Getting to the Bottom Using Domain Wall Fermions

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Getting to the Bottom

- Lattice discretization effects are significant at large quark masses as some cutoff effects go as am.
- The JLQCD collaboration has recently produced very fine Domain Wall(DW) Lattices $a = 0.080$ to 0.044fm
- Can the discretization effects for DW be understood and accounted for at large quark masses
- How far can we push the limits and hopefully extrapolate to the bottom

Project Overview

- $N_f = 2 + 1$ simulations on 15 Esnambles with $10,000$ MD times for each.
- Simulations at three lattice spacing $a^{-1} \approx 2.4, 3.6$ and 4.5GeV
- Pion masses from 230 MeV to 500 MeV
- Domain-Wall (Möbius) fermions
  - Good chiral symmetry with $m_{\text{res}} \ll m_{ud}$. $m_{\text{res}} \approx 1MeV$ on our coarsest lattice; ≈ 0 on the finer lattices.
  - Small residual mass is achieved by the Möbius representation and using stout link-smearing
  - Simpler Renormalization $Z_V = Z_A$
  - Topological charge not fixed
- **Fine lattices for heavy quarks: How well controlled are the discretization effects?**

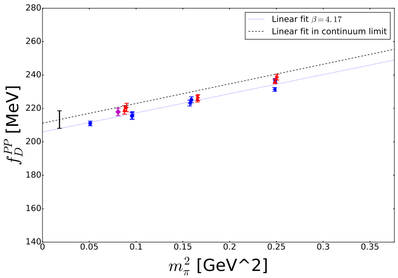
JLQCD Lattices

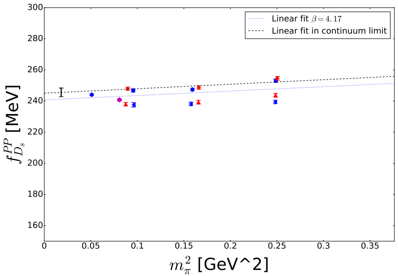
Lattice Spacing	$L^3 \times T$	L_5	am_{ud}	am_s	m_π [MeV]	$m_\pi L$
$\beta = 4.17, a = 0.080$fm	$32^3 \times 64$	12	0.0035	0.040	230	3.0
$a^{-1} = 2.453(4)$ GeV			0.0070	0.030	310	4.0
			0.0070	0.040	310	4.0
			0.0120	0.030	400	5.2
			0.0120	0.040	400	5.2
			0.0190	0.030	500	6.5
			0.0190	0.040	500	6.5
	$48^3 \times 96$	12	0.0035	0.040	230	4.4
$\beta = 4.35, a = 0.055$fm	$48^3 \times 96$	8	0.0042	0.018	300	3.9
$a^{-1} = 3.610(9)$ GeV			0.0042	0.025	300	3.9
			0.0080	0.018	410	5.4
			0.0080	0.025	410	5.4
			0.0120	0.018	500	6.6
			0.0120	0.025	500	6.6
$\beta = 4.47, a = 0.044$fm	$64^3 \times 128$	8	0.0030	0.015	280	4.0
$a^{-1} = 4.496(9)$ GeV						

Measurements

- Correlators measured on each lattice for both smeared and unsmeared Z_2 sources
- Axial and pseudoscalar current correlators were produced on 100 configurations with $6-8$ source points each.
- Sources and sinks were smeared with Gaussian smearing
- Masses and amplitudes were computed with a combined fit to the axial and pseudoscalar correlators with a combination of smeared and unsmeared sources and sinks.
- Chiral fermions means $Z_V = Z_A$ where Z_V was computed using short distance space-like correlators

D and D_s decay constant



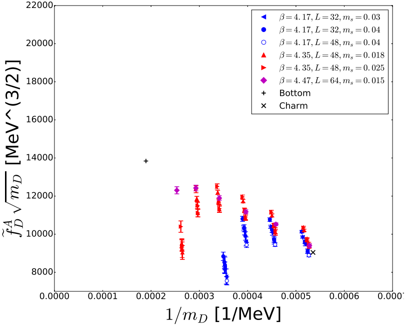


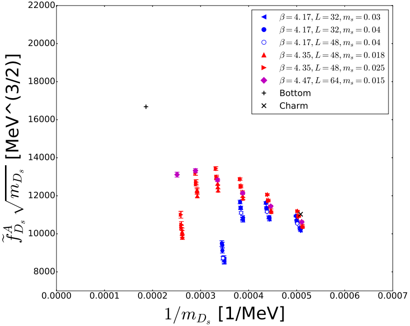
- Similar results were presented recently at the last lattice conference
- The lattice spacing dependence is small with cutoff effects of our coarsest lattice of only about 1%
- $f_D = 212.1 \pm 5.2$ MeV and $f_{D_s} = 245.5 \pm 2.8$ MeV.

- Since cutoff effects at the charm are reasonably controlled, how far above the charm mass can we go?
- Bare quark masses chosen $m_i = (1.25)^i m_c$:
- All heavy quarks treated with DW

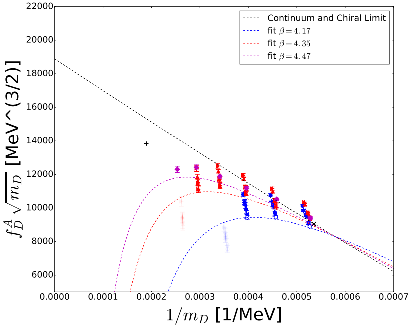
Beta	$m_0 = m_c$	m_1	m_2	m_3	m_4	m_5
4.17	0.4404	0.5505	0.6881	0.8600		
4.35	0.2729	0.3411	0.4264	0.5330	0.6661	0.8327
4.45	0.2105	0.2631	0.3289	0.4111	0.5139	0.6423

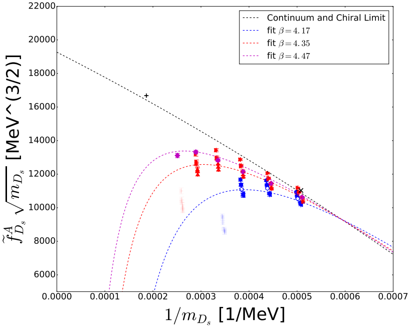
Heavy-light and heavy-strange results





Heavy-light and heavy-strange results





Fit excluding $m_q > 0.7$ assuming
$(F_D\sqrt{m})^\infty(1+bm_\pi^2)(1+c\Delta m_{ss})(1+C_1/m+C_2/m^2+\gamma(a^2m^2)+\mu(a^2))$

Corrections motivated by HQET

- Lattice discritization errors become large when am is large
- We would like to understand what goes wrong in the large mass limit
- Follow the ideas of Lepage and assume small momentum and expand the action
- Use these to determine the low order corrections to the wave function normalization and energies $E = m_1 + \frac{p^2}{2m_2} + \dots$

Wave function renormalization

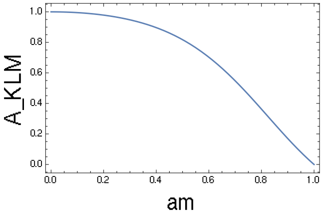
- In the Continuum

$$S(p) = \frac{1}{\not{p} + m} \quad \to C(t, \vec{p} = 0) = \int \frac{dp_0}{2\pi} S(p) e^{ip_0 t} = \frac{1 + \gamma^0}{2} e^{-mt}$$

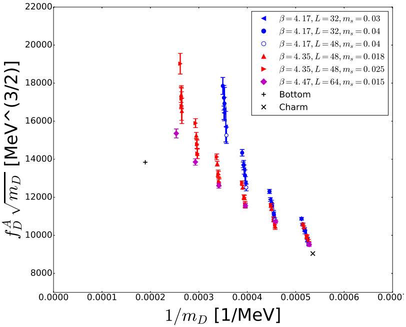
- On the lattice

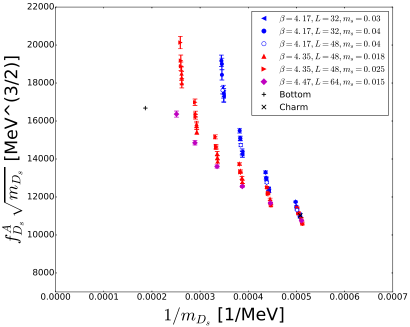
$$S(p) = \text{Complicated} \quad \to C(t, \vec{p} = 0) = A_{KLM} \frac{1 + \gamma^0}{2} e^{-m_1 t}$$

This correction to the normalization known as the Kronfeld-Lepage-Mackenzie factor

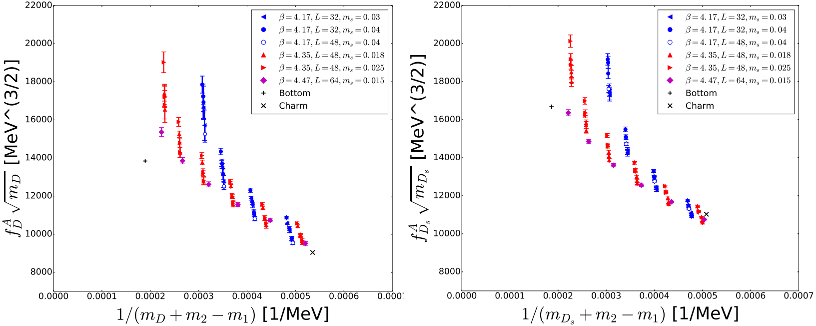


Renormalized





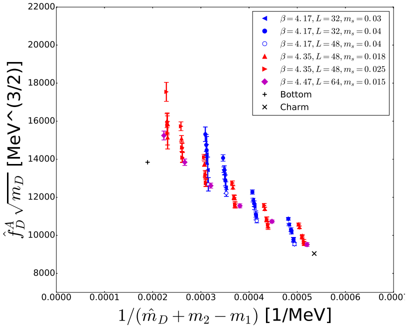
Mass Corrections

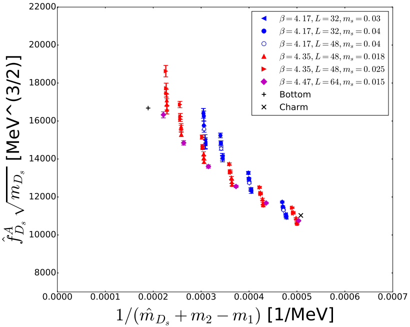


Checking the KLM factor

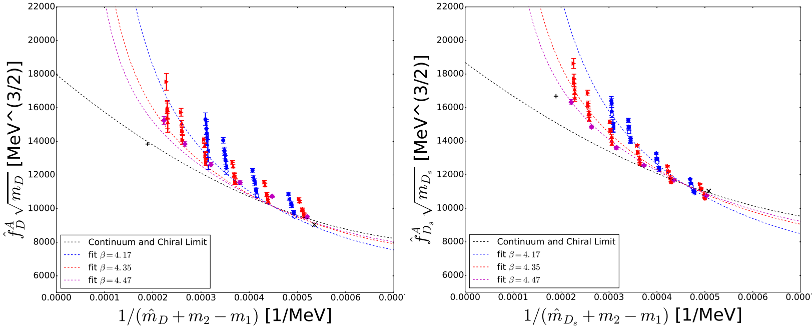
- We checked the KLM factor by numerically integrated the propagator for particular heavy quark masses of interest and compared to the KLM factor.
- This reverse engineering of the KLM factor agreed at large time separations but for large quark masses they disagreed significantly.
- We suspect this may be due to the DW fermions non locality being significant at very large quark masses.
- Compute corrected correlators by dividing the correlators by the integrated lattice propagator and multiply by the continuum result.

Corrected Correlators





Fit to the Corrected Values



Fit assuming

$$(F_D\sqrt{m})^{\infty}(1+bm_{\pi}^2)(1+c\Delta m_{ss})(1+C_1/m+C_2/m^2+\gamma(a^2m^2)+\mu(a^2))$$

Conclusions and Future work

- Results of heavy mesons seem promising and the cutoff effects for heavy domain wall fermions can be partially understood
- Corrections motivated by Heavy Quark Effective theory seems to account for most of the problems as we approach masses of $1/a$
- Extrapolation to the B might not be completely unreasonable

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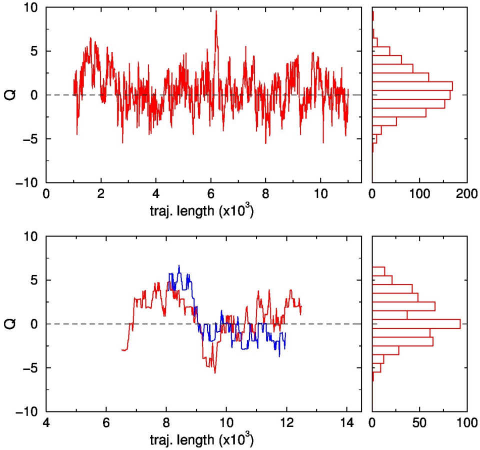
- Better understanding of the correlator normalization and possible effects of non local fermions still need to be worked out
- Determine more appropriate fit functions to extrapolate to the B
- Try the “ratio method” using ratios of successive heavy masses to constrain the extrapolation

Thank You.

Backup Slides

Topological charge



Topological charge for $a^{-1} = 2.4$ GeV (top) and $a^{-1} = 3.6$ GeV (bottom)

Corrections

$$m_1 = \log\left(1 - W_0 + \sqrt{(1 - W_0)^2 - 1}\right)$$
$$m_2 = \sqrt{W_0^2 - 2W_0}\left(\frac{Q + 1 - 2W_0}{(Q+1) + (Q-1)(2W_0 2 + W_0)}\right)$$

$$A_{KLM}^{DW} = \frac{2}{(1 - m^2)\left[1 + \sqrt{\frac{Q}{1+4W_0}}\right]}$$

$$Q = \left(\frac{1 + m^2}{1 - m^2}\right)^2 \quad \text{and} \quad W_0 = \frac{1+Q}{2} - \frac{\sqrt{3Q + Q^2}}{2}$$