Finite Volume Corrections to Electromagnetic Masses

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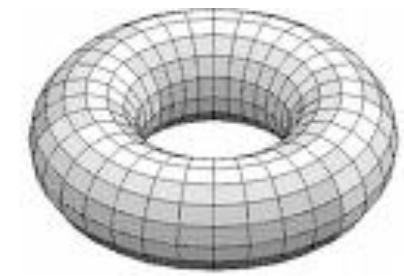
Finite Volume Corrections to Electromagnetic Masses

Outline

- **QCD+QED** computations continue to make impressive progress
- Long-range nature of photon addressed with various approaches
- *Effective Field Theory:* flexible, systematic framework for computing finite volume effects

QED $\rightarrow$ **NRQED** $\rightarrow$ **NRQHED**

QED on a Spatial Torus



- Well-known IR trouble: Gauss and Ampere Laws, …

- What should be an accumulation of soft photons is an isolated singularity

$$\int \frac{d^3\vec{q}}{2\omega_{\vec{q}}} f_+(\vec{q}) = \int_0^\infty 2\pi|\vec{q}|d|\vec{q}|f_+(\vec{q}) \longrightarrow \frac{1}{L^3}\left[\frac{f_+(0)}{2\sqrt{\vec{0}^2}} + \sum_{\vec{n}\neq\vec{0}} \frac{f_+(\vec{n})}{2\omega_{\vec{n}}}\right]$$

- An approach: ***QED(L)*** quenching photon zero modes
  - Isolated poles, all gapped $\frac{2\pi}{L}\vec{n}$
  - non-locality (mild, assuming perturbative QED physics)
    - (unknown, non-perturbative, …)
  - Modified long-range behavior can be accounted for systematically

BMW Collaboration, *Science* 347 (2015) 1452-1455
BMW Collaboration, *Phys.Lett. B755 (2016) 245-248*
Davoudi & Savage, *Phys.Rev. D90 (2014) 5, 054503*
Lee & BCT, *Phys.Rev. D*93 (2016) 3, 034012

Power-Law Volume Effects: QED(L)

- Test Case: electron electromagnetic mass

$$\Delta m_e = \alpha\, m_e \left[\frac{c_2}{2\pi}\xi + c_1 \xi^2 - 3\pi c_0 \xi^3 - \frac{12}{\sqrt{2\pi}} \xi^{3/2} e^{-1/\xi} + \cdots \right]$$

$$\xi = \frac{1}{m_e L}$$

power-law scaling

... further exponentials

BMW Collaboration, *Science* 347 (2015) 1452-1455
BMW Collaboration, *Phys.Lett. B755 (2016) 245-248*

- Shape coefficients: Lüscher, Hassenfratz & Leutwyler

$$c_j = \sum_{\vec{n}\neq\vec{0}} \frac{1}{|\vec{n}\,|^j} - \int \frac{d\vec{n}}{|\vec{n}\,|^j}$$

$$c_0 = -1$$

entirely from quenching zero mode

- Power-Law follow perturbative expansion in Compton wavelength

QED $\Rightarrow$ **NRQED**

Power-Law Volume Effects: NRQED(L)

- Test Case: electron electromagnetic mass

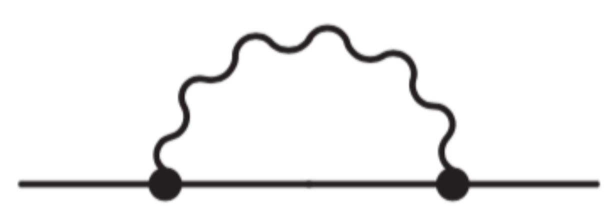
$$\Delta m_e = \alpha\, m_e \left[\frac{c_2}{2\pi}\xi + c_1 \xi^2 - \boxed{3\pi c_0 \xi^3} + \cdots\right]$$

power-law

$$\xi = \frac{1}{m_e L}$$

BMW Collaboration, *Science* 347 (2015) 1452-1455
BMW Collaboration, *Phys.Lett. B755 (2016) 245-248*

$$\boxed{-\frac{3\pi}{2} c_0 \xi^3}$$



QED → **NRQED**

$$\mathcal{L} = \psi^\dagger \left[iD_0 + \frac{\mathbf{D^2}}{2m_e} + e\frac{\sigma\cdot\mathbf{B}}{2m_e} + e\frac{[\nabla\cdot\mathbf{E}]}{8m_e^2} \right]\psi$$

Power-Law Volume Effects: NRQED(L)

- Test Case: electron electromagnetic mass

$$\Delta m_e = \alpha\, m_e \left[\frac{c_2}{2\pi}\xi + c_1\xi^2 - 3\pi c_0\xi^3 + \cdots\right]$$

power-law

$$\xi = \frac{1}{m_e L}$$

BMW Collaboration, *Science* 347 (2015) 1452-1455
BMW Collaboration, *Phys.Lett. B755 (2016) 245-248*

$$-\frac{3\pi}{2}c_0\xi^3 \qquad -\frac{3\pi}{2}c_0\xi^3$$

$$\mathcal{L} = \chi^\dagger i D_0 \chi - \frac{\pi\alpha}{m_e^2}\left(\psi^\dagger\sigma\chi\right)\cdot\left(\chi^\dagger\sigma\psi\right)$$

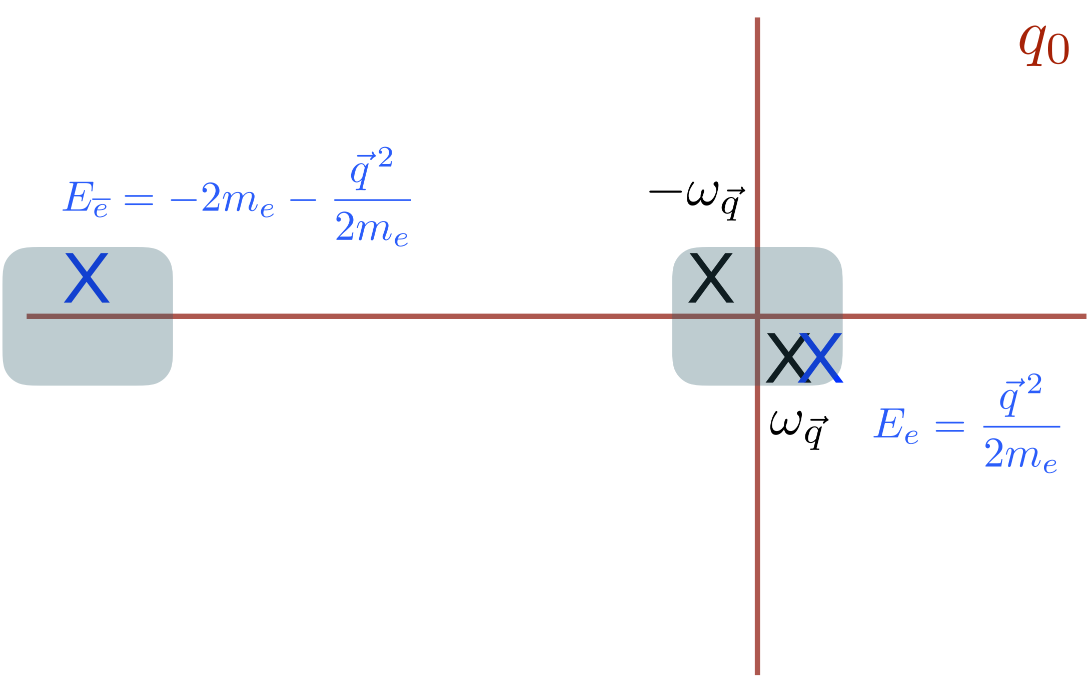
QED → **NRQED**

$$\mathcal{L} = \psi^\dagger\left[iD_0 + \frac{\mathbf{D^2}}{2m_e} + e\frac{\sigma\cdot\mathbf{B}}{2m_e} + e\frac{[\nabla\cdot\mathbf{E}]}{8m_e^2}\right]\psi$$

Power-Law Volume Effects: NRQED(L)

- Test Case: positrons in electron electromagnetic mass

$$\Delta m_e = \alpha\, m_e \left[\frac{c_2}{2\pi}\xi + c_1 \xi^2 - 3\pi c_0 \xi^3 + \cdots \right] \qquad \xi = \frac{1}{m_e L}$$



- High-energy region: positron pole

$$\frac{1}{q^2 + i\varepsilon} \rightarrow \frac{1}{4m_e^2}$$

- Positrons always contribute (UV div)
- Finite contribution in FV

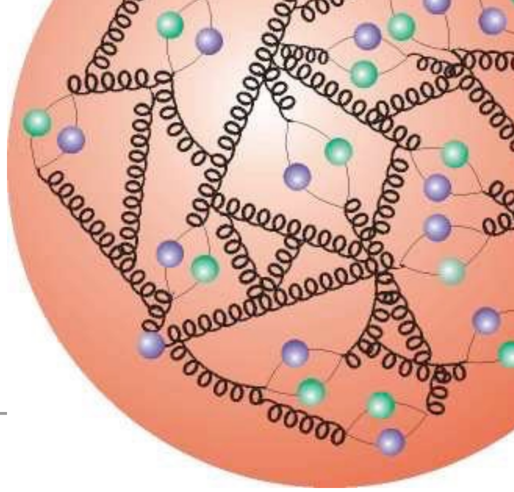
$$-\frac{3\pi}{2} c_0 \xi^3$$

part of quenching zero mode

QED + method of regions

Derive effective theory

Lee & BCT, *Phys.Rev. D*93 (2016) 3, 034012



Power-Law Volume Effects: NRQHED(L)

- Hadron Electrodynamics: emergence of new length scales

$$\mathcal{L} = \psi^\dagger \left[iD_0 + \frac{\mathbf{D^2}}{2M} + ec_F \frac{\sigma \cdot \mathbf{B}}{2M} + ec_D \frac{[\nabla \cdot \mathbf{E}]}{8M^2} \right] \psi$$

$$c_F = Q + \kappa$$

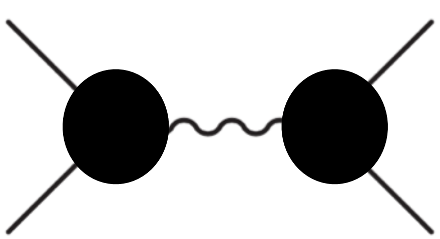
$$c_D = Q + \frac{4}{3} M^2 < r_E^2 >$$

e.g., Hill & Paz, *Phys.Rev.Lett.* 107 (2011) 160402

- Same power counting appears sufficient, even for light nuclei $\xi = \frac{1}{ML}$

NRQHED(L) **Davoudi & Savage, *Phys.Rev. D90 (2014) 5, 054503***

- Addendum: antihadron contributions **Lee & BCT, *Phys.Rev. D*93 (2016) 3, 034012**

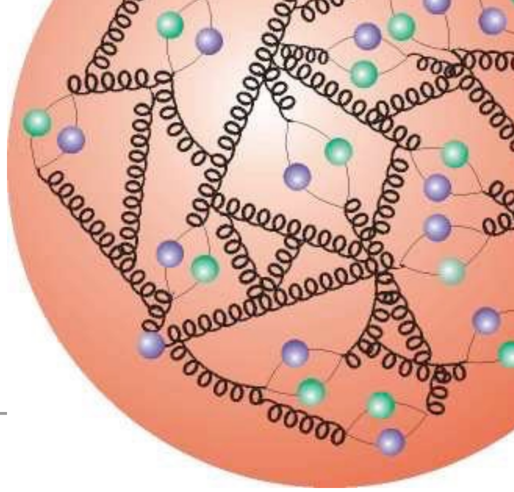


$$\frac{1}{q^2 + i\varepsilon} \to \frac{1}{4M^2}$$

Crazy, but, 30% determination would help phenomenology

Threshold expansion of time-like EM FFs

$$\Delta M_n = \frac{\pi\alpha}{M^2 L^3} \left(\kappa_n^2 + |G_M|^2_{s=4M^2} \right) - \frac{4\pi^2}{L^4} c_{-1}(\alpha_n + \beta_n) + \mathcal{O}(L^{-6})$$



Power-Law Volume Effects: NRQHED(θ)

- Another approach to QED in finite volume: random twist

Lehner & Izubuchi, *PoS LATTICE2014* (2015) 164

$$\psi(x + L_\mu) = e^{i\theta_\mu}\psi(x)$$

- Valence quark propagators computed from stochastic twist average

$$\mathcal{L} = \psi^\dagger \left[iD_0 + \frac{\mathbf{D^2}}{2M} + ec_F \frac{\sigma \cdot \mathbf{B}}{2M} + ec_D \frac{[\nabla \cdot \mathbf{E}]}{8M^2} \right] \psi$$

(Gluons + Sea @ finite volume)

- EFT: hadron source in infinite volume, finite volume dynamics affects couplings

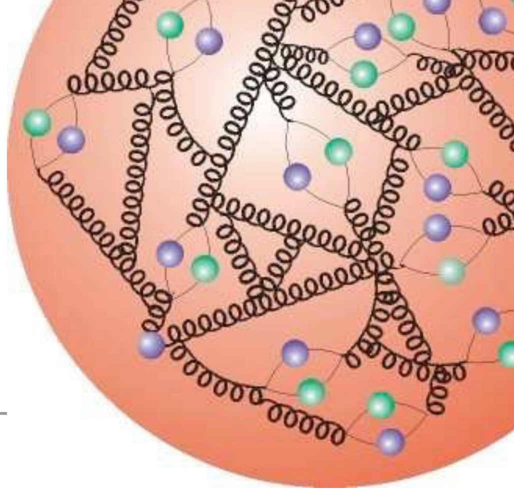
$$L_{\text{sea}} = L$$

$$L_{\text{val}} = NL \to \infty$$

$$\Delta c_F, \Delta c_D \sim e^{-m_\pi L}$$

- Expect only exponentially small volume effects

Need ***PQχPT***: one-pion loop + one-photon loop contributions to EM mass



Power-Law Volume Effects: NRQHED(B)

- Another approach to QED in finite volume: dynamical zero modes

Horsley (*QCDSF-UKQCD*), `arXiv:1508.06401`

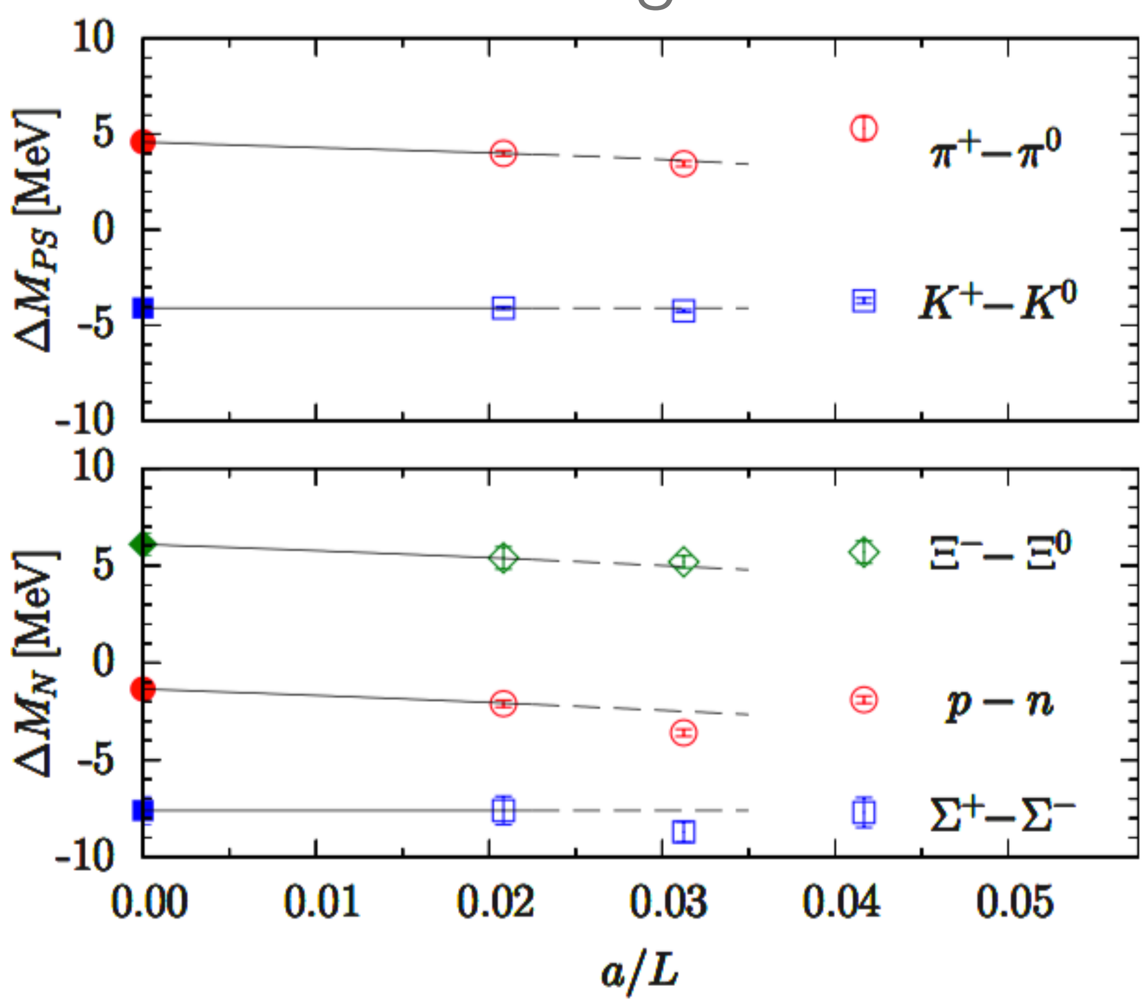
$$0 \leq \theta_\mu \equiv eLB_\mu < 2\pi$$

$$B_\mu = \frac{1}{V}\sum_x A_\mu(x)$$

Compare with zero-mode quenching: $\sum_{\mathbf{x}} A_\mu(\mathbf{x}, x_4) = 0$

- Non-zero average of uniform field & sharp

Göckeler, *et al.*, Nucl.Phys. B371 (1992), 713



$$eA_\mu(x)J_\mu(x) = e\left[B_\mu J_\mu(x) + \mathcal{A}_\mu(x)J_\mu(x)\right]$$

Charged hadron masses

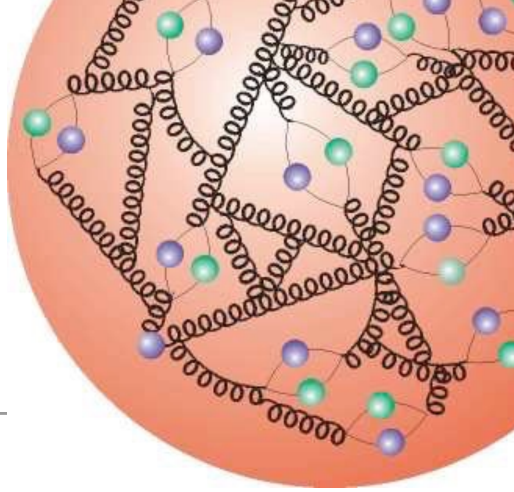
$$M(\mathbf{B}) = M + \frac{e^2\mathbf{B}^2}{2M} + \Delta M(L)$$

from NRQHED(L)

Davoudi & Savage, *PRD90 (2014)*

… work in progress NRQHED(B)

Volume Effects: NRQHED(m_γ)



- Another approach to QED in finite volume: screened photon

Endres (*Collaboration 51*), `arXiv:1507.08916`

- Enables investigation of IR behavior of QED on *fixed-size* lattice $m_\gamma L$
- No quenching of zero modes necessary

$$(m_\gamma \to 0, L \to \infty) \neq (L \to \infty, m_\gamma \to 0) \qquad \varepsilon - \texttt{regime}$$

EFT can economically address long-distance physics

Charged hadron masses

$$M(m_\gamma, L) = M - \frac{1}{2}\alpha\, m_\gamma + 3\,\alpha \frac{e^{-m_\gamma L}}{L} + \cdots$$

$p - \texttt{regime}$ for ease

NRQHED + Higgs + FV

… but *EFT* requires separation of long-distance scales

$m_\gamma \ll 2m_\pi$ else explicit pions are needed …

Summary: Finite Volume Corrections in *QED*+QCD

- **EFT**s provide systematic framework to compute volume dependence
- **NRQHED** can be tailored to lattice *QCD+QED* methods

including C* BCs too…

- Applications beyond electromagnetic masses possible…

$$p + p \to p + p \qquad\qquad \langle H' | \mathcal{O} | H \rangle$$