Domain Wall Fermions as heavy quark discretisation

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RBC and **UKQCD** collaborations

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Team

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Outline

Motivation: Heavy Quark Physics

SM is not the end of the story but: **No smoking gun**.
Instead try to **over constrain** the SM and sharpen predictions

- Heavy quarks sensitive to new physics
- CKM
- Tension between inclusive and exclusive
- Large Experimental Efforts in heavy quark physics: Belle II, LHCb

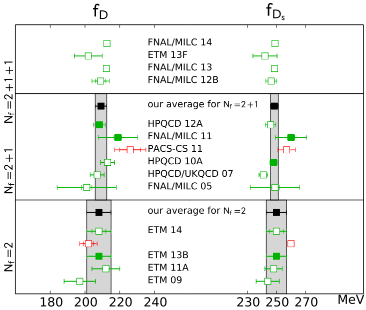
Heavy quarks on the lattice

Problematics

- two largely separated energy scales
- huge cut-off effects
- fine spacings and topological freezing

Fermion discretisations used:

- HISQ
- NRH
- HQET
- TM

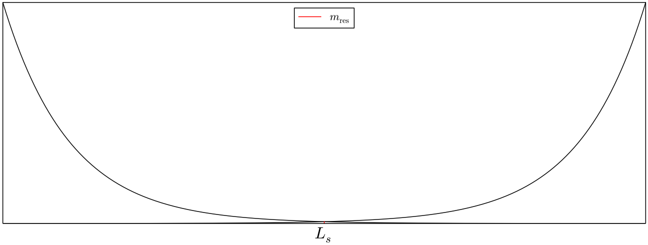


$\Rightarrow$ Add **DWF**

DWF - a brief introduction

DWFs allow for simulations with approximate chiral symmetry.

- Introduce a fifth dimension of length L_s.
- Choose the **Domain Wall Height** M_5
- LH and RH massless modes exponentially localised at boundaries.
- A measure of the **residual chiral symmetry breaking** is given by $m_{\rm res}$.



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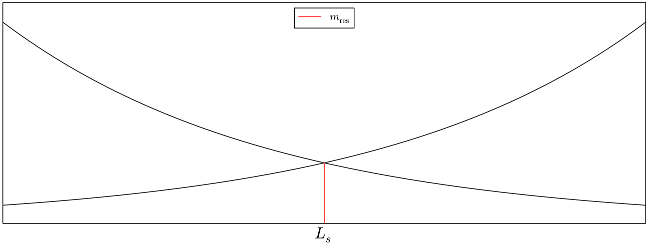


Motivation: Domain Wall Fermions

DWFs provide a method to simulate (approximately) chiral fermions on the lattice

$\Rightarrow$ Automatic $\mathcal{O}(a)$ improvement

$\Rightarrow$ No operator mixing:

$\Rightarrow$ easier renormalisation

BUT:

- More expensive due to fifth dimension

Motivation: Heavy Domain Wall Fermions

For **CHARM**:

- No effective theory needed
  $\Rightarrow$ No tuning of effective theory parameters required
- No rooting required
- Use same discretisation for light and heavy quarks:
  $\Rightarrow$ GIM

For **BOTTOM**:

- Make contact with HQET from data in charm region and beyond.

Study 1: A Quenched Pilot Study

IDEA

- Map out parameter space of DWF suitable for heavy quarks
- Interested in cut-off effects $\Rightarrow$ keep $L \approx \mathrm{const}$
- Test continuum scaling of basic observables

$\Rightarrow$ Quenched Pilot Study: arXiv:1504.01630, arXiv:1602.04118

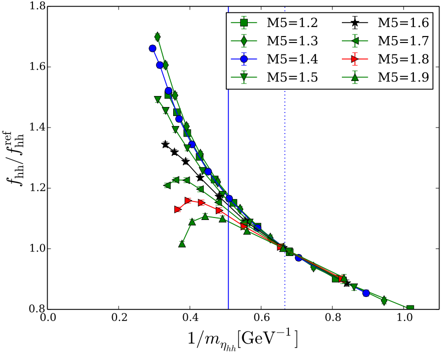
- Hope for qualitatively similar behaviour in the dynamical case

β	L/a	$a^{-1}(\mathrm{GeV})$	$L(\mathrm{fm})$
4.41	16	2.0	1.55
4.66	24	2.9	1.66
4.89	32	3.9	1.63
5.20	48	5.7	1.65

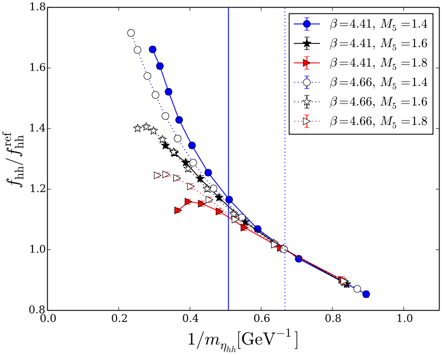
- tree-level Symanzik improved gauge action
- Over-relaxation heat bath

Effects of M_5 - single ensemble

- M_5 and L_s are parameters of the discretisation, i.e. *NOT* of QCD
- Strong dependence on M_5 in the charm region,
- mechanism breaks down beyond charm



Effects of M_5 - two ensembles



- repeat for 2nd ensemble
- similar behaviour, but for heavier **physical** masses
- Indication of cut-off effects

$\Rightarrow$ $M_5 = 1.6$ indicates mild continuum limit scaling.

Residual Mass

Define a measure of the residual chiral symmetry breaking from Axial vector Ward Identity:

$$\left\langle \sum_{\mathbf{x}} \Delta_\mu \mathcal{A}_\mu(x) P(0) \right\rangle = 2m \left\langle \sum_{\mathbf{x}} P(x)P(0) \right\rangle + 2 \left\langle \sum_{\mathbf{x}} J^a_{5q}(x) P(0) \right\rangle$$

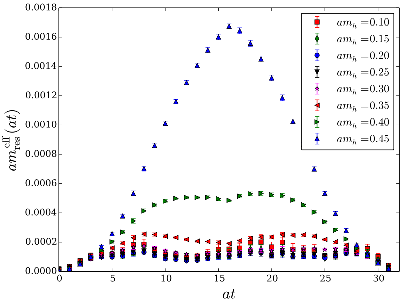
$$\Rightarrow C_{m_{\mathrm{res}}}(t) = \frac{\left\langle \sum_{\mathbf{x}} J^a_{5q}(x) P(0) \right\rangle}{\left\langle \sum_{\mathbf{x}} P(x) P(0) \right\rangle}$$

Residual Mass

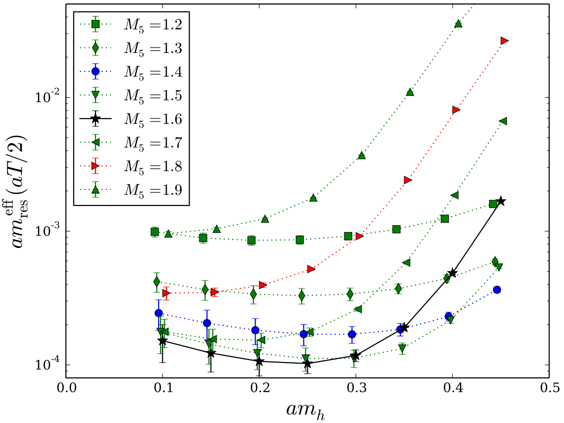
What happens at large input quark masses? Check residual mass

$$C_{m_{\mathrm{res}}}(t) = \frac{\left\langle \sum_{\mathbf{x}} J^a_{5q}(x) P(0) \right\rangle}{\left\langle \sum_{\mathbf{x}} P(x) P(0) \right\rangle}$$

- Expect plateau
- For $M_5 = 1.6$ mechanism seems to break down for $am_h \gtrsim 0.4$
- No longer simulating (approx.) chiral QCD



Residual Mass



- Same qualitative behaviour present for all choices of M_5.
- We choose $M_5 = 1.6$
- Restrict to $am_h \leq 0.4$

Strategy

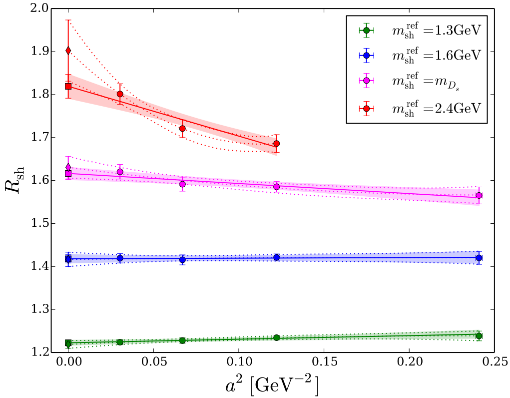
GOAL: Want to investigate **continuum scaling** of observables

- Choose $M_5 = 1.6$
- Restrict to $am \lesssim 0.4$
- Simulate a range of heavy quark masses on all ensembles
- Fit correlation functions $\Rightarrow$ masses, energies, decay constants
- Interpolate observables to common reference masses:

$$m_{\mathrm{sh}}^{\mathrm{ref}} = 1.0, 1.3, 1.6, 1.9685, 2.4\mathrm{GeV}$$

- Take continuum limit of observables

Continuum Limit of decay constants



- Consider

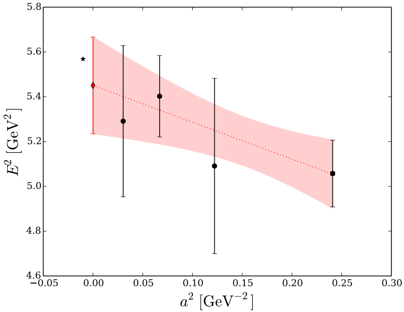
$$R_{sh} = \frac{f_{sh}\sqrt{m_{sh}}}{f_{sh}^{\mathrm{norm}}\sqrt{m_{sh}^{\mathrm{norm}}}},$$

  with $m_{sh}^{\mathrm{norm}} = 1.0\mathrm{GeV}$

- Very flat continuum limit
- can simulate charm even on the coarsest ensemble

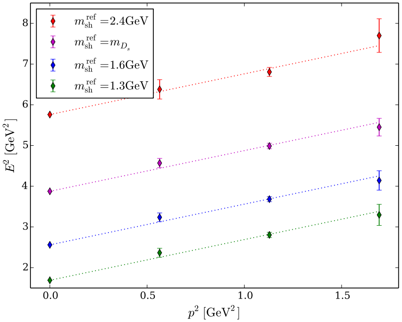
Continuum Limit of dispersion relation

Example Continuum Limit



$\mathbf{p} = \frac{2\pi}{L}(1, 1, 1)$ and $m_{sh} = m_{D_s}$

Comparison to Continuum



Outcome of Study 1

- Can simulate charm quarks with DWFs:
- Found sweet-spot in DWF parameter space to be $\underline{M_5 = 1.6}$
- Limitation: $\underline{am_h \leq 0.4}$
  $\Rightarrow$ Exponentially localised
- Flat CL for decay constants and dispersion relation

$\Rightarrow$ Set up charm program on RBC/UKQCD's dynamical 2+1f ensembles

Study 2: charm phenomenology

GOAL: Do charm phenomenology with DWFs

- D and D_s **decay constants** f_P
- D and D_s bag parameters and ξ
  $\Rightarrow$ Particularly interesting for b (short distance)
- From charm to bottom?
  $\Rightarrow$ Use the ratio method to get to b.
- Semi-Leptonic form factors

$\Rightarrow$ **Determination of the CKM matrix elements**

Our Setup

Name	L/a	a^{-1}[GeV]	m_π[MeV]
C0	48	1.73	**139**
C1	24	1.78	340
C2	24	1.78	430
M0	64	2.36	**139**
M1	32	2.38	300
M2	32	2.38	360
F1	48	2.77	230

RBC/UKQCD's $N_f = 2+1$
Iwasaki ensembles (arXiv:1411.7017)

- sea light and strange have $M_5 = 1.8$
- quenched study suggests $M_5 = 1.6$ for heavy sector

$\Rightarrow$ Mixed Action

	light	heavy
DWF	M,S	M
M_5	1.8	1.6
L_s	12, 16, 24	12

Strategy (as outlined in arXiv:1511.09328)

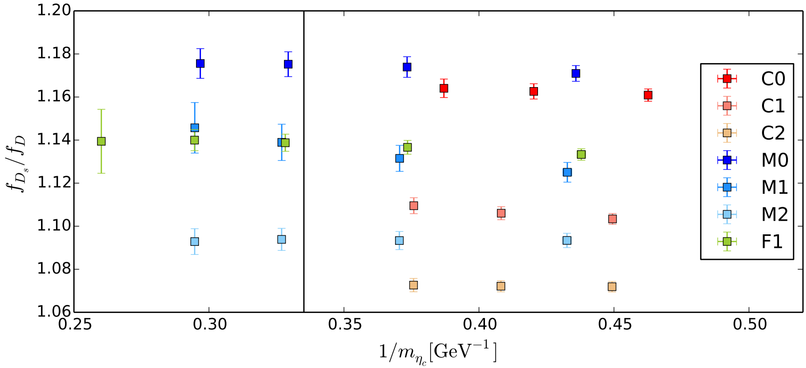
Correlator fits give: $\mathcal{O}(am_l, am_s, am_h, a)$

We want $\mathcal{O}(m_l^{\mathrm{phys}}, m_s^{\mathrm{phys}}, m_h^{\mathrm{phys}}, 0)$

Idea: We fix all parameters one-by-one.

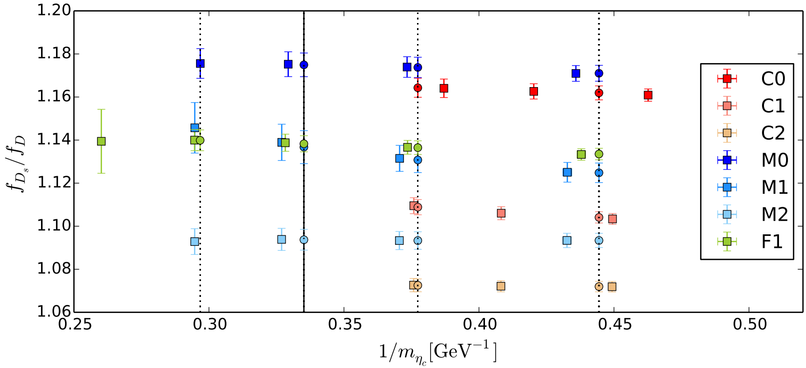
1. am_s^{phys} is known for each ensemble (arXiv:1411.7017)
2. Define am_h^{ref} by fixing reference m_{hh} masses
3. Extrapolate observables to $m_\pi = m_\pi^{\mathrm{PDG}}$
4. Continuum Limit
5. Extrapolate to physical charm mass in the continuum

Data Collection



$\Rightarrow$ Can't reach physical charm (solid line) on the coarse ensembles.

Data Collection



$\Rightarrow$ Can't reach physical charm (solid line) on the coarse ensembles.

Caveat: Renormalisation

Problem: $M_5^l \neq M_5^h$
$\Rightarrow$ We do not have a conserved current for the mixed action and hence no Z_A^{lh} (yet).
Renormalisation of the mixed current is work in progress.

BUT: We **DO** have Z_A for the light-light and heavy-heavy case.

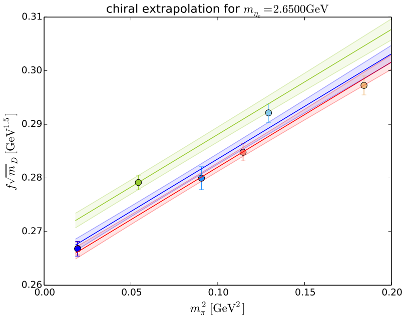
For Now: Use Z_A^{ll}

Chiral Extrapolation

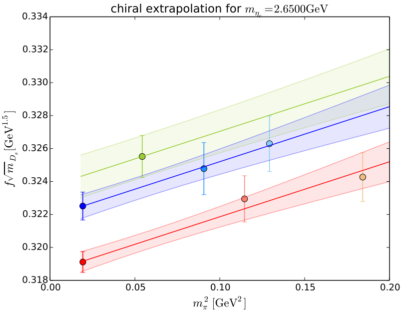
Problem: On the fine ensemble we only have one pion mass.

$$\mathcal{O}(am_l, m_s^{\mathrm{phys}}, m_h^{\mathrm{ref}}, a) = \mathcal{O}(m_\pi^{\mathrm{phys}}, m_s^{\mathrm{phys}}, m_h^{\mathrm{ref}}, a) + C_\chi(m_h^{\mathrm{hh}})m_\pi^2$$

$f_{lh}\sqrt{m_{lh}}$

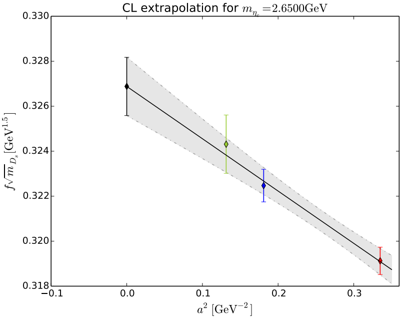


$f_{sh}\sqrt{m_{sh}}$



Continuum Limit

$$\mathcal{O}(m_l^{\mathrm{phys}}, m_s^{\mathrm{phys}}, m_h^{\mathrm{ref}}, a) = \mathcal{O}(m_l^{\mathrm{phys}}, m_s^{\mathrm{phys}}, m_h^{\mathrm{ref}}, a=0) + C_{\mathrm{a}^2}(m_h^{\mathrm{hh}})a^2$$



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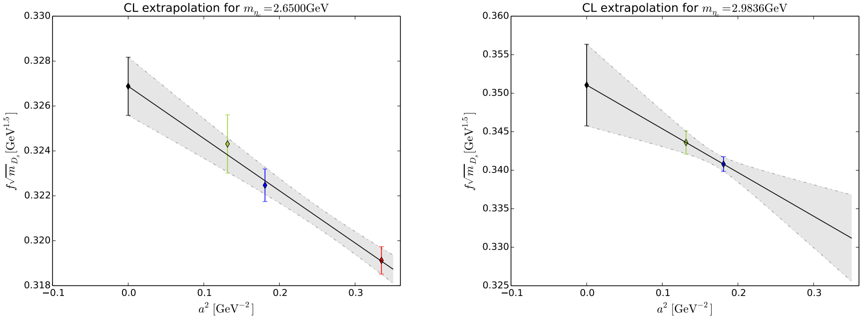

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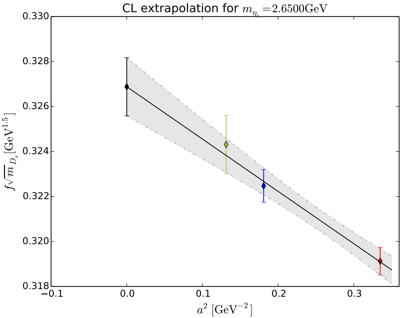
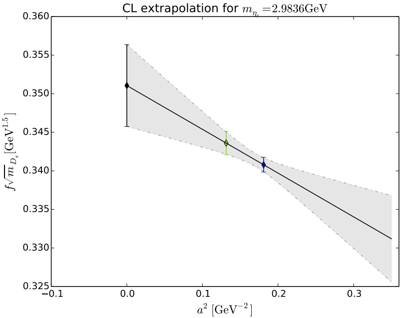
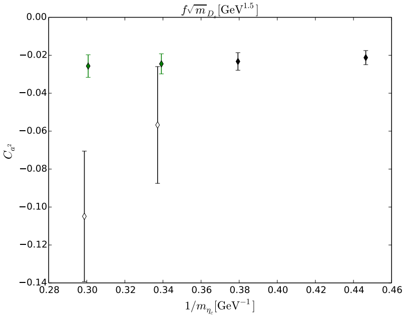




Problem: Only 2 lattice spacings at the physical charm.
Constrain $C_{\mathrm{a}^2}(m_h^{\mathrm{hh}})$ to be a smooth function of m_h^{ref}

Constrained Continuum Limit for physical charm



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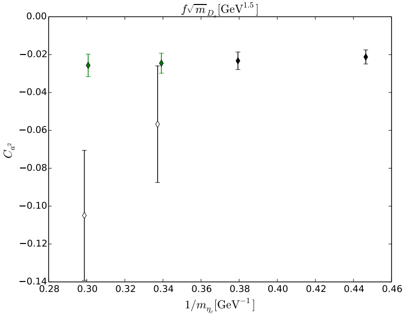
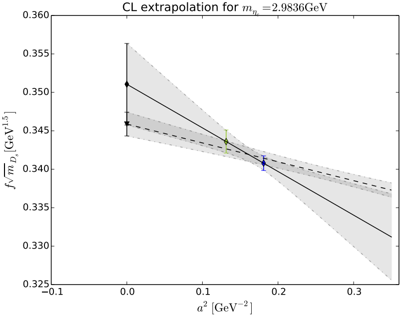




Bag and ξ Parameters

The bag parameter is defined as

$$B_P = \frac{\left\langle P^0 \middle| O_{VV+AA} \middle| \overline{P^0} \right\rangle}{8/3 f_P^2 m_P^2}$$

with the four-quark operator

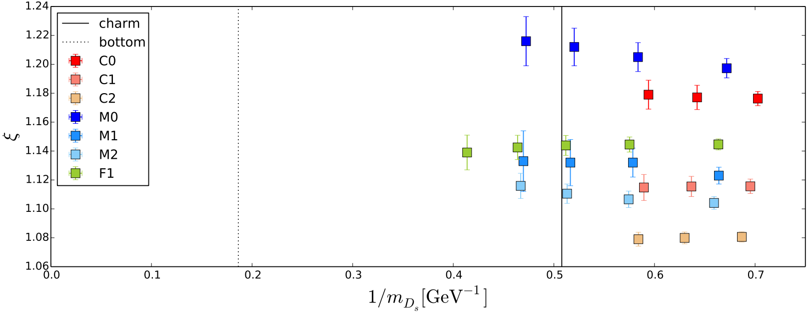
$$O_{VV+AA} = (\bar{h}\gamma_\mu q)(\bar{h}\gamma_\mu q) + (\bar{h}\gamma_5\gamma_\mu q)(\bar{h}\gamma_5\gamma_\mu q)$$

$\Rightarrow$ **mixed current renormalisation** is still work in progress!
For now, just consider

$$\xi = \frac{f_{hs}\sqrt{B_{hs}}}{f_{hl}\sqrt{B_{hl}}}$$

Bag and ξ Parameters

$$\xi = \frac{f_{hs}\sqrt{B_{hs}}}{f_{hl}\sqrt{B_{hl}}}$$



$\Rightarrow$ Same analysis strategy planned as in the above.

Aside: $g-2$

together with the authors of arXiv:1602.01767

Contribution	$a_\mu \times 10^{10}$	Uncertainty $\times 10^{10}$
Exp	11659209.1	6.3
Total	11659184.1	5.0
LO Had. $= a_\mu^{(2)}$	694.9	4.3
HLBL	10.5	2.6

GOAL: 1% error on hadronic contributions to $g-2$.

$$a_\mu^{(2)} = \sum_f a_\mu^{(2),f}$$

Study 3: charm connected contribution to HVP ($a_\mu^{(2),c}$)

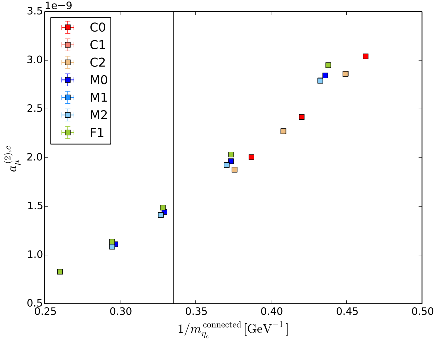
GOAL: 1% error on had. contributions to $g-2$. ($\sim 700 \times 10^{-10}$)

$$a_\mu^{(2)} = a_\mu^{(2),l} + a_\mu^{(2),s} + a_\mu^{(2),h}$$

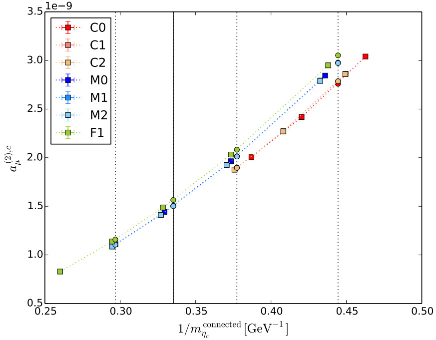
- HPQCD: $a_\mu^{(2),c} = 14.42(39) \times 10^{-10}$ (arXiv:1403.1778)

$\Rightarrow$ At %-level goal the charm contribution becomes relevant.
Calculate using the same strategy as above!

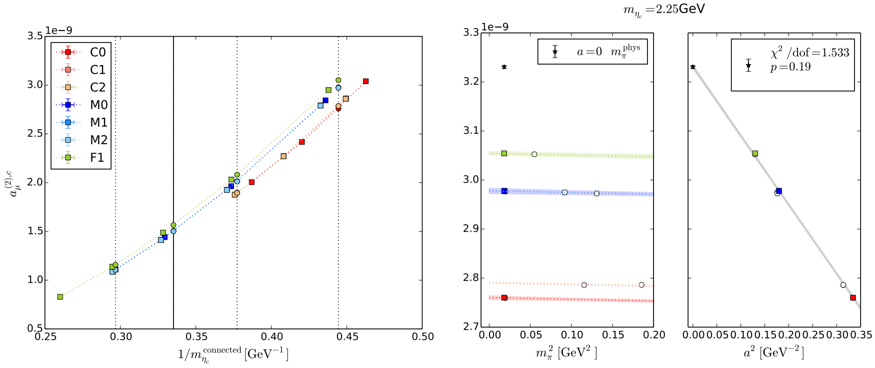
$a_\mu^{(2),c}$


1e−9
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.25
0.30
0.35
0.40
0.45
0.50
C0
C1
C2
M0
M1
M2
F1
$a_\mu^{(2),c}$
$1/m_{\eta_c}^{\mathrm{connected}}\,[\mathrm{GeV}^{-1}]$

$a_\mu^{(2),c}$


1e−9
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.25
0.30
0.35
0.40
0.45
0.50
C0
C1
C2
M0
M1
M2
F1
$a_\mu^{(2),c}$
$1/m_{\eta_c}^{\mathrm{connected}}\,[\mathrm{GeV}^{-1}]$

$a_\mu^{(2),c}$


$m_{\eta_c} = 2.25$GeV
1e−9
C0
C1
C2
M0
M1
M2
F1
$a_\mu^{(2),c}$
$1/m_{\eta_c}^{\text{connected}}\ [\text{GeV}^{-1}]$
$a = 0 \quad m_\pi^{\text{phys}}$
$m_\pi^2\ [\text{GeV}^2]$
$\chi^2/\text{dof} = 1.533$
$p = 0.19$
$a^2\ [\text{GeV}^{-2}]$

Summary

Quenched Pilot Study

- DWF framework to simulate heavy quarks.
- Exponential locality
- Heavy quark discretisation: $M_5 = 1.6$, $am_h \lesssim 0.4$
- Good Continuum Scaling for decay constants and dispersion relation

$N_f = 2 + 1f$ simulations

- Full analysis strategy set up
- D and D_s decay constants, bag parameters and ξ
- 3 lattice spacings
- Physical pion masses.
- Renormalisation of mixed current last missing piece
- charm distribution to HVP

Outlook and applications

Physics Outlook

- global-fit analysis for the above
- charm quark mass determination
- semi-leptonic form factors
- B physics via the ratio method
- GIM mechanism
- Full prediction of $g-2$ at the % level

Strategical Outlook

- Combined Analysis of K.E.K. and RBC/UKQCD data $\Rightarrow$ Very fine lattices **AND** controlled chiral extrapolation
- Link Smearing to increase reach in the heavy quark mass

BACKUP

Domain Wall Fermions I

- $4+1d$ model of free DWFs.
- s dependent mass term

$$M_5(s) = M_5\epsilon(s) = M_5\frac{|s|}{s} = \pm M_5$$

- Dirac equation

$$(\partial\!\!\!/ + \gamma_5\partial_s + M_5(s))\Psi(x,s) = 0$$

- Separation of variables $\Psi(x,s) = \psi(x)b_n(s)$
  $\Rightarrow$ Tower of massive modes propagating in the 5th dim.
  $\Rightarrow$ one **chiral massless** mode exponentially localised at $s=0$

$$b_0(s) \propto e^{-M_5|s|}$$

Domain Wall Fermions II

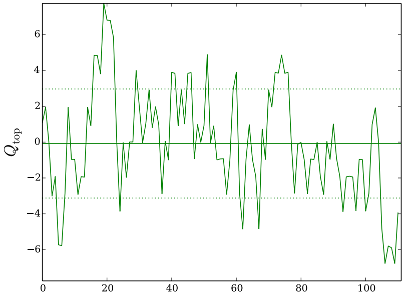
- add Wilson term

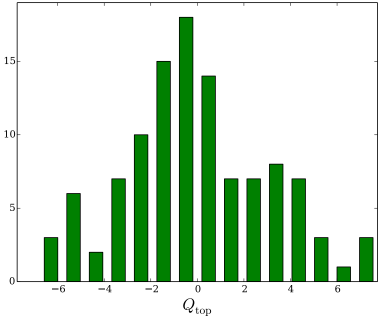
$$b_0(s,k) \propto e^{-\int_0^s (M_5(s')+\Delta m(k))\mathrm{d}s'}$$

- remains normalisable as long as $|\Delta m(k)| < M_5$. Otherwise chiral mode vanishes
- a quark mass term also changes the localisation of the physical mode at the boundary
- Add a 2nd *wall*. $\Rightarrow$ LH and RH modes
- L_s determines the **residual chiral symmetry breaking**, quantified by m_{res}.
- M_5 impacts UV behaviour and localisation of physical modes to DW boundary.

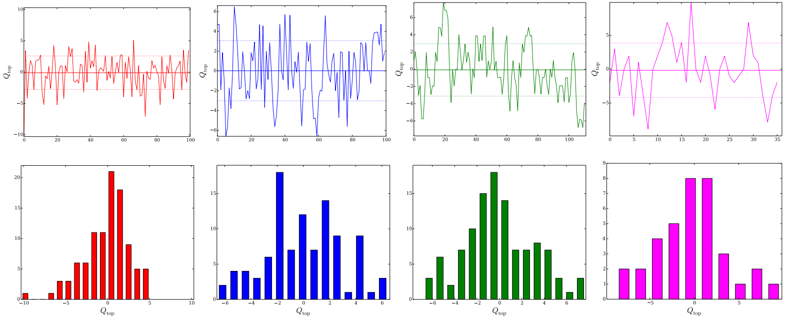
Topological Charge

- Monitored topological charge evolution
- explored different sectors



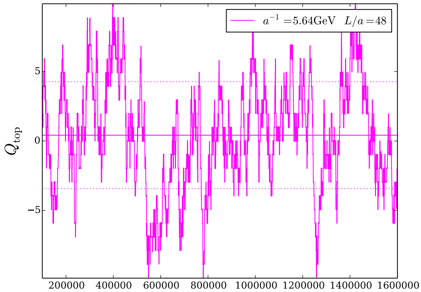


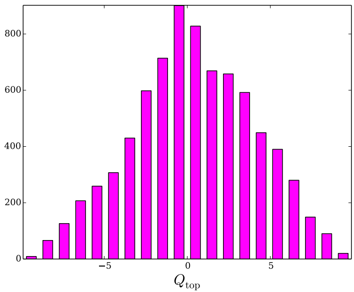
Topological Charge



Topological charge measured on de-correlated configurations

Topological Charge

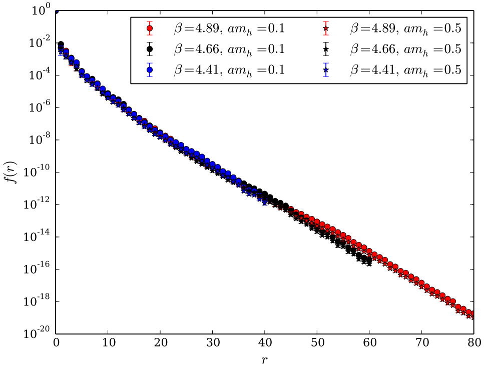




Topological charge measured on all trajectories on the the fine ensemble

Locality

$$f(r) = \max\{|\psi(x)|\,\forall\, x \in \{x\}_r\}$$



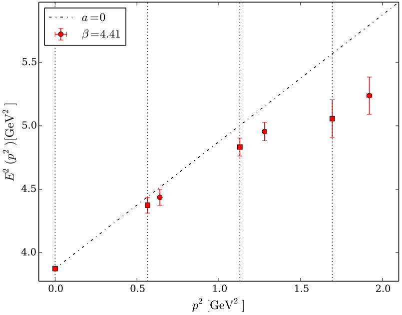
- Exponentially localised, independent of the quark mass

Quenched Dispersion Relation - interpolation to reference momenta

$$\mathbf{p} = \frac{2\pi}{L}\mathbf{n}$$

β	$a^{-1}(\mathrm{GeV})$	$L(\mathrm{fm})$
4.41	2.0	1.55
4.66	2.9	1.66
4.89	3.9	1.63
5.20	5.7	1.65

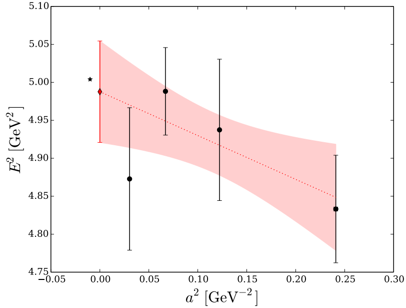
$$L_{\mathrm{ref}} = 1.648\mathrm{fm}$$



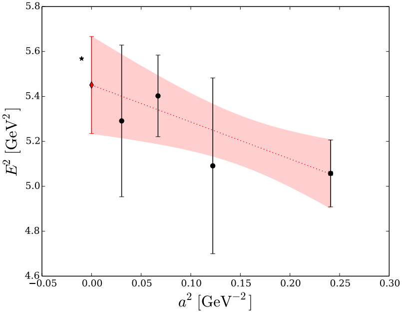
Continuum Limit quenched Dispersion Relation

Dispersion relation at the physical m_{D_s}.

$$\mathbf{p} = \frac{2}{\pi L}(1,1,0)$$



$$\mathbf{p} = \frac{2}{\pi L}(1,1,1)$$



From D to B: The ratio method

arXiv:0909.3187

- Define $\phi \equiv f_{\mathrm{PS}}\sqrt{M}$
- HQET predicts:

$$\lim_{m_h \to \infty} \phi = \mathrm{const.}$$

- Define n reference masses M_i^{ref} and $\lambda > 1$ with $\lambda M_i = M_{i+1}$. Then

$$R(M_i) \equiv \frac{\phi(M_i)}{\phi(M_{i+1})} \to 1$$

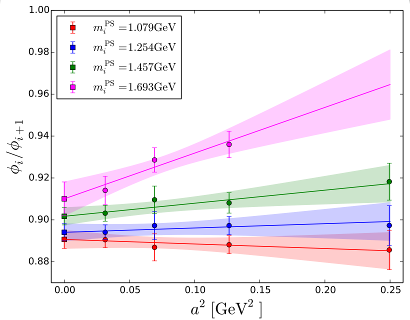
- Expansion around the static limit (HQET):

$$R(M_i) = 1 + \frac{C_1}{M_i} + \frac{C_2}{M_i^2} + \mathcal{O}\left(\frac{1}{M_i^3}\right)$$

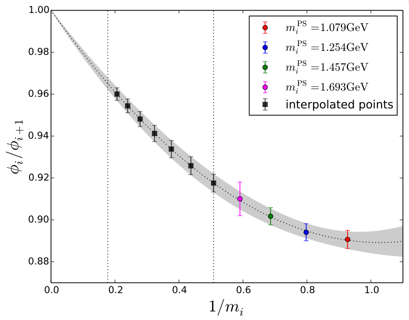
The Ratio Method

1. Define n geometrically spaced reference masses M_i and build $\phi(M_i)$.
2. Interpolate between static limit and $R(M_i)$, to find $C_1, C_2, ...$
3. Reconstruct R_i for $M_i >$simulated data.
4. $$\frac{\phi(M_0)}{\phi(M_m)} = \prod_{i=0}^{m-1} R_i$$

Test the Ratio Method: Quenched Pilot Study



Published (ETMC, $N_f = 2$, arXiv:1308.1851): $f_{B_s}/f_{D_s} = 1.096(49)$



We get: 1.098(31) (quenched) (without heaviest: 1.096(36))