Chiral Perturbation Theory and Domain Wall Fermion QCD

RBRC Workshop on Lattice Gauge Theories 2016
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This work done in conjuction with the RBC and UKQCD Collaborations.
David Murphy (Columbia) has been the prime mover on this project.

- arXiv:1411.7017: physical results for 2+1 flavors using ChPT for small extrapolations from simulation quark masses to physical masses
- arXiv:1511.04419: Lattice 2015 proceedings for NLO and NNLO fits to SU(2) ChPT
- arXiv:1511.01950 and PRD 93 054502 (2016): NLO and NNLO fits to SU(2)
- Lattice 2015 proceedings and manuscript in preparation: NLO and NNLO fits to SU(3) ChPT

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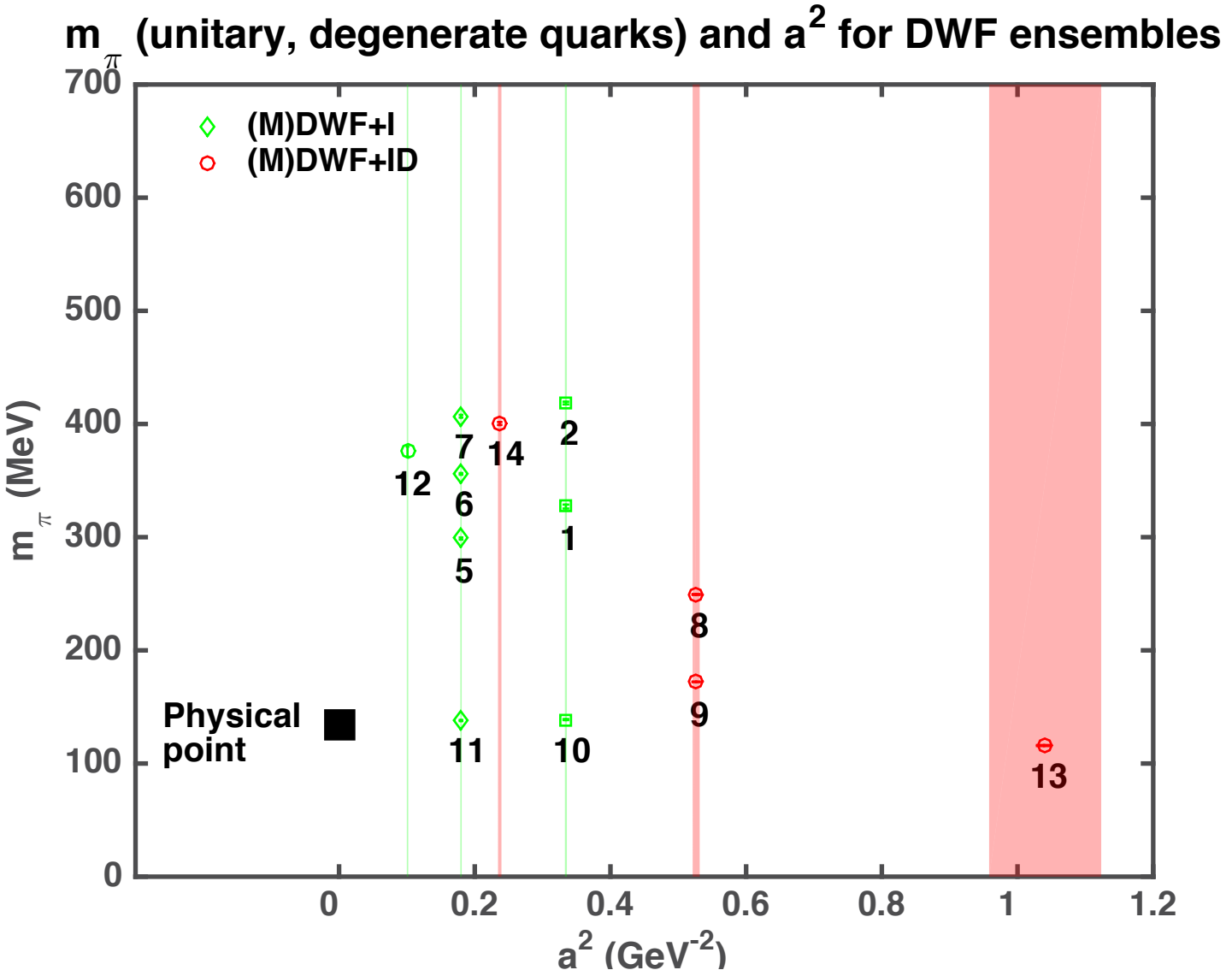
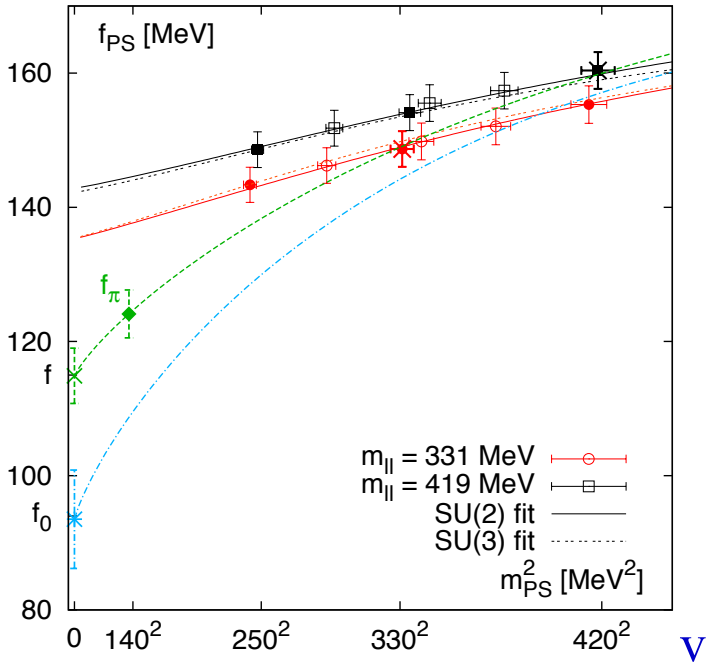
RBC/UKQCD 2+1 Flavor DWF Ensembles



Some ChPT Observations

- The RBC and UKQCD Collaborations have been using NLO SU(2) ChPT to extrapolate to physical quark masses, since observing (PRD 78 (2008) 114509) that SU(3) NLO fits gave small f_0 (and hence large NLO corrections) when fit to m_{PS} in 250-400 MeV range.

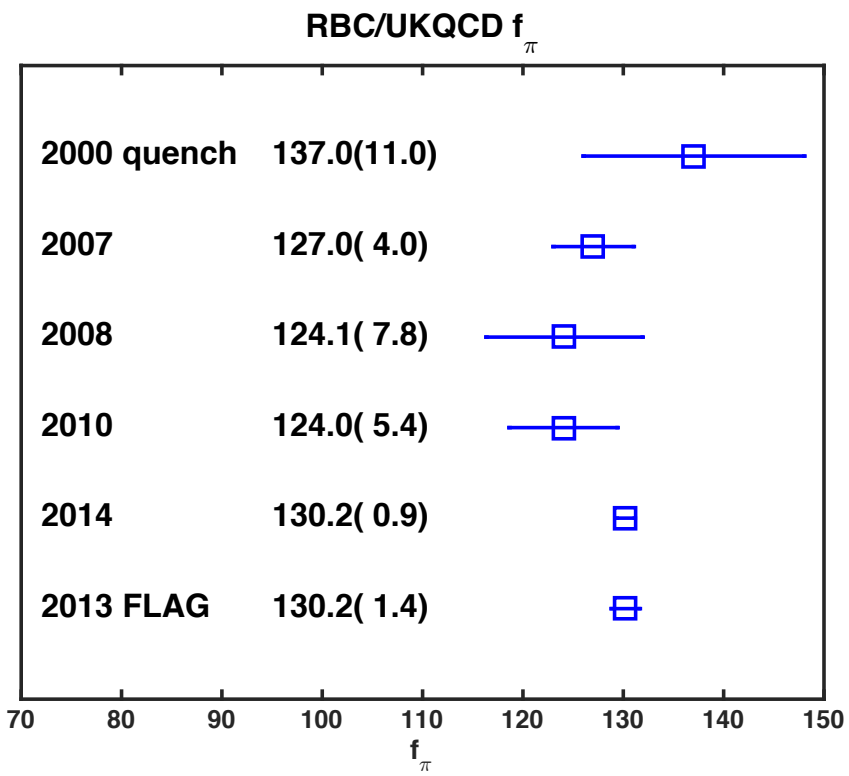


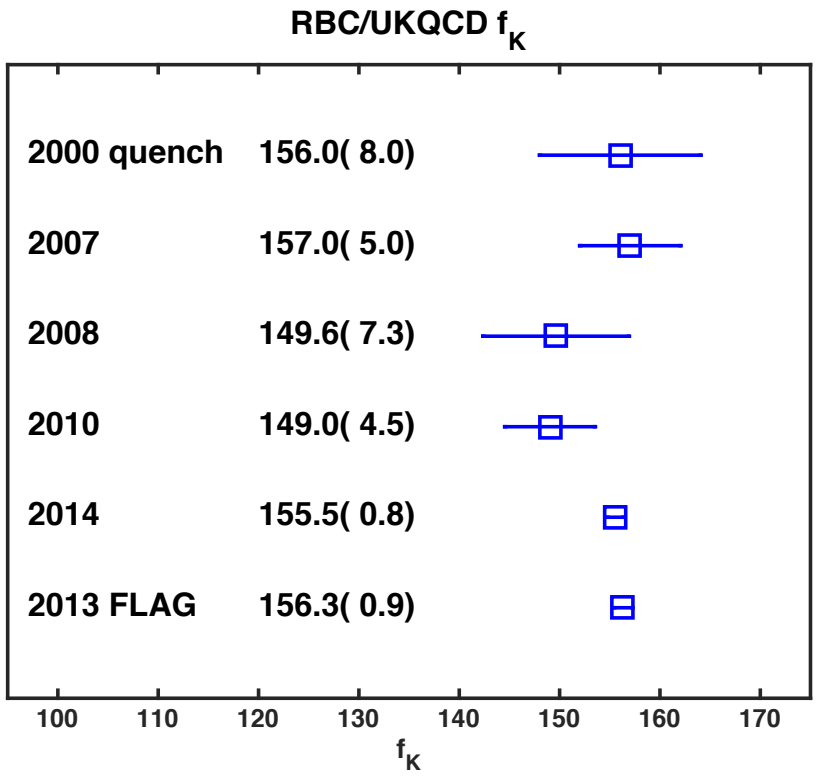
- With extensive new data, test NNLO SU(2) ChPT fits.
- With extensive new data, revisit NLO and NNLO SU(3) ChPT

Essentially Physical Quark Mass Ensembles

- Use SU(2) ChPT to make small extrapolation (arXiv:1411.7017).
- Inputs: m_π, m_K and m_Ω. Outputs: f_π and f_K. Overweight physical pt. ensembles

Quantity	Physical Value	Ens. 10 Value	Deviation	Ens. 11 Value	Deviation
m_π/m_K	0.2723	0.2790	2.4%	0.2742	0.7%
m_π/m_Ω	0.0807	0.0830	2.8%	0.0822	1.9%
m_K/m_Ω	0.2964	0.2974	0.3%	0.2998	1.2%





SU(2) ChPT Fits to m_{PS} and f_{PS}

- We can simultaneously fit lattice data for different lattice spacings, actions and volumes using expansions of the form (SU(2) NLO example):

$$(m_{ll}^{\mathbf{e}})^2 = \chi_l^{\mathbf{e}} + \chi_l^{\mathbf{e}} \cdot \left\{ \frac{16}{f^2}\left((2L_8^{(2)} - L_5^{(2)}) + 2(2L_6^{(2)} - L_4^{(2)})\right)\chi_l^{\mathbf{e}} + \frac{1}{16\pi^2 f^2}\chi_l^{\mathbf{e}} \log\frac{\chi_l^{\mathbf{e}}}{\Lambda_\chi^2} \right\}$$

$$f_{ll}^{\mathbf{e}} = f\left[1 + c_f (a^{\mathbf{e}})^2\right] + f \cdot \left\{ \frac{8}{f^2}(2L_4^{(2)} + L_5^{(2)})\chi_l^{\mathbf{e}} - \frac{\chi_l^{\mathbf{e}}}{8\pi^2 f^2}\log\frac{\chi_l^{\mathbf{e}}}{\Lambda_\chi^2} \right\}$$

with

$$\chi_l^{\mathbf{e}} = \frac{Z_l^{\mathbf{e}}}{R_a^{\mathbf{e}}}\frac{B^{\mathbf{1}}\tilde{m}_l^{\mathbf{e}}}{(a^{\mathbf{e}})^2}$$

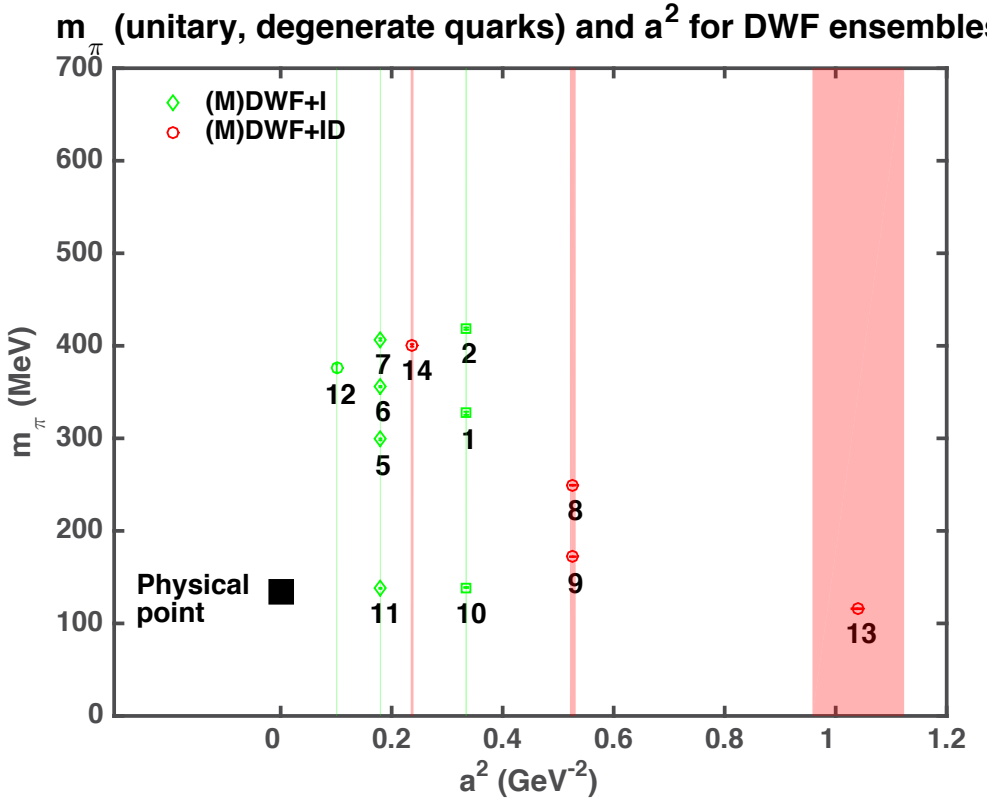
- At NNLO order, using codes from Bijnens and collaborators, we fit to

$$X(\tilde{m}_q, L, a^2) \simeq X_0\left(1 + \underbrace{X^{\mathrm{NLO}}(\tilde{m}_q) + X^{\mathrm{NNLO}}(\tilde{m}_q)}_{\text{NNLO Continuum PQChPT}} + \underbrace{\Delta_X^{\mathrm{NLO}}(\tilde{m}_q, L)}_{\text{NLO FV corrections}} + \underbrace{c_X a^2}_{\text{Lattice spacing}}\right)$$

- For SU(2), we use m_π, m_K and m_Ω to set the scale.
- There are different a^2 corrections to the decay constants for I and ID actions.
- Heavy quark ChPT used for light quark extrapolation of kaon.
- $t_0^{1/2}$ and w_0 are also fit using a linear chiral ansatz.

Some SU(2) ChPT Fit Details

- 2+1 flavor, (M)DWF ensembles of the RBC and UKQCD collaborations used in these fits.
- Volumes: $(2.0\ \text{fm})^3$ to $(5.5\ \text{fm})^3$
- $3.8 \leq m_\pi L \leq 5.8$.
- ~100 quark mass combinations for $m_{PS} \leq 510$ MeV
- m_{PS} and f_{PS} have statistical errors in the 0.1-0.4% range.
- Standard least-squares fitting
- Superjacknife for errors
- w_0 and t_0 not included in fit
- m_π cuts of 370 and 450 MeV used for both NLO and NNLO



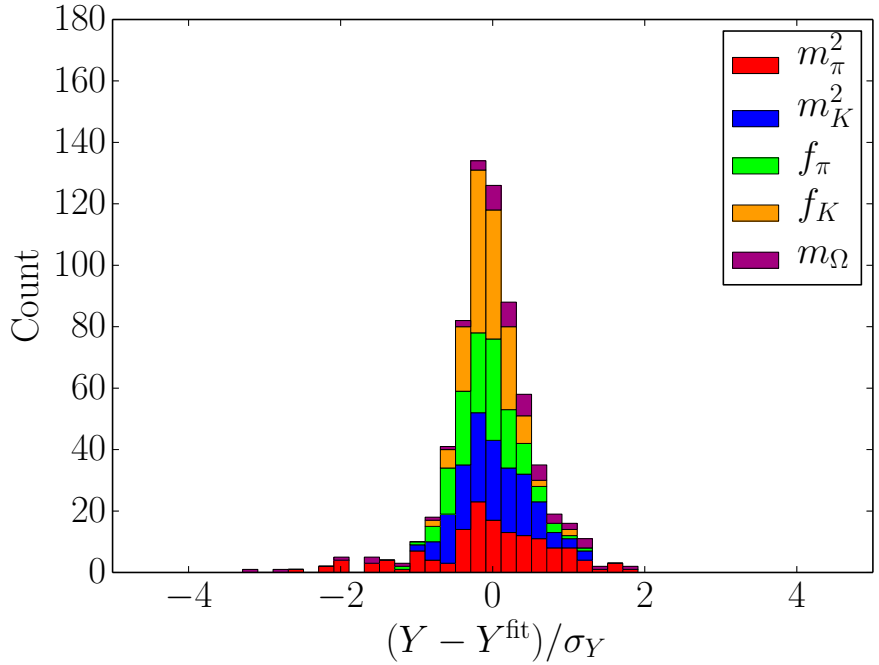
- Data is reweighted in dynamical strange quark mass to the self-consistently determined physical strange quark mass.

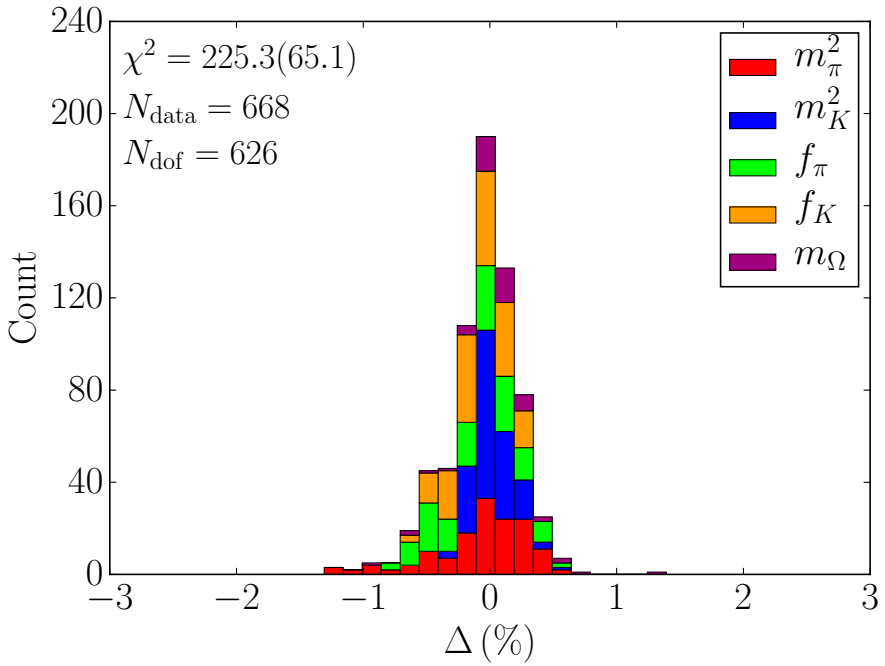
SU(2): Compare NLO @ 370 MeV with NNLO @ 450 MeV

LEC	Λ_χ	NLO (370 MeV cut)	NNLO (450 MeV cut)
B	—	4.229(35) GeV	4.203(44) GeV
f		0.1213(15) GeV	0.1215(16) GeV
$10^3 \hat{L}_0^{(2)}$	1 GeV	—	1.0(1.1)
$10^3 \hat{L}_1^{(2)}$		—	-0.62(52)
$10^3 \hat{L}_2^{(2)}$		—	0.06(74)
$10^3 \hat{L}_3^{(2)}$		—	-1.56(87)
$10^3 \hat{L}_4^{(2)}$		-0.211(79)	-0.56(22)
$10^3 \hat{L}_5^{(2)}$		0.438(72)	0.60(28)
$10^3 \hat{L}_6^{(2)}$		-0.175(48)	-0.38(10)
$10^3 \hat{L}_7^{(2)}$		—	-0.75(27)
$10^3 \hat{L}_8^{(2)}$		0.594(36)	0.69(13)

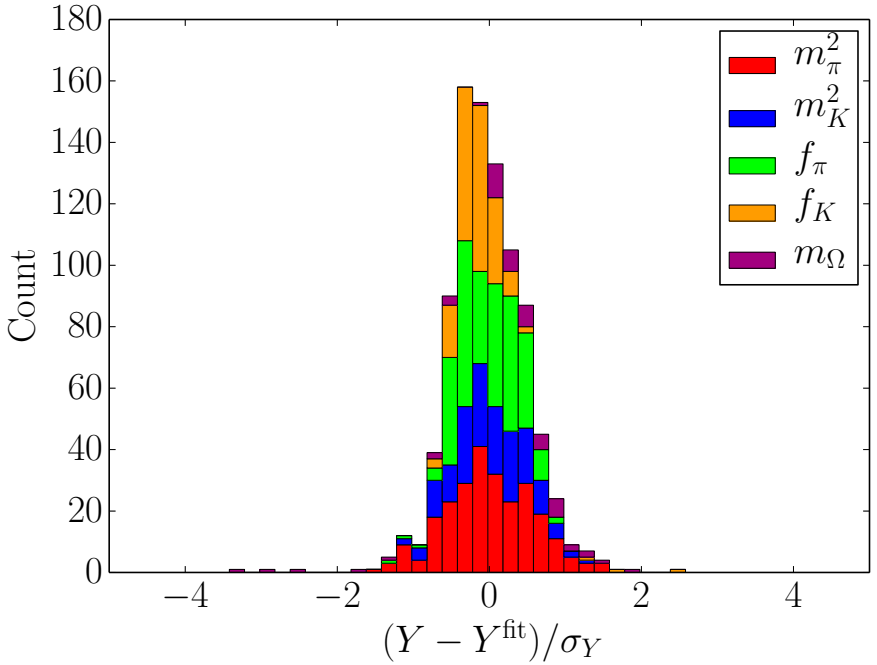
Histograms of Deviations Between Fits and Data

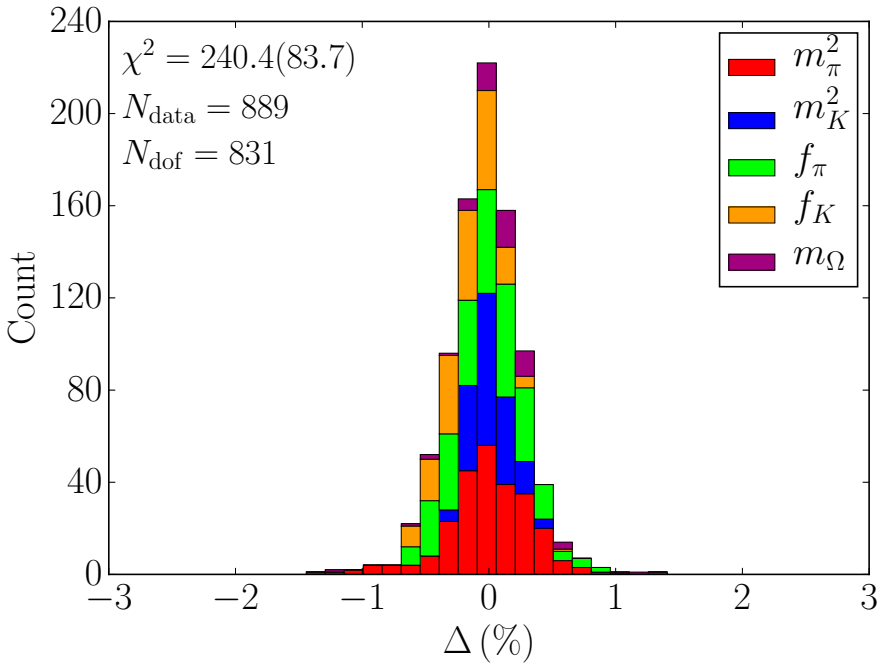
NLO 370 MeV Cut





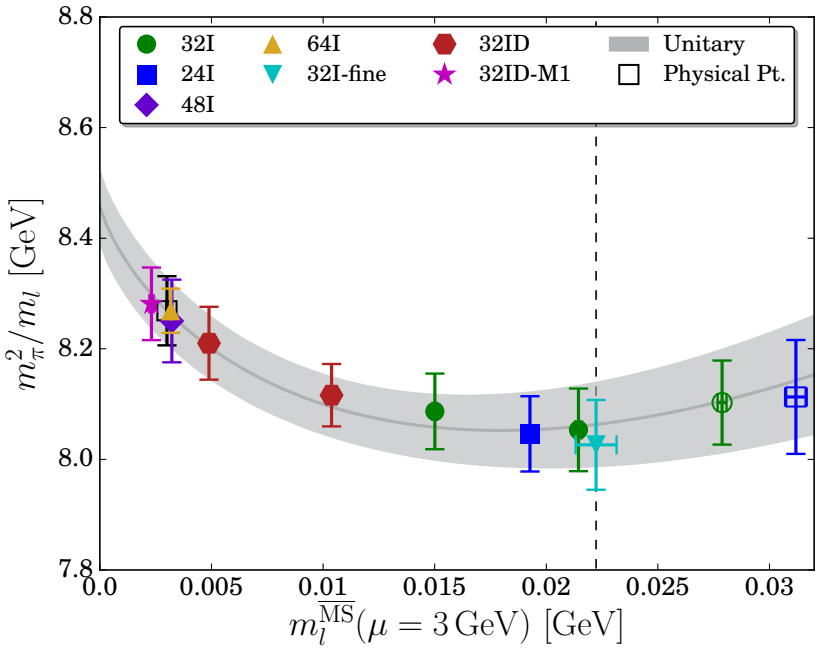
NNLO 450 MeV Cut



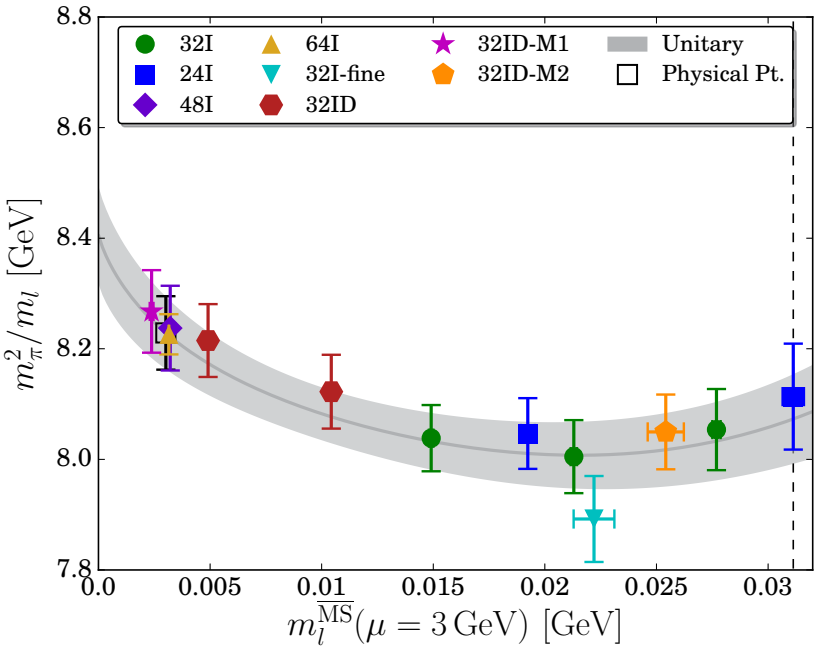


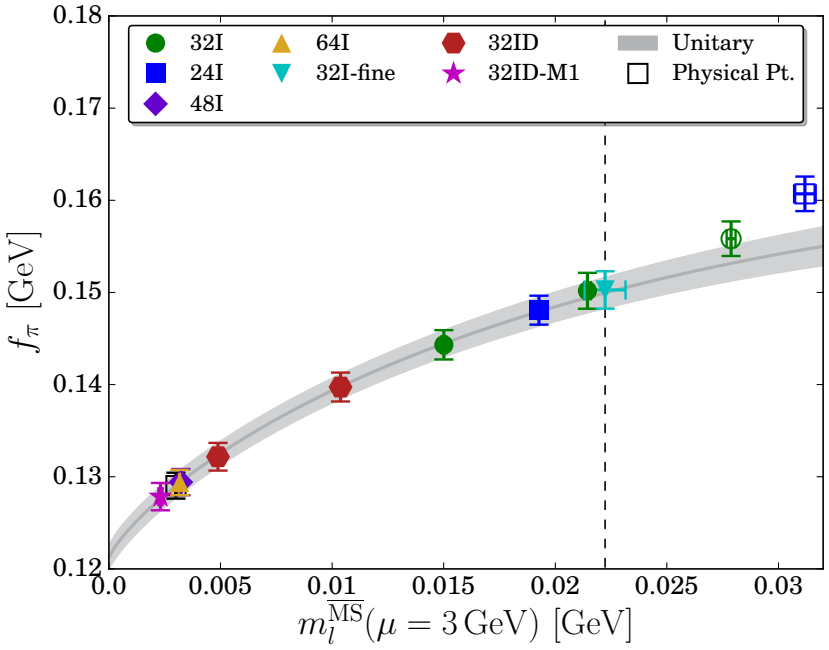
SU(2) Fits and Data for m_π^2 and f_π

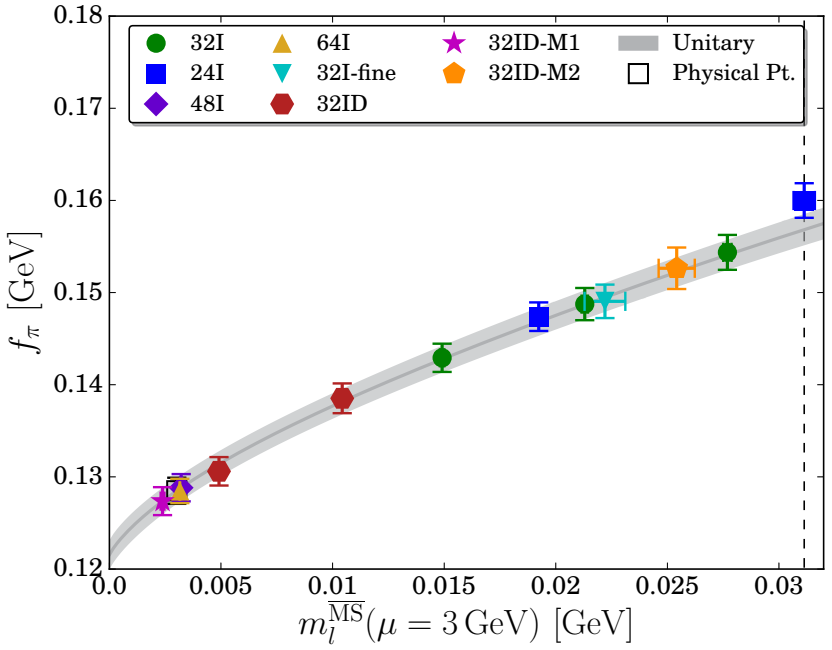
NLO 370 MeV Cut



NNLO 450 MeV Cut



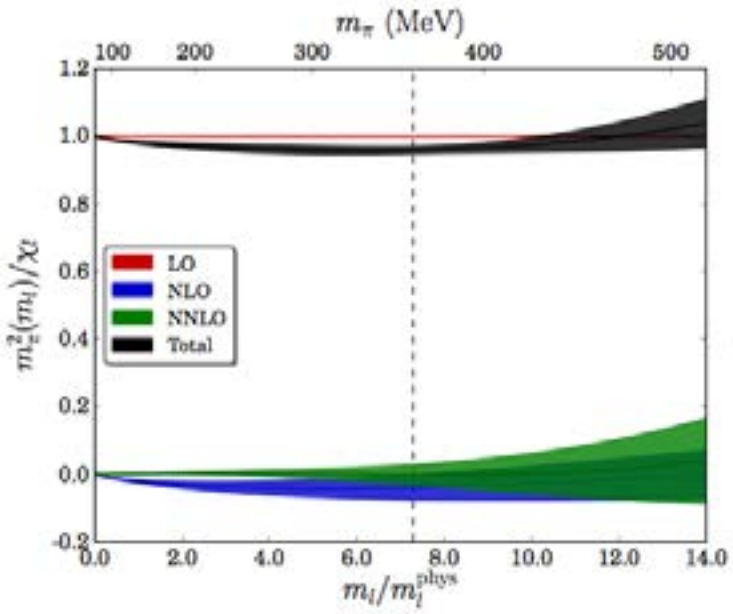


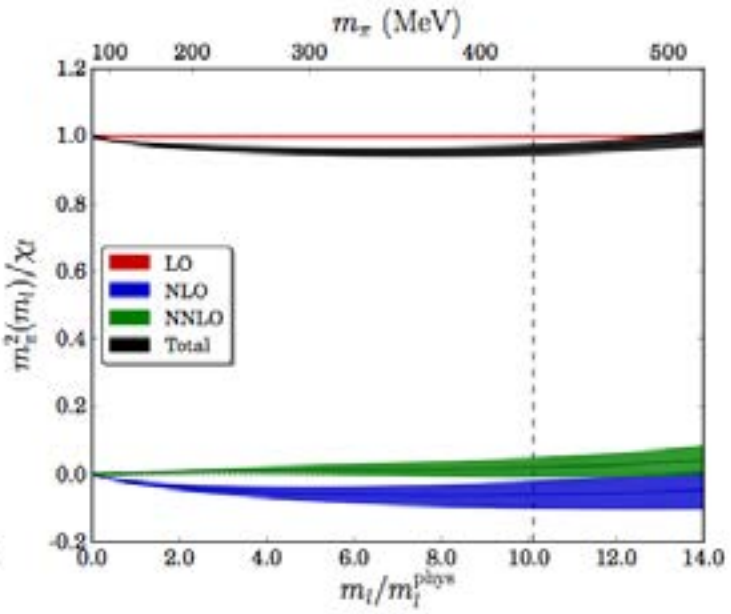


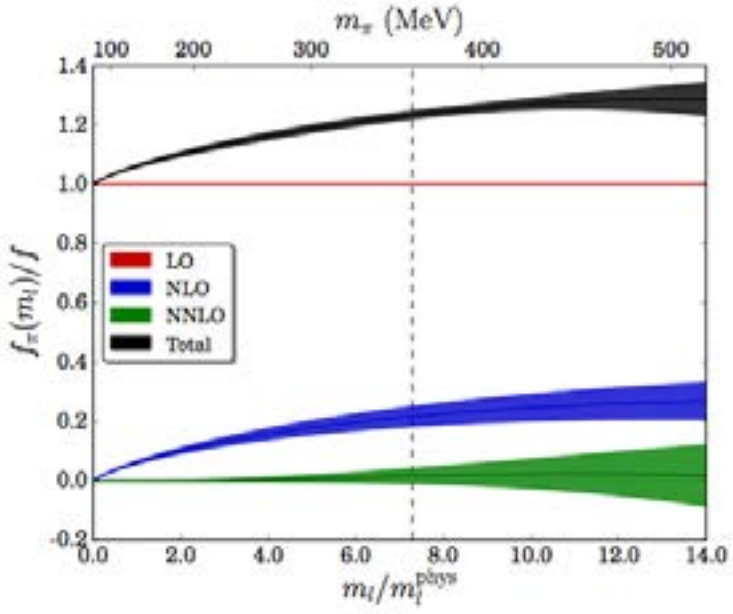
Convergence of SU(2) Fits and Data for m_π^2 and f_π

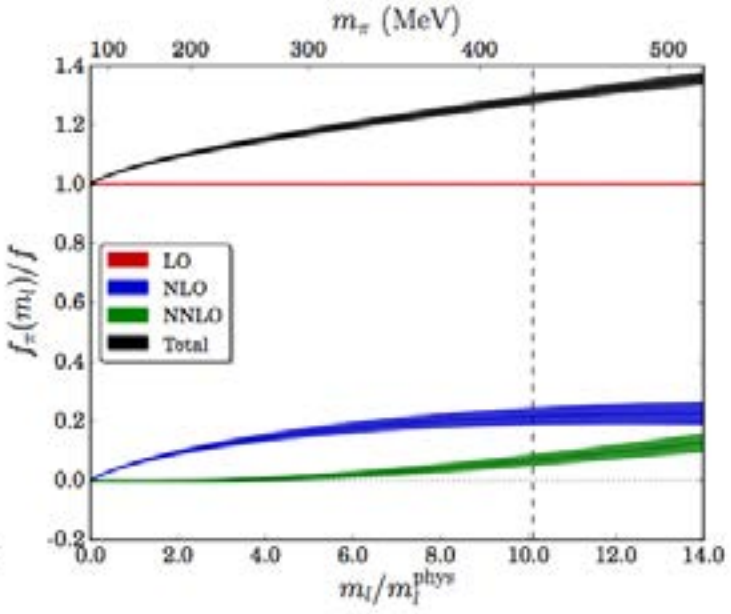
NLO 370 MeV Cut

NNLO 450 MeV Cut

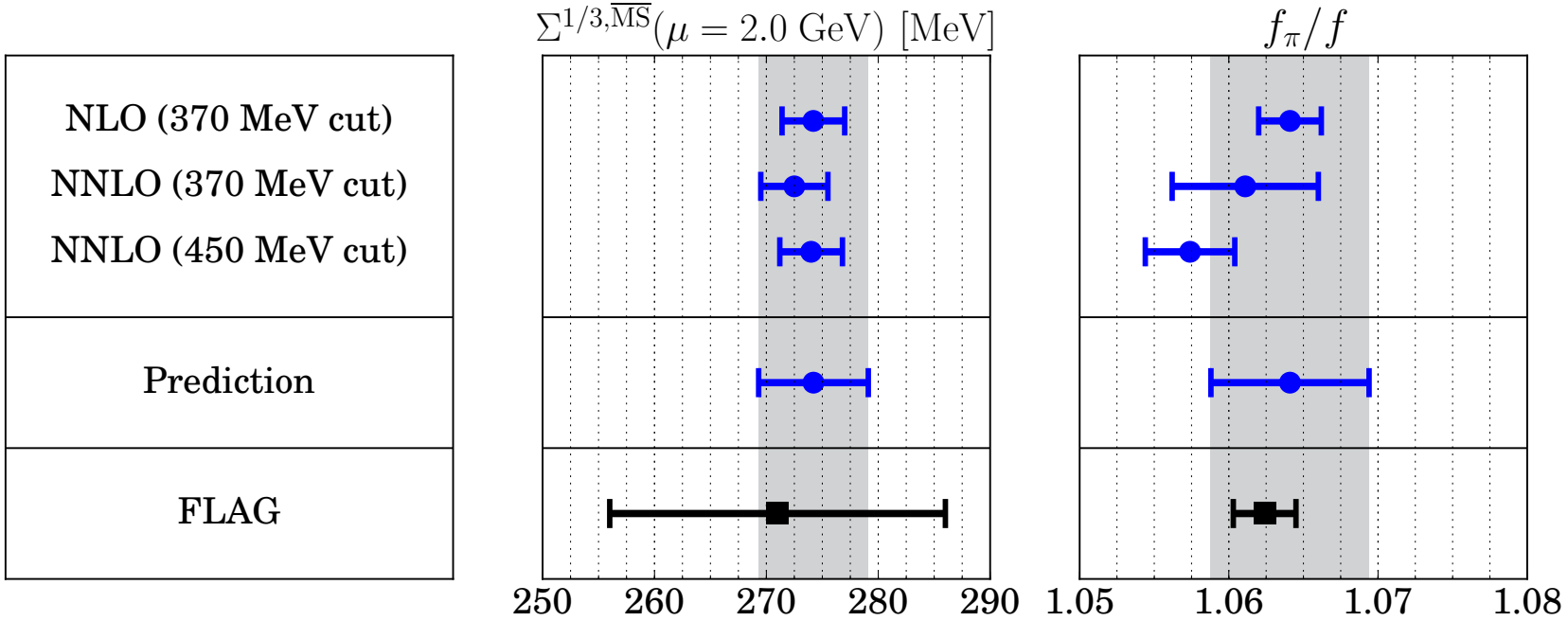


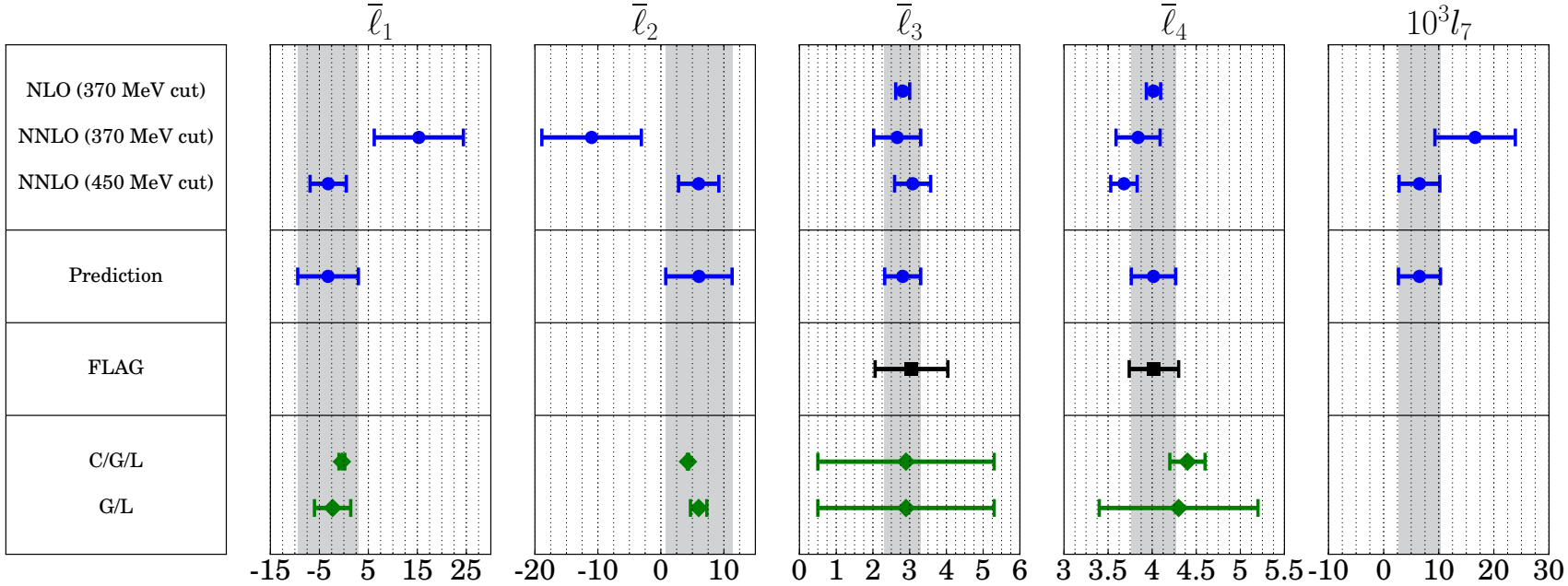






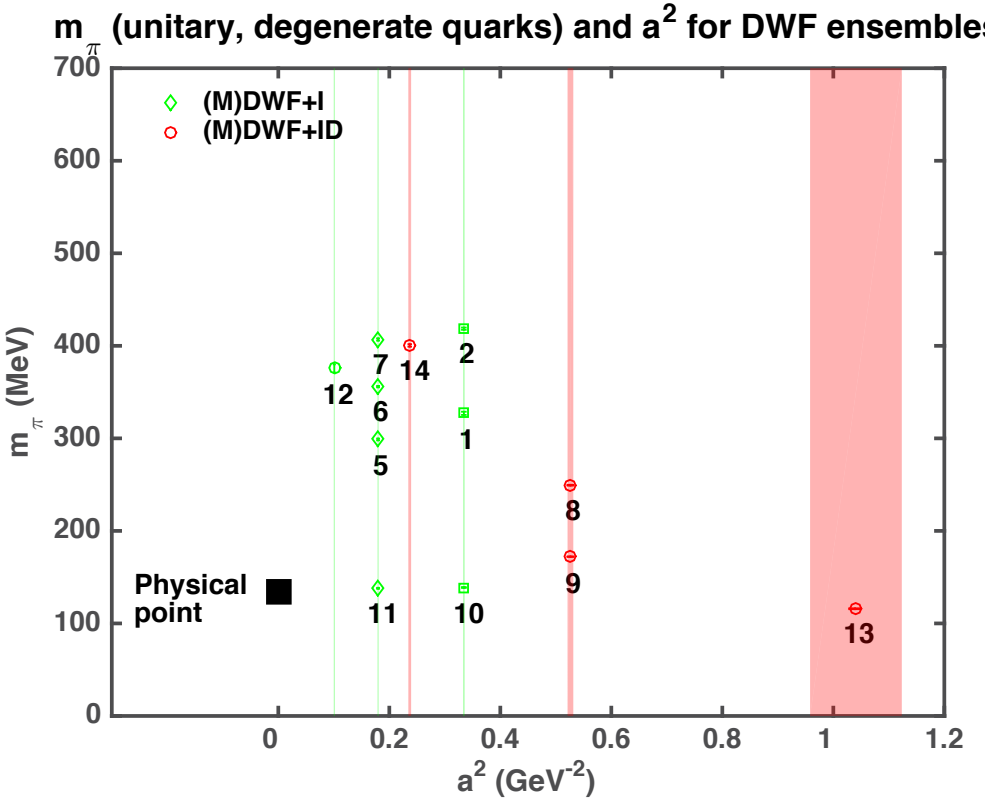
LO and NLO LEC's from SU(2)





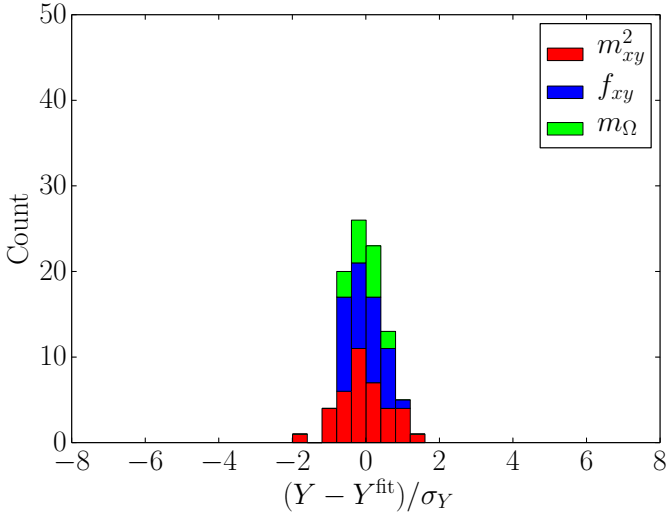
SU(3) Fits

- 2+1 flavor, (M)DWF ensembles of the RBC and UKQCD collaborations used in these fits.
- Volumes: $(2.0\ \text{fm})^3$ to $(5.5\ \text{fm})^3$
- $3.8 \leq m_\pi L \leq 5.8$.
- ~100 quark mass combinations for $m_{PS} \leq 510$ MeV
- m_{PS} and f_{PS} have statistical errors in the 0.1-0.4% range.
- Standard least-squares fitting
- Superjacknife for errors
- w_0 and t_0 not included in fit
- m_π cuts of 370 and 510 MeV used for both NLO and NNLO
- m_π, f_π and m_Ω used to set the scale. No $O(a^2)$ terms included.

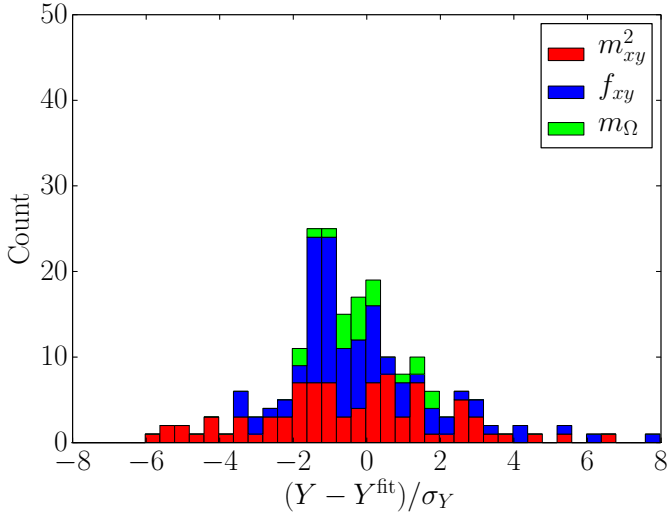


- For NLO fits, $f_0 \sim 113$ MeV
- For NNLO fits, $f_0 \sim 128$ MeV
- Indicates not enough data or not very convergent or both.
- Our choice: freeze NNLO values for f_0 and B_0 to NLO result for each jacknife block

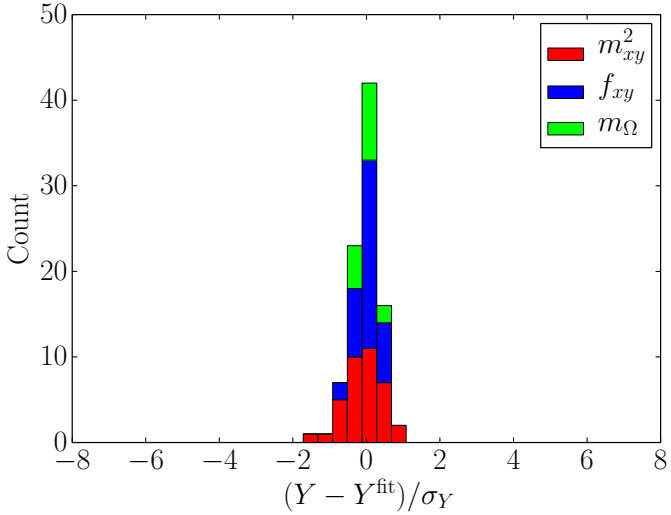
Histograms of Deviations in Units of σ



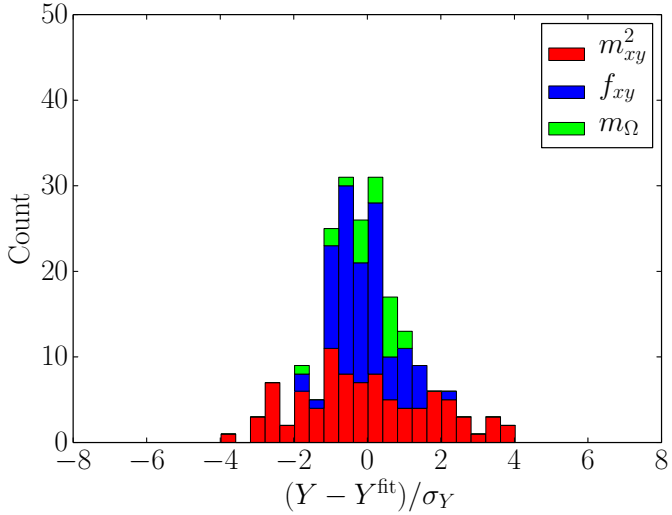
(a) NLO, $m_{xy}^{\mathrm{cut}} = 370\,\mathrm{MeV}$



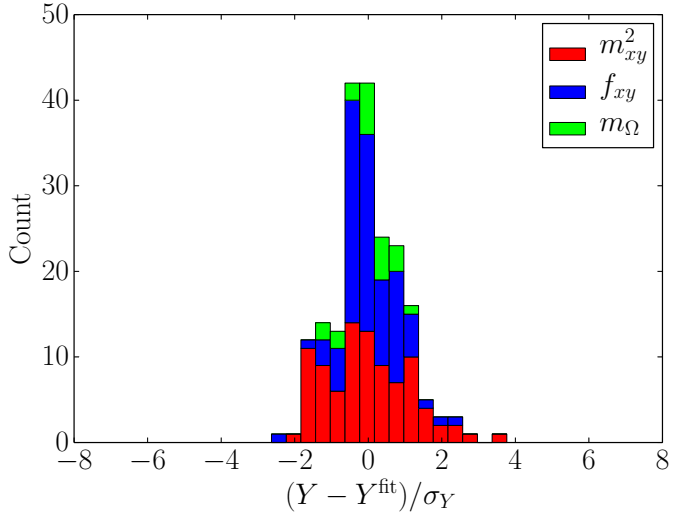
(b) NLO, $m_{xy}^{\mathrm{cut}} = 510\,\mathrm{MeV}$



(c) NNLO, $m_{xy}^{\mathrm{cut}} = 370\,\mathrm{MeV}$

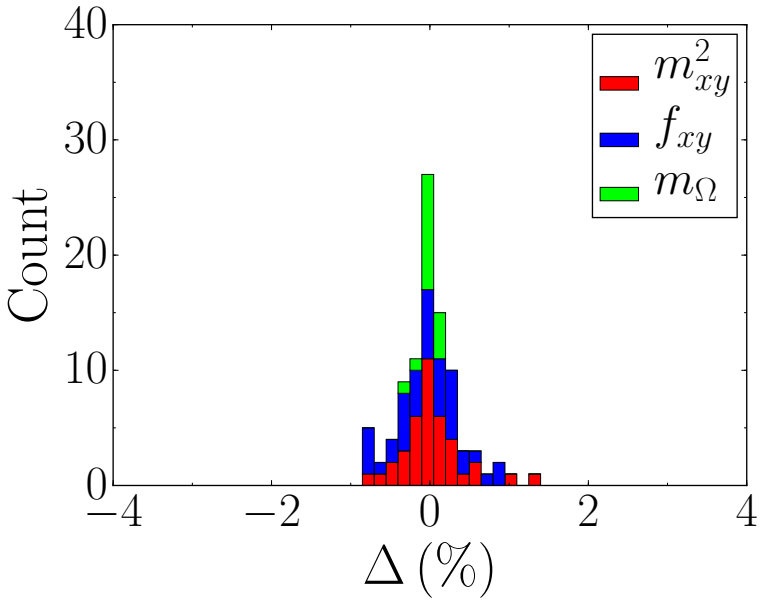


(d) NNLO with frozen LO LECs, $m_{xy}^{\mathrm{cut}} = 510\,\mathrm{MeV}$

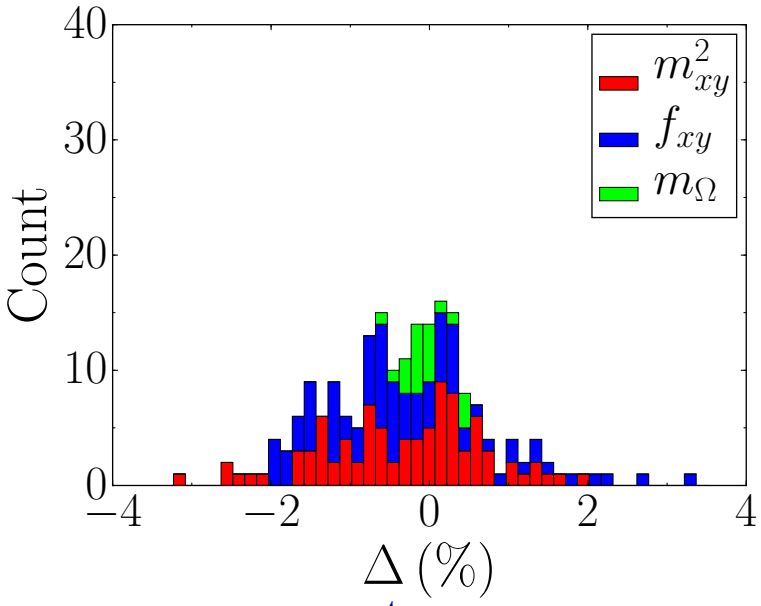


(e) NNLO, $m_{xy}^{\mathrm{cut}} = 510\,\mathrm{MeV}$

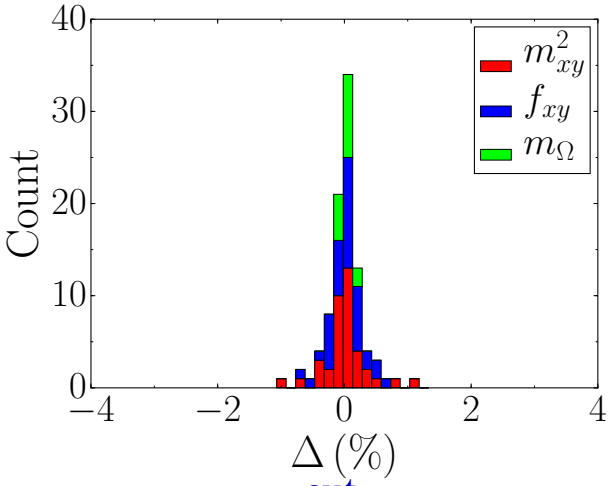
Histograms of Deviations in %



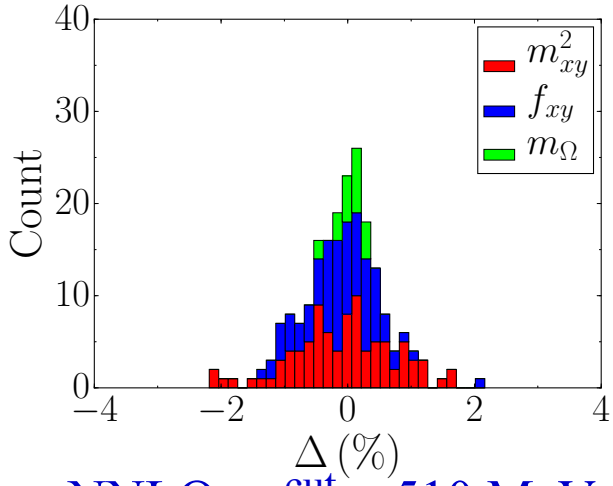
NLO, m^{cut} = 370 MeV



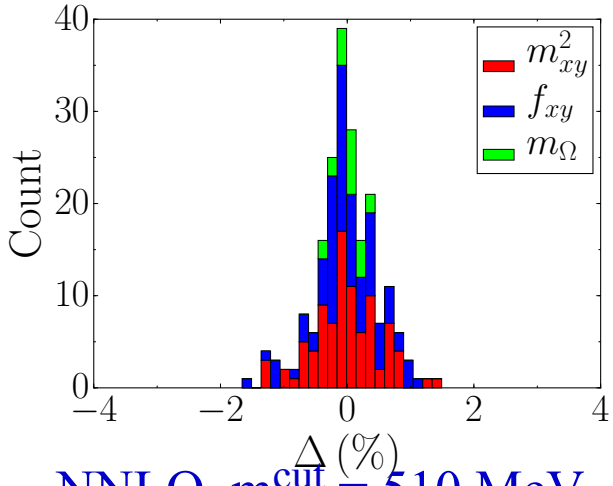
NLO, m^{cut} = 510 MeV



NNLO, m^{cut} = 370 MeV



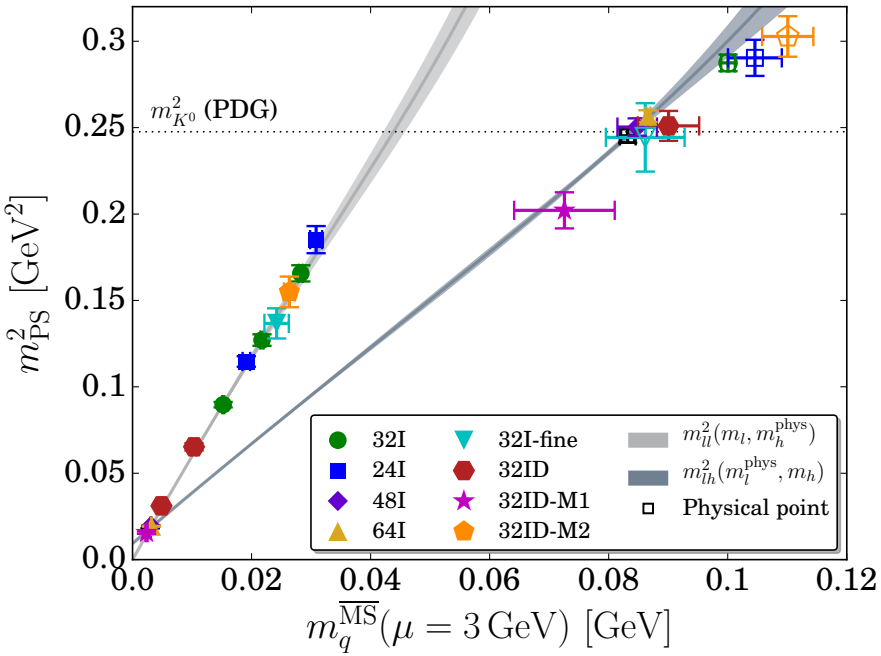
NNLO, m^{cut} = 510 MeV,
Frozen LO LECs

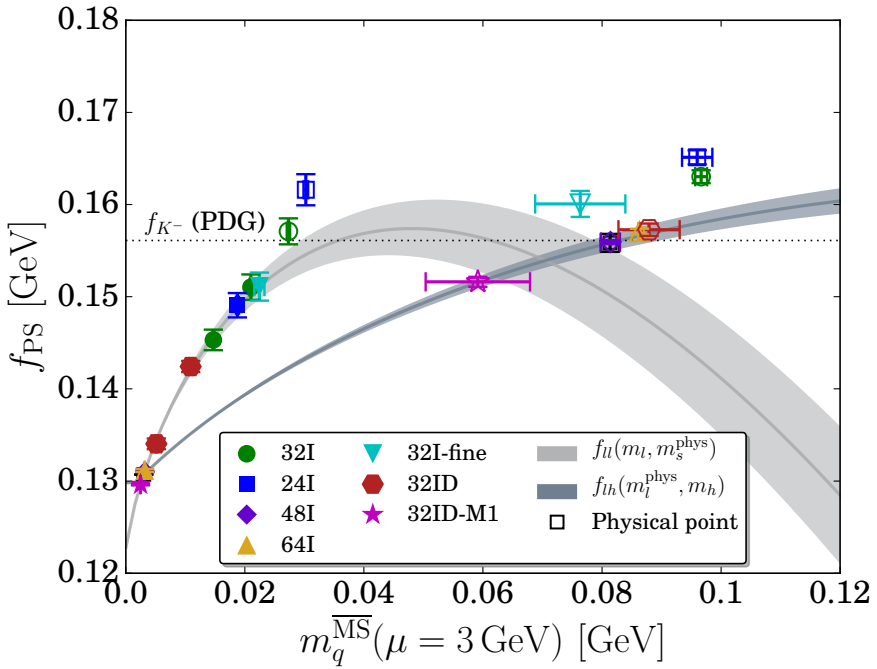


NNLO, m^{cut} = 510 MeV

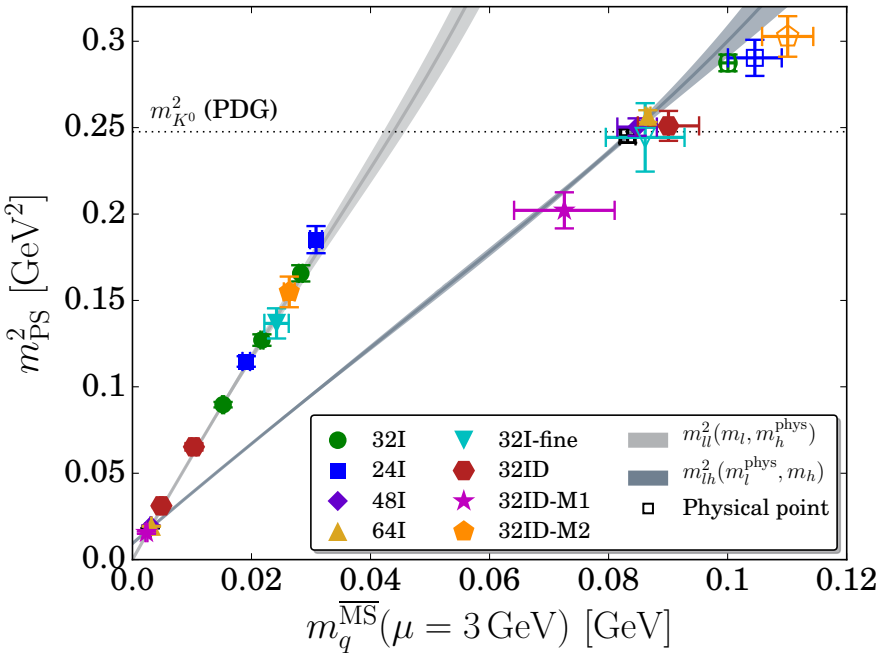
SU(3) Fits and Data for m_{PS}^2 and f_{PS}

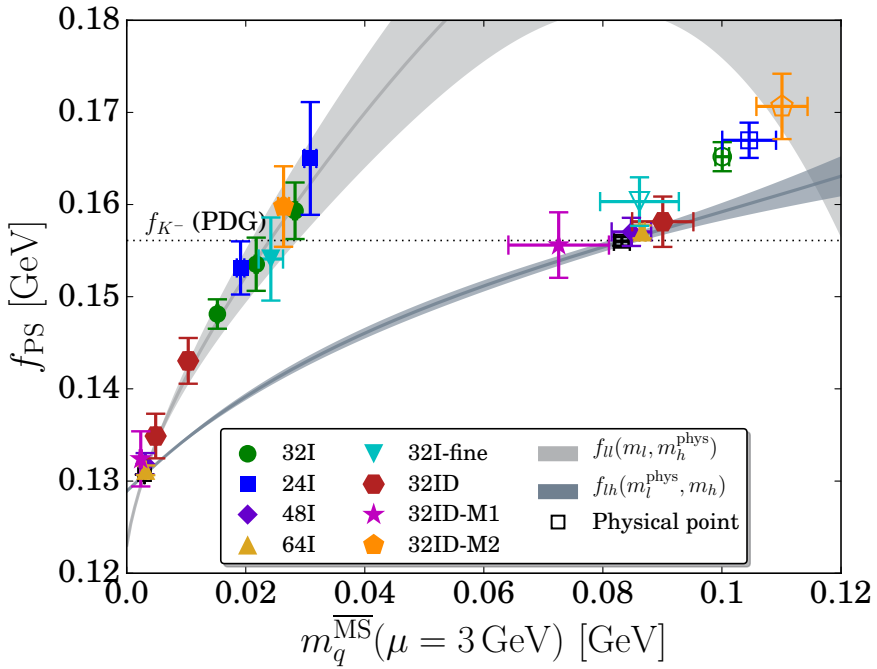
NLO 370 MeV Cut





NNLO 510 MeV Cut (Frozen)





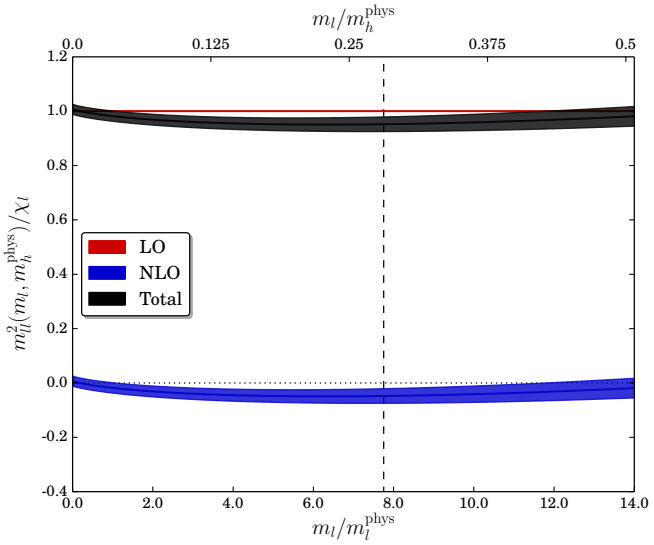
Assessing Reliability of Expansion

- The data shows $m_\pi^2 \sim m_f$ to be very accurate over a large range of masses
- Therefore NLO corrections to m_π^2 must be small
- For NNLO fits must have either or both:
  - NLO terms ~ -(NNLO terms)
  - Both NLO and NNLO terms small
- This means the series for m_π^2 will not have $|\mathrm{LO}| > |\mathrm{NLO}| > |\mathrm{NNLO}|$
- To be reasonably reliable, it should have $|\mathrm{LO}| > |\mathrm{NLO} + \mathrm{NNLO}|$
- f_π can then be used to judge reliability of expansion. If reliable, then should find $|\mathrm{LO}| > |\mathrm{NLO}| > |\mathrm{NNLO}|$ and $|\mathrm{LO}| > |\mathrm{NLO} + \mathrm{NNLO}|$

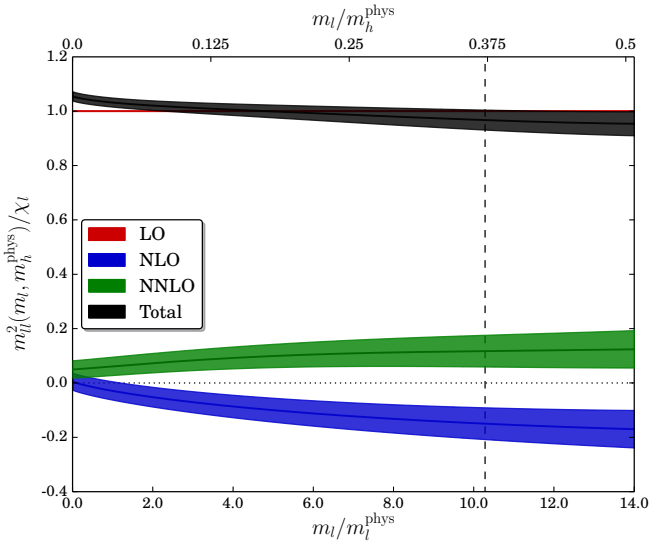
Decomposition of ChPT Expansion: $m_h = m_s$

NLO 370 MeV Cut

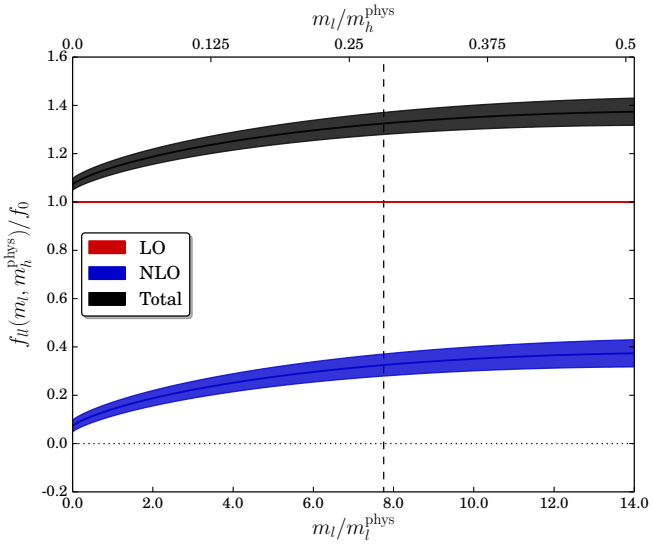
NNLO 510 MeV Cut (Frozen)



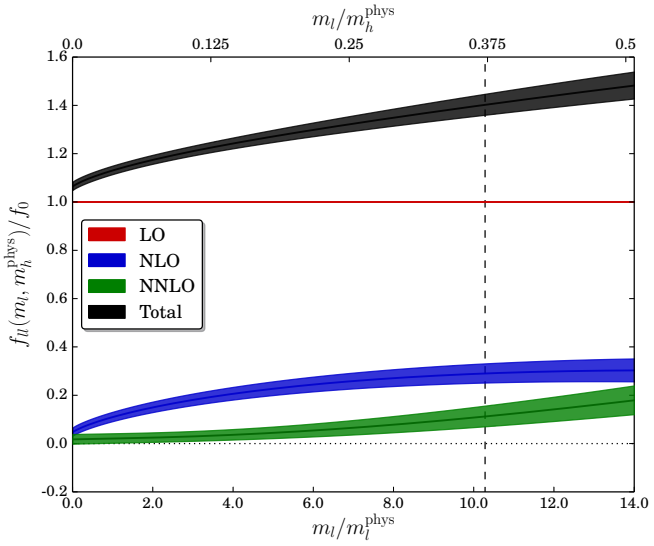
(a) NLO, $m_{xy}^{\rm cut} = 370\,\text{MeV}$



(b) NNLO with frozen LO LECs, $m_{xy}^{\rm cut} = 510\,\text{MeV}$

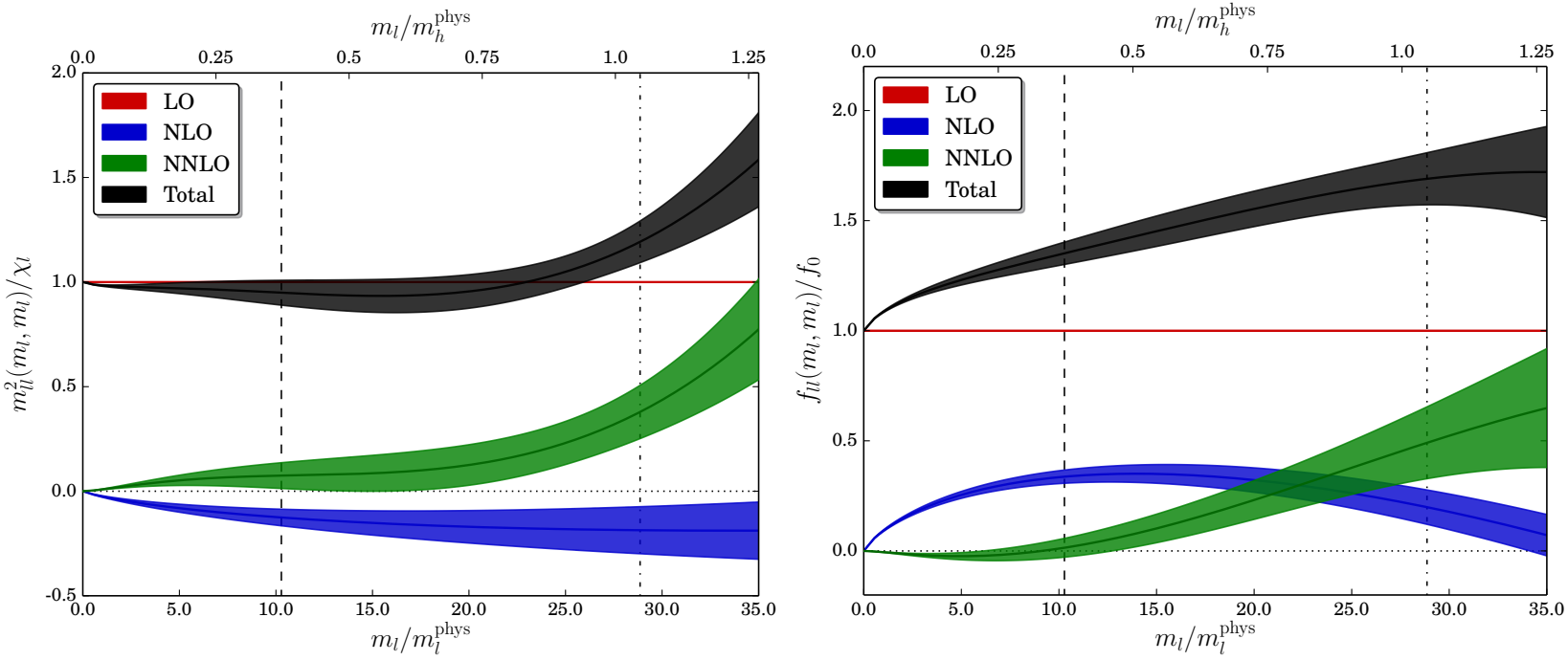


(c) NLO, $m_{xy}^{\rm cut} = 370\,\text{MeV}$



(d) NNLO with frozen LO LECs, $m_{xy}^{\rm cut} = 510\,\text{MeV}$

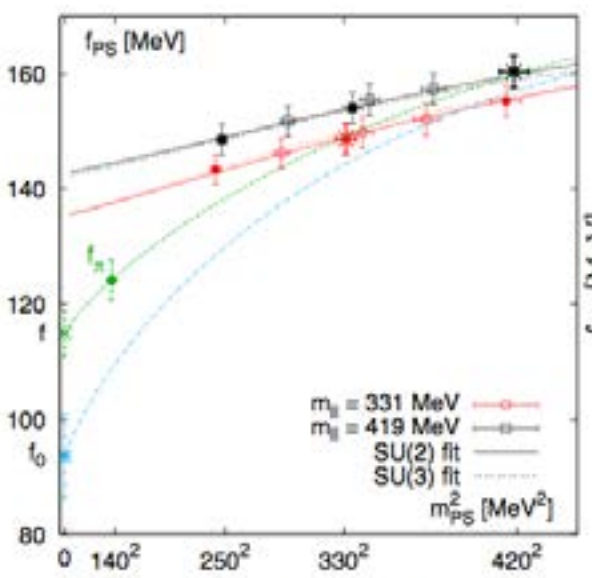
Decomposition of ChPT Expansion: SU(3) symmetric



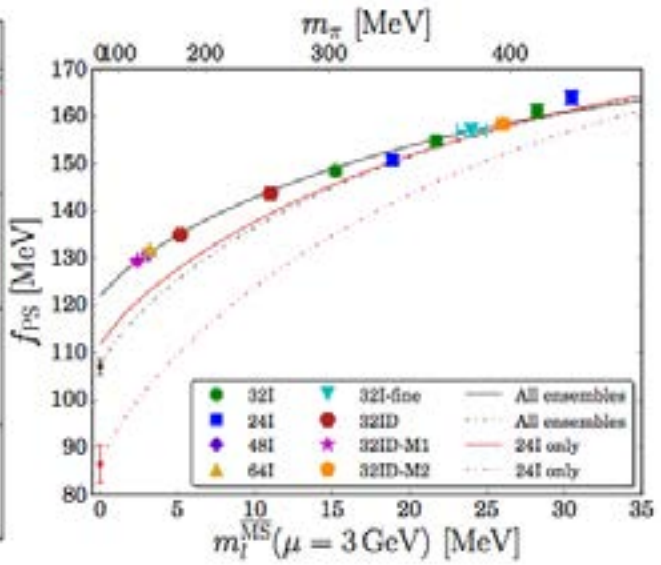
- For f_π see NLO = NNLO at $m_l/m_l^{\rm phys} = 20$.
- Series appears well behaved until $m_l/m_l^{\rm phys} = 15$, which is slightly above m_K.

Comparison with 2008 NLO RBC/UKQCD Fits

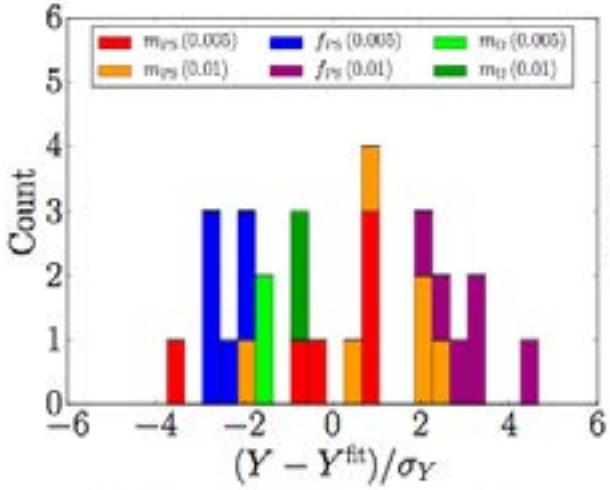
- Top left graph is previous result, with SU(3) chiral limit in blue and SU(2) in green.
- Top right is current NLO fit and also result when removing light masses from fit.
- Lower left histogram shows that f_{PS} is above the fit for one ensemble and below for the other.
- Lower right shows that f_0 varies smoothly with the lowest mass used in the fit.



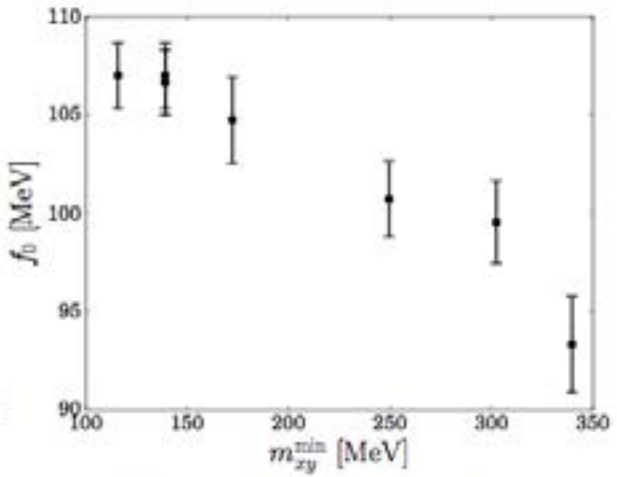
(a) Reprinted from [17]



(b) This work

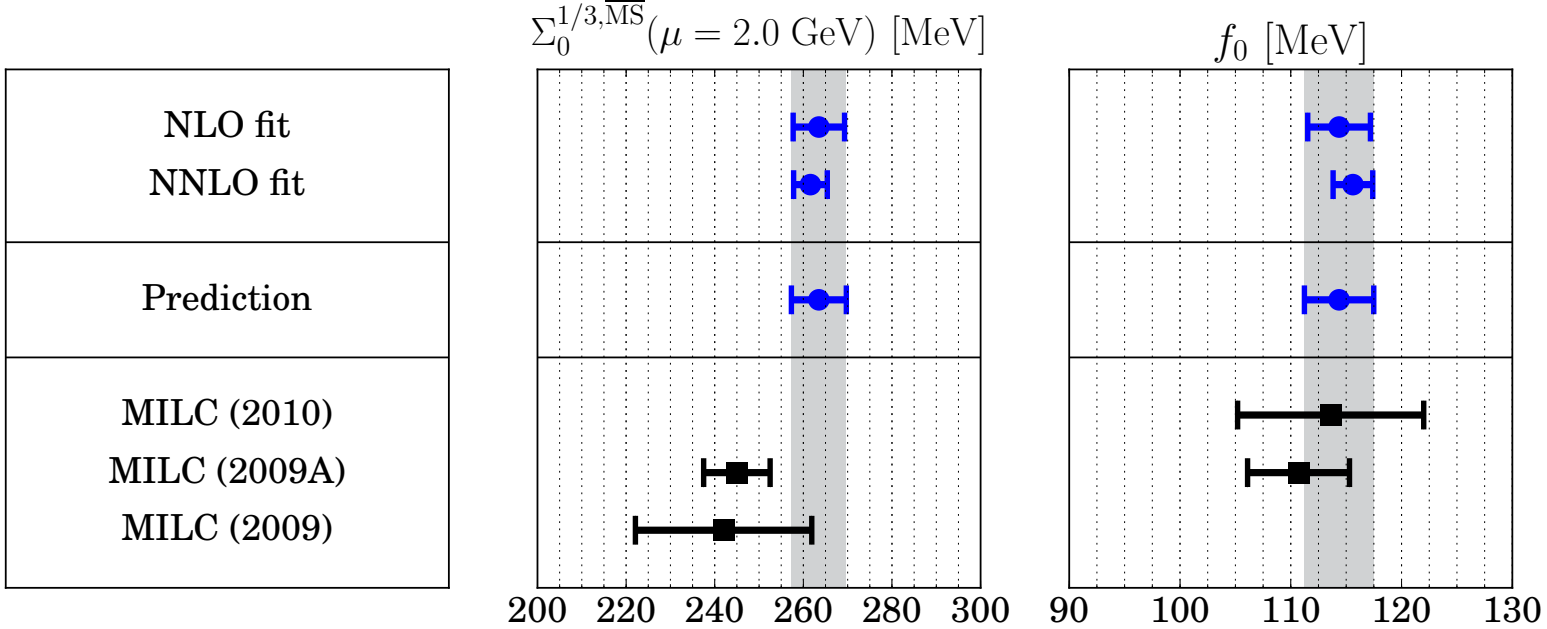


(c) 24I data from the "all ensembles" fit

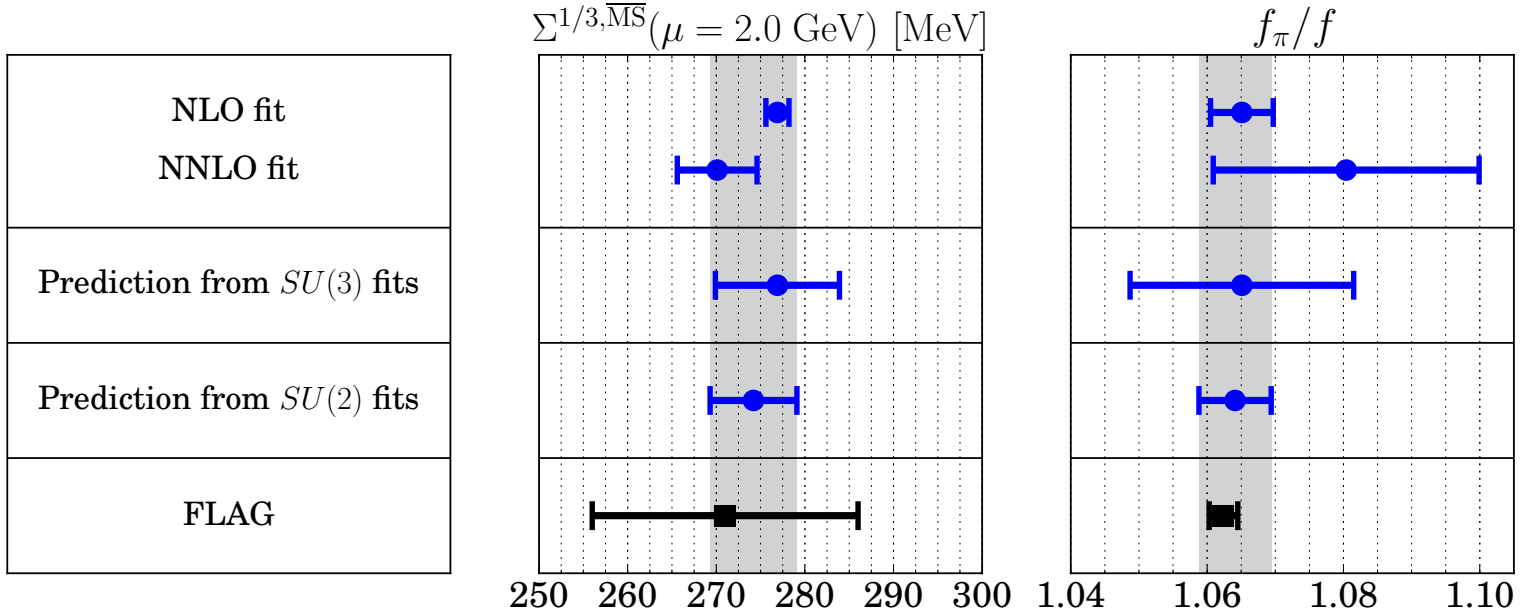


(d) Dependence of f_0 from "all ensembles" fit on lower mass cut

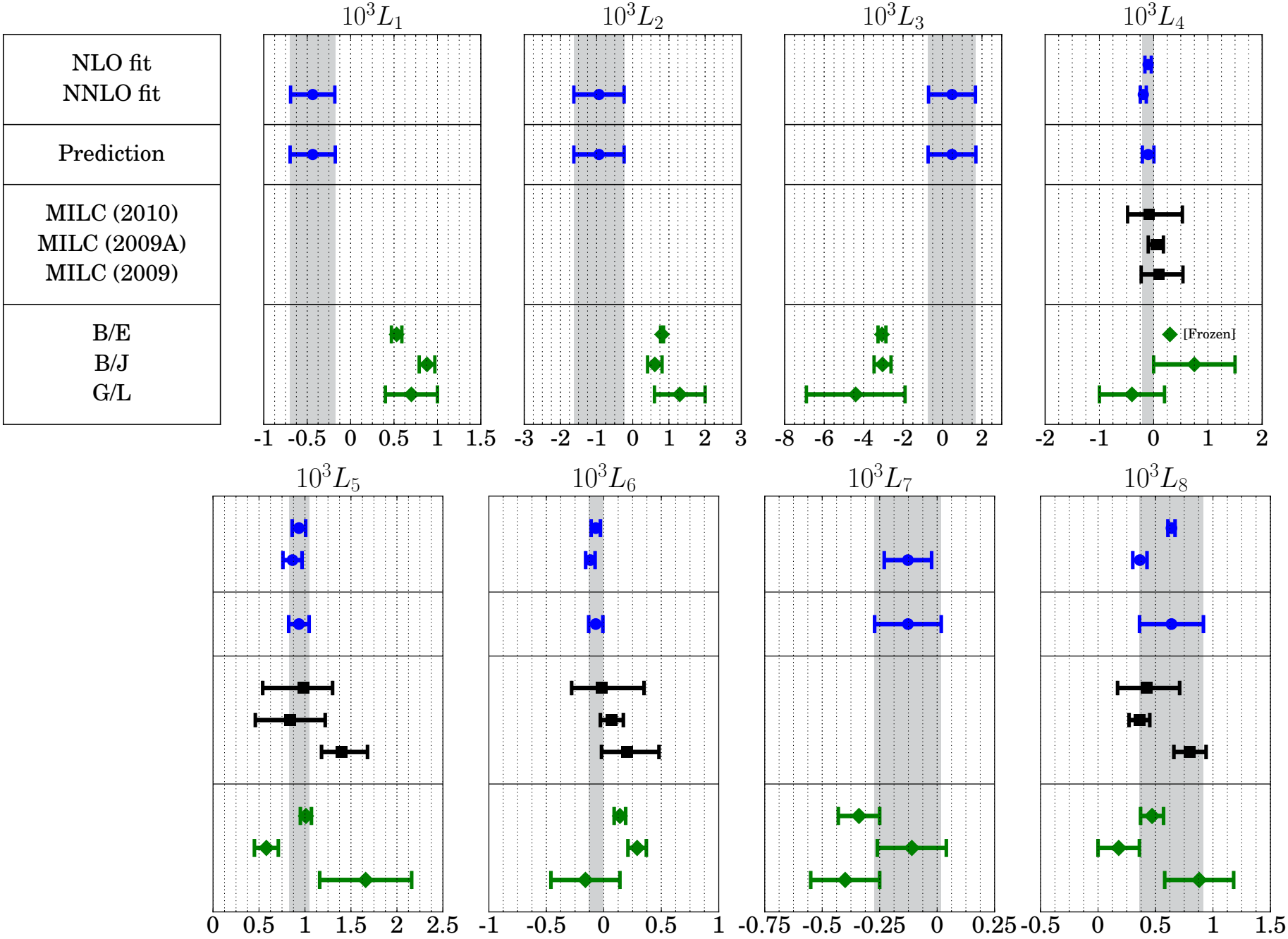
Results for SU(3) LO and SU(2) LO LECs from SU(3) Fits



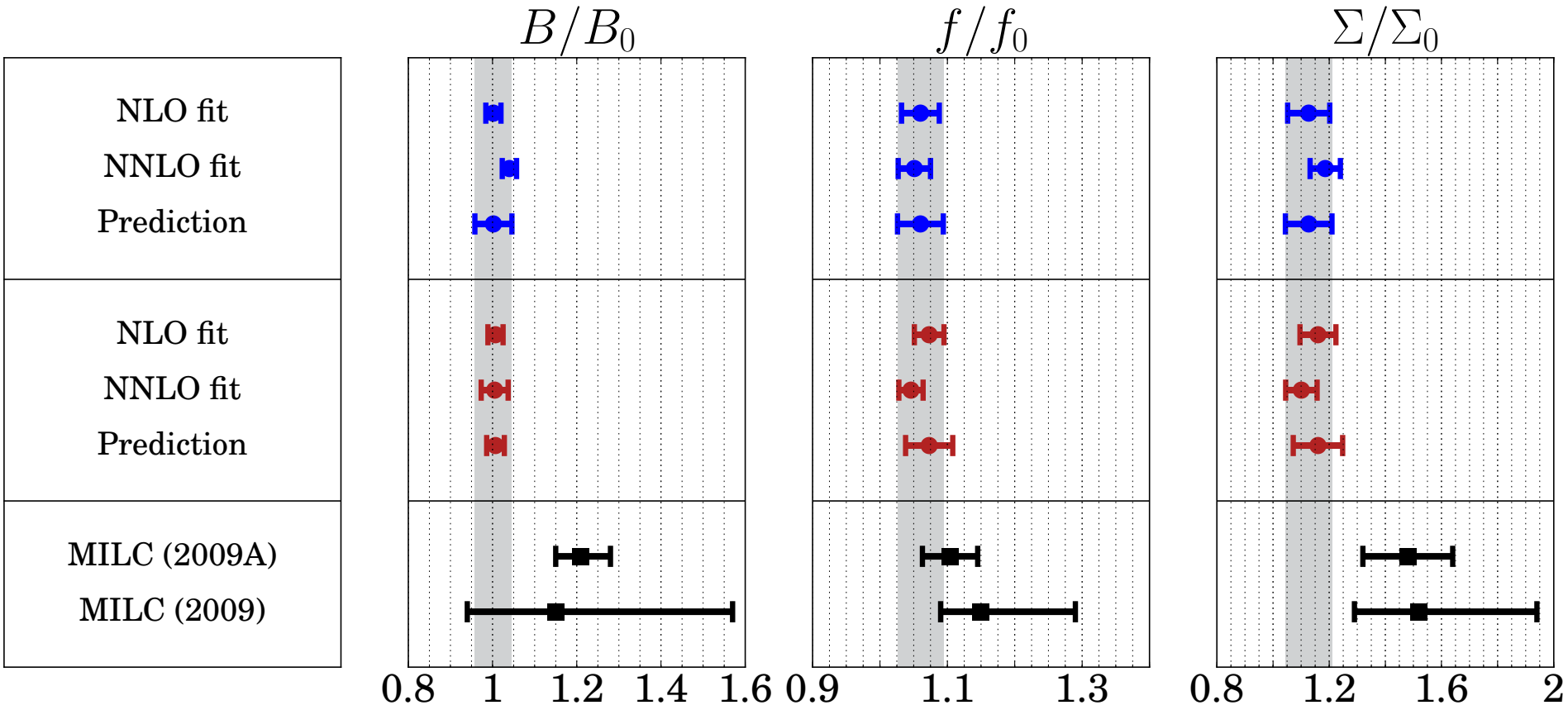
SU(2) LECs from SU(3) fit



Results for SU(3) NLO LECs

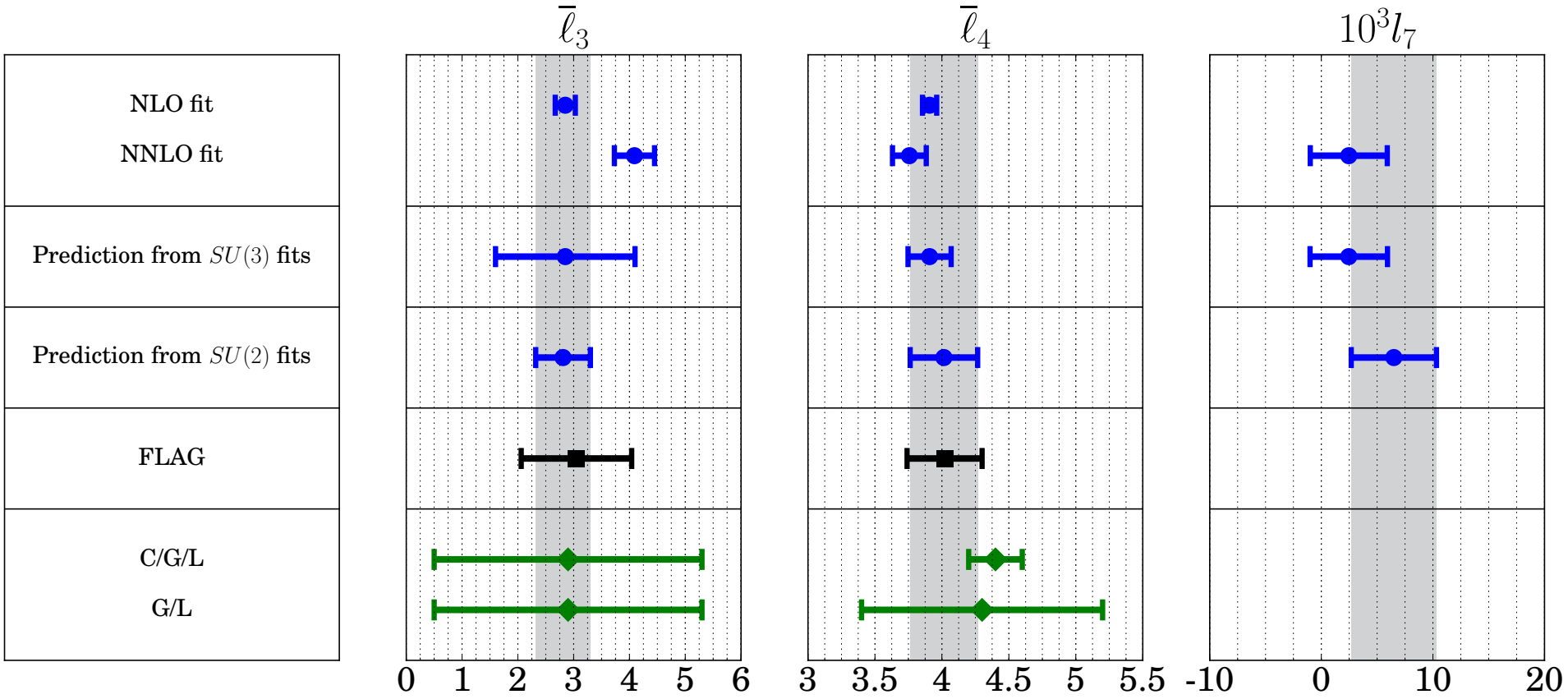


Ratios of SU(2) LO LECs to SU(3)



- Top group of points (blue) are ratios from SU(2) fits and SU(3) fits
- Second group of points (red) are from SU(3) fits only, using 1-loop conversion of SU(3) to SU(2).
- Ratios are closer to 1 than previous estimates
- Should be 1 in large N_c limit

Predictions for NLO SU(2) LECs from SU(3) ChPT Fits



Predictions from SU(3) ChPT Fits

			Free		Frozen LO LECs
	NLO+FV (370 MeV)	NLO+FV (510 MeV)	NNLO+FV (370 MeV)	NNLO+FV (510 MeV)	NNLO+FV (510 MeV)
m_K	0.5171(64) GeV	0.4913(29) GeV	0.479(70) GeV	0.4982(30) GeV	0.4952(41)
f_K	0.15584(97) GeV	0.15566(20) GeV	0.160(42) GeV	0.15562(47) GeV	0.15601(49) GeV
f_K/f_0	1.363(36)	1.390(20)	1.25(39)	1.221(22)	1.349(22)
$[m^2_{K^0} - m^2_{K^\pm}]_{\rm QCD}/\Delta m_{du}$	5.44(24) GeV	3.658(62) GeV	1.75(93) GeV	3.46(28) GeV	2.74(39) GeV
$[\frac{f_{K^0}}{f_{K^\pm}} - 1]_{\rm QCD}/\Delta m_{du}$	3.01(13) GeV^{-1}	3.068(32) GeV^{-1}	1.9(1.9) GeV^{-1}	2.48(19) GeV^{-1}	2.72(27) GeV^{-1}
$[m^2_{\pi^\pm} - m^2_{\pi^0}]_{\rm QCD}/\Delta m^2_{du}$	—	—	45(45)	18(14)	11(16)
$m_\pi a^{I=0}_{\pi\pi}$	—	—	0.153(21)	0.1610(86)	0.1991(65)
$m_\pi a^{I=2}_{\pi\pi}$	—	—	-0.0376(58)	-0.0402(17)	-0.0449(18)
$m_\pi a^{I=1/2}_{\pi K}$	—	—	0.124(18)	0.1435(56)	0.1376(92)
$m_\pi a^{I=3/2}_{\pi K}$	—	—	-0.067(14)	-0.0781(47)	-0.0671(84)

Table 18: Predictions from NLO and NNLO fits and $SU(3)$ ChPT. $\Delta m_{du} \equiv m_d - m_u$.

- Since m_π, f_π and m_Ω are used to set the scale, m_K and f_K are predictions
- π-K scattering lengths are also being calculated directly by RBC/UKQCD - see (T. Janowski, Lattice 2015). The preliminary results are $m_\pi a^{I=1/2}_{\pi K} = 0.16\,(2)$ and $m_\pi a^{I=3/2}_{\pi K} = -0.06\,(1)$. Good agreement between these methods.

Conclusions

- SU(2) NLO (350 MeV cut) and NNLO (450 MeV cut) fits are quite robust and accurate at the 1% level
- SU(3) NLO (370 MeV cut) and NNLO (510 MeVcut) well represent data and appear reasonably reliable.
- Good agreement between SU(3) NLO LECs and SU(2)
- Find ratio of LO SU(2)/SU(3) LECs close to 1
- David has measured ππ scattering amplitudes and is adding these into the fits, to further constrain LECs.

SU(2)

$B^{\overline{\rm MS}}(\mu = 2\,\text{GeV})$	**2.804(34)(40) GeV**
f	**121.3(1.5)(2.1) MeV**
$\Sigma^{1/3,\,\overline{\rm MS}}(\mu = 2\,\text{GeV})$	**274.2(2.8)(4.0) MeV**
f_π/f	**1.0641(21)(49)**
$\bar{\ell}_1$	−3.2(3.7)(5.0)
$\bar{\ell}_2$	6.0(3.2)(4.2)
$\bar{\ell}_3$	**2.81(19)(45)**
$\bar{\ell}_4$	**4.02(8)(24)**
$10^3 l_7$	6.5(3.8)(0.2)

SU(3)

$B_0^{\overline{\rm MS}}(\mu = 2\,\text{GeV})$	**2.80(7)(13) GeV**
f_0	**114.4(2.8)(1.3) MeV**
$\Sigma_0^{\overline{\rm MS}}(\mu = 2\,\text{GeV})$	**263.5(5.8)(2.4) MeV**
$10^3 L_1$	−0.44(25)(5)
$10^3 L_2$	−0.93(69)(4)
$10^3 L_3$	0.5(1.2)(0.2)
$\mathbf{10^3 L_4}$	**−0.102(59)(89)**
$\mathbf{10^3 L_5}$	**0.934(73)(83)**
$\mathbf{10^3 L_6}$	**−0.070(40)(47)**
$10^3 L_7$	−0.13(10)(1)
$\mathbf{10^3 L_8}$	**0.64(3)(28)**