

## Introduction

**Bayesian Optimization (BO)** is a time effective method of automating accelerator tuning for CLS because we sample as few points as possible to find input values that achieve a good optimum. A problem with this BO application is that its performance worsens exponentially in higher dimensions due to poor scaling. A good prior mean in **Gaussian Processes (GPs)** can help with scaling. Therefore, this **study's objective** is to explore cheap and more expressive (non-constant) prior means for Gaussian Processes to optimize Bayesian Optimization for tuning the injector.

## Objective Function

Inputs:

SOL1:solenoid_field_scale (kG*m)
CQ01:b1_gradient (kG)
SQ01:b1_gradient (kG)
QA01:b1_gradient (kG)
QA02:b1_gradient (kG)
QE01:b1_gradient (kG)
QE02:b1_gradient (kG)
QE03:b1_gradient (kG)
QE04:b1_gradient (kG)

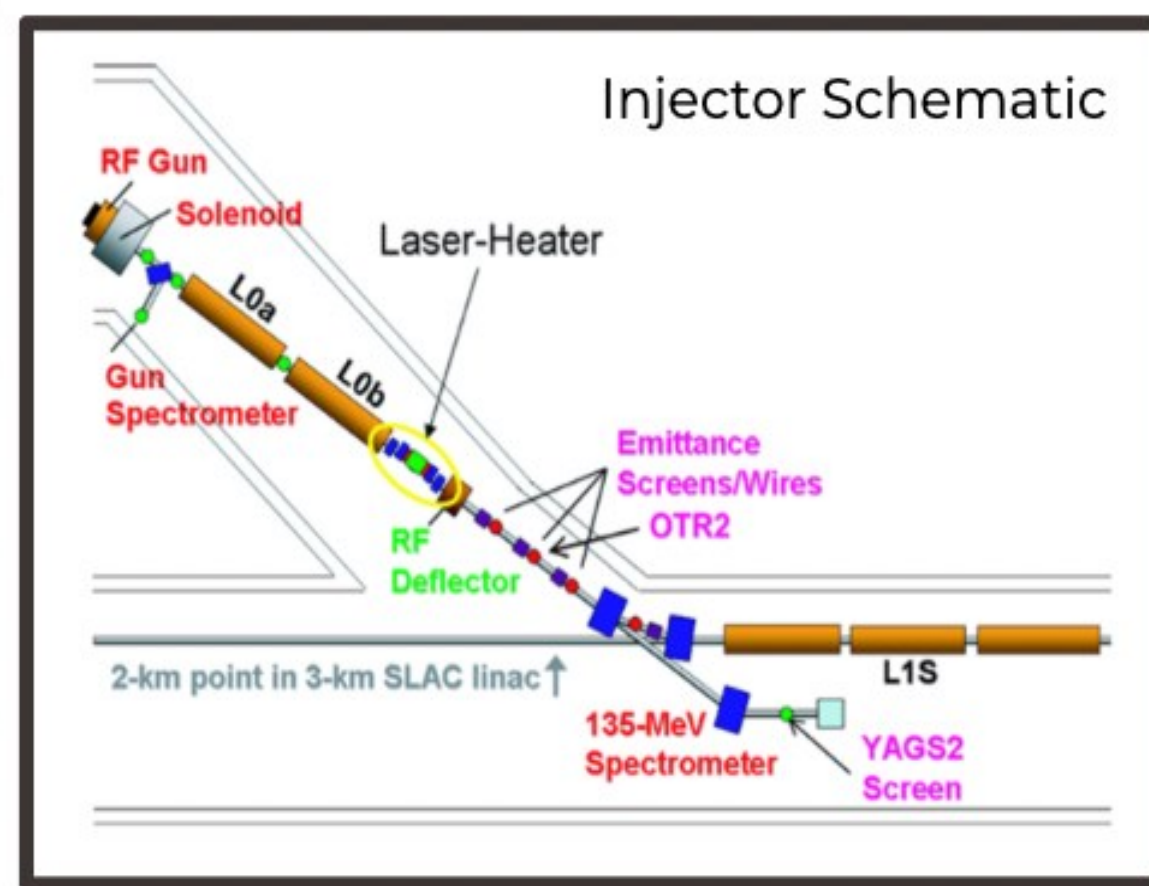
Outputs:

-emittance\*bmag (mm-mrad)

$$\beta_{mag} = \frac{1}{2} \left( \frac{\bar{\beta}}{\beta} + \frac{\beta}{\bar{\beta}} \right) + \frac{1}{2} \left( \alpha \sqrt{\frac{\bar{\beta}}{\beta}} - \bar{\alpha} \sqrt{\frac{\beta}{\bar{\beta}}} \right)^2$$

$$\beta = \frac{\sigma^2}{\epsilon}$$

[1]



- We use BO during accelerator tuning to find input values to the injector that minimize emittance\*bmag (**maximize -emittance\*bmag**).
- The objective function employs a Neural Net (NN) surrogate model of the accelerator injector to prototype this optimization approach.

## Gaussian Processes

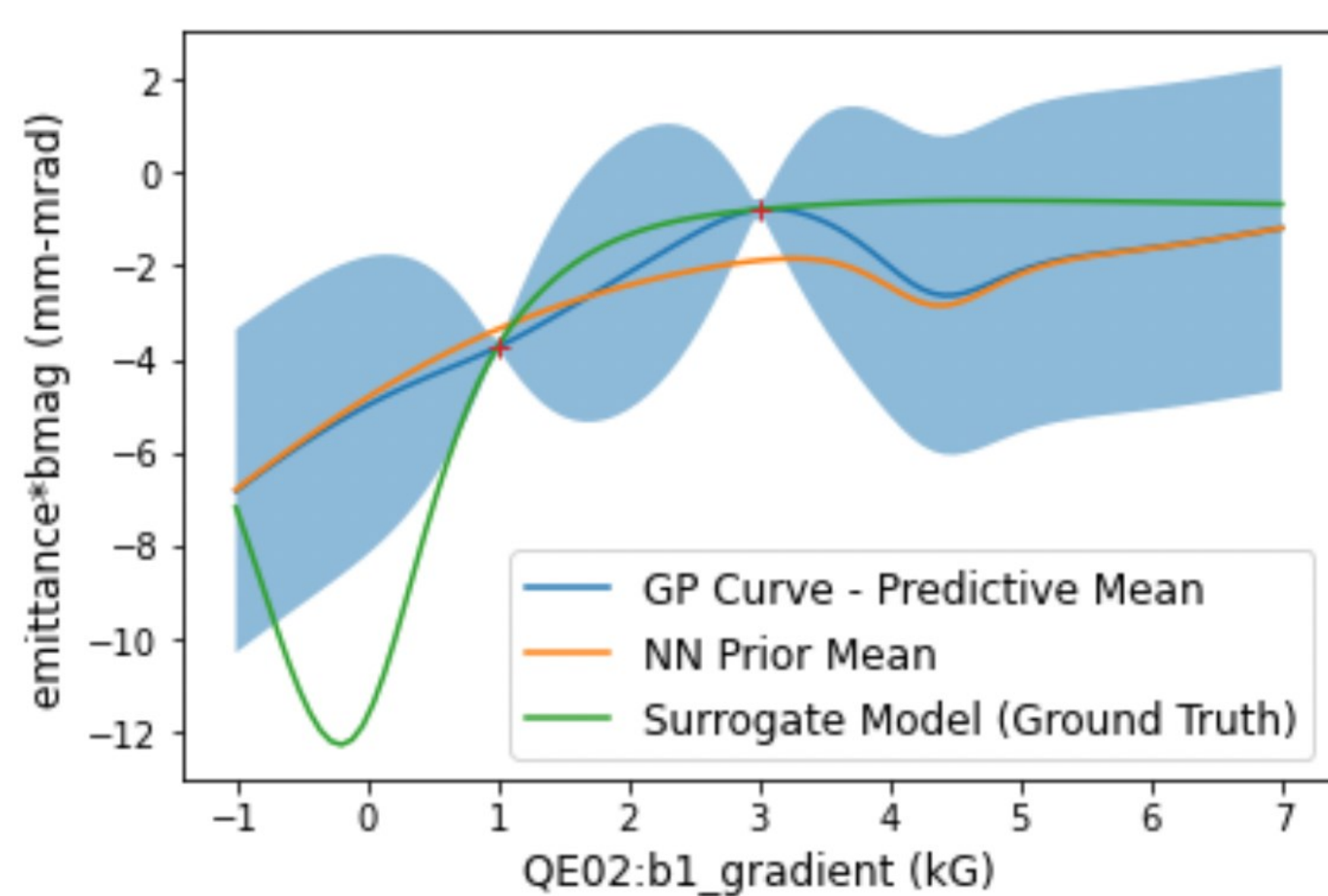


Figure 1: Visualization of a GP's posterior curve (blue) fitted to two data samples (red) compared to the GP's prior mean curve (orange).

The GP and predictive mean function (blue line) are defined as:

$$f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}'))$$

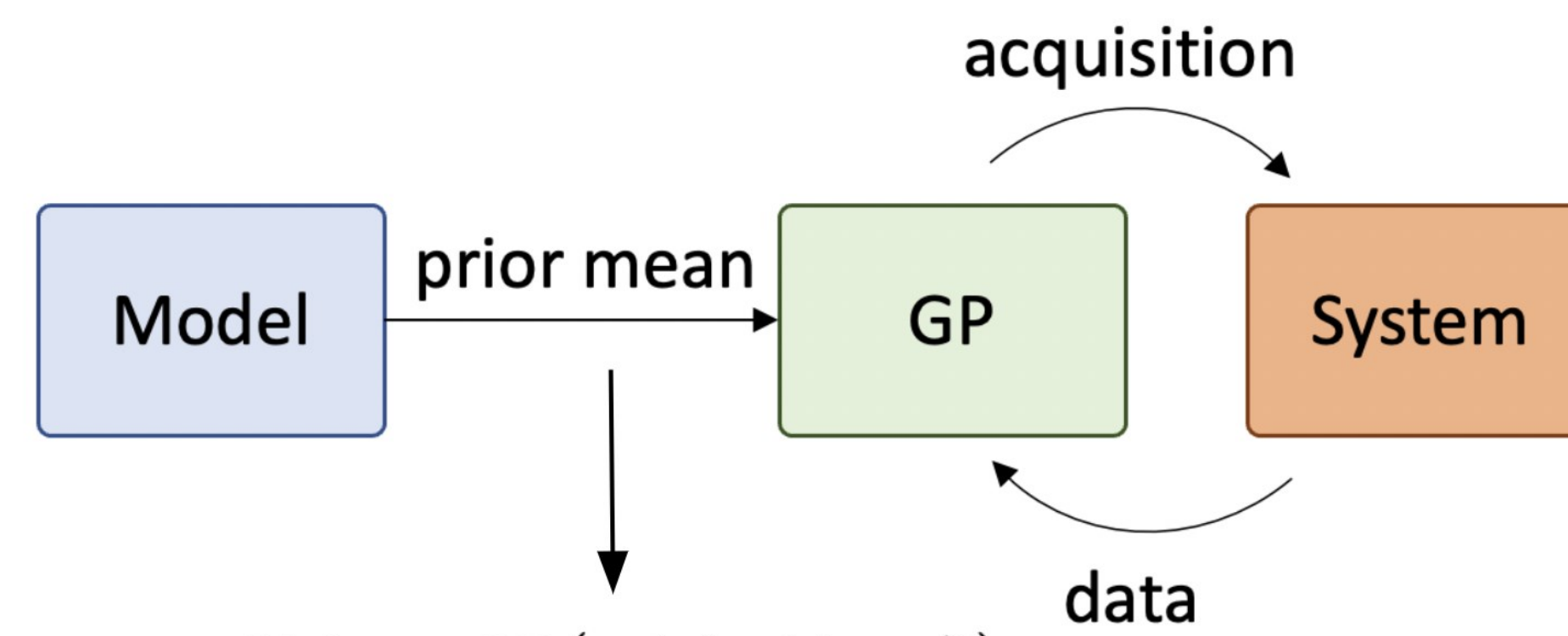
$$\bar{\mathbf{f}}_* = \mathbf{m}(X_*) + K(X_*, X)K_y^{-1}(\mathbf{y} - \mathbf{m}(X))$$

where  $K_y = K + \sigma_n^2 I$  [2]

- The GP's **predictive mean** fits through ground truth points and then returns to the prior mean in areas with no data samples.

## Implementing Custom Priors

- Using a NN model prior mean that includes prior information of the objective function, BO should be able to find better optima faster.



$$f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}'))$$

$$\bar{\mathbf{f}}_* = \mathbf{m}(X_*) + K(X_*, X)K_y^{-1}(\mathbf{y} - \mathbf{m}(X))$$

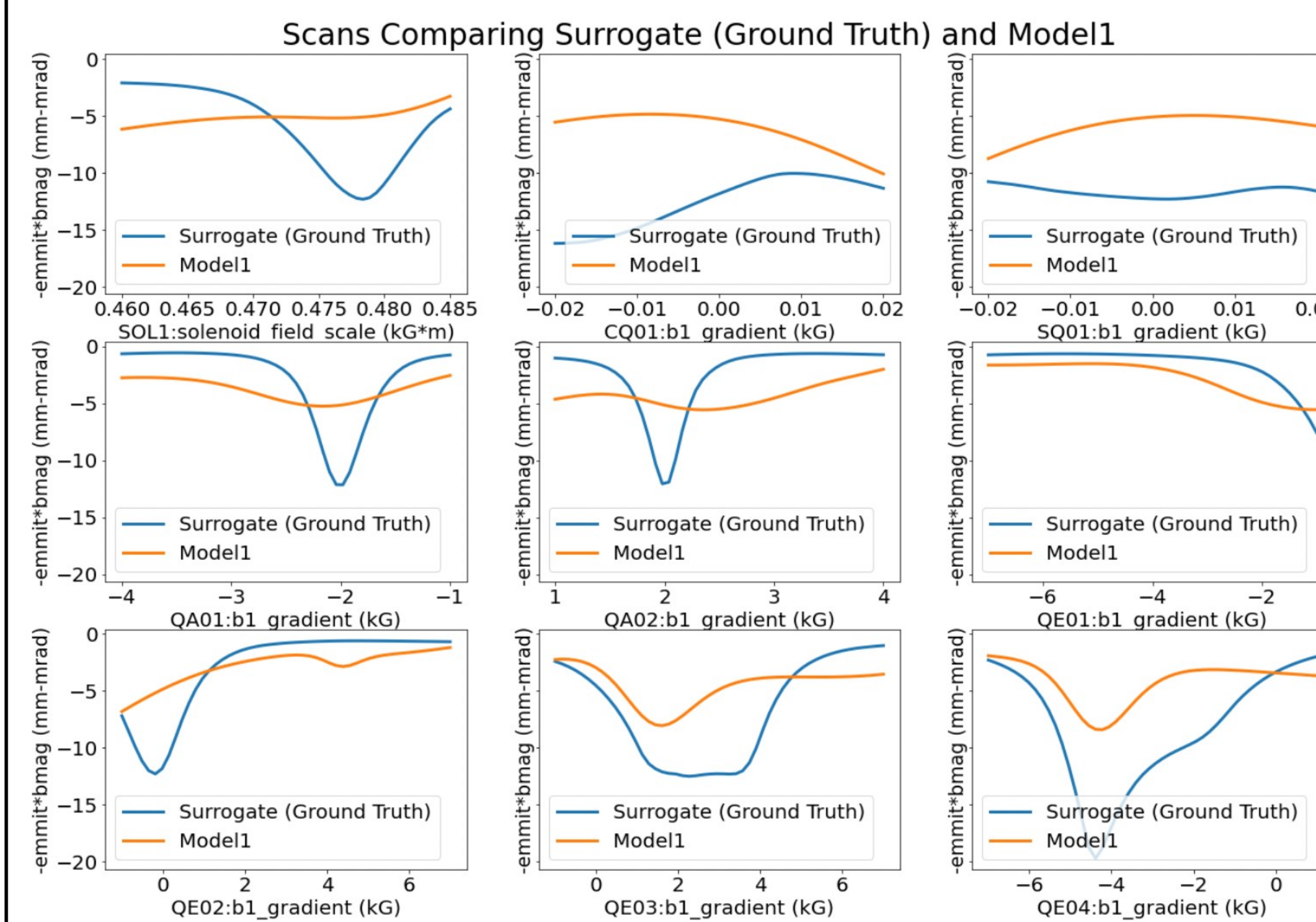
where  $K_y = K + \sigma_n^2 I$  [2]

Figure 2: A model is specified as the mean function of the GP and represents the initial behavior of the objective function. The system gives the GP data samples. The GP then generates a posterior distribution and predictive mean function, which the acquisition function (Upper Confidence Bound) uses to find the inputs most likely to yield an optimum and return it to the system.

- We use PyTorch for all models and BO implementations to aid end-to-end differentiability.

## Neural Net Accuracies

### Model1



### Model2

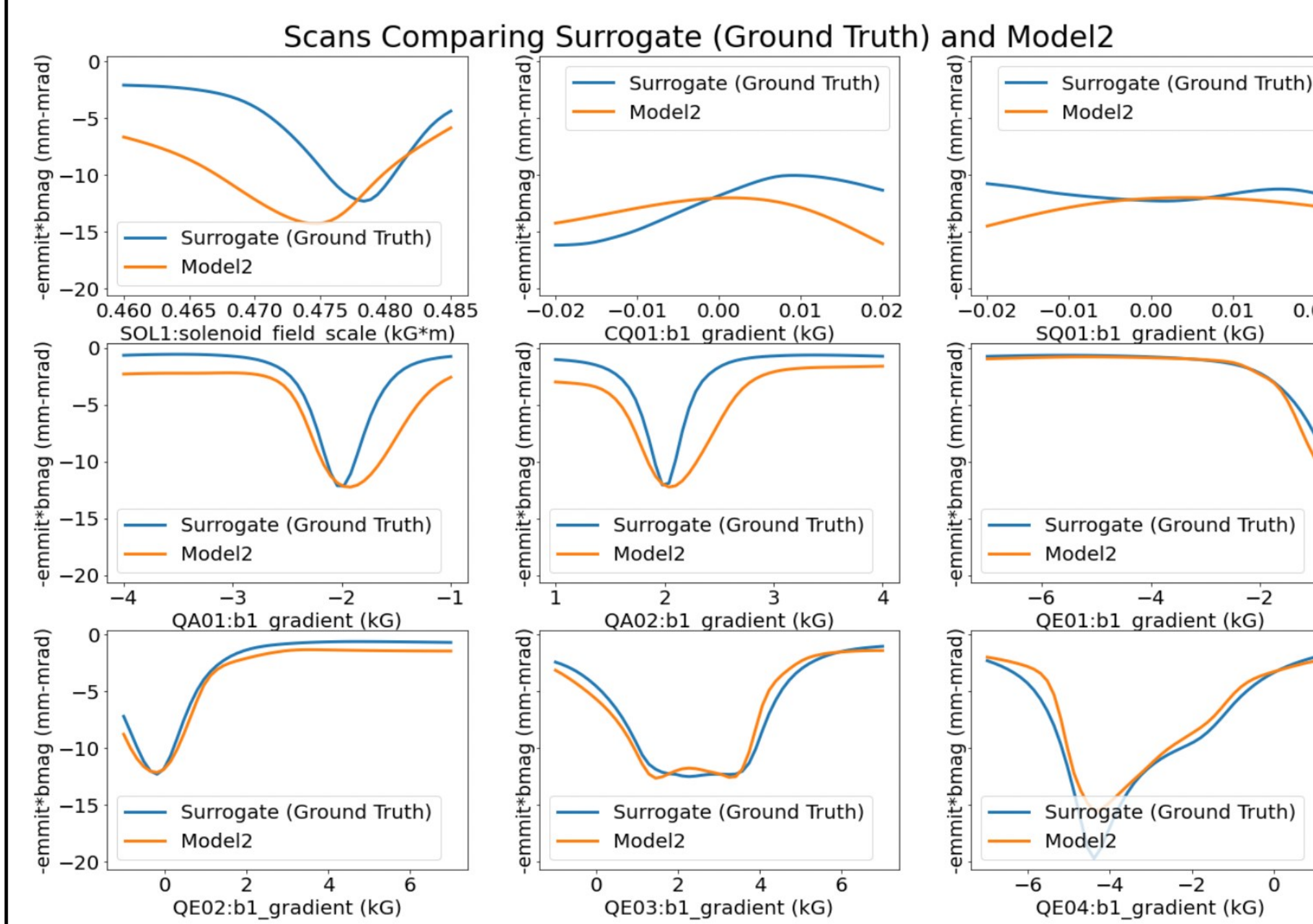
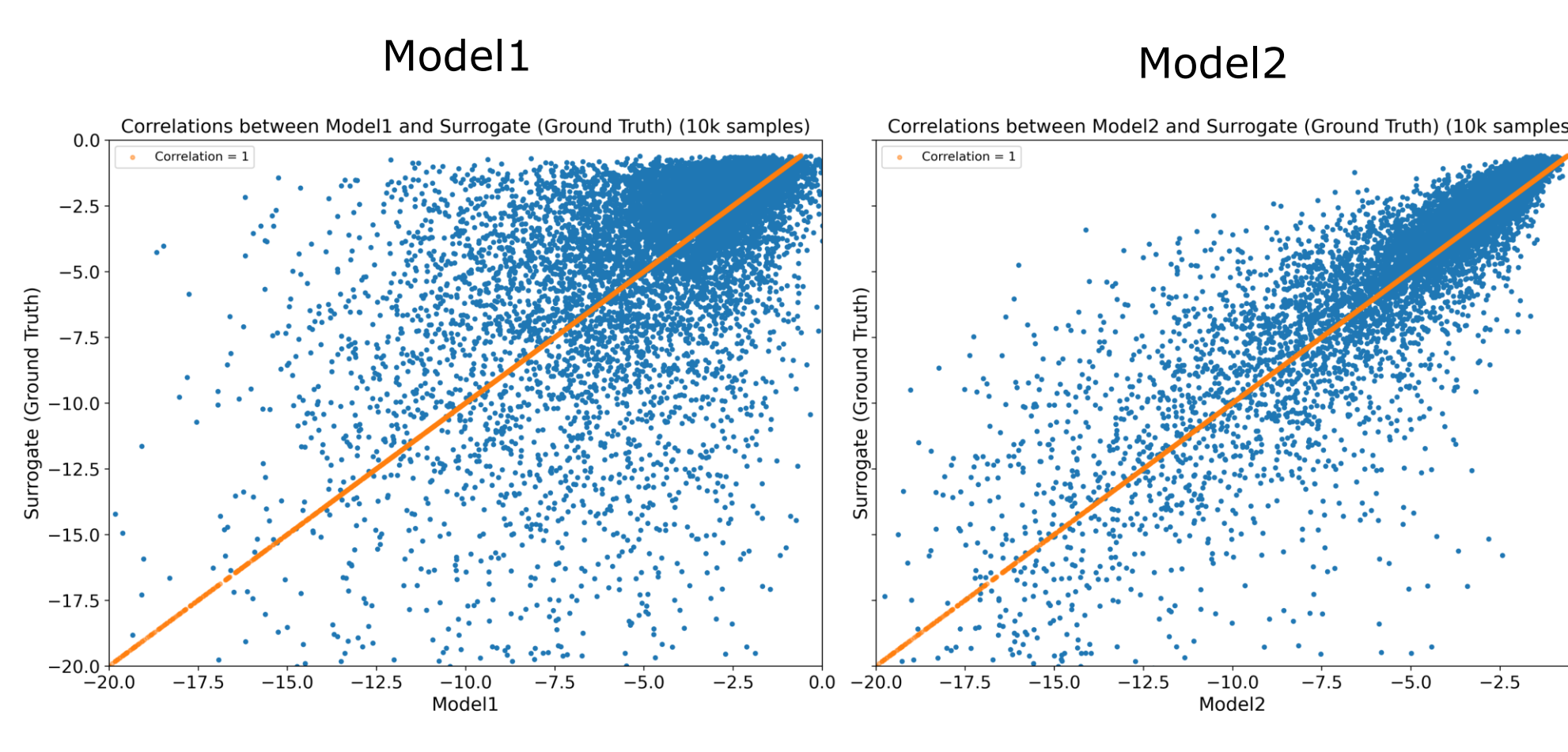


Figure 4: Accuracies of models trained on a mesh grid of  $3^9$  (top - Model1) and  $4^9$  (bottom - Model2) data samples are shown with parameter scans of the 9 input variables as well as the correlation values with the ground truth.



## BO Comparison Results

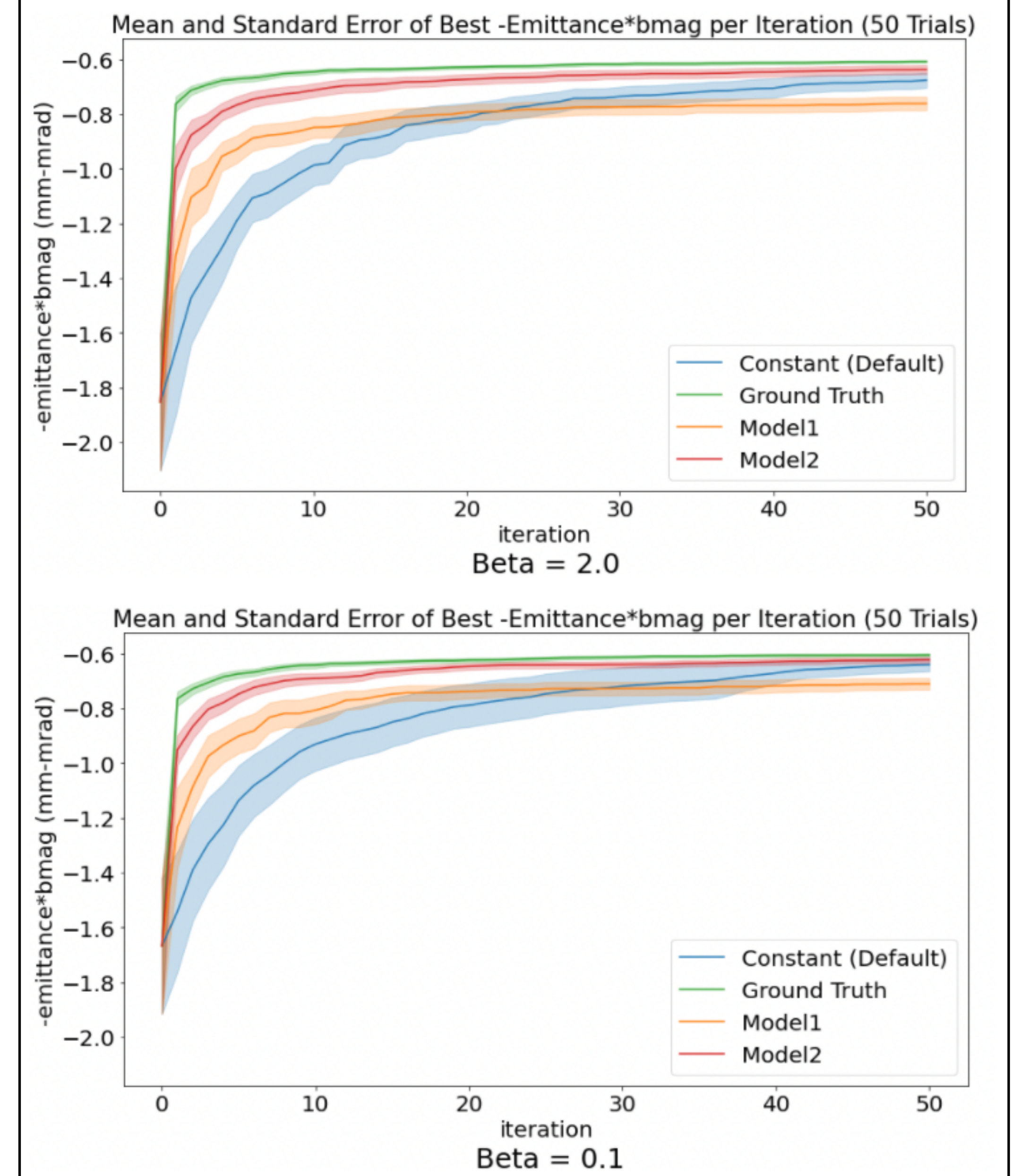


Figure 5: BO comparisons of the constant prior, injector surrogate "ground truth" prior, Model1, and Model2.

- In the trivial case, where the true model is used as a prior, the solution is found almost immediately
- Using better prior models demonstrates better initial performance vs. worse priors
- Both priors initially improve performance over uniform prior
- However, allowing the prior to be retrained at every iteration improves late-stage optimization performance

## Conclusions

- Our goal was to determine how accurate a model must be as a prior mean in order to get a performance gain in BO.
- Including prior information in the GP's prior mean always leads to an improvement in BO performance during **coarse tuning**.
- During **fine tuning**, we need a more accurate model to give better BO performance.

## Next Steps

- We will explore methods of improving BO performance during the fine-tuning stage.
- We will perform an experimental demonstration with the real accelerator.

## References

[1] F. -J. Decker et al., "Dispersion and Betatron Matching into the Linac," Conference Record of the 1991 IEEE Particle Accelerator Conference, 1991, pp. 905-907 vol.2, doi: 10.1109/PAC.1991.164485  
[2] Rasmussen, C. E. and Williams, C. K. I. *Gaussian Processes for Machine Learning*. MIT Press, 2006.