QCD energy momentum tensor at finite temperature using gradient flow

Yusuke Taniguchi
for
WHOT QCD collaboration

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Introduction

$\boxed{\text{Energy momentum tensor}}$

$$T_{\mu\nu}$$

Introduction

Energy momentum tensor ⟺ Poincare symmetry

$T_{\mu\nu}$

Introduction

Energy momentum tensor $\leftrightarrow$ Poincare symmetry

$$T_{\mu\nu}$$

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pressure

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$\Rightarrow$ direct measurement of thermodynamic quantity

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- Fluctuations and correlations of $T_{\mu\nu}$

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  ➡ specific heat, viscosity, $\cdots$

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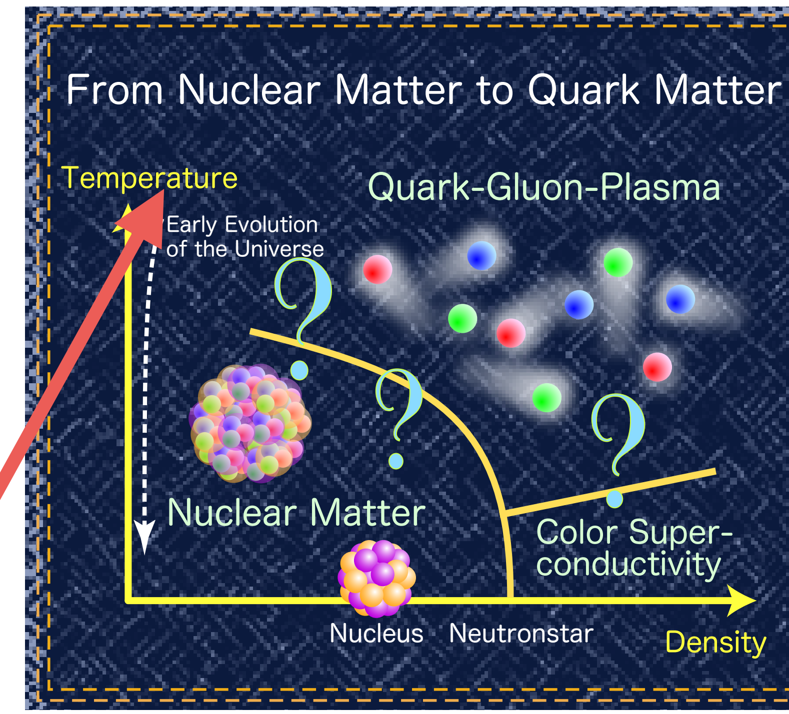

hot topics in QGP

- If we have $T_{\mu\nu}$

➡ direct measurement of thermodynamic quantity

- Fluctuations and correlations of $T_{\mu\nu}$

➡ specific heat, viscosity, $\cdots$

How to calculate T$\mu\nu$ on lattice?

How to calculate $T_{\mu\nu}$ on lattice?

- Measure VEV's on lattice

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- Renormalization

How to calculate $T_{\mu\nu}$ on lattice?

- Measure VEV's on lattice

  $\boxed{\text{terms in QCD Lagrangian}}$

  $\delta_{\mu\nu} F^a_{\rho\sigma}(x) F^a_{\rho\sigma}(x)$ $\quad$ $\delta_{\mu\nu} \bar{\psi}(x) \overleftrightarrow{\not{D}} \psi(x)$ $\quad$ $\delta_{\mu\nu} \bar{\psi}(x) \psi(x)$

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  $\boxed{\text{terms in QCD Lagrangian when trace is taken}}$

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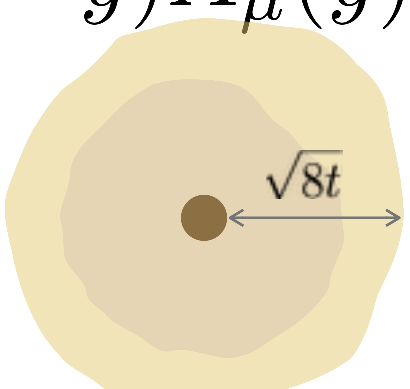

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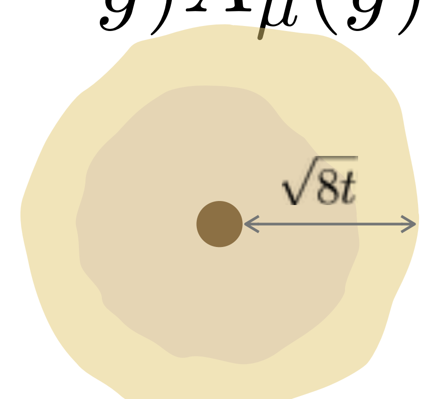

smear field within $\sqrt{8t}$

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A great view point:

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Gradient Flow as a renormalization scheme

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Re<1−□>

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NP renormalized operator

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universal

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$\boxed{\text{From Flow scheme to } \overline{\text{MS}} \text{ scheme}}$

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PT scheme defined to subtract 1/(4-D) for each loop

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scale matching $\quad \mu = \dfrac{1}{\sqrt{8t}}$

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Matching coefficients are calculable perturbatively at small t region

H.Suzuki, PTEP 2013, 083B03 (2013)

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How to calculate T$_{\mu\nu}$ on lattice?

From Flow scheme to $\overline{\mathrm{MS}}$ scheme

Matching coefficients are calculable perturbatively at small t region

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How to calculate T$_{\mu\nu}$ on lattice?

From Flow scheme to $\overline{\text{MS}}$ scheme

- Matching coefficients at one loop

$$c_1(t) = \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2}\left(9\gamma - 18\ln 2 + \frac{19}{4}\right)$$

$$c_2(t) = \frac{1}{(4\pi)^2}\frac{33}{16}$$

Matching coefficients are calculable perturbatively at small t region

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How to calculate T$_{\mu\nu}$ on lattice?

$\boxed{\text{Three steps to calculate } T_{\mu\nu}}$

How to calculate T$\mu\nu$ on lattice?

Three steps to calculate T$\mu\nu$

1. Flow the link variable $\partial_t U_\mu(t,x) U_\mu^\dagger(t,x) = -g_0^2 \partial_{x,\mu} S_{\mathrm{lat}}(U)$

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appropriately defined on lattice

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How to calculate T$\mu\nu$ on lattice?

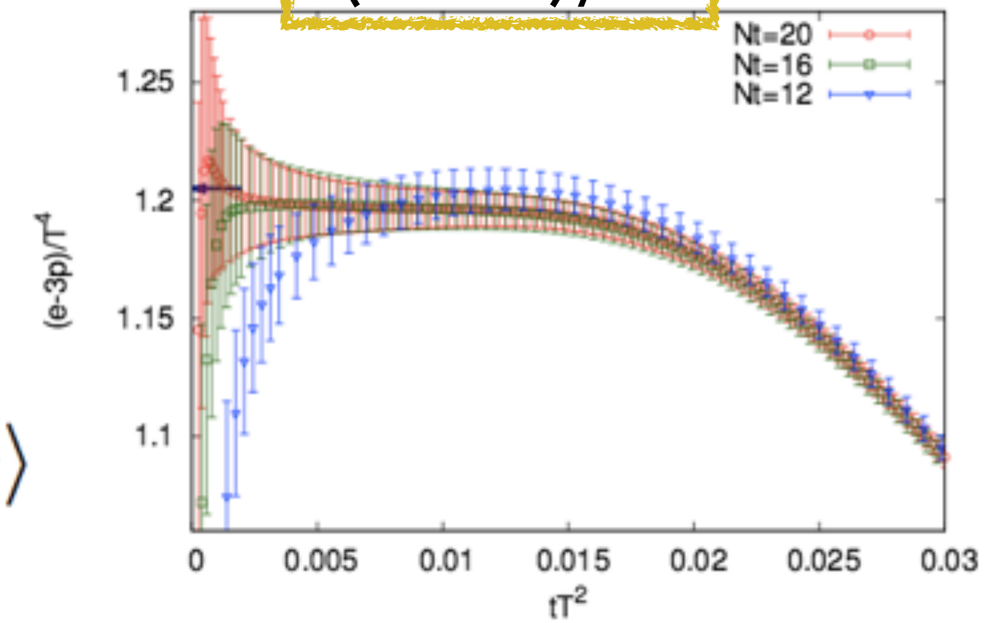
Previous works and lessons

FlowQCD Collaboration
(2014-)

T=1.66Tc

$$\varepsilon = -\langle T_{00}\rangle,\ P = \langle T_{ii}\rangle$$

(E-3P)/T^4



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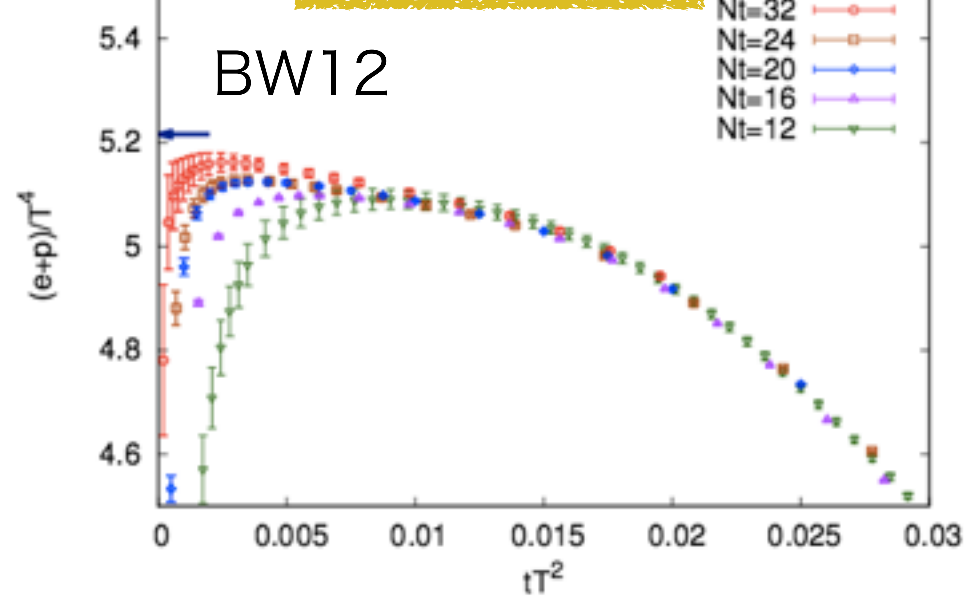

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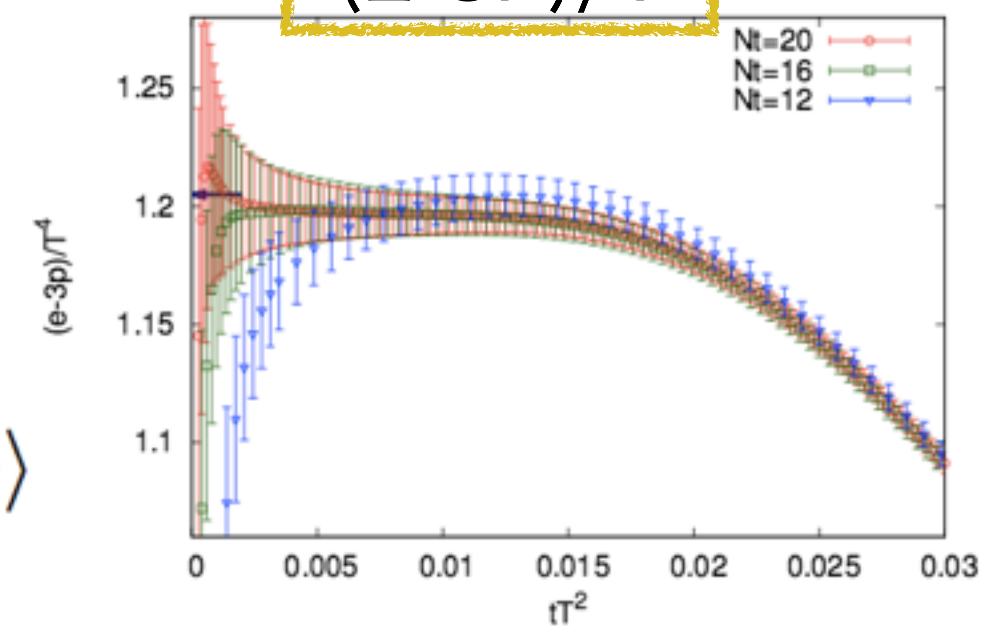

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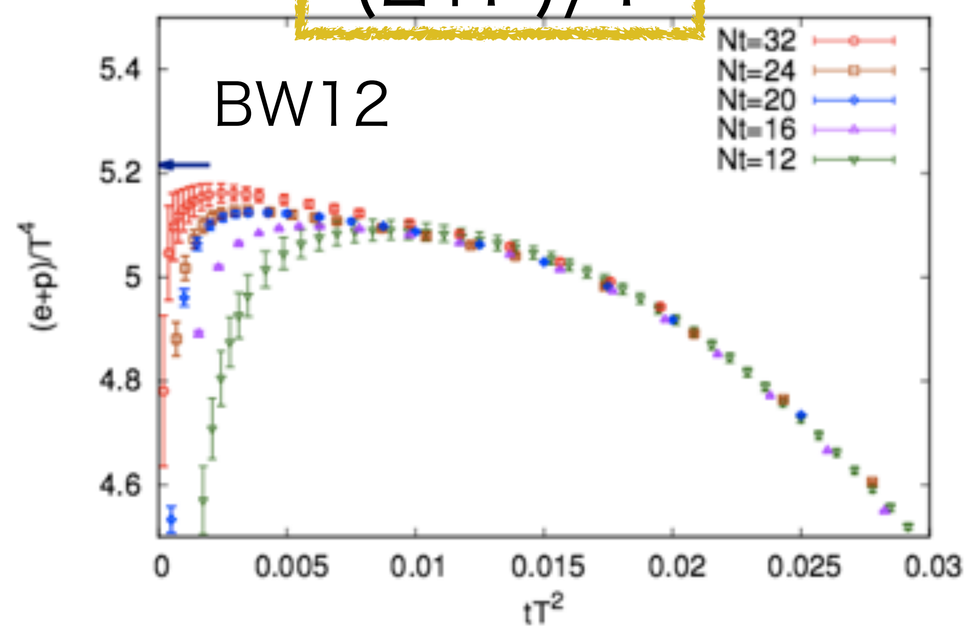

We need a window

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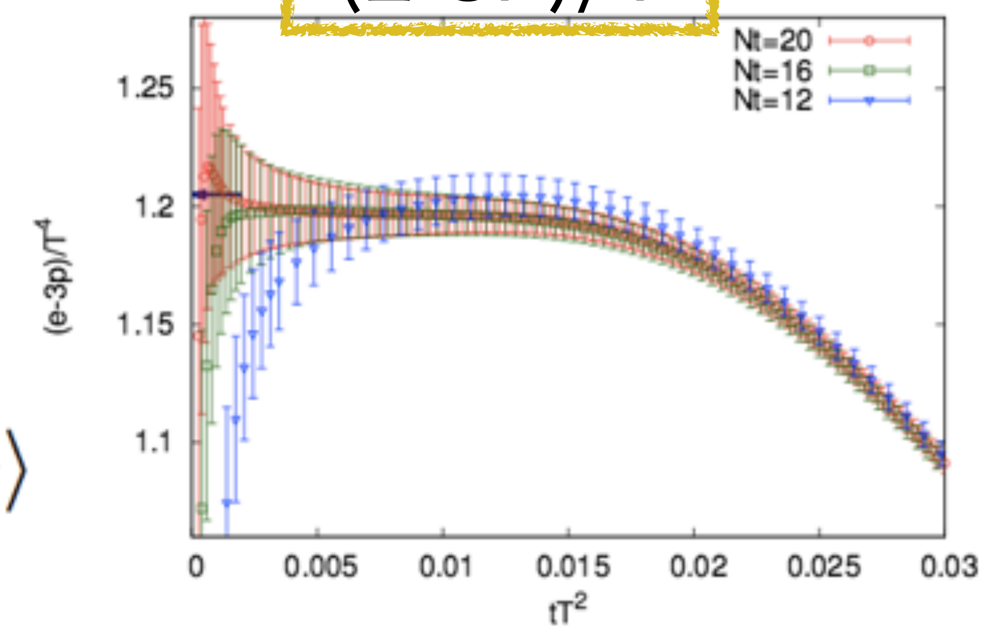

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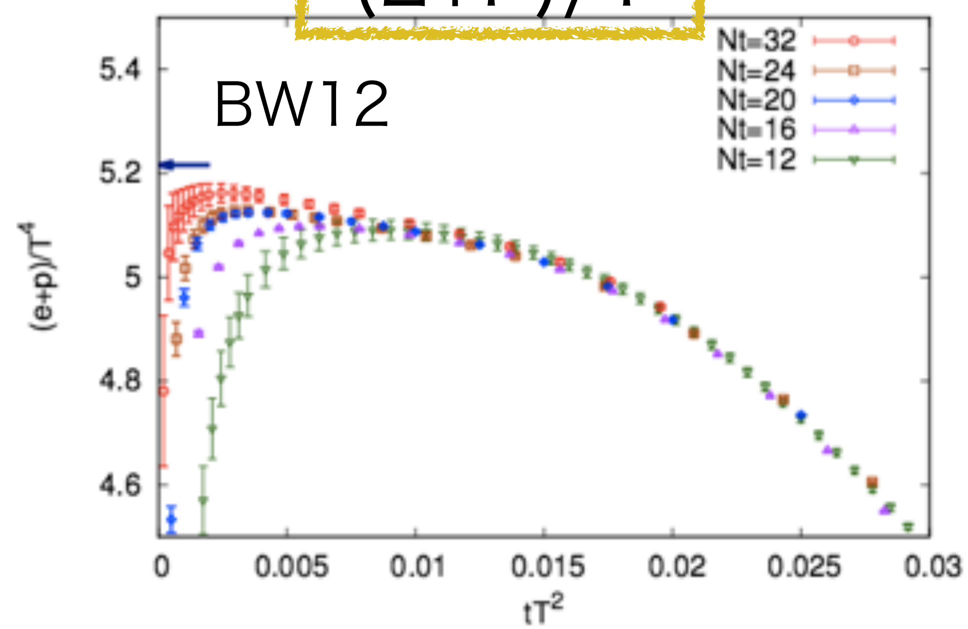

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$$a \ll \sqrt{8t} \ll \frac{1}{2T}$$

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Previous works and lessons

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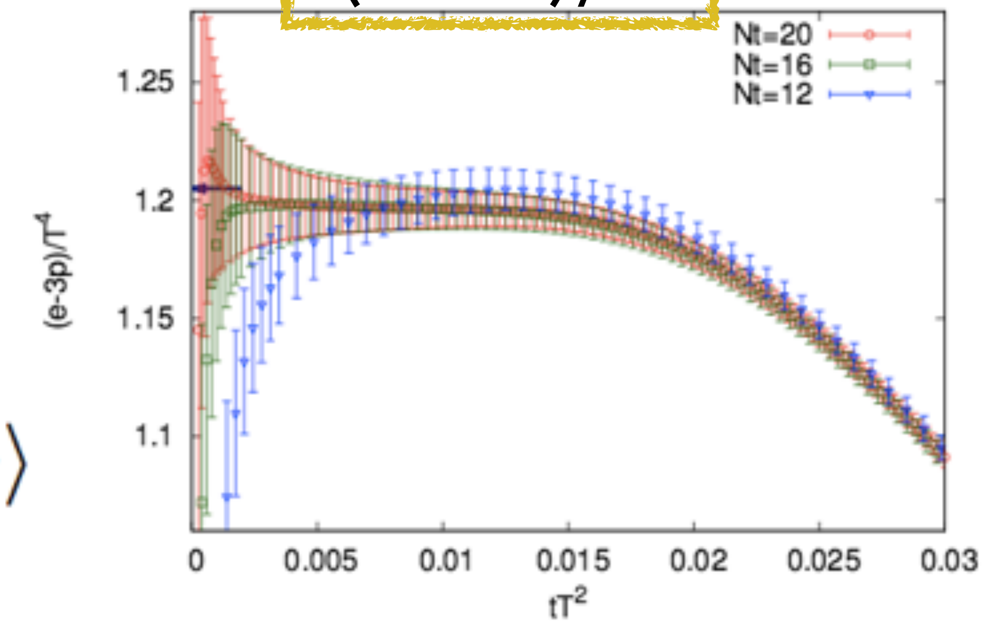

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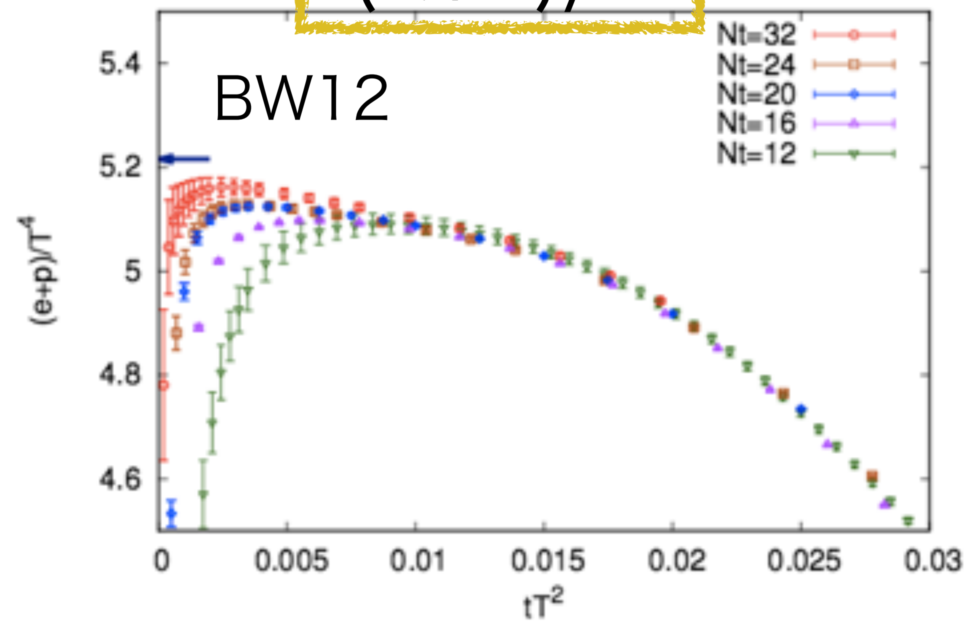

We need a window $a \ll \sqrt{8t} \ll \dfrac{1}{2T}$

lattice artifact

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Previous works and lessons

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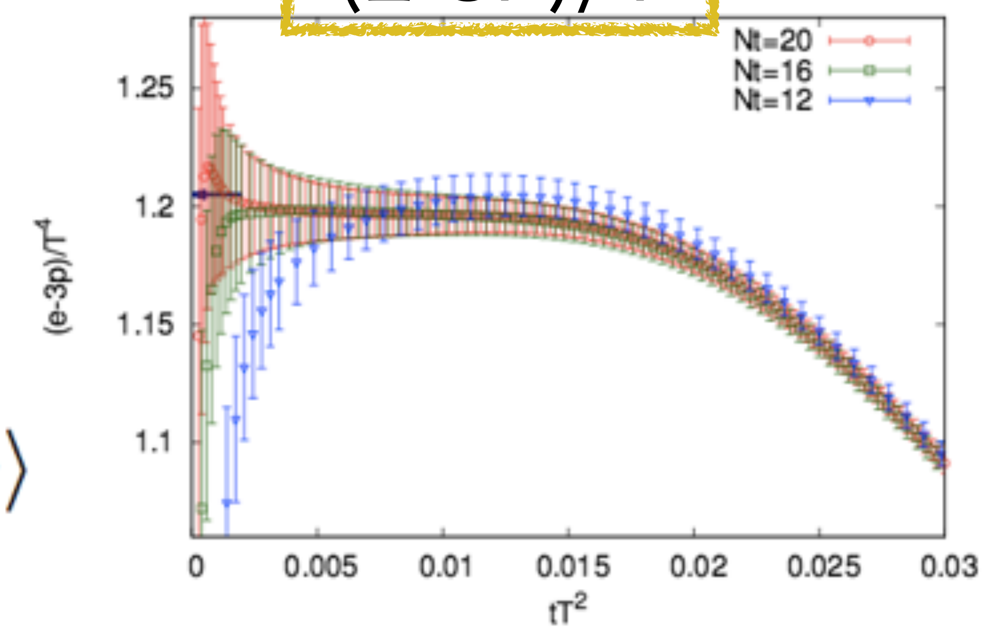

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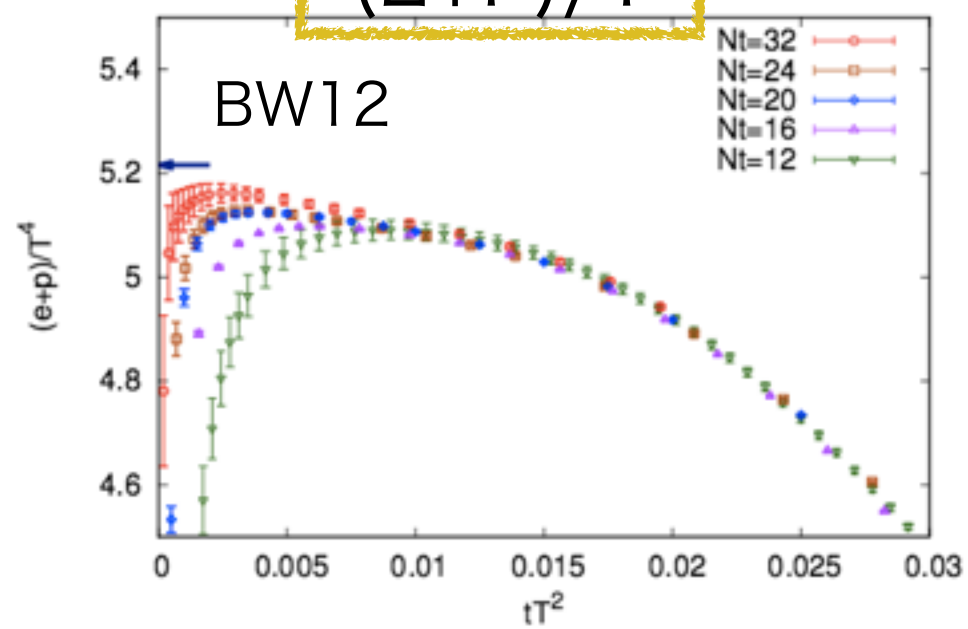

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finite temporal length

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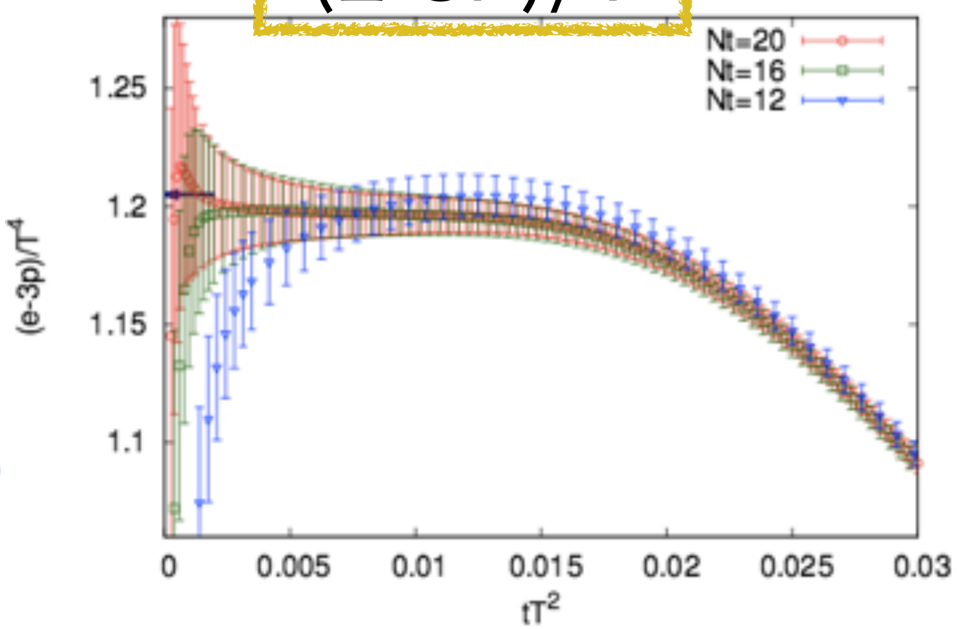

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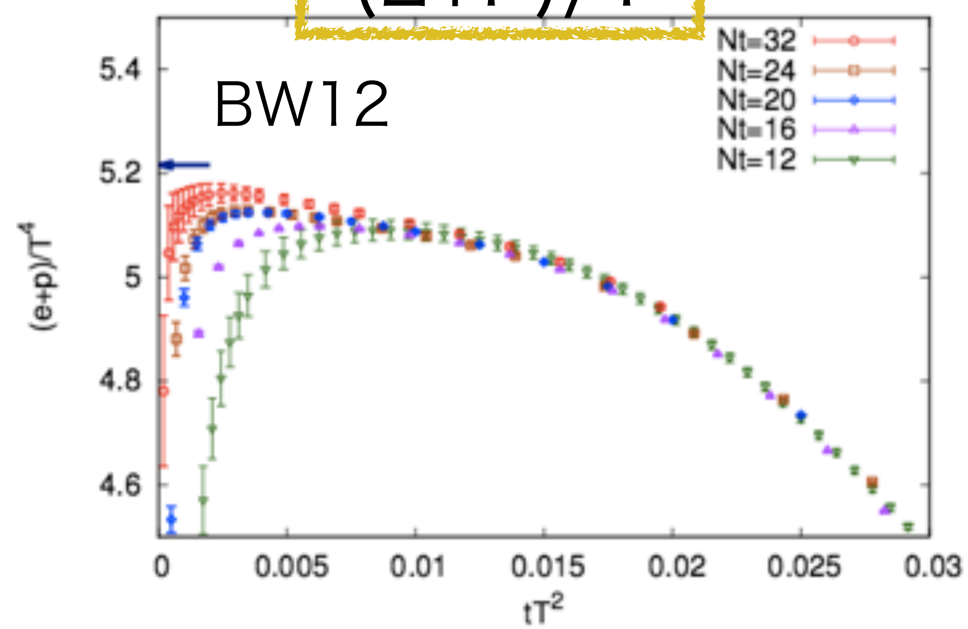

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Is there a small t region within the window where two loops contribution is negligible?

What’s new?

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Quarks included!

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Flow of quark field

Lüscher, JHEP 1304, 123 (2013)

What's new? $\boxed{\text{Quarks included!}}$

$\boxed{\text{Flow of quark field}}$

Lüscher, JHEP 1304, 123 (2013)

$$\partial_t \chi(t,x) = D_\mu D_\mu \chi(t,x) \qquad \chi(t=0,x) = \psi(x)$$

$$\partial_t \bar{\chi}(t,x) = \bar{\chi}(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu \qquad \bar{\chi}(t=0,x) = \bar{\psi}(x)$$

What's new? Quarks included!

Flow of quark field

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Renormalization is needed for quark field

$$\chi_R(t,x) = Z_\chi \chi_0(t,x)$$

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Renormalization is needed for quark field

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No more renormalization is needed for composite op.

$$\left(\bar{\chi}(t,x)\chi(t,x)\right)_R = Z_\chi^2 \left(\bar{\chi}(t,x)\chi(t,x)\right)_0$$

What's new? $\boxed{\text{Quarks included!}}$

$\boxed{\text{Flow of quark field}}$ Lüscher, JHEP 1304, 123 (2013)

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No additive correction in chiral condensate

How to calculate $T_{\mu\nu}$ on lattice?

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$\boxed{\text{Three steps to calculate } T_{\mu\nu}}$

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$$\langle \chi(t,x)\bar{\chi}(t,y)\rangle_{\mathrm{Wick}} \neq (D(A_\mu(t,x))+m)^{-1}$$

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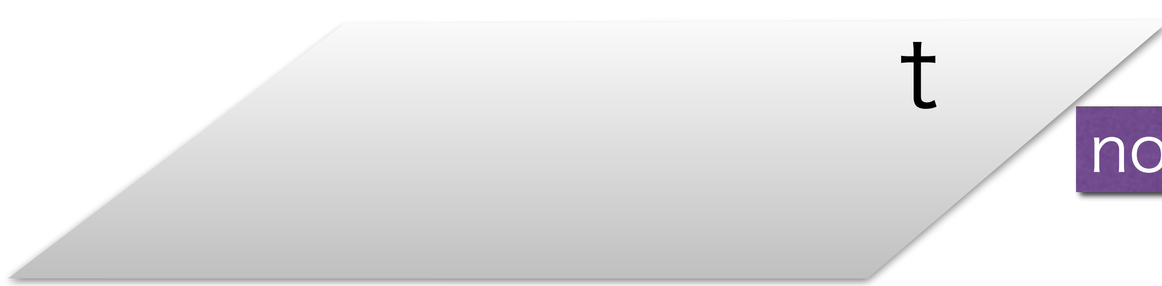

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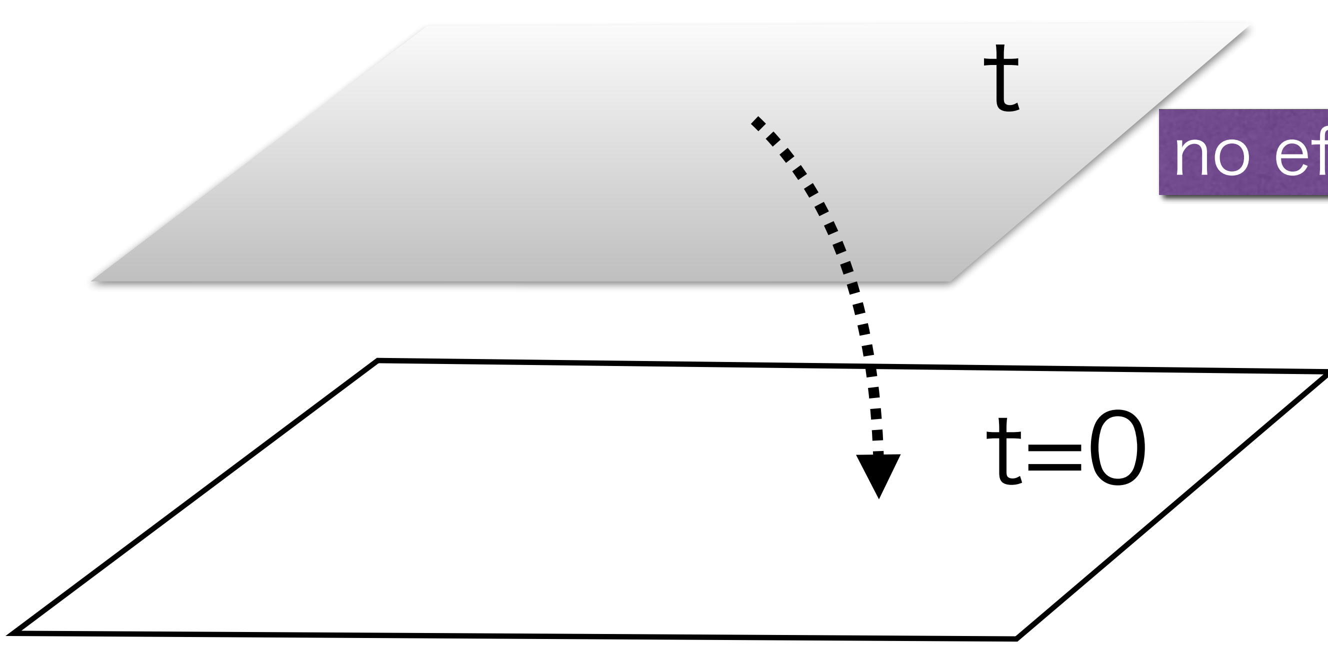

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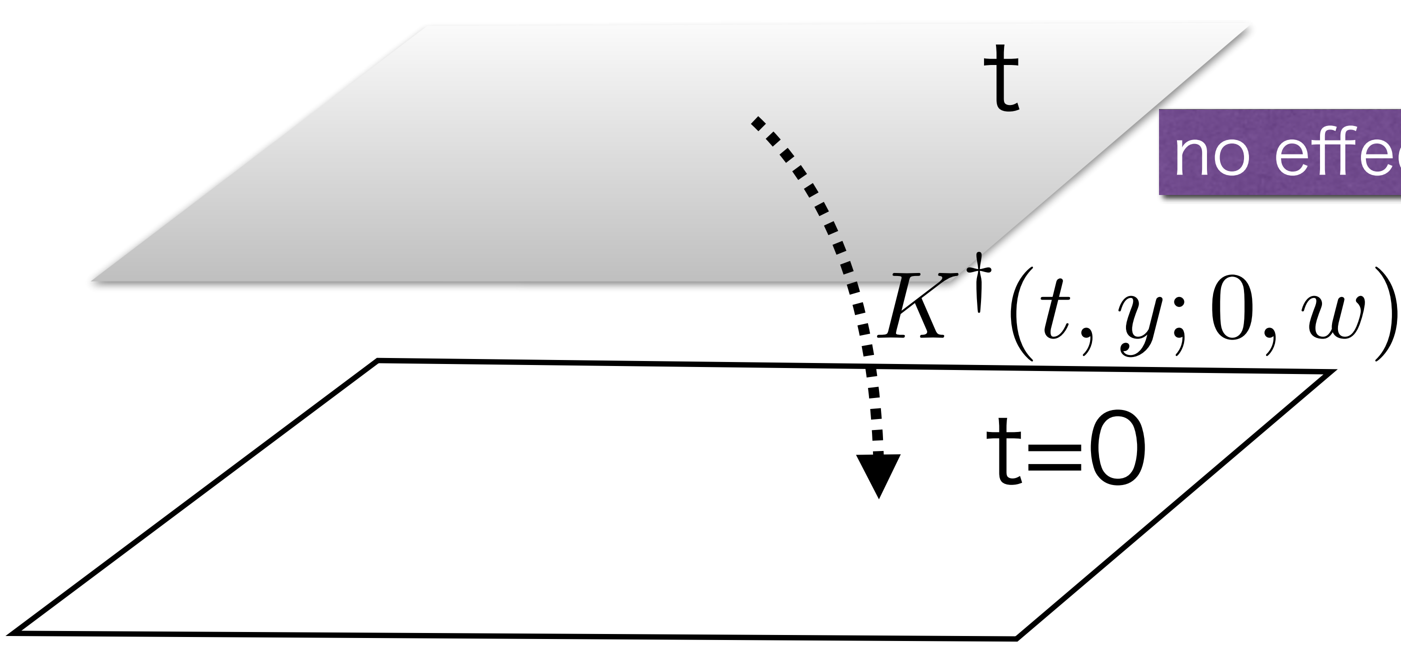

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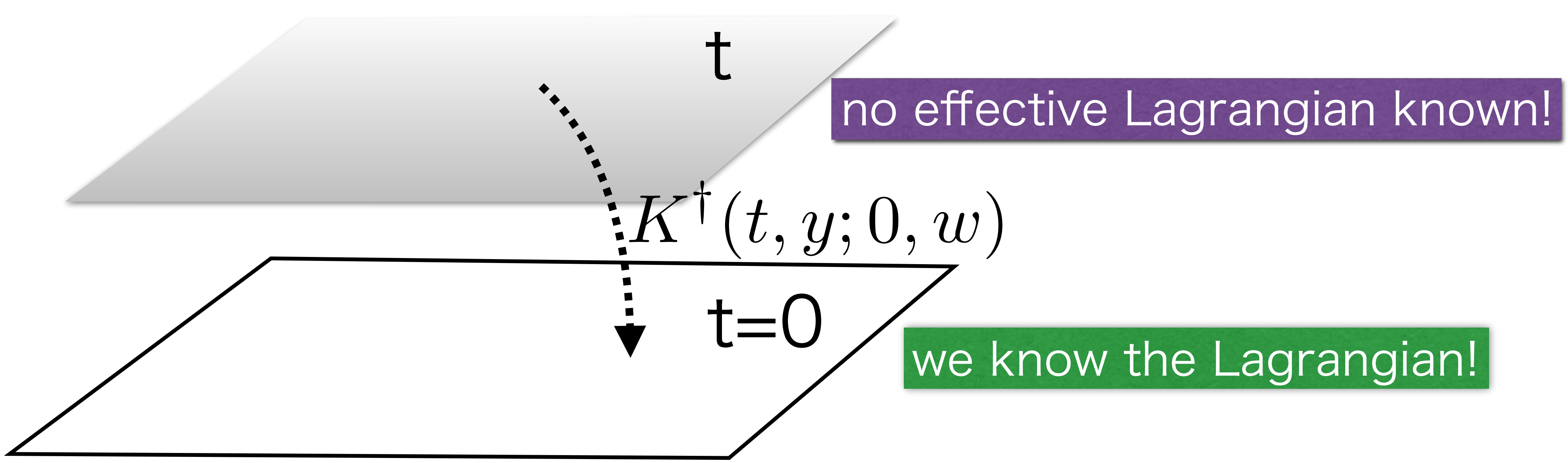

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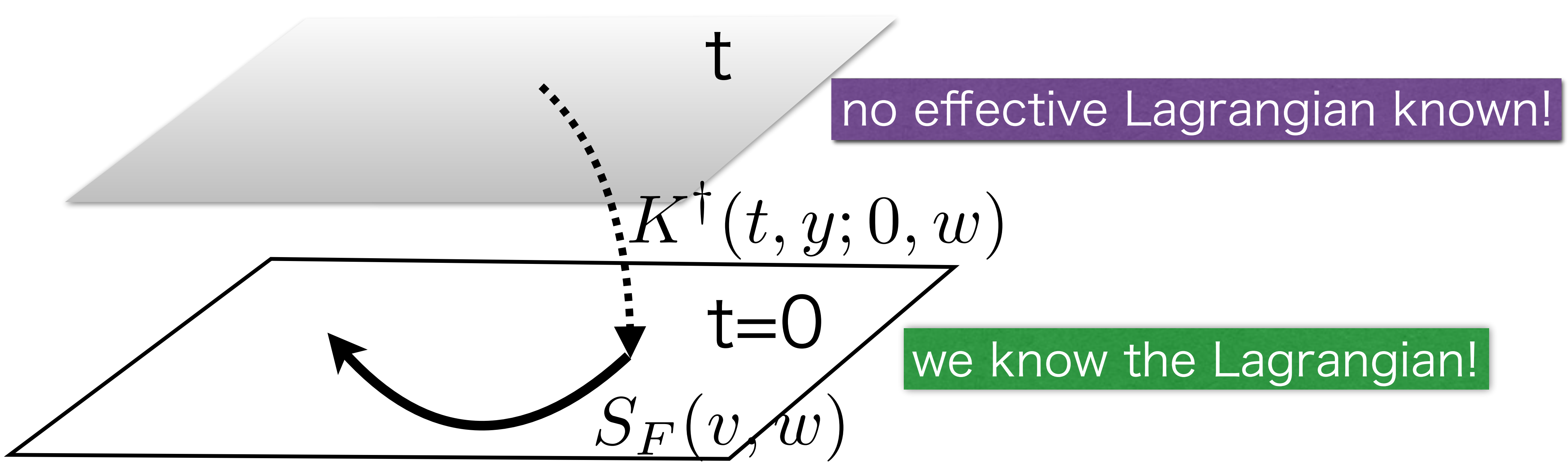

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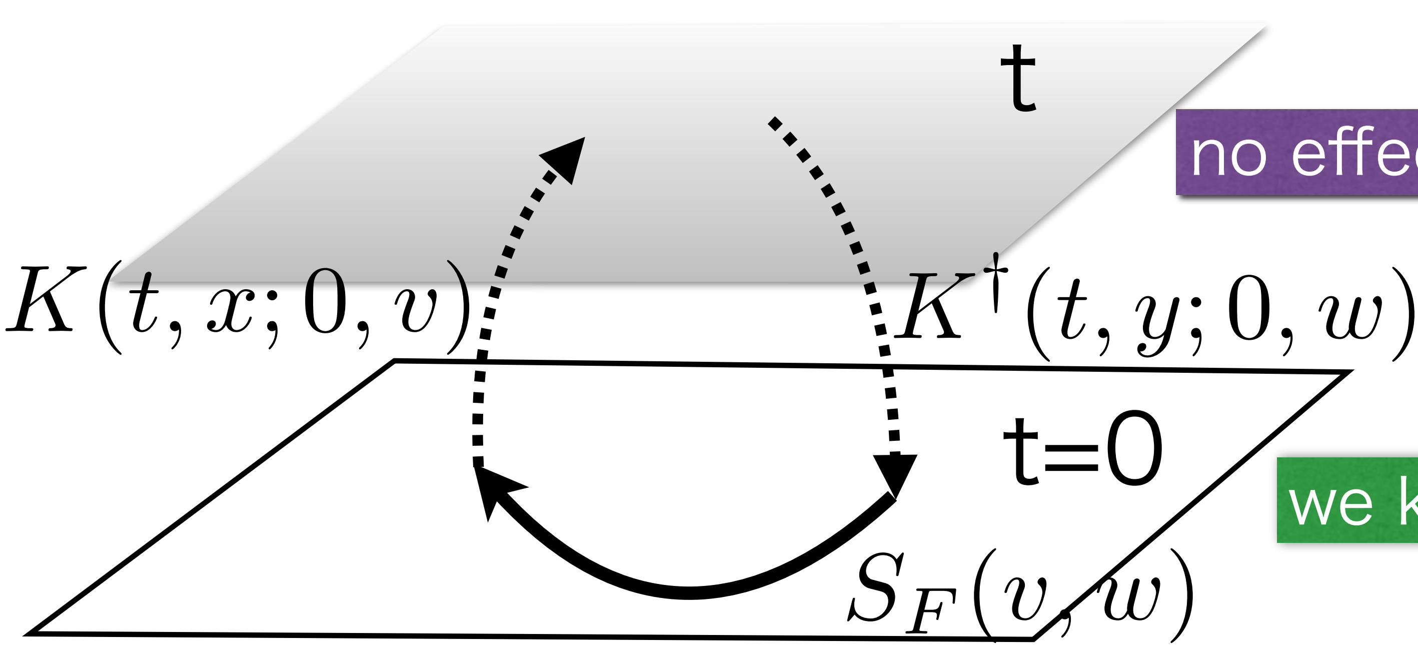

no effective Lagrangian known!

we know the Lagrangian!

How to calculate T$_{\mu\nu}$ on lattice?

3. Multiply the coefficients and visit small t region

$$\{T_{\mu\nu}\}(x)^{q}_{\overline{\mathrm{MS}}} = \lim_{t\to 0}\Bigg\{ c_3(t) \sum_{r=u,d,s} \left(\tilde{\mathcal{O}}^r_{3\mu\nu}(t,x) - 2\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}^r_{3\mu\nu}(t,x) - 2\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x) \right\rangle_{T=0} \right)$$

$$+ c_4(t) \sum_{r=u,d,s} \left(\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}^r_{4\mu\nu}(t,x) \right\rangle_{T=0} \right) + \sum_{r=u,d,s} c^r_5(t) \left(\tilde{\mathcal{O}}^r_{5\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}^r_{5\mu\nu}(t,x) \right\rangle_{T=0} \right) \Bigg\}$$

$$\tilde{\mathcal{O}}^r_{3\mu\nu}(t,x) \equiv \varphi_r(t) \bar{\chi}_r(t,x) \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \chi_r(t,x)$$

$$\tilde{\mathcal{O}}^r_{4\mu\nu}(t,x) \equiv \varphi_r(t) \delta_{\mu\nu} \bar{\chi}_r(t,x) \overleftrightarrow{\not{D}} \chi_r(t,x)$$

$$\tilde{\mathcal{O}}^r_{5\mu\nu}(t,x) \equiv \varphi_r(t) \delta_{\mu\nu} \bar{\chi}_r(t,x) \chi_r(t,x)$$

$$\varphi_r(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_r(t,x) \overleftrightarrow{\not{D}} \chi_r(t,x) \right\rangle_{T=0}}$$

$$2\left\langle \tilde{\mathcal{O}}^r_{3\mu\nu}(t,x) \right\rangle_{T=0} = \left\langle \tilde{\mathcal{O}}^r_{4\mu\nu}(t,x) \right\rangle_{T=0} = \frac{-6}{(4\pi)^2 t^2} \delta_{\mu\nu}$$

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3. Multiply the coefficients and visit small t region

$$c_3(t) = \frac{1}{4}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left(2 + \frac{4}{3}\ln(432)\right)\right\}$$

$$c_4(t) = \frac{1}{(4\pi)^2}\bar{g}(1/\sqrt{8t})^2$$

$$c_5^r(t) = -\bar{m}_r(1/\sqrt{8t})\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left(4\gamma - 8\ln 2 + \frac{14}{3} + \frac{4}{3}\ln(432)\right)\right\}$$

Makino-Suzuki, PTEP 2014, 063B02 (2014)

Numerical setups

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- Iwasaki gauge action

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  - β =2.05 : a～0.07 [fm]

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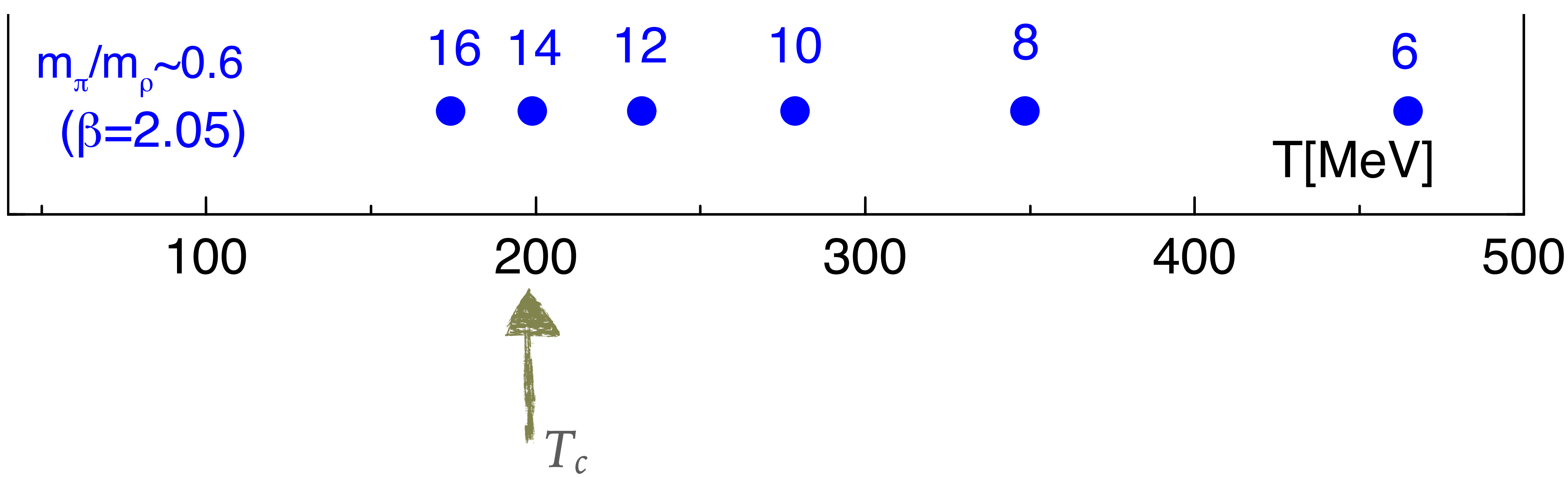
  - T=1/(aNt), Nt=16, 14, 12, 10, 8, 6, 4



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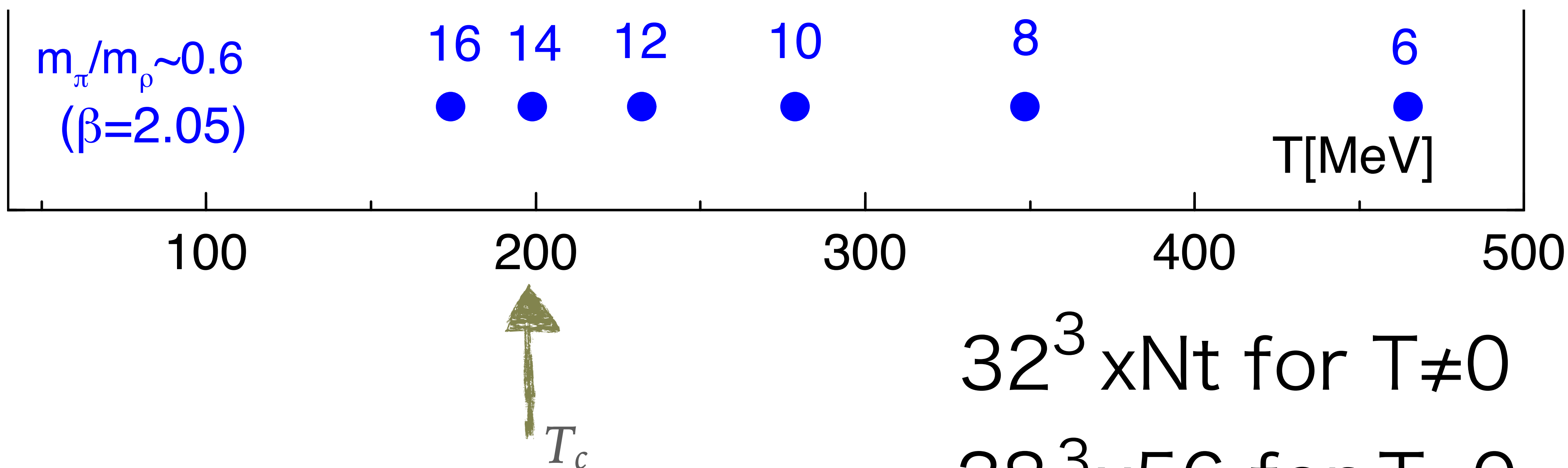

32^3xNt for T≠0

28^3x56 for T=0

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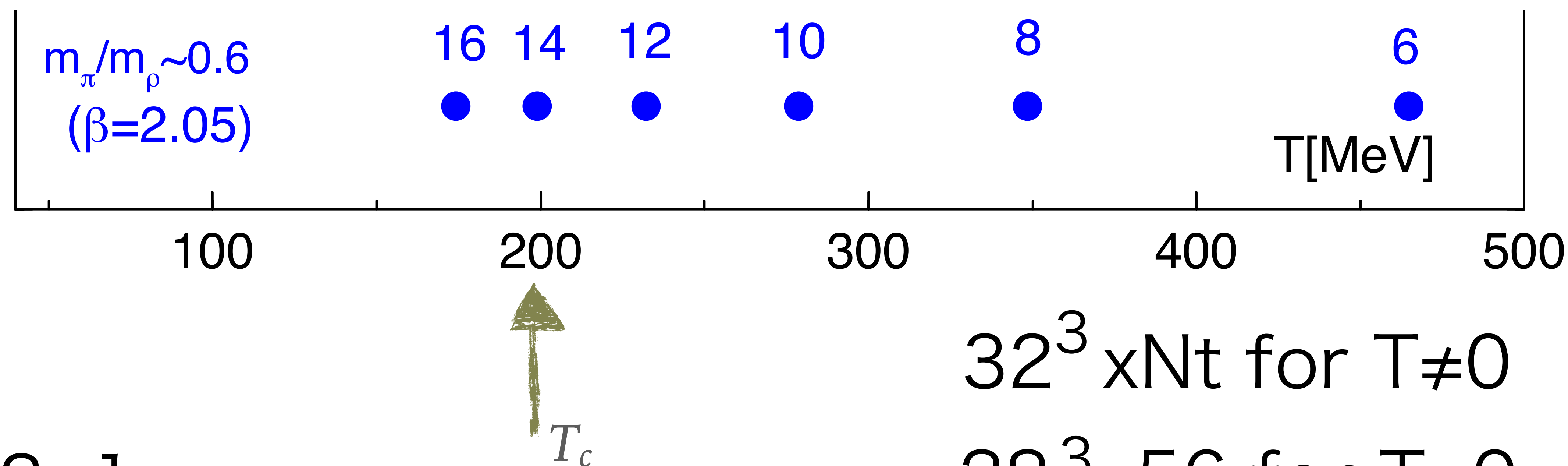


32^3xNt for T≠0

28^3x56 for T=0

- Nf=2+1
- NP improved Wilson fermion
- On an equal quark mass line

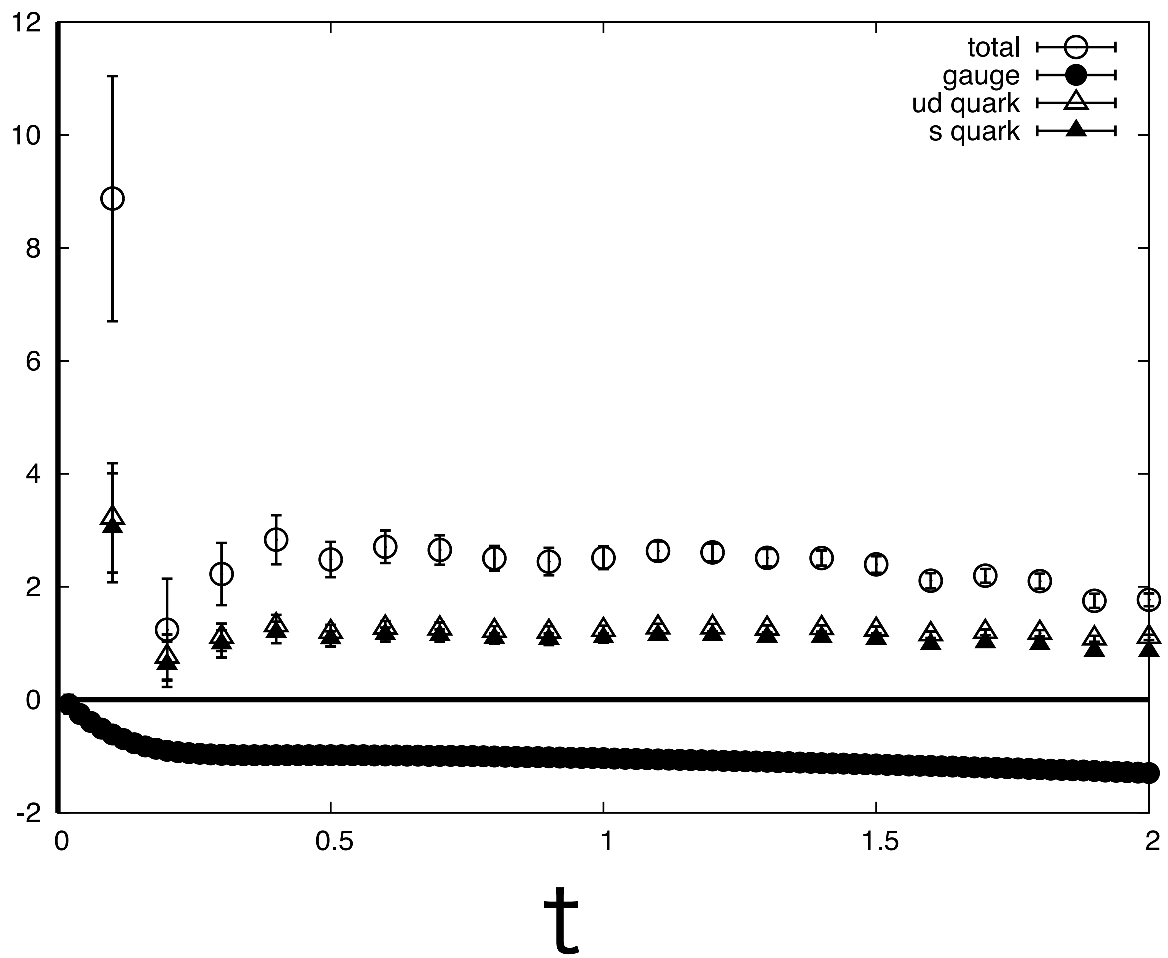
$$\frac{m_\pi}{m_\rho} \sim 0.6$$

Energy and Pressure (preliminary)

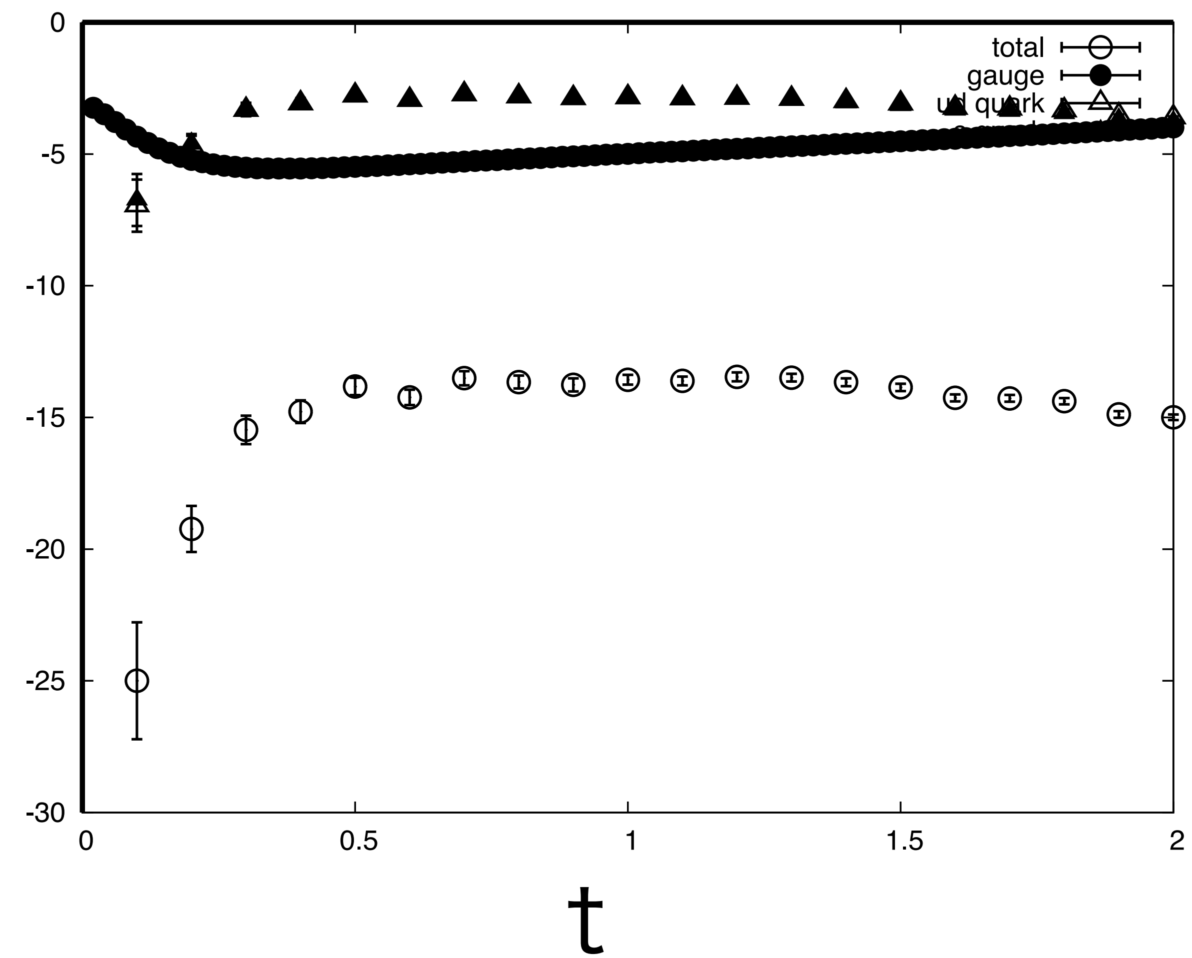
T=279MeV

contributions from gauge and quarks

$$\frac{P}{T^4}$$



$$-\frac{E}{T^4}$$

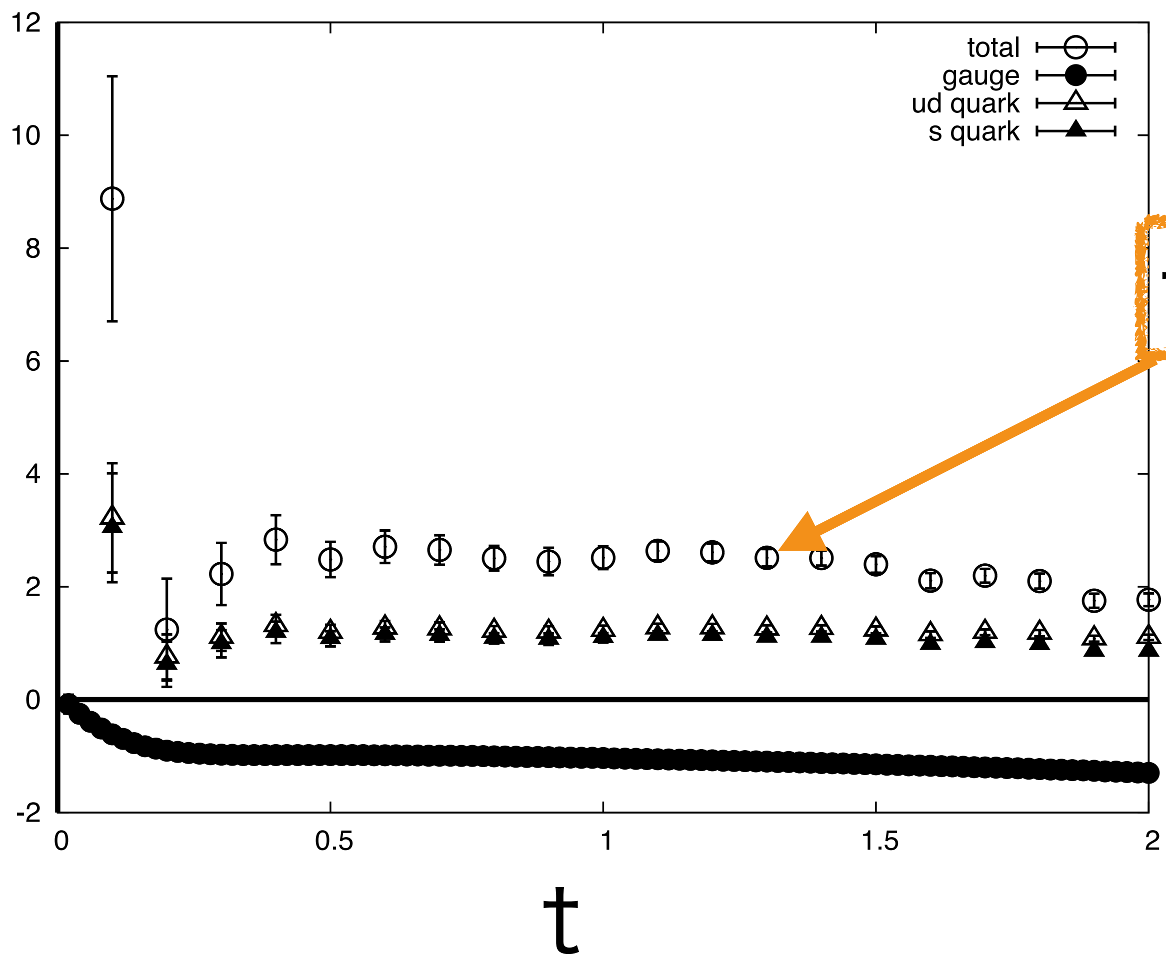


Energy and Pressure (preliminary)

T=279MeV

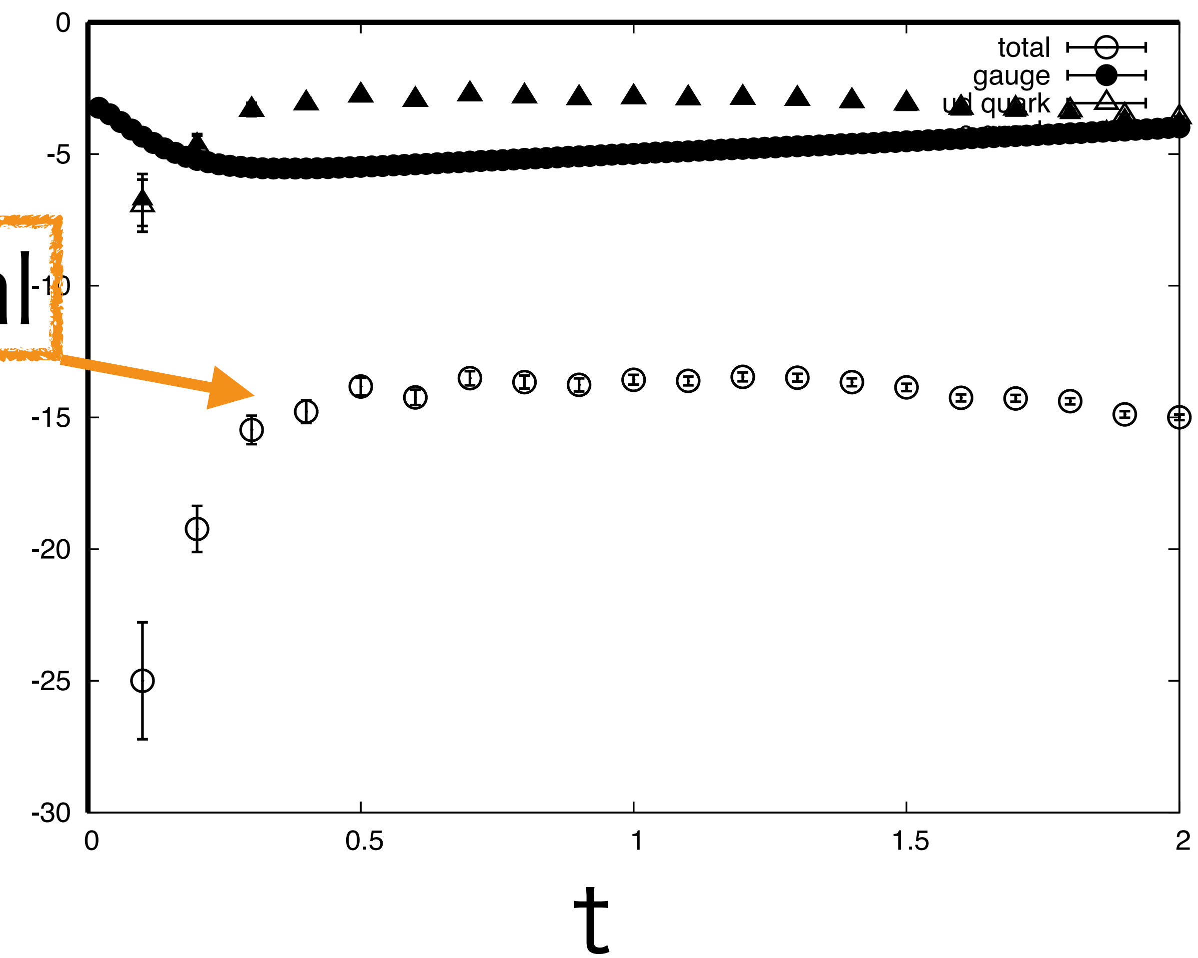
contributions from gauge and quarks

$$\frac{P}{T^4}$$



total

$$-\frac{E}{T^4}$$

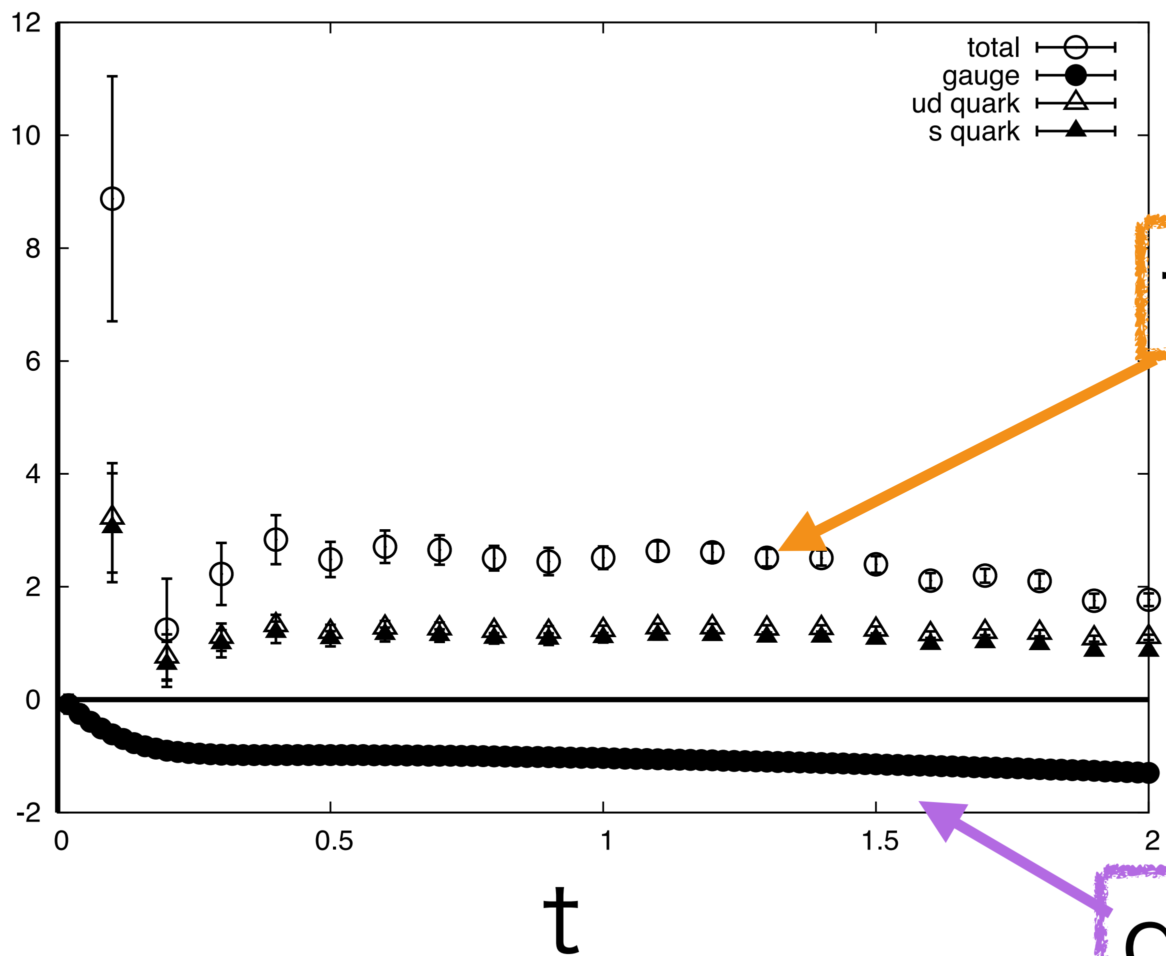


Energy and Pressure (preliminary)

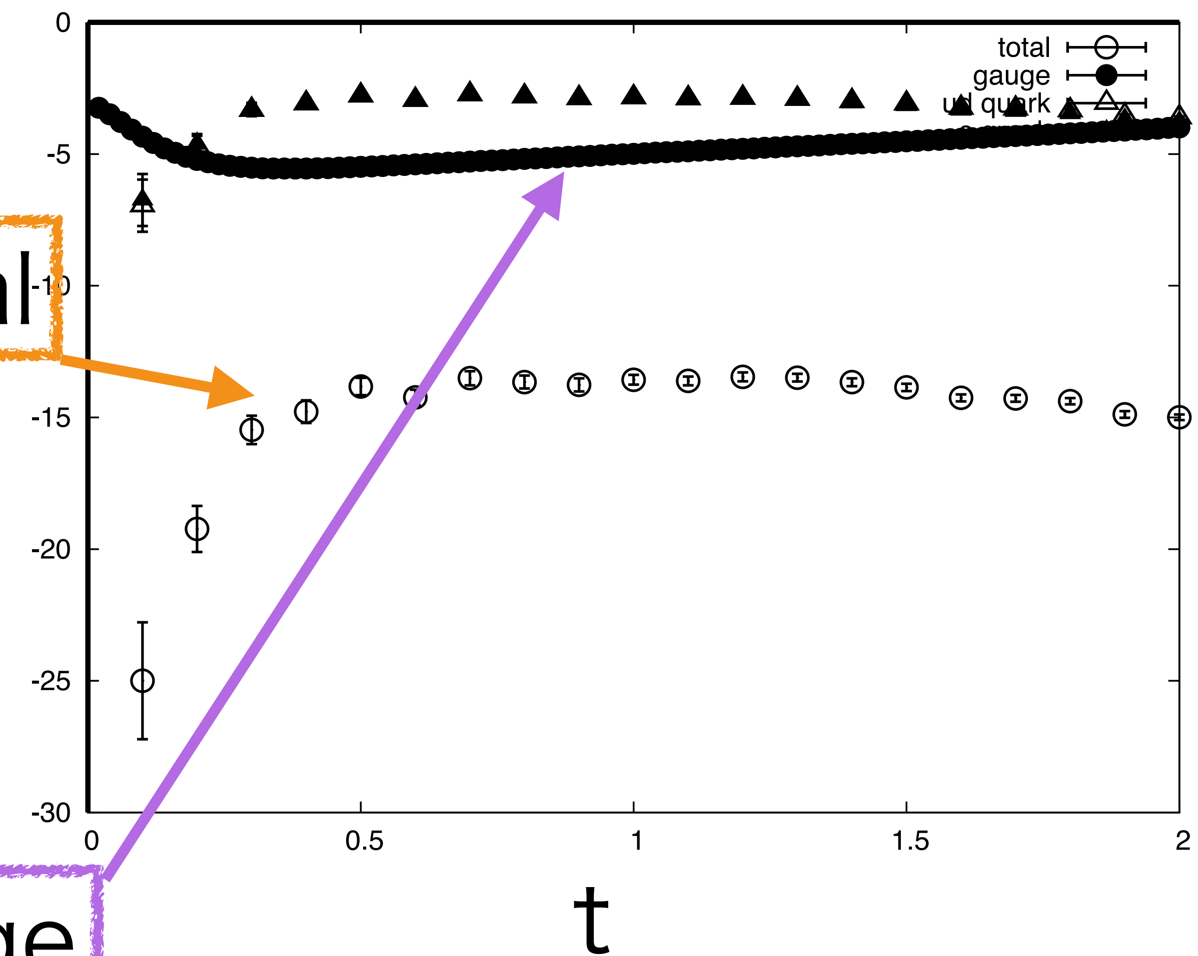
T=279MeV

contributions from gauge and quarks

$$\frac{P}{T^4}$$



$$-\frac{E}{T^4}$$



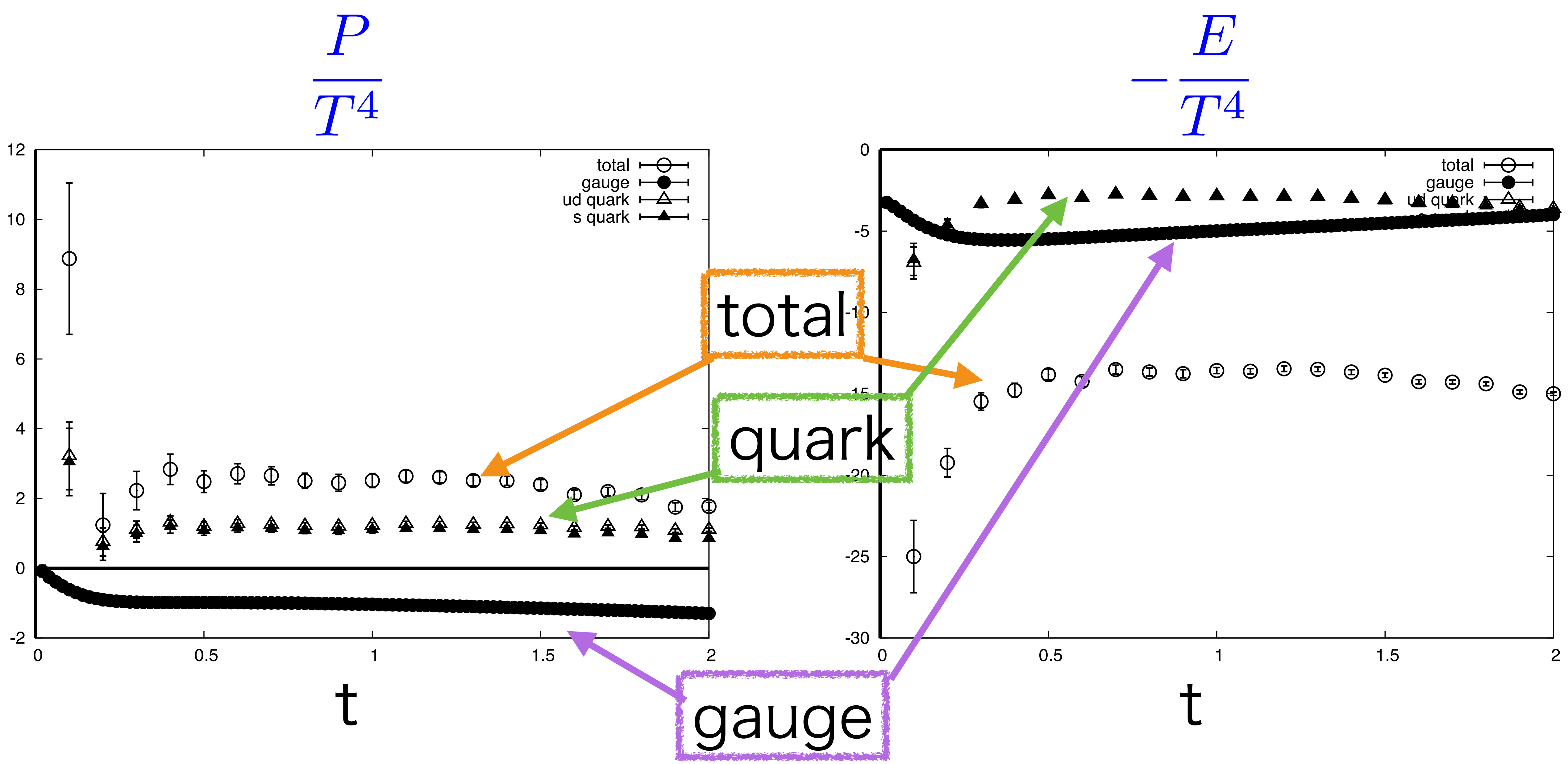
total

gauge

Energy and Pressure (preliminary)

T=279MeV

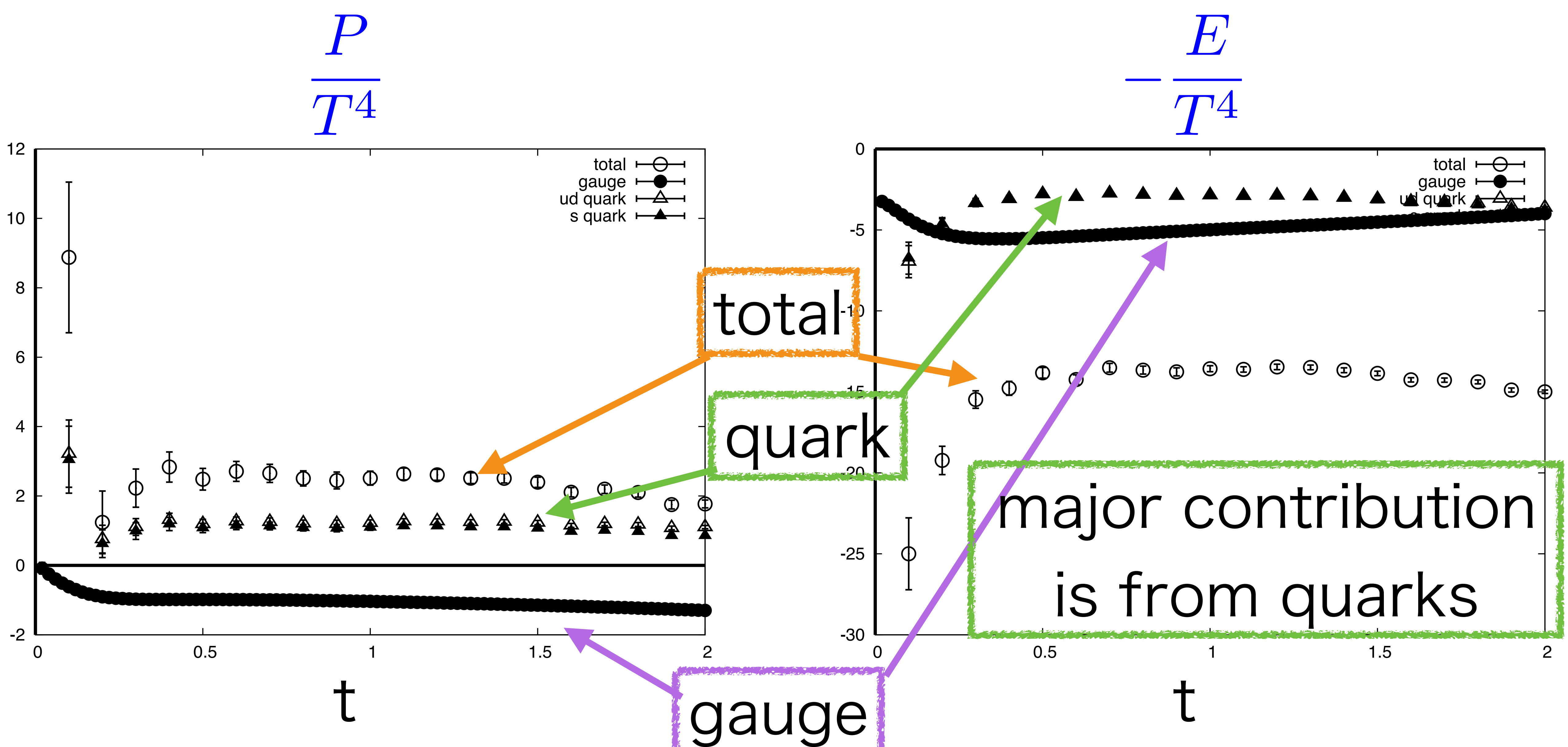
contributions from gauge and quarks



Energy and Pressure (preliminary)

T=279MeV

contributions from gauge and quarks

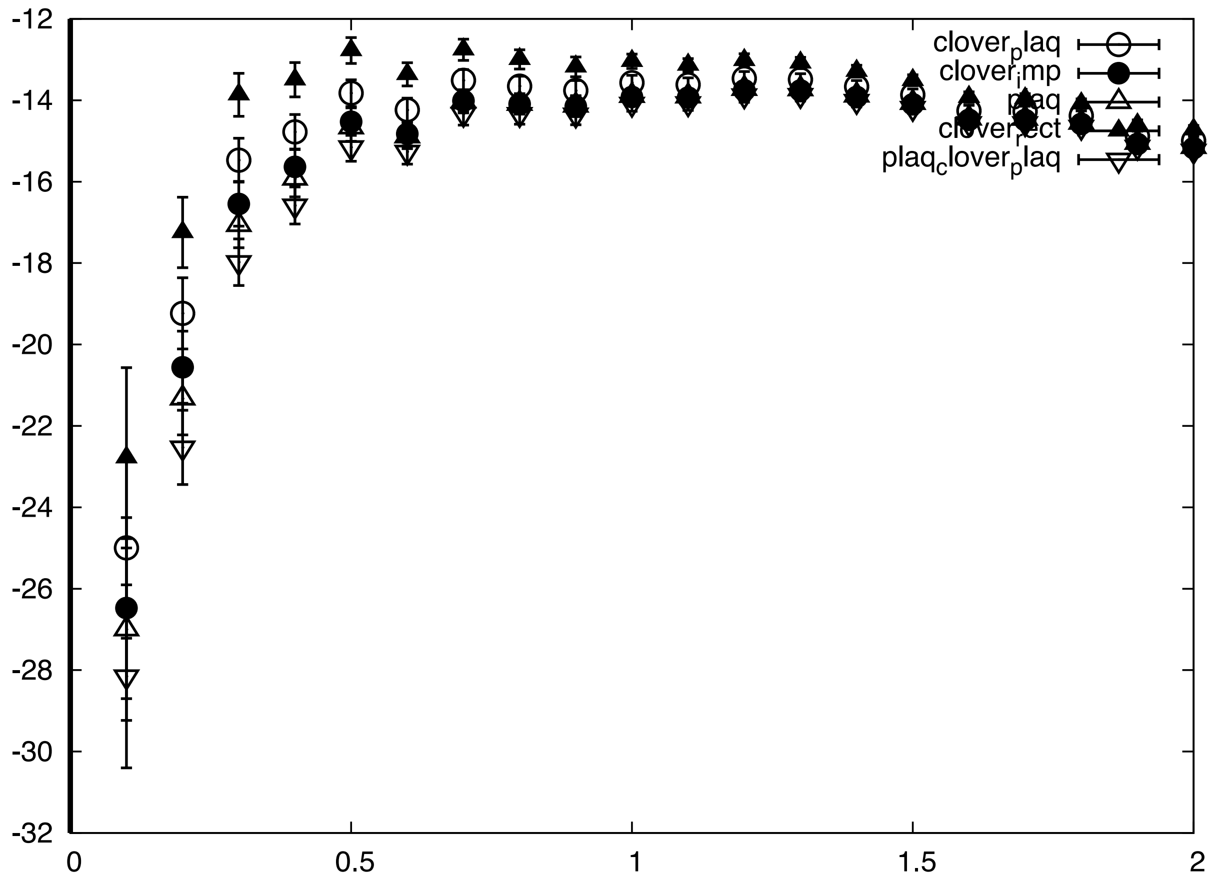


Energy and Pressure (preliminary)

T=279MeV

dependence on lattice operators

$$-\frac{E}{T^4}$$



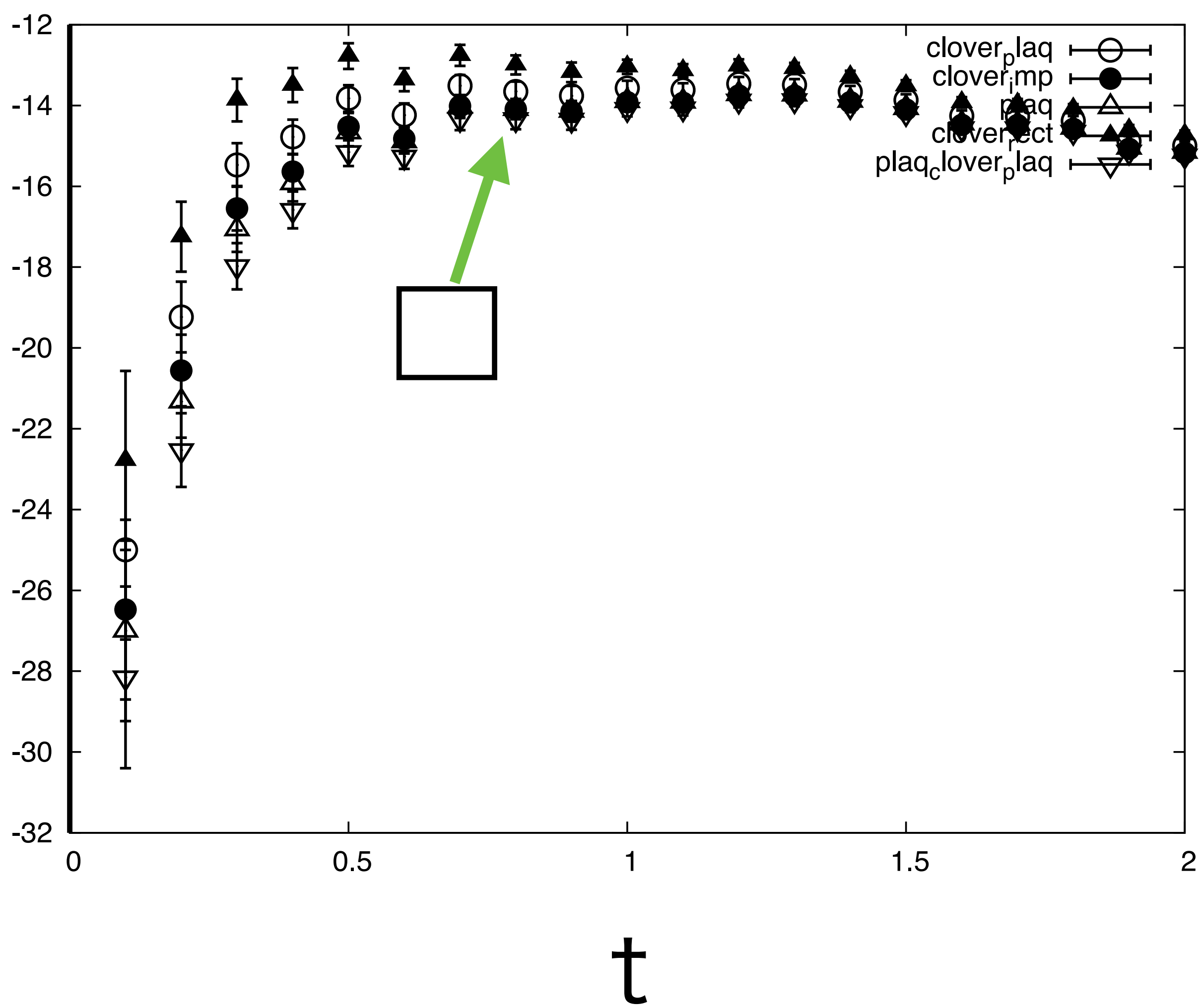
t

Energy and Pressure (preliminary)

T=279MeV

dependence on lattice operators

$$-\frac{E}{T^4}$$

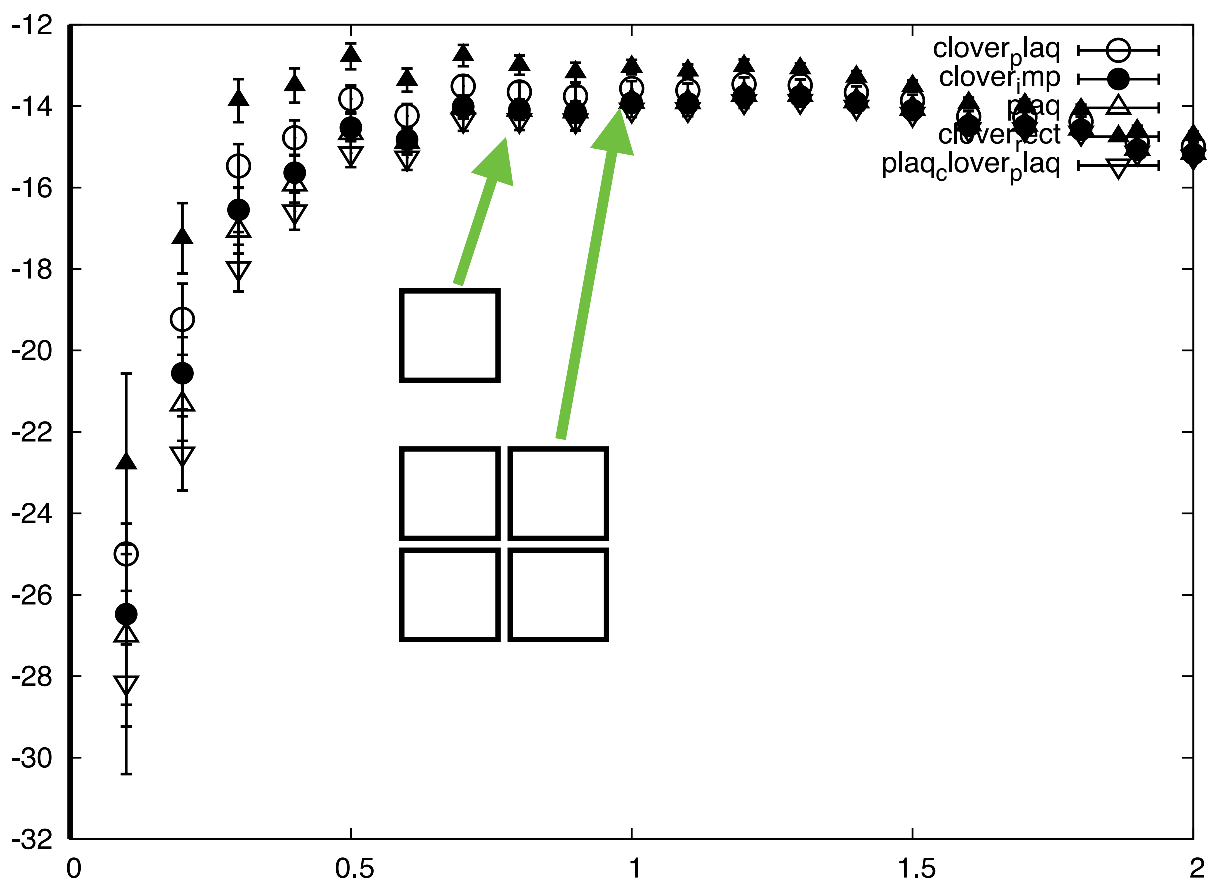


Energy and Pressure (preliminary)

T=279MeV

dependence on lattice operators

$$-\frac{E}{T^4}$$



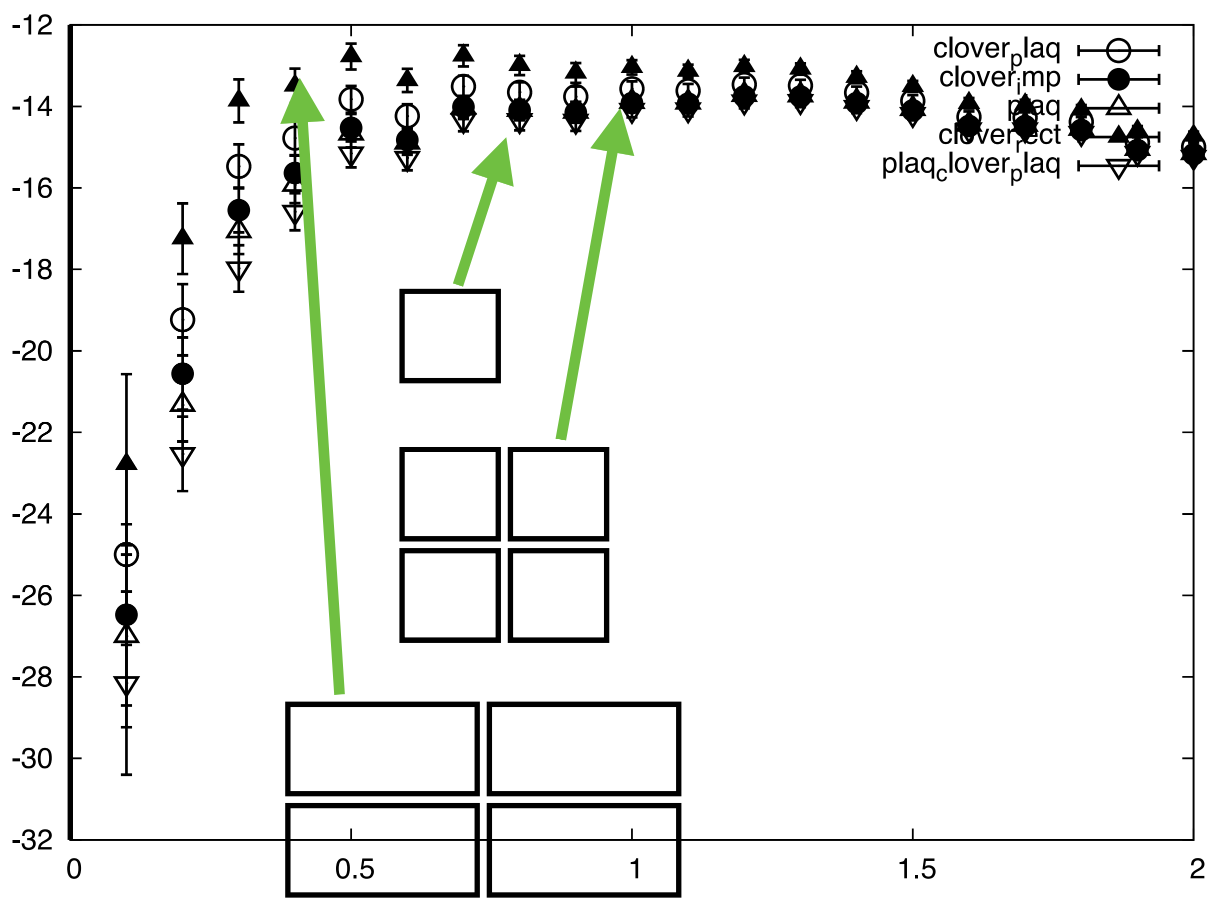
t

Energy and Pressure (preliminary)

T=279MeV

dependence on lattice operators

$$-\frac{E}{T^4}$$



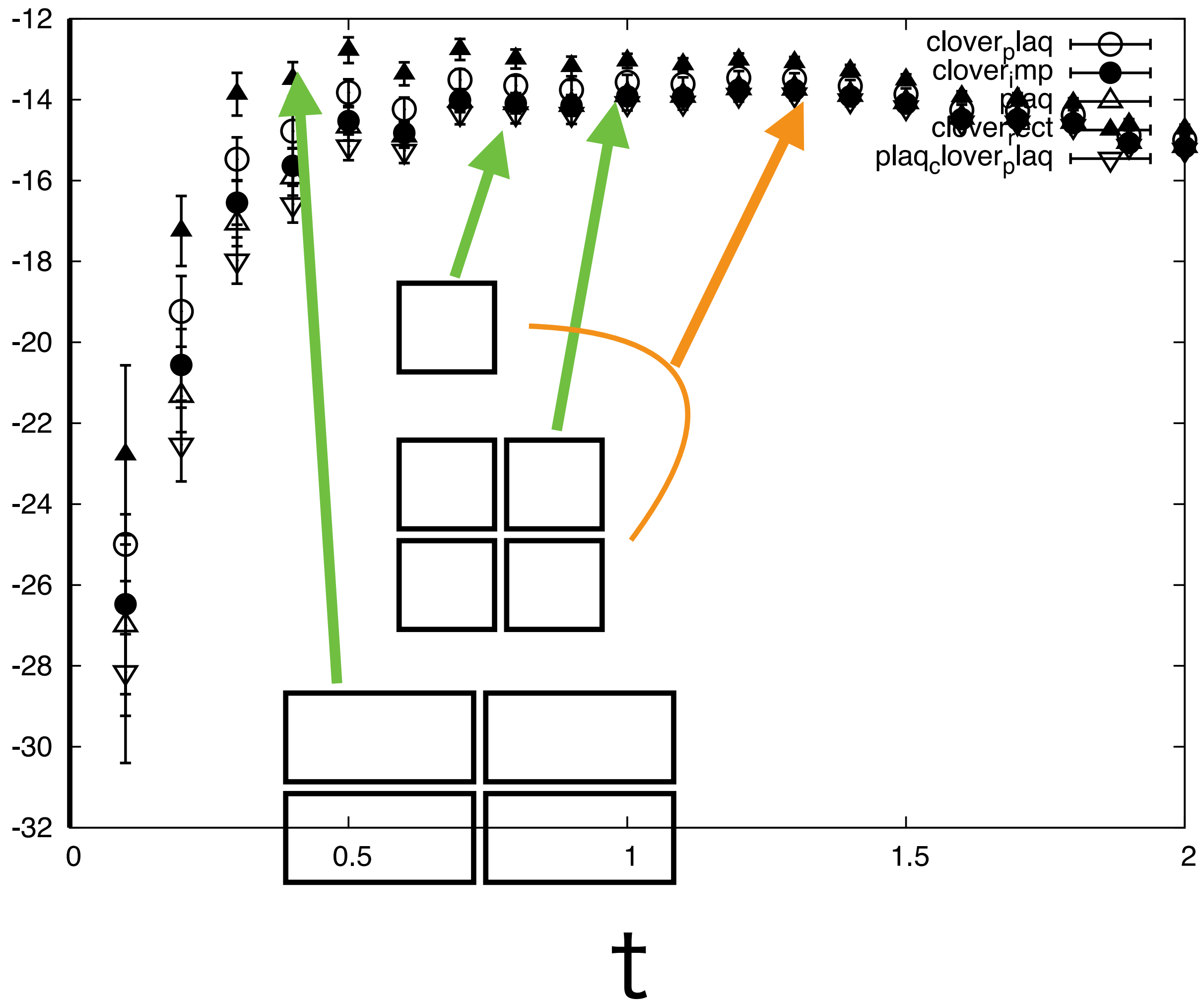
t

Energy and Pressure (preliminary)

T=279MeV

dependence on lattice operators

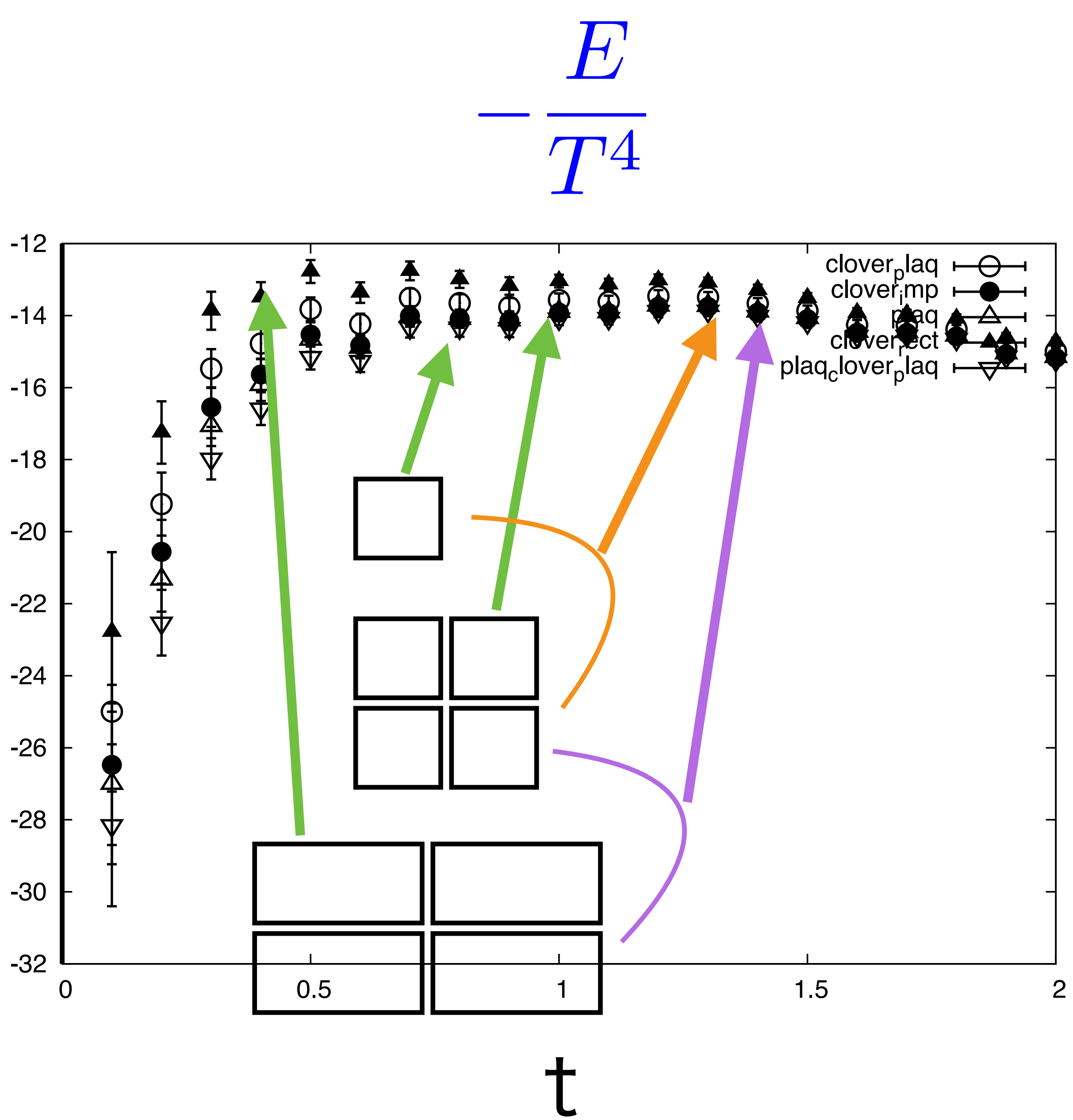
$$-\frac{E}{T^4}$$



Energy and Pressure (preliminary)

$\boxed{\text{T=279MeV}}$

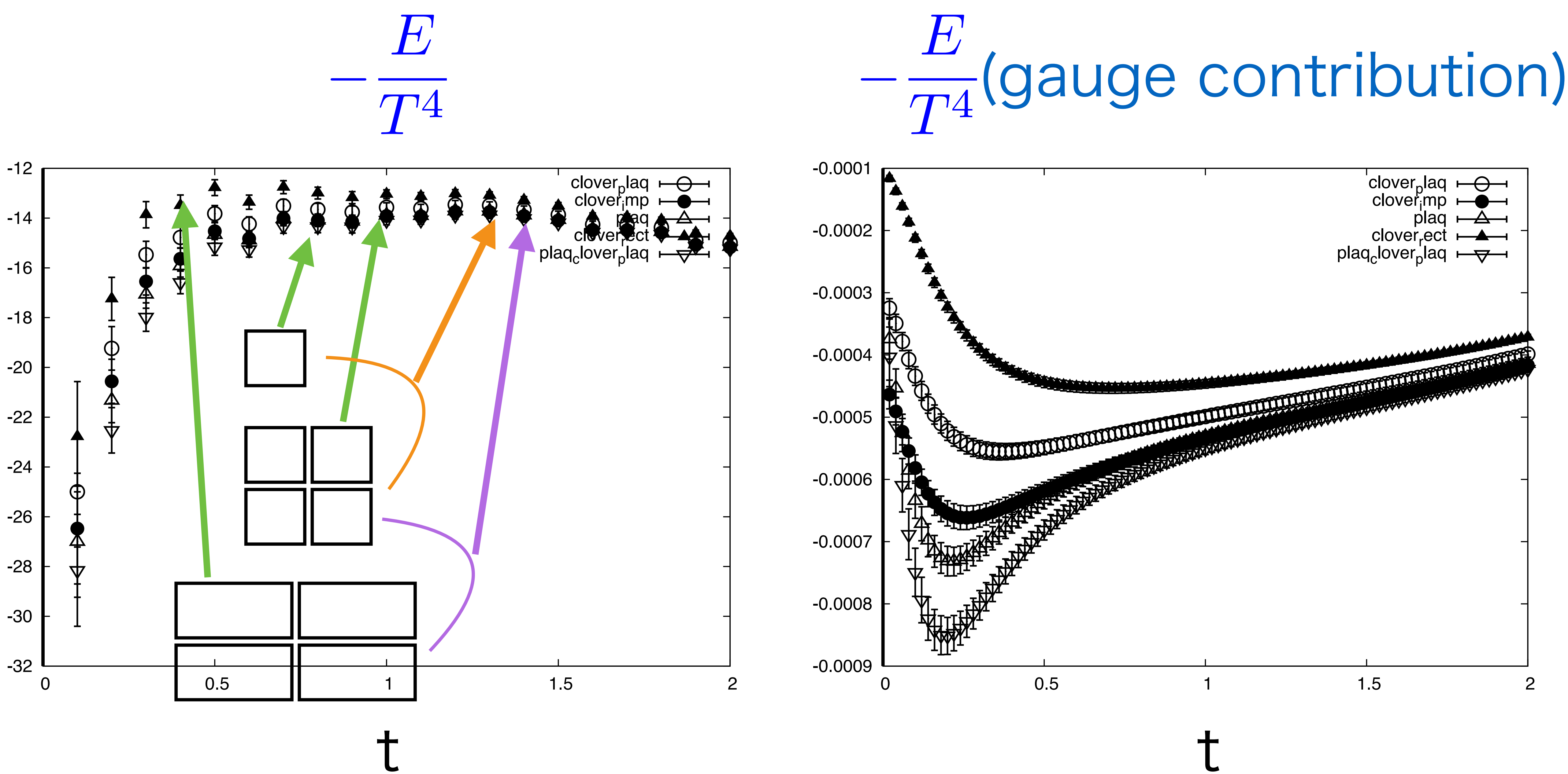
dependence on lattice operators



Energy and Pressure (preliminary)

T=279MeV

dependence on lattice operators



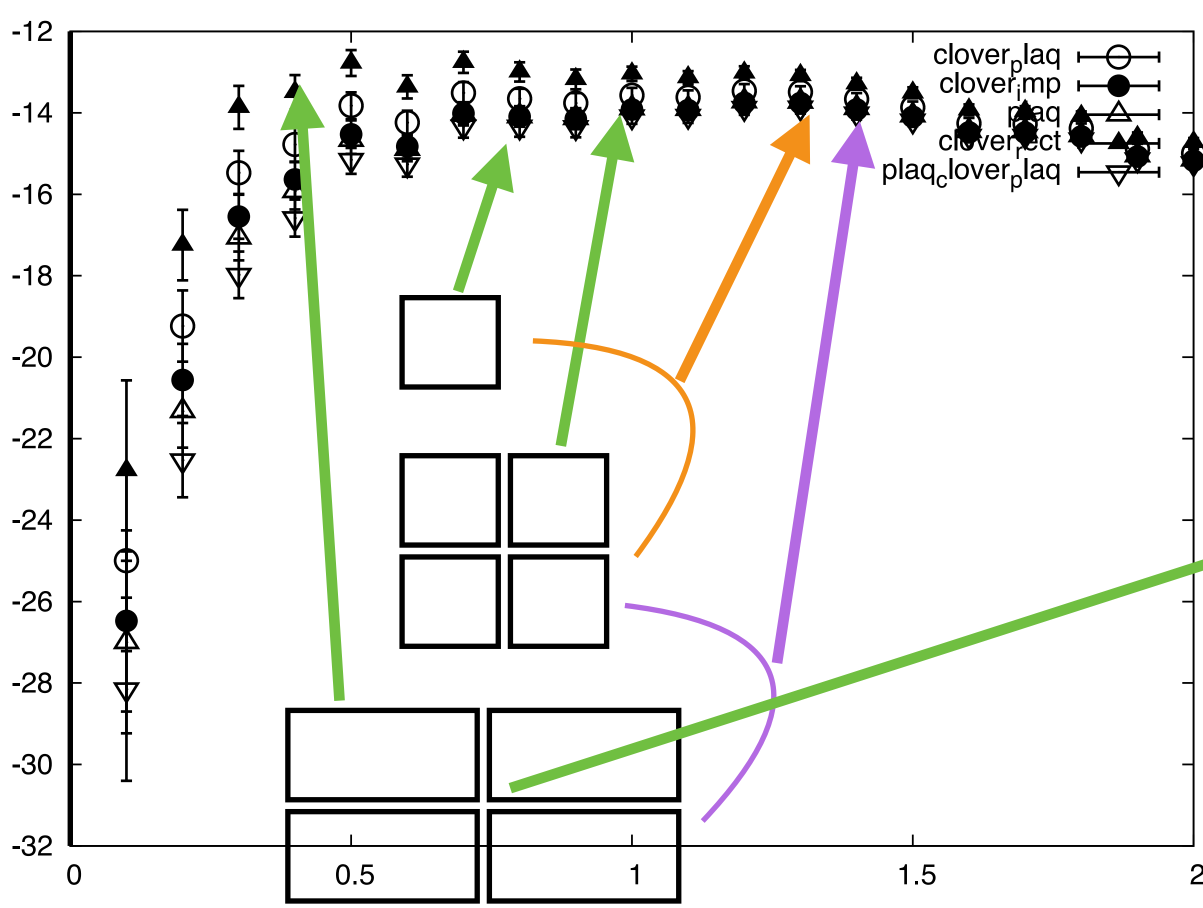
Energy and Pressure (preliminary)

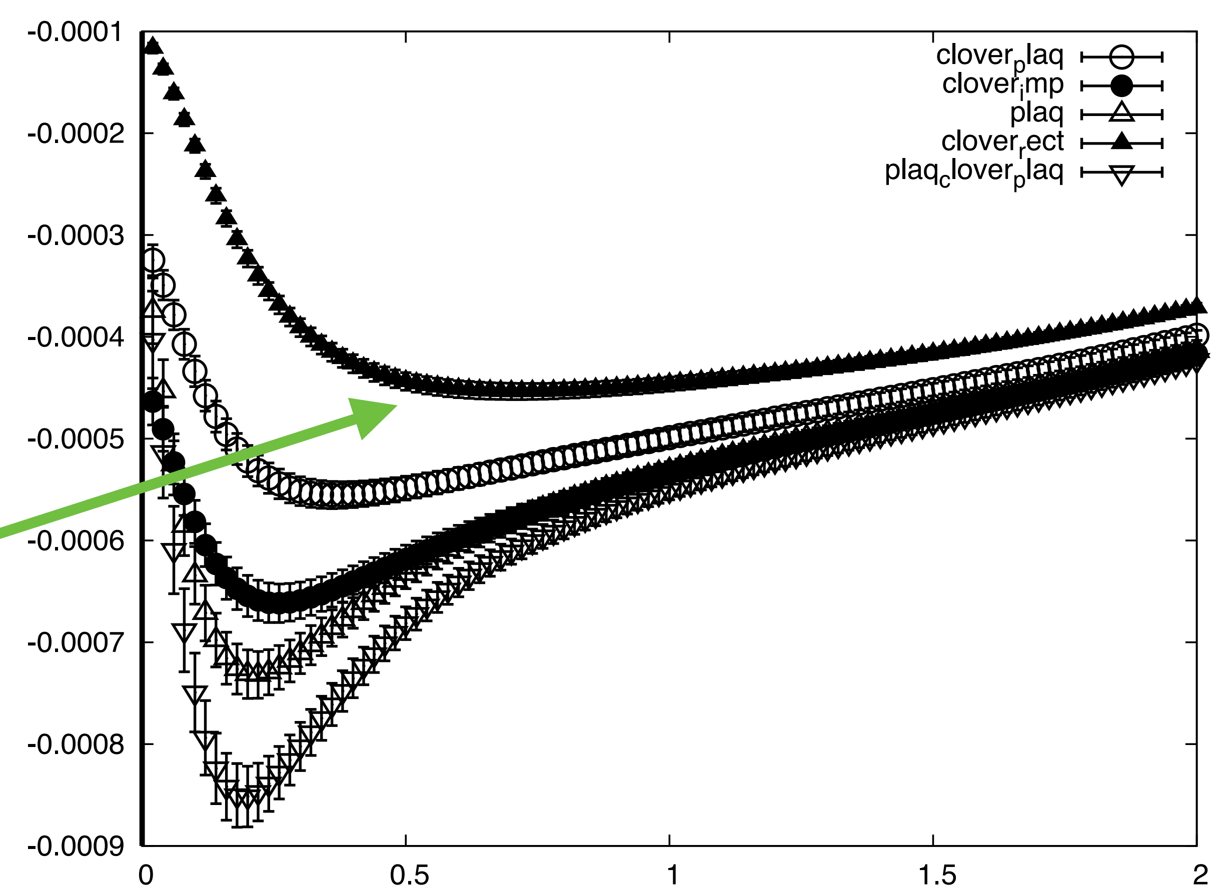
T=279MeV

dependence on lattice operators

$-\frac{E}{T^4}$

$-\frac{E}{T^4}$(gauge contribution)



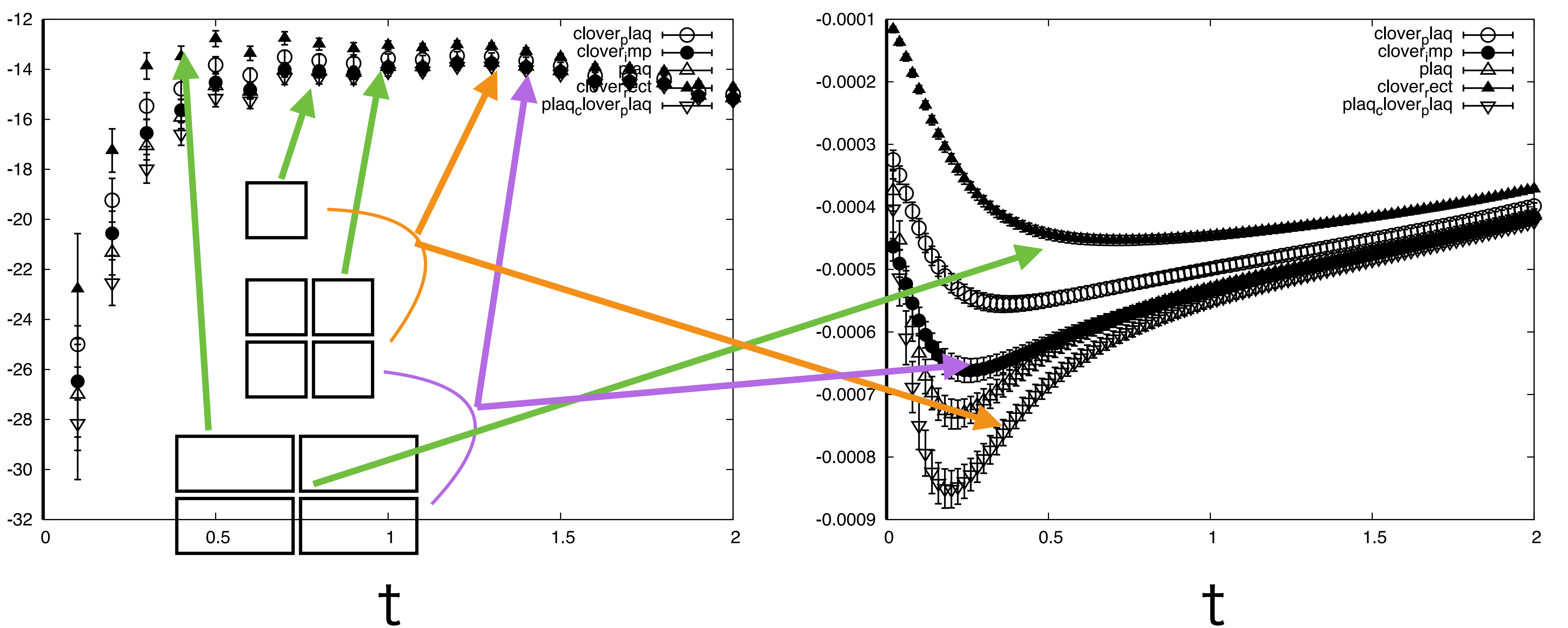


Energy and Pressure (preliminary)

T=279MeV

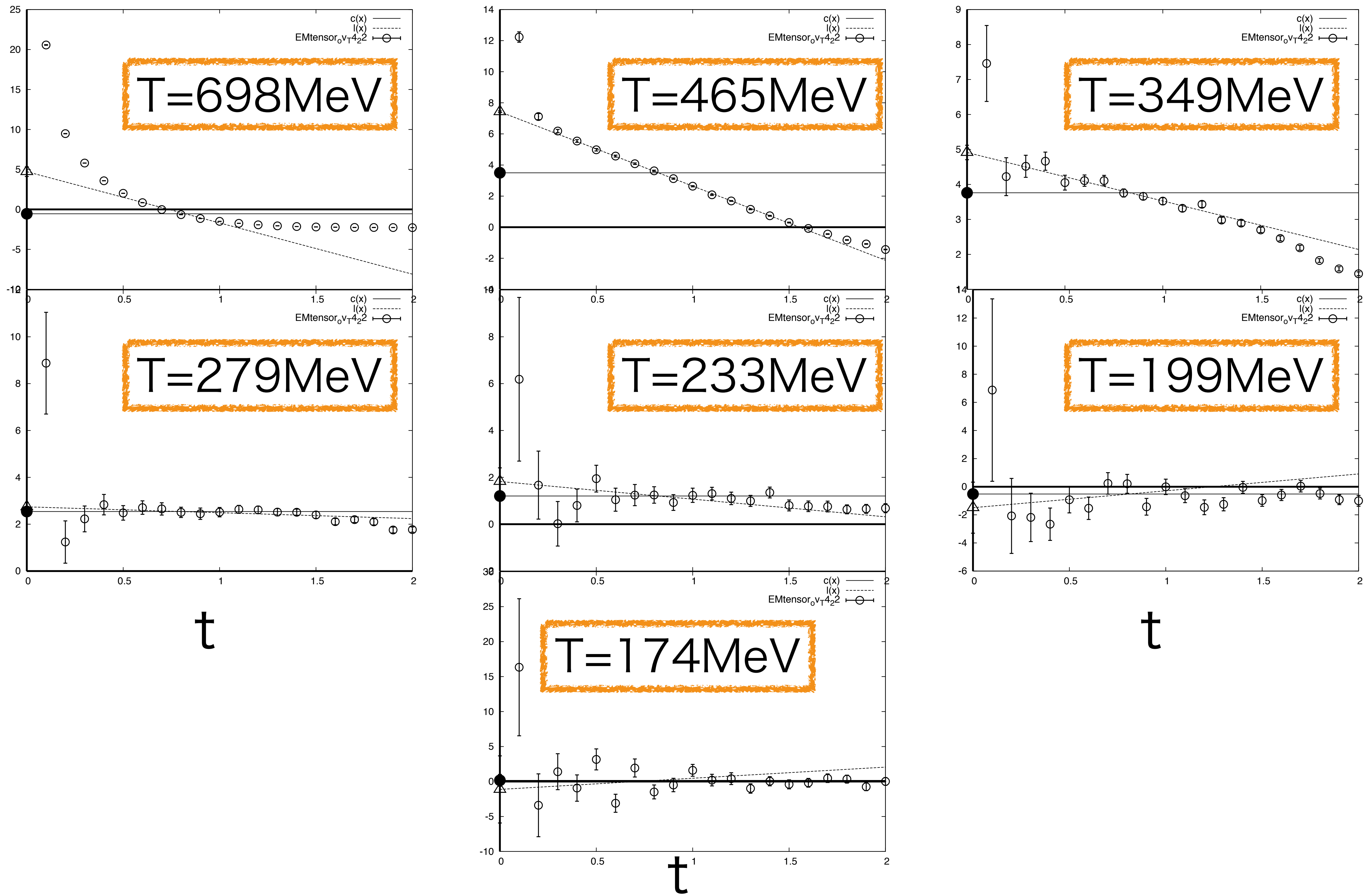
dependence on lattice operators

$$-\frac{E}{T^4} \qquad\qquad -\frac{E}{T^4}\text{(gauge contribution)}$$



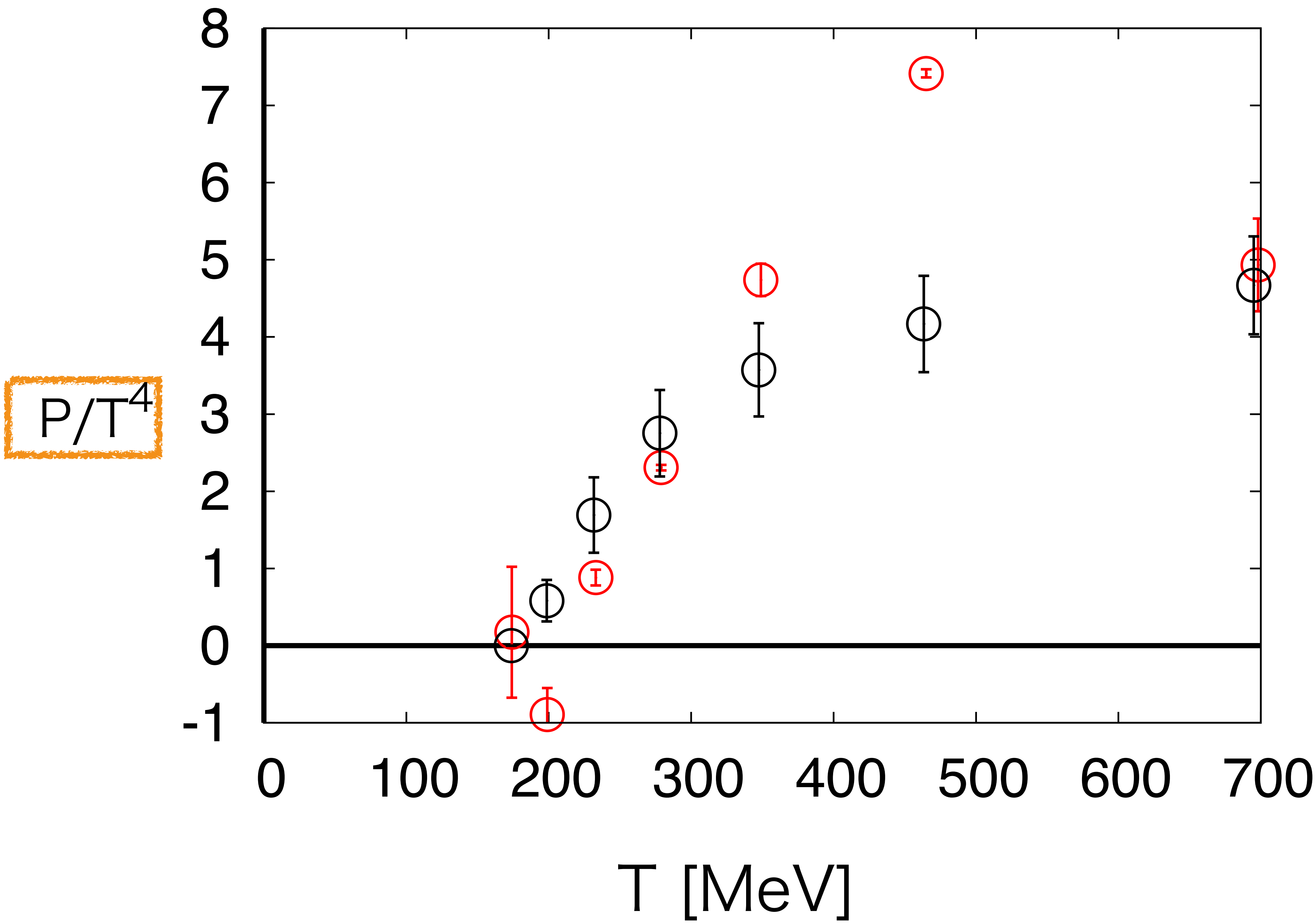
Pressure (preliminary)

small t limit



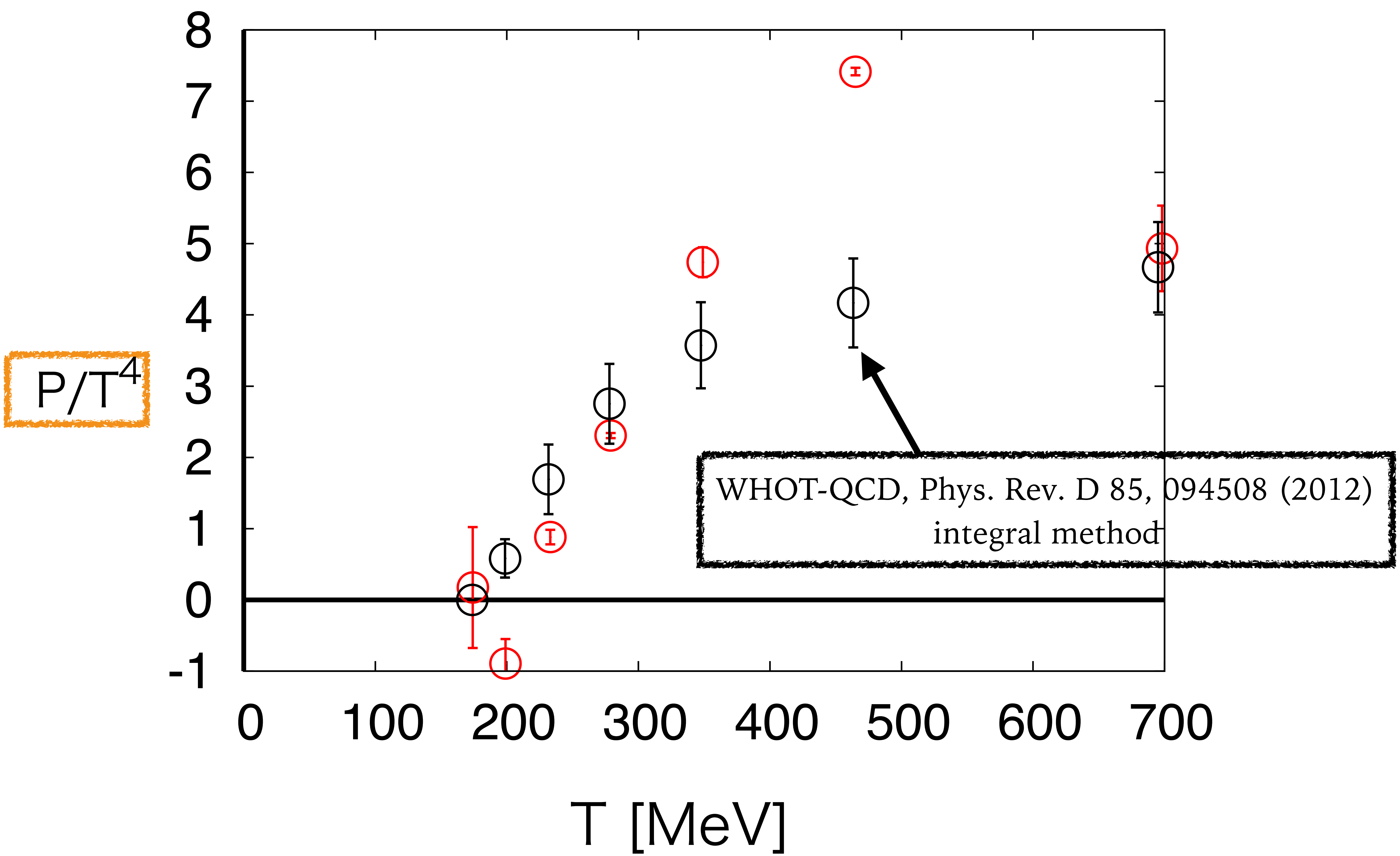
Pressure (preliminary)

as a function of T

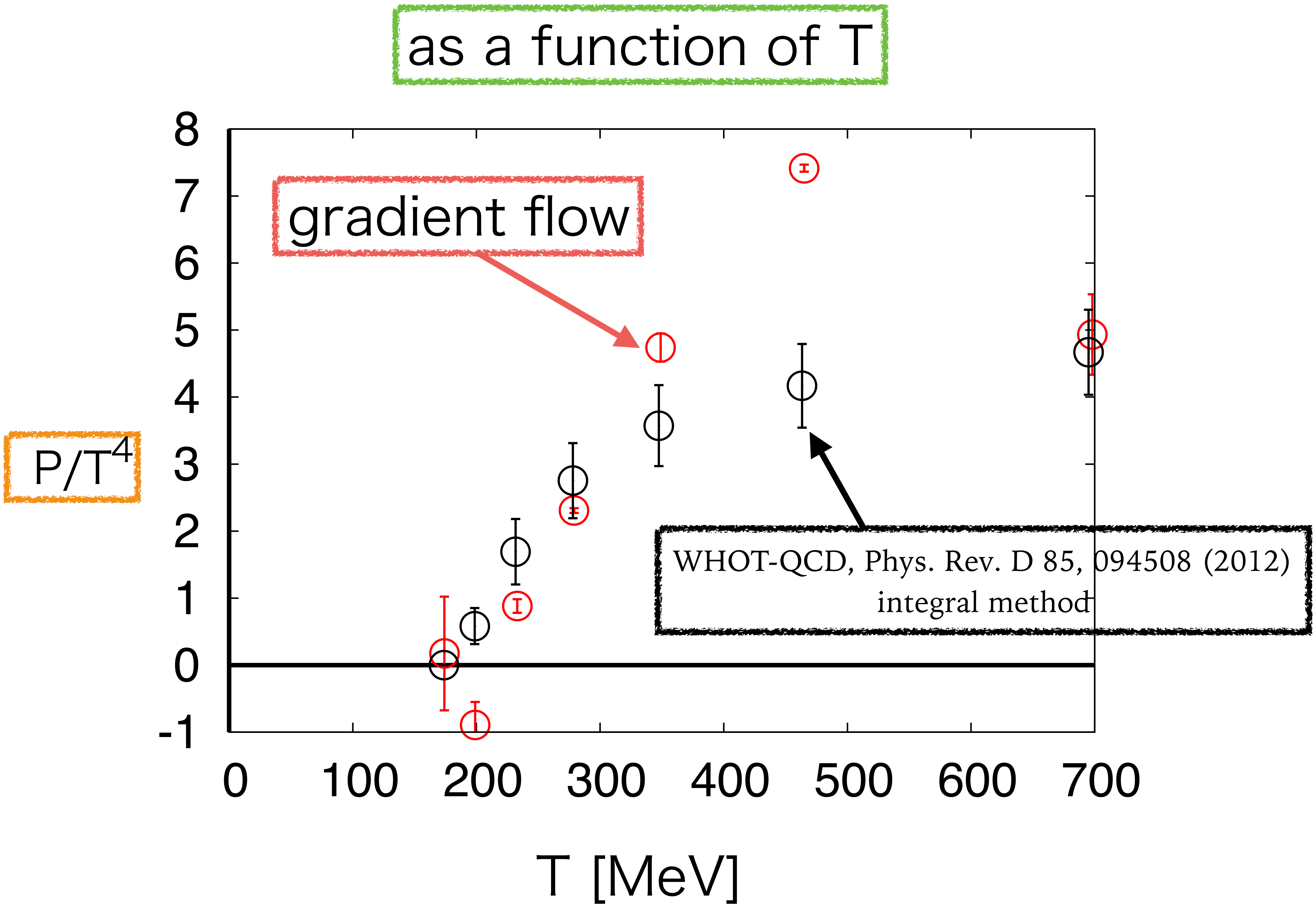


Pressure (preliminary)

as a function of T

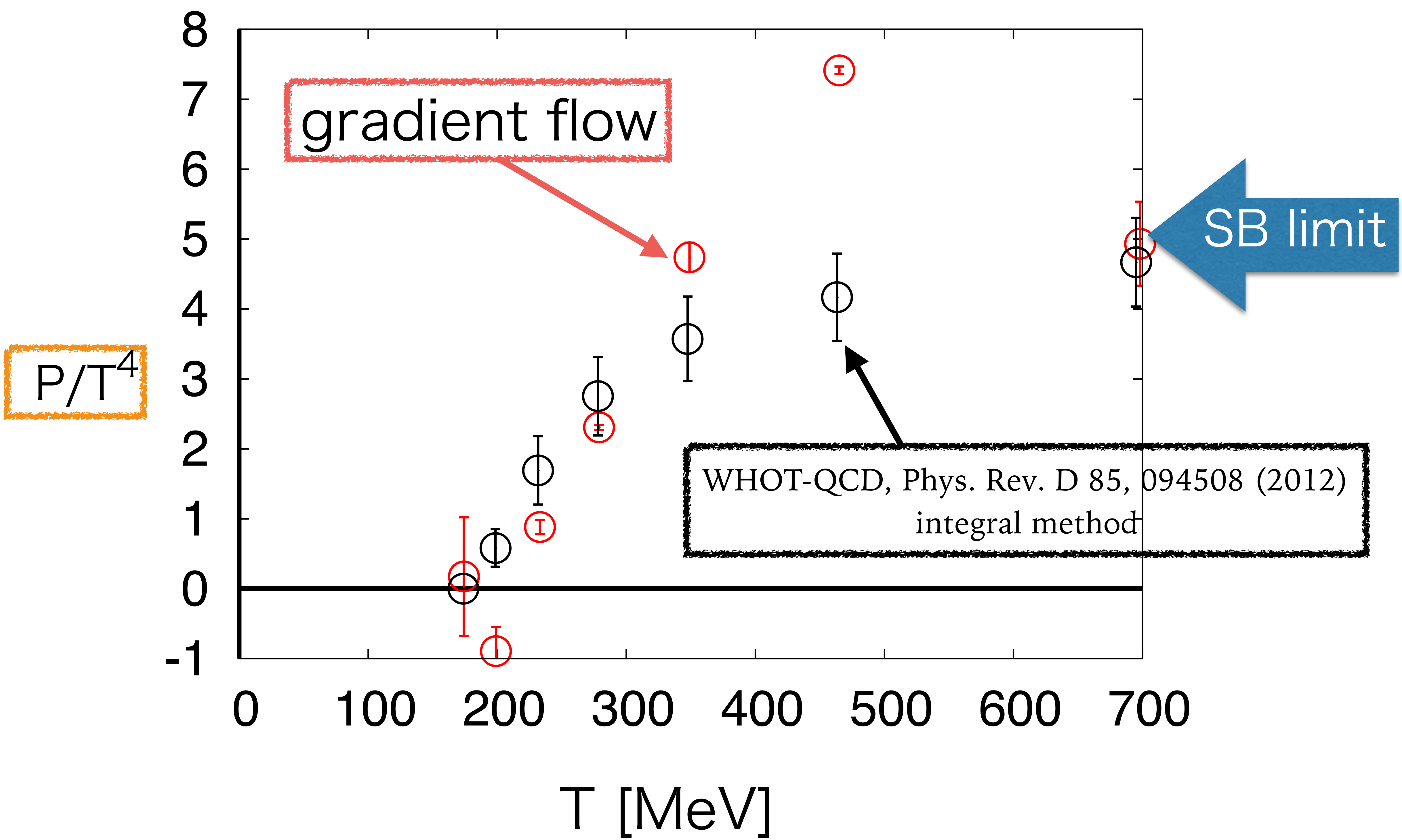


Pressure (preliminary)



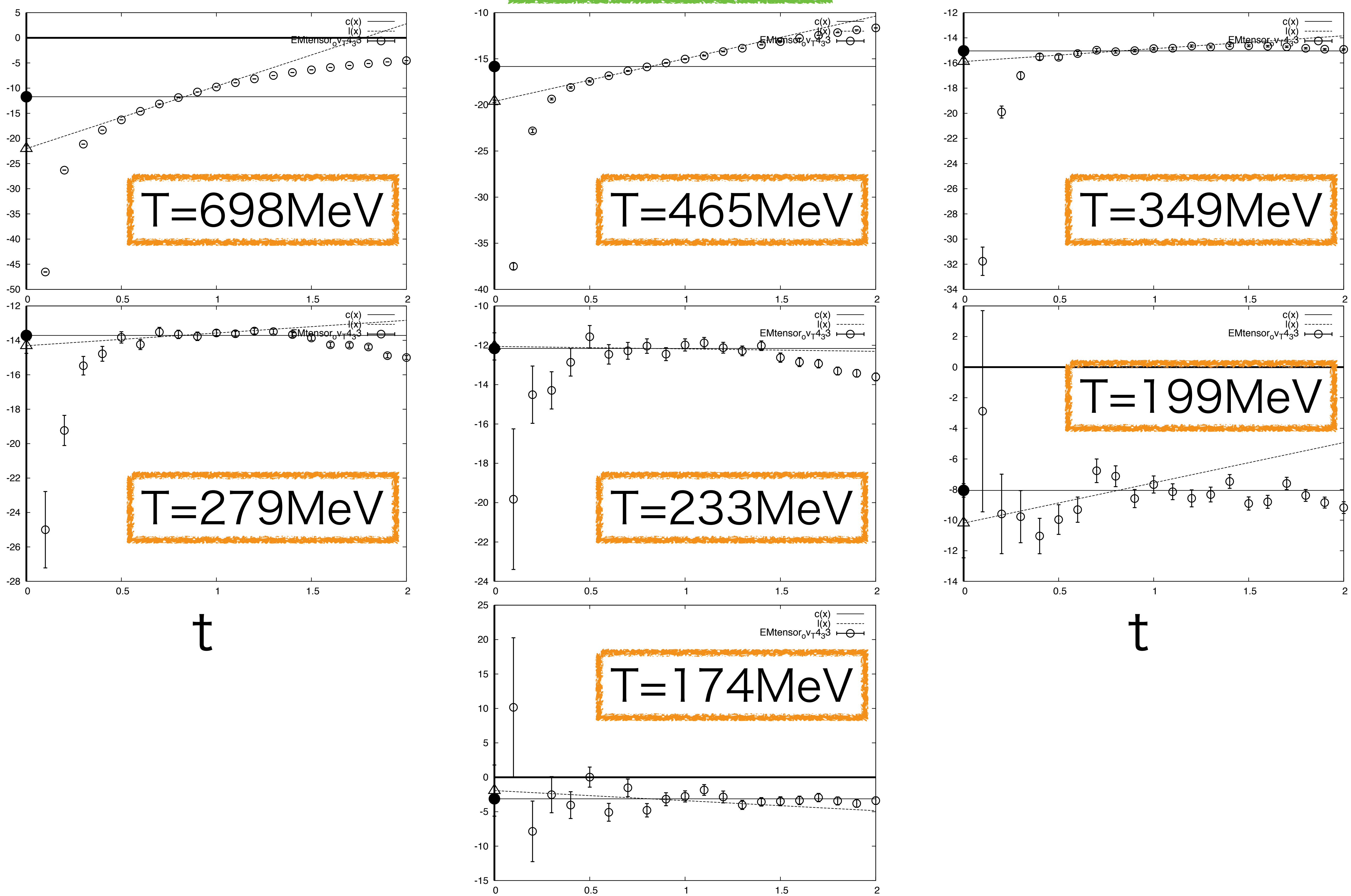
Pressure (preliminary)

as a function of T



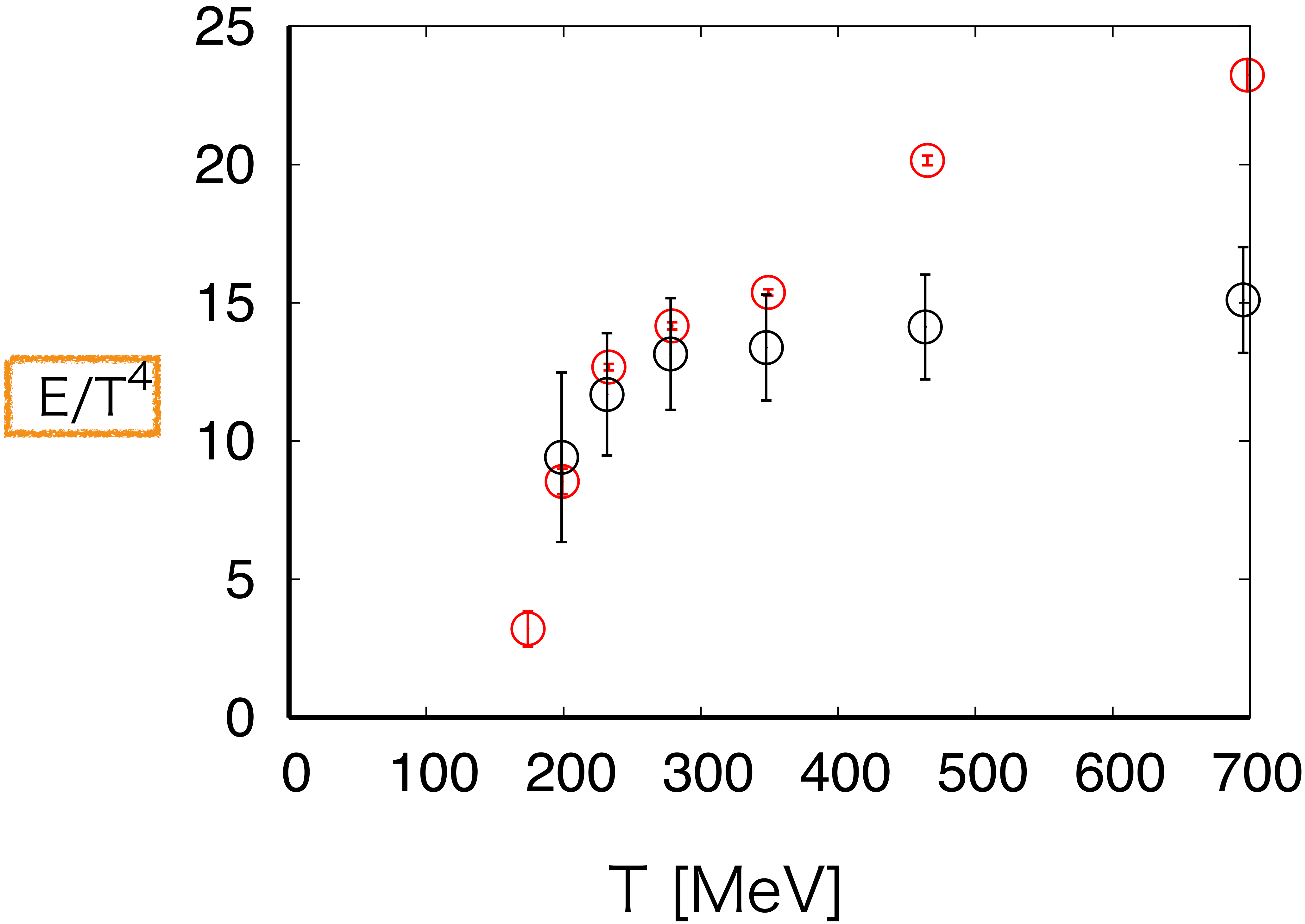
Energy (preliminary)

small t limit



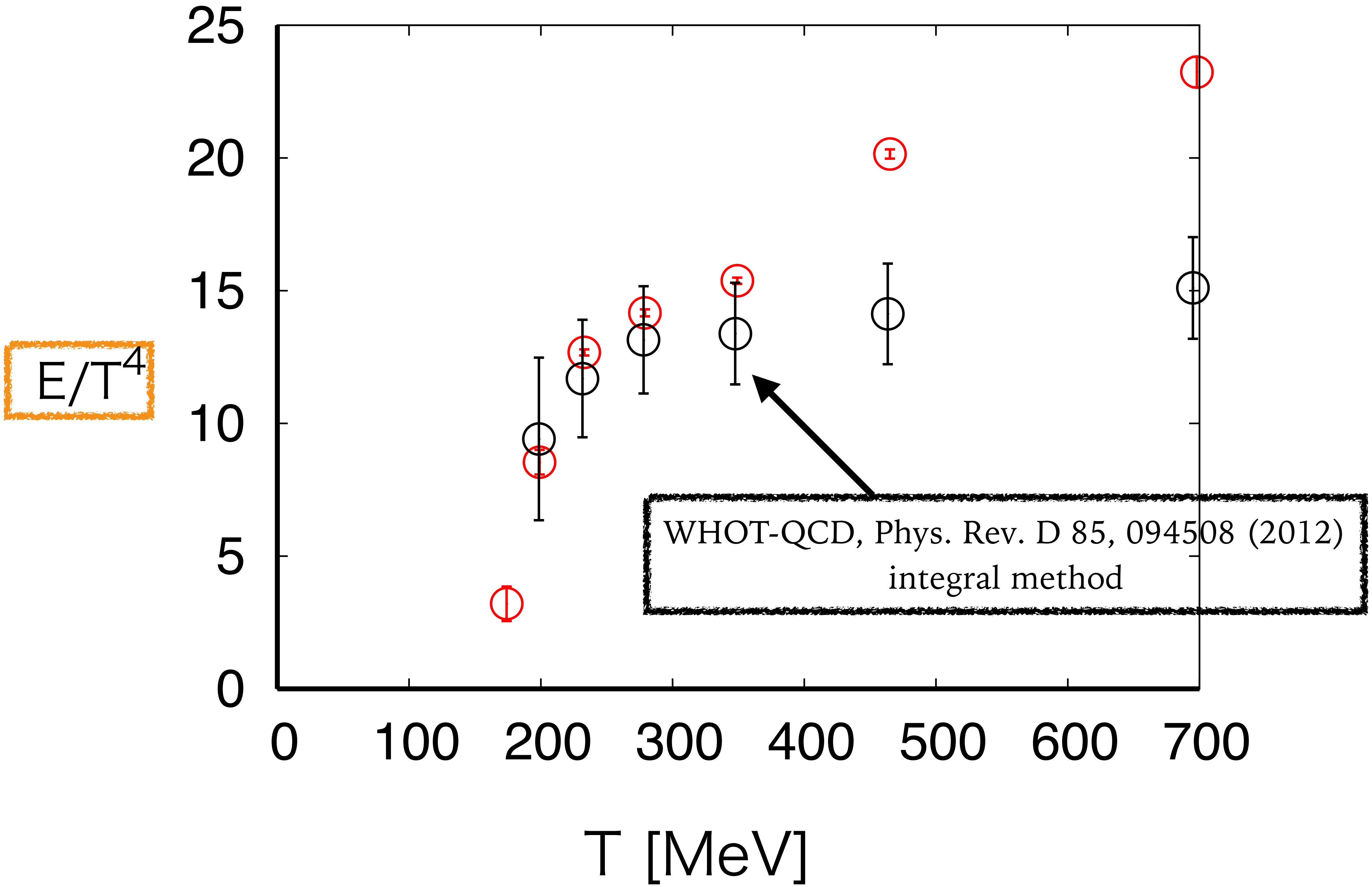
Energy (preliminary)

as a function of T

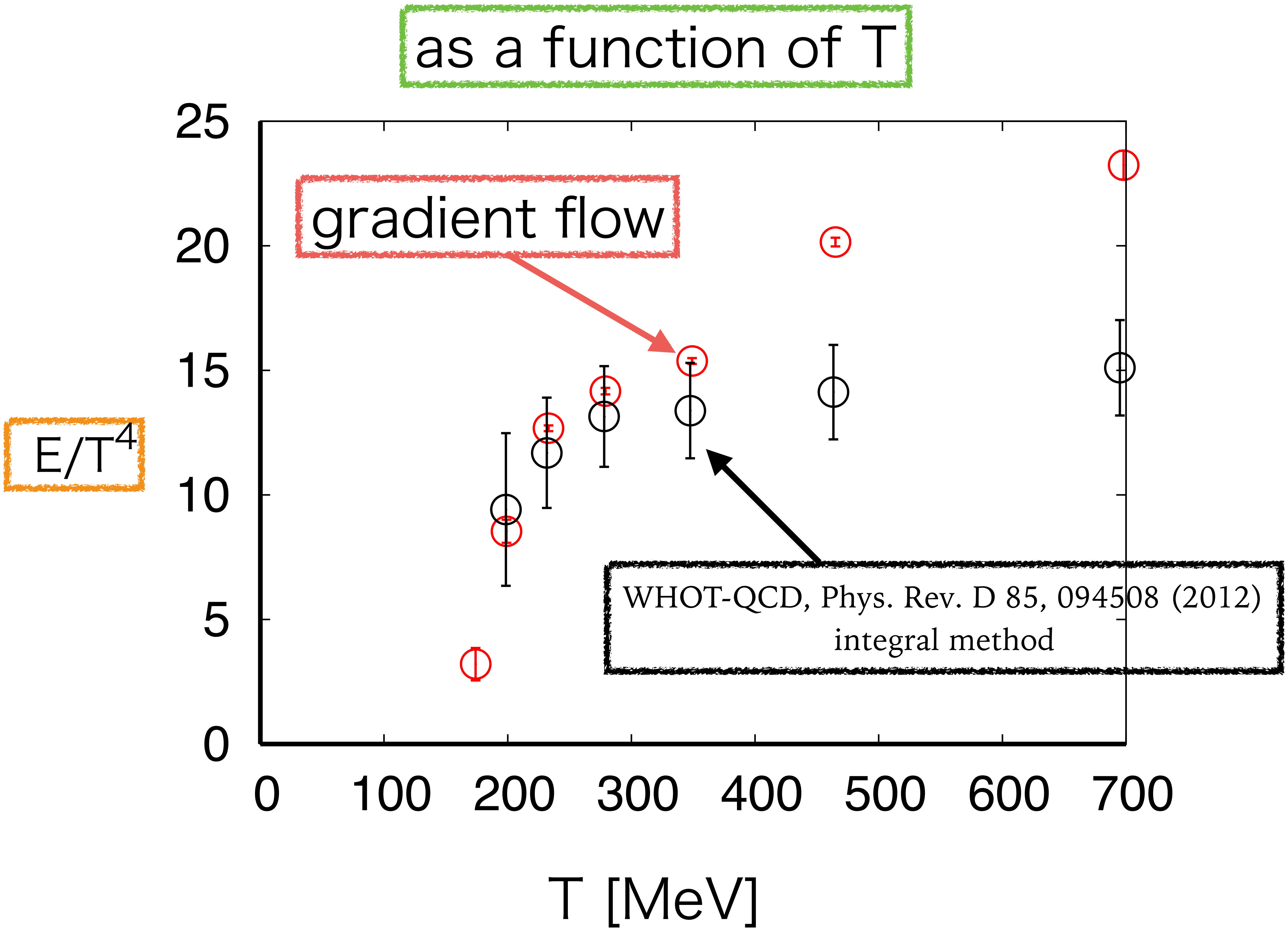


Energy (preliminary)

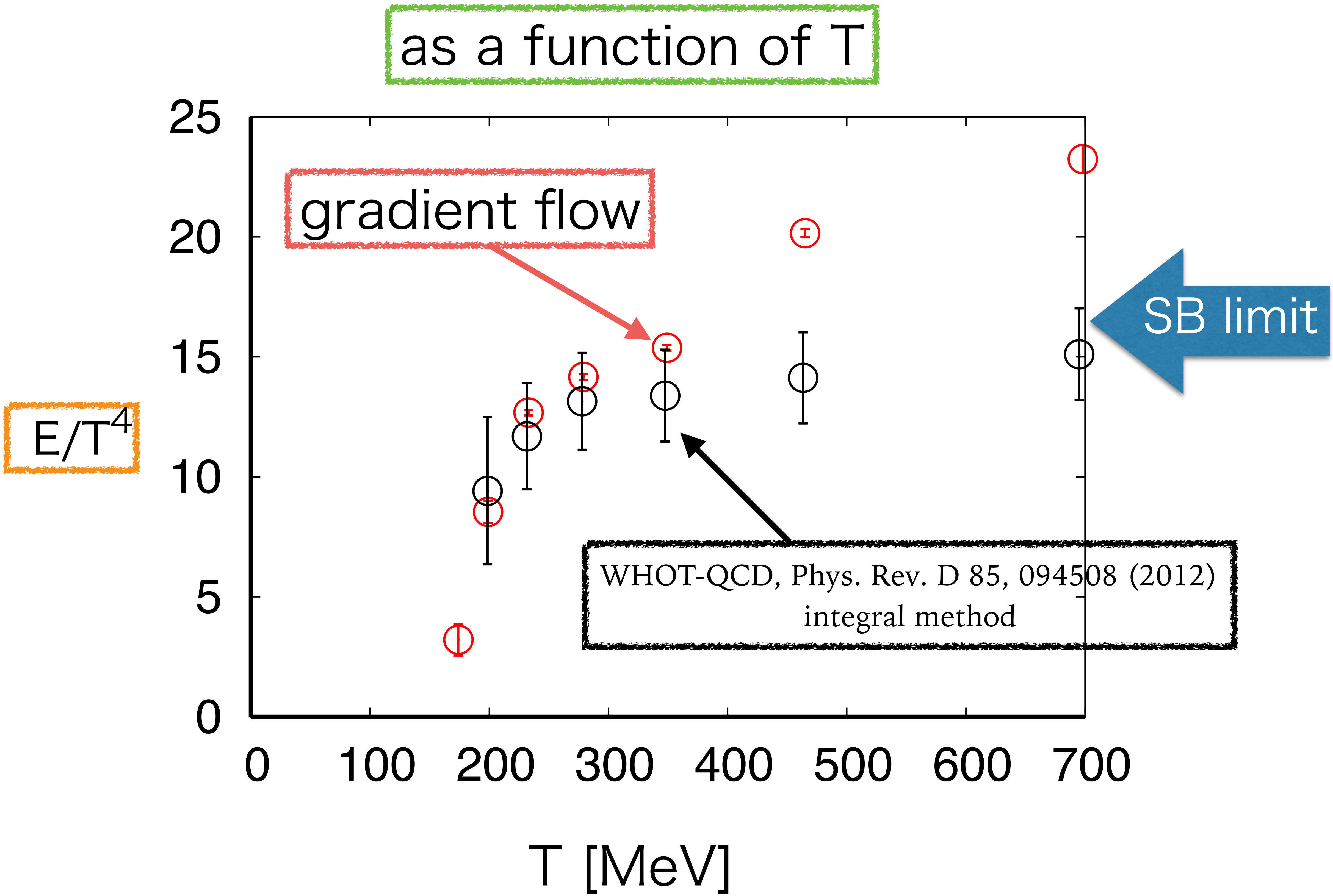
as a function of T



Energy (preliminary)

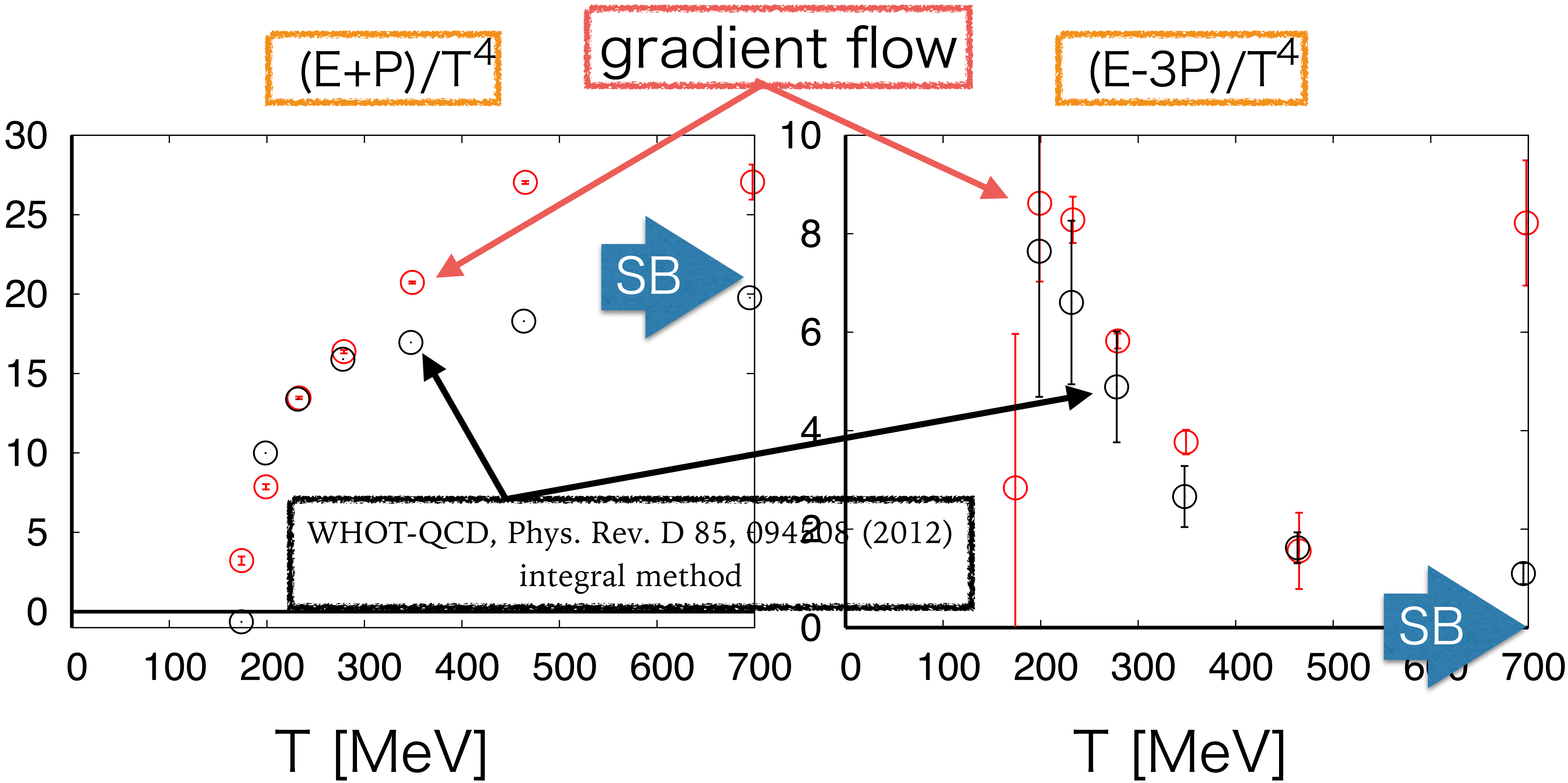


Energy (preliminary)

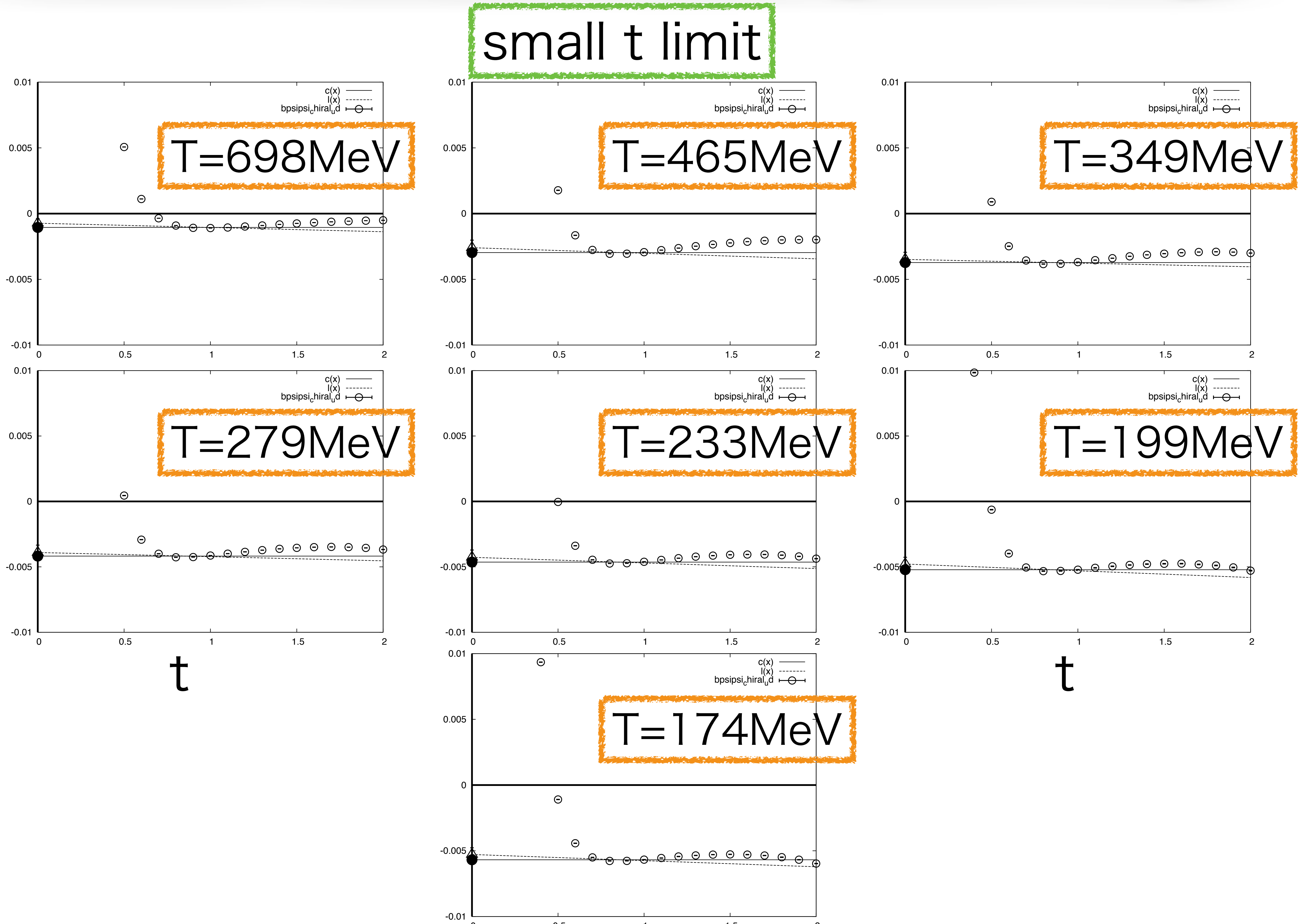


(E+P) and (E-3P) (preliminary)

as a function of T

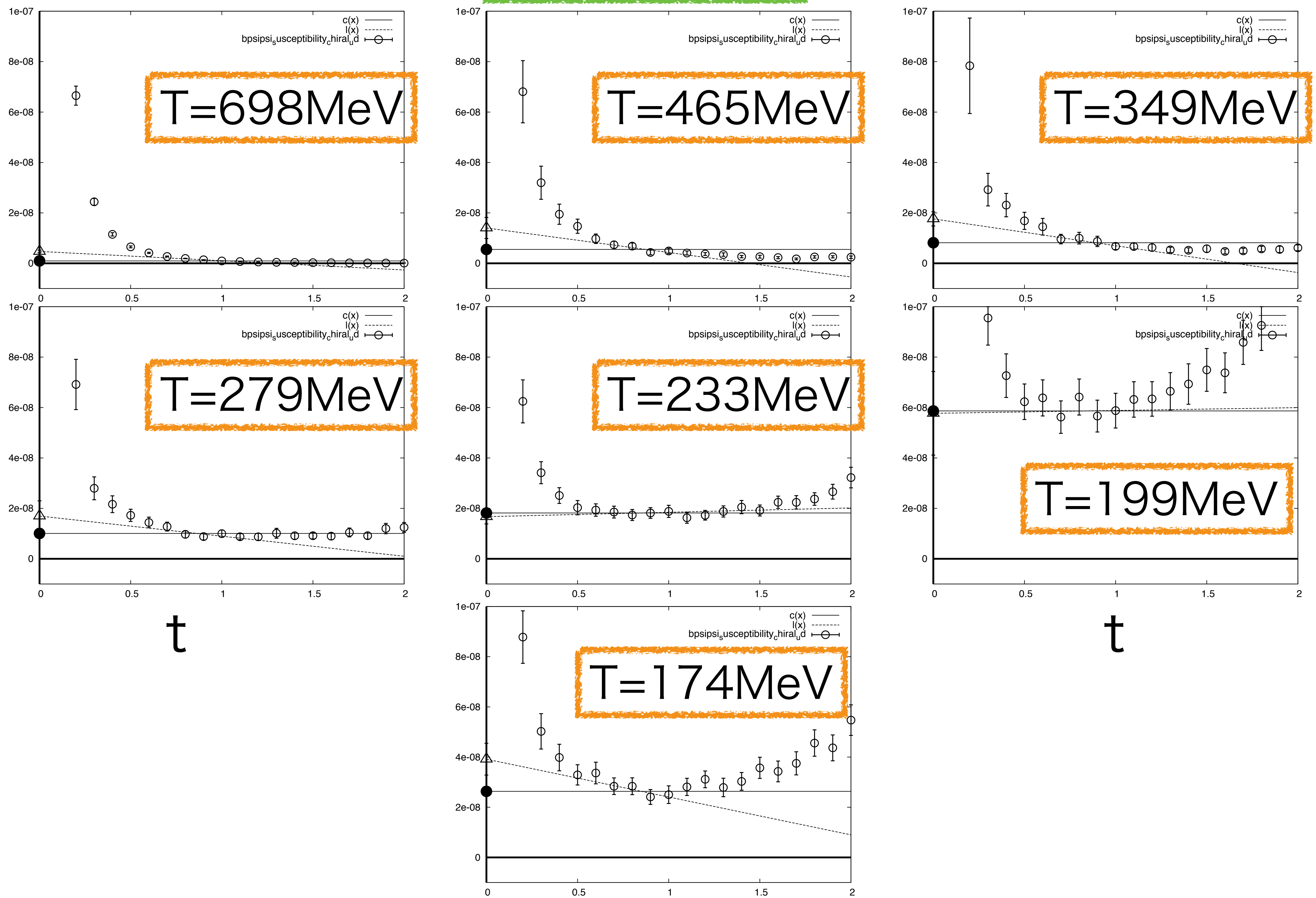


Chiral condensate (preliminary)



Chiral susceptibility (disconnected)

small t limit

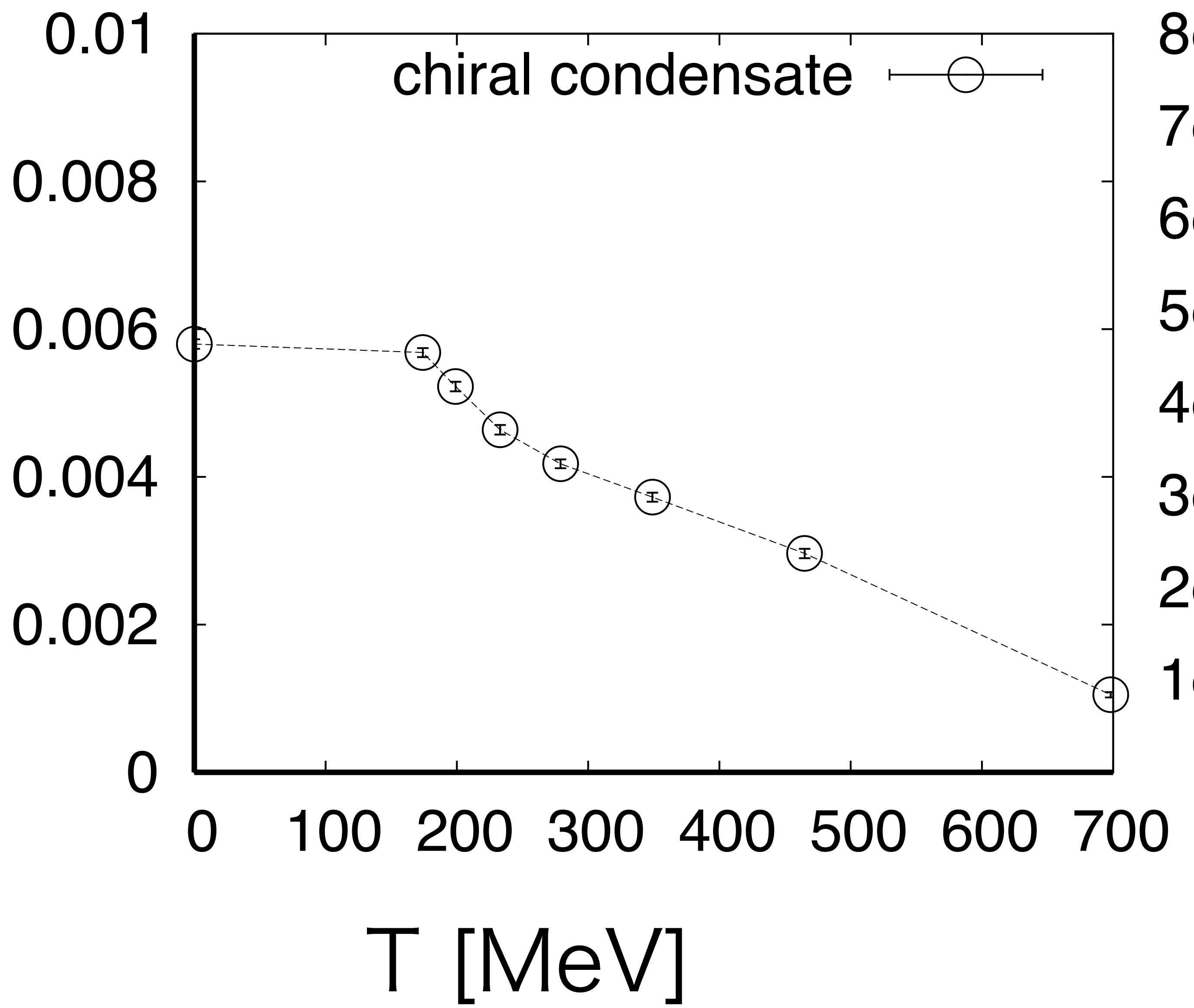


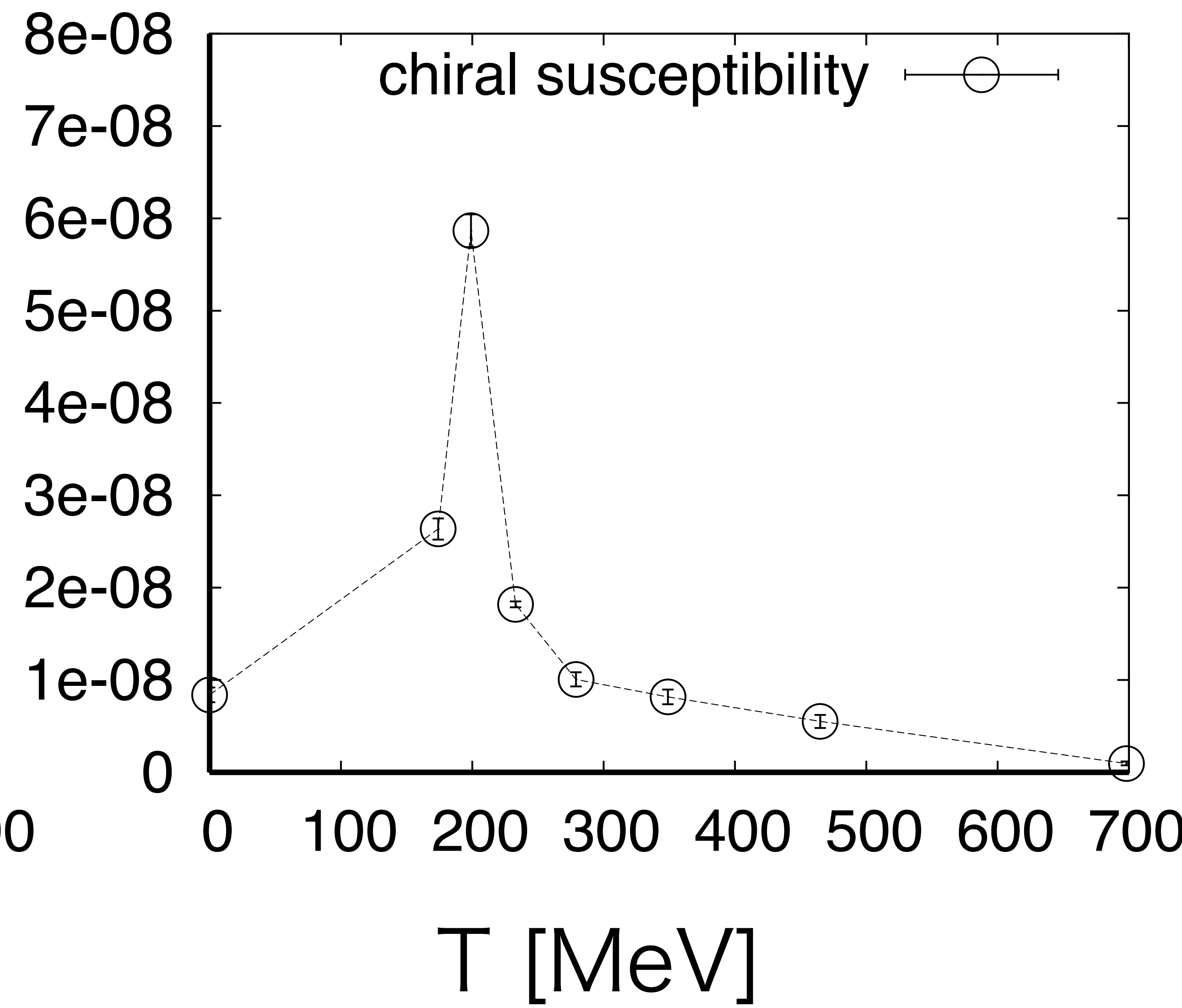
Chiral condensate (preliminary)

as a function of T

chiral condensate

chiral susceptibility
(disconnected)

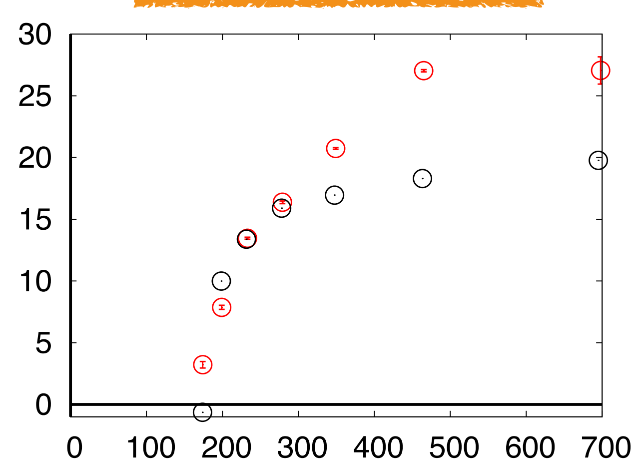




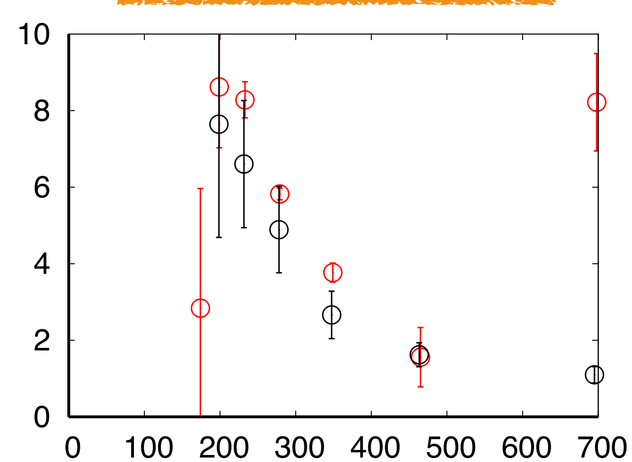
Summary

- Flow method works well for EM tensor!
  - as powerful as the derivative method.
- More suitable for Wilson fermion.
- We have exciting results:

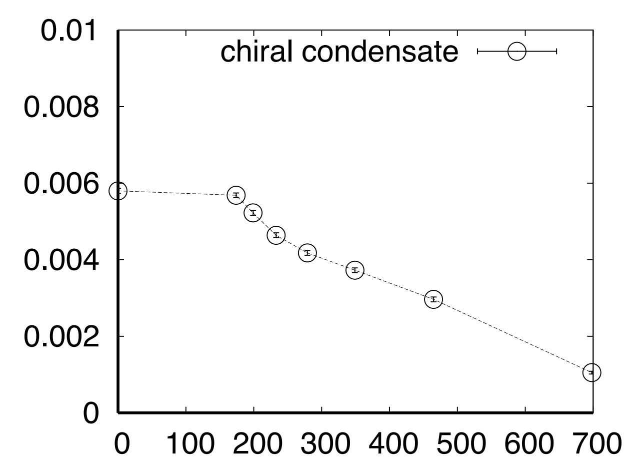
(E+P)/T^4



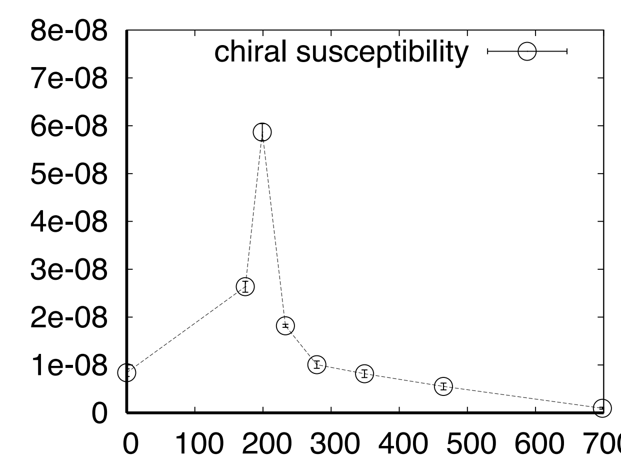
(E-3P)/T^4



chiral condensate



chiral susceptibility



- Lattice artifact is severe for Nt=4, 6
- We want work with fluctuation and correlator using the flow!