Heavy quark momentum diffusion from lattice QCD

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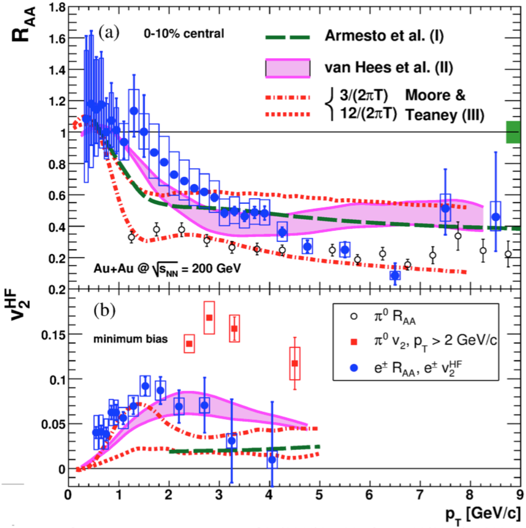
Outline

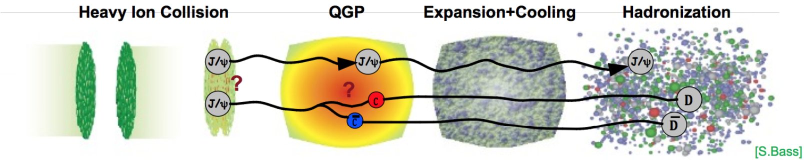
Transport and dissociation in HICs

- **Transport coefficients** enter the hydro/transport evolution of the system
- Usually determined by matching different models to experiment (e.g. in R_{AA})
- Ab initio determination?
  $\Rightarrow$ Lattice QCD

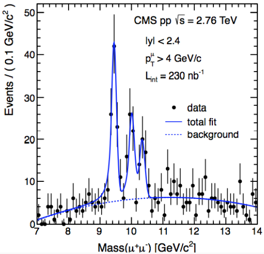


[PHENIX Collaboration, Adare et al.,PRC84(2011)044905 & PRL98(2007)172301]

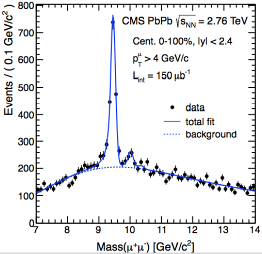
Transport and dissociation in HICs



- (Heavy) quarkonium is produced in the early stage of the collision
- Depending on its **dissociation temperature**
  - remains as bound state for the whole evolution
  - releases its constituents into the plasma



Sequential suppression for bottomonium observed at CMS



Spectral functions from lattice correlators

- The (real-time) information on transport coefficients and dissociation temperatures is encoded in spectral functions $\rho_{\mu\nu}(\omega, \vec{p}, T)$
- Connection to lattice:

$$G_{\mu\nu}(\tau, \vec{p}, T) = \int_0^\infty \frac{d\omega}{\pi}\, \rho_{\mu\nu}(\omega, \vec{p}, T)\, \frac{\cosh(\omega(\tau - \beta/2))}{\sinh(\omega\beta/2)}$$

- with the mixed representation correlator:

$$G_{\mu\nu}(\tau, \vec{p}, T) = \sum_{\vec{x}} G_{\mu\nu}(\tau, \vec{x}, T)\, e^{i\vec{p}\vec{x}}$$

$$G_{\mu\nu}(\tau, \vec{x}, T) = \left\langle \left(\bar{\psi}(\tau, \vec{x})\Gamma_\mu \psi(\tau, \vec{x})\right)\left(\bar{\psi}(0, \vec{0})\Gamma_\nu \psi(0, \vec{0})\right)^\dagger \right\rangle$$

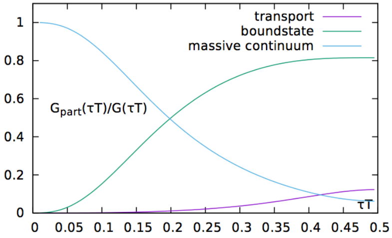
- Transport coefficient: Kubo formula and the SPF:

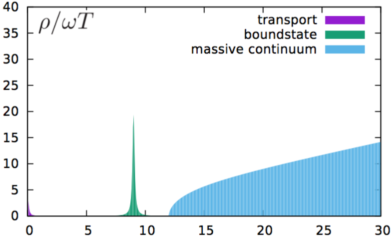
$$D \sim \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T)}{\omega T}$$

- Dissociation: disappearance of bound state peaks in the SPF

Spectral functions from lattice correlators

- The inverse transform from $G(\tau)$ to $\rho(\omega)$ is a (numerically) ill-posed problem
  - Systematics of the reconstruction algorithm?
- The Euclidean lattice correlator is largely insensitive to the detailed properties of the SPF line shape
  - Statements beyond the area underneath it?



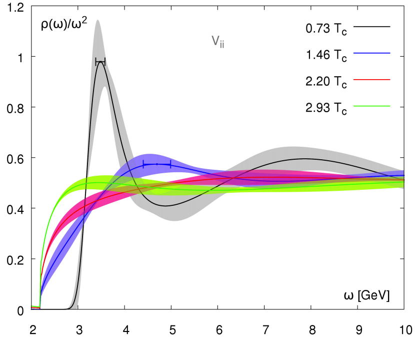


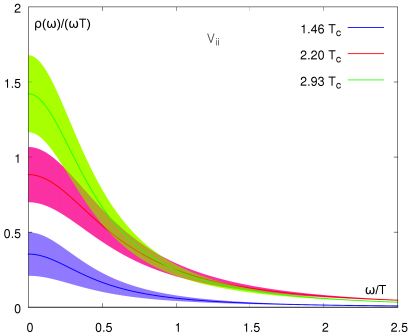
Goal: Reconstruct the SPF from the lattice data

1. Maximum Entropy Method (MEM)
   - Most probable SPF given data, errors and default model
   - Dependence on default model? Systematics due to algorithm (basis functions)?
2. Reconstruction via fit ansatz
   - SPF from fitting a phenomenological model to data
   - Model dependence?
3. Backus Gilbert Method (BGM)
   - Compute a smeared or filtered SPF in the local vicinity of some ω in a model independent way
   - How local is it? How close to the true SPF is the result?

Example: Charmonium spectral functions

- For **charmonium** both transport and dissociation was studied in **[H.T. Ding, A. F., O. Kaczmarek, F. Karsch, H. Satz, W. Soeldner, 1204.494]**
  - large, isotropic, quenched QCD ensembles
  - MEM reconstruction
- Dissociation of J/Ψ and η_c around $1.5\,T_c$
- Diffusion $2\pi TD \sim 1...3$



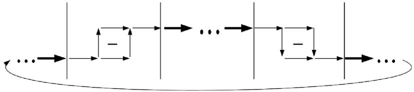


- It is difficult to extract the transport from the full (vector) correlator
  - Difficulty to perform the lattice calculation for (very) heavy quarks
  - Insensitivity of the correlator to the low ω region
  - Signal gets mixed up with bound state dissociation
- Is there perhaps an exclusive correlator for heavy quark diffusion?

- Using Heavy Quark Effective Theory, the force felt by a heavy quark as it propagates through a gluon plasma can be related to a colour-electric correlator **[S. Caron-Huot, M.Laine, G. D. Moore, 0901.1195], [J. Casalderrey-Solana, D. Teaney, 0605199]**

$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{ReTr}\left[U(\tfrac{1}{T};\tau)\, gE_i(\tau,\vec{0})\, U(\tau;0)\, gE_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{ReTr}[U(\tfrac{1}{T};0)]\right\rangle}$$

  where gE_i denotes the colour-electric field, T the temperature, and $U(\tau_2;\tau_1)$ a Wilson line in the Euclidean time direction.
- The lattice discretization of this correlator is not unique, we implement this correlator as:



- The heavy quark momentum diffusion coefficient is linked to this correlator via its spectral function $\rho_E(\omega)$ and the Kubo formula:

$$\kappa = \lim_{\omega \to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} \quad , \qquad D = \frac{2T^2}{\kappa}$$

- **Hope:** There is a strong effect on the correlator due to κ and the analytic continuation is straight forward, once accurate lattice data is gathered.

- In the following: All data is normalized via the LO correlator **[S. Caron-Huot, M.Laine, G. D. Moore, 0901.1195]**

$$G_{\mathrm{norm}}(\tau) \;\equiv\; \pi^2 T^4 \Big[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)}\Big]$$

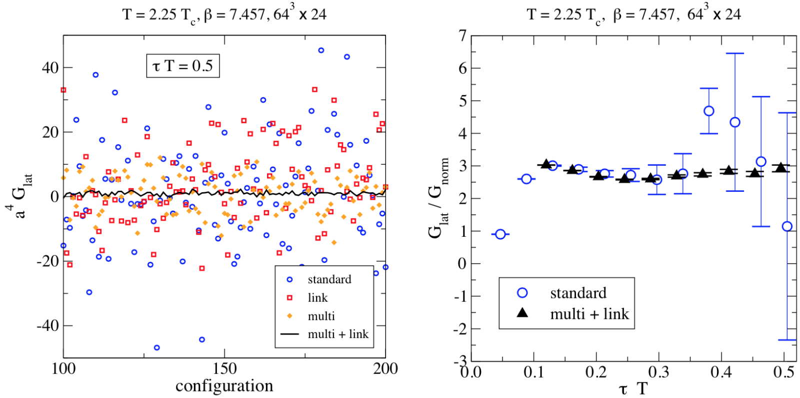
- **Goal:** Give a "minimal" practical answer to heavy quark diffusion using lattice techniques with extrapolation to the continuum
- **Setup:** Quenched lattice QCD, employing the standard Wilson gauge action, at a temperature corresponding to about $\simeq 1.5\,T_c$

β_0	$N_s^3 \times N_\tau$	confs	$T\sqrt{t_0}^{(\mathrm{imp})}$	$T/T_c\|_{t_0}^{(\mathrm{imp})}$	$T\sqrt{t_0}^{(\mathrm{clov})}$	$T/T_c\|_{t_0}^{(\mathrm{clov})}$	Tr_0	$T/T_c\|_{r_0}$
6.872	$64^3 \times 16$	172	0.3770	1.52	0.3805	1.53	1.116	1.50
7.035	$80^3 \times 20$	180	0.3693	1.48	0.3739	1.50	1.086	1.46
7.192	$96^3 \times 24$	160	0.3728	1.50	0.3790	1.52	1.089	1.46
7.544	$144^3 \times 36$	693	0.3791	1.52	0.3896	1.57	1.089	1.46
7.793	$192^3 \times 48$	223	0.3816	1.53	0.3955	1.59	1.084	1.45

- Scale setting is performed using $T\sqrt{t_0}$ **[1503.05652]**
- Aspect ratio $N_s/N_\tau = 4$ is kept fixed

Measurements - Algorithmic improvement

- Use the multilevel algorithm with $N_{multi} = 1000$ per configuration for the electric field insertion
- Use semi-analytical link integration for the straight link lines

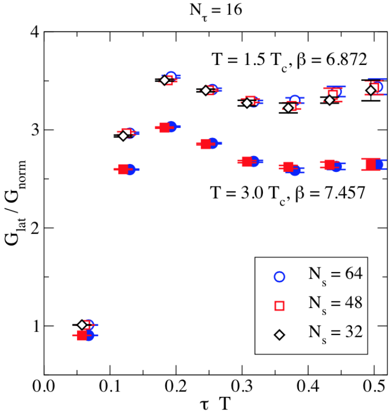


$\Rightarrow$ We obtain signal with errors at the $\simeq 1\%$-level

[1109.3941]

Measurements - Volume effects

- We checked the volume dependence of our results for $N_\tau = 16$

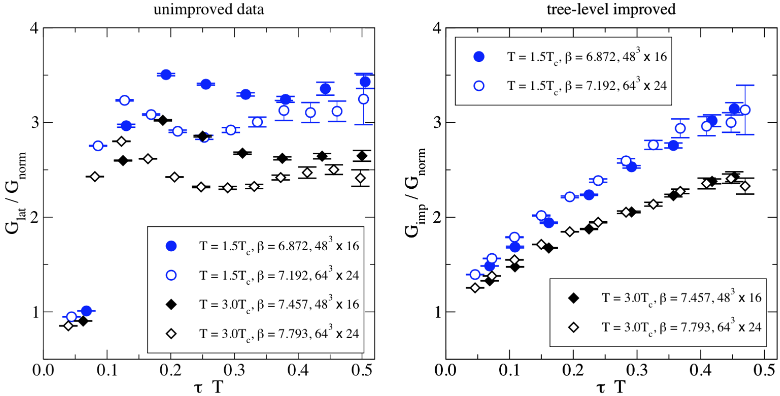


$\Rightarrow$ Volume effects are below our statistical precision
[1109.3941]

Measurements - Discretization effects

- In order to reduce discretization effects, the imaginary-time separations are tree-level improved via

$$G^{LO}_{cont.}(\overline{\tau T}) = G^{LO}_{lat}(\tau T)$$



⇒ Discretization effects are highly reduced

[1109.3941]

- To carry out a continuum limit the lattice correlators need to be multiplied by a renormalization factor:

$$G_{\mathrm{E,cont}}(\tau) \equiv \mathcal{Z}_{\mathrm{E}} G_{\mathrm{E,latt}}(\tau)$$

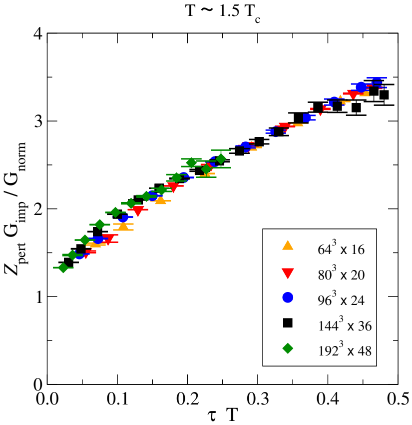
- A 1-loop perturbative computation yields **[C. Christensen, M. Laine, 1601.01573]**

$$\mathcal{Z}_{\mathrm{E,pert}} = 1 + 0.079 \times \frac{6}{\beta_0} + \mathcal{O}\Big(0.079 \times \frac{6}{\beta_0}\Big)^2$$

where $\beta_0 \equiv 6/g_0^2$ is the coupling of the plaquette term in the Wilson action

- The small coefficient of the 1-loop term suggests perturbative renormalization should give a reasonable approximation to the full non-perturbative result $\Rightarrow$ Non-perturbative check in the future?

Continuum limit - Data

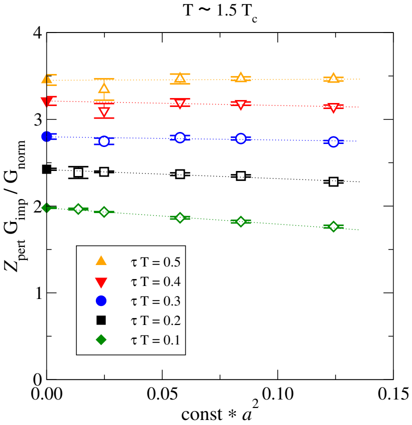


- After renormalization the lattice spacing dependence seems mild
- Signal deteriorates for the $N_\tau = 48$ ensemble around $\tau T = 0.3$

[1508.04543]

Continuum limit - Extrapolation

- Results on all 4 or 5 values of N_τ are interpolated to the values of τT determined by the $N_\tau = 48$ ensemble
- Extrapolation to the continuum is carried out for fixed τT in a^2



[1508.04543], Note: correlator data and covariances are provided

Spectral function reconstruction

- Connecting the lattice data and the spectral function we have:

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\,\rho_{\mathrm{E}}(\omega)\,\frac{\cosh[\omega(\frac{\beta}{2}-\tau)]}{\sinh[\frac{\omega\beta}{2}]}$$

- Large variations of ρ_{E} may lead to only small changes of G_{E}.
- Constrain the allowed form of ρ_{E} from general considerations
- **Idea:**
  - Fix the functional form of ρ_{E} at small ($\omega \ll T$) and large frequencies ($\omega \gg T$)
  - Impose theoretically motivated interpolations between the two regimes

SPF reconstruction - IR asymptotics

- In the IR regime ($\omega \ll T$), the heavy quark momentum diffusion coefficient can be defined as

$$\kappa \equiv \lim_{\omega \to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}$$

- The approach to this limit appears to be smooth suggesting that ρ_{E} has no transport peak but is rather a monotonically increasing function
- (checked in resummed PT, numerically in classical lattice theory and at strong coupling **[0901.1195], [0902.2856], [hep-ph/0605199], [hep-th/0612143]**)
- Define the infrared asymptotics through the simplest consistent form

$$\phi_{\mathrm{IR}}(\omega) \equiv \frac{\kappa\omega}{2T}$$

SPF reconstruction - UV asymptotics

- In the UV regime ($\omega \gg T$) the SPF can be computed via perturbation theory, one finds:

$$\rho_{\mathrm{E}}(\omega) \overset{\omega \gg T}{=} \phi^{(a)}_{\mathrm{UV}}(\omega)\left[1 + \mathcal{O}\left(\frac{1}{\ln(\omega/\Lambda_{\overline{\mathrm{MS}}})}\right)\right]$$

- where we defined the asymptotic form:

$$\phi^{(a)}_{\mathrm{UV}}(\omega) \equiv \frac{g^2(\bar{\mu}_\omega)\, C_F\, \omega^3}{6\pi}\,, \quad \bar{\mu}_\omega \equiv \max(\omega, \pi T)$$

SPF reconstruction - Interpolating the IR and the UV

- Between the two regimes we interpolate using different combinations of polynomials:
  - model 1:

$$\rho_{\mathrm{E}}^{(1\mu i)}(\omega) \equiv \left[1 + \sum_{n=1}^{n_{\max}} c_n e_n^{(\mu)}(y)\right]\left[\phi_{\mathrm{IR}}(\omega) + \phi_{\mathrm{UV}}^{(i)}(\omega)\right]$$

  - model 2 (more rapid crossover from IR to UV):

$$\rho_{\mathrm{E}}^{(2\mu i)}(\omega) \equiv \left[1 + \sum_{n=1}^{n_{\max}} c_n e_n^{(\mu)}(y)\right]\sqrt{\left[\phi_{\mathrm{IR}}(\omega)\right]^2 + \left[\phi_{\mathrm{UV}}^{(i)}(\omega)\right]^2}$$

  - model 3 (simple 2-parameter Ansatz):

$$\rho_{\mathrm{E}}^{(3i)}(\omega) \equiv \max\left[\phi_{\mathrm{IR}}(\omega), c\, \phi_{\mathrm{UV}}^{(i)}(\omega)\right]$$

$\Rightarrow$ We can now fit these SPF parametrizations to the data and extract κ

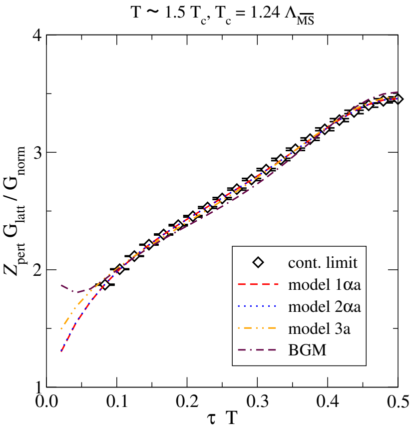
- As parametrization-independent cross-check, we also perform an analysis using the Backus-Gilbert method.
- In the BGM the goal is not to reconstruct the spectral function itself, but rather an averaged version thereof given a resolution function determined from the precision of the data.

$$\rho_{\mathrm{BGM}}(\omega) = \int d\omega' \delta_{\mathrm{resolution}}(\omega, \omega') \rho_{\mathrm{E}}(\omega')$$

- Bonus: If the true SPF is flat, the BGM result is close to the true SPF, even for wide resolution functions
- Caveat: If the width of the resolution function is wide, the BGM result does not necessarily yield a small χ^2

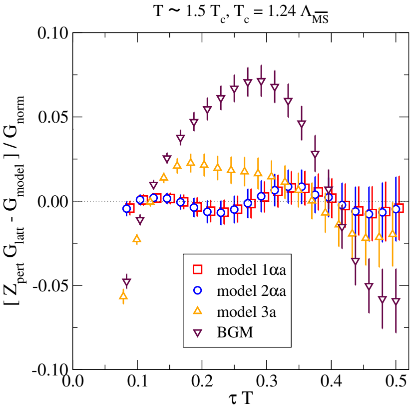
$\Rightarrow$ The method can only be used as cross-check and not stand-alone. It is complimentary to a fit or MEM analysis.

Estimation of κ - Correlator



⇒ At first glance all models provide an excellent description of the data

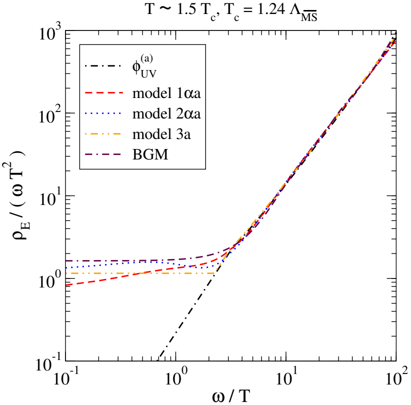
Estimation of κ - Relative deviation



$\Rightarrow$ Forming the difference $Z_{\rm pert}G_{\rm lat} - G_{\rm model}/G_{\rm norm}$

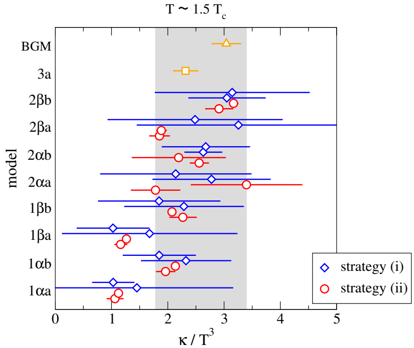
- The BGM leads to the largest deviations
- Model 3 works best at large τT
- Model 1 and 2 give consistently good results for all τT

Estimation of κ - Spectral functions



$\Rightarrow$ The spectral functions share the same qualitative features, especially in the intercept $\omega \to 0$ region.

Estimation of κ - Final results



$\Rightarrow$ Given the spread of our results we estimate:

$$\kappa/T^3 = 1.8...3.4 \quad \text{so that} \quad DT = 0.59...1.1$$

and

$$\tau_{\text{kin}} = \frac{1}{\eta_{\text{D}}} = (1.8\ldots3.4)\left(\frac{T_c}{T}\right)^2\left(\frac{M}{1.5\ \text{GeV}}\right)\ \text{fm/c}$$

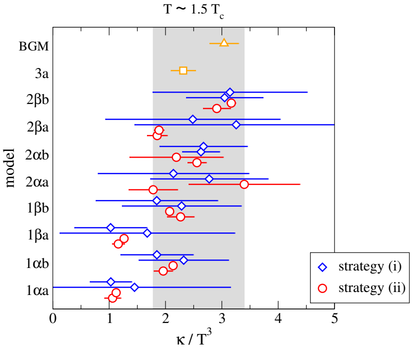
Conclusions

What was done:

- Heavy quark momentum diffusion was computed at $\simeq 1.5\,T_c$
- A result in the continuum limit of quenched QCD by extrapolating a large set of imaginary-time correlators was obtained
- Robust results for κ were determined using SPF parametrizations and the Backus-Gilbert method

What was learnt:

- $\kappa/T^3 = 1.8 \ldots 3.4$ strongly constrains the magnitude of the heavy quark momentum diffusion coefficient
- The estimated kinetic equilibration time indicates heavy quarks equilibrate almost as fast as light quarks



Thanks for listening