Notes by Les Fishbone for Summarizing the Zel’dovich and Novikov Textbook, *The Structure and Evolution of the Universe*.

Introduction

A. Well-Established

Observed isotropy of Universe to <~0.1 - 1.0% via relic radiation

Homogeneity deviations <~0.1 - 1.0% on a scale of 10**10 ly

GTR with cosmo constant is the best basis

Steady state and changing G are not valid

Hubble parameter is 50 km/sec/Mpc to within 50%

Uniform density and pressure

With lambda ~ 0, critical density is 0.5 x 10**-29 g /cm**3

If density < critical, Universe will expand unbounded and is infinite

Density is important to know

The use of celestial objects of a given type to determine the structure of the Universe is complicated by their intrinsic evolution and the evolution of their number as a function of time

“But the distances over which galaxies can be observed are small compared to cosmological scales. To this day, therefore, the structure of the Universe has not been established through observations of ordinary galaxies either.”

Important to know average density and particle types

Luminous matter has an average density of ~10**-31 g/cm**3, suggesting average number density of baryons ~6 x 10**-8 /cm**3

Galaxy motions suggest dark matter

Antimatter absence suggests charge asymmetry

RR photons now have an average number density of ~400/cm**3, 10**8 - 10**10 more than the number density of baryons. Their T of 2.7 K corresponds to an energy of 0.0007eV, yielding an overall photon mass-energy density now of 5 x 10**-34 g/cm**3, much lower than that of baryons now.

Density of neutrinos and gravitational waves is difficult to determine.

Thus there is as yet no answer to whether total density now is greater or less than critical density, and consequently whether Universe is finite or infinite.

Going back in time, T increases and radiation and matter are in thermo equilibrium because matter density ~ V**-1, while radiation density ~V**-4/3

At t~ 1 sec in Friedmann solution, T~10**10 or 10**6 eV and matter density ~10**6 g /cm**3. There would have been photons plus electrons and positrons and protons and neutrons.

Expansion leads to disappearance of positrons, while neutrons decay or combine with protons, forming 70% hydrogen and 30% helium by mass, but almost no heavier elements. Also remaining were neutrinos and antineutrinos.

Further expansion means matter mass density exceeds photon mass-energy density of photons.

All RR now seen is from z~1000, the time of last scattering. The corresponding distance is about 97% of the distance to the singularity, the horizon distance. The observable volume is then about 90% of the maximum possible volume.

Existing structure indicates early deviations from homogeneity and isotropy.

B. Not Well-Established

Use perturbation theory by modes on simple time-dependent solution rather than exact solutions of four dimensional spacetime, especially since initial conditions are unknown.

But how large were density perturbations?

Present average density of galaxy clusters is roughly characteristic of the average overall density at their formation time; this leads to an estimation of the formation time. For the plasma state in the RD era, this leads to fractional density oscillations of 10**-3 for delta rho/rho

With theory, observations of RR fluctuations then permit estimation perturbation magnitude was functions of the scale or mass, i.e., the perturbation spectrum.

Summary of Important RecentResult: Universe picture represents a weakly perturbed (almost homogeneous) expanding Universe with a definite initial (and large) entropy. Measurements of the spectrum and spatial distribution of the RR support this picture.

But can this picture explain galaxy rotation, magnetic fields and the origin of quasars?

Primordial magnetic fields are not necessary; plasma motions can generate observed fields.

But galaxy rotation given vortex-free initial perturbations? Possible given galaxy interactions.

Another theory is that galaxies formed from explosions of hyper dense bodies, but this violates known physics.

<u>C. Beginning of Expansion</u>

Anisotropic expansion before t ~ 1 sec?

Is there infinite density at the beginning or is that a characteristic of the isotropic homogeneous model?

There is proof a singularity even if expansion was not homogeneous and isotropic?

Details later, but here consider here aforementioned models plus perturbations. With these bases, do present observations and the laws of physics permit the establishment of the history of the Universe, including after and before (if meaningful) the singularity and the nature of the singularity itself?

Approach this via thermodynamics: many initial states can lead to a the same final state, which can serve as the new initial state for further evolution; the actual initial state is *forgotten*.

Thus find that cosmo model which arises from a wide class of initial early states.

Many anisotropic lead to isotropic expansion. But are such statistical arguments applicable?

Why is the entropy of the Universe large? Why hot at the start of expansion? Why are perturbations leading to observed structure of just the correct magnitude?

Laws of physics seem sufficient to explain all. Intense particle creation can occur from intense gravitational field close to the time right after the singularity, but only given anisotropic expansion.

Finally, there can be new phenomena given quantization of the metric.

Historical remark: Friedmann theory 1922-1924; Einstein mention thereof; Lemaitre 1927. Thus Lemaitre did not “independently” establish the laws of the expanding Universe.

After Hubble discovery in 1929, math solutions became established theory. Einstein remarked in 1931 that Friedmann was the first to follow this way;

I. The Homogeneous, Isotropic Universe: Its Expansion and Geometrical Structure

1. Local Properties of the Homogeneous, Isotropic Cosmological Model

Standard exposition based on Newtonian theory for Hubble expansion, age of the universe, and matter density and pressure

2. Relativistic Theory of the Homogeneous, Isotropic Universe

GTR needed to analyze large regions

See Vol. 1 for a sufficient exposition of GTR; Theory of Fields by Landau and Lifschitz for a complete GTR >> Friedmann eqns, with results the same as for the Newtonian description

Various models for open, closed, and flat (critical density) geometry

3. The Propagation Of Photons And Neutrinos; Observational Methods For Testing Cosmological Theories

Significant effects of relativistic matter on early expansion; cosmological neutrinos would not be observable today, though photons are

As density becomes infinite as size and age approach zero, visibility to an earlier stage is not possible because the optical depth, dependent on particle density, diverges.

Whereas the theoretical particle horizon is at t=0, practically it is at a later time when the optical depth is of order unity.

Observational quantities: red shift, angular size and luminosity of distant objects, amount of matter as a function of red shift, apparent magnitude

Deceleration parameter and the first approximation

Impossibility of determining the cosmo model if sources evolve in an unknown way

Distance ladder to far-away objects

Redshift vs. apparent magnitude observations rule out steady-state universe

No Olbers paradox in an expanding universe

4. The Cosmological Constant

Cosmo constant would only be manifest on the scale of the universe

History of cosmo constant starting with opinion that universe is static; Einstein desire for corresponding GTR solution and ideas of Mach

Hubble observation of expansion and Friedmann non-static GTR solutions Realization that cosmo constant is not needed, especially given new Hubble value of 75 and longer age for universe

Various cosmo models with nonzero cosmo constant.

II. Physical Processes in the Hot Universe

5. Intro to Part II

Relic radiation (RR = CMB) at T=2.7 K is the most important observational fact, and this RR (nor the equivalent background neutrinos) could not have been produced by astronomical objects

Also, there are about 10**(9+-1) photons per baryon

These two data allow characterization of the composition of the Universe at earlier time given thermodynamic equilibrium with specific entropy of matter conserved and volume changing smoothly during expansion

In later stages, nuclear reactions cease and nucleosynthesis takes place, with only photons, electrons, nuclei, neutrinos and gravitons surviving, with the last two undetectable

Hot universe proved by observations for the period 10 y <= t <= 10**10 y, and likely only consisting of mattering the large, not antimatter too.

Short historical review of RR prediction and discovery

Complete EM spectrum in the universe, of which a small section is the RR (Fig. 27, p. 126)

6. Thermodynamic Equilibrium…

Early radiation-dominated era with matter and antimatter

Ratio of photons to baryons hardly changes during expansion: thermo equilibrium during early stages and conservation of RR photons later

Given kT > mc**2 , the number of particles and antiparticles of each kind equals the number of photons. Thus ~10**8 nucleon-antinucleon pairs in the early universe for each nucleon today. This suggests that the present nucleons result from a small excess (10**-8) of nucleons over anti nucleons early.

Expansion eras are therefore:

1. Hadron era: with nucleons and antinucleons and ordinary and anti versions of all other particles; t~< 10**-6 s and T>10**13 K
2. Lepton era: with only a small remainder of nucleons, electron and

positrons annihilate by the end, leaving a small remainder, and neutrinos decouple; 10**-6 s < t < 10 s and 10**13 K > T > 5 10**9 K

3. Photon-Plasma era: plasma and radiation in equilibrium; 10 s < t < 10**12 s and 5x10**9 K > T > 10**4 K

4. Post-recombination era: t > 10**12 s and T < 10**4 K when the RR becomes transparent

Gravitons, if they exist, would always be present but would not interact with other particles after Planck time ~10**-43 s.

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At a sufficiently high temperature T such that kT > Mc**2, where M is the mass of the most massive particle, photons and other relativistic particles dominate

P = e /3 = rho x c**2/3 and e = kappa x sigma x T**4 to take account .of all kinds of relativistic particles

And n ~ e /(3kT) for the particle density

Consider T ~ 1 MeV, t ~ 1 sec, and n(electron) ~ n(positron) ~ 10**-31 cm**-3 and annihilation cross section sigma(Annihilation) ~ 10**-24 and particles move at c, then time to establish equilibrium is

.Tau ~ 10**-17 small compared to 1 sec

Similarly for higher mass particles at higher temperatures and .correspondingly earlier times

When kT > m(nuc),   n(nuc then) - n(antinuc then) = n(nuc now) ~

(10**-8 x n(photons now)

n(nuc then) ~ n(photons then)

One could apply the same considerations to quarks and so on and so forth if kT > m(q) c**2

Hot vs. cold matter as n >> infinity ? Different models of Hagedorn and Omnes

Quark theory for nucleons

Conservation of energy and baryon charge and entropy for slow, .adiabatic processes => evolution can be described

Particle-antiparticle annihilation requires binary collisions, increasingly .unlikely as the Universe expands

Residual n(antiparticle) in charge symmetric theory is very small at •
the end of hadron era, T ~ 1 MeV because annihilation sigma is large and nucleon excess => exponentially small n(antiparticle) .when their creation ceases

Residual n(q) is large . With respect to photons, it is ~[Gm**2/ .2
.hc]**1/2 ~10**-18; with respect to nucleons, it is about 10**-9

In spatially homogeneous, charge-sym universe, nucleon problem .3
similar to quarks and leads to 10**-18 nucleons/photon, disagreeing with observations by 10**10. So we should consider charge-.asymmetric universe

This leads to Omnes theory. Charge symmetry of primordial homogeneity is spontaneously broken on the microscopic scale. Strong interaction leads to separation of matter and antimatter drops of size

~10**-3 cm at 10**-6 sec

This separation tendency stops as T decreases and annihilation occurs as usual. But spatial separation means annihilation occurs mainly at the .boundary of regions

There exist regions with 10**-9 nucleons /photon and regions with 10**-9 antinuc/photon => galaxies and antigalaxies. Omnes calculations lead to two characteristic quantities: average n(nuc or antinuc) / .n(photons). And characteristic size of matter or antimatter region
But a consistent calc of separation and following annihilation leads to a much smaller concentration of nucleons and antinucleons, disagreeing with present density of nucleons

Annihilation continues during radiation-dominated stage, but expected consequences of prolonged annihilation are not observed. Thus, even with account of phase separation, charge-symmetric theory does not .agree with observations

Consider therefore charge-asymmetric Universe with excess baryons always. Early, excess of baryons is small given number of pairs, so Omnes phase separation is plausible then

For T = 300 MeV, charge asymmetry manifests itself only in at T > 1 MeV, when there is an abundance of electrons and positrons and RR .spectrum takes equilibrium form

Finally, charge asymmetry leads to n(baryon today) = n(baryon charge .density initially)

It would nevertheless be very interesting to find evidence now of hadron .era phase separation

Hagedorn theory that the number of charged particles is infinite is

.contradicted by experimental results of QED

7. Kinetics of Elementary Particle Processess

In earliest stages of the hot Universe, neutrinos (+anti) are in thermo equilib with other particles. Creation of neutrinos mainly by e- + e+ >> neutrino (+anti) with relativistic cross section

Sigma ~ g**2 x E**2 / h**4 x c**4, where g ~ 10**-49 ergs cm**3 is the weak interaction constant

Given particle energy of kT, time to reach equilibrium tau = 1 / (sigma x n x c) , and previous relation between universe time t and temperature T, :we obtain dependent of tau and t

tau ~ [G**5/4 x h ** 13/4] / [g**2 x c**1/4] x t**5/2 (Landau & Lifschitz for statistical factors). When tau is greater than t, neutrinos no longer interact either with other particles or with one another. Equating them leads to t ~ 0.1 s without consideration of numerical factors but showing the dependencies of G and g . More accurate calculations .follow, including consideration of mu neutrinos

Present temperatures neutrinos compared to photons is then (Peebles)

T(neutrino) = (4/11)**(1/3) x T(photon) ~ 0.7 x 2.7K ~ 2K

But mass of neutrinos could be ~<100 eV, while background cosmic neutrinos would have energy 5 x 10**-4 eV , so observation of the .background would require measurement improvement by 10**6

RR spectrum tells us about z ~ 10**6 at t ~ 1 yr; neutrino spectrum ! could tell us abut z ~ 10**10 at t ~ 0.1 s

Particles that decay spontaneously disappear exponentially as Universe expands. Stable particles would remain if annihilation reactions do not .occur

YFIGURE 30 FOR TEMPERATURE-DEPENDENCE OF PARTICLE RELATIVE ENERGY DENSITY

⁇ MISSING TEXT FOR APRIL AND MAY RESULTING FROM IMAC FAILURE

May 12 Introductory Remarks2

a.. Concerning *Gravitation* by MTW, a new edition (printing) was published in 2017 by Princeton University Press after the original in 1973 by Freeman. This new edition contains by David Kaiser describing the style of the original, the publishing history, and reactions to it. The new edition also contains an additional preface by MT that focuses on the status of the material in the text in light of developments subsequent to the original—chapter by chapter. Perturbations in the early Universe that could lead to structure formation are not addressed in MTW, though the papers by Lifschitz and Khalatnikov are cited (see point b).s

b. The 1961 textbook by Landau and Lifschitz entitled the *The Classical Theory of Fields* does not contain material about GTR cosmology that goes beyond the simple Friedmann models, even though Lifschitz himself in 1946 and with Khalatnikov in 1964 addressed GTR perturbations.

c. The 1972 monograph by Weinberg, *Gravitation and Cosmology: Principles and Applications of the GTR*, does contain material about GTR cosmology that goes beyond the simple Friedmann models, in particular descriptions of the early hot Universe and perturbations that could lead to galaxy formation. There are citations to work by Z&N and to Lifschitz, among many, many others.

May 12 Summary

11: Instability in the Hot Model

Approach of last chapter here applied to RD period, when matter completely ionized, the radiation density dominates, and the matter is coupled to the radiation.   For omega0 = 1, there is a short period, when the matter is still completely ionized, and the sound speed varies as b = (c/3**0.5) [1 + 3 rho(m) / 4 rho(r)]**-0.5. Matter and radiation densities become equal when z = 10**4 omega0 or t ~ 2 * 10**11 * (omega0)**-2 s.

?????During the period the Universe is filled with a medium whose equation of state is, P = e/3 ~ a**-4 ~ t**-2, and sound speed is b = c / 3**0.5   A definite value for the temperature follows, while the matter density still requires specification of several parameters.

e

With the help of the Jeans criterion, let us find the conditions dividing regions of stability and instability.

Perturbations are of the form delta = deltakappa(t) * exp(k dot x *i) , where k = k0 * a(t0)**-1 (1 + z) and lamda = lamda0 / (1 + z). K0 and lamda0 refer to the present. With the Jeans criterion taking the form (b * k)**2 = 4 pi G rho, and substituting values of b and rho for the RD era, we obtain

kJeans = 3 /(8**0.5 * c t) and lamdaJeans = 2 pi /kJeans = c t * 4 pi* 2**0.5 / 3

The Jeans length is therefore of the order of the distance over which pressure gradients (sound waves) can equalize density.

The regions of stability and instability are conveniently seen in Figures 43 &44 Also see Weinberg, Figure 15.6, n

See slides PPT 36 & 38

Other points in this chapter:
a. as a result of dissipative processes, decay of a wave is determined by conditions during the last part of every period considered because the increase in photon MFP overpowers the increase in the wavelength;
b. conclusion that difficult-to-observe particles give rise to a rather small decrease in the amplitude of oscillations compared to neglecting them;
c hypotheses that supermassive stars or globular clusters result from entropy perturbs;
d.. conservation of vortex velocity upon early stages of expansion, matching perturbs when the eqn of state change; and
e .Sakharov oscillations

12: Gravitational Instability in the GTR

GTR approach necessary for perturbations with lambda >~ c t in a fluid .with eqn of state P = e / 3

Method is take homogeneous, isotropic Friedmann model but then to replace metric g by g0 + h , where h represents perturbations; analogously, stress-energy tensor changes from e to e0 + delta e, with delta P determined by eqn of state, and finally perturbed velocities u are assumed small, with u0 fixed via the identity u dot u = 1 . These expressions are then substituted into GTR eqns, relating h, delta e, and u , and yielding their time evolution once initial perturbations are .specified

e slides PPT 41 & 42 for the metric and the resulting eqns for the perturbs from the GTR eqns. All computations are done in the linear approx in which the quantities h are first-order. Then perturbed values of e, P and. U occur only in second order.

Two caveats:
1. GTR eqns put some restrictions on these initial values.
2. Coordinate system choice allows apparent unphysical results; distinguishing between them is important.

Follow approach of Lifschitz and many others: synchronous reference system is used to study the perturbations, and their consequences are studied in other coord systems. Nonlinear approximations likely lead to important changes to be examined in later chapters.

So consider perturbations in spatial regions which may be large with respect to c*t but small with respect to radius of curvature during the period studied; a >> c*t during the early stages of evolution, conditions equivalent to perturbs in a flat model with rho(critical). This will yield possible galactic evolution paths; Lifschitz studied more general cases.

Solutions of the eqns will be as plane waves of the form Q = exp [i kappa dot x] on the background of a spatially homogeneous and isotropic evolving Universe with the same invariances as the unperturbed solution, with kappa a certain vector and Q a scalar.

Then tensors can be constructed from kappa and Q. There will also be a vector P = kappa * Q, and still another vector Sigma = Es * Q , with Es perpendicular to kappa.

These are scalars, vectors and tensors only in 3-d space.

The scalar Q will describe density perturbations and Es will describe velocity perturbs. Another tensor with a plane wave dependence will describe gravitational waves.

Scalar : The main term for very early times is that the fractional energy density varies linearly with time.

For late times, the fractional energy density would vary as cos [(kappa * eta) / 3**0.5], corresponding to acoustic oscillations with a speed of c / 3**0.5 and to constant-amplitude density perturbations. This exact result supports the results of the intuitive analysis. The exact theory also works with metric perturbs, which would be finite as t tends to zero. {There was a line missing in the translation on page 292.}.

The independence of the metric perturbations with respect to time during early stages of the expansion accords completely with the idea underscored by the intuitive analysis of long-wave perturbs. This independence is also in accord with the independent evolution of different regions of the Universe with different initial conditions.

For lambda << c * t , the metric perturbs tend to zero and we are left with a description of sound waves. But this result applies to the eqn of state of a RD plasma, where b = c / 3**0.5, with the pressure gradient smoothing perturbs, so the "sound" horizon is important. For dust, P = 0, there is no sound propagation to smooth geometry and density perturbs. Density perturbs continue to grow for lambda < c * t, but metric perturbs remain constant.

One very important conclusion of Lifschitz's work that remains valid is that to explain finite perturbs today (galaxies!), it is sufficient for delta to tend to 0 as t tends to 0, whereas the metric perturb must remain nonzero as t tends to 0. Novikov found h ~ 10**-2 to 10**-3 as t tends to 0, corresponding to galaxy clusters.

Vector (Rotational) perturbs case exhibits differences from Hubble expansion. The solutions show that metric perturbs grow as time approaches zero, leading to the conclusion that initial rotational perturbs are incompatible with a Friedmann model. This conclusion is important in any discussion of galaxy formation in the vortex theory.

Tensor perturbs have two independent polarizations for a given wave vector. If the wavelength is less that c * t , the solution describes a wavelike gravitational field. So when the wavelength becomes less than the horizon size, usual energy density computations apply:

Amplitude h ~ (1 + z) ~ a**-1, and energy density e ~ (1 + z)**4 ~ a**-4 . The wave velocity is c . The density and velocity perturbs are not connected with gravitational waves.

Matter (coming?) velocity relative to perturbed coordinates is zero in the field of a gravitational wave, but particle velocities do change, so a sphere becomes a time-varying ellipsoid in the directions perpendicular to the velocity of the wave. If the wave passes through an ideal fluid, energy is not dissipated, so entropy does not grow and new waves are not created. Viscosity would change this

Entropy perturbs could arise as inhomogeneities in the eqn of state, with one approximate form being

,P = e/3 [1 - B(x) e**-1/4]

with consequent effects on the metric and motion. Initial entropy perturbs would give rise to adiabatic density perturbs, and, in particular, to growing-mode adiabatic perturbs if the wavelength is sufficiently large, though this is not a relativistic effect. Entropy perturbs corresponding to masses between one solar mass and 10**4 solar masses would only cause decaying RD plasma oscillations before recombination. For masses greater than 10**4 solars masses, the entropy irregularities are preserved.Such perturbs could be related to the formation and evaporation of n PBHs, leading to entropy production, !possibly all entropy!y

One interesting idea is a quasi-isotropic solution with a uniform :distribution of perturbs described by this metric ds**2 = (c dt)**2 - [t* a(x) + t**2 b(x) + …] dx dx

The functions a and b are of the spatial-metric type, with 3x3 indices, .and x is a spatial vector

A general result is that Friedmann behavior near the singularity is compatible with density perturbs and gravitational waves, which may not be small, but not with vortex perturbs. Thus the quasi-isotropic solution, or entropy perturbs, represent cosmo solutions not in conflict with the present state of the Universe, deviating least from the strictly ,homogeneous solution

Perturbs whose wavelength is comparable with the size of the Universe requires analysis with math beyond plane-wave theory. Assume omega unequal to unity to exclude the flat model. The ratio of the wavelength of the perturb to the radius of the model is constant during the expansion. Solutions are constructed similar to the methods for .constructing spherical harmonics. Scalar functions are considered

Some remarks about the possible periodic distribution of quasars as a .function of redshift

:The concluding remark in this chapter is as follows

ZN: "One indeed ought to expect that the spherical-wave method will find wide application in the theory of perturbations of the homogeneous, isotropic Universe in the very near future." N.B.:This is a prediction from 1975!**.**

LF: See slide 43 interpreting the CMB Planck observations as a function of the spherical harmonic parameter l, a result from 2009 and later, as well as from WMAP from 2001 and later.

June 15 Summary

13: Statistical Theory

Any small perturbation can be represented as a linear combo of independent plane waves. Further steps are to examine wave interactions and to solve nonlinear problems.

Galaxy forms and locations are random, suggesting random initial perturbations and a statistical description of the Universe subject to the fundamental laws of physics.

An initial assumption of density perturbations in boxes fails because of interactions among neighboring boxes.

Consider instead a plane wave expansion of the density perturbations:

Deltarho = Sumk Ak * Psik(x) where Psik(x) = V**-0.5 * exp(i*k dot x)

The Psi functions satisfy orthonormal conditions. A convenient dimensionless quantity is

Deltak = [k**3 / (2 pi**2 * n)] * Sumk (abs Ak)**2.

If for all k Delta is small, then the density perturb is small.

For a bounded volume V = L**3, then kx = 2 pi nx / L, etc., where the n values are whole numbers.

A reasonable definition of a random function is one whose Fourier coeffs are random, and the randomness is not resolved by the physical interaction while the perturb is small. The randomness hypothesis is connected with the idea that we can choose many different volumes in the Universe. Each has a definite density function and a single set of .amplitudes Ak. How often is a given Ak value encountered? l

Let Ak = Bk + iCk and consider over N volumes and respective k values the probability P(B,C,...) for the appearance of given values of the Fourier coeffs. A natural form for this probability is proportional to

exp(-Bk**2 / [2 * betak**2]) * exp(-Ck**2 / [2 * gammak**2])

At an early stage near the singularity, the integral which determines the Fourier coeffs reduces to a sum over causally diisconnected regions (if there is no period "before the singularity"). Hence the assumption of a normal distribution for the Ak is natural.

Even for small inhomogeneities of 10-20%, the astronomer wants to know their form and amplitude, not Fourier coeffs. While the average value of delta vanishes at each point of space, the average of its square does not. The properties of the Fourier series leads to

Avg[delta**2] = Sumk Avg[Ak**2] / V

With normal distributions for Bk and Ck,

we then find for the prob that a given value of delta is obtained at any given point, with DELTA the same for all points and independent of time,f

P(delta) = [1 / (2 * pi * DELTA)**0.5] * exp(-delta**2 / [2 * DELTA])

Note well: delta and DELTA represent different quantities; delta is the dimensionless density amplitude.

This function describes the amplitude of the inhomogeneities, but says ..nothing about their spatial structure

The correlation function f(r) characterizes their spatial structure, with

f(r) = Avg [deltax * deltay] / Avg [delta**2] and where r = x - y

This correlation function will be positive for small values of r , but its sign will vary for larger values. The natural conclusion is that the first zero of this function will define regions with the same sign of delta. For example, if f(r) is given in terms of the spectral function betak characterizing the amplitude of waves of various length and is concentrated in a narrow interval around a wave number k0, then the (.first zero is at r0 = pi / k0 , half a wavelength

When has a significant fraction of the mass passed into gravitationally bound objects? Suppose that the betak with k < kJeans grow with time as a consequence of gravitational instability. Fragmentation will have occurred when the growing DELTA is of order unity.

Agood and simple description of the matter inhomogeneity is given by average mass and its deviation through

mu = Avg[delta M]/Avg M = (Avg [M**2] - [Avg M]**2)**0.5 / Avg M

Assume that all matter is distributed in the form of isolated bodies of mass M1 with average density rho1, so the number density of bodies is Avg (rho) / M1 and the volume of each body is v = M1 / rho1. Then mu has the following behavior:

1. For Avg M >> M1, many bodies are found in the volume under consideration, so mu is small and tends to zero as Avg M tends to infinity;

2. If (Avg [rho)] / rho1) * M1 < Avg M < M1, then sometimes there is only only one or not even one body in the volume under consideration and
mu ~ (M1 / Avg [M])**0.5 > 1

For Avg [M] < Avg [rho] * M1 / rho1, i.e., for Avg [M] < Avg [rho] * .3 v, the volume under consideration is less than that of a single body and

mu ~ (rho1 / Avg [rho])**0.5 > 1

How does mu behave for Avg[M]>>M1, for objects containing many of the smaller bodies? Naively, much as does a statistically independent distribution of particles. The average number of little bodies in such an object is   Avg[N] = Avg [M] / M1, while

mu = Avg[deltaN]/Avg[N] = 1 / N**.5 = (M1/Avg[M])**0.5 < 1 for Avg .[M] > M1

But this is not always correct. There is no universal description of the behavior of the function mu since it depends on the isolation process. Indeed, a study of the processes in large volumes containing many objects and leading to small mu values can give valuable info about the .universe and its large-scale structure

The law delta N = N**0.5 is only obtained given a random—not correlated—arrangement of discrete objects in space, corresponding to the hypothesis of a God who, from outside, sows space with galaxies and that they fall into regions independent of how the preceding ones are distributed. But this hypothesis is evidently unacceptable, since gravity from existing objects affects the growth of small perturbs. An evolutionary formulation is necessary from a uniform distribution. Seemingly perturbs grow from the inflow of matter from neighboring regions. But this reasoning is not valid in the case of gravitational .instability, taking account of the long-range nature of gravity

Considerations of Jeans theory allows one to say that the increase of matter at the center (in a spherical configuration) is not from neighboring regions, but from infinity! Thus there is not an anticorrelation among .neighboring galaxies
The final conclusion is that the fluctuation law delta N = f(N) for the distribution of discrete objects depends on the law for the original small perturbs. In principle, delta N ~ 1 / N**1/6 or delta N ~ N**2/3 are .possible, depending on the spectrum of small perturbs
.Observational studies can give insight into the initial state

Concerning limitations of the linear theory, the first is that it is practically difficult to calculate the properties of a surface of given delta for a distribution function whose Fourier expansion is specified. Second, there are conceptual problems in matching the topology of regions with particular delta values, as small islands say, with the topology of known .astronomical objects

The root of these difficulties is that astronomical objects result from .strong nonlinearities, which are addressed in the next chapter

14: Nonlinear Theory and Thermal Instability
Three ways to approach the nonlinear problem, possibly in combination:

1. Exact solution with special initial conditions;
2. Approximation method for extrapolating the linear solution to the general case; and
Q3.Qualitative explanation of the properties of the general exact .3 solution.3

Spherically symmetric perturbs can be analyzed exactly because the effect of neighboring perturbs vanishes. Second method takes account of the tidal action of neighboring perturbs, but this is an approx.

Analyses are only for dust, for which pressure vanishes. The simplest spherically symmetric case is of a sphere with perturbed density Omega' on a Friedmann background with Omega, where Omega' > Omega. With no extra mass, there is a hole in the shell outside the higher density region.

If Omega <= 1, then the two cases Omega' <= 1 and Omega' > 1 are possible. In the first case, the perturbed sphere expands indefinitely and a bound object does not form. In the second the perturb behaves as a closed Friedmann model, going from expansion to contraction and eventual recollapse. In this "Swiss-cheese model", the many such perturbs do not affect the average expansion. Differences occur if there is not initial symmetry or inhomogeneity. If there is pressure, then infinite density does not occur over the whole volume simultaneously, leading to shock waves and nonzero pressure and entropy.

In the early moments of the solution, when the unperturbed density is large and perturbations small, what critical perturb amplitude leads to the formation of gravitationally bound objects. The answer is

Deltacrit = (3/5) * (1 - Omega0) / [Omega0 * (1 + z)]

From observations in the expanding Universe, the amplitude of density perturbs become of order unity when the linear size of inhomogeneities is much less than the horizon ct and the radius of curvature a . Thus a linear perturb theory loses its validity when the use of Newtonian physics remains valid, i.e. when relativistic effects are insubstantial. Additionally, there are cases where gas pressure negligibly affects perturb growth, especially adiabatic perturbs during the prerecombination era RD era. Below we consider post recombination effects of such perturbs.

Eulerian coords r of particles are written as functions of their Lagrangian coords s as

r = a(t) * s + b(t) x(s),

where the first term corresponds to unperturbed motion. Neglecting the second term, we find

u = dr/dt =s * da/dt = r *1/a * da/dt ,

which is the Hubble expansion law. Thus the Lagrangian coords s are defined as the comoving cords of the unperturbed motion. The second term describes perturbs, exact for small, growing perturbs; we shall use it even for large density contrasts.

Earlier it was shown that in the linear approx and when P=0, a perturb of any form grows, but its form remains unchanged:

delta = delta0* (r / a) * phi1(t)   and w = w0 * (r/a) * phi2(t)

But we also stipulate that, while the density distribution is arbitrary, the peculiar velocity w is vortex-free. A reformulation incorporating this condition is that w0 is derivable from an arbitrary potential, w0 = Del phi;   then Del x w0 = 0. We assume too that the delta0 = - Del dot w0.

In the construction of the approx nonlinear theory, we select as an extrapol the linear formula w = w0(s) * phi2(t). The peculiar velocity is then

W = dr/dt - H*r = (1/a) * [a* (db/dt) - b * (da/dt)] * x

The Hubble parameter is (1/a) * (da/dt), x(s) is vortex-free, and phi2 and b re related. This variant of the linear approx
is useful for the following qualitative reason. In the absence of other

forces, the exact solution takes the form

r = a0 * t * s + t v(s) + s

Then particle trajectories intersect and infinite density is achieved. Clearly the perturbs are large near this singularity. But in the general case, the sing. Takes the form of a 2D surface. Only for degenerate cases does the intersection occur along a line or at a point. Now take into account grav. forces: near the 2D sing these forces are finite, so they do not exert a drastic influence on the perturb growth and do not seriously affect the general picture. Thus it is reasonable to to seek a solution for large perturbs in a form valid for small perturbs and small gravitational forces. And it becomes easy to calculate other quantities such as the density and the velocity. For a given r(s), the density equation in Lagragian coords is exactly soluble.

In the linear approx, b(t) is well known and density perturbs are proportional to the ratio b(t) / a(t), also well known.

The density can be rewritten in this form:

rho = Avg(rho) / {[1 - (b / a) * alpha] * [1 - (b / a) * beta] * [1 - (b / a) * gamma]}

Here, alpha, beta and gamma are functions of s only, while b / a is a universal function of t (depending on omega0 also), conveniently expressed in terms of z . The generally unequal functions alpha, beta and gamma depend on the specific form x(s) of the initial perturb and thus characterize the deformation along the three orthogonal axes of the deformation tensor. For definiteness, choose alpha > beta > gamma. V v

While the case alpha < 0 is possible, if alpha > 0, then [alpha * (b / a)] grows and can reach unity in the course of the evolution. Then it follows from the density equation that the density becomes infinite there.

This arises as a result of the 1D contraction along the axis related to alpha. The picture that results is that when the perturbs become sufficiently large, flat pancakes of collapsed dust form in various places.

This general picture is supported more generally. It is necessary to note that contraction along one coord dan be accompanied by contraction or expansion in the plane of the pancake. Based on a complicated probability distribution function (pdf) for alpha, beta and gamma obtained by Doroshkevich [LF interpretation of the text; note too that this pdf is given in the text], only 8% of the matter contracts along all three axes, while 84% contracts in one direction but expands in one or two.

The value 8% is close to Oort's estimate of ~6% that matter is compressed in all three directions. It is however not evident that only this 8% eventually becomes gravitationally bound. [see Oort, J.H. 1958, in La structure et l'evolution de l'univers, 11 Conseil de Physique Solvay (Brussels: Stoops) and 1970, Astronomy. Ap., **7**, 384]. e

Physical applications of this pancake picture will be discussed in the next chapter. r