Quantum Computing

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About me

My name: João Barata

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If you have questions or want to chat, you can find me in 2.42
Why Quantum Computing?

We believe Nature is fundamentally quantum

Simulating Physics with Computers
Richard P. Feynman

“Nature isn’t classical
... and if you want to make a simulation of Nature,
you’d better make it quantum mechanical,
and by golly it’s a wonderful problem,
because it doesn’t look so easy.”

Richard Feynman
Why Quantum Computing?

We believe Nature is fundamentally quantum

5. CAN QUANTUM SYSTEMS BE PROBABILISTICALLY SIMULATED BY A CLASSICAL COMPUTER?

In practice for QCD: $$$$$

In principe yes, but

"... and if you want to make a simulation of Nature, you’d better make it quantum mechanical, and by golly it’s a wonderful problem, because it doesn’t look so easy."

Richard Feynman
What is Quantum Computing?

“Infant” field in the intersection of many sub-fields

Quantum algorithms
Quantum devices
Quantum problems
What is Quantum Computing?

“Infant” field in the intersection of many sub-fields

Quantum algorithms
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What is Quantum Computing?

What I mean by infant

Moore’s law

According to Google this is a 60s computer

Quantum Volume: The New Moore’s Law

Quantum Volume ~ quality of the quantum computer
What is Quantum Computing?

What I mean by infant
Overview

1. From Classical to Quantum Mechanics

2. From Classical to Quantum Computing

3. Application to High Energy Physics (HEP)
Consider $\omega$ a classical distribution

This satisfies a Liouville equation

$$\partial_t \omega(x, p) = \mathcal{L} \omega$$

Equivalent to Newton's laws

$$\vec{F} = m \vec{a}$$
Classical mechanics

The state of the system can be identified with $\omega \geq 0$

Some classical axioms:

- The state of the system can be identified with $\omega \geq 0$
- $\partial_t \omega(x, p) = \mathcal{L} \omega$
- Probabilities: $\delta p \delta x \omega(x, p)$
- The system can be measured trivially
Quantum mechanics

Quantum case: state described by wavefunction $\psi$

This satisfies a **Schrodinger equation**

$$i \partial_t \psi(x, p) = \mathcal{H} \psi$$

And classical probabilities are related to

$$|\psi(x, p)|^2$$
The state of the system is described by $\psi \in \mathbb{C}$.

Some quantum axioms:

- The state of the system is described by $\psi \in \mathbb{C}$
- $i\hbar \frac{\partial}{\partial t} \psi(x, p) = \mathcal{H} \psi$
- Probabilities: $|\psi|^2$
- The system can be measured but
The state of the system is described by $\psi \in \mathbb{C}$.

**Quantum Superposition:**

- The state of the system is described by $\psi \in \mathbb{C}$

**Any combination of $\psi$ is still a valid state!**

**but probabilities:**

$$\text{Prob} \sim |\sum \psi|^2$$

**Quantum interference!**

Remember: probabilities add to 1
Discrete quantum mechanics

Example: electrons have electric charge -1, mass $m_e$ and spin 1/2

If we ignore all other dynamics, then this is a 2 state quantum system
Discrete quantum mechanics

An aside: Stern–Gerlach experiment

[Diagram of the Stern–Gerlach experiment with labels: Classical prediction, What was actually observed, Silver atoms, Inhomogeneous magnetic field, Furnace, Walter Gerlach & Otto Stern]
What is the relation to computation?
Classical digital computation

Modern computers are made of basic information units: bits

Let’s call these: $|0\rangle$ and $|1\rangle$
Classical digital computation

Modern computers are made of basic information units: bits

Example:

0 1 1 0
Classical digital computation

Time evolution of information: just draw some lines

Example:

\[ A = \{0, 1\} \]
\[ B = \{0, 1\} \]
Classical digital computation

I will rewrite this in a more convenient notation

\[ \psi = \{0,1\} \text{ (bit). Only two possible operations} \]

\[ \{0,1\} \quad \{0,1\} \]

\[ \{0,1\} \quad \{0,1\} \]

\[ \psi = \{0,1,2,3\} \text{ (in binary)} \]

\[ \{0,1\} \quad \{0,1\} \]

\[ \{0,1\} \quad \{0,1\} \]

a.k.a NOT

\[ \{0,1\} \quad \{1,0\} \]

\[ \sigma_{\text{class.}}^{x} \]

\[ U \]
Classical digital computation

I will rewrite this in a more convenient notation

Measurement:

Non-trivial topologies:
Quantum digital computation

Quantum computers are made of basic information units: qubits

The simplest example: 1 qubit

$$|\psi\rangle = a |0\rangle + b |1\rangle \equiv a |\uparrow\rangle + b |\downarrow\rangle$$
Quantum digital computation

Quantum computers are made of basic information units: qubits

Example: 4 qubits

\[ |\psi\rangle = a_1 + a_2 + a_3 + \cdots \]

How many bits to get the same computing power?
Quantum digital computation

Quantum computers are made of basic information units: qubits

Example: 4 qubits

\[ |\psi\rangle = a_1 + a_2 + a_3 + \cdots \]

How many bits to get the same computing power? $2^4$
Quantum digital computation

Let's revisit the previous diagrams

For 1 qubit

Infinite set of operations: \( |\psi\rangle = a |0\rangle + b |1\rangle \equiv a |\uparrow\rangle + b |\downarrow\rangle \)

\[ |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \]

\[ |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]

\[ |\psi\rangle \]

\[ \sigma^{x,y,z} \]

\[ H \]

\[ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} |\psi\rangle \]

\[ S \]

\[ \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} |\psi\rangle \]

Remember: probabilities add to 1
Quantum digital computation

Let’s revisit the previous diagrams

For 2 qubits

$$|\psi\rangle = \sum_{i,j} c_{ij} |x_i, x_j\rangle$$

Single qubit operations generalize in a simple way, for example:

$$1 \otimes H |\psi\rangle = H$$
Quantum digital computation

Let’s revisit the previous diagrams

For 2 qubits

\[ |\psi\rangle = \sum_{i,j} c_{ij} |x_i, x_j\rangle \]

But things can get interesting

\[ \text{CNOT} |\psi\rangle = C\sigma^x |\psi\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix} \]
Quantum digital computation

Let’s revisit the previous diagrams

Example and first quantum circuit:

\[ |\downarrow\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}} (|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle) \]

Bell state

SOLUTION?
Quantum digital computation

Let’s revisit the previous diagrams

Example and first quantum circuit:

\[|\downarrow\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle)\]  Bell state

\[H \otimes 1\]

\[|\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}(|\downarrow\rangle + |\uparrow\rangle)\]
Quantum digital computation

Let’s revisit the previous diagrams

Example and first quantum circuit:

$$|↓↓↓⟩ \rightarrow \frac{1}{\sqrt{2}}(|↓↓↓⟩ + |↑↑⟩) \quad \text{Bell state}$$

$$|0⟩ \quad \sigma^x$$

$$|0⟩ \quad H$$

$$|↓↓⟩ \rightarrow \frac{1}{\sqrt{2}}(|↓↓⟩ + |↑⟩) \rightarrow \frac{1}{\sqrt{2}}(|↓↓⟩ + |↑⟩)$$
Quantum digital computation

Finally we have

Measurement:

\[ |\psi\rangle \longrightarrow P_{|x\rangle} |\psi\rangle \longrightarrow P_{|x\rangle} |\psi\rangle \]

Non-trivial topologies:

(No cloning) (Unitarity) (Linearity)
What is the relation to HEP?
Simulating RHIC in a Qcomputer

Challenge: Simulate Pancake-Pancake event at RHIC

Relativistic Heavy Ion Collider
Simulating RHIC in a Qcomputer

Challenge: Simulate Pancake-Pancake event at RHIC

You saw this figure in R. Pisarski lecture a couple of weeks ago
Simulating RHIC in a Qcomputer

Pancake-Pancake event at RHIC
Simulating RHIC in a Qcomputer

**Strategy:** We know that

$$i \partial_t |\psi\rangle = H |\psi\rangle \quad \text{so} \quad |\psi\rangle(t) = \exp(-iHt) |\psi\rangle(0)$$

1. Map dofs to qubits
2. Write evolution operator in terms of gates
3. Measure the state
Suppose: $H = \sigma^x \otimes \sigma^z$ and $\psi(0) = |\downarrow\downarrow\rangle$

$H = (\text{Had} \otimes 1)(\sigma^z \otimes \sigma^z)(\text{Had} \otimes 1)^\dagger$

Exercise: check this

$\sigma^x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
$\sigma^z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Exercise: check this

Disclaimer: Going fast here, so don’t worry if you don’t follow 100%
In reality: scattering in scalar QFT

Introduce lattice

Two particles scattering to four particles

\[ H = \sum_{x \in \Omega} a^d \left[ \frac{1}{2} \pi(x)^2 + \frac{1}{2} (\nabla a \phi)^2(x) + \frac{1}{2} m_0^2 \phi(x)^2 + \frac{\lambda_0}{4!} \phi(x)^4 \right] \]

Meaningful simulations will require something in the order of **thousands of high quality qubits**!
Some key ideas

1. Quantum computing is picking up steam
Some key ideas

Quantum computing is only starting in HEP/NP

Submitted to the Proceedings of the US Community Study on the Future of Particle Physics (Snowmass 2021)

Quantum Simulation for High Energy Physics

I. Physics drive: Collider phenomenology
II. Physics drive: Matter in and out of equilibrium
III. Physics drive: Neutrino (astro)physics
IV. Physics drive: Cosmology and early universe
V. Physics drive: Nonperturbative quantum gravity

To find the full document Google: arxiv 2204.03381
Some key ideas

Quantum computing has a wide range of applications
Some places to look for more info

Open-Source Quantum Development

Excellent introduction to QM and QC

Software to play around

Global introduction to QC

https://qiskit.org

qiskit 0.37.0
see release notes