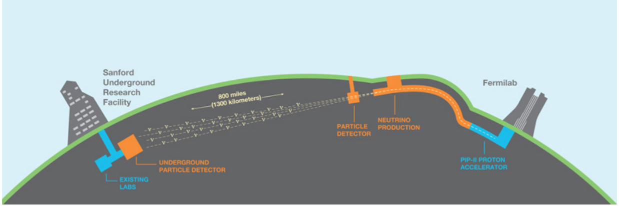
$$\mathcal{A}_{\alpha\beta} = \sum_i \widehat{U}^*_{\alpha i}\, e^{-i\widehat{m^2}_i L/2E}\, \widehat{U}_{\beta i}$$

- Eigen**values** give the frequencies of the oscillations

  Where should DUNE be?

- Eigen**vectors** give the amplitudes of the oscillations

  How many events will DUNE see?



Exact Neutrino Oscillations in Matter: Mixing Angles

$$s^2_{\widehat{12}} = \frac{-\left[(\widehat{m^2}_2)^2 - \alpha\widehat{m^2}_2 + \beta\right]\Delta\widehat{m^2}_{31}}{\left[(\widehat{m^2}_1)^2 - \alpha\widehat{m^2}_1 + \beta\right]\Delta\widehat{m^2}_{32} - \left[(\widehat{m^2}_2)^2 - \alpha\widehat{m^2}_2 + \beta\right]\Delta\widehat{m^2}_{31}}$$

$$s^2_{\widehat{13}} = \frac{(\widehat{m^2}_3)^2 - \alpha\widehat{m^2}_3 + \beta}{\Delta\widehat{m^2}_{31}\Delta\widehat{m^2}_{32}}$$

$$s^2_{\widehat{23}} = \frac{s^2_{23}E^2 + c^2_{23}F^2 + 2c_{23}s_{23}c_\delta EF}{E^2 + F^2}$$

$$e^{-i\widehat{\delta}} = \frac{c_{23}s_{23}\left(e^{-i\delta}E^2 - e^{i\delta}F^2\right) + \left(c^2_{23} - s^2_{23}\right)EF}{\sqrt{\left(s^2_{23}E^2 + c^2_{23}F^2 + 2EFc_{23}s_{23}c_\delta\right)\left(c^2_{23}E^2 + s^2_{23}F^2 - 2EFc_{23}s_{23}c_\delta\right)}}$$

$$\alpha = c^2_{13}\Delta m^2_{31} + \left(c^2_{12}c^2_{13} + s^2_{13}\right)\Delta m^2_{21}\,,\ \ \beta = c^2_{12}c^2_{13}\Delta m^2_{21}\Delta m^2_{31}$$

$$E = c_{13}s_{13}\left[\left(\widehat{m^2}_3 - \Delta m^2_{21}\right)\Delta m^2_{31} - s^2_{12}\left(\widehat{m^2}_3 - \Delta m^2_{31}\right)\Delta m^2_{21}\right]$$

$$F = c_{12}s_{12}c_{13}\left(\widehat{m^2}_3 - \Delta m^2_{31}\right)\Delta m^2_{21}$$

H. Zaglauer, K. Schwarzer Z.Phys. C40 (1988) 273

New Physics

DUNE and T2HK will unprecedented capabilities to test the three-neutrino oscillation picture

Extend DMP to new physics progress report:

- ☑ Sterile

  S. Parke, X. Zhang 1905.01356

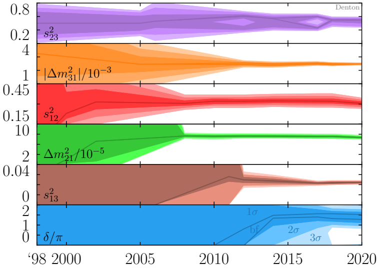
- ☑ NSI

  S. Agarwalla, et al. 2103.13431

- ☐ Neutrino decay
- ☐ Decoherence
- ☐ ...

Given Rosetta, extensions should be considerably simpler

References



SK hep-ex/9807003

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T2K 1106.2822

D. Forero, M. Tortola, J. Valle 1205.4018

D. Forero, M. Tortola, J. Valle 1405.7540

P. de Salas, et al. 1708.01186

CP violation in matter

The CPV Term in Matter

The amount of CPV is

$$P_{\alpha\beta} - \bar{P}_{\alpha\beta} = \pm 16 J \sin\Delta_{21} \sin\Delta_{31} \sin\Delta_{32} \qquad \alpha \neq \beta$$

where the Jarlskog is

$$J \equiv \Im[U_{\alpha i} U_{\beta j} U^*_{\alpha j} U^*_{\beta i}] \qquad \alpha \neq \beta,\ i \neq j$$

$$J = c_{12} s_{12} c_{13}^2 s_{13} c_{23} s_{23} \sin\delta$$

C. Jarlskog PRL 55 (1985)

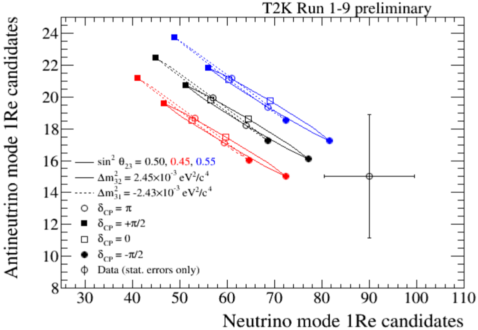
The exact term in matter is known to be

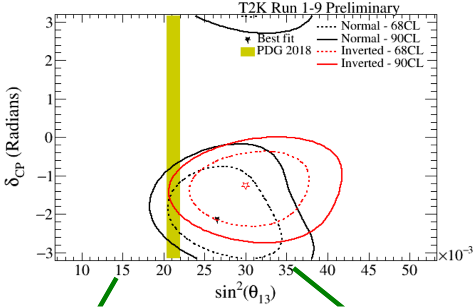
$$\frac{\widehat{J}}{J} = \frac{\Delta m^2_{21} \Delta m^2_{31} \Delta m^2_{32}}{\Delta\widehat{m^2}_{21} \Delta\widehat{m^2}_{31} \Delta\widehat{m^2}_{32}}$$

V. Naumov IJMP 1992

P. Harrison, W. Scott hep-ph/9912435

CPV Tension at T2K





$$J = c_{12}s_{12}c_{13}^2s_{13}c_{23}s_{23}\sin\delta$$

CPV in Matter

CPV in matter can be written sans $\cos(\frac{1}{3}\cos^{-1}(\cdots))$ term.

$$\frac{\widehat{J}}{J}=\frac{\Delta m^2_{21}\Delta m^2_{31}\Delta m^2_{32}}{\Delta\widehat{m^2}_{21}\Delta\widehat{m^2}_{31}\Delta\widehat{m^2}_{32}}$$

$$\left(\Delta\widehat{m^2}_{21}\Delta\widehat{m^2}_{31}\Delta\widehat{m^2}_{32}\right)^2=(A^2-4B)(B^2-4AC)+(2AB-27C)C$$

$$A\equiv\sum_j\widehat{m^2}_j=\Delta m^2_{31}+\Delta m^2_{21}+a$$

$$B\equiv\sum_{j>k}\widehat{m^2}_j\widehat{m^2}_k=\Delta m^2_{31}\Delta m^2_{21}+a(\Delta m^2_{ee}c^2_{13}+\Delta m^2_{21})$$

$$C\equiv\prod_j\widehat{m^2}_j=a\Delta m^2_{31}\Delta m^2_{21}c^2_{13}c^2_{12}$$

This is the only oscillation quantity in matter that can be written exactly without $\cos(\frac{1}{3}\cos^{-1}(\cdots))$!

H. Yokomakura, K. Kimura, A. Takamura hep-ph/0009141

CPV Factorizes

Thus $\widehat{J}^{-2}$ is fourth order in matter potential:
only two matter corrections are needed.

$$\frac{\widehat{J}}{J} = \frac{1}{|1-(a/\alpha_1)e^{i2\theta_1}||1-(a/\alpha_2)e^{i2\theta_2}|}$$

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CPV in matter can be well approximated:

$$\frac{\widehat{J}}{J} \approx \frac{1}{|1-(a/\Delta m^2_{ee})e^{i2\theta_{13}}||1-(c^2_{13}a/\Delta m^2_{21})e^{i2\theta_{12}}|}$$

PBD, Parke 1902.07185

See also X. Wang, S. Zhou 1901.10882

Precise at the $< 0.04\%$ level!

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One caveat in support of δ

If the goal is **CP violation** the Jarlskog should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12}, θ_{13}, θ_{23}, and J, I can't determine the sign of $\cos\delta$ which is physical

e.g. $P(\nu_\mu \to \nu_\mu)$ depends on $\cos\delta$ a tiny bit

- As T2(H)K has almost no $\cos\delta$ sensitivity, they should focus on J
- NOvA/DUNE has some $\cos\delta$ sensitivity, so both J and δ should be reported