Charmonium and bottomonium spectral functions at finite temperature

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in collaboration with

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Plan of this talk

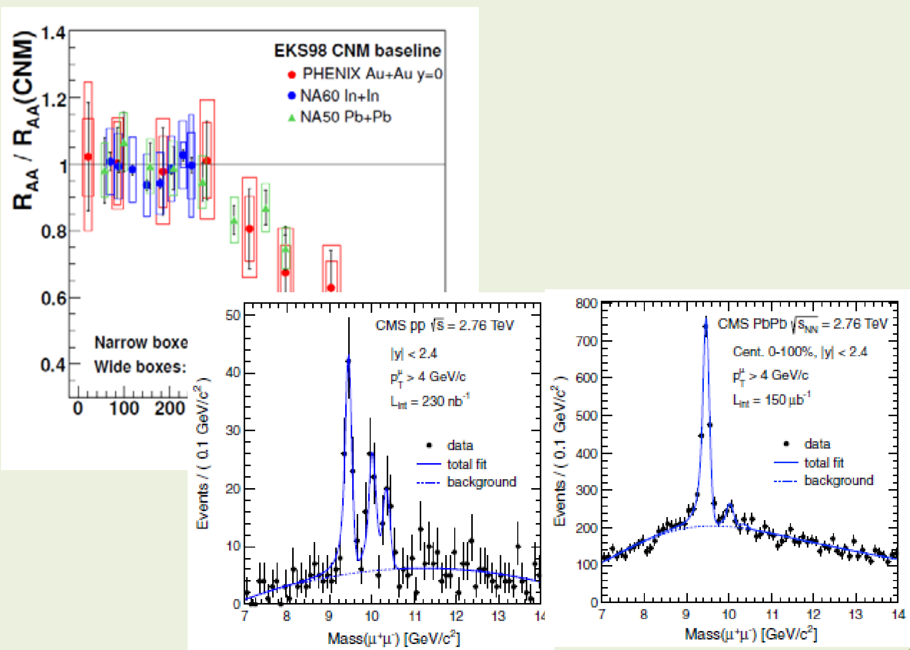
- Introduction
- Stochastic methods to reconstruct spectral functions
- Numerical Results
  - Mock data tests
  - Charmonium and bottomonium SPF at $T > 0$
- Conclusions and outlook

Motivation

- Quarkonium spectral functions (SPF)
  - have all information about in-medium properties of quarkonia

Quarkonium dissociation temperature

→ Important to understand quarkonium suppression

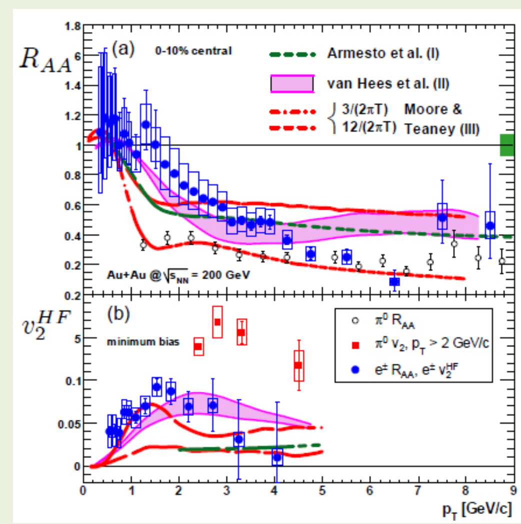


Heavy quark diffusion coefficient

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \to 0} \sum_{i=1}^{3} \frac{\rho_{ii}^{V}(\omega, \mathbf{0})}{\omega}$$

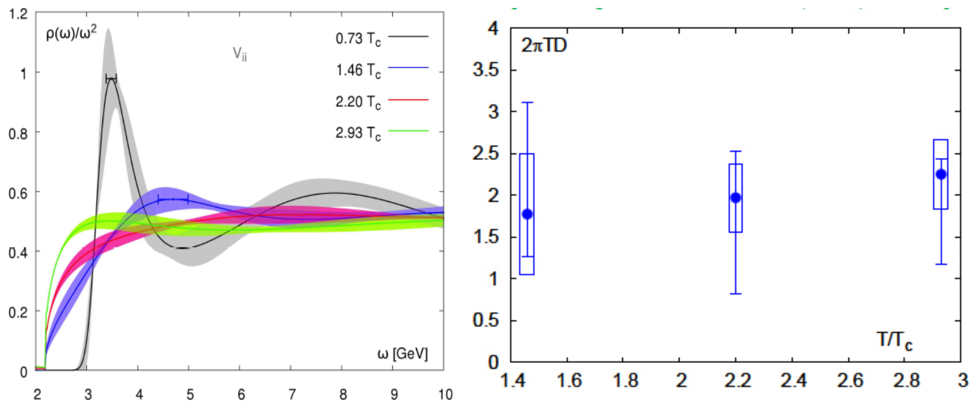
$\rho_{ii}^{V}(\omega)$: vector SPF

→ Important input for hydro models



Recent lattice studies

- Charmonia
  - Several studies both in quenched QCD and with dynamical quarks
  - Dissociation temperatures are still not conclusive
  - A transport coefficient has been computed
  - **More precise determination of the SPFs on larger and finer lattice is needed**

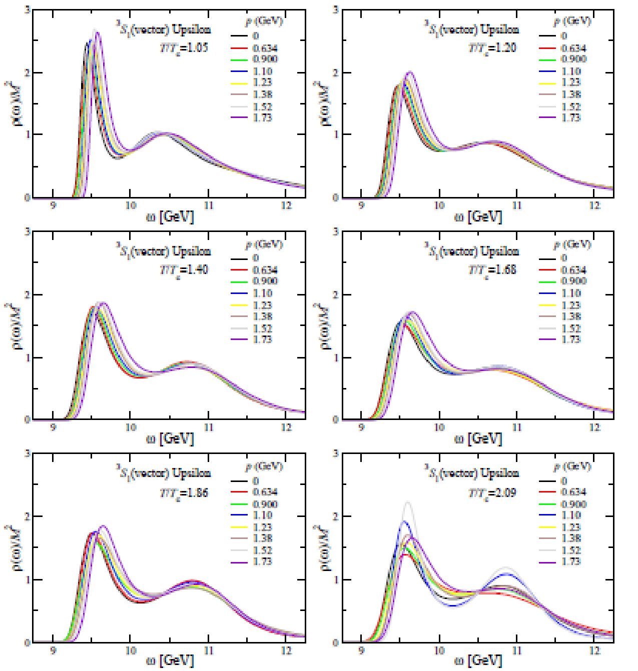


H.-T. Ding *et al.*, PRD 86 (2012) 014509

- Bottomonia
  - NRQCD
  - **Melting of P-wave states is still not conclusive.**
  - **It is good to crosscheck without NRQCD**

G.Aarts *et al.*, PRL 106 (2011) 061602

G.Aarts *et al.*, JHEP 1303 (2012) 084



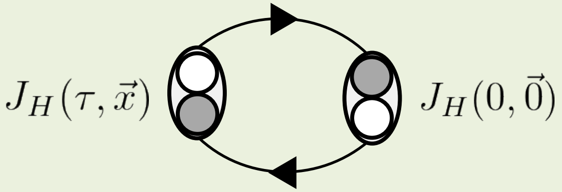
Reconstruction of SPF

Euclidian (imaginary time) meson correlation function

$$G_H(\tau,\vec{p}) \equiv \int d^3x e^{-i\vec{p}\cdot\vec{x}} \langle J_H(\tau,\vec{x}) J_H(0,\vec{0}) \rangle$$

$$= \int_0^\infty \frac{d\omega}{2\pi} \rho_H(\omega,\vec{p}) K(\omega,\tau)$$

Spectral function (SPF)



$K(\omega,\tau) \equiv \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$ $\quad J_H(\tau,\vec{x}) \equiv \bar{\psi}(\tau,\vec{x})\Gamma_H \psi(\tau,\vec{x})$

- Computing SPF → **Ill-posed problem**
  - # of data points of a correlator is *O*(10) while a SPF needs O(1000) data points.
  - In general, simple χ^2 fitting does not work!
- Several ways to reconstruct SPF
  - Maximum entropy method (MEM) M. Asakawa, T. Hatsuda and Y. Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459-508
  - A new Bayesian method Y. Burnier and A. Rothkopf, PRL 111 (2013) 18, 182003
  - Stochastic methods

Stochastic analytical inference (SAI)

- Introducing a mapping $\phi : \mathbb{R} \mapsto [0,1]$

$$\phi(\omega) = \frac{1}{\mathcal{N}} \int_{-\infty}^{\omega} D(\nu) d\nu$$

Positive-definite
Same normalization to a spectral function

K.S.D. Beach
arXiv:cond-mat/0403055

- Regularization

$$A(\omega) \equiv \frac{1}{2\pi} \rho(\omega) \coth(\omega/2T) \quad \tilde{K}(\omega,\tau) \equiv K(\omega,\tau) \tanh(\omega/2T) = \frac{\cosh(\omega(\tau - 1/2T))}{\cosh(\omega/2T)}$$

- Normalization

$$G(0) = \int_0^{\infty} d\omega A(\omega) = \int d\phi(\omega) \frac{A(\omega)}{D(\omega)} = \int_0^1 dx n(x) \qquad n(x) = \frac{A(\phi^{-1}(x))}{D(\phi^{-1}(x))}$$

- Averaging over all possible spectra weighted by $w \sim e^{-\chi^2/2\alpha}$

$$\langle n(x) \rangle = \frac{1}{Z} \int \mathcal{D}n \, n(x) e^{-\chi^2/2\alpha} \Rightarrow \langle A(\omega) \rangle = \langle n(\phi(\omega)) \rangle D(\omega)$$

$$\int \mathcal{D}n = \int_0^{\infty} \left(\prod_x dn(x) \right) \Theta(n) \delta \left(\int_0^1 dx n(x) - G(0) \right) \qquad \chi^2 = \sum_{\tau,\tau'} \Delta(\tau) C^{-1}_{\tau,\tau'} \Delta(\tau')$$

$$\Delta(\tau) \equiv \int_0^1 dx n(x) \hat{K}(x,\tau) - G(\tau)$$

$$\hat{K}(\phi(\omega),\tau) = \tilde{K}(\omega,\tau)$$

Eliminating α

1. Plotting χ^2 as a function of α and choosing α at a kink in the χ^2 curve

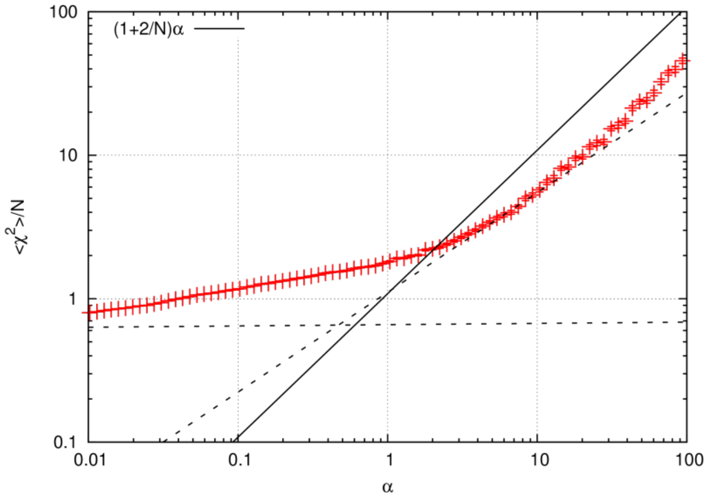
   K.S.D. Beach, arXiv:cond-mat/0403055

2. Calculating a posterior probability $P[\alpha|G]$ from a Bayesian inference as MEM and choosing α which maximizes $P[\alpha|G]$ or averaging over all $\langle n(x)\rangle_\alpha$ weighted by $P[\alpha|G]$ (similarly to MEM)

   S. Fuchs *et al.*, PRE81, 056701 (2010)

$$P[\alpha|G] \propto P[\alpha] \boxed{\int \mathcal{D}n\, e^{-\chi^2/2\alpha}} \qquad P[\alpha] = 1 \text{ or } 1/\alpha$$

Partition function → Density of states calculation is needed. (e.g. Wang-Landau algorithm)



One can also estimate the peak position of P[α|G] by using a relation below.

$$\frac{\partial P[\alpha|G]}{\partial \alpha} = 0 \quad \Rightarrow \quad \frac{\langle \chi^2 \rangle_\alpha}{N} - \left(1 + \frac{2p}{N}\right)\alpha = 0$$

p= 0 or 1

Relation to MEM

- A mean-field treatment of the system of *n*(*x*) is equivalent to MEM

– Entropy

$$S = -\int_0^1 dx \bar{n}(x) \ln \bar{n}(x) = -\int d\omega \bar{A}(\omega) \ln \left(\frac{\bar{A}(\omega)}{D(\omega)} \right)$$

– Free energy

$$F\mathcal{N} = \frac{1}{2}\chi^2 - \alpha S - \mu \mathcal{N}$$

Default model
= prior information of SPF

MEM minimizes the free energy at the mean field level

Monte Carlo evaluation

1. Generating a configuration as superposition of δ functions

$$n_C(x) = \sum_\gamma r_\gamma \delta(x - a_\gamma)$$

   - Update scheme:
     a. Shifting δ functions
     b. Changing residues of δ functions, keeping $\sum_\gamma r_\gamma = G(0)$
   - Updating with probability $P = \min\{1, e^{-\Delta\chi^2/2\alpha}\}$

2. Taking ensemble average at a certain α $\langle n(x) \rangle_\alpha = \frac{1}{N} \sum_C n_c(x)$

3. Converting to the SPF $\langle A(\omega) \rangle_\alpha = D(\omega) \langle n(\phi(\omega)) \rangle_\alpha$

Repeat 1-3 for various αs

Stochastic optimization method (SOM)

- Essentially a special case of SAI

A. S. Mishchenko et al., Phys. Rev. B62, 6317 (2000)

  - No mapping ω→x
  - Same to $D(\omega) = K^{-1}(\omega, \tau_0)$ for our normalization
  - Using boxes instead of delta functions
  - Taking different procedure to determine final solution, which doesn't rely on α

  (Counting how many times difference between input and output correlation functions crosses zero)

  - After sampling some number of independent solutions, just takes simple average over those solutions

Mock data test 1

- A resonance peak + continuum
  - Resonance peak

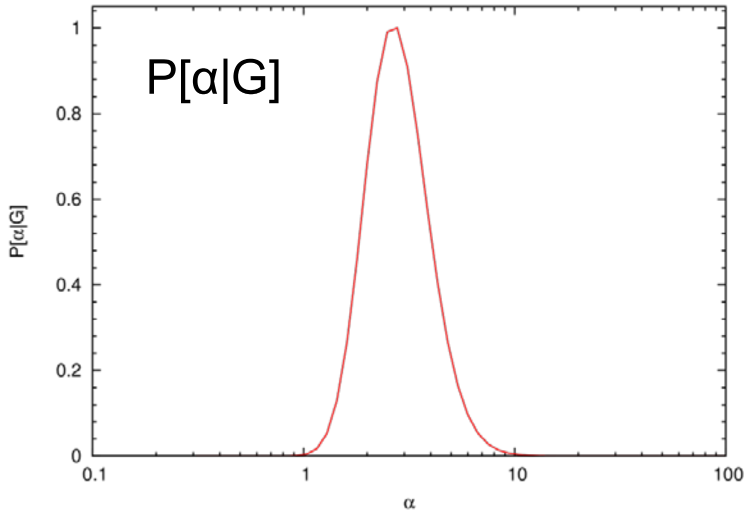
$$\rho(\omega) = \frac{\Gamma M}{(\omega^2 - M^2)^2 + M^2\Gamma^2}\frac{\omega^2}{\pi}$$

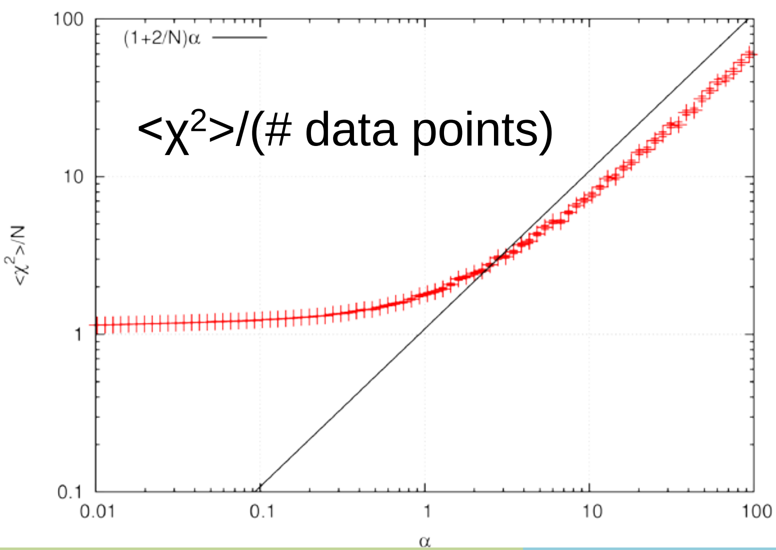
  - Continuum

$$\rho(\omega) = \frac{N_c}{8\pi^2}\Theta(\omega^2 - 4m^2)\omega^2 \tanh\left(\frac{\omega}{4T}\right)\sqrt{1 - \left(\frac{2m}{\omega}\right)^2}\left[a^{(1)} + a^{(2)}\left(\frac{2m}{\omega}\right)^2\right].$$

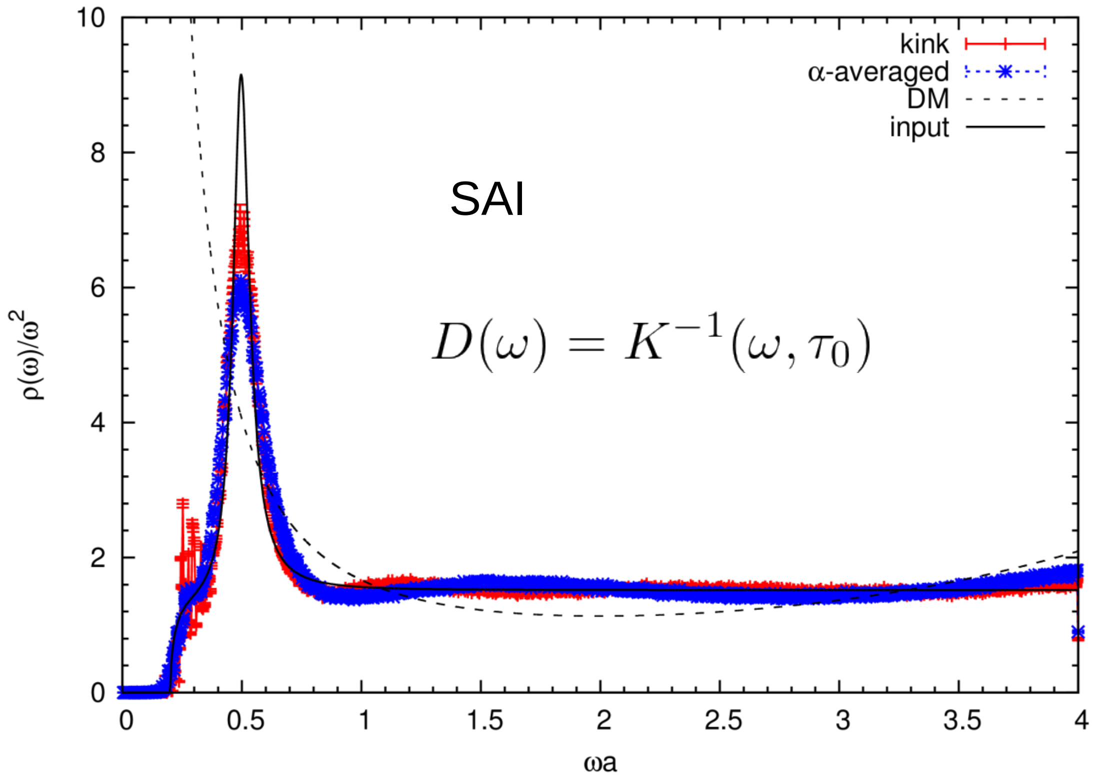
  - # of data points was set to 48.
  - Mock correlator data were created by adding a Gaussian random noise with variance of a form $\sigma(\tau) = \epsilon\tau G(\tau)$, where ε was set to 10^{-4}.

Mock data test 1: α dependence

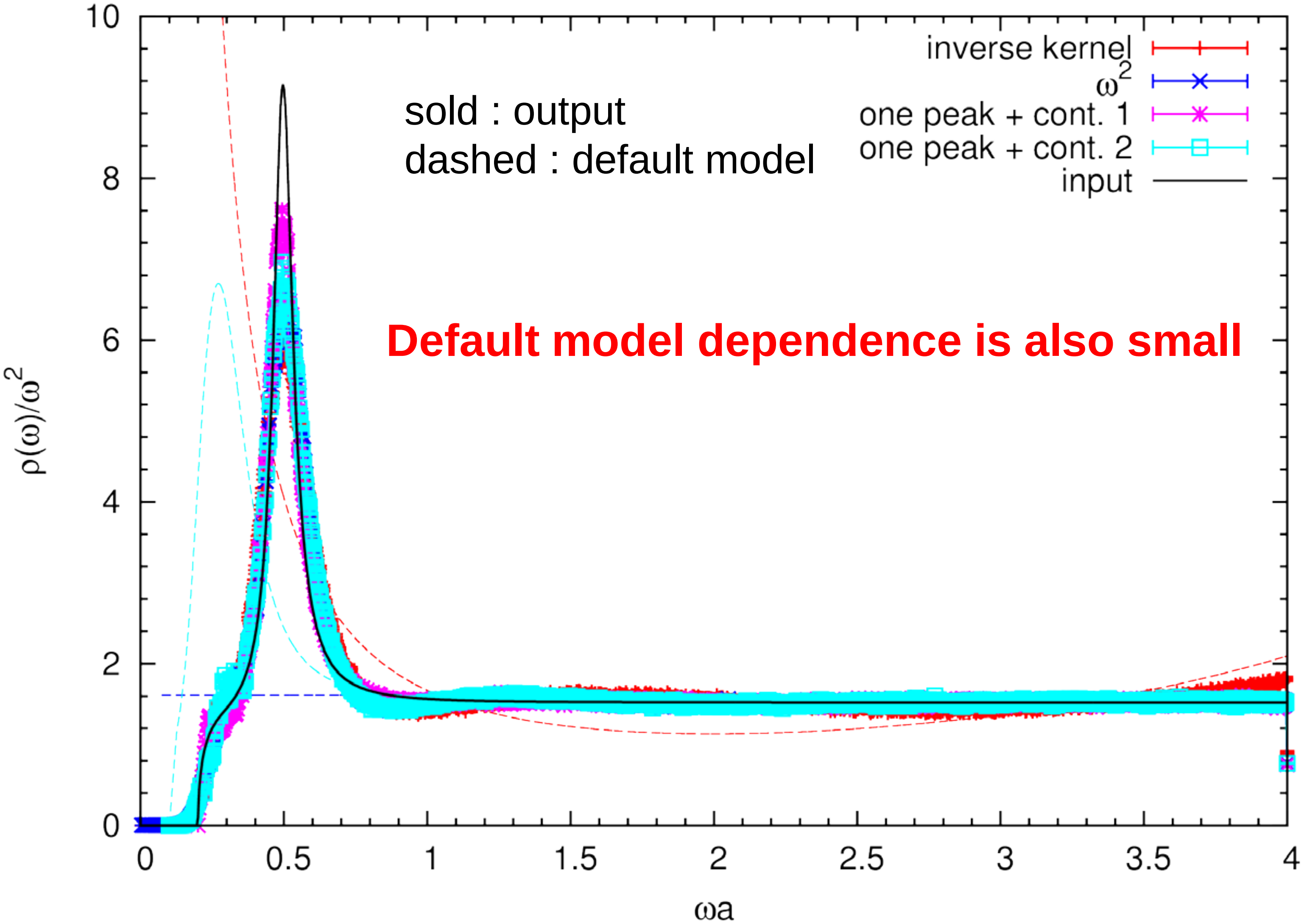




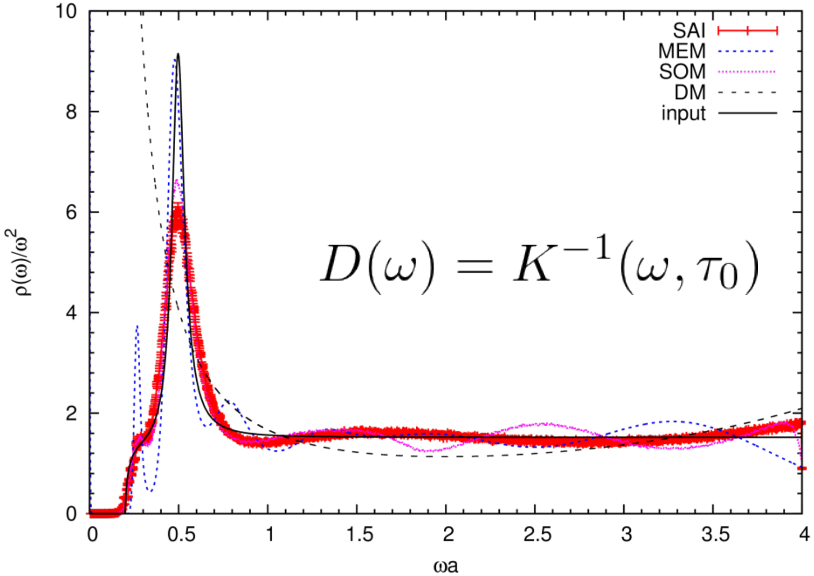
α dependence is small

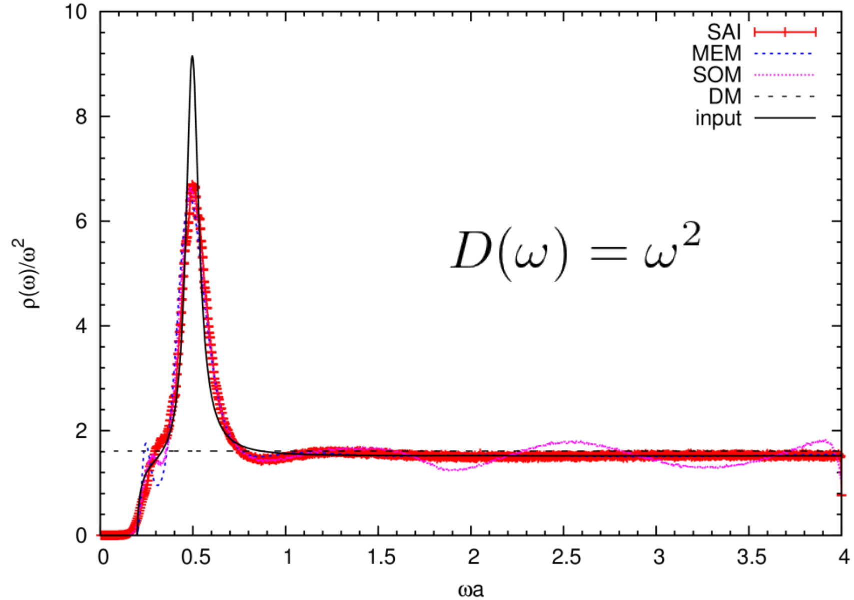


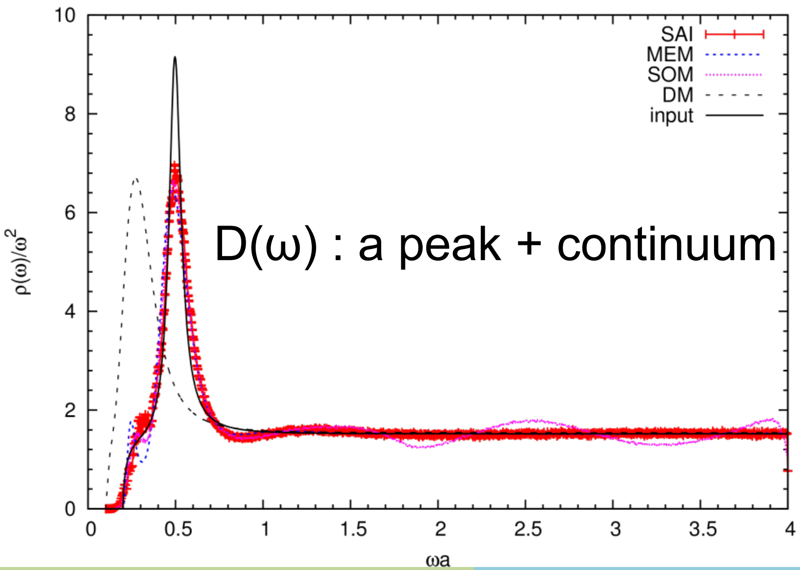
Mock data test 1: default model dependence



Mock data test 1 :comparison with MEM and SOM







SAI and SOM work well for all
MEM doesn't work for some case

Mock data test 2

- A transport peak + continuum
  - Transport peak

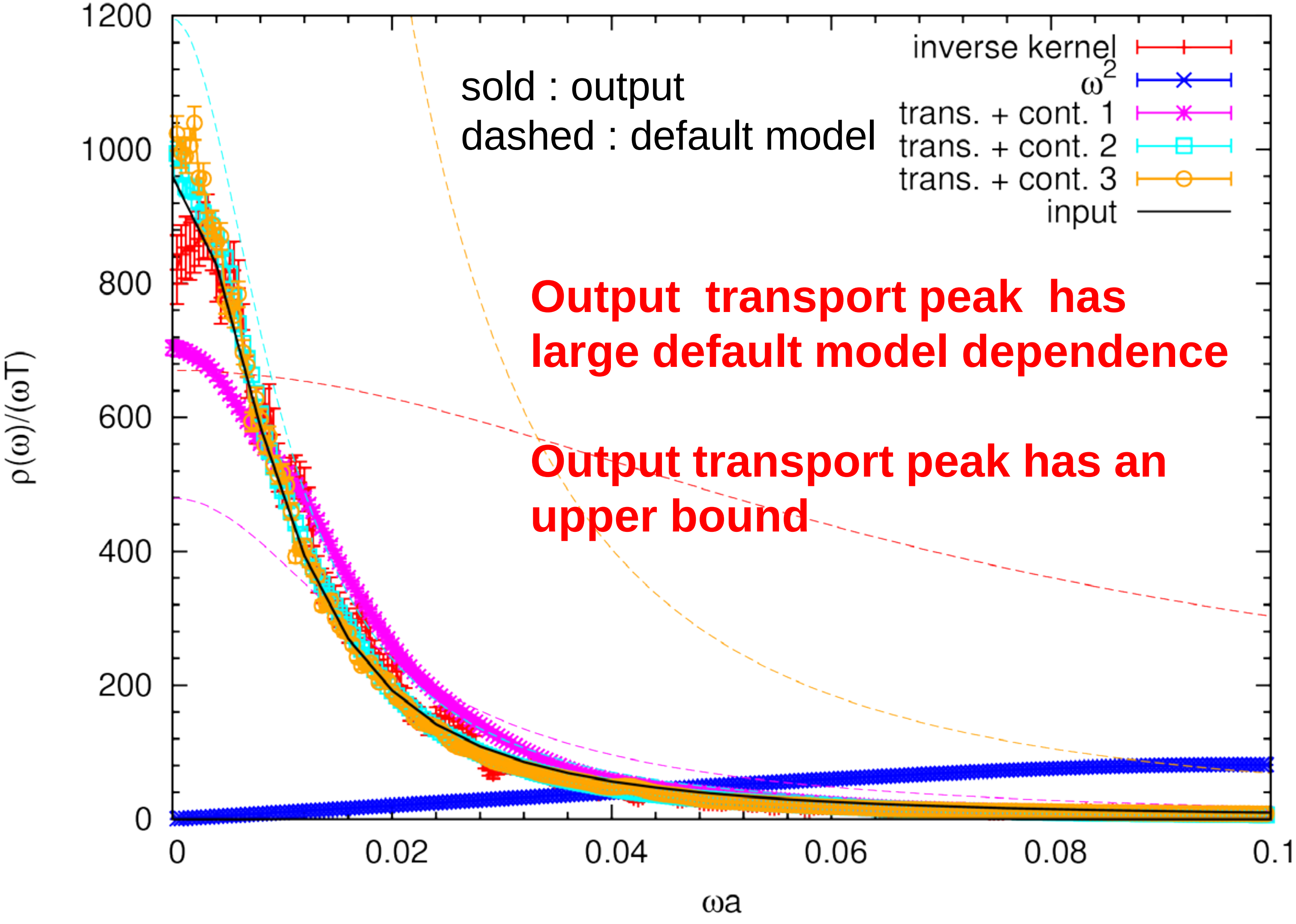
$$\rho(\omega) = \frac{\omega\eta}{\omega^2 + \eta^2}$$

  - Continuum

$$\rho(\omega) = \frac{N_c}{8\pi^2}\Theta(\omega^2 - 4m^2)\omega^2 \tanh\left(\frac{\omega}{4T}\right)\sqrt{1 - \left(\frac{2m}{\omega}\right)^2}\left[a^{(1)} + a^{(2)}\left(\frac{2m}{\omega}\right)^2\right].$$

  - Mock correlator data were created similarly as Test 1.

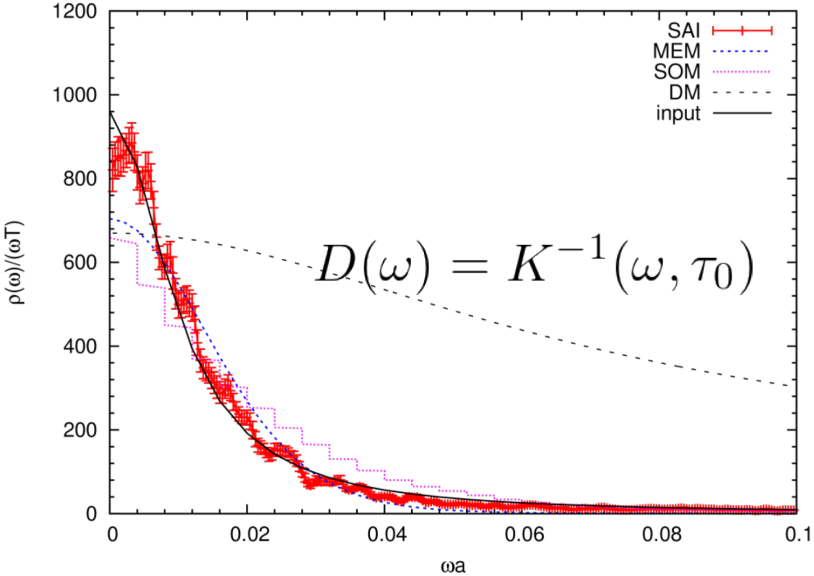
Mock data test 2: default model dependence

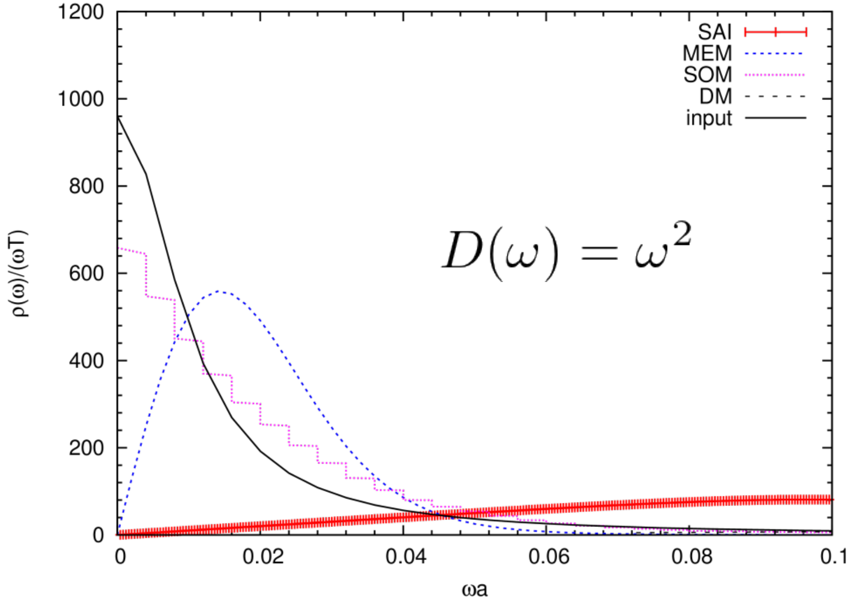


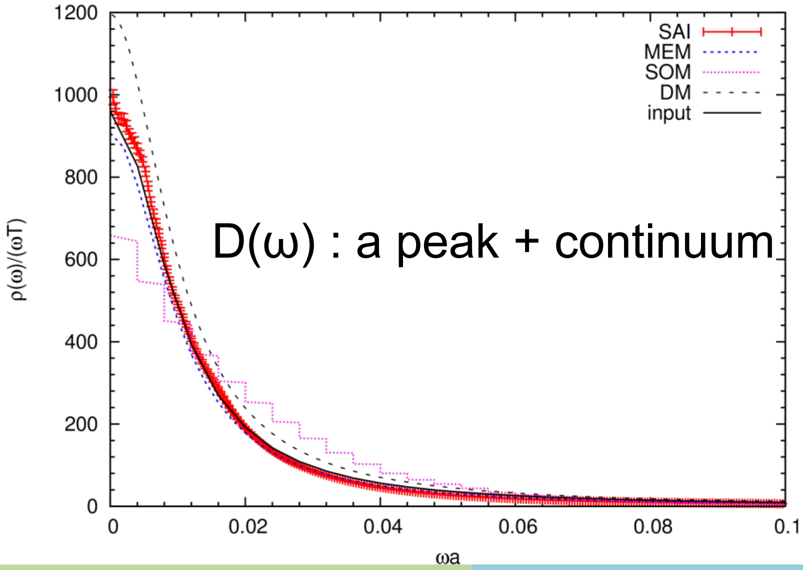
Output transport peak has large default model dependence

Output transport peak has an upper bound

Mock data test2: comparison with MEM and SOM







All methods work similarly.

Lattice setup

- Standard plaquette gauge & O(a)-improved Wilson quark actions
- In quenched QCD
- On fine and large isotropic lattices
- $T = 0.73 - 2.2T_c$
- Both charm & bottom
- Vector SPFs have been calculated by SAI
- So far only $D(\omega) = K^{-1}(\omega, \tau_0)$ was used.

β	N_σ	N_τ	T/T_c	# confs.
7.192	96	48	0.73	259
		32	1.1	476
		28	1.25	336
		24	1.5	336
		16	2.2	239

β	a [fm]	a^{-1} [GeV]	$\kappa_{\rm charm}$	$\kappa_{\rm bottom}$	$m_{J/\Psi}$ [GeV]	m_Υ [GeV]
7.192	0.0188	10.5	0.13194	0.12257	3.140(3)	9.574(3)

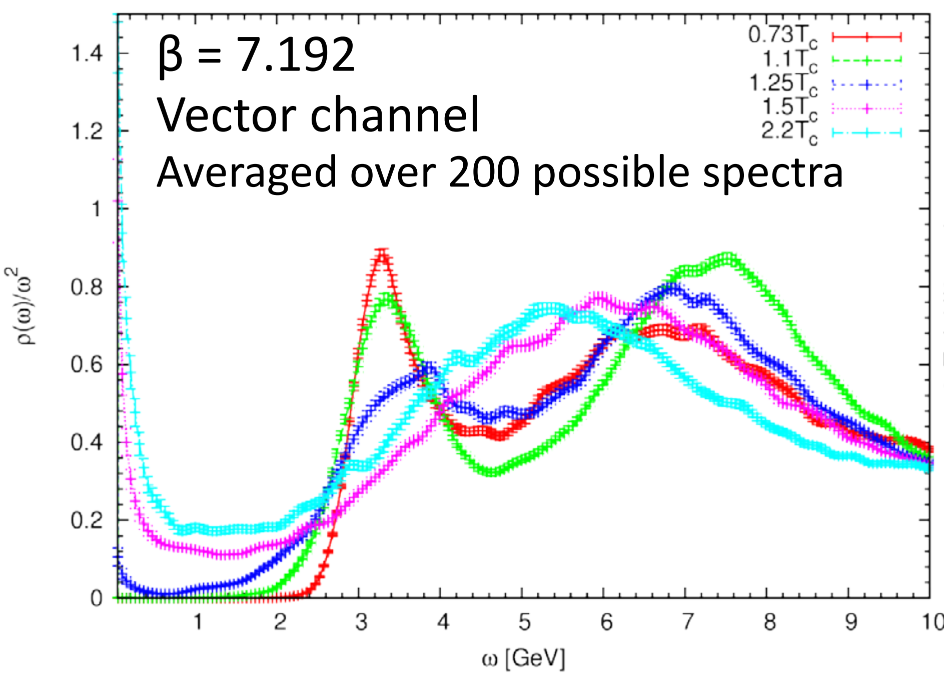
The scale has been set by r_0=0.49fm and with a formula for r_0/a in

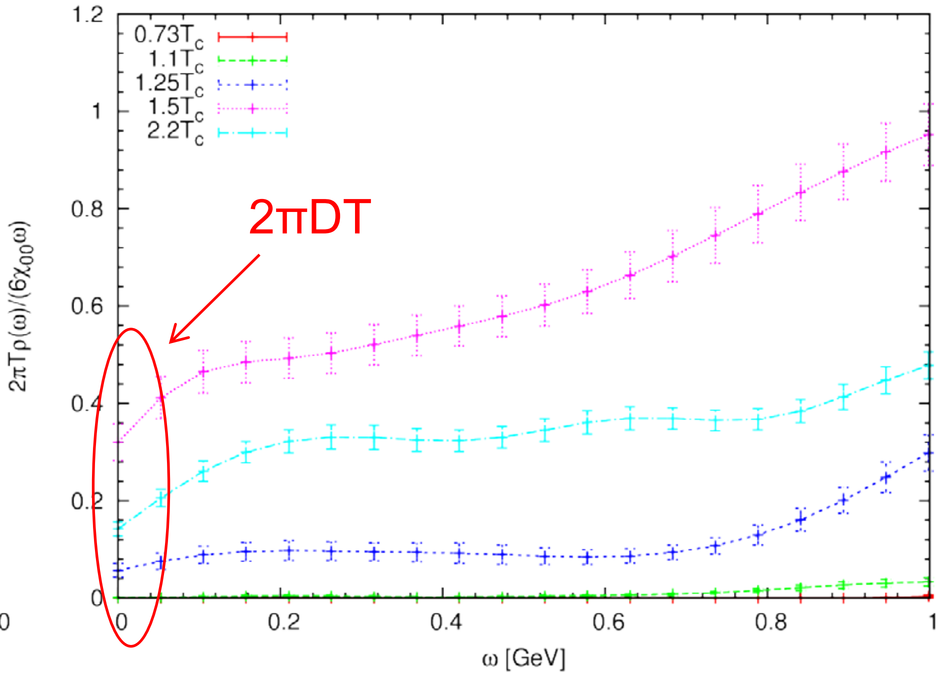
A. Francis, O. Kaczmarec, M. Laine, T. Neuhaus, HO, PRD 91 (2015) 9, 096002

Experimental values: $m_{J/\Psi}$ = 3.096.916(11) GeV, m_Υ = 9.46030(26) GeV

J. Beringer *et al*. [PDG], PRD 86 (2012) 010001

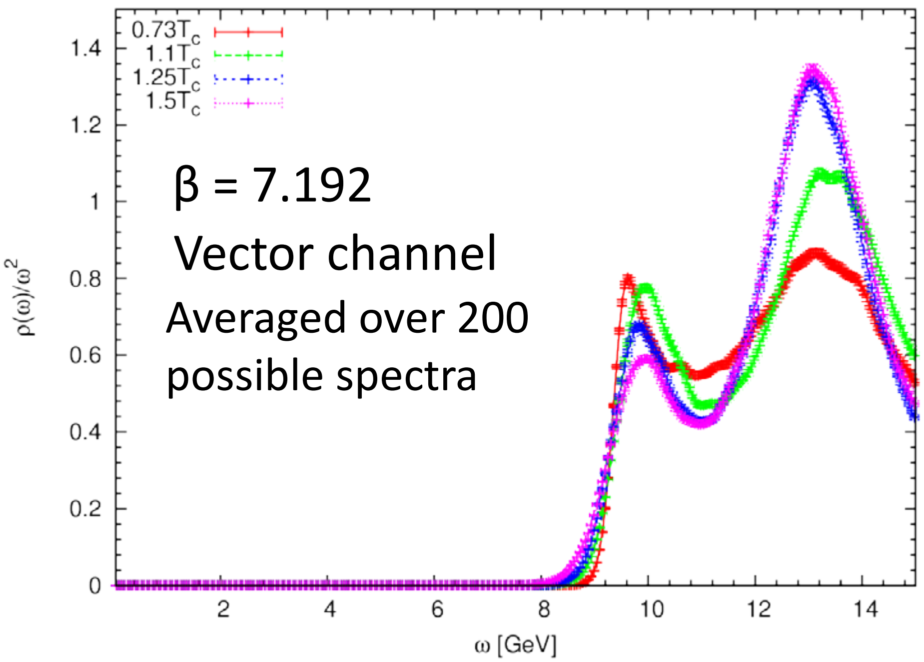
Charmonium SPF at $T > 0$

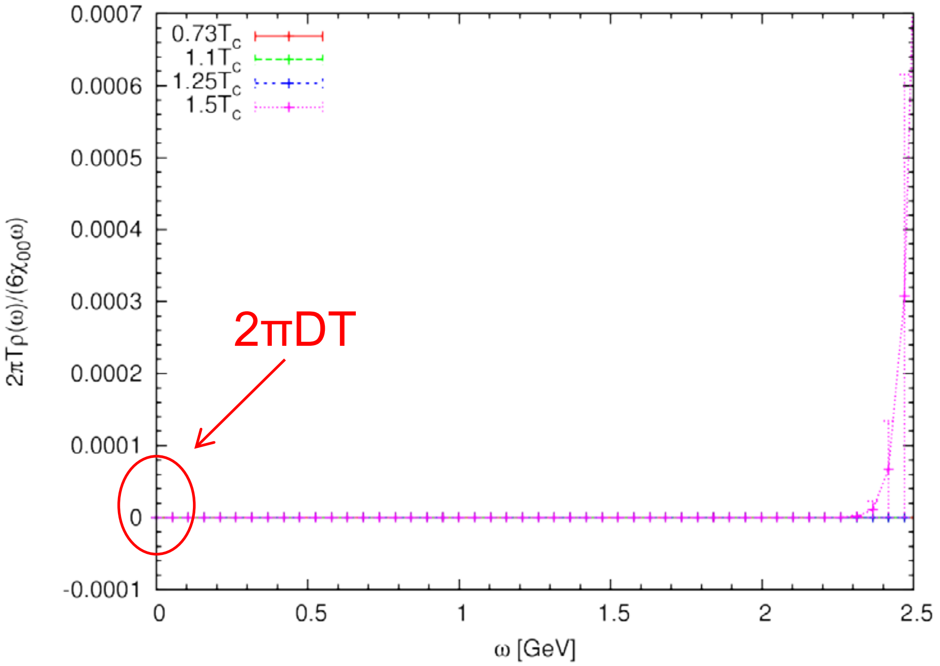




The J/Ψ peak might exist up to 1.25Tc.
A transport peak appears at T > 1.25Tc.

Bottomonium SPF at *T* > 0





Data at 2.2Tc is not shown since it is unstable.

The Υ(1S) peak might exist up to 1.5Tc.

A transport peak is not visible up to 1.5Tc.

Summary

- Stochastic methods to reconstruct SPFs have been tested.
  - SAI and SOM work similarly to MEM.
- Charmonium and bottomonium spectral functions at finite temperature have been studied
  - with a stochastic method
  - J/Ψ seems to survive at $T < 1.25T_c$
  - A transport peak appears at $T > 1.25T_c$ for charm
  - Υ(1S) seems not to melt $T < 1.5T_c$
  - There is no clear signal of a transport peak for bottom even at $1.5T_c$

Outlook

- More studies for SPFs
  - Checking systematic uncertainty more carefully
- Estimating the heavy quark diffusion coefficient precisely
- P-wave states
- Finite momentum
- Taking continuum limit

End