Inclusive τ decay analysis with lattice HVPs

Hiroshi Ohki

RIKEN BNL Research Center

In collaboration with R. J. Hudspith, T. Izubuchi, A. Juettner, C. Lehner, R. Lewis, K. Maltmann, …

@ RBRC Workshop on Lattice Gauge Theories 2016, March 9-11, 2016

outline

- Introduction
  Inclusive tau decay experiment
  Finite energy sum rule and |Vus| determination

- Lattice HVPs and tau decay

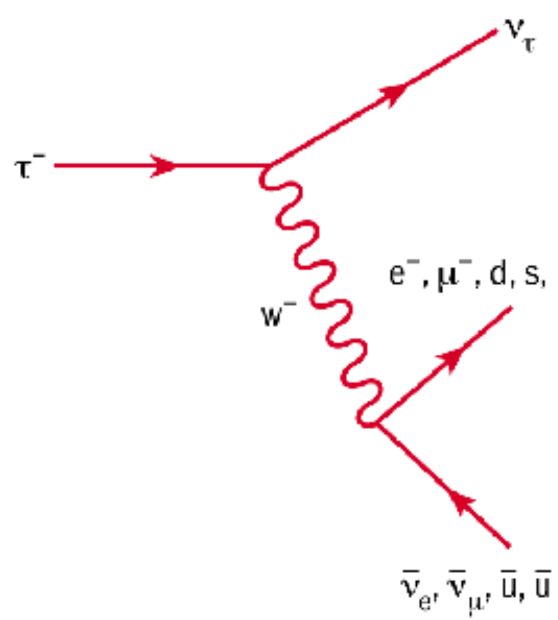
- Result of |Vus|

- Summary

Intruduction

- Lattice QCD calculation can apply to the exclusive modes:
  $f\pi$, fK: $K \to \pi$
- How about inclusive hadronic decay?
  We use τ inclusive Kaon decay experiments -> $|V_{us}|$ determination

- Using optical theorem and dispersion relation,
  τ decay differential cross section
  (τ hadronic decay/τ leptonic decay)
  and the hadronic vacuum polarization
  (HVP) function are related.
  -> We can use lattice HVP calculations.



Optical theorem

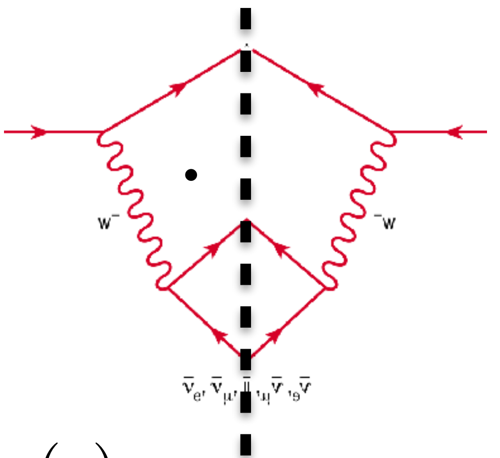
- From unitarity of S matrix, invariant matrix elements are related to the total scattering cross section σ

$$\mathrm{Im}\mathcal{M}(\mathrm{k_1 k_2} \to \mathrm{k_1 k_2}) \propto \sum_{\mathrm{X}} \mathrm{d}\Pi_{\mathrm{X}} |\mathcal{M}(\mathrm{k_1 k_2} \to \mathrm{X})|^2 = \sigma_{\mathrm{tot}}(\mathrm{k_1 k_2} \to \mathrm{any})$$

- Using analytic of M(s) for $s = (k_1 + k_2)^2$

and above multi particles threshold $s > s_{th}$

a branch cut is formed, then



$$2i\mathrm{Im}\mathcal{M}(s + i\epsilon) = \mathcal{M}(s + i\epsilon) - \mathcal{M}(s - i\epsilon) = \sigma_{\mathrm{tot}}(s)$$

- Im M for s0 < s < sth is read off from experimental result.

Tau decay experiment

τ → ν + hadrons decay through V-A current (weak decay)

For the final states with strangeness -1,

R ratio(hadron/lepton) is given in terms of CKM matrix elements Vus and hadron vacuum polarization functions,

$$R_{ij;V/A} \equiv \frac{\Gamma[\tau^- \to \nu_\tau H_{ij;V/A}(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]}$$

$$\frac{dR_{us;V/A}}{ds} = \frac{12\pi^2 |V_{us}|^2 S_{EW}}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\Pi^1_{us;V/A}(s) + \mathrm{Im}\Pi^0_{us;V/A}(s)\right]$$

The spin 0, and 1, hadronic vacuum polarization, V/A current-current

$$\begin{aligned}\Pi^{(\mu\nu)}_{ij;V/A}(q^2) \equiv& i \int d^4x e^{iqx} \langle 0| T\left(J^\mu_{ij;V/A}(x) J^{\dagger\nu}_{ij;V/A}(0)\right)|0\rangle \\ =&(q^\mu q^\nu - q^2 g^{\mu\nu})\Pi^{(1)}_{ij;V/A}(Q^2) + q_\mu q_\nu \Pi^{(0)}_{ij;V/A}(Q^2)\end{aligned}$$

Previous study

|Vus| determination from finite energy sum rule

Finite energy sum rule

- The finite energy sum rule (FESR)

$$\int_0^{s_0} \omega(s)\rho(s)ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} \omega(s)\Pi(s)ds,$$

S0 … finite energy,

w(s) is an arbitrary **analytic** function with polynomial in s.

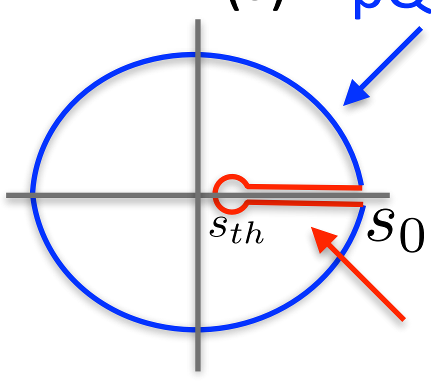
- LHS … ρ(s) is related to the experimental τ inclusive decays

$$\frac{dR_{us;V/A}}{ds} = \frac{12\pi^2|V_{us}|^2 S_{EW}}{m_\tau^2}(1-y_\tau)^2 \times \left[(1+2y_\tau\rho^{(0+1)}_{us;V/A} - 2y_\tau\rho^0_{us;V/A})\right]$$

- RHS … Analytic calculation

with perturbative QCD (pQCD) and OPE

(s0 should be large enough)



|Vus| determination from FESR

[E. Gamiz, et al. PRL 94, 011803, 2005]

- **Inclusive** τ decay rates with ud and s quark final states,

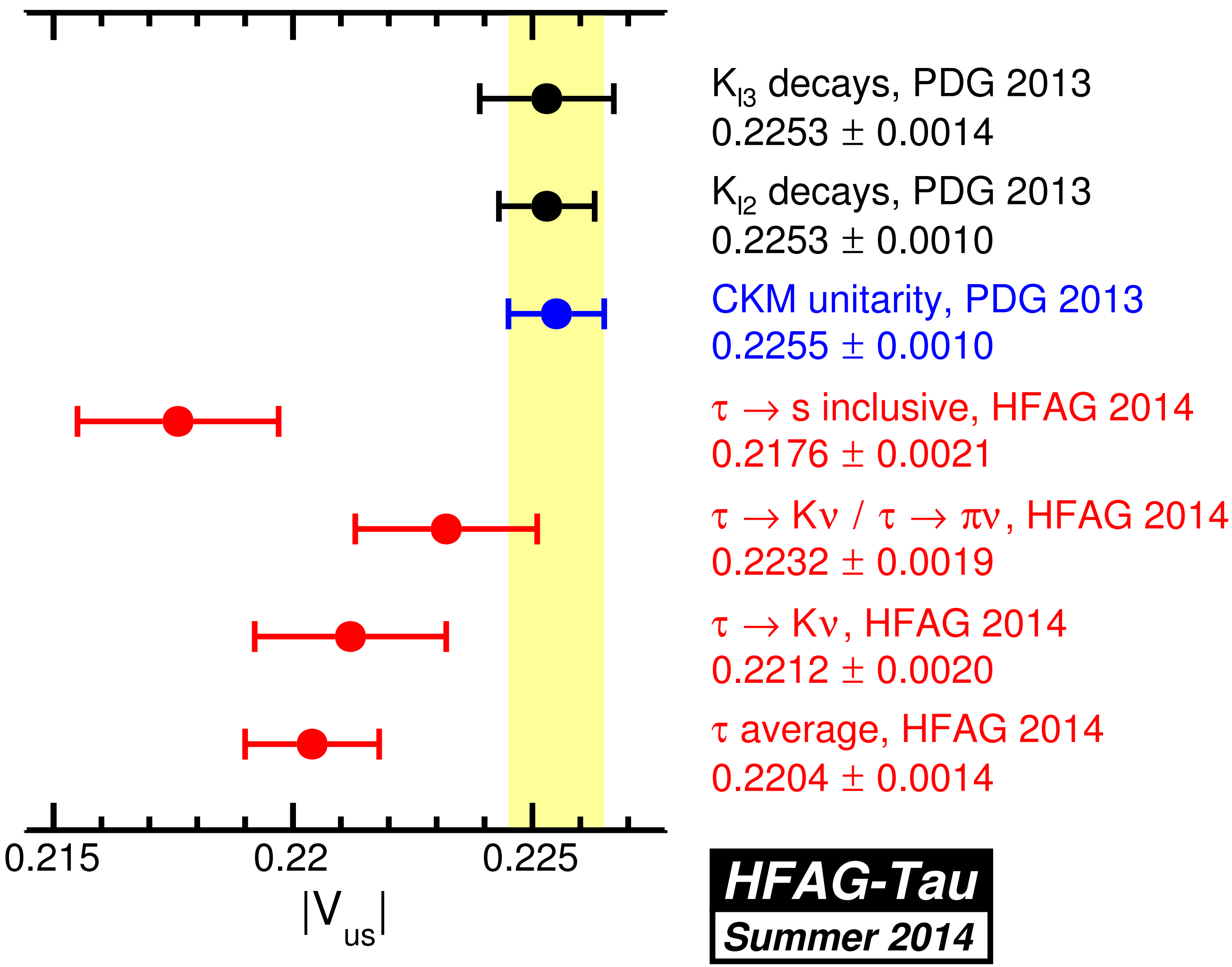
$$R_\tau^{kl} \equiv \int_0^{M_\tau^2} ds \left(1 - \frac{s}{M_\tau^2}\right)^k \left(\frac{s}{M_\tau^2}\right)^l \frac{dR_\tau}{ds} = R_{\tau,NS}^{kl} + R_{\tau,S}^{kl} \,,$$

- Taking the differences, $\delta R \equiv \frac{R_{NS}}{|V_{ud}|^2} - \frac{R_S}{|V_{us}|^2}$

- Use perturbative OPE with D > 2, since these observables vanish in the SU(3) symmetry limit.

  Many theoretical uncertainties may drop out.

- |Vus| is 3+σ lower than Kl3, Kl2 determinations.



- |Vus| from inclusive τ decay -> 3 σ deviation from CKM unitarity
- pQCD and high order OPE -> problematic uncertainties?

This work

- So far we do not know if 3σ discrepancy may be explained
  by new physics beyond the SM.

- We would like to propose an alternative method to calculate
  |Vus| from the inclusive τ decay.

- By combing both the lattice data and pQCD,
  we could expect more precise determination of |Vus|.
- As a result, pQCD uncertainty can be suppressed.

- We aim to elucidate a possible origin of the so-called |Vus| puzzle.

Our strategy

- Using a different type of the weight function w(s) which has residues

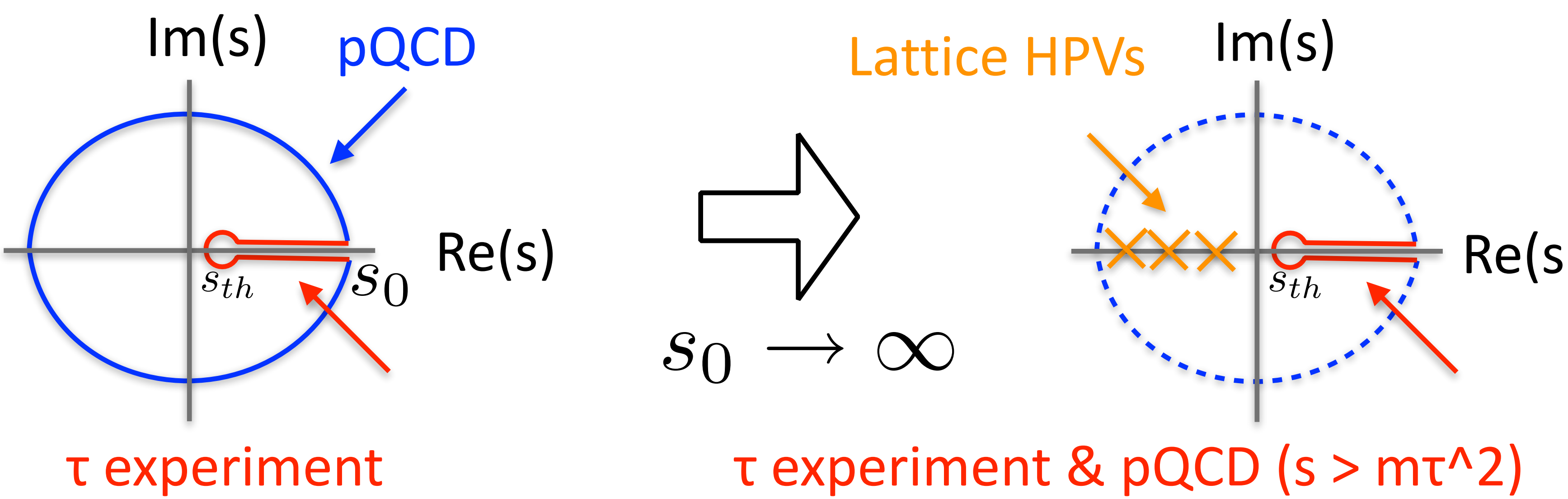
$$\omega(s) = \frac{1}{(s+Q_1^2)(s+Q_2^2)\cdots(s+Q_N^2)}$$

and taking S0 -> ∞,

$$\int_0^{\infty} \rho(s)\omega(s)ds = \sum_k^N \mathrm{Res}\left(\Pi(-Q_k^2)\omega(-Q_k^2)\right)$$

LHS … Experimental data and pQCD

RHS … Lattice HPVs Π(Q) at Euclidean momentum region

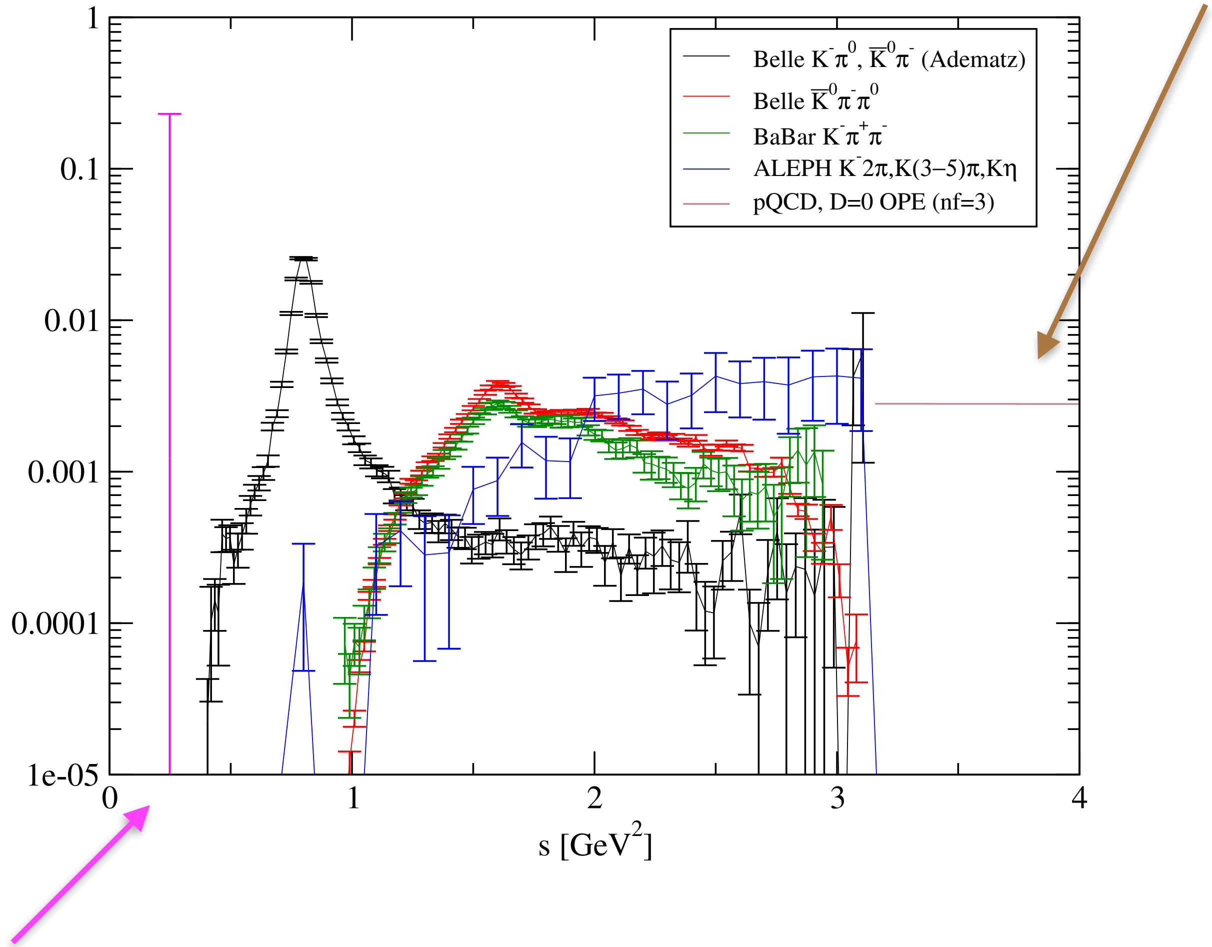


τ inclusive decay experiment

$$|V_{us}|^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\Pi^1(s) + \mathrm{Im}\Pi^0(s) \right]$$

To compare with experiments,

a conventional value of |Vus|=0.2253 is used



For K pole, we assume a delta function form with kaon decay experiments,

$$\delta(s - m_k^2)0.0012299(46)$$

Weight function

- we use pole-type weight function;

$$\omega(s) = \prod_{k}^{N} \frac{1}{(s+Q_k^2)}, \quad (Q_k^2 > 0)$$

(Number of poles: N)

For convergence of contour integral,

a weight function with N ≧ 3 is required, which suppresses

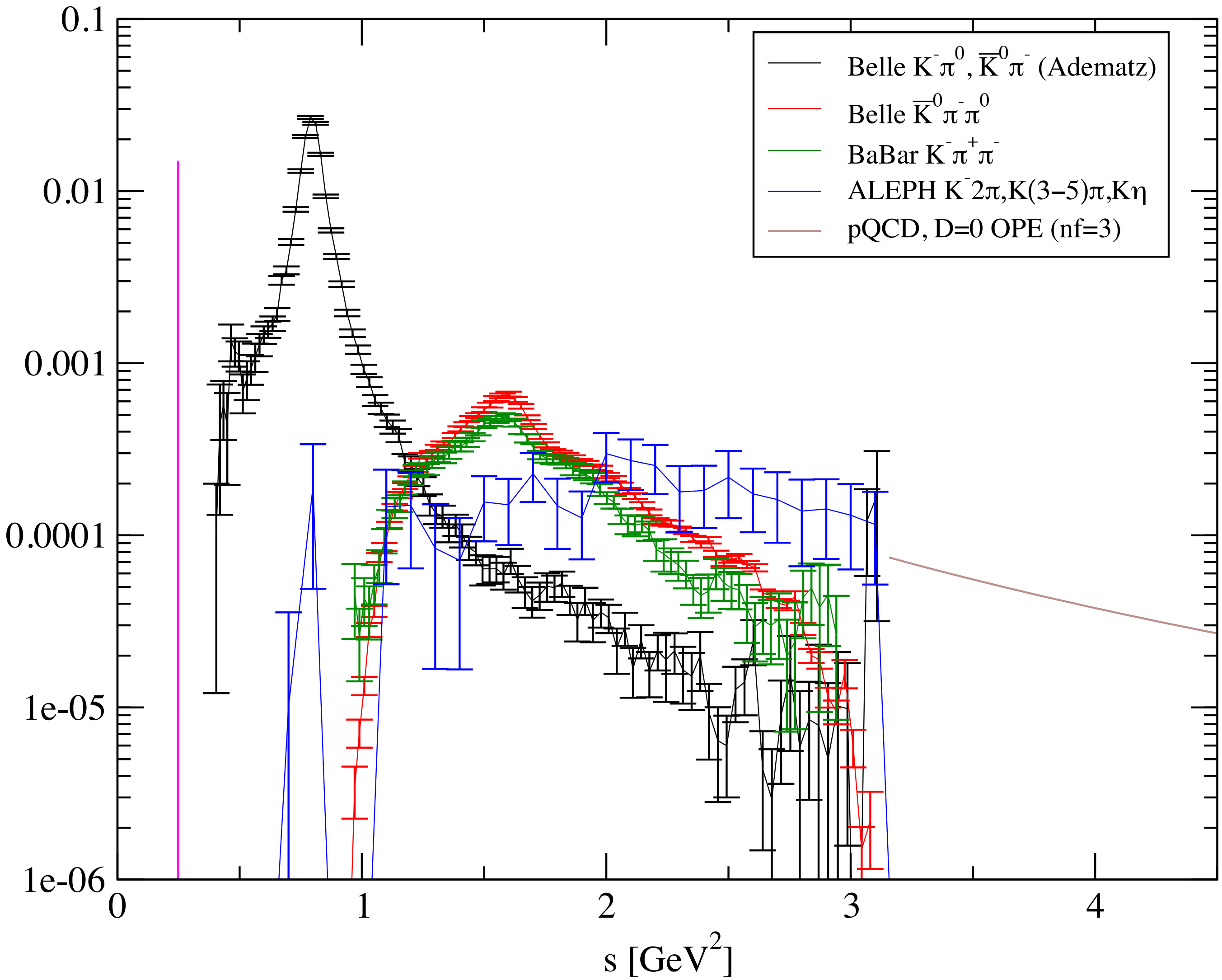
- large error from higher multi hadron final states at s > mk^2
- contributions from pQCD with OPE at s > mτ^2

For lattice HVPs,

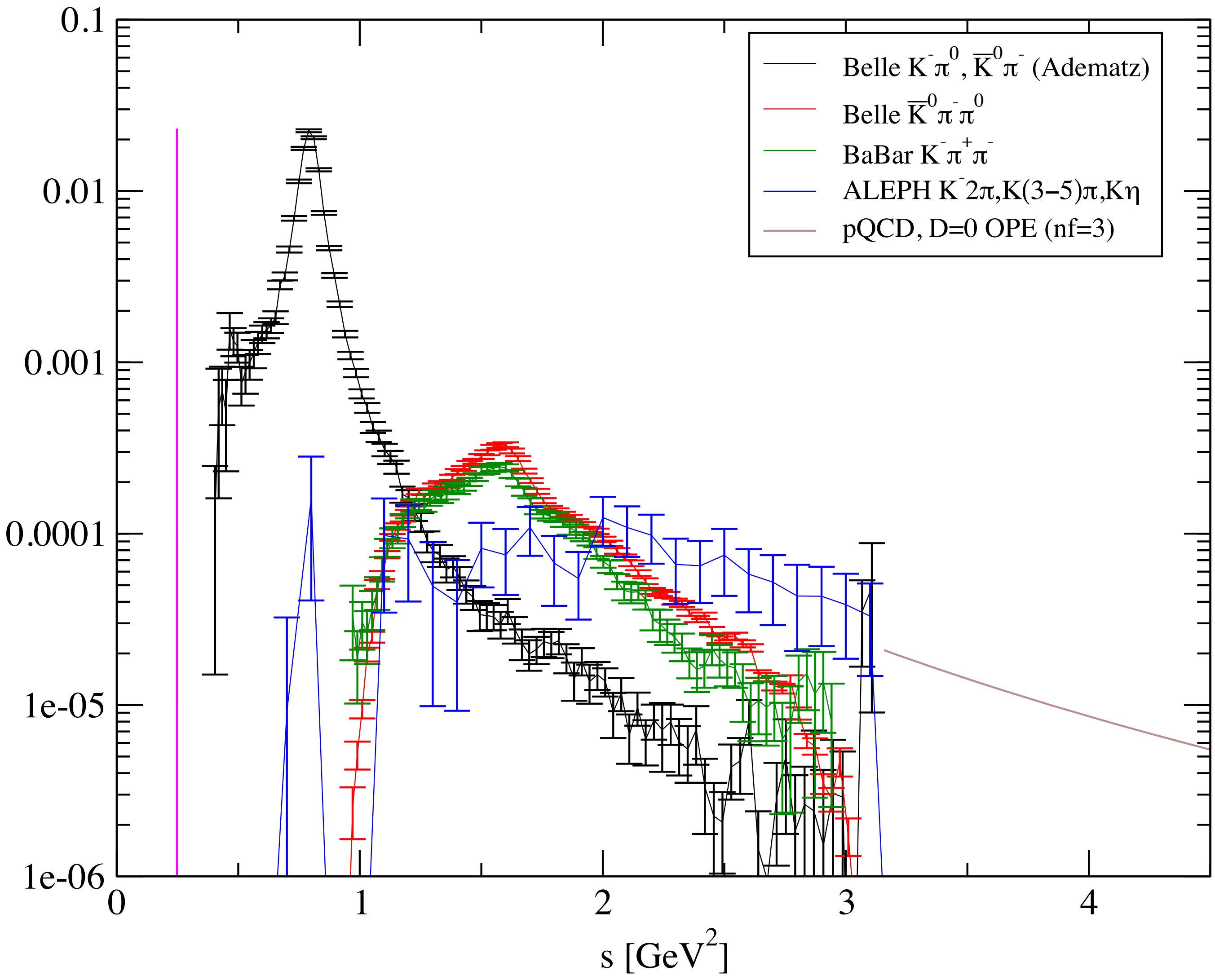
Q^2 values should not be too small to avoid finite size(time) effect,

and not to be large to avoid large discretization error.

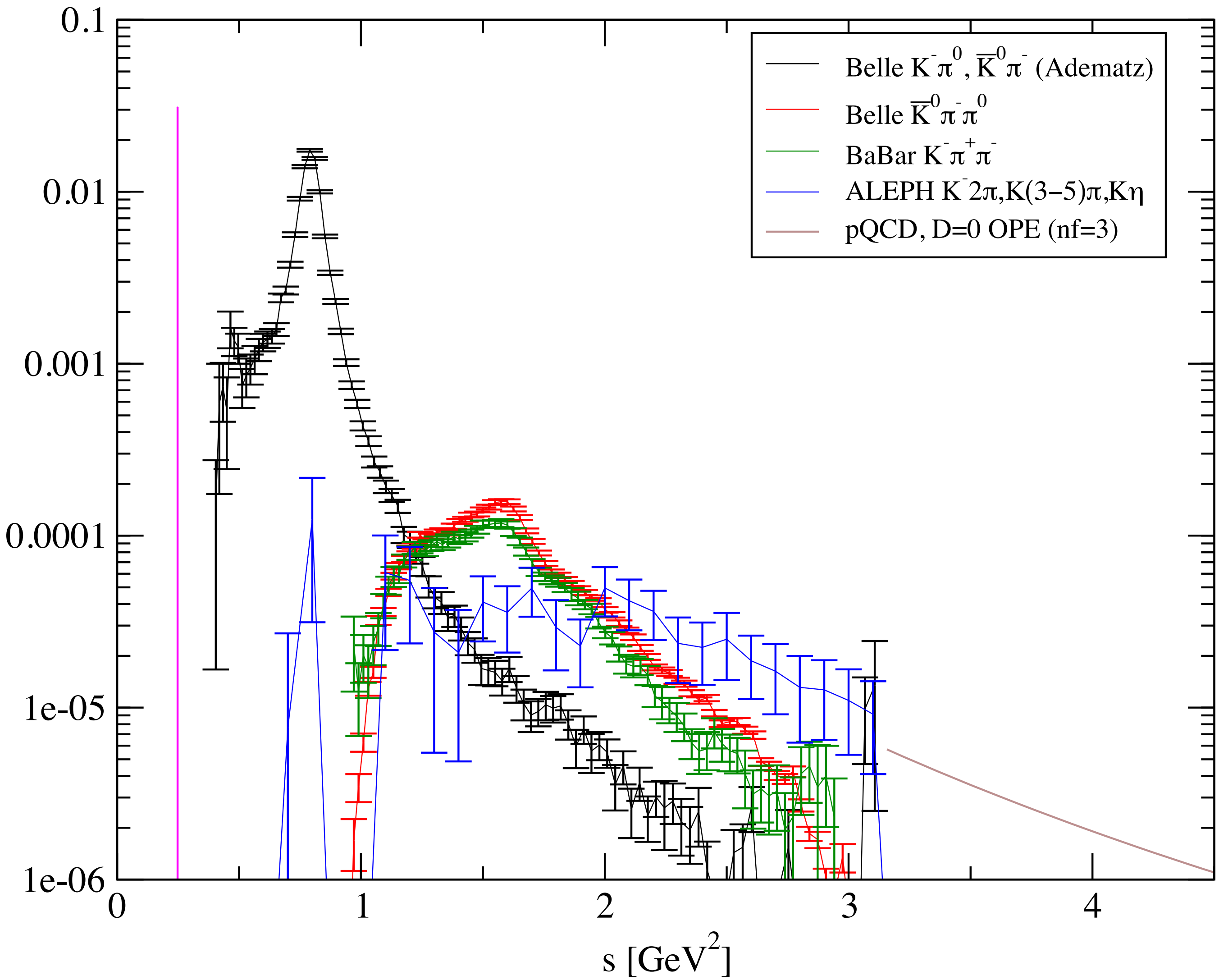
- example: N=3, $\{Q_1^2, Q_2^2, Q_3^2\} = \{0.1, 0.2, 0.3\}$



- example: N=4, $\{Q_1^2, Q_2^2, Q_3^2, Q_4^2\} = \{0.1, 0.2, 0.3, 0.4\}$



- example: N=5, $\{Q_1^2, Q_2^2, Q_3^2, Q_4^2, Q_5^2\} = \{0.1, 0.2, 0.3, 0.4, 0.5\}$



Lattice calculation

Lattice HVPs

HVPs from V/A current-current correlation functions with u s flavors,

we consider zero-spatial momentum

$$\Pi_{\mu\nu}^{V/A}(t) = \frac{1}{V}\sum_{\vec{x}}\langle J_{\mu}^{V/A}(\vec{x},t) J_{\nu}^{V/A}(\vec{x},0)\rangle$$

Spin =1, 0 components can be obtained in momentum space as

$$\Pi_{\mu\nu}(q) = (q^2\delta_{\mu\nu} - q_\mu q_\nu)\Pi^{(1)}(q^2) + q_\mu q_\nu \Pi^{(0)}(q^2),$$

On the lattice, those with subtraction of unphysical zero-mode can be obtained by discrete Fourier transformation,

(direct double subtraction, sine cardinal Fourier transformation.)

$$\hat{\Pi}(q^2) = \sum_{t=-T/2}^{t=T/2-1}\left(\frac{e^{i\tilde{q}t}-1}{q^2} + \frac{t^2}{2}\right)\Pi(t)$$

$$\tilde{q}_\mu = 2\sin\left(q_\mu/2\right)$$

lattice QCD ensemble and parameters

2+1 flavor domain-wall fermion gauge ensemble generated by RBC-UKQCD

Vol.	a^{-1}[GeV]	m_π[GeV]	m_K[GeV]	stat.
$24^3 \times 64$	1.785(5)	0.340	0.533	450
		0.340	0.593	450
$32^3 \times 64$	2.383(9)	0.303	0.537	372
		0.303	0.579	372
		0.360	0.554	207
		0.360	0.596	207
• $48^3 \times 96$	1.730(4)	0.139	0.499	4224
		$0.135^\dagger$	$0.4937^\dagger$	5 PQ-correction, (4224)
$64^3 \times 128$	2.359(7)	0.139	0.508	2560

- Our main analysis is done on L=48 and 64,
  at almost physical quark mass region, L=5 fm.
- **PQ-correction**: partially quench (PQ) corrected HVP data at the physical point (†)
- L=24 and 32 have heavier kaon masses, which will be used
  to see general tendency.

Lattice HVPs and inclusive τ decay

$$\Pi(s) \equiv \left[\left(1 + 2\frac{s}{m_\tau^2}\right)\mathrm{Im}\Pi^1(s) + \mathrm{Im}\Pi^0(s)\right]$$

$$\sum_{k=1}^{N} \mathrm{Res}\left(\omega(-Q_k^2)\Pi_{lat}(-Q_k^2)\right)$$

$$|V_{us}|^2 \int_0^\infty ds\omega(s)\Pi(s)$$

- s < mτ^2, experimental data is used for spectrum integral.
- s > mτ^2, we use D=0, OPE result. For comparison with experiments,
- a conventional value of |Vus|=0.2253 is used.

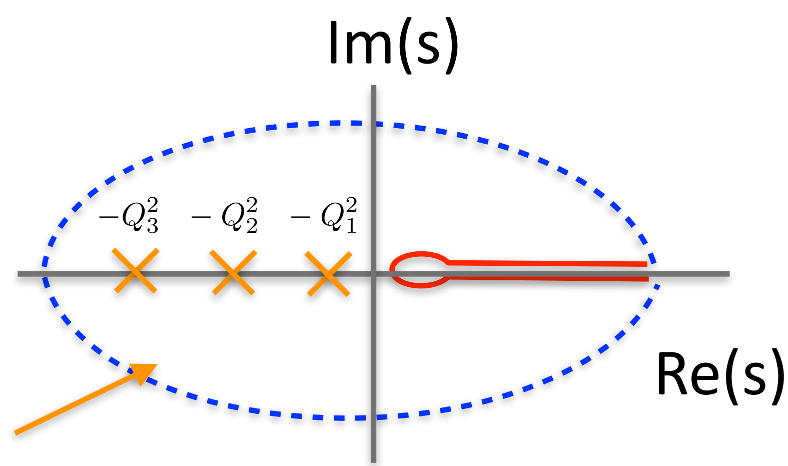
A systematic study of weight function dependence

$$\omega(s) = \prod_{k}^{N} \frac{1}{(s+Q_k^2)}, \quad (Q_k^2 > 0)$$

- C (center value of weights),
- Δ (separation of the pole position),
- N (the number of the poles).

$$\{Q_1^2, Q_2^2, \cdots, Q_N^2\} = \{C-(N/2+1)\Delta, \cdots, C-\Delta, C, C+\Delta, \cdots, C+(N/2+1)\Delta\}$$

$$C = \frac{Q_1^2 + Q_2^2 + \cdots + Q_N^2}{N}$$

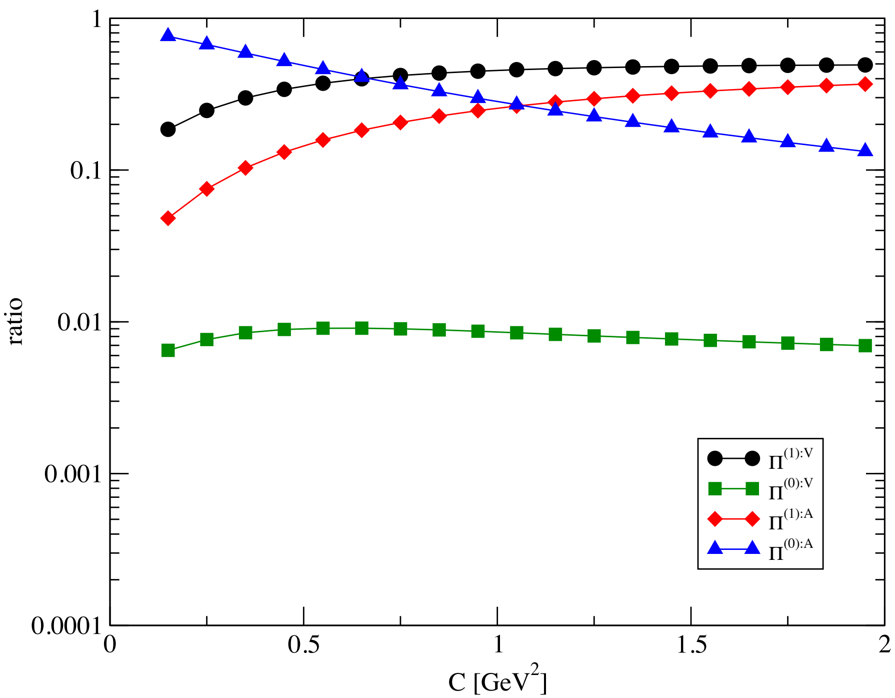


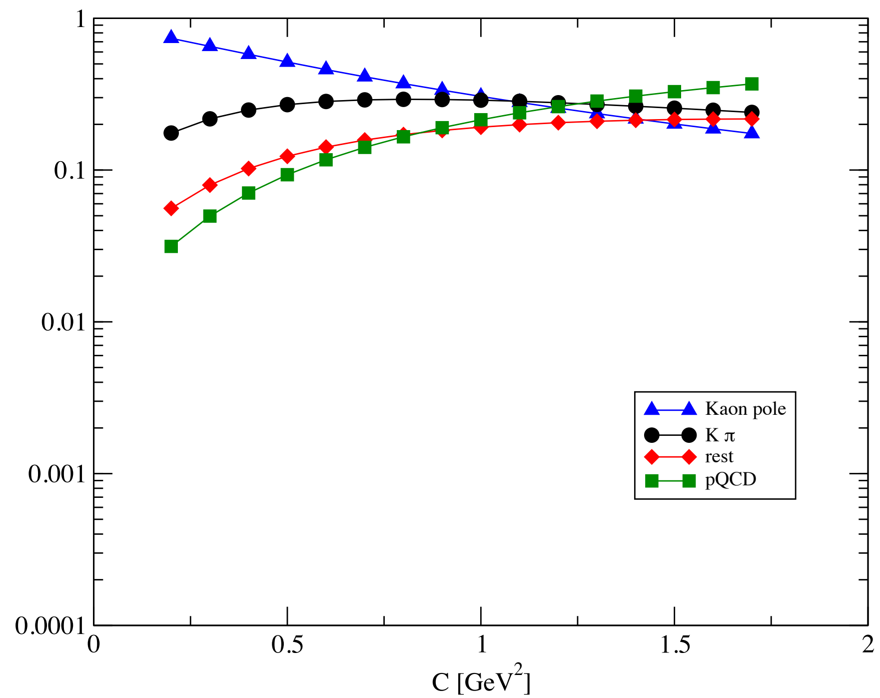
pole positions (N=3 case)

• N=3,   Δ=0.1 [GeV^2]

$L = 48,\ a^{-1} = 1.73[\mathrm{GeV}],\ \ m_\pi = 0.139[\mathrm{GeV}]$

Experiment





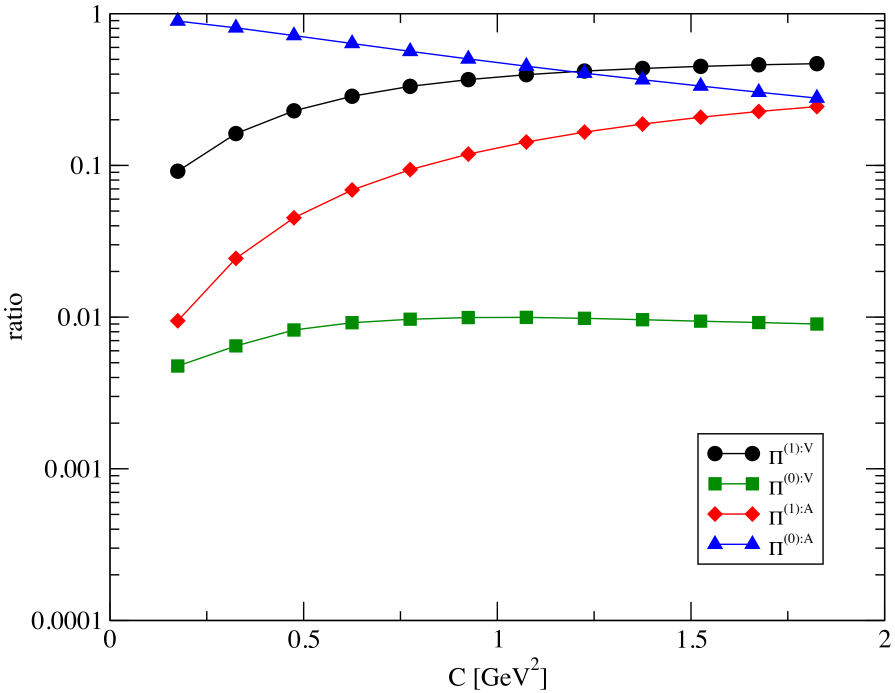
Left : Ratios of each contribution of V/A with spin=0, 1 to the total residue. (Lattice)

Right: Ratios of each decay modes to total cross section. (Experiments)

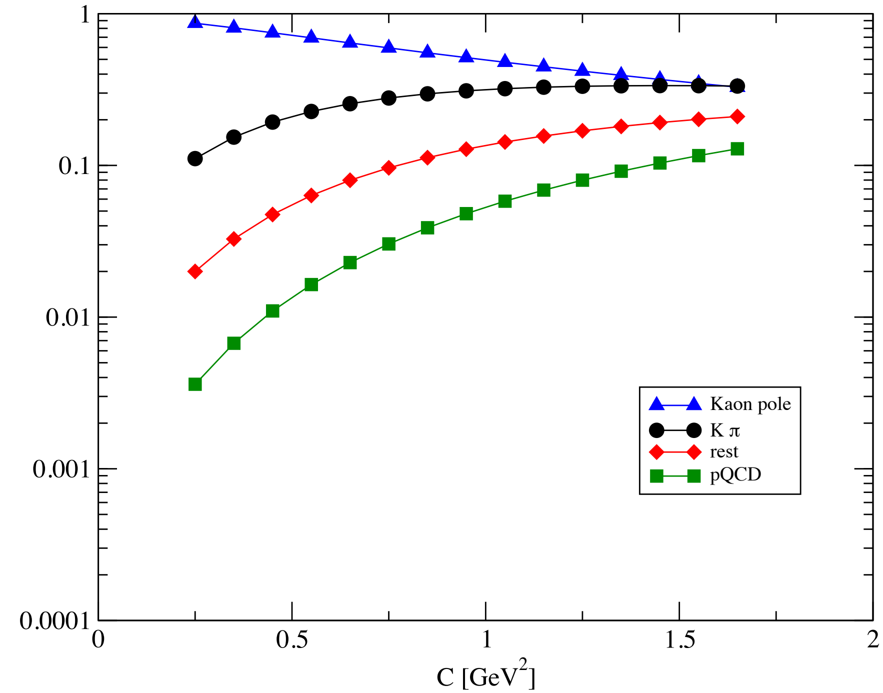
rest : multi π channels, K η

- N=4,   Δ=0.1 [GeV^2]

$$L = 48,\ a^{-1} = 1.73[\mathrm{GeV}],\ \ m_\pi = 0.139[\mathrm{GeV}]$$

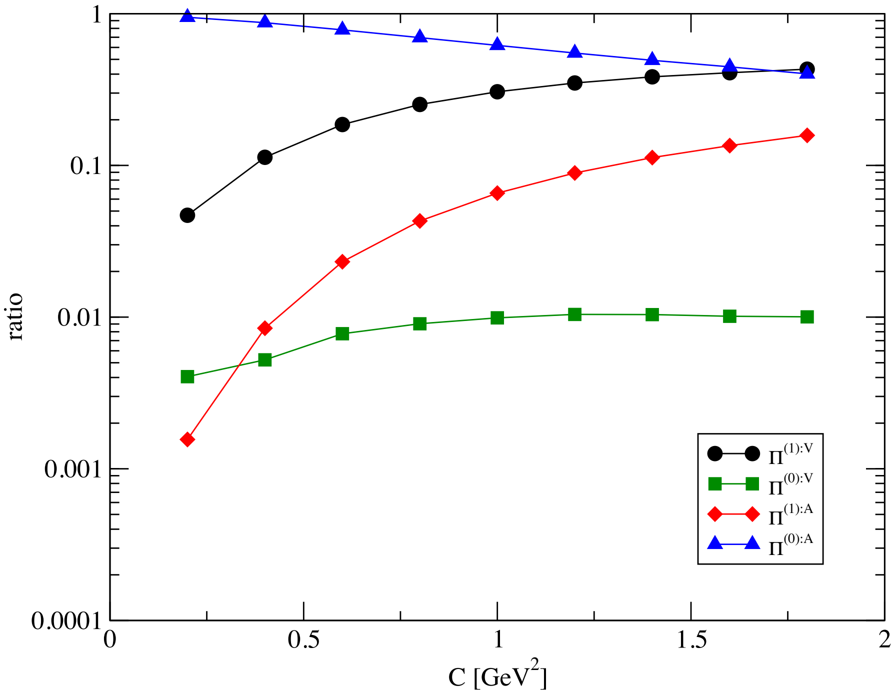


Experiment

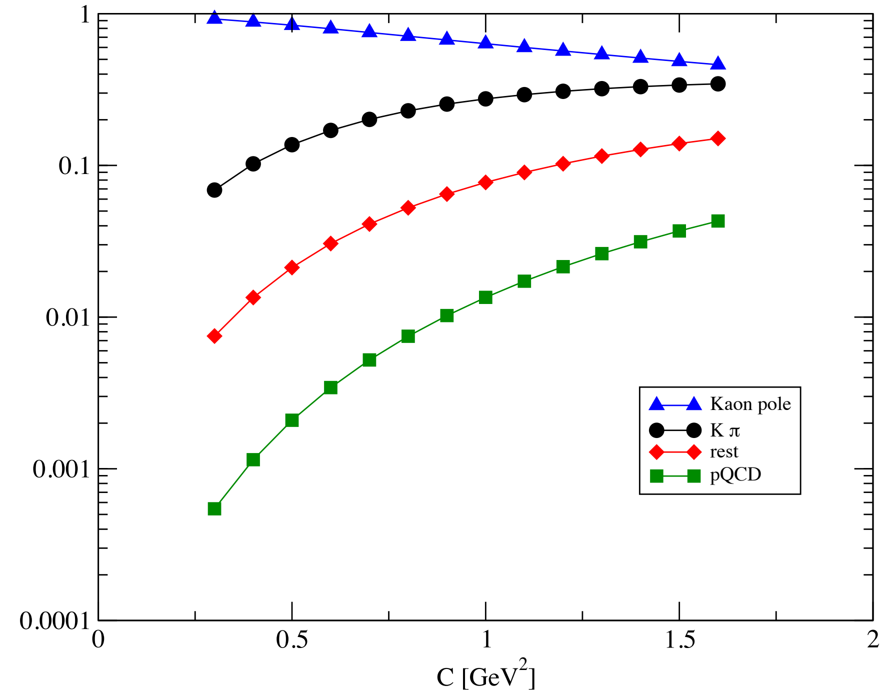


- N=5,   Δ=0.1 [GeV^2]

$L = 48,\ a^{-1} = 1.73[\mathrm{GeV}],\ \ m_\pi = 0.139[\mathrm{GeV}]$



Experiment



- For larger N with smaller Q^2, Kaon pole is the most dominant contribution.
- pQCD and rest modes are highly suppressed.

|Vus| from lattice HVPs

- |Vus| can be determined from K pole channel only (exclusive mode).
- Since τ -> K decay mode is dominated by axial spin = 0 channel, so we have

$$|V_{us}^{\mathrm{K-pole}}| = \sqrt{\frac{\rho_{exp}^{\mathrm{K-pole}}}{F_{lat}(\Pi^{(0):A})}}$$

- We can also determine |Vus| using all inclusive decay modes and lattice results;

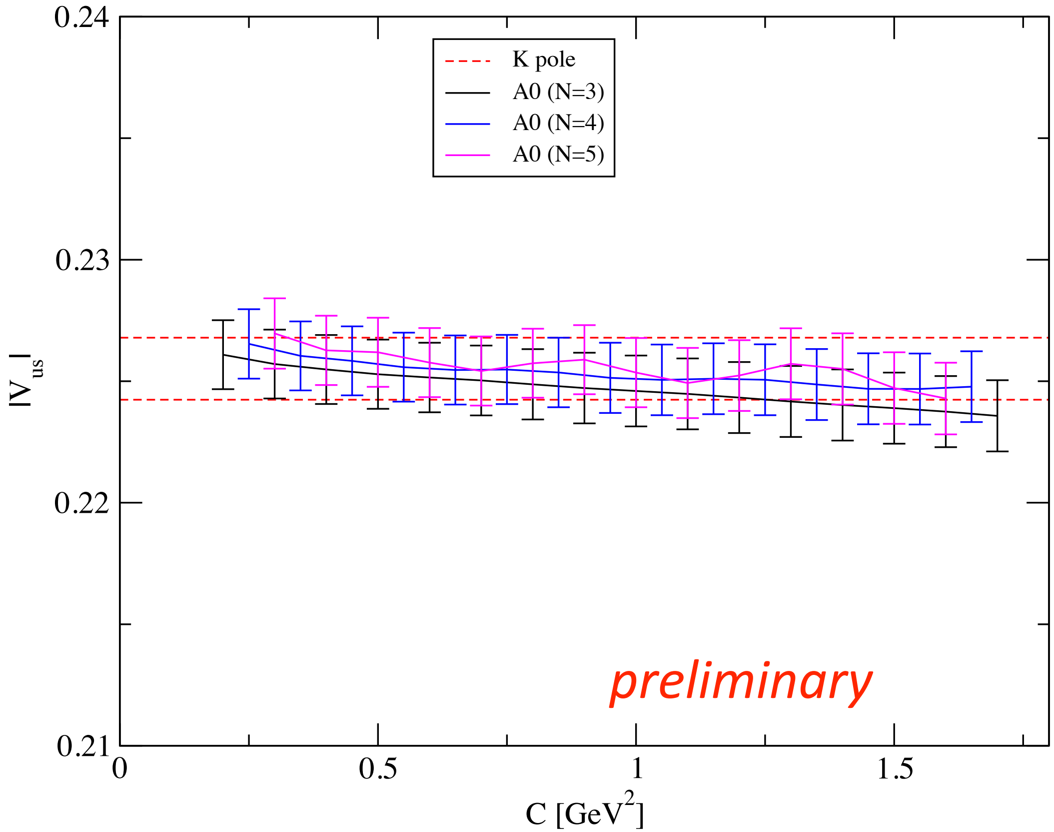
$$|V_{us}| = \sqrt{\frac{\rho_{exp}}{F_{lat} - \rho_{pQCD}}}$$

$$\rho_{exp} = |V_{us}|^2 \int_0^{m_\tau^2} ds \omega(s) \Pi(s) \qquad \rho_{pQCD} = \int_{m_\tau^2}^{\infty} ds \omega(s) \Pi_{OPE}(s)$$

$$F_{lat} = \sum_{k=1}^{N} \mathrm{Res}\left(\omega(-Q_k^2)\right) \Pi_{lat}(-Q_k^2)$$

$|V_{us}^{\mathrm{K-pole}}|$ Result

$|V_{us}^{\mathrm{K-pole}}|$ from L=48 lattice at physical quark mass



N : number of poles

K pole: determined from fK (K decay constant)

|Vus| is universal and consistent with fK determination (mild dependence of C, N)

Our result suggests : A0 channels is dominated by K pole

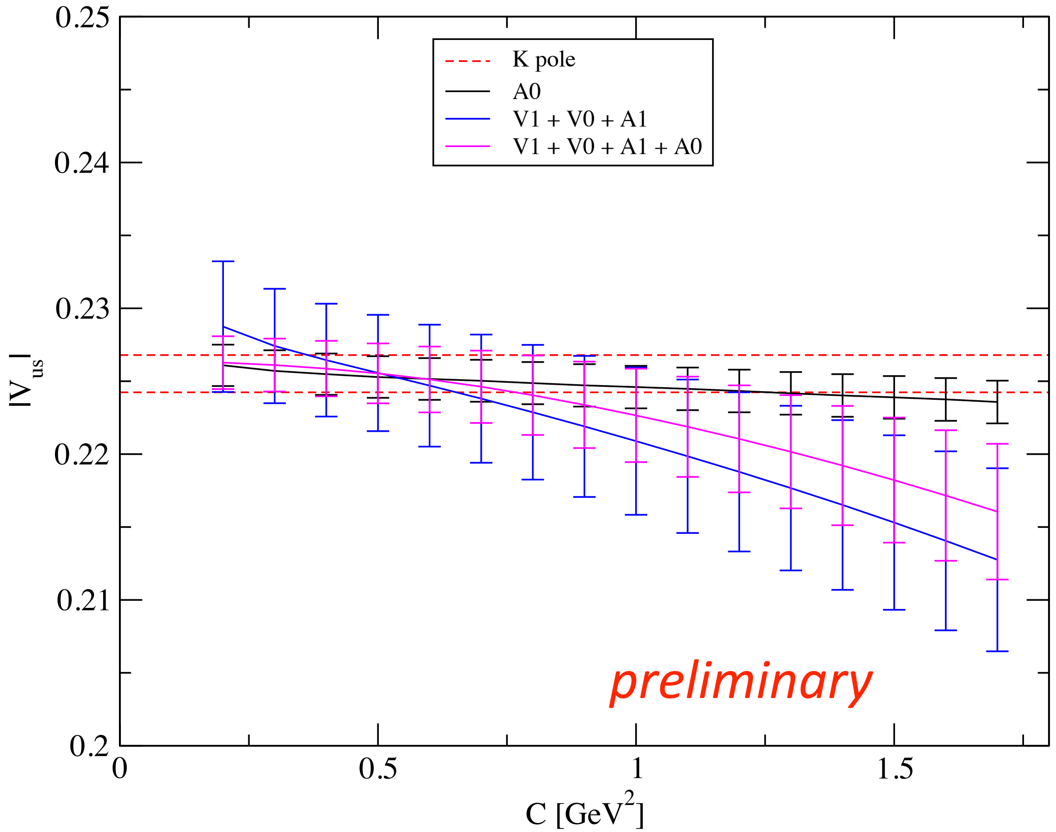
(Excited mode contributions and lattice discretization error are small

in this momentum region.)

|Vus| from other channels

- A0 channel is dominated by K pole.
- How about other channels?
- Lattice HVPs for A1, V1, V0   < - > multi hadron states & pQCD

|Vus| : weight function with N=3



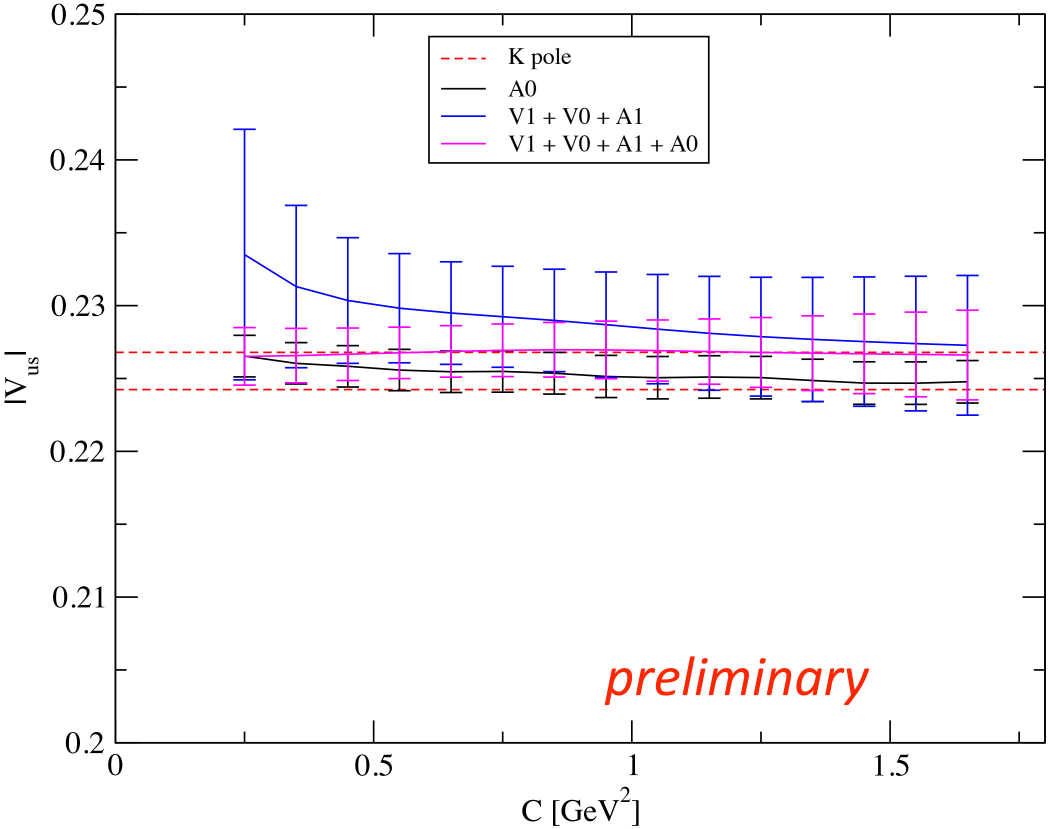
V1 + V0 + A1: Result in the continuum limit using L=48 and L=64 lattice data.

(We omit mK and mπ mass correction, which are multi hadron states and less sensitive to the quark mass compared to single K state.

For larger C > 1 region, |Vus| is different from K pole determination.

Is it due to large uncertainties from pQCD? **(Remember ratio analysis.)**

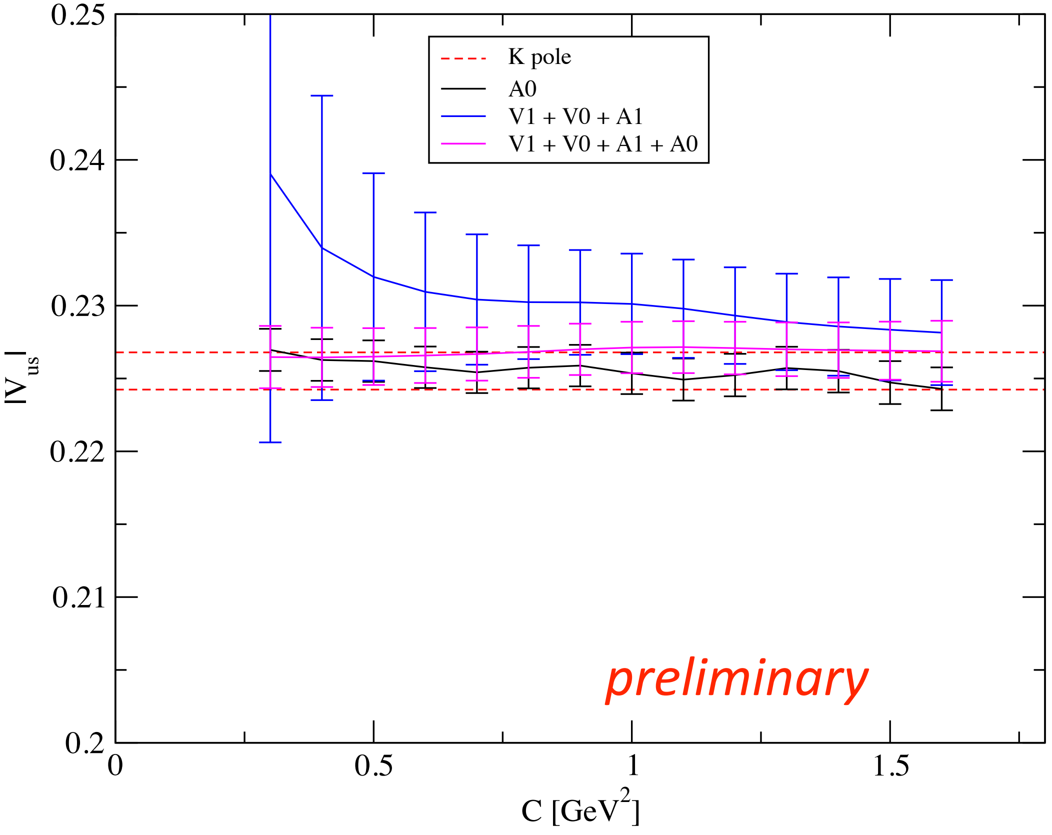
|Vus| : weight function with N=4



V1 + V0 + A1: consistent with K pole determination with larger error.

Full result (V1 + V0 + A1+A0) is stable against the change of C.

|Vus| : weight function with N=5



The error becomes larger due to noisy signal of vector channels (multi hadron states).

Full result is competitive with the result of K pole determination.

Summary

Summary

Precise determination of CKM matrix elements is very important.

We have demonstrated how the inclusive τ decay experiments and the lattice observables can be related, from which we can determine the CKM matrix element $|Vus|$.

Thanks to the physical point lattice, we can obtain better signal from A0 channel, whose grand state is K which is most sensitive to the quark mass among four channels.
From A0 analysis, we obtain an universal value of $|Vus|$.
This result suggests that excited states contributions and discretization error are negligible for A0 channel.

We also found discrepancy between K pole determination and other channels in the case of N=3, where OPE become to dominantly contribute to total decay rate.
N=4, 5 the results are consistent with K pole determination, but larger statistical error.

Several systematic uncertainties need to be investigated, e.g. quark mass effect near physical point, sea quark mass effect, perturbative OPE.

Thank you

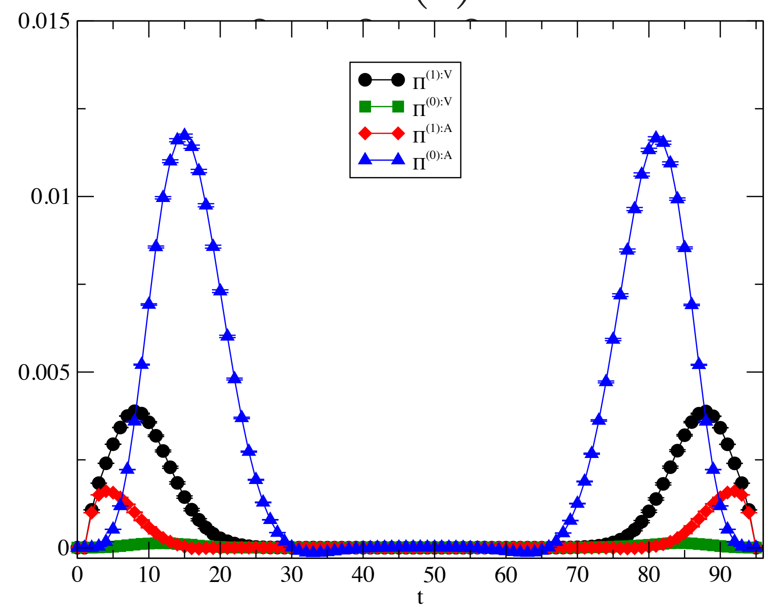
Backup

Fourier decomposition of residue

$$\tilde{F}^{(1):V/A}(t) = \sum_{i=1}^{N} \left(\frac{e^{i\tilde{Q}_i^2 t} - 1}{Q_i^2} + \frac{t^2}{2} \right) \left(1 - 2\frac{Q_i^2}{m_\tau^2} \right) \mathrm{Res}\left(\omega(Q_i^2)\Pi^{(1):V/A}(Q_i^2) \right),$$

$$\tilde{F}^{(0):V/A}(t) = \sum_{i=1}^{N} \left(\frac{e^{i\tilde{Q}_i^2 t} - 1}{Q_i^2} + \frac{t^2}{2} \right) \mathrm{Res}\left(\omega(Q_i^2)\Pi^{(0):V/A}(Q_i^2) \right).$$

$\tilde{F}^{(1/0):V/A}(t)$ for 48



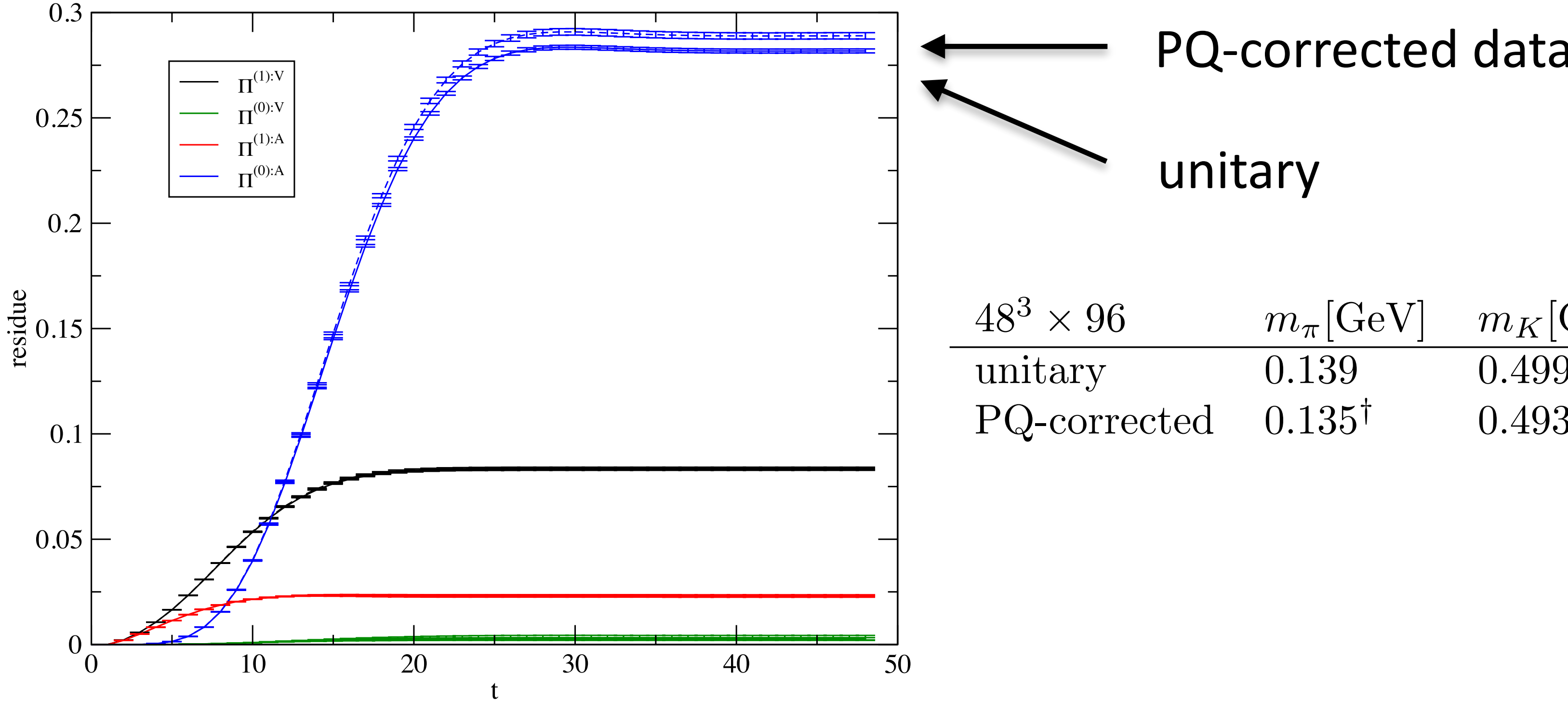
$N = 3$, and $(Q_1^2, Q_2^2, Q_3^2) = (0.1, 0.2, 0.3)$.

Total residue (t-> T)

$$\tilde{G}^{(1/0):V/A}(t) = \sum_{l=-t}^{t} \tilde{F}^{(1/0):V/A}(l).$$

Comparison of unitary and PQ-corrected data on L=48

Effective residue

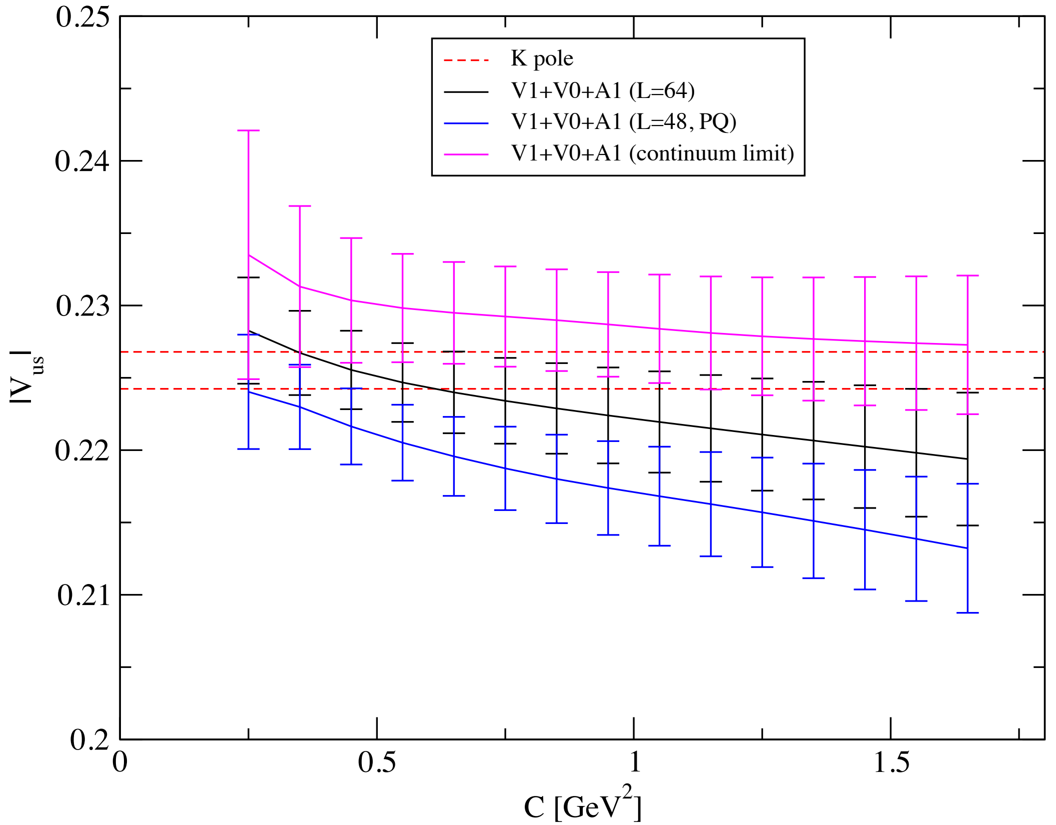


$N = 3$, and $(Q_1^2, Q_2^2, Q_3^2) = (0.1, 0.2, 0.3)$.

$48^3 \times 96$	m_π[GeV]	m_K[GeV]
unitary	0.139	0.499
PQ-corrected	$0.135^\dagger$	$0.4937^\dagger$

Only A0 has visible difference (Kaon),
other channels are consistent with each other (quark mass effect is negligible for multi hadron states).

Continuum limit of V1+V0+A1



vol.	$a^{-1}[GeV]$	m_π[GeV]	m_K[GeV]
$48^3 \times 96$	1.730(4)	0.135	0.4937
$64^3 \times 128$	2.359(7)	0.139	0.508

Continuum extrapolation by a^2 linear fit using L=48 (PQ) and L=64.