Standard-model prediction for direct CP violation in kaon decays

Christopher Kelly
(RBC & UKQCD Collaboration)

Talk, LGT 2016, BNL, USA
March 10th 2015

RIKEN BNL
Research Center

BROOKHAVEN
NATIONAL LABORATORY

The RBC & UKQCD collaborations

BNL and RBRC

Tomomi Ishikawa
Taku Izubuchi
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Taichi Kawanai
Christopher Kelly
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

CERN

Marina Marinkovic

Columbia University

Ziyuan Bai
Norman Christ
Xu Feng
Luchang Jin
Bob Mawhinney
Greg McGlynn
David Murphy
Daiqian Zhang

University of Connecticut

Tom Blum

Edinburgh University

Peter Boyle
Luigi Del Debbio
Julien Frison
Richard Kenway
Ava Khamseh
Brian Pendleton
Oliver Witzel
Azusa Yamaguchi

Plymouth University

Nicolas Garron

University of Southampton

Jonathan Flynn
Tadeusz Janowski
Andreas Juettner
Andrew Lawson
Edwin Lizarazo
Antonin Portelli
Chris Sachrajda
Francesco Sanfilippo
Matthew Spraggs
Tobias Tsang

York University (Toronto)

Renwick Hudspith

Introduction and Motivation

Motivation for studying K→ππ Decays

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP in decays (direct CPV).

- Direct CPV first observed in late 90s at CERN and Fermilab in $K_0 \to \pi\pi$:

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}.$$

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3)\times 10^{-4} \qquad \text{(experiment)}$$

measure of direct CPV

measure of indirect CPV

- In terms of isospin states: ΔI=3/2 decay to I=2 final state, amplitude A_2
  ΔI=1/2 decay to I=0 final state, amplitude A_0

$$\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$$

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2},$$
$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2 e^{i\delta_2}.$$

$$\Longrightarrow \quad \epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

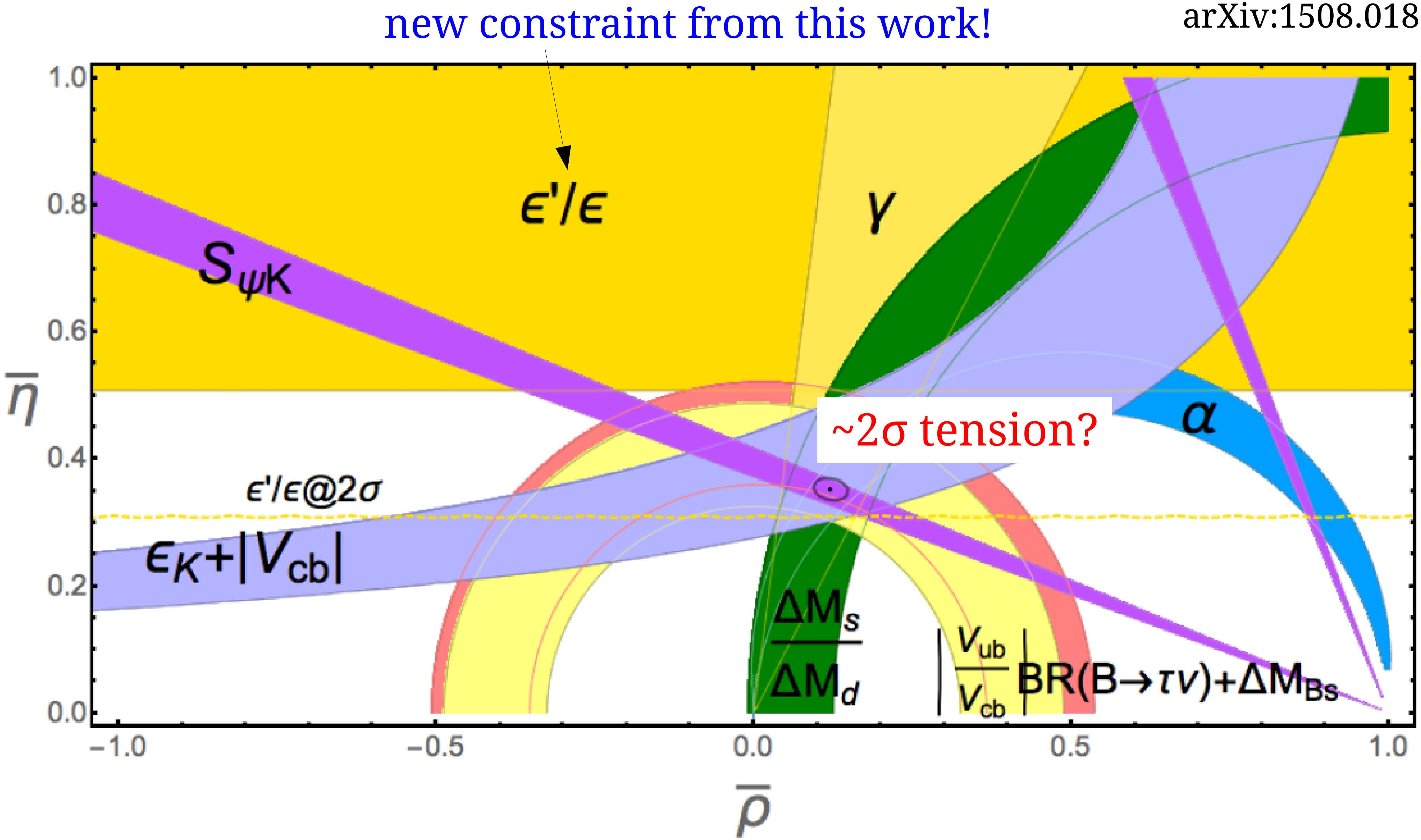
(δ_I are strong scattering phase shifts.)

- Amount of direct CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.

- Small size of ε' makes it particularly sensitive to new direct-CPV introduced by most BSM models.

- ε' also provides a new horizontal band constraint on CKM matrix:

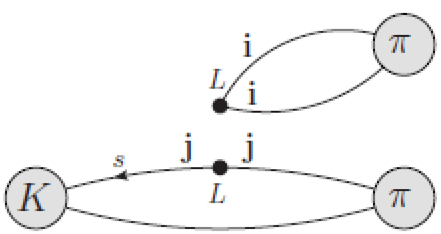
[Lehner et al arXiv:1508.01801]



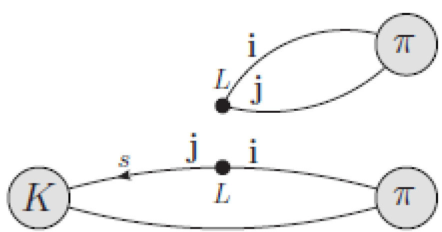
The role of the lattice

- In experiment kaons approx 450x (!) more likely to decay into I=0 pi-pi states than I=2. $$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \simeq 22.5$$ (the ΔI=1/2 rule)
- Perturbative running to charm scale accounts for about a factor of 2. Is the remaining 10x non-perturbative or New Physics?
- The answer is **low-energy *QCD!*** RBC/UKQCD [arXiv:1212.1474, arXiv:1502.00263]

  Strong cancellation between the two dominant contractions



Contraction ①.



Contraction ②.

$$\mathrm{Re}(A_2) \sim ① + ②$$

$$② \approx -0.7①$$

  heavily suppressing $\mathrm{Re}(A_2)$.

- Lattice QCD only *ab initio*, systematically improvable technique for studying QCD at hadronic scale.

Standard Model Physics and Lattice Determination

Weak Effective Theory

- At energy scales μ«M_W, K→ππ decays accurately described by weak effective theory.

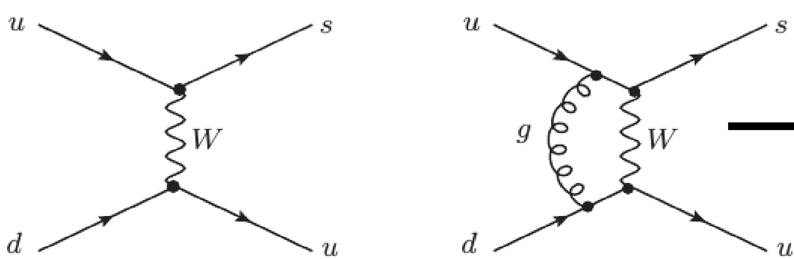
$$H_W^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{j=1}^{10} [z_j(\mu) + \tau y_j(\mu)] Q_j$$

perturbative Wilson coeffs.

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i$$

Imaginary part solely responsible for CPV
(everything else is pure-real)

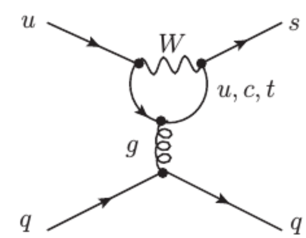
- Q_j are 10 effective four-quark operators:



(a) current-current

$\longrightarrow Q_1, Q_2$

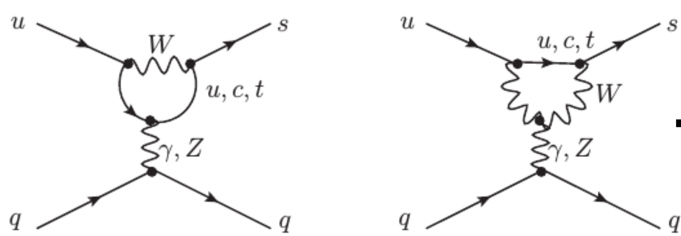
dominate
Re(A_0), Re(A_2)



(b) QCD penguin

$\longrightarrow Q_3 - Q_6$

Q_4, Q_6 dominate
Im(A_0)



(c) Electro-Weak penguin

$\longrightarrow Q_7 - Q_{10}$

Q_7, Q_8 dominate
Im(A_2)

Lattice Determination of K→ππ

- On the lattice compute $M_j = \langle (\pi\pi)_I | Q_j | K \rangle$
- Operators must be renormalized into same scheme as Wilson coeffs: Use RI-(S)MOM NPR and perturbatively match to MSbar at high scale.
- Mixing under renormalization, hence Z is a matrix.

$$A_{2/0} = F \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[\Big(z_i(\mu) + \tau y_i(\mu) \Big) Z_{ij}^{\mathrm{lat}\to\overline{\mathrm{MS}}} M_j^{\frac{3}{2}/\frac{1}{2},\mathrm{lat}} \right] ,$$

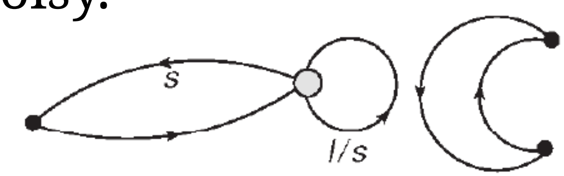
- F is finite-volume correction calculated using LL method.

[Phys.Rev. D91 (2015) 7, 074502]

- A_2 computable using standard lattice techniques. Most recent determination ~12% total error (3% stat) dominated by PT truncation in NPR.
- A_0 considerably more difficult for 2 reasons:

Disconnected Diagrams

- "Type-4" disconnected diagrams (coupling between subdiagrams only via sea gluons) are *very* noisy.



- Use computationally expensive (and non-trivial to implement) Trinity-style all-to-all (A2A) propagators:
  - 900 exact low-eigenmodes computed using Lanczos algorithm
  - Stochastic high-modes with full dilution of indices
- Allows us to tune ππ source shape to minimize vacuum overlap.
- Also to perform all spatial and temporal translations to boost statistics.

Physical Kinematics

- Important to calculate with physical (energy-conserving) kinematics.
- With physical masses: $2 \times m_\pi \sim 270 \text{ MeV} \ll m_K \sim 500 \text{ MeV}$
- Requires moving pions!
- This is excited state of the ππ-system. Possibilities:
    - try to perform multi-state fits to very noisy data (esp. A_0 where there are disconn. diagrams)
    - modify boundary conditions to remove the ground-state
- Second approach optimal. Straightforward for A_2 (APBC on d-quark) but additional requirements for A_0 not satisfied by APBC: must conserve isospin and apply momentum to both charged and neutral pions.
- Solution: Use G-parity BCs:

$$\hat{G} = \hat{C} e^{i\pi \hat{I}_y} \quad : \quad \hat{G}|\pi^{\pm}\rangle = -|\pi^{\pm}\rangle \qquad \hat{G}|\pi^0\rangle = -|\pi^0\rangle$$

- As a boundary condition: (i=+, -, 0)

$$\pi^i(x+L) = \hat{G}\pi^i(x) = -\pi^i(x) \longrightarrow |p| \in (\pi/L,\, 3\pi/L,\, 5\pi/L \ldots)$$

(moving ground state)

- Technically very challenging to implement.

A_0 Calculation

Phys.Rev.Lett. 115 (2015) 21, 212001

(arXiv:1505.07863)

Ensemble

- 32^3x64 Mobius DWF ensemble with IDSDR gauge action at β=1.75. Coarse lattice spacing (a^{-1}=1.378(7) GeV) but large, (4.6 fm)^3 box.
- Using Mobius params (b+c)=32/12 and L_s=12 obtain same explicit χSB as the L_s=32 Shamir DWF + IDSDR ens. used for ΔI=3/2 but at reduced cost.
- Performed 216 independent measurements (4 MDTU sep.).
- Cost is ~0.9 BG/Q rack-day per complete measurement (4 configs generated + 1 set of contractions).
- G-parity BCs in 3 spatial directions results in close matching of kaon and $\pi\pi$ energies:

$$m_K = 490.6(2.4)\text{ MeV}$$

$$E_{\pi\pi}(I=0) = 498(11)\text{ MeV}$$

$$E_{\pi\pi}(I=2) = 573.0(2.9)\text{ MeV}$$

$$E_{\pi} = 274.6(1.4)\text{ MeV} \quad (m_{\pi} = 143.1(2.0)\text{ MeV})$$

Issue with ensemble generation

- Recently discovered mistake with RNG seeding used in ensemble generation:

  - With GPBC we have independent u and d quarks fields.

  - Dirac matrix is 2x2 in flavor space with components spanning boundary.

  - Pseudofermion field

$$\phi = (M^\dagger[U])^{1/2} \begin{pmatrix} \eta_d \\ \eta_u \end{pmatrix} \quad \text{where} \quad P[\eta_i] \sim e^{-\eta_i^\dagger \eta_i}$$

independent for each flavor

  - Due to coding error, identical random numbers were used for η_u and η_d up to a relative shift of 12 sites in the y-direction:

$$\eta_u(x) = \eta_d(x + 12\hat{y})$$

  - Persists through entire ensemble.

- At present have not found theoretical interpretation that would allow effect to be estimated.
- However, strong empirical evidence that effect is negligible for present calculation.

Evidence from 32^3x64 calculation

- Measured plaquette vs. value obtained from non-GPBC ensemble (with extrap to same quark mass):

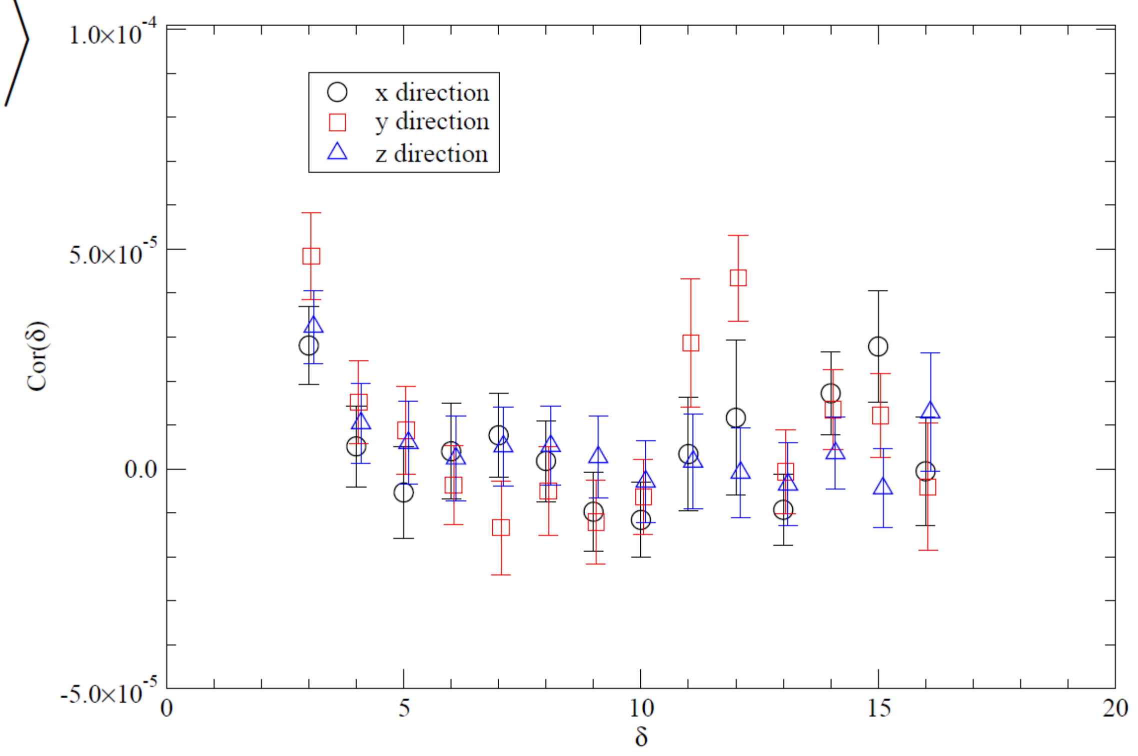
GPBC, incorrect ensemble	Standard
0.512239(6)	0.512239(3)(7)

- More sensitive test: as u, d fields couple to same gauge field we should observe correlations between observables separated by 12 in y-direction.

$$\mathrm{Cov}(\delta) = \left\langle \frac{1}{6V} \sum_{x,\mu<\nu} \left[P_{\mu\nu}(x) P_{\mu\nu}(x+\delta) - \mathcal{P}^2 \right] \right\rangle$$

$$\mathrm{Cor}(\delta) = \frac{\mathrm{Cov}(\delta)}{\mathrm{Cov}(0)}$$

- Statistically significant (3 sigma) correlation between plaquettes seen at sep 12.
- However effect is tiny, ~5x10^{-5}, unlikely to have strong effect on paper results where errors are 100x – 1000x larger.

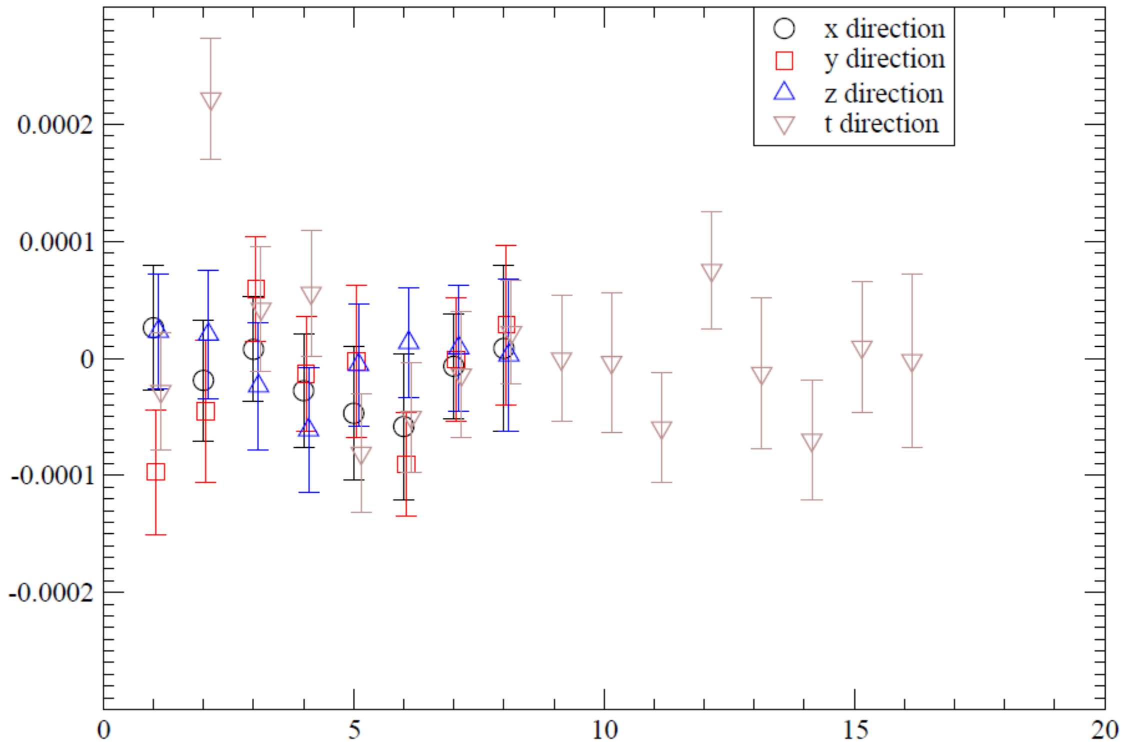


Evidence from 16^3x32 calculation

- 16^3x32 DWF+Iwasaki (β=2.13) test ensembles.

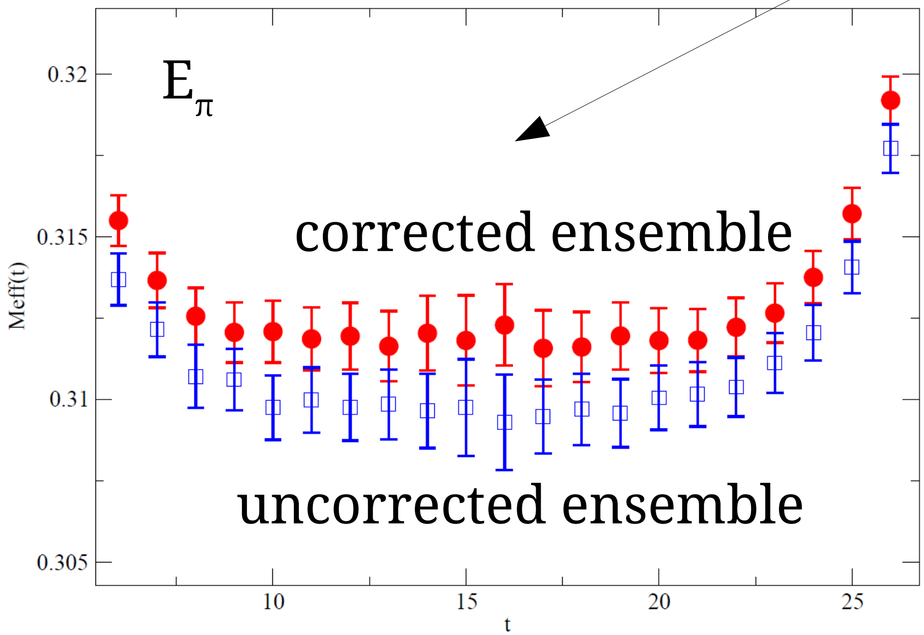
$$\eta_u(x) = \eta_d(x + 2\hat{t})$$

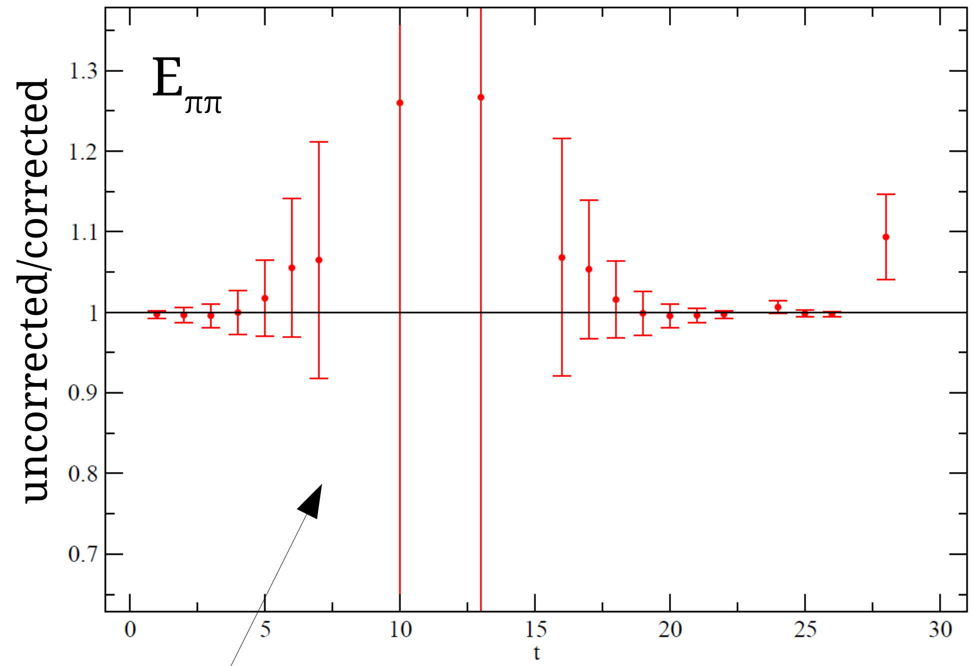
- Smaller lattice separation between correlated sites likely enhances effect.
- Generated an ensemble without error for comparison.
- Presently ~860 meas on corrected ensemble and 670 on uncorrected.
- Cannot see correlation in plaquette due to natural correlation between neaby sites. However evidence in link trace:



- Here at $2\text{x}10^{-4}$ level.

- Inconclusive, ~1.5 sigma, ~0.8% discrepancy in pion energy





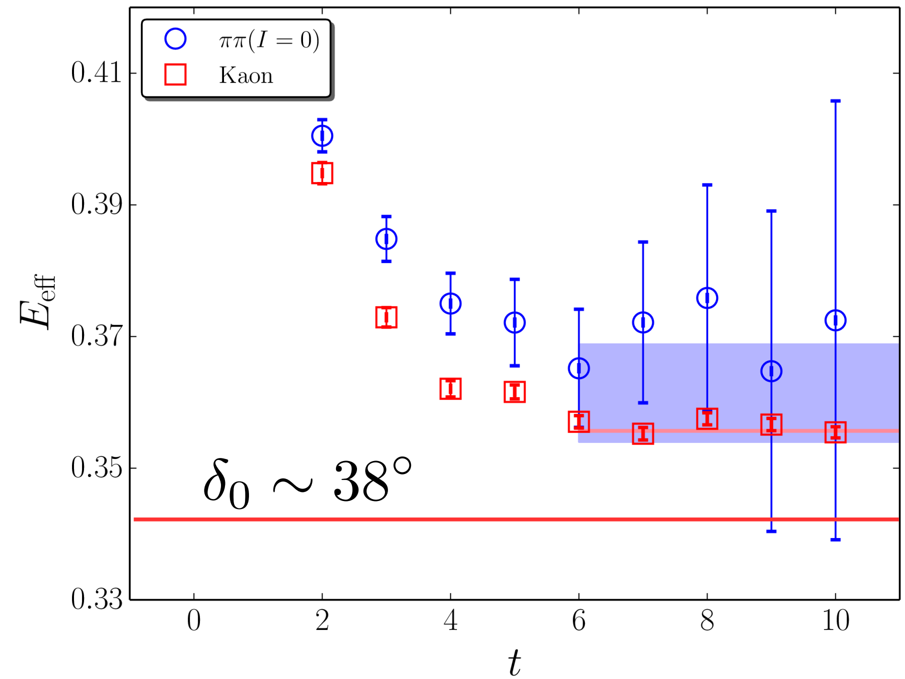
- No presently measurable difference between ππ(I=0) effective energies (important for validity of K→ππ calculation)
- While apparently negligible, this error is uncontrolled theoretically and detracts from our claim of a first-principles calculation.
- Error will be corrected as part of our plans to extend the present calculation in near future.

Results of A_0 Calculation

- Our phase shift $\delta_0 = 23.8(4.9)(1.2)^\circ$ ~2.7σ below conventional Roy equation determination of $\delta_0 = 38.0(1.3)^\circ$

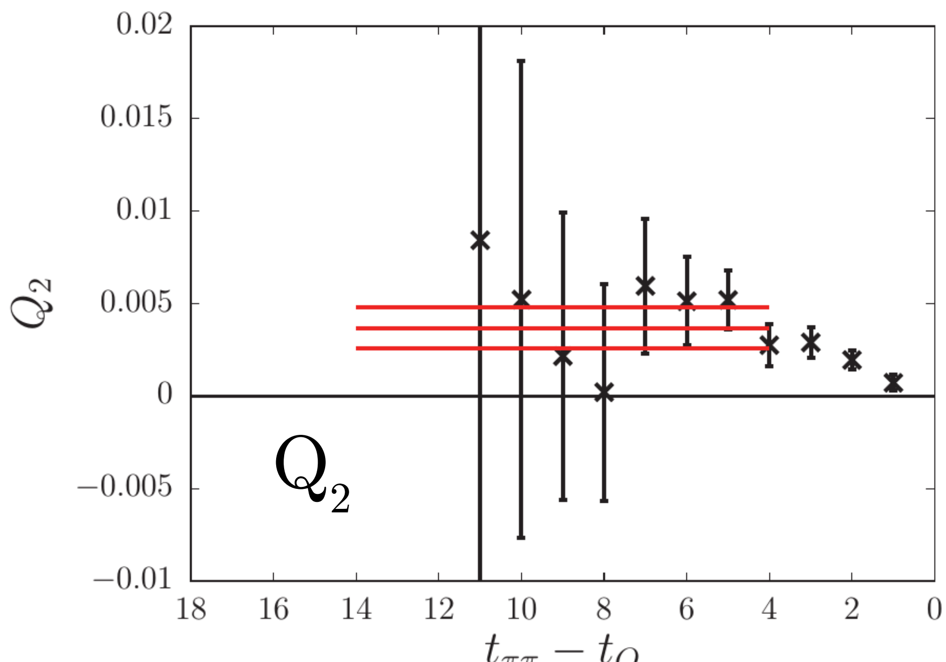
[G.Colangelo, private communication]

- Possibly low statistics concealing delayed plateau start?

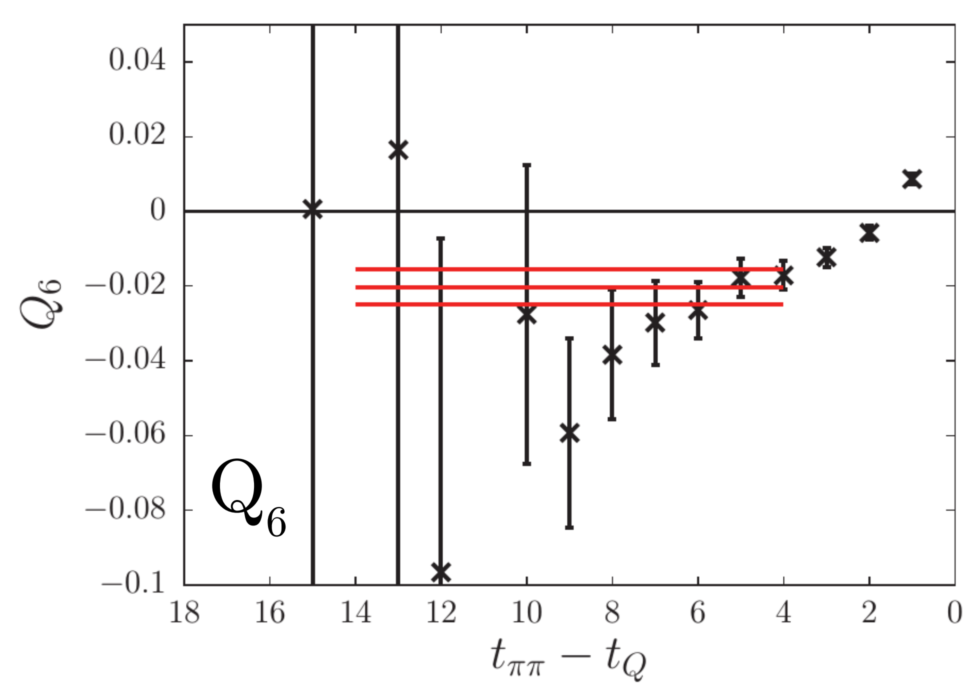


- Matrix elements:

[Dominant contribution to Re(A_0)]



[Dominant contribution to Im(A_0)]



$$\mathrm{Re}(A_0) = 4.66(1.00)_{\mathrm{stat}}(1.21)_{\mathrm{sys}} \times 10^{-7}\ \mathrm{GeV} \quad \text{(This work)}$$
$$\mathrm{Re}(A_0) = 3.3201(18) \times 10^{-7}\ \mathrm{GeV} \quad \text{(Experiment)}$$

- Good agreement for Re(A_0) serves as test for method.

$$\mathrm{Im}(A_0) = -1.90(1.23)_{\mathrm{stat}}(1.04)_{\mathrm{sys}} \times 10^{-11}\mathrm{GeV} \quad \text{(This work)}$$

- First ab initio prediction of Im(A_0).
- ~85% total error on the predicted Im(A_0) due to strong cancellation between dominant Q_4 and Q_6 contributions:

$$\Delta[\mathrm{Im}(A_0), Q_4] = \quad 1.82(0.62)(0.32) \times 10^{-11}$$
$$\Delta[\mathrm{Im}(A_0), Q_6] = -3.57(0.91)(0.24) \times 10^{-11}$$

despite only 40% and 25% respective errors for the matrix elements.

- Dominant systematic (15%) is due to PR truncation errors in the NPR exacerbated by low renormalization scale 1.53 GeV.

Results for ε'

- $\mathrm{Re}(A_0)$ and $\mathrm{Re}(A_2)$ from expt.
- Lattice values for $\mathrm{Im}(A_0)$, $\mathrm{Im}(A_2)$ and the phase shifts,

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{ReA}_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$\boxed{\begin{array}{ll} = \quad 1.38(5.15)(4.43)\times 10^{-4}, & \text{(this work)} \\ \quad 16.6(2.3)\times 10^{-4} & \text{(experiment)} \end{array}}$$

- Find discrepancy between lattice and experiment at the 2.1σ level.

Conclusions and Outlook

- First direct, lattice computation of A_0 performed.
- Ensemble generation error recently discovered, evidence suggests effect negligible.
- Measured $\mathrm{Re}(A_0)$ in good agreement with experiment.
- 85% total error on $\mathrm{Im}(A_0)$ despite 25% and 40% errors on dominant Q_6 and Q_4 contributions resp., due to strong mutual cancellation.
- Total error on $\mathrm{Re}(\varepsilon'/\varepsilon)$ is ~3x the experimental error, and we observe a 2.1σ discrepancy. Strong motivation for continued study!
- Sys. errors dominated by perturbative truncation errors on the renormalization and Wilson coeffs due to low, 1.53 GeV scale.
- Currently computing NPR running to higher energies in order to reduce this systematic.
- On final result, stat. error currently dominant. Plan to shortly begin programme to greatly increase statistics, thus reducing stat. error and enabling better sys. error estimates.
- Existing, flawed data will be corrected as part of this programme.

Thank you!

$\Delta I=3/2$ Calculation

Phys.Rev. D 91 (2015) 7, 074502
[arXiv:1502.00263 [hep-lat]].

Calculation Strategy

- A_2 can be computed directly from charged kaon decay:

$$\langle (\pi\pi)^{I=2}_{I_3=1} | H_W | K^+ \rangle = \sqrt{2} A_2 e^{i\delta_2}$$

- Remove stationary (charged) pion state using antiperiodic BCs on d-quark propagator: $d(x+L) = -d(x) \longrightarrow |p| \in (\pi/L,\, 3\pi/L,\, 5\pi/L \ldots)$

$\pi^+(x+L) = [\bar{u}d](x+L) = -\pi^+(x)$ Moving ground state!

$\pi^0(x+L) = [\bar{u}u - \bar{d}d](x+L) = +\pi^0(x)$ Stationary ground state....

- Use Wigner-Eckart theorem to remove neutral pion from problem

$$\langle (\pi^+\pi^0)_{I=2} | Q^{\Delta I_z = 1/2} | K^+ \rangle = \frac{\sqrt{3}}{2} \langle (\pi^+\pi^+)_{I=2} | Q^{\Delta I_z = 3/2} | K^+ \rangle$$

- APBCs on d-quark break isospin symmetry allowing mixing between isospin states: however $\pi^+\pi^+$ is the only charge-2 state with these Q-numbers hence it cannot mix.

- Calculation performed on RBC & UKQCD 48^3x96 and 64^3x128 Mobius DWF ensembles with (5 fm)^3 volumes and a=0.114 fm, a=0.084 fm. Continuum limit computed.
- Make full use of eigCG and AMA to translate over all timeslices. Obtain 0.7-0.9% stat errors on all bare matrix elements!
- Results:

$$\mathrm{Re}(A_2) = 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{sys}} \times 10^{-8}\ \mathrm{GeV}$$
$$\mathrm{Im}(A_2) = -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{sys}} \times 10^{-13}\ \mathrm{GeV}$$

10%, 12% total errors on Re, Im!

- Systematic error completely dominated by perturbative error on NPR and Wilson coefficients.

An old homework problem

- 1964: CP-violation (indirect) first observed at BNL (Cronin, Fitch et al → 1980 Nobel prize)
- 1973: Framework for Standard Model CPV established (Kobayashi, Maskawa)
- 1993: Publication of first evidence of direct CPV from NA31 expt at CERN.
- 1999: KTeV at FermiLab and NA48 at CERN confirm direct CPV.
- 2001: First quenched calculations of ε' performed by CP-PACS and RBC using single particle amplitudes and LO ChPT to correct for missing pion.
- 2001: Technique established for lattice measurement of decays (Lellouch, Luscher)
- 2011: First full threshold (stationary, unphysically-heavy pions) calc. of A_0 and A_2 using dynamical domain wall fermions performed by RBC/UKQCD.
- 2012: First calculation of A_2 performed by RBC/UKQCD using DWF with physical kinematics, pion masses and large physical volume but single lattice spacing.
- 2015: Continuum calculation of A_2 performed by RBC/UKQCD
- 2015: Full threshold calculation of A_0 and A_2 using Wilson fermions by Ishizuka *et al* [arXiv:1505.05289]
- 2015: (This work) First complete, *ab initio* determination of ε' with physical kinematics and pion masses.

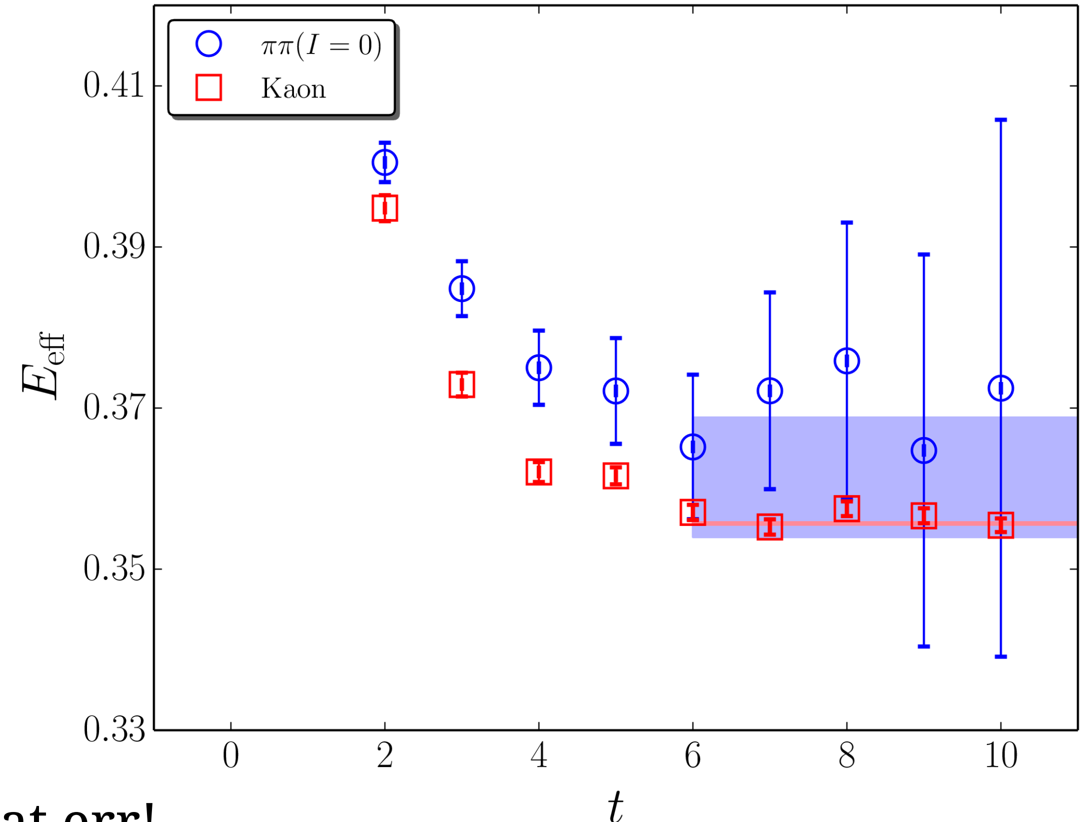
I=0 ππ energy

- Signal/noise deteriorates quickly due to vacuum contrib.
- Difficult to determine plateau start. Performed both 1- and 2-state fits.

$t_{\min}$	$E_{\pi\pi}$	E_{exc}	χ^2/dof
2	0.363(9)	1.04(17)	1.7(7)
3	0.367(11)	1.27(73)	1.8(8)
4	0.364(12)	0.86(39)	1.9(8)

$t_{\min}$	$E_{\pi\pi}$	χ^2/dof
5	0.375(6)	2.2(9)
6	0.361(7)	1.6(7)
7	0.380(11)	0.9(7)

2% stat err!



- Our phase shift $\delta_0 = 23.8(4.9)(1.2)^\circ$ lower than most pheno estimates, which prefer $\delta_0 \sim 35^\circ$.
- Luscher formula very steep in $E_{\pi\pi}$: small shifts energy translate to large (fractional) errors in δ_0. More statistics needed to resolve.
- Using 35° → ~3% change in A_0; much smaller than other errs. For consistency we choose to use our lattice value.