Nucleon form factors and light nuclei in $N_f = 2+1$ lattice QCD

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for PACS Collaboration

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1. Nucleon form factors

collaboration with

K.-I. Ishikawa, Y. Kuramashi, S. Sasaki and A. Ukawa

for PACS Collaboration

Ref: PoS(LATTICE 2015)081

2. Light nuclei

collaboration with

K.-I. Ishikawa, Y. Kuramashi, and A. Ukawa for PACS Collaboration

Refs: PRD81:111504(R)(2010); PRD84:054506(2011); PRD86:074514(2012)

PRD92:014501(2015); PoS(LATTICE 2015)081

Outline

- Introduction
- Nucleon form factors
  - Simulation parameters
  - Axial charge g_A
  - F_1 and F_2
  - dipole fit of F_1 and F_2
- Light nuclei
- Summary and future work

Introduction

$$\text{Binding force} \begin{cases} \text{protons and neutrons} & \rightarrow & \text{nuclei} \\ \text{quarks and gluons} & \rightarrow & \text{protons and neutrons} \end{cases}$$

both from fundamental strong interaction of quark and gluon

well known, but hard to prove

quark and gluon $\rightarrow$ proton and neutron$\rightarrow$ nucleus

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well known, but hard to prove

Spectrum of proton and neutron (nucleons)

success of non-perturbative lattice QCD calculation

degrees of freedom of quarks and gluons

$$\overbrace{\text{quark and gluon} \rightarrow \text{proton and neutron}} \rightarrow \text{nucleus}$$

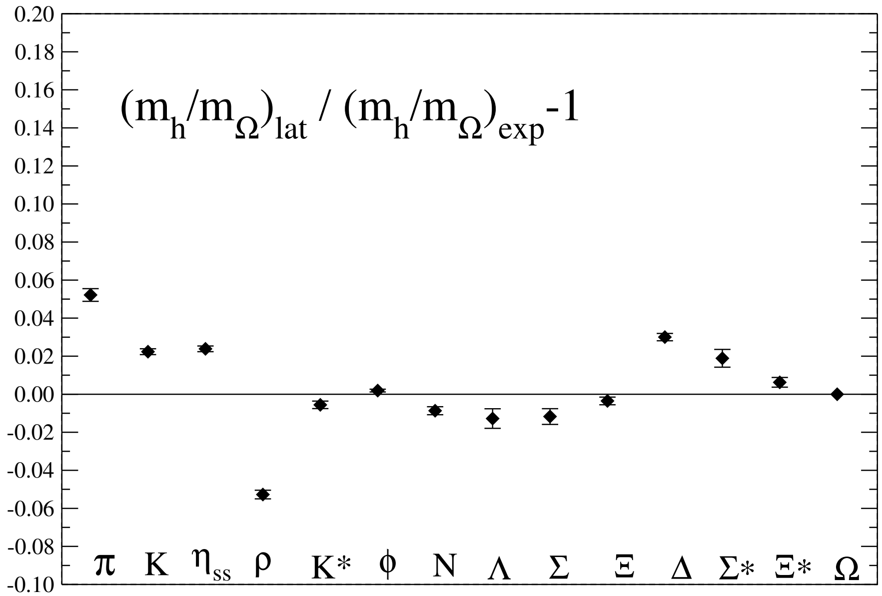
Hadron spectrum in $N_f = 2+1$ QCD

Lattice 2015, Ukita for PACS Collaboration PoS(LATTICE2015)075

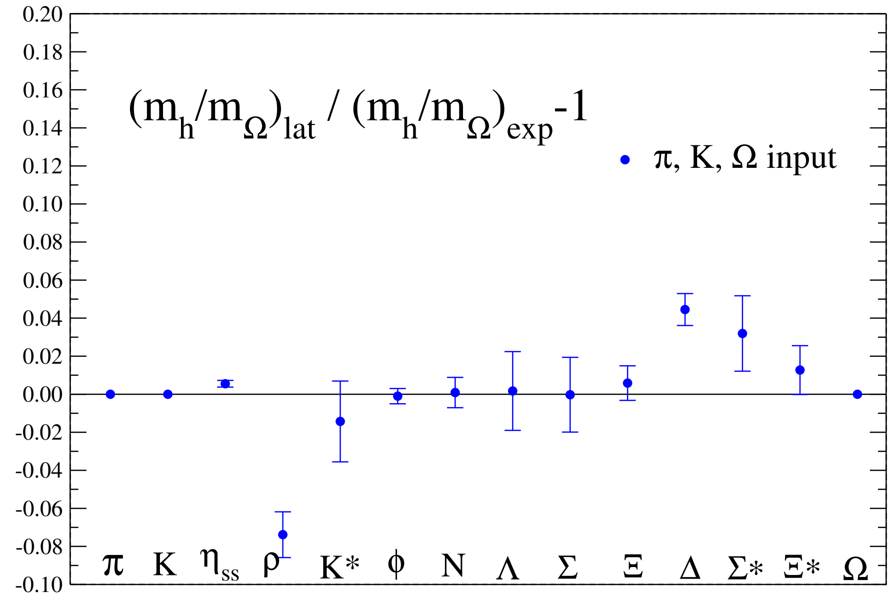
$m_\pi \sim 0.145$ GeV on $L \sim 8$ fm at $a^{-1} = 2.33$ GeV (SPIRE Field 5)

using reweighting m_{ud}, m_s + extrapolation $\rightarrow$ physical m_π and m_K

$m_\pi \sim 0.145$ GeV



physical point



Hadron spectrum in $N_f = 2+1$ QCD

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using reweighting m_{ud}, m_s + extrapolation $\to$ physical m_π and m_K

$$\bar{l}_3 = 2.87(62), \quad \bar{l}_4 = 4.38(33)$$

FLAG2013: $\bar{l}_3 = 3.05(99)$, $\bar{l}_4 = 4.02(28)$ at $\mu = m_\pi^{\text{phys}}$

$$m_{ud}^{\overline{\text{MS}}} = 3.142(26)(35)(28)\text{MeV}, \quad m_s^{\overline{\text{MS}}} = 88.59(61)(98)(79)\text{MeV}$$

FLAG2013: $m_{ud}^{\overline{\text{MS}}} = 3.42(6)(7)$MeV, $m_s^{\overline{\text{MS}}} = 93.8(1.5)(1.9)$MeV

$$f_\pi = 131.79(80)(90)(1.25)\text{MeV}, \quad f_K = 155.55(68)(1.06)(1.48)\text{MeV}$$

FLAG2013: $f_\pi = 130.2(1.4)$MeV, $f_K = 156.3(0.9)$MeV

reasonably consistent

investigation of $a \to 0$ limit necessary

Introduction

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both from fundamental strong interaction of quark and gluon

well known, but hard to prove

Spectrum of proton and neutron (nucleons)

success of non-perturbative lattice QCD calculation

degrees of freedom of quarks and gluons

$$\underbrace{\overbrace{\text{quark and gluon} \rightarrow \text{proton and neutron}}^{\text{1st part: Nucleon form factors not well understood}} \rightarrow \text{nucleus}}_{\text{2nd part: nucleus from lattice QCD}}$$

Nucleon form factors at almost physical m_π

in collaboration with

K.-I. Ishikawa, Y. Kuramashi, S. Sasaki, and A. Ukawa

for PACS Collaboration

Computational resources (the HPCI System Research Project: hp140155, hp150135)

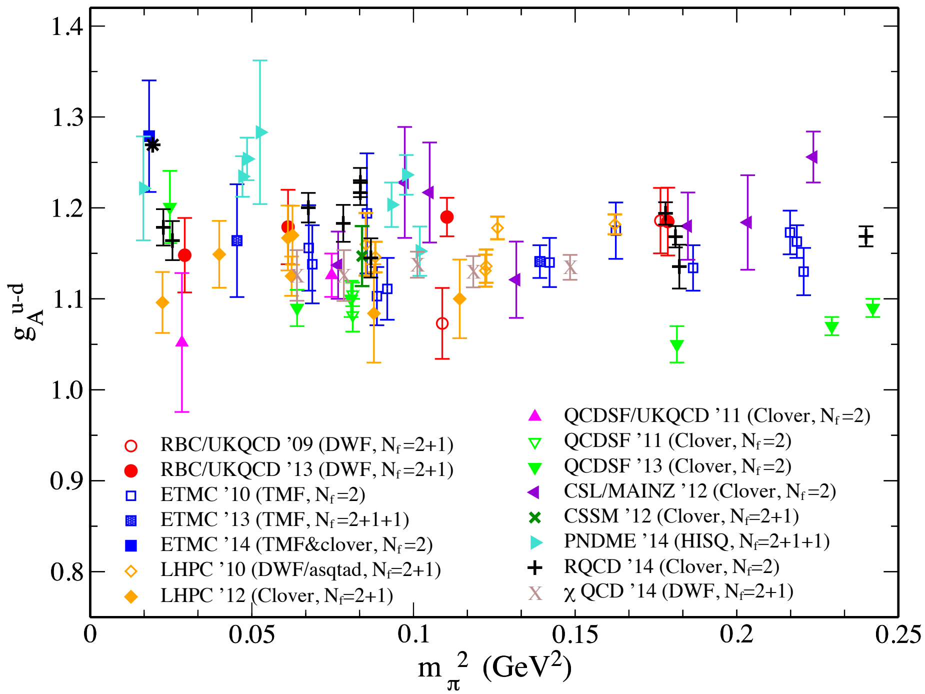
COMA @Univ. of Tsukuba, FX10 @Univ. of Tokyo,

FX100 @RIKEN, System E @Kyoto Univ., FX100 @Nagoya Univ.

Current status of g_A from lattice QCD

most fundamental quantity $(\sim f_\pi)$ $\Leftarrow$ important to check

Isovector Axial charge g_A (Constantinou, Lat14 plenary)



c.f.) see also Zanotti Lat15 plenary

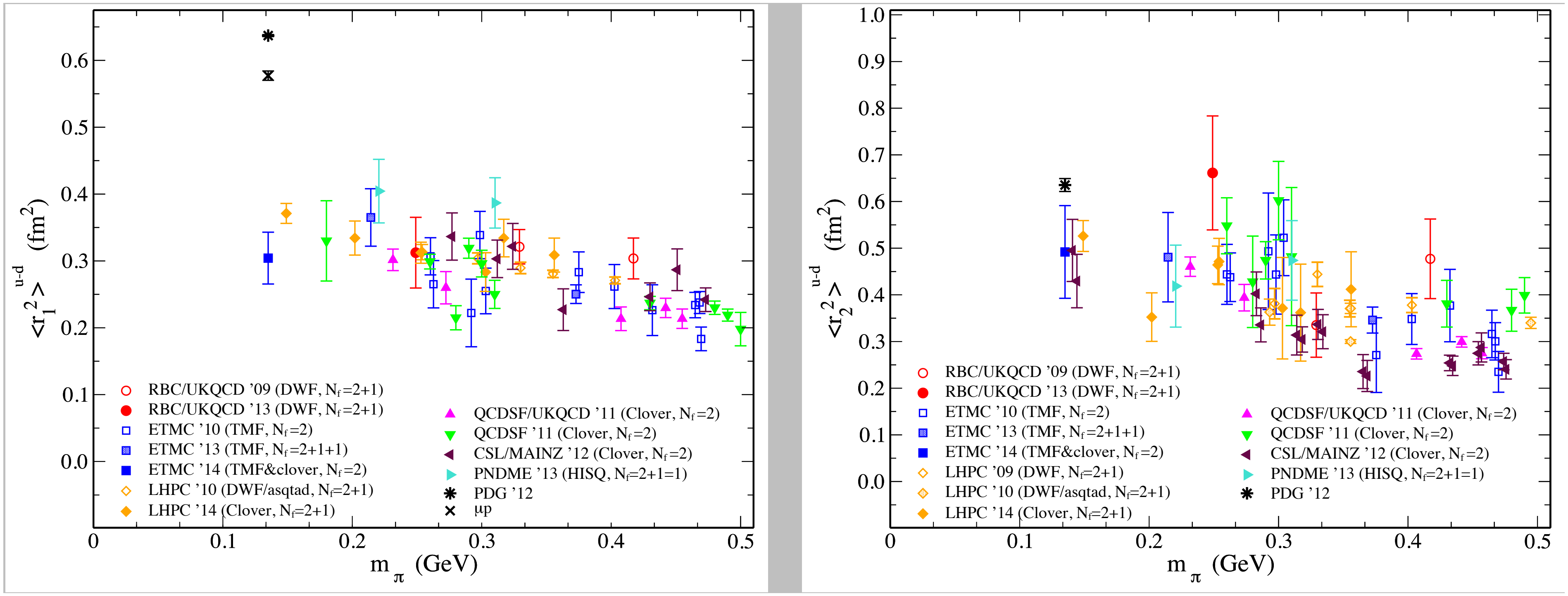
- Mild m_π dependence + roughly 10% small at large m_π

  $\rightarrow$ close to physical m_π calculation necessary
- Z_A in Wilson type action

Current status of radii from lattice QCD

Isovector radii from form factors F_1 and F_2

Dirac and Pauli radii $\langle r_1^2 \rangle$ and $\langle r_2^2 \rangle$ (Constantinou, Lat14 plenary)



c.f.) see also Zanotti Lat15 plenary

$\langle r_1^2 \rangle$: almost half of experiment at larger m_π

$\rightarrow$ close to physical m_π calculation necessary

c.f.) '14 LHP, '15 Capitani *et al.*,'15 ETM

Current status of lattice QCD

Fundamental physical quantities of nucleon

Isovector Axial charge g_A and Dirac and Pauli radii $\langle r_1^2 \rangle$ and $\langle r_2^2 \rangle$

Not well understood

Motivation: reproduce experiments (from one simulation)

Configuration with stout-smeared Clover quark action

$m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Systematic errors of nucleon form factors

- large m_π
- finite volume effect
- chiral symmetry breaking
- excited states

Current status of lattice QCD

Fundamental physical quantities of nucleon

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Configuration with stout-smeared Clover quark action

$m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Systematic errors of nucleon form factors

- large m_π $\rightarrow$ $m_\pi \sim 0.145$ GeV
- finite volume effect $\rightarrow$ $L \sim 8$ fm, $Lm_\pi = 6$
- chiral symmetry breaking $\rightarrow$ stout-smeared Clover
- excited states $\rightarrow$ large (not small) $|t_{\text{sink}} - t_{\text{src}}|$

further investigation necessary in future

Simulation parameters

$N_f = 2+1$ QCD $L^3 \times T = 96^3$ PoS(LATTICE2015)075

Iwasaki gauge action at $\beta = 1.82$ $\quad a^{-1} \sim 2.33$ GeV with m_Ω

non-perturbative $O(a)$-improved Wilson quark action $c_{\mathrm{SW}} = 1.11$

Taniguchi, PoS(LATTICE2012)236

with stout smearing $(\rho, N_\rho) = (0.1, 6)$ '04 Morningstar and Peardon

$m_\pi \sim 0.145$ GeV and little larger m_s than m_s^{phys}

Measurement of Isovector form factors

- 104 conf (every 20 τ) $\times$ 64 meas/conf
- exponential smeared N operator in t_{src} and t_{sink}
- $|t_{\mathrm{sink}} - t_{\mathrm{src}}| = 15$ (~ 1.27 fm)
- $n^2 = (pL/2\pi)^2 = 0, 1, 2, 3, 4, 5, 6, 8, 9(3,0,0), 9(2,2,1)$ with PBC

$$\rightarrow 0 \le q^2 < 0.2 \text{ GeV}^2$$

All results are preliminary.

Computational resources for Measurements

FX10 @Univ. of Tokyo, System E @Kyoto Univ., FX100 @Nagoya Univ.

(the HPCI System Research Project: hp140155, hp150135)

COMA @Univ. of Tsukuba, FX100 @RIKEN

Isovector form factors

- Vector and induced tensor form factors

(elastic proton-electron scattering)

$$\langle N, p | V_\mu(q) | N, p' \rangle = \overline{u}_N(p) \left(\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2M_N} \right) u_N(p')$$

$$q_\nu = p'_\nu - p_\nu$$

$F_1(q^2), F_2(q^2) \rightarrow$ $F_1(0) = F_1^p(0) - F_1^n(0) = 1$

$F_2(0) = \mu_p - \mu_n - 1$ (μ_i : magnetic moment)

$\langle r_1^2 \rangle, \langle r_2^2 \rangle$ related to charge radii $\langle r_p^2 \rangle, \langle r_n^2 \rangle$

- Axial-vector and induced pseudoscalar form factors

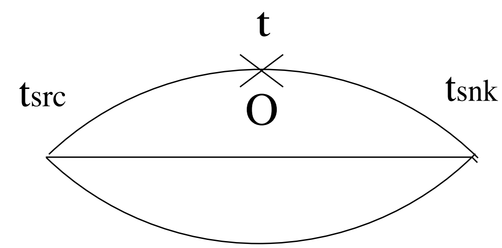
(β decay; muon capture on proton; neutrino-nucleon scattering; pion electroproduction)

$$\langle N, p | A_\mu(q) | N, p' \rangle = \overline{u}_N(p) \left(i\gamma_5 \gamma_\mu G_A(q^2) + i\gamma_5 q_\mu G_P(q^2) \right) u_N(p')$$

$G_A(q^2), G_P(q^2) \rightarrow$ $\underline{G_A(0) = g_A}$: axial charge

$g_{\pi NN}$: pion-nucleon coupling

g_P : pseudoscalar coupling for muon capture



Matrix elements '03 LHPC, '05 QCDSF

$$\frac{C_3^{\mathcal{P}V}(t,p)}{C_2^S(t_{\mathsf{sink}},0)}\left[\frac{C_2^L(t_{\mathsf{sink}}-t+t_{\mathsf{src}},p)C_2^S(t,0)C_2^L(t_{\mathsf{sink}},0)}{C_2^L(t_{\mathsf{sink}}-t+t_{\mathsf{src}},0)C_2^S(t,p)C_2^L(t_{\mathsf{sink}},p)}\right]^{1/2}$$

$$\propto \quad \langle N(0)|V(q)|N(p)\rangle \qquad (t_{\mathsf{src}} \ll t \ll t_{\mathsf{sink}})$$

cancel normalization of nucleon operators

$C_3^{\mathcal{P}V}(t,p)$: 3-point function of V current with p and projector $\mathcal{P}$
exponential smeared quarks in $t_{\mathsf{src}}, t_{\mathsf{sink}}$

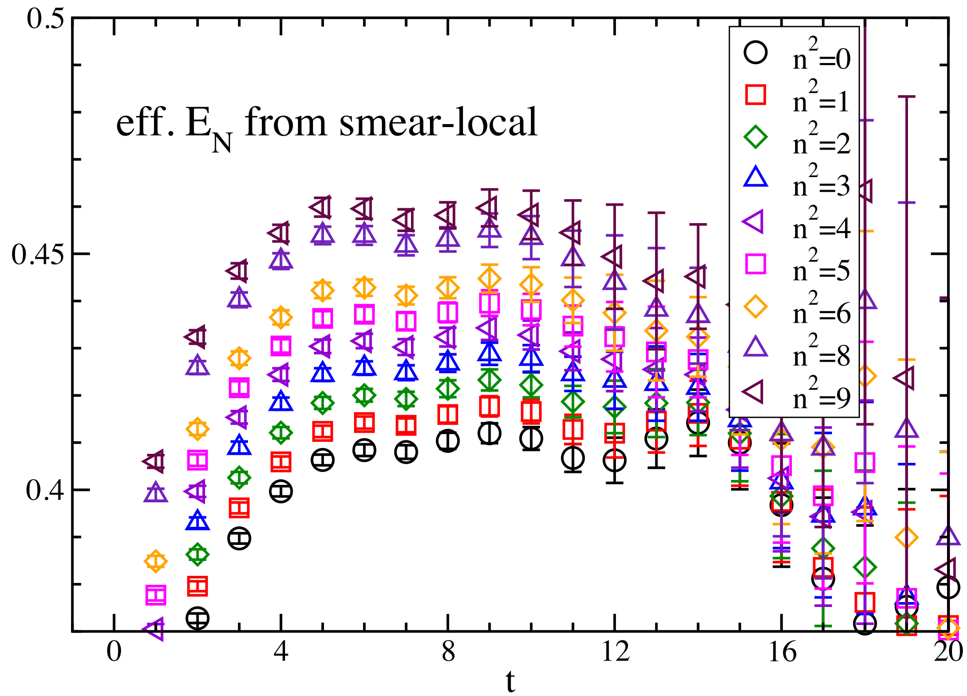
$G_2^{S,L}(t,p)$: 2-point function with p and smear(S) or local(L) sink
exponential smeared source

$$\mathcal{P}V = P_4V_4 \rightarrow \quad G_E(q^2) = F_1(q^2) - \frac{q^2}{(2M_N)^2}F_2(q^2) \quad P_4 = (1+\gamma_4)/2$$

$$\mathcal{P}V = P_{12}V_1 \rightarrow \quad G_M(q^2) = F_1(q^2) + F_2(q^2) \quad P_{12} = (1+\gamma_4)\gamma_1\gamma_2/2$$

F_1 and F_2 are obtained by solving linear equation.

Effective energy from $C_2^L(t,p)$ Preliminary result

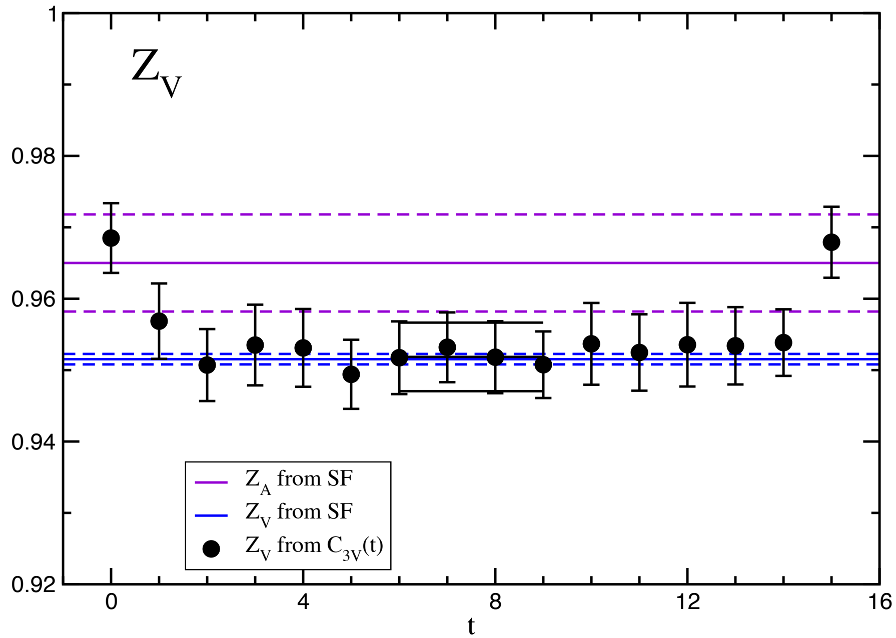


Clear signal in all p up to $t \sim 12$
Reasonable plateau in $t \gtrsim 6$

$|t_{\text{sink}} - t_{\text{src}}| = 15$ could be acceptable in this smearing parameter.

Z_V from nucleon 3pt   Preliminary result

$$Z_V = 1/F_1^{\text{bare}}(0) = C_2^S(t_{\text{sink}}, 0)/C_3^{P_4V_4}(t, 0)$$



Consistent with Z_V from SF scheme @ $m_\pi = 0$

also agrees with Z_A in SF scheme within 1–2%

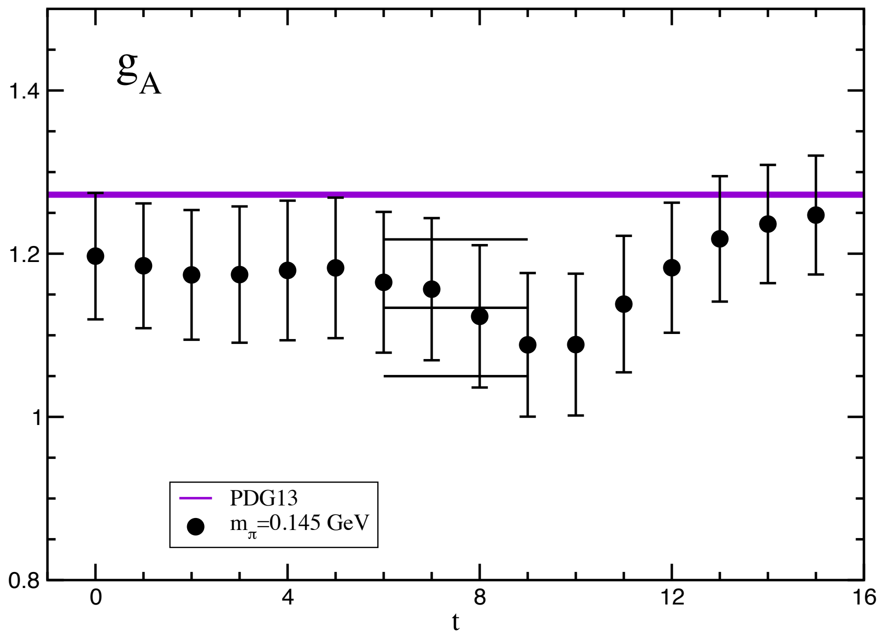
Ishikawa *et al.*, PACS Collaboration, PoS(LATTICE2015)271

$\rightarrow$ small chiral symmetry breaking effect for Z_A

Axial charge $g_A = Z_A g_A^{\text{bare}}$ Preliminary result

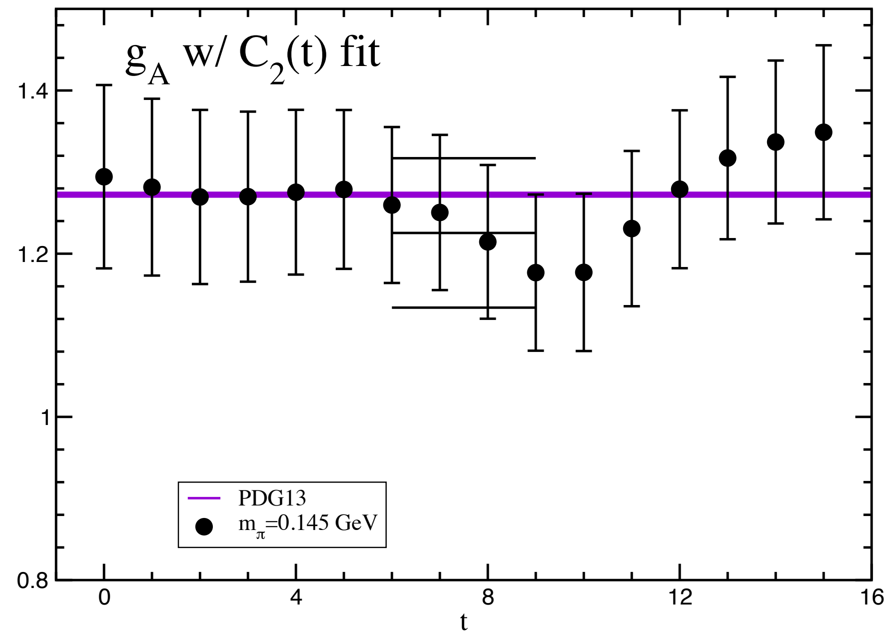
Z_A from SF scheme (Lattice 2015, Ishikawa for PACS Collaboration)

$g_A^{\text{bare}} = C_3^{P_{12}A_3}(t,0)/C_2^S(t_{\text{sink}},0)$



$g_A^{\text{bare}} = C_3^{P_{12}A_3}(t,0)/(Z_N^2 e^{-M_N t_{\text{sink}}})$

$Z_N(M_N)$ from fit of $C_2^{S(L)}(t,0)$

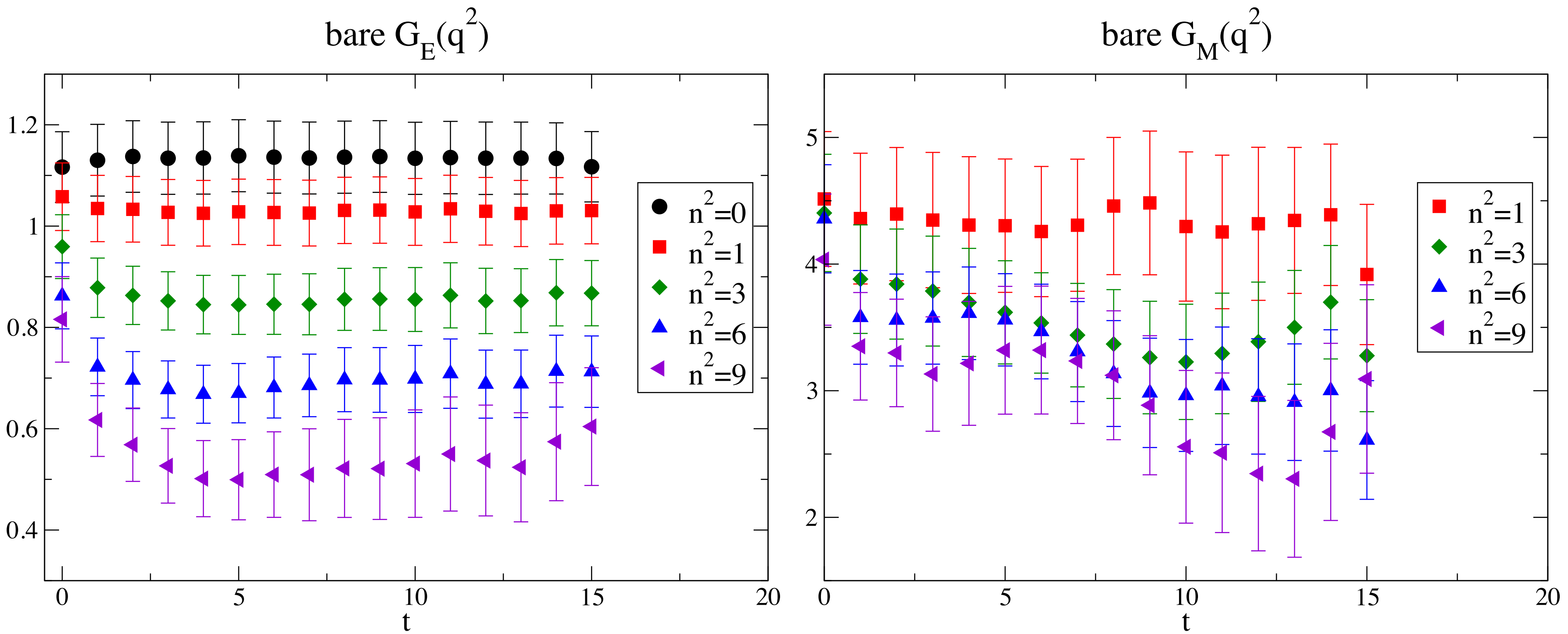


Difference of two results $\rightarrow$ systematic error of g_A

roughly consistent with experiment,

but need much more statistics for stringent test

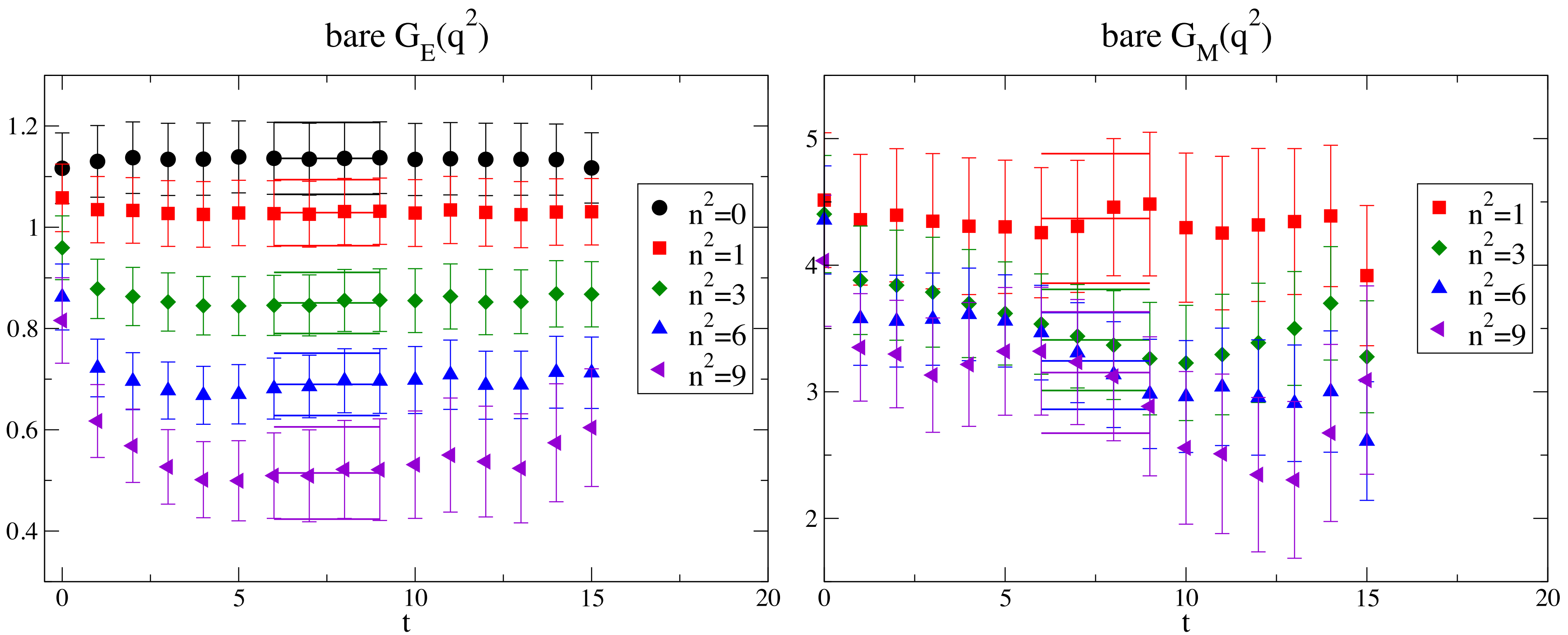
Isovector G_E and G_M form factors Preliminary result



G_E: Clear signal and plateau seen

G_M: Large statistical fluctation, but plateau in small q^2

Isovector G_E and G_M form factors Preliminary result



G_E: Clear signal and plateau seen

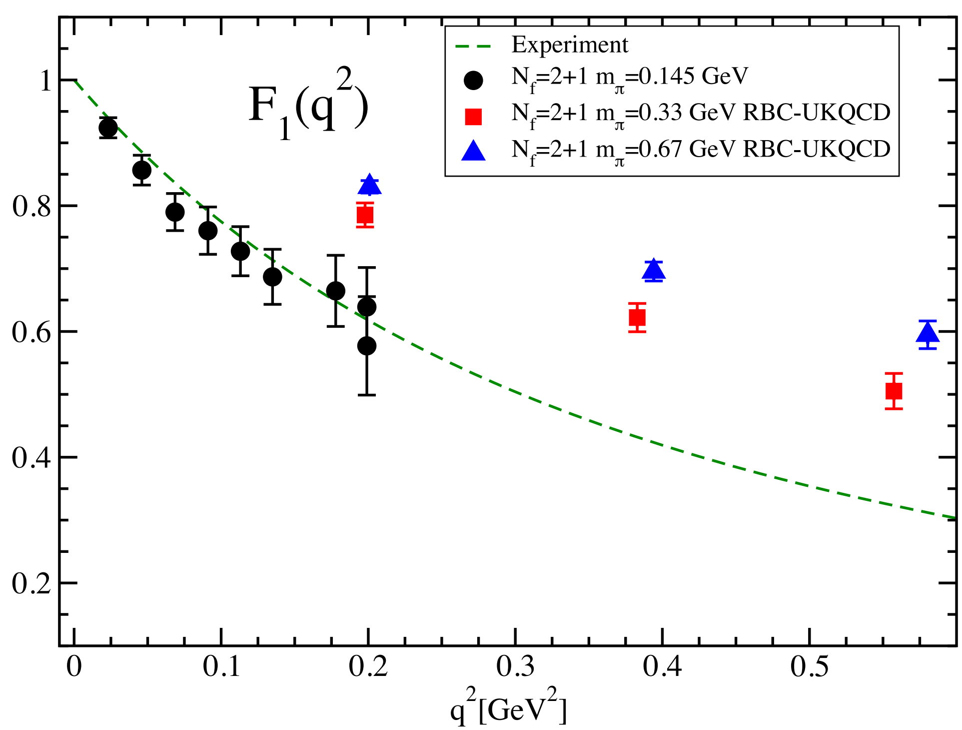
G_M: Large statistical fluctation, but plateau in small q^2

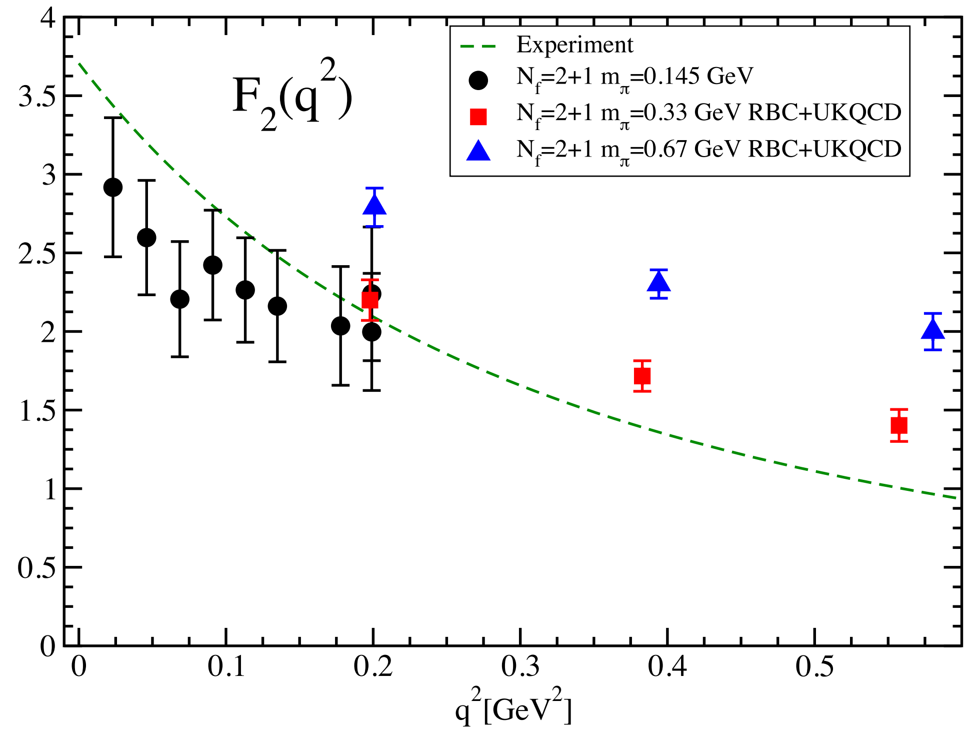
constant fit in $6 \leq t \leq 9 \rightarrow G_E(q^2)$ and $G_M(q^2)$

$\rightarrow F_1(q^2)$ and $F_2(q^2)$ by solving linear equation

Isovector F_1 and F_2 form factors Preliminary result

renormalized by $Z_V = 1/F_1(0)$





large statistical error comparing to $m_\pi > 0.3$ GeV data

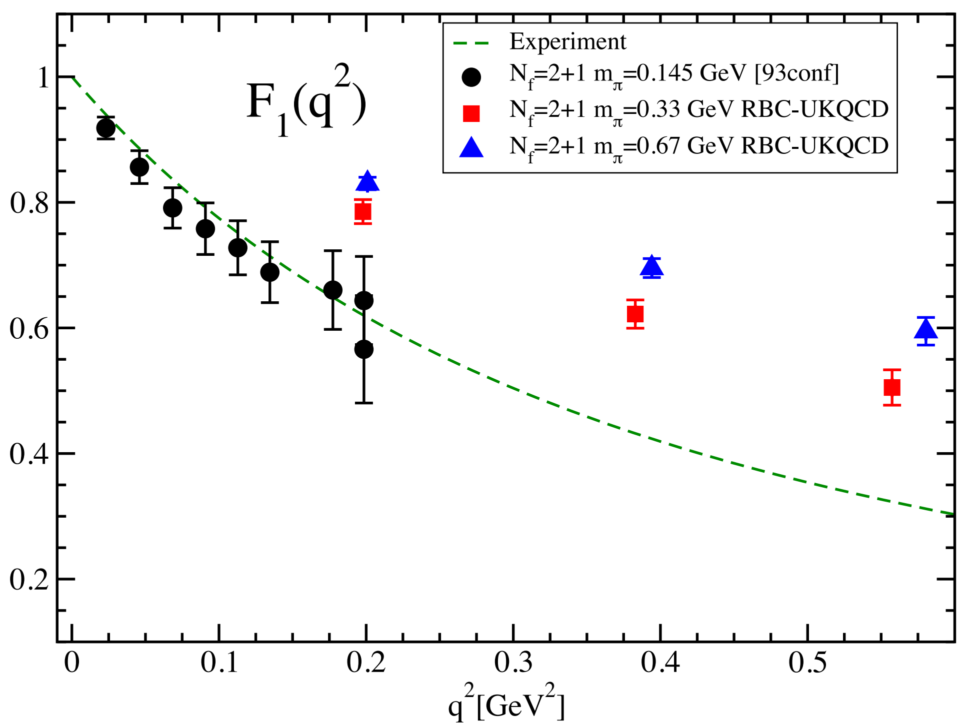
smaller values in $F_1(q^2)$ than $m_\pi > 0.3$ GeV

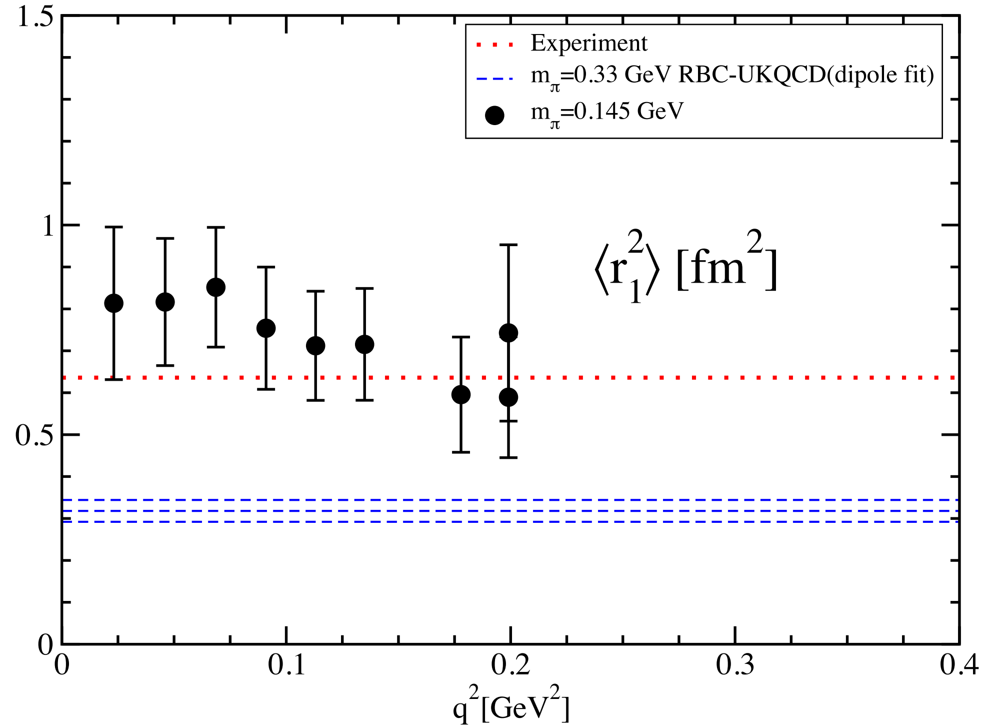
close to experimental curves

Dirac radius $\langle r_1^2 \rangle$ Preliminary result

$$\text{Dipole form } F_1(q^2) = \left(1 + \frac{q^2}{12}\langle r_1^2 \rangle\right)^{-2}$$

$$\text{Effective Dirac radius } \langle r_1^2 \rangle = \frac{12}{q^2}\left(\sqrt{\frac{1}{F_1(q^2)}} - 1\right)$$





Reasonably consistent with dipole form in all q^2

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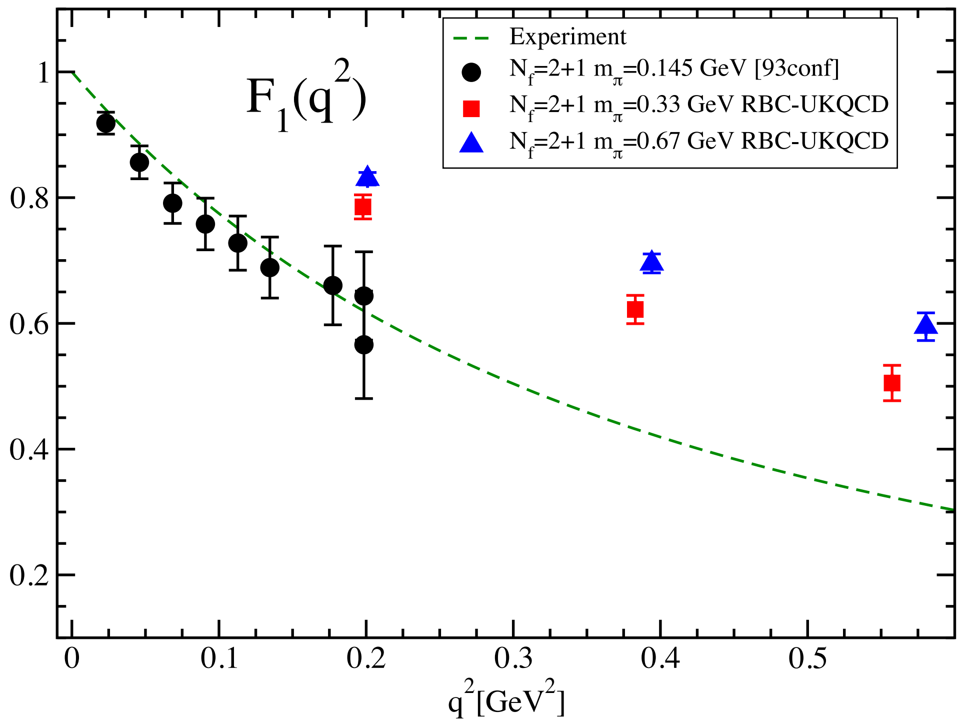
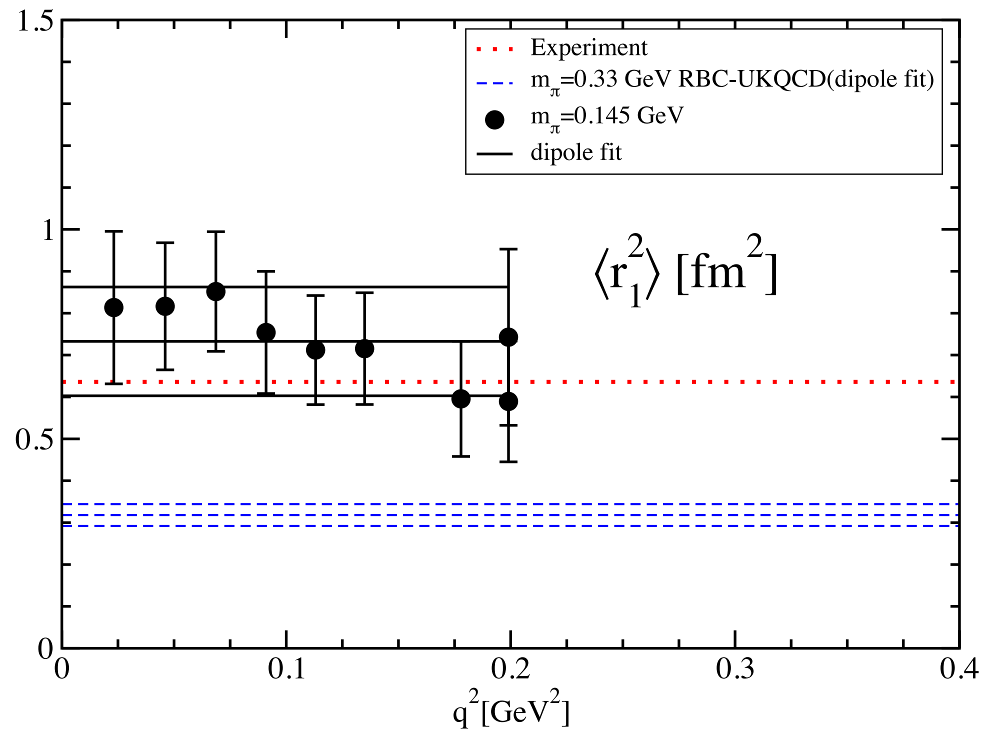


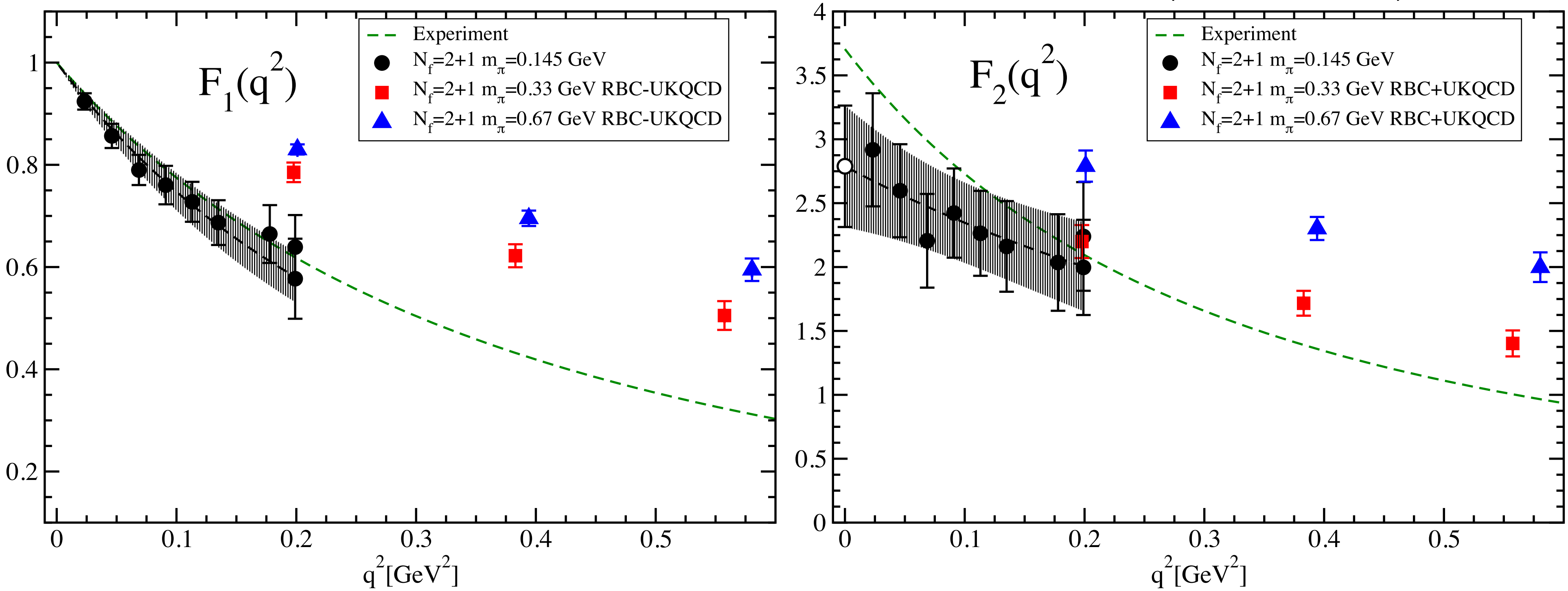


Reasonably consistent with dipole form in all q^2

Dipole fit of F_1 and F_2 Preliminary result

renormalized by $Z_V = 1/F_1(0)$

$$F_1(q^2) = \left(1 + \frac{q^2}{12}\langle r_1^2\rangle\right)^{-2} \qquad F_2(q^2) = \frac{F_2(0)}{\left(1 + \frac{q^2}{12}\langle r_2^2\rangle\right)^2}$$

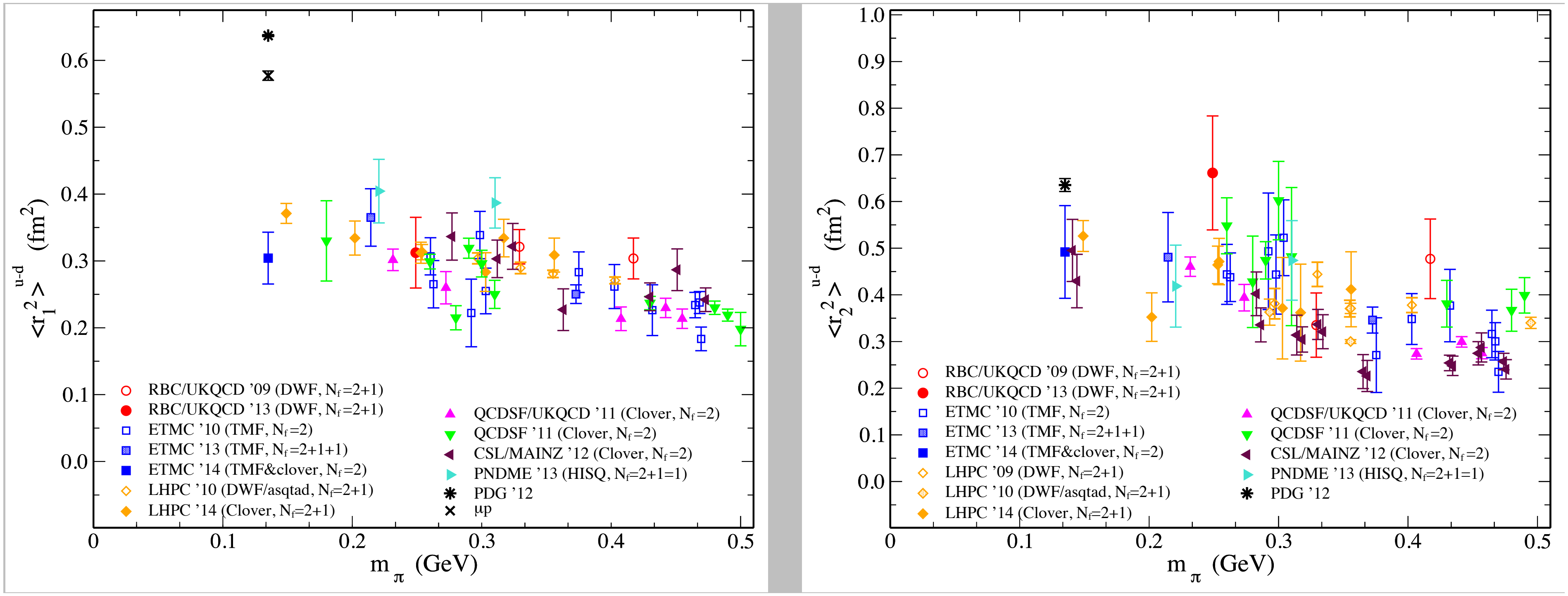


$\langle r_1^2\rangle = 0.73(13)\ \mathrm{fm}^2$

$\langle r_2^2\rangle = 0.42(30)\ \mathrm{fm}^2$

$F_2(0) = 2.79(48)$

Dirac and Pauli radii $\langle r_1^2 \rangle$ and $\langle r_2^2 \rangle$ Preliminary result



$\langle r_1^2 \rangle = 0.73(13)$ fm^2 $\qquad$ $\langle r_2^2 \rangle = 0.42(30)$ fm^2, $F_2(0) = 2.79(48)$

Need much more statistics for especially $F_2(q^2)$

but encouraging results

Light nuclei from lattice QCD

Current purpose: reproduce binding energy for light nuclei

Method: ΔE from nucleus 2-point function (Lüscher's method)

Calculation at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

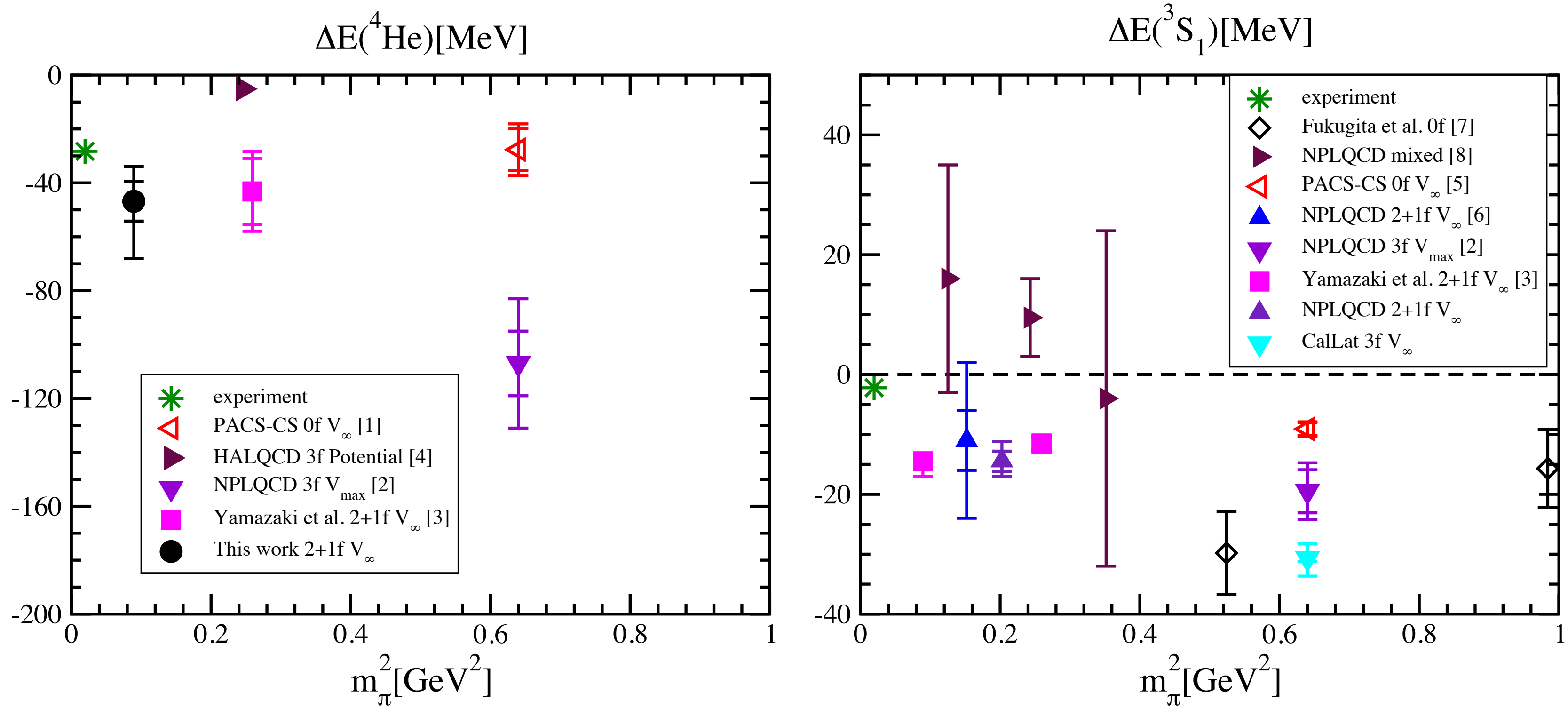
in collaboration with

K.-I. Ishikawa, Y. Kuramashi, and A. Ukawa for PACS Collaboration

Computational resources

HA-PACS, COMA @Univ. of Tsukuba, FX100 @RIKEN, K @AICS (SPIRE Field 5)

Current status of ^{4}He and NN ^{3}S$_0$ channels



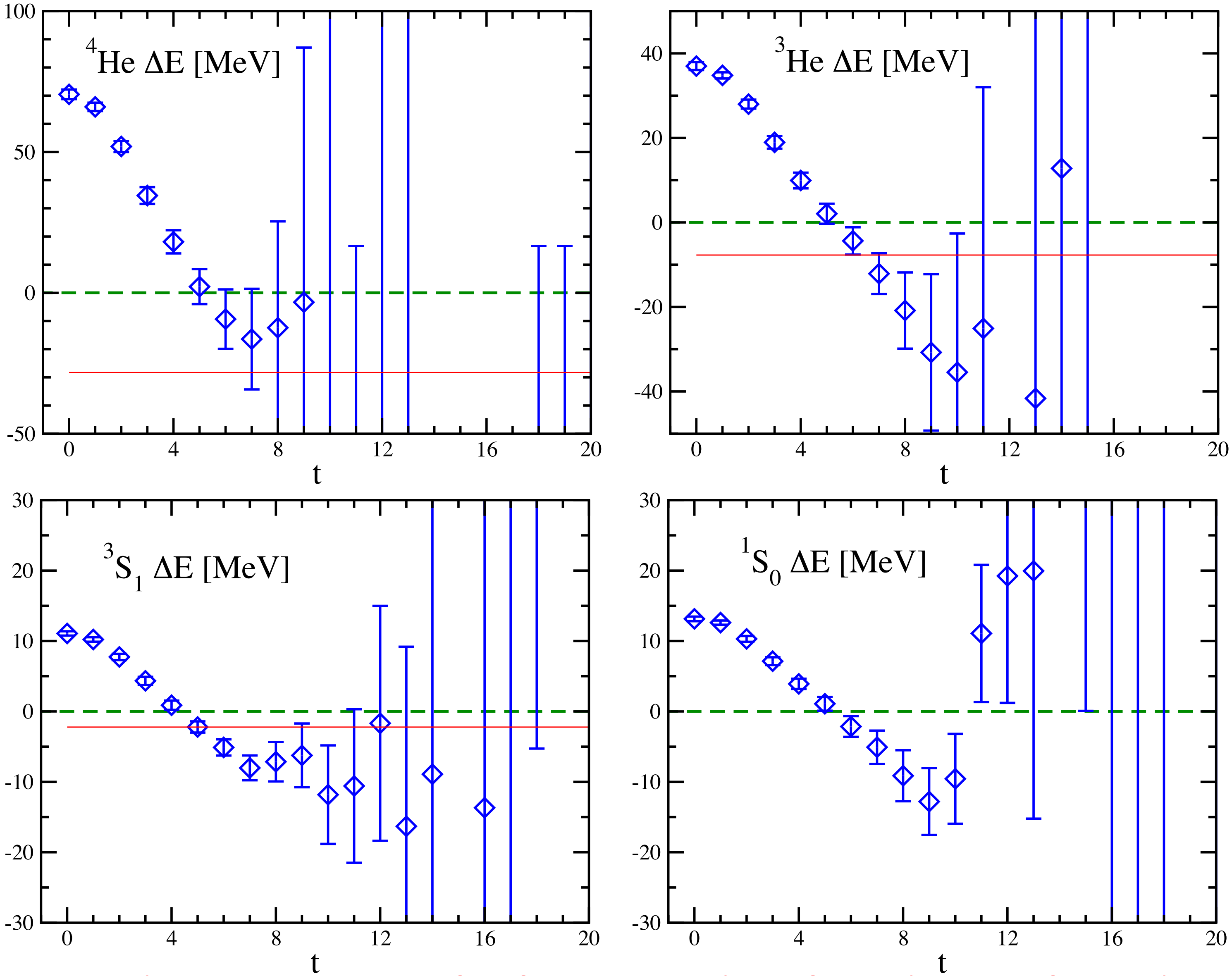
^{4}He: large uncertaintiy and close to experiment

NN ^{3}S$_0$: larger binding energy at large m_π

do not approach to experiment as m_π decreases

near physical m_π calculation necessary

Preliminary results of ΔE at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm



Much more statistics to obtain clear signals

Summary

$N_f = 2+1$ lattice QCD at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Nucleon form factors $\cdots$ still large statistical error

g_A: roughly consistent with experiment
F_1 and F_2 : different behavior from data at $m_\pi > 0.3$ GeV
$\langle r_1^2 \rangle$: consistent with experiment

Future work

increasing statistics, investigation of excited states contribution

Light nuclei

Need much more statistics

Need further investigations

e.g. systematic error from large m_π and finite lattice spacing

Bound state $\cdots$ PACS, NPLQCD and CalLat

No bound state $\cdots$ HALQCD

variational method could give hint to solve the difference