Unveiling Nucleon 3D Chiral-Odd Structure with Jet Axes

Wai Kin Lai

University of California, Los Angeles
South China Normal University

WKL, Xiaohui Liu, Manman Wang, Hongxi Xing, arXiv:2205.04570 [hep-ph]

July 19, 2022

California EIC Consortium Meeting
UC Davis

Motivation

- 3D structures of proton were studied typically using
  - semi-inclusive DIS/Drell-Yan
    [Mulders, Tangerman (1996), Brodsky, Hwang, Schmidt (2002), Bacchetta *et al.*(2007), Bacchetta *et al.*, MAP Collaboration (2022)]
  - jet production/hadron in jet
    [Kang, Metz, Qiu, Zhou (2011), Liu, Ringer, Vodelsang, Yuan (2019), Kang, Lee, Shao, Zhao (2021)]

Motivation

- 3D structures of proton were studied typically using
  - semi-inclusive DIS/Drell-Yan
    [Mulders, Tangerman (1996), Brodsky, Hwang, Schmidt (2002), Bacchetta *et al.*(2007), Bacchetta *et al.*, MAP Collaboration (2022)]
  - jet production/hadron in jet
    [Kang, Metz, Qiu, Zhou (2011), Liu, Ringer, Vodelsang, Yuan (2019), Kang, Lee, Shao, Zhao (2021)]
- Jet was thought to be able to probe only a subset of TMD PDFs (4 out of 8 at leading twist).

Motivation

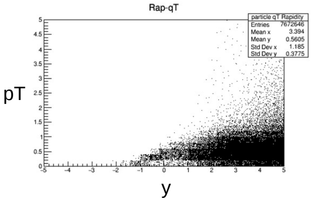
- 3D structures of proton were studied typically using
  - semi-inclusive DIS/Drell-Yan
    [Mulders, Tangerman (1996), Brodsky, Hwang, Schmidt (2002), Bacchetta *et al.*(2007), Bacchetta *et al.*, MAP Collaboration (2022)]
  - jet production/hadron in jet
    [Kang, Metz, Qiu, Zhou (2011), Liu, Ringer, Vodelsang, Yuan (2019), Kang, Lee, Shao, Zhao (2021)]
- Jet was thought to be able to probe only a subset of TMD PDFs (4 out of 8 at leading twist).
- We will demonstrate how all TMD PDFs at leading twist can be accessed by jets by including the T-odd jet function.
  [**WKL**, Liu, Xing, Wang, arXiv:2205.04570 [hep-ph]]

  T-odd jet function was introduced by [X. Liu and H. Xing (2021)]

Role of jet algorithm

Consider $l + p(P,S) \to l' + J(P_J) + X$ at EIC

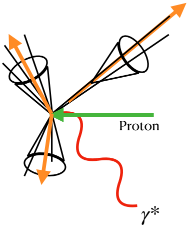
- A lot of statistics at small p_T in the forward region.

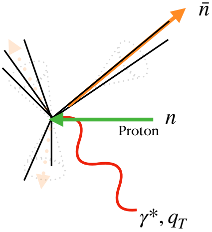


- Focus on the region $\Lambda_{\rm QCD} \sim p_T \ll Q$.
  This is unlike LHC, for which only jets with $p_T \gg \Lambda_{\rm QCD}$ are of interest.
- Still get jets if we use jet algorithms that involve energy (e.g. spherically-invariant jet algorithm [Cacciari, Salam, Soyez (2012)]) instead of k_T.
  Low p_T ($\sim \Lambda_{\rm QCD}$) and low Q^2 ($\sim 5 - 100\ \mathrm{GeV}^2$) is not a problem.

Role of jet axis definition

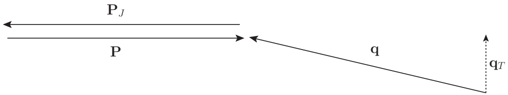
- Probes of TMD PDFs amount to measuring q_T of the virtual photon w.r.t. two pre-defined axes:
  - Drell-Yan: Two nucleon beams define two axes.
  - SIDIS: Nucleon beam and momentum of tagged hadron define two axes.
- In DIS, with a specific recombination scheme, a jet axis can be defined for a given jet. Once the axis is defined, we can forget about the fact that it's a jet. We thus get a nucleon beam axis and a jet axis, w.r.t. which q_T of the virtual photon can be defined. Therefore, jet probes of nucleon structure in DIS are as differential as SIDIS or Drell-Yan.





Factorization

Factorization frame:



For $Q \gg |\boldsymbol{q}_T|$ and $Q \gg \Lambda_{\mathrm{QCD}}$, factorization from SCET:

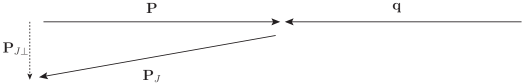
$$\sigma = H \otimes \Phi \otimes \mathcal{J}$$

H: hard function, Φ: TMD PDFs, $\mathcal{J}$: TMD jet functions (JFs)
[Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi (2018)]

$$\Phi^{ij}(x, p_T) = \int \frac{dy^- d^2\boldsymbol{y}_T}{(2\pi)^3} e^{ip\cdot y} \langle P|\bar{\chi}_n^j(0)\chi_n^i(y)|P\rangle|_{y^+=0}$$

$$\mathcal{J}^{ij}(z, k_T) = \frac{1}{2z}\sum_X \int \frac{dy^+ d^2\boldsymbol{y}_T}{(2\pi)^3} e^{ik\cdot y} \langle 0|\chi_{\bar{n}}^i(y)|JX\rangle\langle JX|\bar{\chi}_{\bar{n}}^j(0)|0\rangle|_{y^-=0}$$

GNS frame: $(\boldsymbol{P}_{J\perp} = -z\boldsymbol{q}_T)$



Azimuthal asymmetries

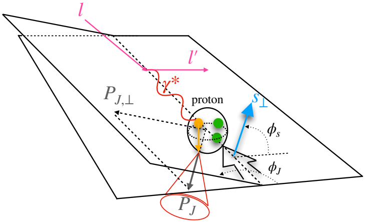
TMD PDFs and TMD JFs encoded in azimuthal asymmetries (leading power in $1/Q$):

$$\frac{d\sigma}{dx\,dy\,dz\,d\psi\,d\phi_J\,dP_{J\perp}^2} = \frac{\alpha^2}{xyQ^2}\Bigg\{\left(1-y+\frac{y^2}{2}\right)F_{UU,T} + (1-y)\cos(2\phi_J)F_{UU}^{\cos(2\phi_J)}$$
$$+ S_\parallel(1-y)\sin(2\phi_J)F_{UL}^{\sin(2\phi_J)} + S_\parallel\lambda_e y\left(1-\frac{y}{2}\right)F_{LL}$$
$$+|\boldsymbol{S}_\perp|\Bigg[\left(1-y+\frac{y^2}{2}\right)\sin(\phi_J-\phi_S)F_{UT,T}^{\sin(\phi_J-\phi_S)} + (1-y)\sin(\phi_J+\phi_S)F_{UT}^{\sin(\phi_J+\phi_S)}$$
$$+(1-y)\sin(3\phi_J-\phi_S)F_{UT}^{\sin(3\phi_J-\phi_S)}\Bigg] + |\boldsymbol{S}_\perp|\lambda_e y\left(1-\frac{y}{2}\right)\cos(\phi_J-\phi_S)F_{LT}^{\cos(\phi_J-\phi_S)}\Bigg\}$$

F's: convolutions of TMD PDFs and TMD JFs.

F's: accessible by traditional jet function

F's: inaccessible by traditional jet function



TMD PDFs at leading twist

$$\Phi = \frac{1}{2}\left\{ f_1 \not{n} - f_{1T}^{\perp} \frac{\epsilon_{\alpha\beta} p_T^{\alpha} S_T^{\beta}}{M} \not{n} + \left(S_L g_{1L} - \frac{p_T \cdot S_T}{M} g_{1T} \right) \gamma_5 \not{n} \right.$$
$$\left. + h_{1T} \frac{[\not{S}_T, \not{n}]\gamma_5}{2} + \left(S_L h_{1L}^{\perp} - \frac{p_T \cdot S_T}{M} h_{1T}^{\perp} \right) \frac{[\not{p}_T, \not{n}]\gamma_5}{2M} + i h_1^{\perp} \frac{[\not{p}_T, \not{n}]}{2M} \right\}$$

quark / hadron	unpolarized	longitudinal	transverse
U	f_1		$h_1^{\perp}$ (Boer-Mulders)
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$ (Sivers)	g_{1T}	$h_{1T}, h_{1T}^{\perp}$ (transversity)

[Angeles-Martinez *et al.*(2015)]

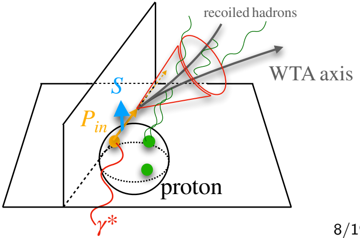
- 8 TMD PDFs at leading twist, functions of x and p_T^2
- 3 functions f_1, g_{1L}, h_{1T} survive after p_T integration giving collinear PDFs
- T-even: $f_1, g_{1L}, g_{1T}, h_{1T}, h_{1L}^{\perp}, h_{1T}^{\perp}$
  T-odd: $f_{1T}^{\perp}, h_1^{\perp}$
- Chiral-even (accessible by traditional jet function): $f_1, f_{1T}^{\perp}, g_{1L}, g_{1T}$
  Chiral-odd (inaccessible by traditional jet function): $h_1^{\perp}, h_{1L}^{\perp}, h_{1T}, h_{1T}^{\perp}$

T-odd jet function

- Traditionally, only jets with high p_T ($\gg \Lambda_{\rm QCD}$) were of interest. Production of high-p_T jets is perturbative. Since massless perturbative QCD is chiral-symmetric, only chiral-even (and T-even) jet functions appear.
- At low p_T ($\sim \Lambda_{QCD}$), the jet is sensitive to nonperturbative physics. In particular, spontaneous chiral symmetry breaking leads to a nonzero chiral-odd (and T-odd) jet function when the jet axis is different from the direction of the fragmenting parton. (This is similar to Collins effect in fragmentation functions of hadrons [Collins (2002)].) [Liu and Xing (2021)]

$$\mathcal{J}(z, k_T) = \mathcal{J}_1(z, k_T)\frac{\not{n}}{2} + i\mathcal{J}_T(z, k_T)\frac{\not{k}_T\not{n}}{2}$$

- $\mathcal{J}_1$: chiral-even, T-even, traditional jet function
- $\mathcal{J}_T$: chiral-odd, T-odd, encodes correlations of quark transverse spin with quark transverse momentum (analogue of Collins function)



Advantages of T-odd jet function

- **Universality**
  Like the T-even $\mathcal{J}_1$, T-odd $\mathcal{J}_T$ is process independent.
- **Flexibility**
  Flexibility of choosing jet recombination scheme and hence the jet axis
  $\Rightarrow$ Adjust sensitivity to different nonperturbative contributions
  $\Rightarrow$ Provide opportunity to "film" the QCD nonperturbative dynamics, if one continuously change the axis from one to another.
- **High predictive power**
  - *Perturbative predictability*. Since a jet contains many hadrons, the jet function has more perturbatively calculable degrees of freedom than the fragmentation function. For instance, in the WTA scheme, for $R \sim \mathcal{O}(1) \gg |\boldsymbol{q}_T|/E_J$, the z-dependence in the jet function is completely determined:

    $$\mathcal{J}(z, k_T, R) = \delta(1-z)J(k_T) + \mathcal{O}\left(\frac{k_T^2}{E_J^2 R^2}\right)$$

    [Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi (2018)]
  - *Nonperturbative predictability*. Similar to the study in [Becher and Bell (2014)], $\mathcal{J}_T$ can be factorized into a product of a perturbative coefficient and a nonperturbative factor. The nonperturbative factor has an operator definition [Vladimirov (2020)], and as a vacuum matrix element can be calculated on the lattice. This is unlike the TMD fragmentation function, which is an operator element of $|h+X\rangle$.

Probing transverity

$$A^{\sin(\phi_J+\phi_S)}(|\boldsymbol{P}_{J\perp}|) = \frac{2}{|\boldsymbol{S}_\perp| \int d\sigma\epsilon} \int d\sigma \sin(\phi_J+\phi_S) = \frac{\langle \epsilon F_{UT}^{\sin(\phi_J+\phi_S)} \rangle}{\bar{\epsilon}\langle F_{UU,T} \rangle}$$

- $F_{UT}^{\sin(\phi_J+\phi_S)} \sim h_1 \otimes J_T$, probes transversity $h_1 \equiv h_{1T} + \frac{\boldsymbol{p}_T^2}{2M^2} h_{1T}^\perp$
- We simulate using PYTHIA 8.2+STRINGSPINNER [Kerbizi, Loennblad (2021)], with jet charge [Kang, Liu, Mantry, Shao (2020)] measured to enhance flavor separation (not mandatory), with EIC kinematics.
- Use the spherically-invariant jet algorithm [Cacciari, Salam, Soyez (2012)]:

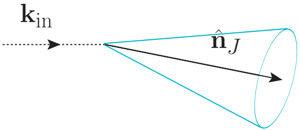
$$d_{ij} = min(E_i^{-2}, E_j^{-2}) \frac{1-\cos\theta_{ij}}{1-\cos R}\ , \quad d_{iB} = E_i^{-2}$$

(Conventional anti-k_T algorithms using k_T instead of E not good for low-p_T jets)

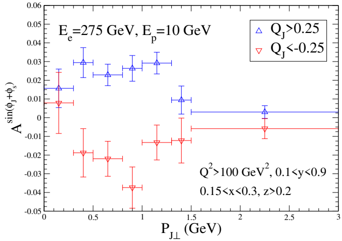
- **Change the jet axis from one to another (WTA → E-scheme), "film" nonperturbative physics.**

WTA scheme:

$$\hat{\boldsymbol{n}}_r = \begin{cases} \hat{\boldsymbol{n}}_1\ , & \text{if } E_1 > E_2 \\ \hat{\boldsymbol{n}}_2\ , & \text{if } E_2 > E_1 \end{cases}$$



e+p→e+J$_{WTA}$+X (EIC)



Probing transversity

$$A^{\sin(\phi_J+\phi_S)}(|\boldsymbol{P}_{J\perp}|) = \frac{2}{|\boldsymbol{S}_\perp| \int d\sigma \epsilon} \int d\sigma \sin(\phi_J+\phi_S) = \frac{\langle \epsilon F_{UT}^{\sin(\phi_J+\phi_S)} \rangle}{\bar{\epsilon}\langle F_{UU,T} \rangle}$$

- $F_{UT}^{\sin(\phi_J+\phi_S)} \sim h_1 \otimes J_T$, probes transversity $h_1 \equiv h_{1T} + \frac{\boldsymbol{p}_T^2}{2M^2} h_{1T}^\perp$
- We simulate using Pythia 8.2+StringSpinner [Kerbizi, Loennblad (2021)], with jet charge [Kang, Liu, Mantry, Shao (2020)] measured to enhance flavor separation (not mandatory), with EIC kinematics.
- Use the spherically-invariant jet algorithm [Cacciari, Salam, Soyez (2012)]:

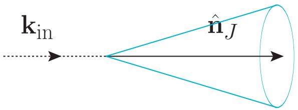
$$d_{ij} = min(E_i^{-2}, E_j^{-2}) \frac{1-\cos\theta_{ij}}{1-\cos R}, \quad d_{iB} = E_i^{-2}$$

(Conventional anti-k_T algorithms using k_T instead of E not good for low-p_T jets)

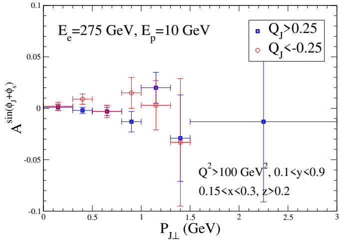
- **Change the jet axis from one to another (WTA $\rightarrow$ E-scheme), "film" nonperturbative physics.**

E-scheme:

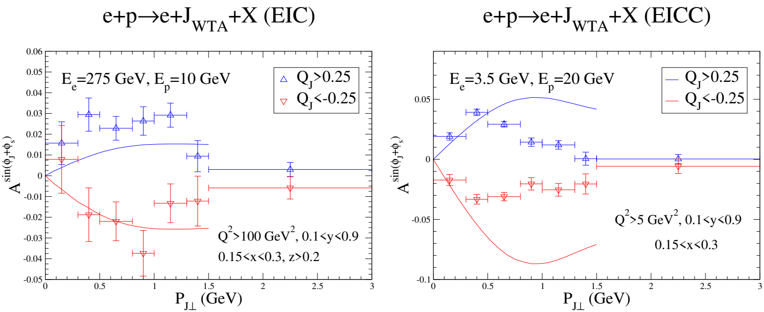
$k_r = k_1 + k_2$



e+p→e+J$_{\text{E-scheme}}$+X (EIC)



Probing transverity



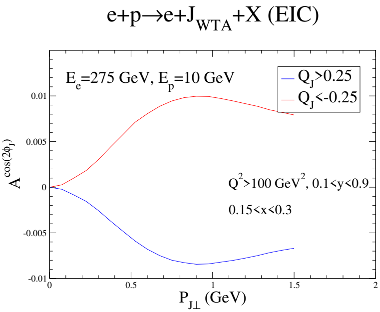
- Data points: from Pythia simulations
  Lines: from factorization formula (including evolution via Sudakov factors, normalization of J_1 fixed by jet charge bins, k_T-dependence of J_1 and J_T from pion FFs, ratio of normalization of J_T to that of J_1 set equal to that for pions)
- Asymmetries at lower $\sqrt{s}$ are generally larger, owing to the perturbative Sudakov factor.

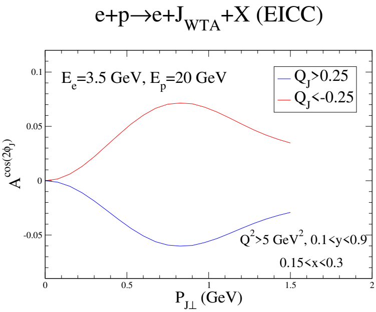
Probing Boer-Mulders function

$$A^{\cos(2\phi_J)}(|\boldsymbol{P}_{J\perp}|) = \frac{2}{\int d\sigma\epsilon}\int d\sigma \cos(2\phi_J) = \frac{\langle \epsilon F_{UU}^{\cos(2\phi_J)}\rangle}{\bar{\epsilon}\langle F_{UU,T}\rangle}$$

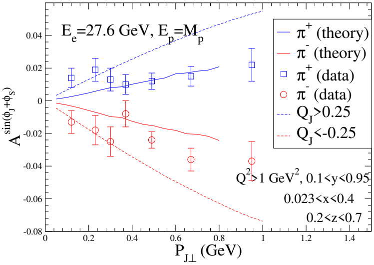
$F_{UU}^{\cos(2\phi_J)} \sim h_1^\perp \otimes J_T$, probes Boer-Mulders function

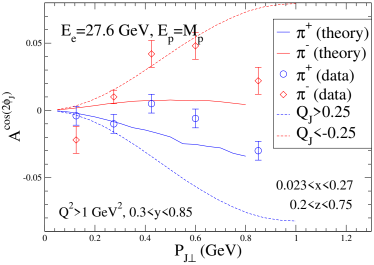
Predictions from factorization formula:





Comparison with Hermes SIDIS data



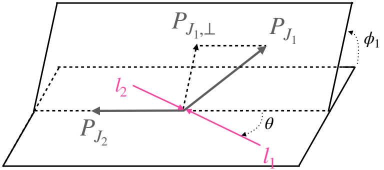


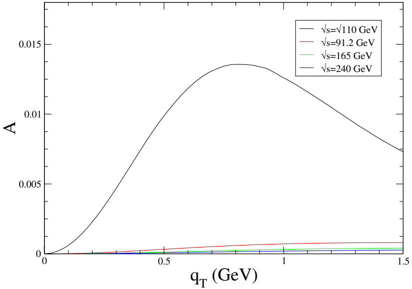
We give prediction on azimuthal asymmetry in $e^+e^- \to J + X$ at $\sqrt{s} = \sqrt{110}$ GeV (Belle, BaBar), $91.2, 165, 240$ GeV (CEPC) with WTA scheme:

$$A = 2\int d\cos\theta \,\frac{d\phi_1}{\pi}\cos(2\phi_1) A^{J_1 J_2}$$

$$A^{J_1 J_2} = 1 + \cos(2\phi_1)\frac{\sin^2\theta}{1+\cos^2\theta}\frac{F_T}{F_U}$$

$$F_T \sim J_T \otimes J_T$$





Summary and outlook

- With spherically-invariant jet algorithms, we can study jets at low p_T ($\sim \Lambda_{\mathrm{QCD}}$), e.g. at EIC.
- Specification of a jet axis makes jet probes of TMD PDFs in DIS as differential as SIDIS or Drell-Yan.
- Using the T-odd jet function, together with the traditional T-even one, we can probe all 8 TMD PDFs at leading twist using jets.
- T-odd jet function has the advantages of universality, flexibility, and high predictive power.
- We have shown that the T-odd jet function gives rise to sizable azimuthal asymmetries at EIC, which help probe the chiral-odd TMD PDFs, such as the quark transversity and the Boer-Mulders function, which the traditional jet function is unable to access.
- T-odd jet function provides new input to the global analysis of nonperturbative proton structure.

Thank you.

Backup slides

Azimuthal asymmetry at leading twist

TMD PDFs and TMD JFs encoded in azimuthal asymmetries:

$$\frac{d\sigma}{dx\,dy\,dz\,d\psi\,d\phi_J\,dP_{J\perp}^2} = \frac{\alpha^2}{xyQ^2}\Bigg\{\left(1-y+\frac{y^2}{2}\right)F_{UU,T} + (1-y)\cos(2\phi_J)F_{UU}^{\cos(2\phi_J)}$$
$$+ S_\parallel (1-y)\sin(2\phi_J)F_{UL}^{\sin(2\phi_J)} + S_\parallel \lambda_e y\left(1-\frac{y}{2}\right)F_{LL}$$
$$+ |\boldsymbol{S}_\perp|\Bigg[\left(1-y+\frac{y^2}{2}\right)\sin(\phi_J-\phi_S)F_{UT,T}^{\sin(\phi_J-\phi_S)} + (1-y)\sin(\phi_J+\phi_S)F_{UT}^{\sin(\phi_J+\phi_S)}$$
$$+(1-y)\sin(3\phi_J-\phi_S)F_{UT}^{\sin(3\phi_J-\phi_S)}\Bigg] + |\boldsymbol{S}_\perp|\lambda_e y\left(1-\frac{y}{2}\right)\cos(\phi_J-\phi_S)F_{LT}^{\cos(\phi_J-\phi_S)}$$

The F's are convolutions of TMD PDFs and TMD JFs:

$$\mathcal{C}[wfJ] \equiv x \sum_a e_q^2 \int d^2\boldsymbol{p}_T \int d^2\boldsymbol{k}_T \delta^{(2)}\left(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T) f(x, p_T^2) J(z, k_T^2)$$

$$F_{UU,T} = \mathcal{C}[f_1 \mathcal{J}_1], \quad F_{LL} = \mathcal{C}[g_{1L} \mathcal{J}_1]$$

$$F_{UT,T}^{\sin(\phi_J-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T}{M} f_{1T}^{\perp} \mathcal{J}_1\right], \quad F_{LT}^{\cos(\phi_J-\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T}{M} g_{1T} \mathcal{J}_1\right],$$

$$F_{UU}^{\cos(2\phi_J)} = \mathcal{C}\left[-\frac{(2(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T)(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T) - \boldsymbol{k}_T \cdot \boldsymbol{p}_T)}{M} h_1^{\perp} \mathcal{J}_T\right]$$

$$F_{UL}^{\sin(2\phi_J)} = \mathcal{C}\left[-\frac{(2(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T)(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T) - \boldsymbol{k}_T \cdot \boldsymbol{p}_T)}{M} h_{1L}^{\perp} \mathcal{J}_T\right]$$

$$F_{UT}^{\sin(\phi_J+\phi_S)} = \mathcal{C}\left[-\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T h_1 \mathcal{J}_T\right]$$

$$F_{UT}^{\sin(3\phi_J-\phi_S)} = \mathcal{C}\left[\frac{2(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T)(\boldsymbol{p}_T \cdot \boldsymbol{k}_T) + \boldsymbol{p}_T^2(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T) - 4(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T)^2(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T)}{2M^2} h_{1T}^{\perp} \mathcal{J}_T\right]$$

where $\hat{\boldsymbol{h}} \equiv \boldsymbol{P}_{J\perp}/|\boldsymbol{P}_{J\perp}|$ and $h_1 \equiv h_{1T} + \frac{\boldsymbol{p}_T^2}{2M^2} h_{1T}^{\perp}$