Neutron Electric Dipole Moment from Dimension 5 Operators

Tanmoy Bhattacharya[a,b]

Vincenzo Cirigliano[a] Rajan Gupta[a]
Emanuele Mereghetti[a] Boram Yoon[a]

[a]Los Alamos National Laboratory

[b]Santa Fe Institute

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Introduction
Standard model

Standard model of particle physics with neutrino mass

- Explains all laboratory experiments.
- Is in violent contradiction with cosmology.

The universe:

- Can't be big, empty and homogeneous ("inflation")
- Can't have much matter ("baryogenesis")
- Can't clump into clusters of galaxies ("dark matter")
- Can't be accelerating ("dark energy")

Baryogenesis related to matter-antimatter symmetry.

Introduction
CP violation and nEDM

CP violation needed in the universe.
Observed baryon asymmetry: $n_B/n_\gamma = 6.1^{+0.3}_{-0.2} \times 10^{-10}$.

WMAP + COBE 2003

Without CP violation, freezeout ratio: $n_B/n_\gamma \approx 10^{-20}$.

Kolb and Turner, *Front. Phys.* **69** (1990) 1.

Either asymmetric initial conditions or baryogenesis!
Sufficiently asymmetric initial conditions kills inflation.

Sakharov Conditions

Sakharov, *Pisma Zh. Eksp. Teor. Fiz.* **5** (1967) 32.

- Baryon Number violation
- C and CP violation
- Out of equilibrium evolution

Introduction
Standard Model CP Violation

Two sources of CP violation in the Standard Model.

- Complex phase in CKM quark mixing matrix.
  - Too small to explain baryon asymmetry
  - Gives a tiny ($\sim 10^{-32}$ e-cm) contribution to nEDM

Dar arXiv:hep-ph/0008248.

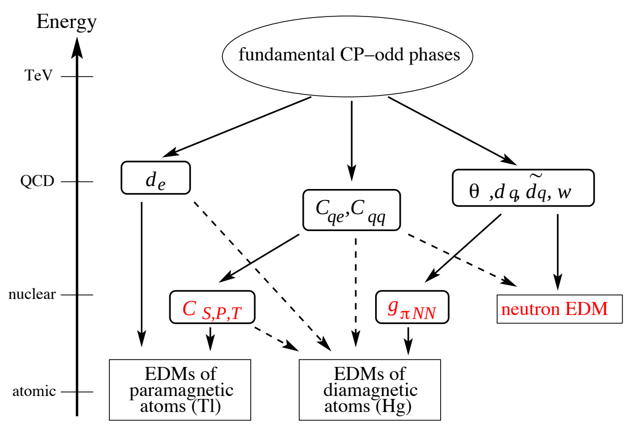
- CP-violating mass term and effective $\Theta G\tilde{G}$ interaction related to QCD instantons
  - Effects suppressed at high energies
  - nEDM limits constrain $\Theta \lesssim 10^{-10}$

Crewther *et al.*, *Phys. Lett.* **B88** (1979) 123.

Contributions from beyond the standard model

- Needed to explain baryogenesis
- May have large contribution to EDM

Introduction
Effective Field Theory



Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119.

Introduction
Form Factors

Vector form-factors ($V_\mu \equiv \delta\mathcal{S}/\delta A_\mu$)

Dirac F_1, Pauli F_2, Electric dipole F_3, and Anapole F_A

Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

$$
\begin{aligned}
\langle N|V_\mu(q)|N\rangle \quad = \quad \overline{u}_N \Bigg[& \gamma_\mu\ F_1(q^2) + i\frac{[\gamma_\mu, \gamma_\nu]}{2} q_\nu\ \frac{F_2(q^2)}{2m_N} \\
& + (2i\, m_N \gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2)\ \frac{F_A(q^2)}{m_N^2} \\
& + \frac{[\gamma_\mu, \gamma_\nu]}{2} q_\nu \gamma_5\ \frac{F_3(q^2)}{2m_N} \Bigg] u_N
\end{aligned}
$$

- The charge $G_E(0) = F_1(0) = 0$.
- $G_M(0)/2M_N = F_2(0)/2M_N$ is the (anomalous) magnetic dipole moment.
- $F_3(0)/2m_N$ is the electric dipole moment.
- F_A and F_3 violate P; F_3 violates CP.

Introduction
Projection

The three point function we calculate is

$$N \equiv \bar{d}^c \gamma_5 \frac{1+\gamma_4}{2} u \; d$$

$$\langle\Omega|N(\vec{0},0)V_\mu(\vec{q},t)N^\dagger(\vec{p},T)|\Omega\rangle \quad = \quad u_N e^{-m_N t} \, \langle N|V_\mu(q)|N'\rangle \; e^{-E_{N'}(T-t)} \overline{u}_N$$

We project onto only one component of the neutron spinor with

$$\mathcal{P} = \frac{1}{2}(1+\gamma_4)(1+i\gamma_5\gamma_3)$$

Fermion action is a ‘constrained action’: equation of motion first order. Mode expansion depends on Lagrangian parameters. On symmetry grounds: $u_N \overline{u}_N = e^{i\alpha_N \gamma_5}(i\not{p} + m_N)e^{i\alpha_N \gamma_5}$ To extract:

$$\begin{aligned} \operatorname*{Lim}_{T-t\to\infty} \; \mathrm{Tr}\,\mathcal{P}\langle\Omega|NV_3N^\dagger|\Omega\rangle \quad &\propto \quad i m_N q_3 G_E \\ &+ \alpha_N m_N(E_N - m_N)F_1 + \alpha_N[m_N(E_N - m_N) + \frac{q_3^2}{2}]F_2 \\ &- 2i\,(q_1^2 + q_2^2)F_A - \frac{q_3^2}{2} F_3 \end{aligned}$$

Introduction
BSM Operators

Standard model CP violation in the weak sector.
Strong CP violation from dimension 3 and 4 operators anomalously small.

- Dimension 3 and 4:
  - CP violating mass $\bar{\psi}\gamma_5\psi$.
  - Toplogical charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.
- Suppressed by $v_{\rm EW}/M^2_{\rm BSM}$:
  - Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Chromo Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.
- Suppressed by $1/M^2_{\rm EW,BSM}$ or higher:
  - Gluon chromo-electric moment: $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.

Renormalization and Mixing

Vacuum Alignment and Phase Choice

CP and chiral symmetry do not commute. Outer automorphism: $\mathrm{CP}_\chi \equiv \chi^{-1}\mathrm{CP}\chi$ also a CP.

- $\psi_L^{CP} = i\gamma_4 C\bar{\psi}_L^T$ and $\psi_R^{CP} = i\gamma_4 C\bar{\psi}_R^T$.
- $\psi_L^\chi = e^{i\chi}\psi_L$ and $\psi_R^\chi = e^{-i\chi}\psi_R$
- $\psi_L^{CP_\chi} = e^{-2i\chi} i\gamma_4 C\bar{\psi}_L^T$ and $\psi_R^{CP_\chi} = e^{+2i\chi} i\gamma_4 C\bar{\psi}_R^T$

Consider the chiral and CP violating parts of the action

$$\mathcal{L} \supset d_i^\alpha O_i^\alpha$$

where i is flavor and α is operator index.

Consider only one chiral symmetric CP violating term: $\Theta G\tilde{G}$

Convert to polar basis

$$d_i \equiv |d_i| e^{i\phi_i} \equiv \frac{\sum_\alpha d_i^\alpha \langle \Omega | \mathcal{I}m\, O_i^\alpha | \pi \rangle}{\sum_\alpha \langle \Omega | \mathcal{I}m\, O_i^\alpha | \pi \rangle}$$

Then CP violation is proportional to:

$$\bar{d}\bar{\Theta}\, \mathcal{R}e\, \frac{d_i^\alpha}{d_i} - |d_i|\, \mathcal{I}m\, \frac{d_i^\alpha}{d_i}$$

with

$$\frac{1}{\bar{d}} \equiv \sum_i \frac{1}{d_i} \qquad \bar{\Theta} = \Theta - \sum_i \phi_i$$

CP violation depends on $\bar{\Theta}$ and on a *mismatch* of phases between d_i^α and d_i.

Renormalization and Mixing

Operator Basis

$$ig\bar\psi\tilde\sigma^{\mu\nu}G_{\mu\nu}t^a\psi \qquad \partial^2\left(\bar\psi i\gamma_5 t^a\psi\right)$$
$$\tfrac{ie}{2}\bar\psi\tilde\sigma^{\mu\nu}F_{\mu\nu}\left\{Q,t^a\right\}\psi$$

$$\mathrm{Tr}\left[MQ^2t^a\right]\tfrac{1}{2}\tilde F_{\mu\nu}F^{\mu\nu} \qquad \mathrm{Tr}\left[Mt^a\right]\tfrac{1}{2}\tilde G^a_{\mu\nu}G^{\mu\nu a}$$
$$\mathrm{Tr}\left[Mt^a\right]\partial_\mu\left(\bar\psi\gamma^\mu\gamma_5\psi\right) \qquad \tfrac{1}{2}\partial_\mu\left(\bar\psi\gamma^\mu\gamma_5\left\{M,t^a\right\}\psi\right)\Big|_{\rm traceless}$$

$$\tfrac{1}{2}\bar\psi i\gamma_5\left\{M^2,t^a\right\}\psi \qquad \mathrm{Tr}\left[M^2\right]\bar\psi i\gamma_5 t^a\psi$$
$$\mathrm{Tr}\left[Mt^a\right]\bar\psi i\gamma_5 M\psi$$

$$i\bar\psi_E\gamma_5 t^a\psi_E \qquad \mathrm{Re}\,\partial_\mu\left[\bar\psi_E\gamma^\mu\gamma_5 t^a\psi\right]$$
$$\mathrm{Re}\,\bar\psi\gamma_5\not{\partial}t^a\psi_E \qquad \mathrm{Re}\,\tfrac{ie}{2}\bar\psi\left\{Q,t^a\right\}\not{A}^{(\gamma)}\gamma_5\psi_E$$

Renormalization and Mixing

RI-$\tilde{\mathrm{S}}$MOM scheme

$$\begin{pmatrix} O \\ N \end{pmatrix}_{\mathrm{ren}} = \begin{pmatrix} Z_O & Z_{ON} \\ 0 & Z_N \end{pmatrix} \begin{pmatrix} O \\ N \end{pmatrix}_{\mathrm{bare}}$$

O: Gauge-invariant operators, does not vanish by equation of motion.

N: Gauge-dependent operators, restricted by BRST, vanish by equation of motion.

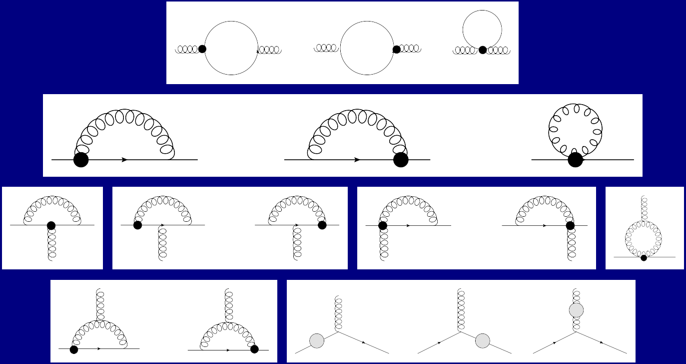
Impose conditions on matrix elements of quarks and gluons:

- Use $\overline{\mathrm{MS}}$ quark masses in the expansion.
- Three point functions at $p^2 = p'^2 = q^2 = -\Lambda^2 \ll 0$ (RI-SMOM).
- Four point functions at $p^2 = p'^2 = k^2 = q^2 = s = u = t/2 = -\Lambda^2$.

This choice eliminates non-1PI contributions. (See arXiv:1502.07325 [hep-ph]).

Renormalization and Mixing

Connection to $\overline{\mathrm{MS}}$ scheme



QEDM
Tensor charge

$$\begin{aligned} \langle N|\frac{\delta\bar\psi\Sigma_{\mu\nu}\tilde F^{\mu\nu}\psi}{\delta A_\mu}|N\rangle &\propto \epsilon_{\kappa\lambda\mu\nu}\, q_\kappa\langle N|\bar\psi\Sigma_{\lambda\nu}\psi|N\rangle \\ &\supset \epsilon_{\kappa\lambda\mu\nu}\, q_\kappa \bar u_N \Sigma_{\lambda\nu} u_N \,\frac{F_3}{2m_N} \end{aligned}$$

Noting that

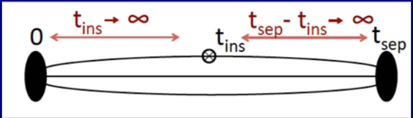
- $\langle N|\bar\psi\Sigma_{\lambda\nu}\psi|N\rangle \equiv g_T\, \bar u_N \Sigma_{\lambda\nu} u_N$
- $F_3/2m_N \equiv d_N$,

we have

$$d_N \propto g_T\,.$$

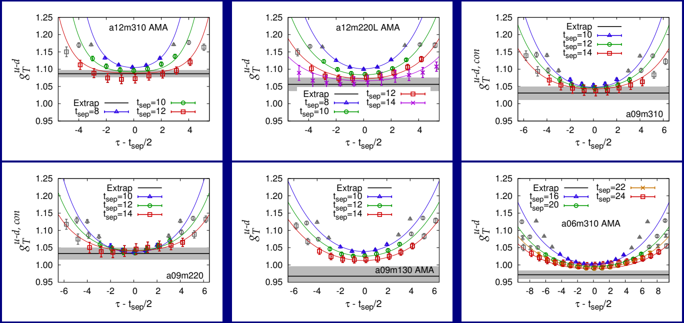
Note that $u_N \bar u_N = \not p + m + O(\alpha_{\rm EW})$

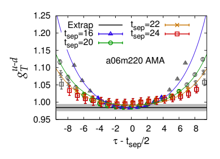
QEDM
Excited states



$$
\begin{aligned}
C^{3pt} \quad = \quad & |\mathcal{A}_0|^2 \langle 0|T|0\rangle e^{-M_0 t_{sep}} + |\mathcal{A}_1|^2 \langle 1|T|1\rangle e^{-M_1 t_{sep}} + \\
& \mathcal{A}_0^* \mathcal{A}_1 \langle 0|T|1\rangle e^{-M_0 t_{ins} - M_1 (t_{sep} - t_{ins})} + \\
& \mathcal{A}_1^* \mathcal{A}_0 \langle 1|T|0\rangle e^{-M_1 t_{ins} - M_0 (t_{sep} - t_{ins})}
\end{aligned}
$$

Top line has no dependence on t_{ins}: need multiple t_{sep}.
$\mathcal{A}$ and M can be obtained from 2-pt functions.





PRELIMINARY

Clover on 2+1+1 HISQ (MILC).

$M_\pi \approx 135,\ 225,\ 315$ MeV.

$a \approx 0.06,\ 0.09,\ 0.12$ fm.

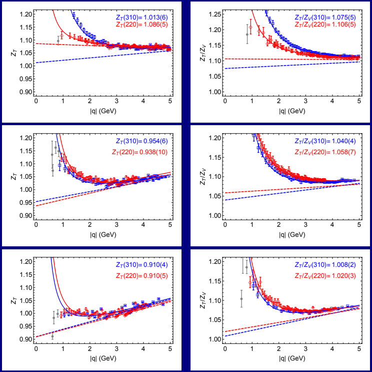
$M_\pi L \approx$ 3.3–5.5.

$t_{\rm sep} \approx$ 0.9–1.7 fm.

QEDM
Renormalization

PRELIMINARY

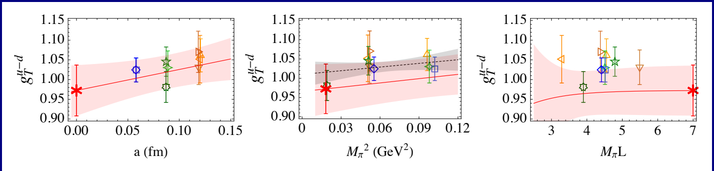
RI-SMom scheme: Nonexceptional symmetric momentum matrix element has tree-level value.



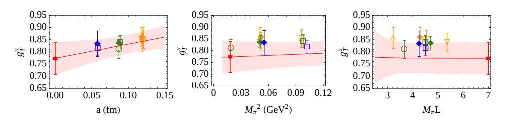
QEDM
Extrapolation

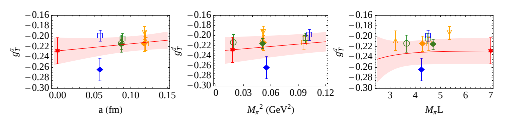
PRELIMINARY

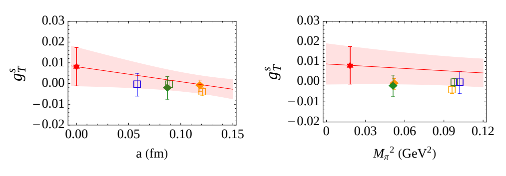
$$g_T = c_1 + c_2 a + c_3 M_\pi^2 + c_4 e^{-M_\pi L}$$



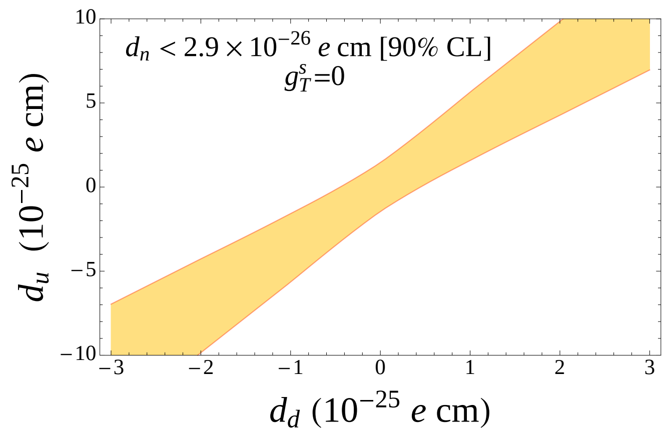
OLD DATA


g_T^u
a (fm)
M_π^2 (GeV^2)
M_πL


g_T^d
a (fm)
M_π^2 (GeV^2)
M_πL

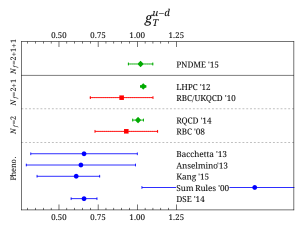

g_T^s
a (fm)
M_π^2 (GeV^2)

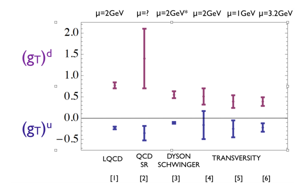
OLD DATA


$d_n < 2.9 \times 10^{-26}$ e cm [90% CL]
g_T^s=0
d_u (10^{-25} e cm)
d_d (10^{-25} e cm)
10
5
0
−5
−10
−3
−2
−1
0
1
2
3

QEDM
Comparison

OLD DATA





[1] Bhattacharya *et al.* 2015
[2] Pospelov-Ritz 2000
[3] Pitschmann *et al.* 2014
[4] Bacchetta *et al.* 2013
[5] Anselmino *et al.* 2013
[6] Kang *et al.* 2015

QCEDM: Lattice Calculation

Technique

The quark chromo-EDM operator is a quark bilinear.
Schwinger source method: Add it to the Dirac operator in the propagator inversion routine:

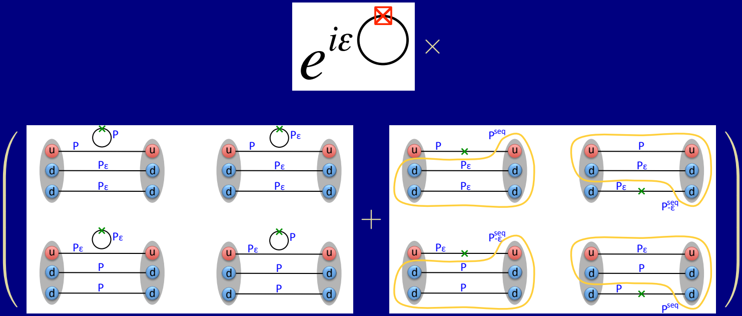
$$\not{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu} \longrightarrow \not{D} + m - \frac{r}{2} D^2 + \Sigma^{\mu\nu} (c_{sw} G_{\mu\nu} + i\epsilon \tilde{G}_{\mu\nu})$$

The fermion determinant gives a ‘reweighting factor’

$$\begin{aligned} & \frac{\det(\not{D} + m - \frac{r}{2} D^2 + \Sigma^{\mu\nu}(c_{sw} G_{\mu\nu} + i\epsilon \tilde{G}_{\mu\nu})}{\det(\not{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu})} \\ = & \exp \operatorname{Tr} \ln \left[1 + i\epsilon\, \Sigma^{\mu\nu} \tilde{G}_{\mu\nu} (\not{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu})^{-1} \right] \\ \approx & \exp \left[i\epsilon \operatorname{Tr} \Sigma^{\mu\nu} \tilde{G}_{\mu\nu} (\not{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu})^{-1} \right] . \end{aligned}$$

QCEDM: Lattice Calculation

Three-point function



The chromoEDM operator is dimension 5.
Uncontrolled divergences unless $\epsilon \lesssim 4\pi a \Lambda_{\mathrm{QCD}} \sim 1$.
Need to check linearity.

QCEDM: Lattice Calculation

Propagator inversion

Using BiCGStab in Chroma (Clover on HISQ $a \approx 0.12$fm, $m_\pi \approx 310$MeV)

- Cost of $\not{D}$ increases by about 7%.
- Condition number changes by less than 5%.
- Can use $\epsilon = 0$ as initial guess.

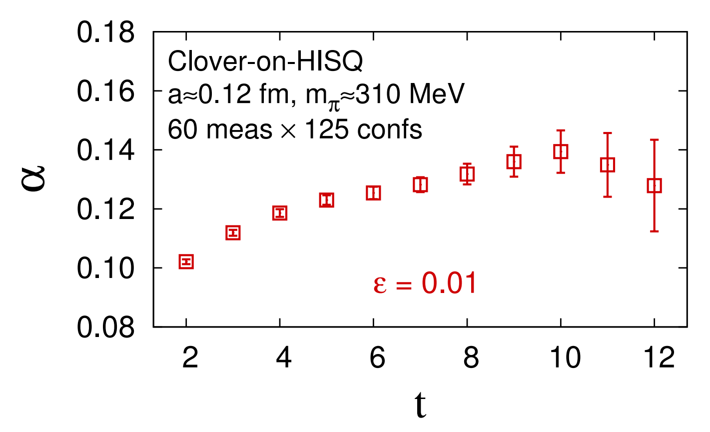
Each extra inversion less than the cost of the $\epsilon = 0$ inversion.

Accuracy	$\epsilon = 0.005$	$\epsilon = 0.01$
10^{-8}	85%	86%
10^{-3}	51%	66%
5×10^{-3}	28%	45%

Calculation of connected EDM measurement on each configuration is about 1.5 times the cost of V/A form factors measurements.

QCEDM: Numerical Tests

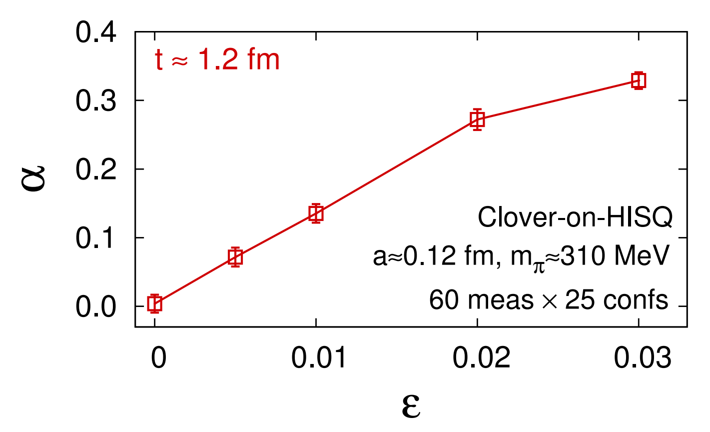
Propagator



Preliminary; Connected Diagrams Only

QCEDM: Numerical Tests

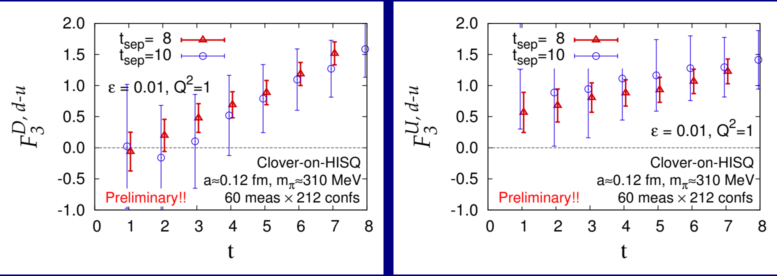
Linearity



Preliminary; Connected Diagrams Only

QCEDM: Numerical Tests

F_3 Form factor



Preliminary; Connected Diagrams Only

QCEDM: Conclusions

Future

- Disconnected diagrams.
- Continuum limit.

  Most divergent mixing with $\frac{\alpha_s}{a^2}\bar{\psi}\gamma_5\psi$.

  nEDM due to this same as due to $\frac{\alpha_s}{ma^2}G\cdot\tilde{G}$.

  Current estimates of nEDM due to
  - $\mathrm{CEDM}^{\overline{\mathrm{MS}}} \Rightarrow O(1)$
  - $\dfrac{\alpha_s}{ma^2}\Theta G\cdot\tilde{G} \Rightarrow \dfrac{O(0.1)}{5\mathrm{MeV}a^2}O(10^{-3})$e-fm $= O(1)$

    at $a \approx 0.1\mathrm{fm}$.

  Expect O(1–10) cancellation.
- Fermions with better chiral symmetry.