Charm quark mass extraction from charmonium current correlators with Mobius Domain Wall fermion

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for JLQCD Collaboration

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◇ Motivation

- Essential input parameters to precisely determine…

(e.g.) Higgs partial widths

Future experiments (ILC,HL-LHC)

$$\delta m_c \sim 0.6\%, \delta\alpha_s \sim 0.5\%$$

[LHC Higgs Cross Section Working Group (2013)]

Previous lattice works (HPQCD)

$$\delta m_c \sim 0.6\%, \delta\alpha_s \sim 0.6\%$$

[G. P. Lepage, P. B. Macknzie, and E. Peskin (2015)]
[HPQCD (2015)]

- Precise lattice calculation helps the future experiments.

◇ Motivation

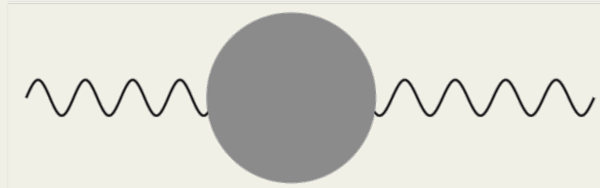
<u>Previous works (HPQCD)</u>

- Moment method (pseudoscalar-current correlators)
- Staggered fermion formalism
- Lattice spacings: 0.15 ~ 0.06 fm
- (Bayesian like fitting)

◇ We provide independent result with DW fermion (0.083 ~ 0.044 fm).

◇ Moment method

Current correlator



$$q^2\Pi(q^2) = i\int \mathrm{d}x e^{iqx}\langle j_5(x) j_5(0)\rangle$$

Moment: Derivative of the correlators

$$g_{2n} = \frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \left(q^2\Pi(q^2)\right)_{q^2=0}$$

Calculable perturbatively as a function of $m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}}$.

[K. G. Chetyrkin et al. (2006)]
[R. Boughezal et al. (2006)]
[A. Maier et al. (2009)]

Moment on the lattice

- Replace $i\int \mathrm{d}x \frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \mathrm{e}^{iqt} \longrightarrow a^4 \sum_x t^{2n}$

Correlator $G(t)$ on the lattice

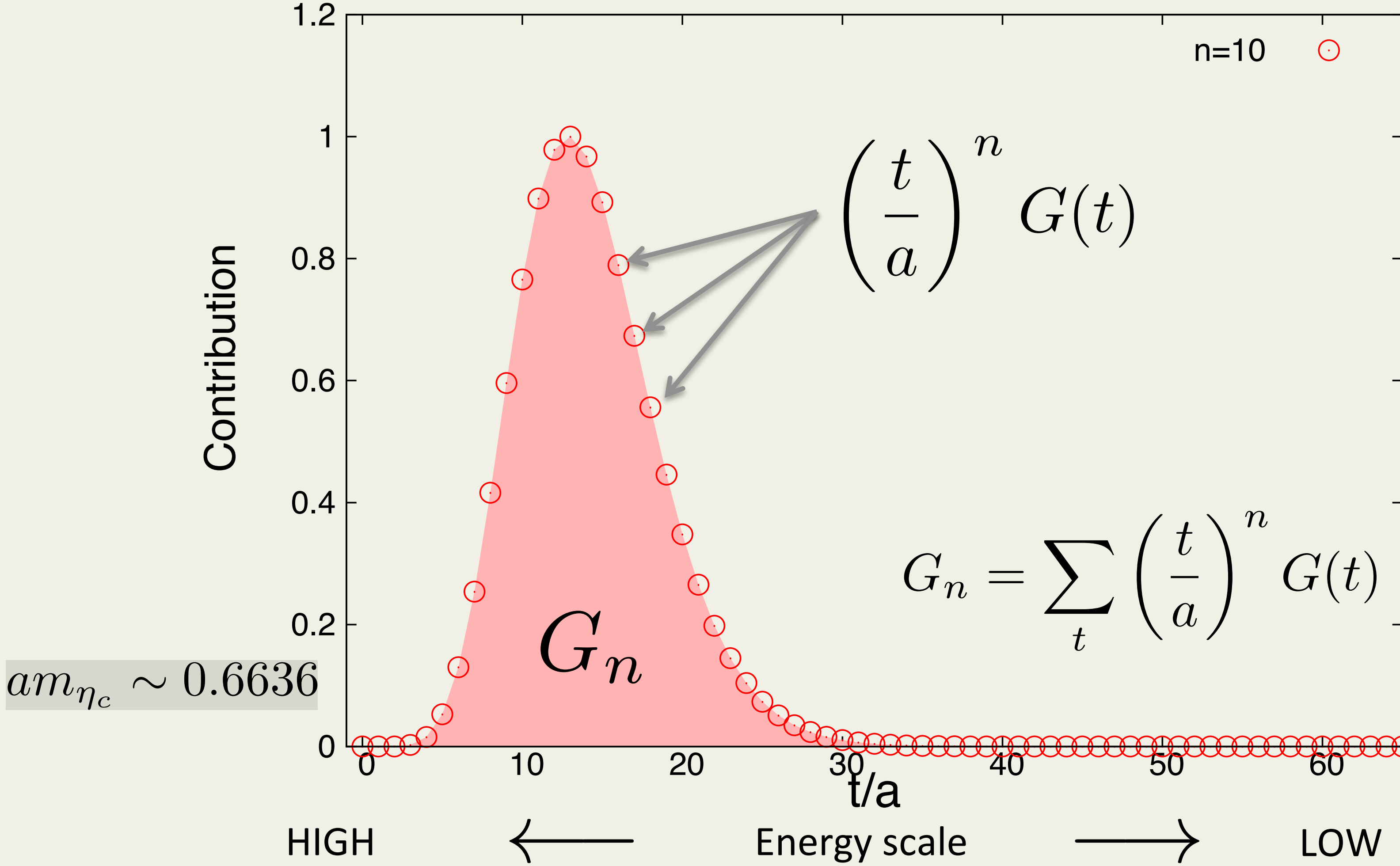
$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0c})^2 \langle j_5(x) j_5(0) \rangle$$

- Moment is easily calculated from $G(t)$

$$G_n = \sum_t \left(\frac{t}{a}\right)^n G(t)$$

What's the Moment?

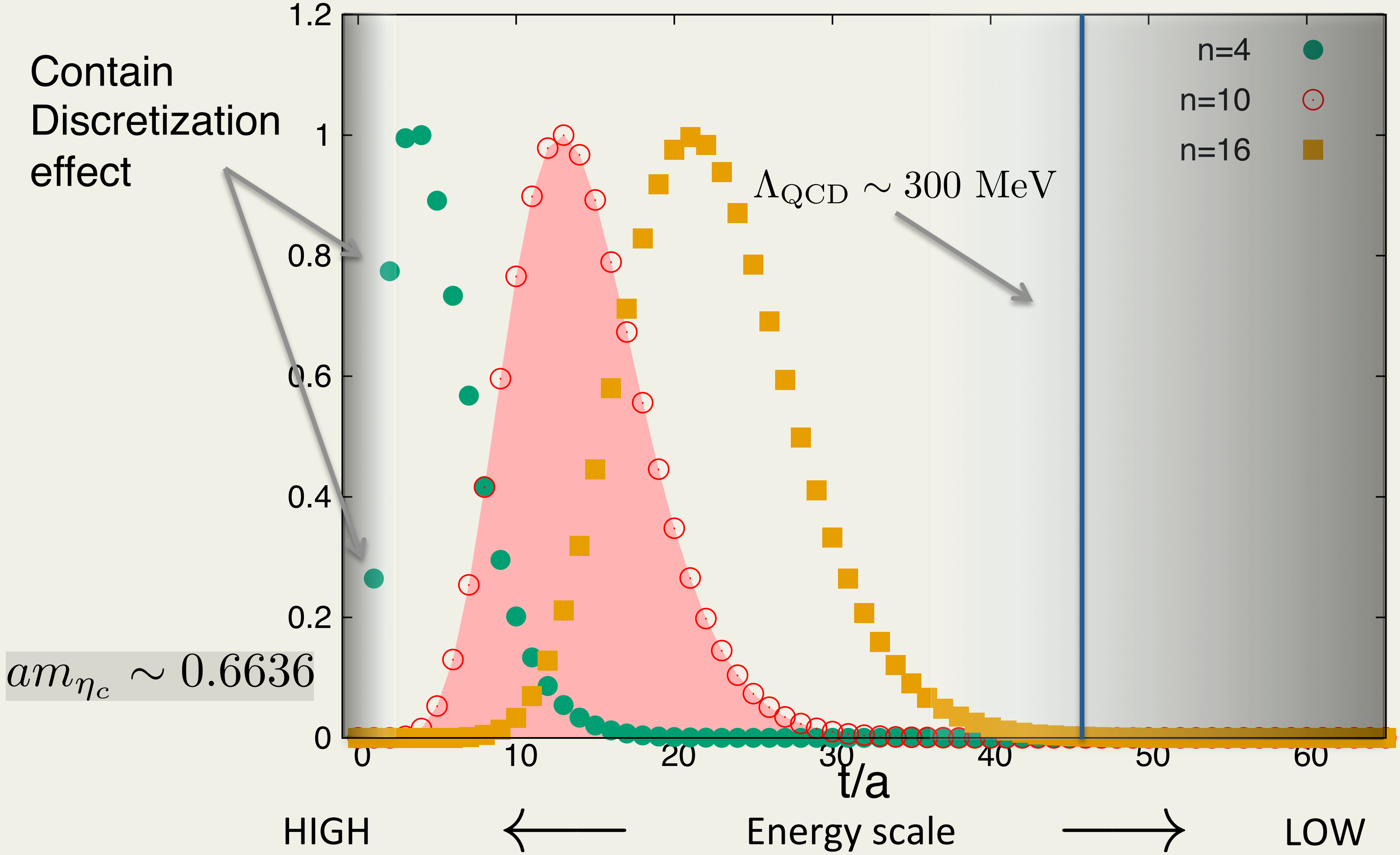
- Moment is defined by $G(t)$ <u>at typical energy scale</u>



- Typical energy scale depends on the weight factor n

"Window"

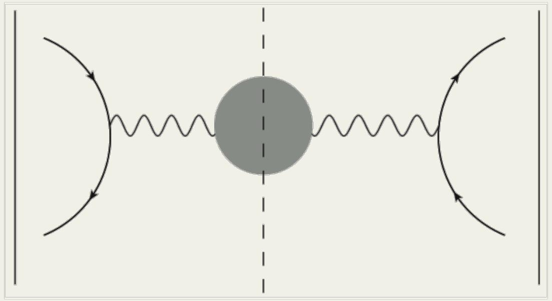
$a^{-1} \gg (\text{Energy scale}) \gg \Lambda_{\text{QCD}} \longrightarrow 6 \leq n \ll 20$

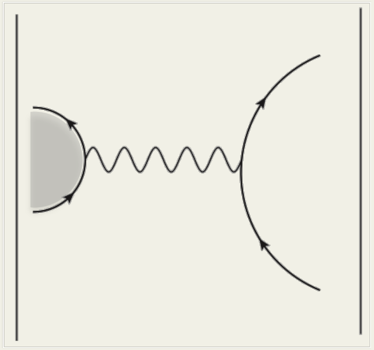


What's the Moment? (From comparison with experiment)

- Vector moment can be measured in the experiment.

$$\frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \left(\Pi(q^2)\right)_{q^2=0} = \frac{1}{12\pi Q_f^2}\int \mathrm{d}s \frac{1}{s^{n+1}} R(s)_{e^+e^-\to \text{ hadron}}$$

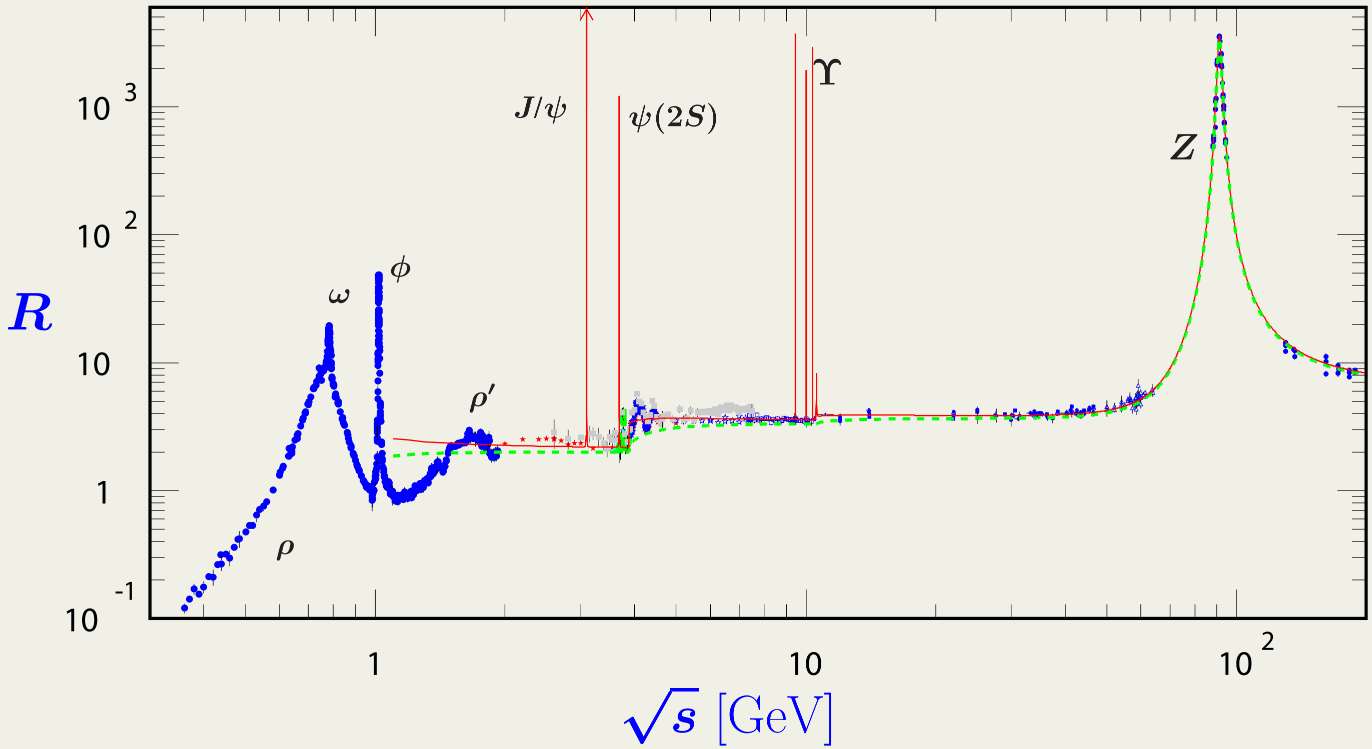




- Moment is a weighted integral of the R-ratio.

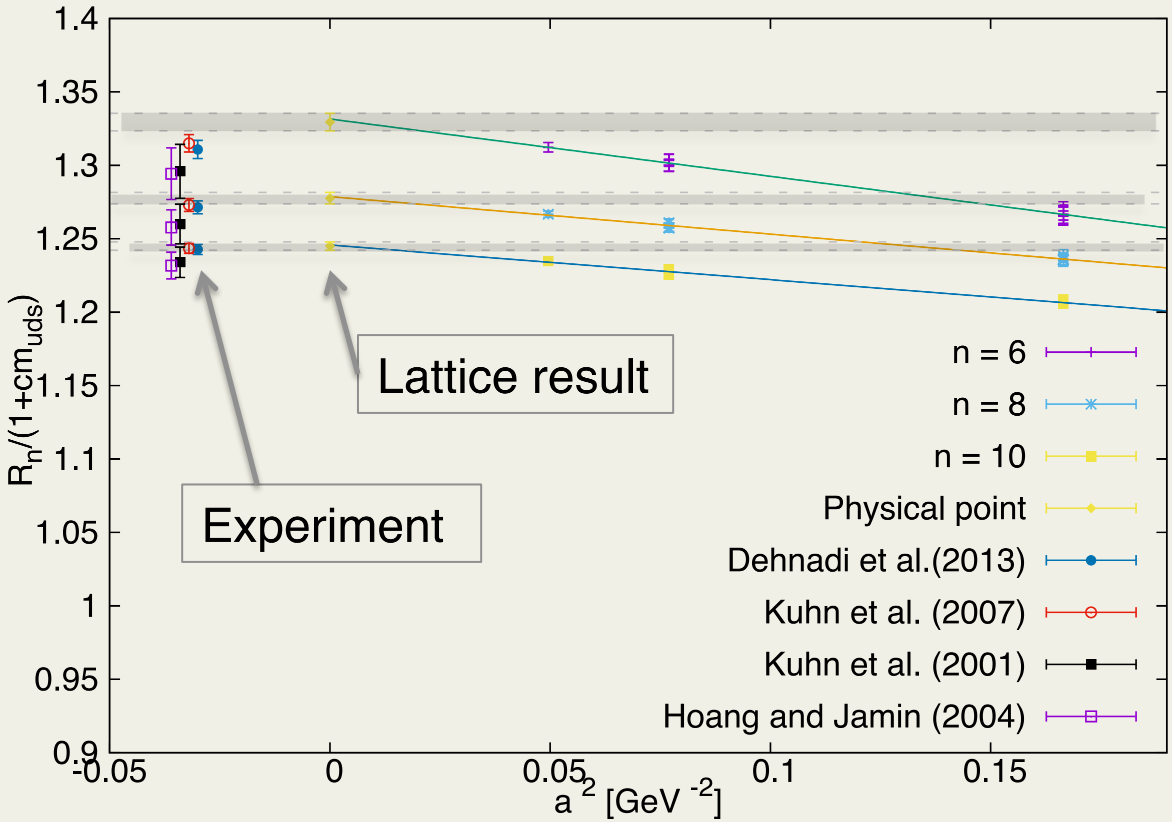
What's the Moment? (From comparison with experiment)

- Vector moment can be measured in the experiment.



- Moment is a weighted integral of the R-ratio.

Consistency with experiment (Vector)



- Same analysis with the pseudo-scalar.
- It has bigger fluctuation (since we have to need Z_V)

(A little discrepancy at n = 6, from EM and Disconnected contribution (?))

Correspondence between lattice and continuum

$$G_n^{(\mathrm{Lat})} = \frac{g_n^{(\mathrm{conti})}(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})}{(am_c^{\overline{\mathrm{MS}}})^{n-4}}$$

- Using reduced moment R_n …

  with G_n and $G_n^{(0)}$, the counterparts at the tree-level,

Lattice

$$R_n = \frac{am_{\eta_c}}{2am_c}\left(\frac{G_n}{G_n^{(0)}}\right)^{\frac{1}{n-4}}$$

Continuum

$$r_n = \left(\frac{g_n}{g_n^{(0)}}\right)^{\frac{1}{n-4}}$$

$$R_n = \frac{m_{\eta_c}^{\mathrm{exp}}}{2m_c^{\overline{\mathrm{MS}}}} r_n(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})$$

Lattice setup

- Mobius Domain Wall Fermion formalism
- $n_f = 2+1$ in the sea;
  3 values of (valence) charm quark mass to interpolate

β	a^{-1} [GeV]	$L \times T$	L_5	am_{ud}	am_s	congfigs	m_π [MeV]	$m_\pi L$	Id.
4.17	2.4531(40)	$32^3 \times 64$ ($L = 2.6$ fm)	12	0.0035	0.040	300	230	3.0	32-0
				0.007	0.030	300	310	4.0	32-1
				0.007	0.040	300	310	4.0	32-2
				0.012	0.030	300	400	5.2	32-3
				0.012	0.040	300	400	5.2	32-4
				0.019	0.030	300	500	6.5	32-5
				0.019	0.040	300	500	6.5	32-6
		$48^3 \times 96$ ($L = 3.9$ fm)	12	0.0035	0.040	401	230	4.4	32-7
4.35	3.6097(89)	$48^3 \times 96$ ($L = 2.6$ fm)	8	0.0042	0.0180	300	300	3.9	48-0
				0.0042	0.0250	300	300	3.9	48-1
				0.0080	0.0180	301	410	5.4	48-2
				0.0080	0.0250	297	410	5.4	48-3
				0.0120	0.0180	298	500	6.6	48-4
				0.0120	0.0250	300	500	6.6	48-5
4.47	4.4961(92)	$64^3 \times 128$ ($L = 2.8$ fm)	8	0.0030	0.015	397	280	4.0	64-0

Steps toward the $m_c^{\overline{\rm MS}}$ extraction

Lattice: Calculate reduced moment R_n ,

and extrapolate to continuum limit.

Continuum: Calculate r_n perturbatively

(we use n = 6, 8, 10).

- Solving the simultaneous equations for different n's,

  we obtain $m_c^{\overline{\rm MS}}, \alpha_{\overline{\rm MS}}$.

Equality

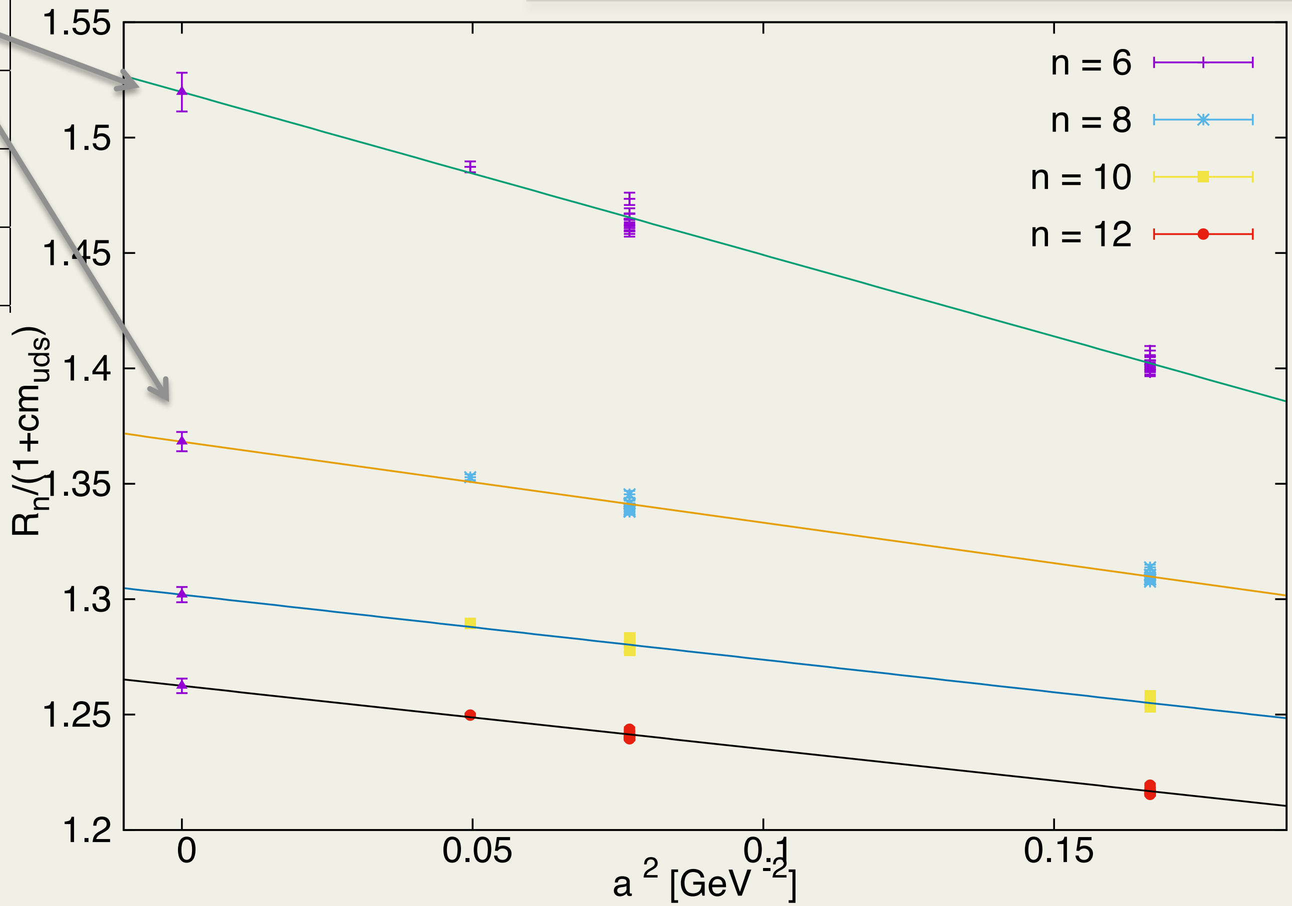
$$R_n = \frac{m_{\eta_c}^{\rm exp}}{2m_c^{\overline{\rm MS}}} r_n(m_c^{\overline{\rm MS}}, \alpha_{\overline{\rm MS}})$$

Extrapolation to contimuum ($a = 0, m_{uds} = 0$)

$$R_n(a) = R_n(0)\left(1 + c_1 (a m_c)^2\right) \times \left(1 + f_1 \frac{m_u + m_d + m_s}{m_c}\right)$$

Essentially flat (~1) term

	(Stat.)(a)(O(a^4))(Vol.)
R_6	1.520(2)(1)(8)(5)
R_8	1.368(1)(1)(4)(2)
R_{10}	1.302(1)(0)(3)(1)
R_{12}	1.262(1)(0)(3)(0)



χ^2 fitting.

Error ❶ Finite volume effect

- Prepare two ensembles (same setup except for the volume)

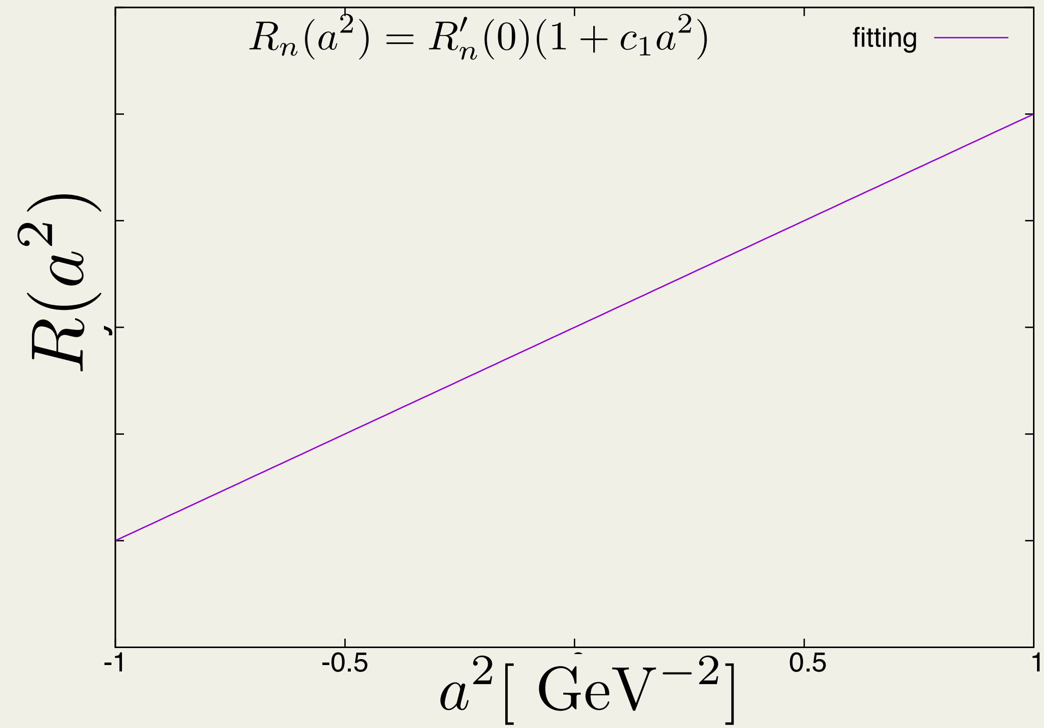
$$R_n(L = 32) \qquad m_\pi L \sim 3.0$$

$$R_n(L = 48) \qquad m_\pi L \sim 4.4$$

Finite volume error

$$\delta_L R_n = |R_n(L = 48) - R_n(L = 32)|$$

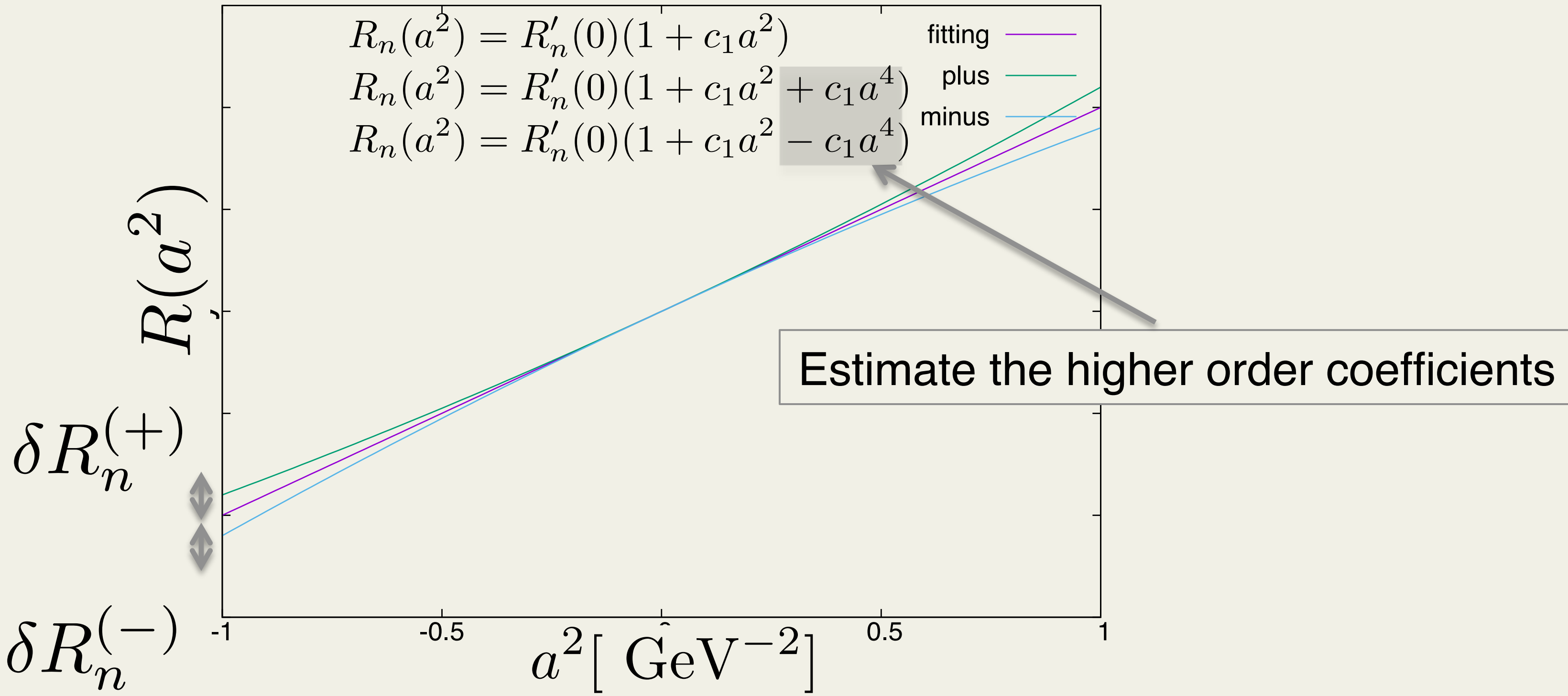
Error ❷ $O(a^4)$ truncation error



$O(a^4)$ truncation error

$$\delta_{O(a^4)} R_n = \max(\delta R_n^{(+)}, \delta R_n^{(-)})$$

Error ❷ $O(a^4)$ truncation error



$O(a^4)$ truncation error

$$\delta_{O(a^4)} R_n = \max(\delta R_n^{(+)}, \delta R_n^{(-)})$$

Any other sources of errors?

- Possible sources of systematic error…

(1): Input meson mass $m_{\eta_c}^{\exp}$ error (Almost negligible)

After correcting for… (a) Electromagnetic effect,
(b) Disconnected diagram contributions.

(2): Gluon condensate contribution

(3): Truncation error from perturbative expansion of r_n

Equality

$$R_n = \frac{m_{\eta_c}^{\exp}}{2m_c^{\overline{\mathrm{MS}}}} r_n(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})$$

Error ❷ Gluon condensate

◉ Perturbative calculation does not contain gluon condensate.

It is known to 2-loop by OPE. [M. A. Shifman, A. I. Vaientshtein, & V. I. Zakharov (1979)] [D. J. Broadhurst, et al (1994)]

$$r_n^{\mathrm{glue}} = \left(\frac{\langle (\alpha_s/\pi) G^{\mu\nu} G_{\mu\nu} \rangle}{(2m)^4} A_n(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}}) \right)^{\frac{1}{n-4}}$$

◉ New parameter arise… $\frac{\langle (\alpha_s/\pi) G^{\mu\nu} G_{\mu\nu} \rangle}{(2m)^4}$

◇ We may extract it as a solution of the equations, n = 6, 8, & 10.

Error ❸ Perturbative truncation

Equality

$$R_n = \frac{m_{\eta_c}^{\mathrm{exp}}}{2m_c^{\overline{\mathrm{MS}}}} r_n(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})$$

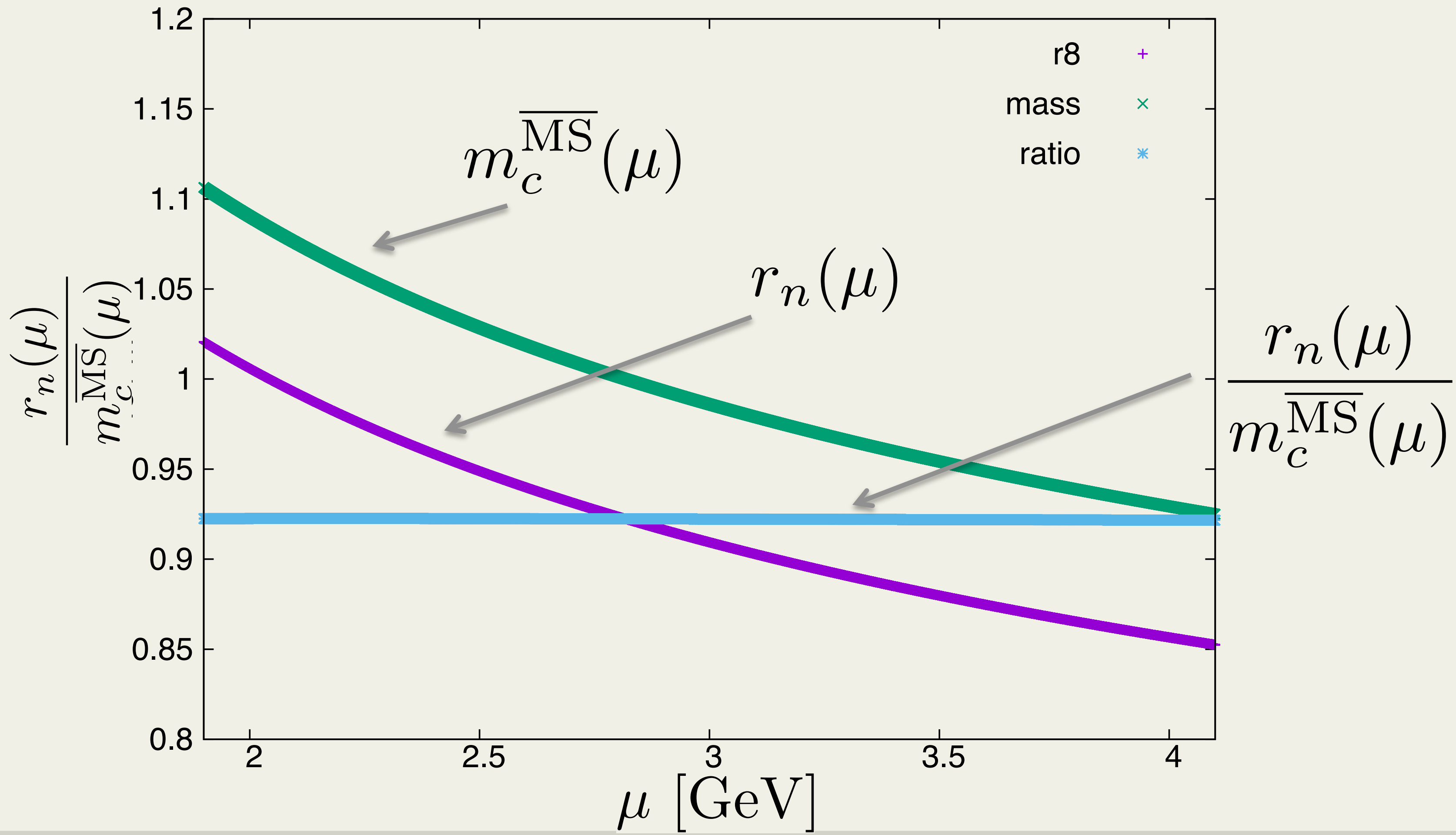
- R_n does not depend on the renormalization scale μ .

$$\downarrow$$

$\frac{r_n(\mu)}{m_c^{\overline{\mathrm{MS}}}(\mu)}$ does not depend on the scale μ at the all order of perturbative expantions.

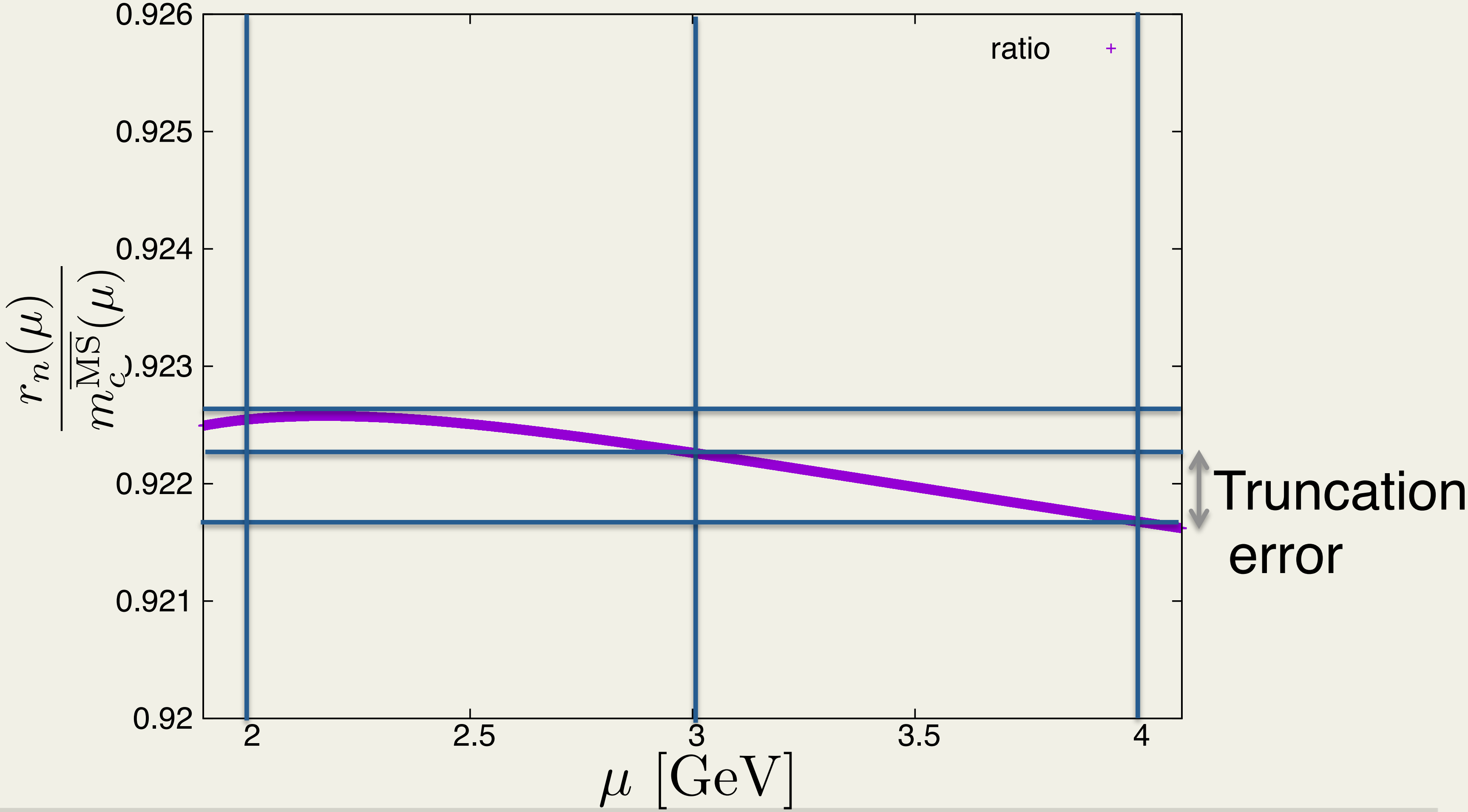
- Estimate the truncation error from μ dependence of $\frac{r_n(\mu)}{m_c^{\overline{\mathrm{MS}}}(\mu)}$

Error ❸ Perturbative truncation

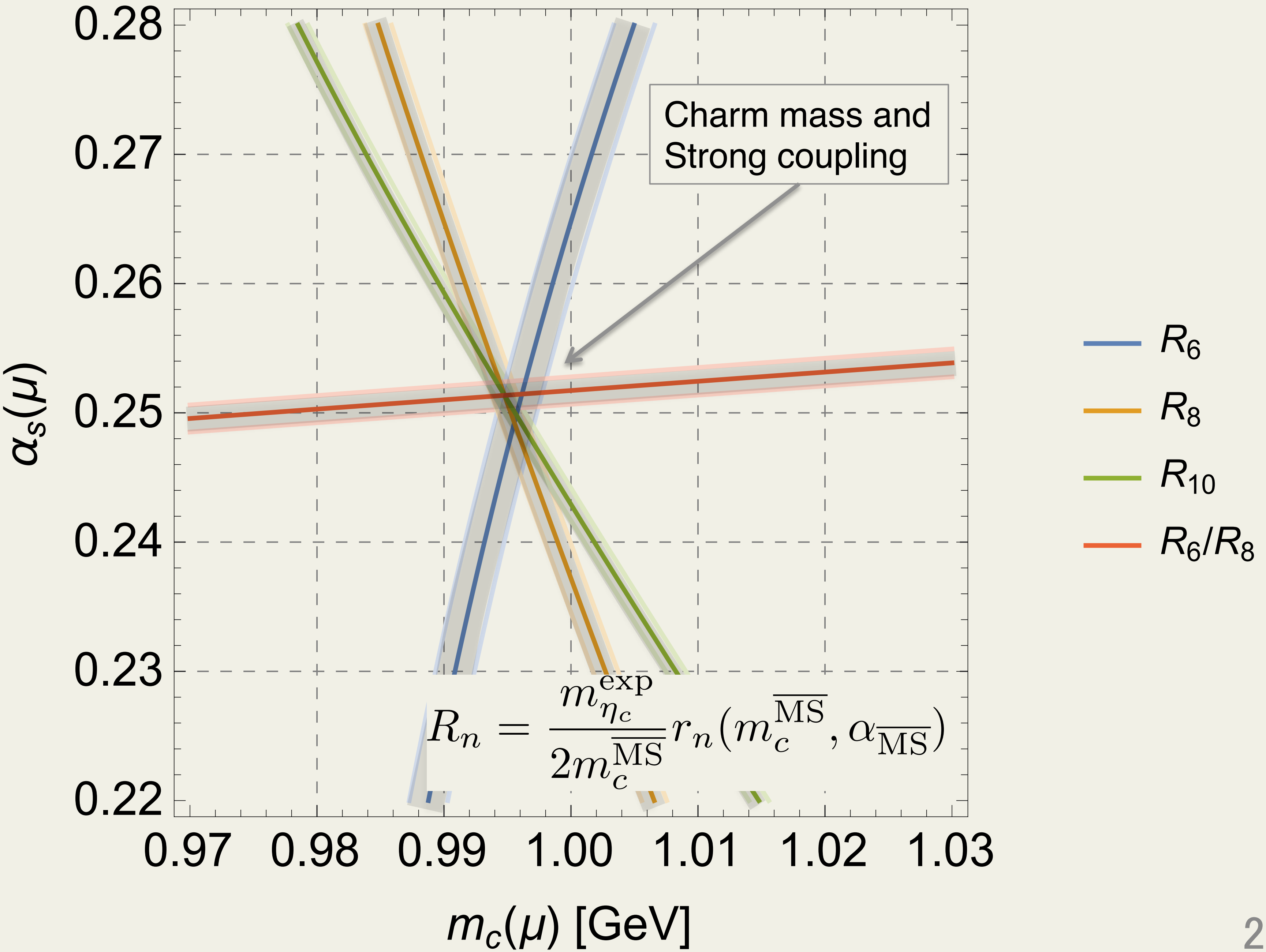


- Dependence is almost canceled out.

Error ❸ Perturbative truncation



◉ Actually, we consider $\mu_\alpha \neq \mu_m$ $(\alpha(\mu_\alpha), m_c(\mu_m))$, not only $\mu_\alpha = \mu_m$ [B. Dehnadi, A. H. Hoang, and V. Mateu (2015)]



0.28
0.27
0.26
0.25
0.24
0.23
0.22
$\alpha_s(\mu)$
0.97
0.98
0.99
1.00
1.01
1.02
1.03
$m_c(\mu)$ [GeV]
Charm mass and
Strong coupling
$R_n = \frac{m_{\eta_c}^{\exp}}{2m_c^{\overline{\rm MS}}} r_n(m_c^{\overline{\rm MS}}, \alpha_{\overline{\rm MS}})$
R_6
R_8
R_{10}
R_6/R_8

Result ($\mu_m = \mu_\alpha$)

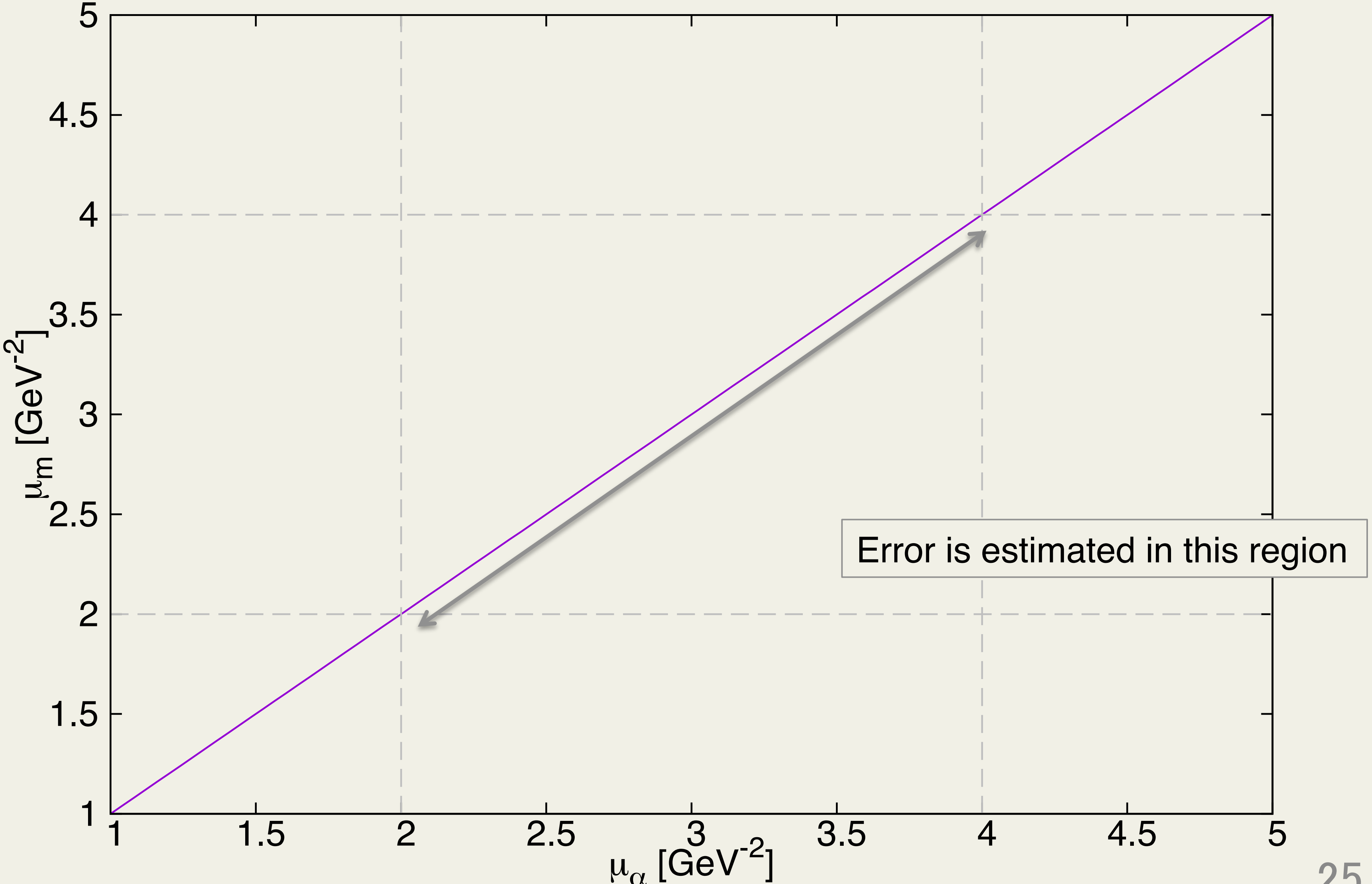
- With a condition $\mu_m = \mu_\alpha$,

$R_6/R_8, R_8, \& \ R_{10}$

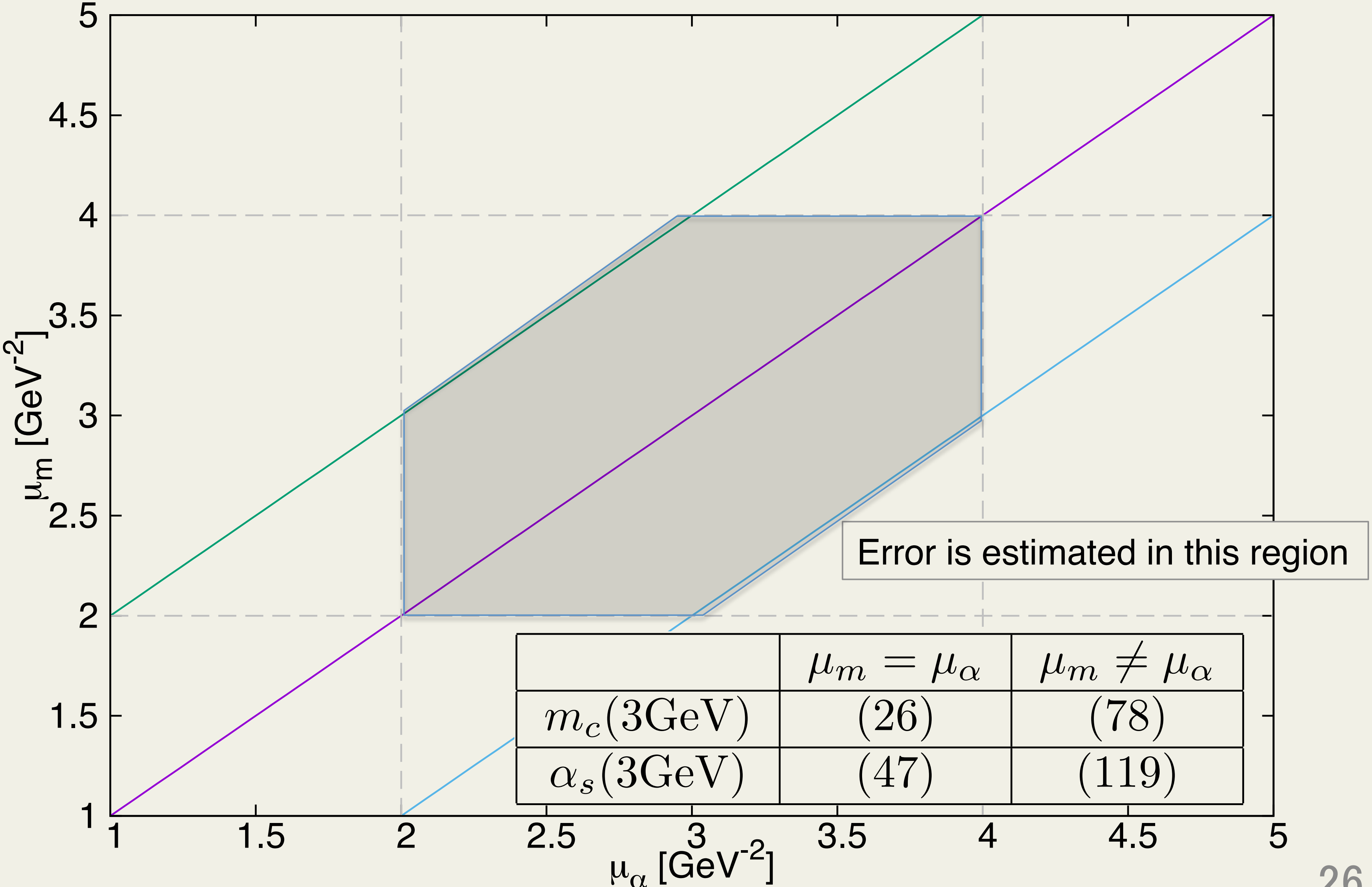
			Lattice				m_{η_c} (negligible)				
		Trunc.	Stat.	a	$O(a^4)$	FV	$m_{\eta c}^{\rm exp}$	Disc	EM	$\eta_c J/\psi$	ALL
$m_c(3{\rm GeV})$	0.9944 GeV	(26)	(14)	(7)	(53)	(33)	(3)	(3)	(5)	(10)	=(70)
$\alpha_s(3{\rm GeV})$	0.2534	(47)	(10)	(4)	(45)	(33)	(0)	(0)	(0)	(1)	=(74)
$\frac{\langle \alpha/\pi G^2 \rangle}{m^4}$	-0.0019	(38)	(0)	(0)	(3)	(0)	(0)	(0)	(0)	(0)	=(38)

- We select $R_6/R_8, R_8, \& \ R_{10}$ since it is perturbatively stable.
- Errors from perturbative and lattice $O(a^4)$ truncation are significant.

Estimation with $\mu_m = \mu_\alpha$.



Estimation with $\mu_m \neq \mu_\alpha$.



	$\mu_m = \mu_\alpha$	$\mu_m \neq \mu_\alpha$
$m_c(3\mathrm{GeV})$	(26)	(78)
$\alpha_s(3\mathrm{GeV})$	(47)	(119)

Result ($\mu_m \neq \mu_\alpha$)

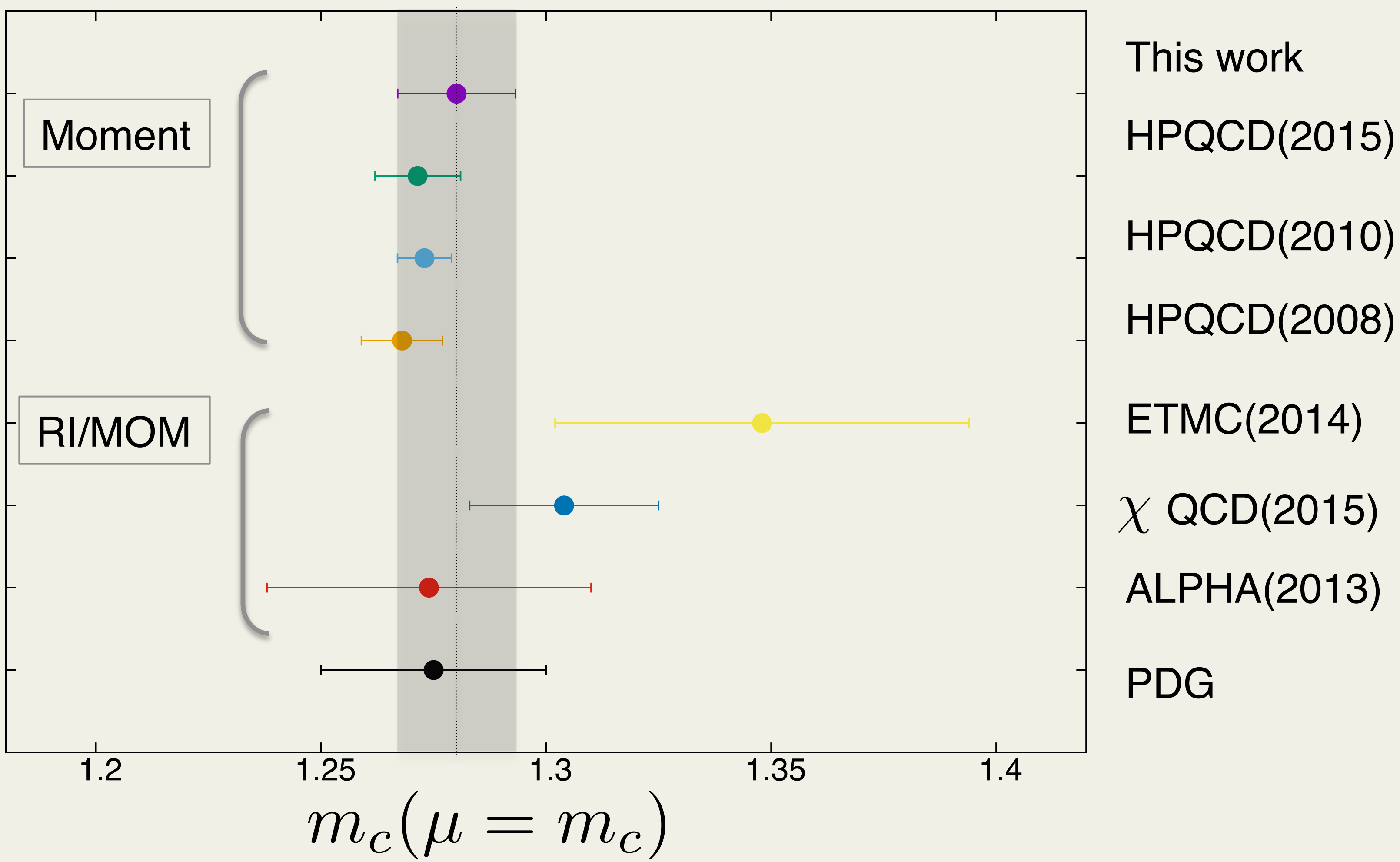
- More conservative estimation of the perturbative error.

$R_6/R_8, R_8, \& \ R_{10}$

			Lattice				m_{η_c} (negligible)				
		Trunc.	Stat.	a	$O(a^4)$	FV	$m_{\eta c}^{\exp}$	Disc	EM	$\eta_c J/\psi$	ALL
m_c(3GeV)	0.9944 GeV	(78)	(14)	(7)	(53)	(33)	(3)	(3)	(5)	(10)	=(102)
α_s(3GeV)	0.2534	(119)	(10)	(4)	(45)	(33)	(0)	(0)	(0)	(1)	=(132)
$\frac{\langle\alpha/\pi G^2\rangle}{m^4}$	-0.0019	(69)	(0)	(0)	(3)	(0)	(0)	(0)	(0)	(0)	=(69)

- Perturbation error is $\sim \times 2$ larger than that of $\mu_m = \mu_\alpha$
- Continuum uncertainty become more important.
- ~1% precision is achieved for $m_c^{\overline{\rm MS}}$.

Result ($\mu_m \neq \mu_\alpha$)



Result ($\mu_m \neq \mu_\alpha$)

	Result	PDG
$m_c^{\overline{\mathrm{MS}}}(\mu = 3\ \mathrm{GeV})$	$0.9944(102)\ \mathrm{GeV}$	
$m_c^{\overline{\mathrm{MS}}}(\mu = m_c^{\overline{\mathrm{MS}}})$	$1.280(13)\ \mathrm{GeV}$	$1.275(25)\ \mathrm{GeV}$
$\alpha_{\overline{\mathrm{MS}}}(\mu = 3\ \mathrm{GeV})$	$0.2534(132)$	$0.2567(34)$
$\alpha_{\overline{\mathrm{MS}}}(\mu = M_Z)$	$0.1178(27)$	$0.1185(6)$
$\Lambda_{\overline{\mathrm{MS}}}^{n_f=4}$	$288(39)\ \mathrm{MeV}$	$297(8)\ \mathrm{MeV}$
$\Lambda_{\overline{\mathrm{MS}}}^{n_f=5}$	$206(34)\ \mathrm{MeV}$	$214(7)\ \mathrm{MeV}$

Consistency with experiment (Vector)

Pseudo $\longrightarrow$ Vector

$$G_n^{(\mathrm{Lat})} = \frac{g_n^{(\mathrm{conti})}(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})}{(am_c^{\overline{\mathrm{MS}}})^{n-4}}$$

$$Z_V^2 G_{Vn}^{(\mathrm{Lat})} = \frac{g_{Vn}^{(\mathrm{conti})}(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})}{(am_c^{\overline{\mathrm{MS}}})^{n-2}}$$

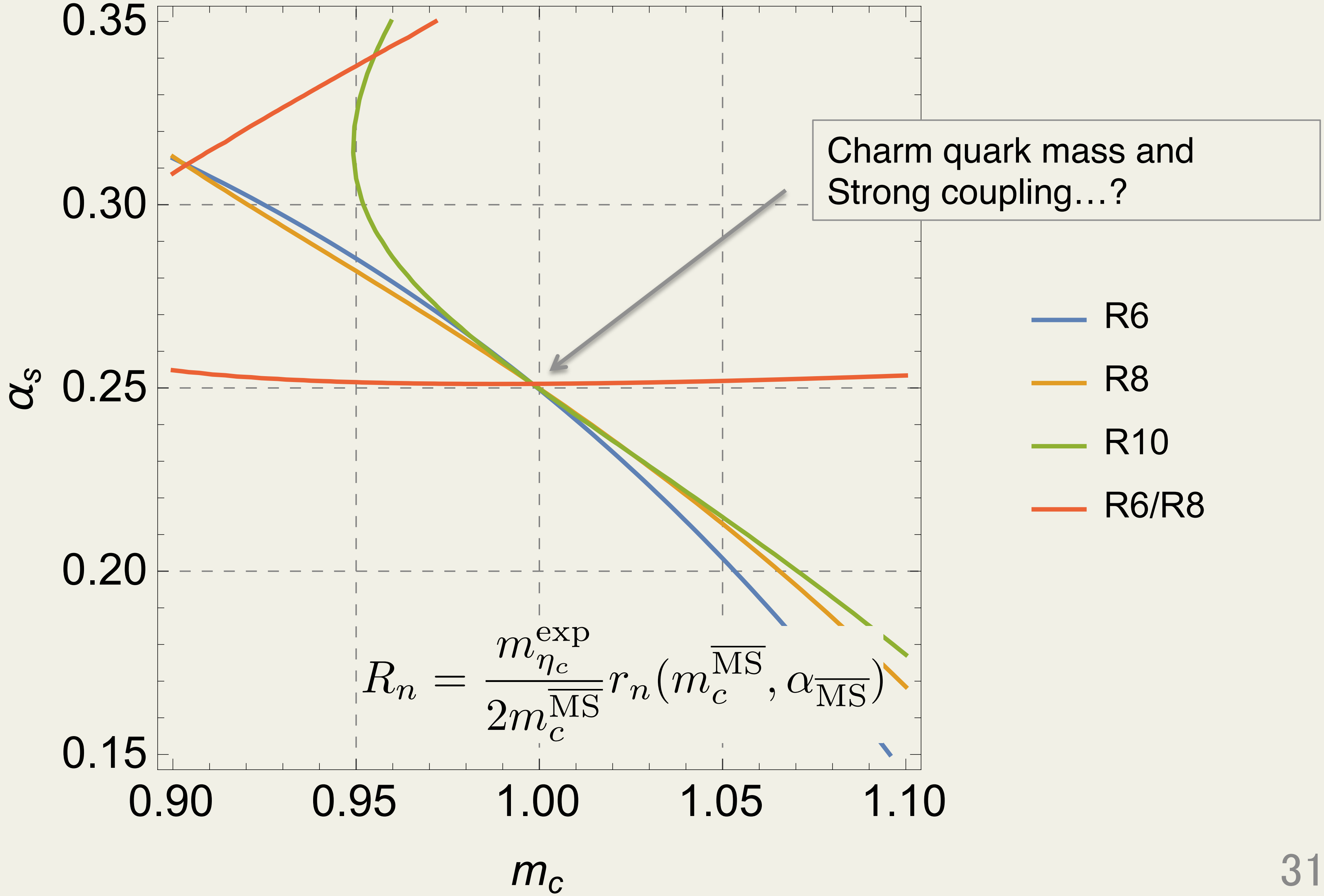
- Input Z_V from $\Pi_V^{\overline{\mathrm{MS}}}(x)$ analysis with OPE.

(JLQCD, M. Tomii et al.)

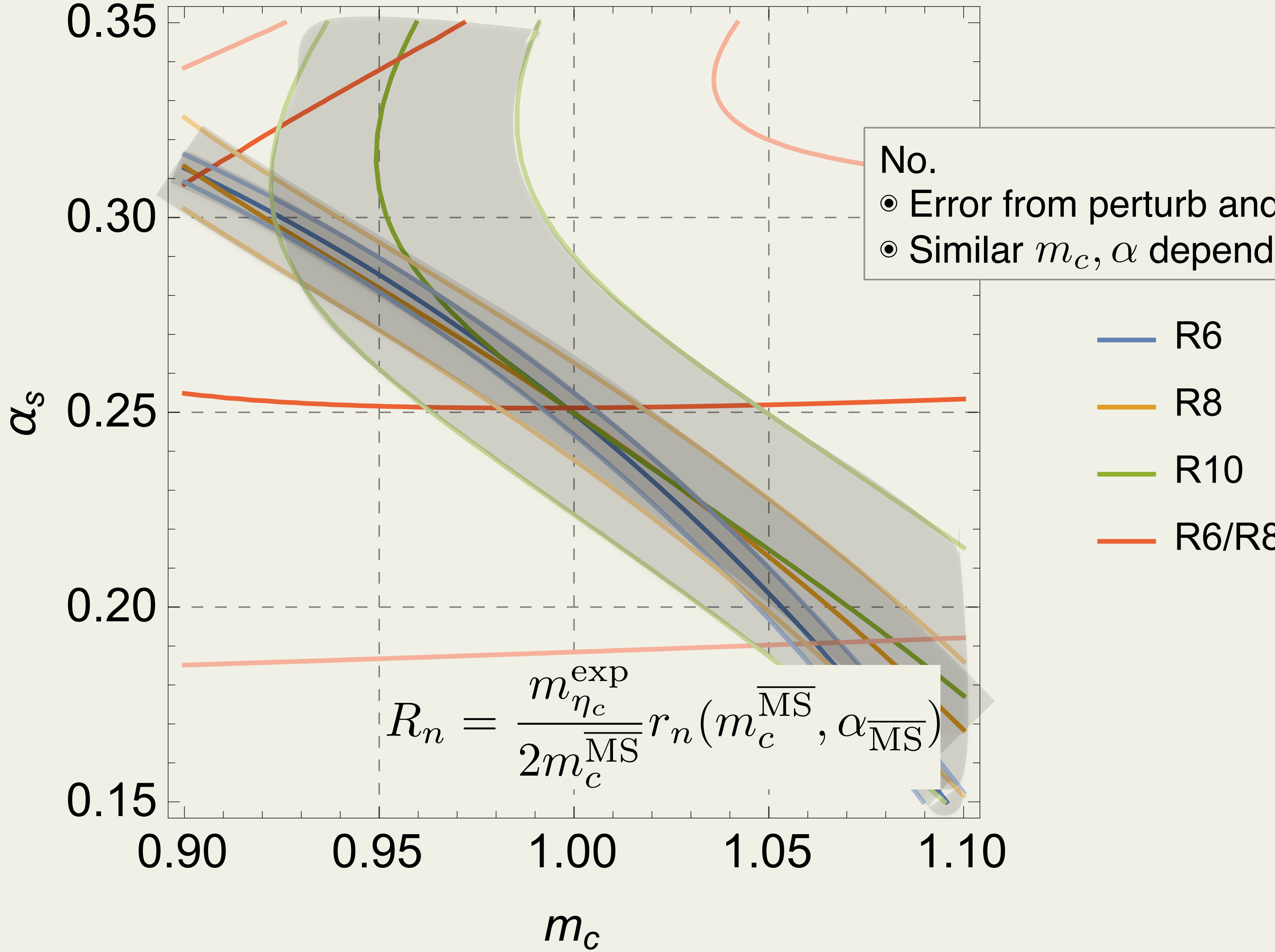
$$\tilde{Z}_\mathrm{V}(x) = Z_V + c_{-2}x^{-2} + c_4 x^4 + c_6 x^6 + O(x^8)$$

(e.g.) $Z_V(a^{-1} = 4.47\ \mathrm{GeV}) = 0.9651(46)$

Vector current Moment



Vector current Moment



No.
- Error from perturb and Z_V
- Similar m_c, α dependence

$$R_n = \frac{m_{\eta_c}^{\mathrm{exp}}}{2m_c^{\overline{\mathrm{MS}}}} r_n(m_c^{\overline{\mathrm{MS}}}, \alpha_{\overline{\mathrm{MS}}})$$

Summary

- We extract $m_c^{\overline{\rm MS}}$ and $\alpha_{\overline{\rm MS}}$ from the time moments of charmonium current correlators.
- Take continuum limit by the result at $a^{-1} = 2.4, 3.6, 4.5\ \mathrm{GeV}$, and carefully discuss the error.
  Discretization effect is significant but controllable, and perturbative truncation is important.
- We also use moment of vector correlator. The moments consistent with the experimental R-ratio.

$$m_c^{\overline{\rm MS}}(3\ \mathrm{GeV}) = 0.9944(102)\ \mathrm{GeV}$$
$$\alpha_{\overline{\rm MS}}(3\ \mathrm{GeV}) = 0.2534(132)$$

Backup slides

Error ❶ Input meson mass $m_{\eta_c}^{\exp}$ error

- We use PDG value, $m_{\eta_c}^{\exp} = 2.9836(7)\ \mathrm{GeV}$, after correcting for…
  - (a) Electromagnetic effect,
  - (b) Disconnected diagram contributions.

- Estimates from previous works (lattice, pheno):

Electromagnetic

$$m_{\eta_c} - m_{\eta_c}^{\text{no EM}} = -2.4(8)\ \mathrm{MeV}$$

[E. Follana, et al. (2007)]

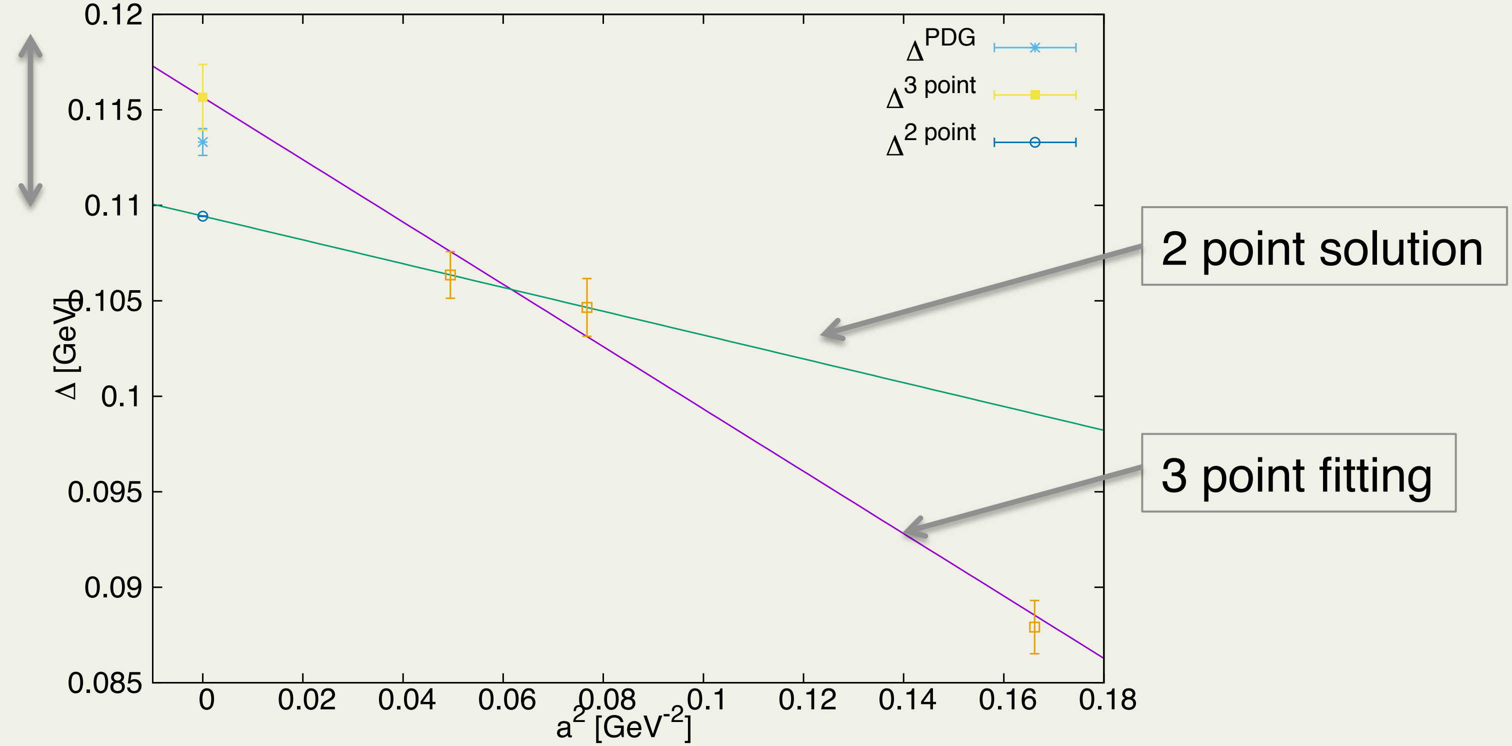
Disconnected

$$m_{\eta_c} - m_{\eta_c}^{\text{no Disconnect}} = -2.6(13)\ \mathrm{MeV}$$

[C. T. H. Davis, et al. (2007)]

Error ❶ Input meson mass $m_{\eta_c}^{\exp}$ error

$$\delta\Delta_{J/\psi-\eta_c}$$



Hyperfine splitting error

$$\delta\Delta_{J/\psi-\eta_c} = |\Delta_{J/\psi-\eta_c}^{3\ \mathrm{point}} - \Delta_{J/\psi-\eta_c}^{2\ \mathrm{point}}|$$

Error ❶ Input meson mass $m_{\eta_c}^{\exp}$ error

- Finally we use…

$$m_{\eta_c}^{\text{modified}} = 2983.6 + 2.4_{\text{Disc.}} + 2.6_{\text{EM}} \pm (0.7)_{\text{PDG}} \pm (0.8)_{\text{Disc.}} \pm (1.3)_{\text{EM}} \pm (2.3)_{\text{split}}$$

◆Note: All of these error sources are negligible.

Result ($\mu_m = \mu_\alpha$)

	Result	PDG
$m_c^{\overline{\mathrm{MS}}}(\mu = 3\ \mathrm{GeV})$	$0.9944(91)\ \mathrm{GeV}$	
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Moment and R-ratio

Residue theorem (or Disparsion relation)

$$\frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \left(\Pi(q^2)\right)_{q^2=Q_0^2} = \oint \frac{\mathrm{d}q^2}{2\pi i}\frac{1}{(q^2-Q_0^2)^{n+1}}\Pi(q^2)$$

Contour integral $$= \int \frac{\mathrm{d}q^2}{2\pi i}\frac{1}{(q^2-Q_0^2)^{n+1}} 2i\mathrm{Im}\left[\Pi(q^2)\right]$$

Optical theorem $$= \int \frac{\mathrm{d}q^2}{\pi}\frac{q^2}{(4\pi\alpha)^2 Q_f^2}\frac{1}{(q^2-Q_0^2)^{n+1}}\sigma_{e^+e^-\to\mathrm{hadron}}(q^2)$$

$$= \frac{1}{12\pi Q_f^2}\int \mathrm{d}q^2 \frac{1}{(q^2-Q_0^2)^{n+1}}\frac{\sigma_{e^+e^-\to\mathrm{hadron}}(q^2)}{\sigma_{e^+e^-\to\mu^+\mu^-}(q^2)}$$

Take $Q_0 = 0$

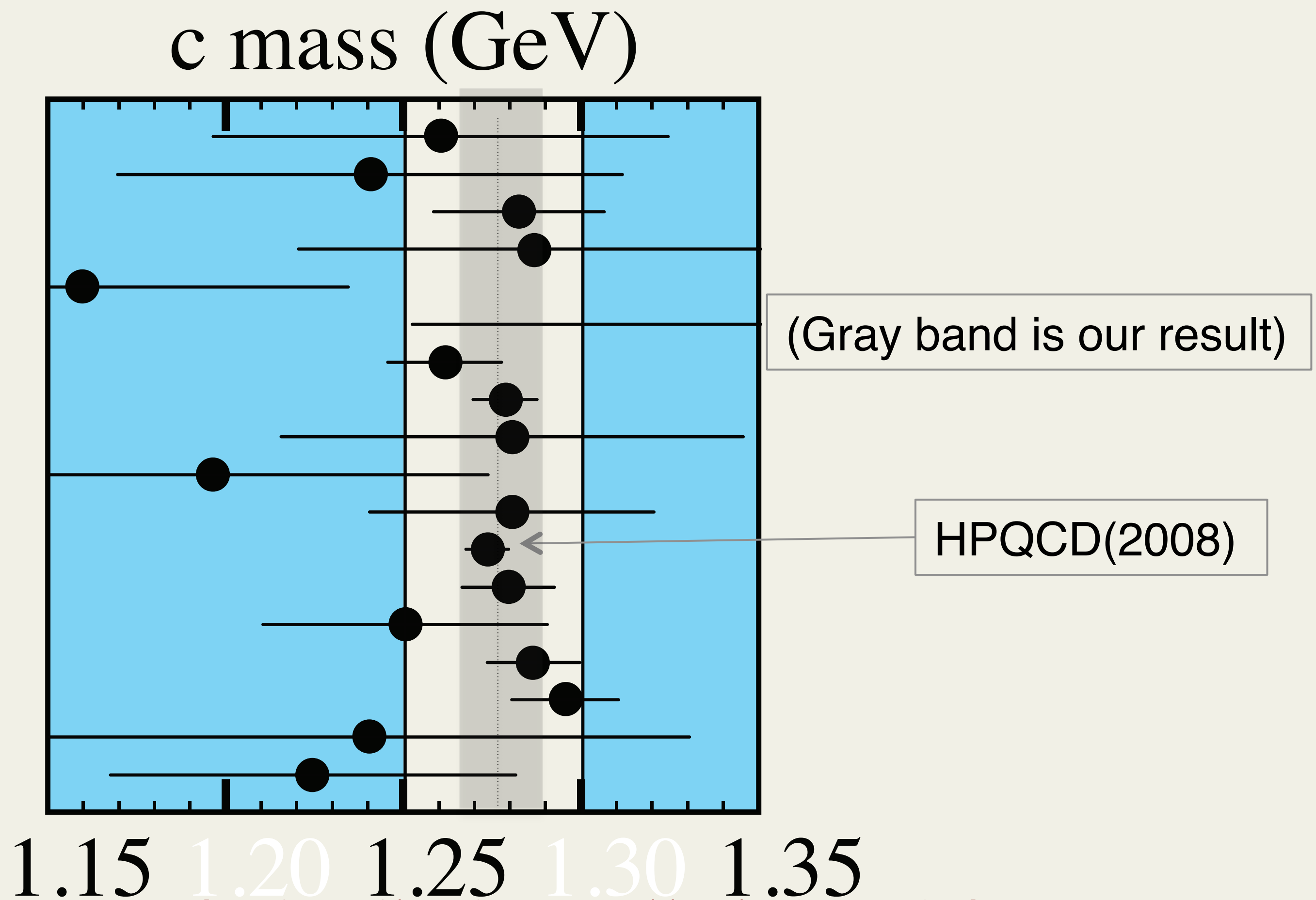
$$\boxed{\frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \left(\Pi(q^2)\right)_{q^2=0} = \frac{1}{12\pi Q_f^2}\int \mathrm{d}s \frac{1}{s^{n+1}} R(s)_{e^+e^-\to\ \mathrm{hadron}}}$$

Perturbative moment

k	$C_k^{(0)}$	$C_k^{(10)}$	$C_k^{(11)}$	$C_k^{(20)}$	$C_k^{(21)}$	$C_k^{(22)}$	$C_k^{(30)}$	$C_k^{(31)}$	$C_k^{(32)}$	$C_k^{(33)}$
1	1.3333	3.1111	0.0000	0.1154	-6.4815	0.0000	-1.2224	2.5008	13.5031	0.0000
2	0.5333	2.0642	1.0667	7.2362	1.5909	-0.0444	7.0659	-7.5852	0.5505	0.0321
3	0.3048	1.2117	1.2190	5.9992	4.3373	1.1683	14.5789	7.3626	4.2523	-0.0649
4	0.2032	0.7128	1.2190	4.2670	4.8064	2.3873	13.3285	14.7645	11.0345	1.4589
5	0.1478	0.4013	1.1821	2.9149	4.3282	3.4971		16.0798	16.6772	4.4685
6	0.1137	0.1944	1.1366	1.9656	3.4173	4.4992		14.1098	19.9049	8.7485
7	0.0909	0.0500	1.0912	1.3353	2.2995	5.4104		10.7755	20.3500	14.1272
8	0.0749	-0.0545	1.0484	0.9453	1.0837	6.2466		7.2863	17.9597	20.4750

n=4,6,8,10

Result and PDG



[K.A.Olive et al (Particle Data Group) (2014) and 2015 update]

Z_V factor extraction

Input Z_V	$\longrightarrow$	Predict R_n

then invert it…

Input R_n	$\longrightarrow$	Predict Z_V

- Moment is known perturbatively (and experimentally).

(Input Experiment	$\longrightarrow$	$\delta Z_V \sim 1\%$)
Input Perturbation	$\longrightarrow$	$\delta Z_V \sim 3\%$

(or 2% with PDG $\alpha_{\overline{\rm MS}}$)

	$m_c(3\mathrm{GeV})$	Trunc.	Stat.	a	$O(a^4)$	FV	ALL
R_6, R_8, R_{10}	0.9945 GeV	(83)	(13)	(7)	(52)	(32)	=(105)
R_6, R_6/R_8, R_{10}	0.9948 GeV	(176)	(13)	(7)	(50)	(32)	=(187)
R_6/R_8, R_8, R_{10}	0.9944 GeV	(78)	(14)	(7)	(53)	(33)	=(102)
	$\alpha_s(3\mathrm{GeV})$	Trunc.	Stat.	a	$O(a^4)$	FV	ALL
R_6, R_8, R_{10}	0.2530	(212)	(9)	(5)	(41)	(31)	=(219)
R_6, R_6/R_8, R_{10}	0.2534	(120)	(10)	(4)	(45)	(33)	=(133)
R_6/R_8, R_8, R_{10}	0.2534	(119)	(10)	(4)	(45)	(33)	=(132)
	$\langle G^2\rangle/m^4$	Trunc.	Stat.	a	$O(a^4)$	FV	ALL
R_6, R_8, R_{10}	-0.0019	(88)	(0)	(0)	(3)	(1)	=(88)
R_6, R_6/R_8, R_{10}	-0.0017	(132)	(0)	(0)	(4)	(0)	=(132)
R_6/R_8, R_8, R_{10}	-0.0019	(69)	(0)	(0)	(3)	(0)	=(69)