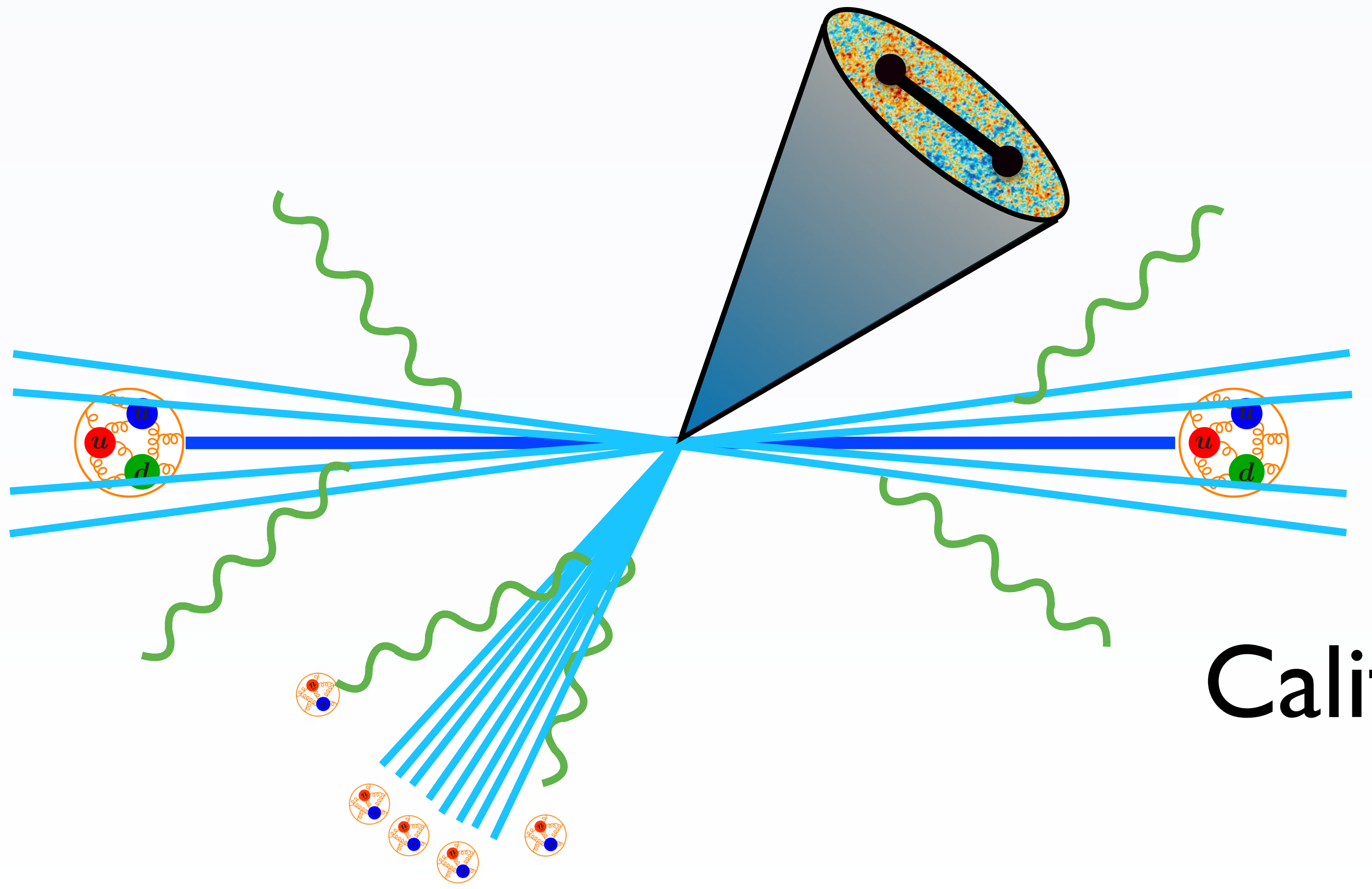
Conformal Colliders Meet the LHC (and the EIC)

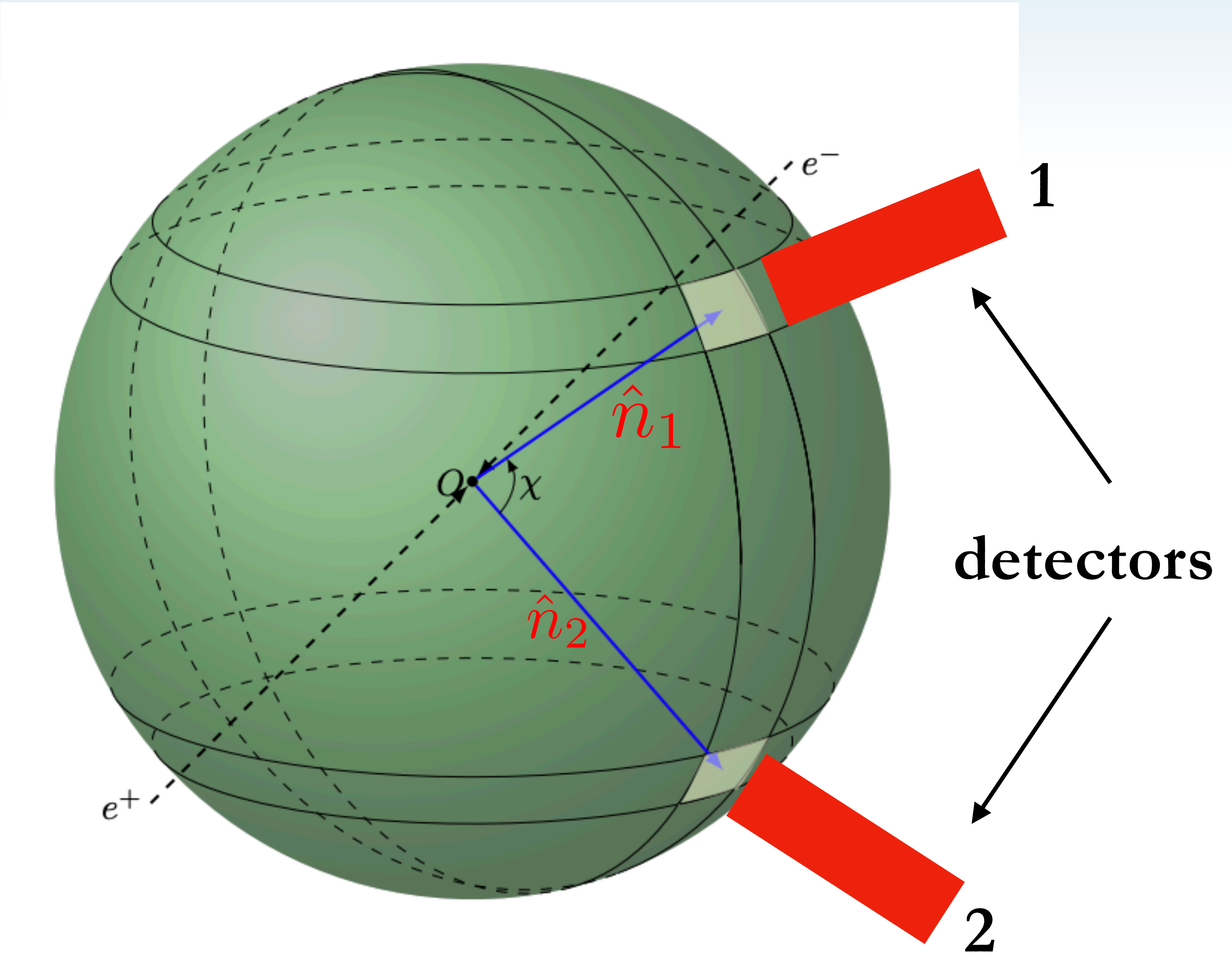


Kyle Lee
LBNL

California EIC Consortium Collaboration Meeting
July 18th - July 19th, 2022

BERKELEY LAB

EECs (Energy-Energy Correlators)



$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

↑inclusive

- IRC safe, weighted cross-section
- Observable in the detector space, a direct correspondence with "calorimeter cells" — an observable which is intuitively simple and obvious
- It has an operator definition

$$\mathcal{E}(\hat{n})|X\rangle = \sum_a E_a \delta^{(2)}\left(\Omega_{\vec{p}_a} - \Omega_{\hat{n}}\right)|X\rangle$$

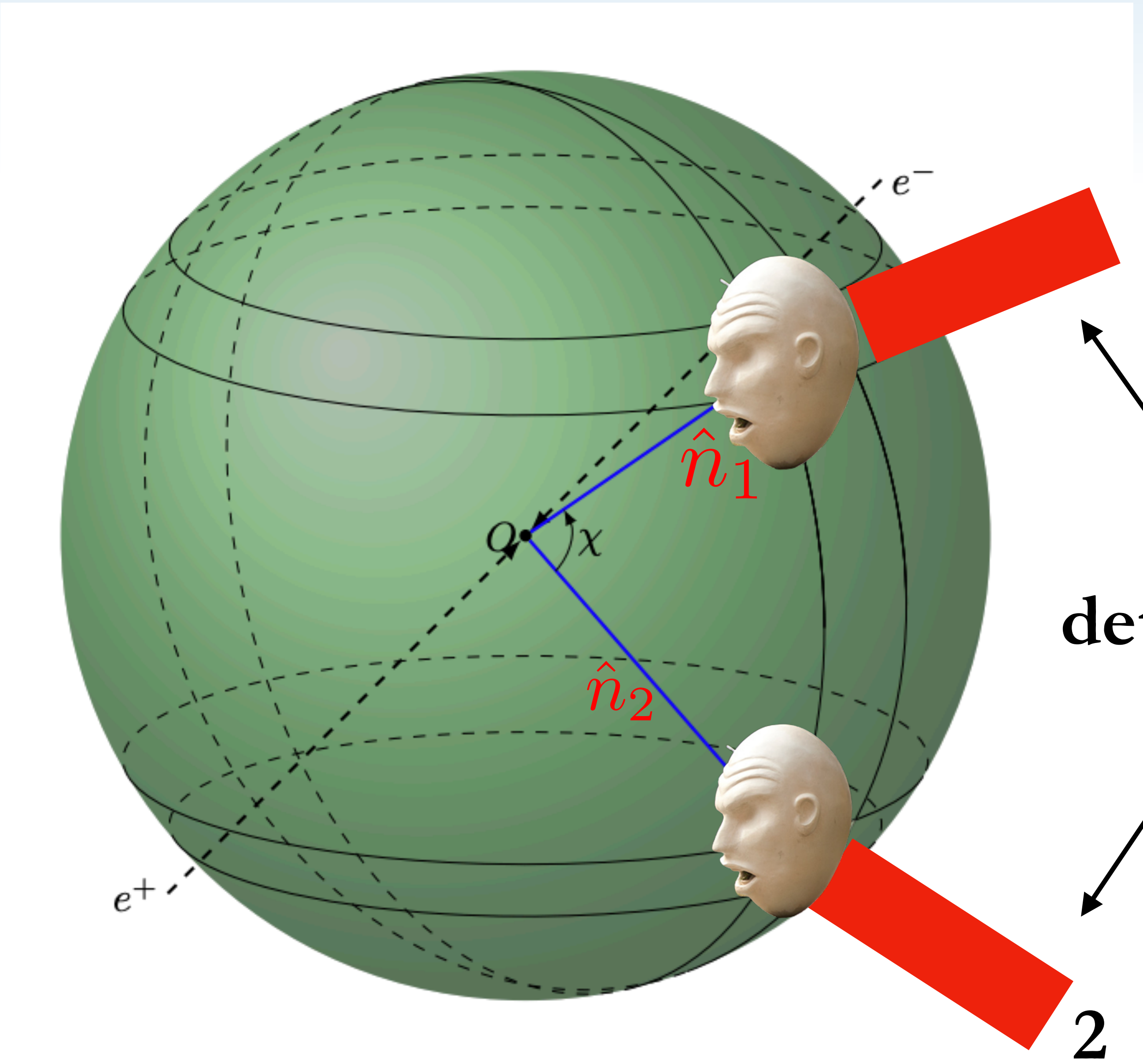
/ Energy Flow /
ANEC / Light-ray Operator /

$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \overbrace{\lim_{r\to\infty} r^2}^{\text{Detector at } \infty} \underbrace{n^i T_{0i}(t, r\hat{n})}_{\text{Energy flux in } \hat{n}}$$

$$\frac{d\sigma}{dz} \sim \langle \Psi | \mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) | \Psi \rangle$$

Basham, Brown, Ellis, Love, `78-79
Sveshnikov, Tkachov, `95
Hofman, Maldacena, `08

EECs (Egghead-Egghead Correlators)



$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

↑inclusive

- IRC safe, weighted cross-section
- Observable in the detector space, a direct correspondence with "calorimeter cells" — an observable which is intuitively simple and obvious
- It has an operator definition

$$\mathcal{E}(\hat{n})|X\rangle = \sum_a E_a \delta^{(2)}\left(\Omega_{\vec{p}_a} - \Omega_{\hat{n}}\right)|X\rangle$$

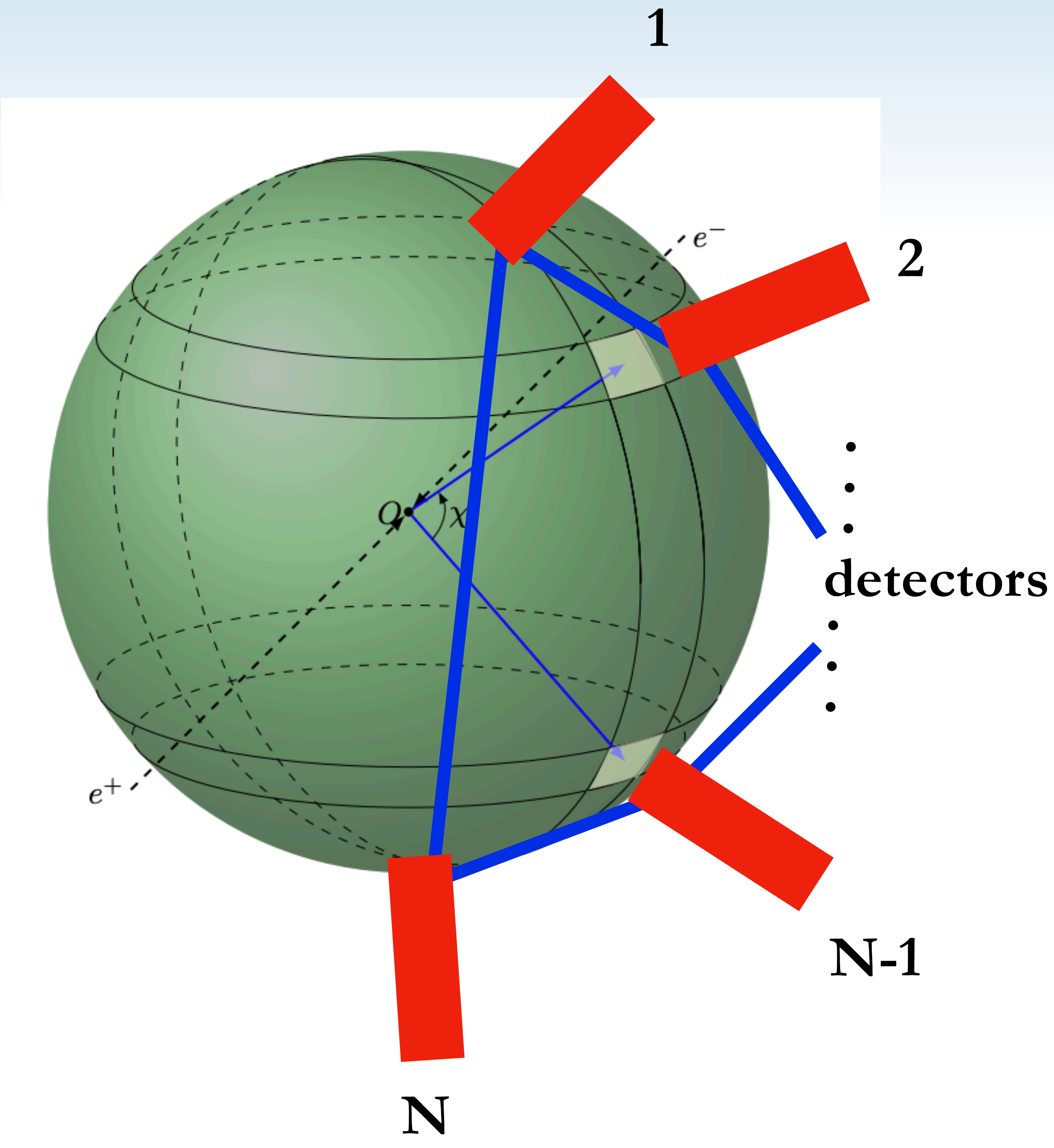
/ Energy Flow /
ANEC / Light-ray Operator /

$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \overbrace{\lim_{r\to\infty} r^2}^{\text{Detector at } \infty} \underbrace{n^i T_{0i}(t, r\hat{n})}_{\text{Energy flux in } \hat{n}}$$

$$\frac{d\sigma}{dz} \sim \langle\Psi|\mathcal{E}(\hat{n}_1)\,\mathcal{E}(\hat{n}_2)|\Psi\rangle$$

Basham, Brown, Ellis, Love, `78-79
Sveshnikov, Tkachov, `95
Hofman, Maldacena, `08

N-Point Energy Correlators



$$\frac{d\sigma}{d\{z\}} = \sum_{i_1, i_2, \ldots, i_N} \int d\sigma \frac{E_{i_1} E_{i_2} \cdots E_{i_N}}{Q^2} \prod_{j=1}^{N-1} \delta\left(z_{j,j+1} - \frac{1-\cos\chi_{i_j i_{j+1}}}{2} \right)$$

inclusive

- IRC safe, weighted cross-section
- Observable in the detector space, a direct correspondence with "calorimeter cells" — an observable which is intuitively simple and obvious
- It has an operator definition

$$\mathcal{E}(\hat{n})|X\rangle = \sum_a E_a \delta^{(2)}\left(\Omega_{\vec{p}_a} - \Omega_{\hat{n}}\right)|X\rangle$$

/ Energy Flow /
ANEC / Light-ray Operator /

$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \overbrace{\lim_{r\to\infty} r^2}^{\text{Detector at } \infty} \underbrace{n^i T_{0i}(t, r\hat{n})}_{\text{Energy flux in } \hat{n}}$$

$$\frac{d\sigma}{d\{z\}} \sim \langle \Psi | \mathcal{E}(\hat{n}_1) \cdots \mathcal{E}(\hat{n}_N) | \Psi \rangle$$

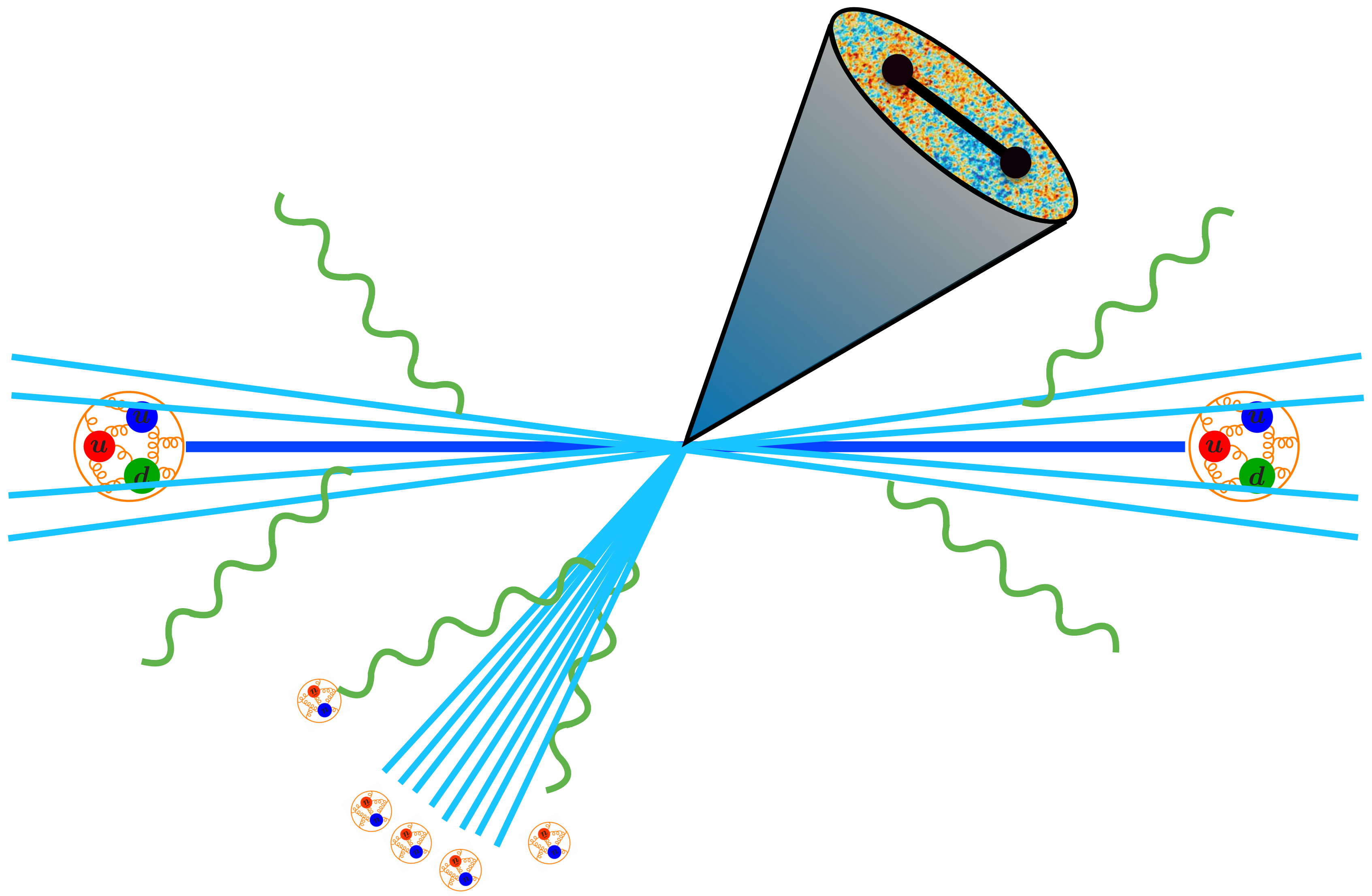
Basham, Brown, Ellis, Love, `78-79
Sveshnikov, Tkachov, `95
Korchemsky, Sterman, `01
Hofman, Maldacena, `08


Img: Quanta Magazine

Collinear Limit of Energy Correlators

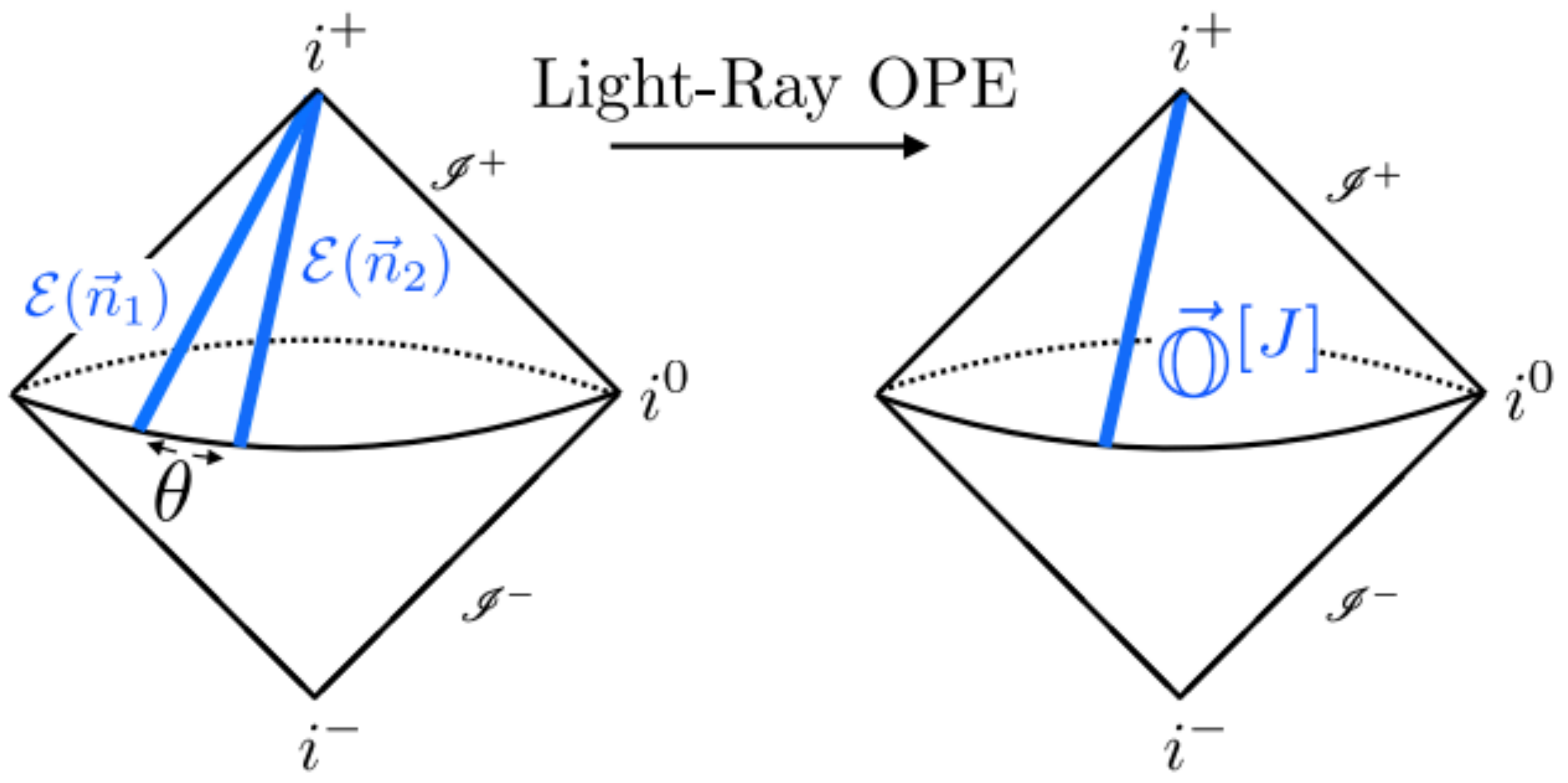
- In the collinear limit, $z_{ij} \to 1$ (i.e. $\theta_{ij}^2 \to 0$), give rise to

Phenomenological tools



- Jet substructure study

Theoretical tools



- Light-ray Operator Product Expansion (OPE)

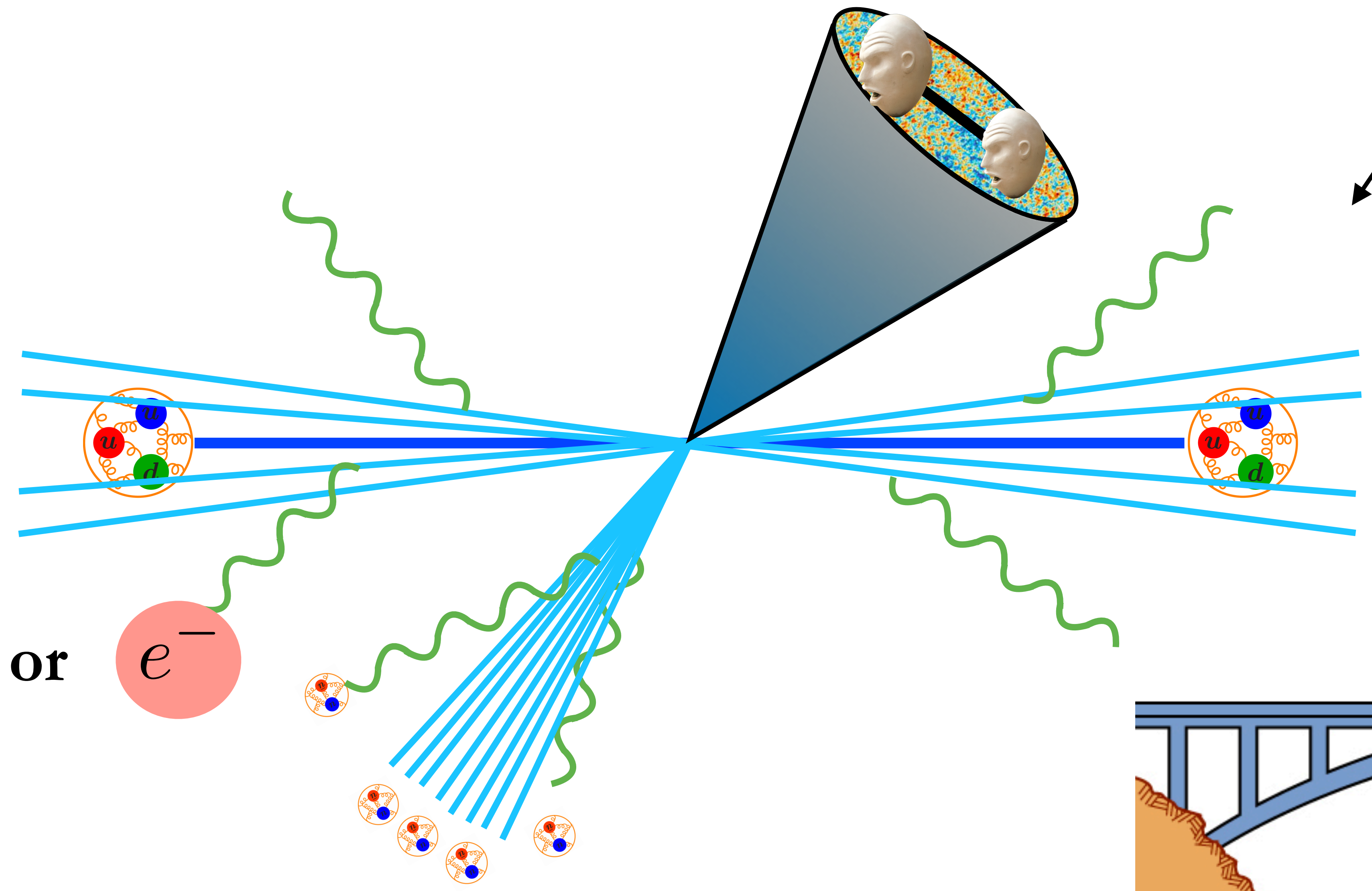
Collinear Limit of Energy Correlators

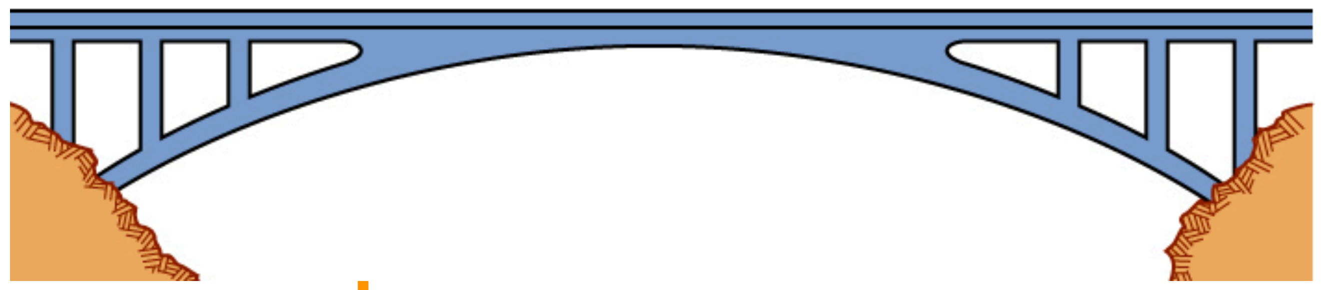
- In the collinear limit, $z_{ij} \to 1$ (i.e. $\theta_{ij}^2 \to 0$), give rise to

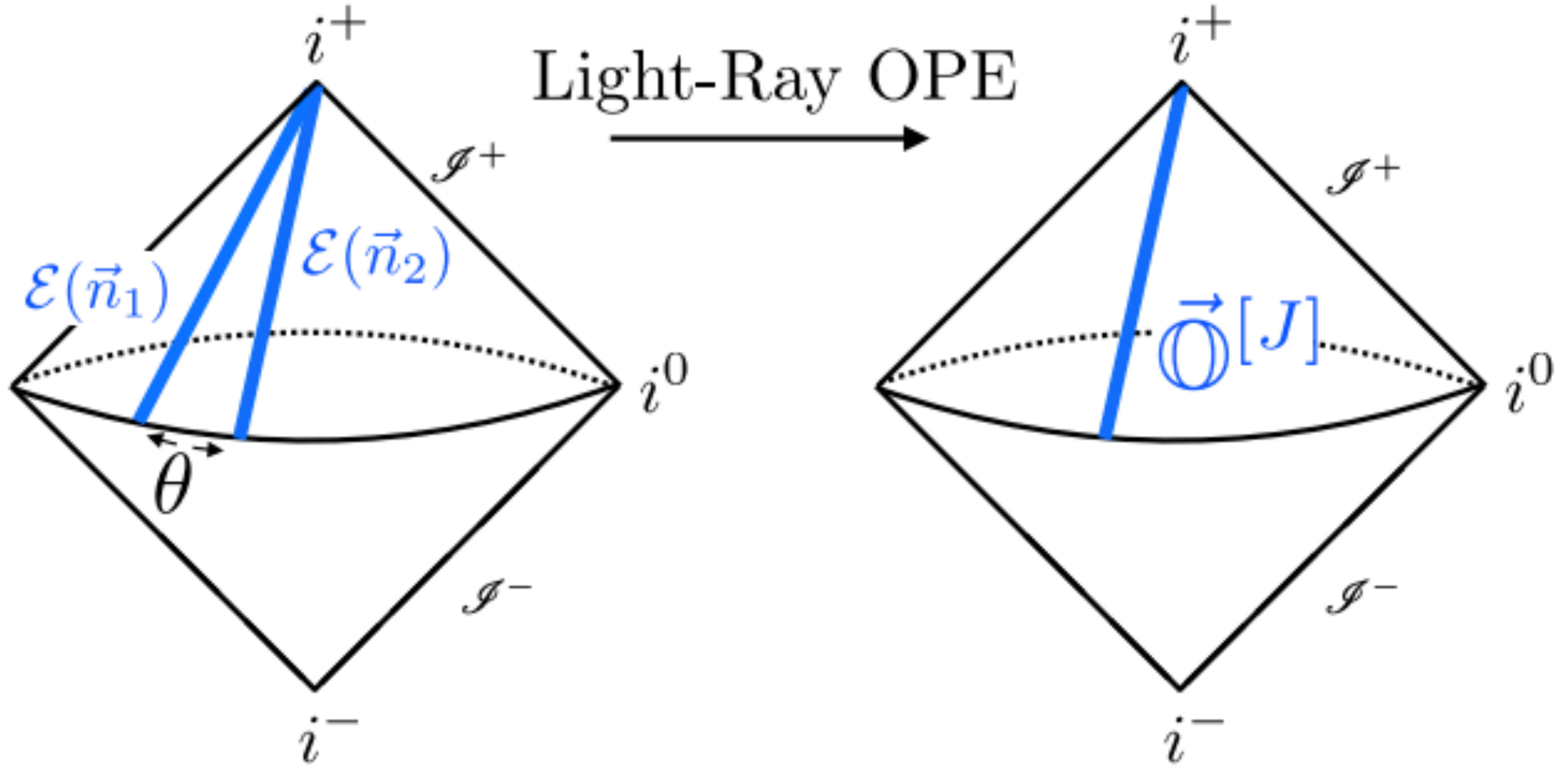
Phenomenological tools

Theoretical tools

Focus of this talk







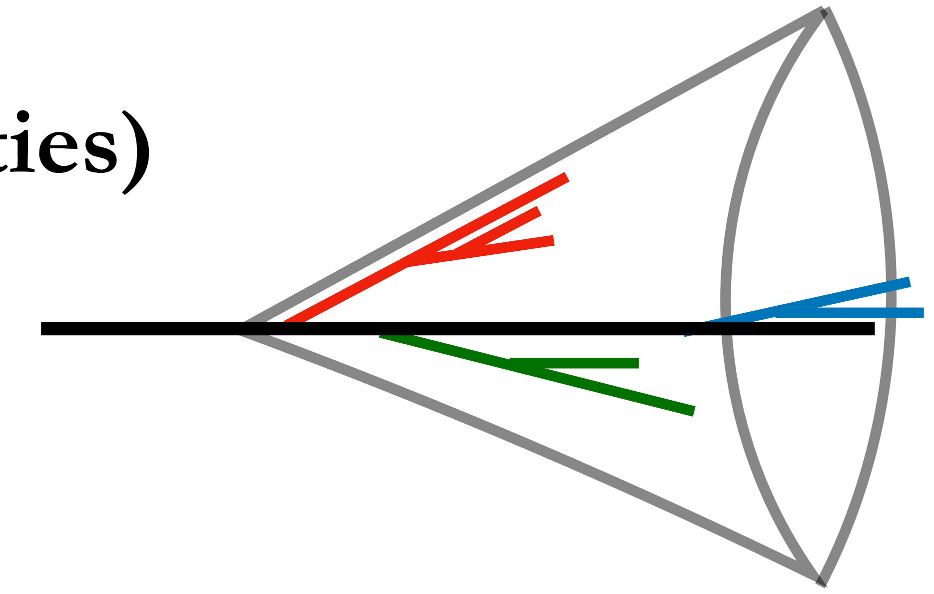
- Jet substructure study

- Light-ray Operator Product Expansion (OPE)

Energy correlators as jet substructure

Jet Shape

Example: Jet mass (angularities)



$$\delta(\tau_a - \hat{\tau}_a(X_J))$$

$$\tau_a \sim z\,\theta^{2-a}$$

Collinear

$$z_c \sim 1 \qquad \theta_c \sim \tau_a^{\frac{1}{2-a}}$$

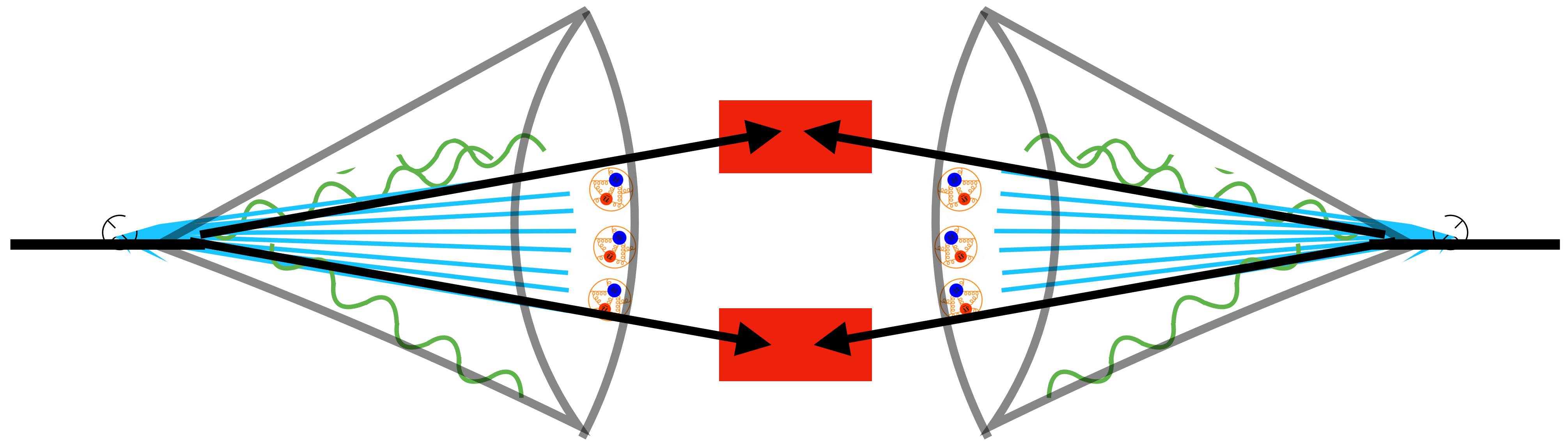
(Collinear-)soft

$$\theta_s \sim R \qquad z_{cs} \sim \frac{\tau_a}{R^{2-a}}$$

- The measured value of e constrains the radiation pattern inside the jet, X_J

Energy Correlators

$$\omega(X_J)$$



- Weighted cross-section, or, ensemble averaged observable.
- It does not constrain the phase space of an individual jet

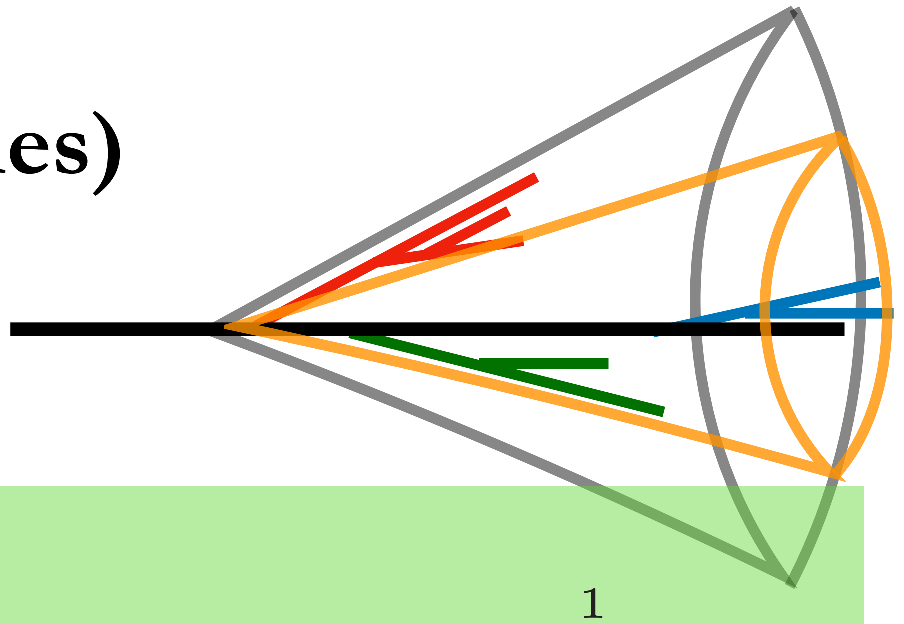
Lessons:

1. space of detectors **vs** space of the states
2. Probes fixed scale

Energy correlators as jet substructure

Jet Shape

Example: Jet mass (angularities)



$$\delta(\tau_a - \hat{\tau}_a(X_J))$$

$$\tau_a \sim z\,\theta^{2-a}$$

Collinear

$$z_c \sim 1 \qquad \theta_c \sim \tau_a^{\frac{1}{2-a}}$$

$\notin$ *gr* **soft**

$$\theta_{\notin \mathrm{gr}} \sim R \qquad z_{\notin \mathrm{gr}} \sim z_{\mathrm{cut}} \left(\frac{\theta}{R}\right)^{\beta} = z_{\mathrm{cut}}$$

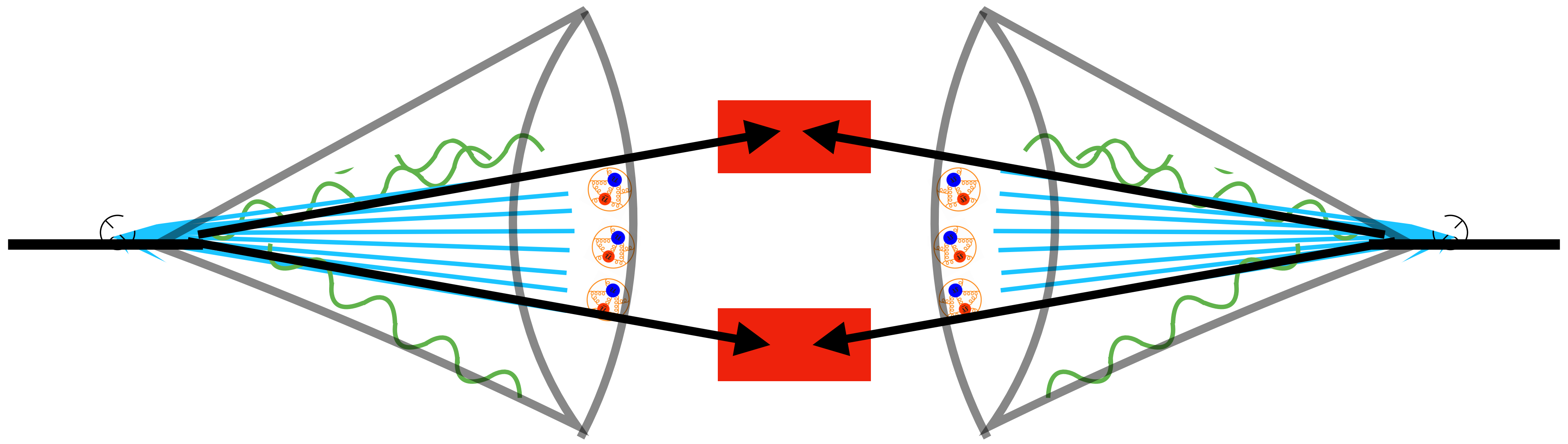
$\in$ *gr* **soft (collinear-soft)**

$$z_{\in \mathrm{gr}} \sim z_{\mathrm{cut}} \left(\frac{\theta}{R}\right)^{\beta} = z_{\mathrm{cut}}^{\frac{2-a}{2-a+\beta}} \left(\frac{\tau_a}{R^{2-a}}\right)^{\frac{\beta}{2-a+\beta}} \qquad \theta_{\in \mathrm{gr}} \sim \left(\frac{\tau_a R^{\beta}}{z_{\mathrm{cut}}}\right)^{\frac{1}{2-a+\beta}}$$

- The measured value of e constrains the radiation pattern inside the jet, X_J
- Still sensitive to soft radiation, complicated NP structure.

Energy Correlators

$$\omega(X_J)$$



- Weighted cross-section, or, ensemble averaged observable.
- It does not constrain the phase space of an individual jet

Lessons:

1. space of detectors **vs** space of the states
2. Probes fixed scale

Energy correlators as jet substructure

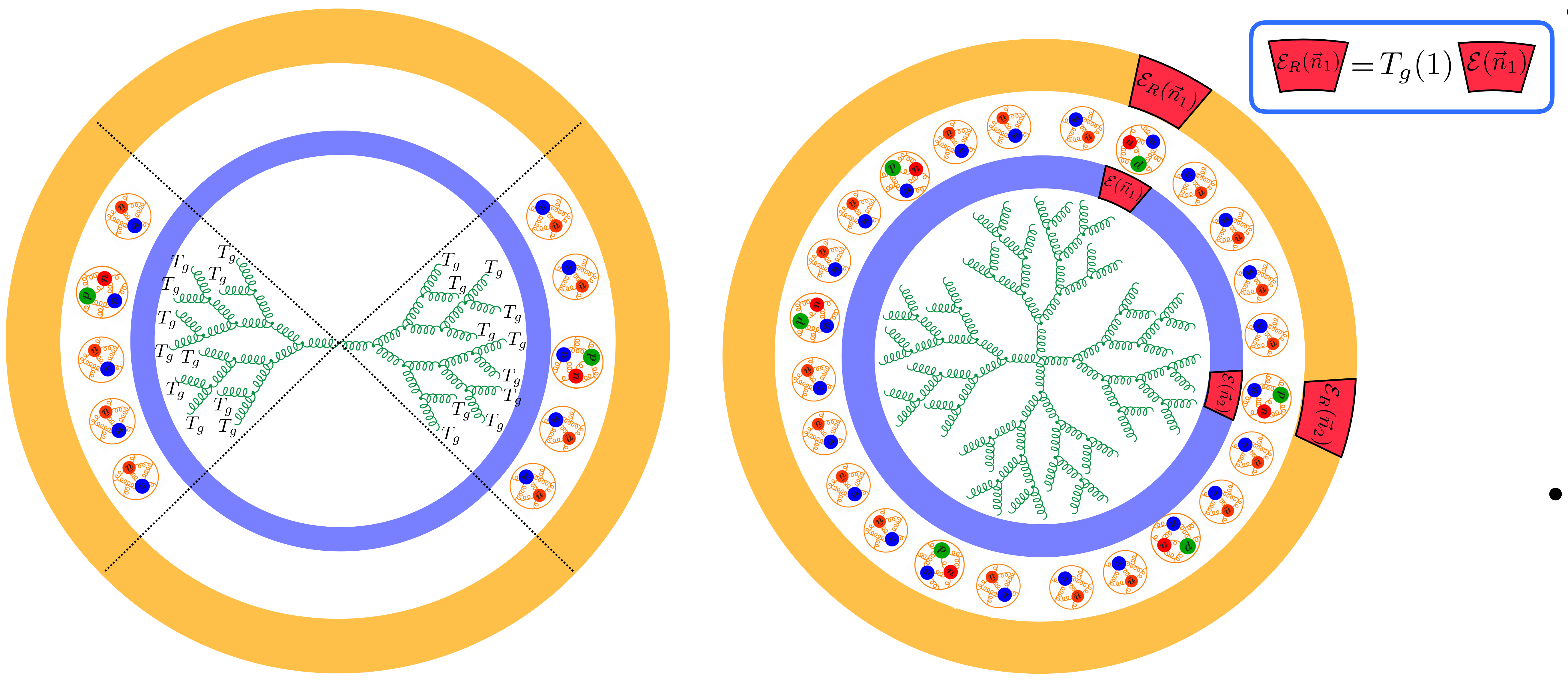
- Fixed number of detectors

space of detectors **vs** space of the states

- Makes incorporation of NP dynamics much simpler

Example: Track observable

Chang, Procura, Thaler, Waalewijn, `13



Chen, Moult, Zhang, Zhu, `20
Li, Moult, van Velzen, Waalewijn, Zhu, `21
Jaarsma, Li, Moult, Waalewijn, Zhu, `22

- $T_i(x_i, \mu)$ describes momentum fraction of initial parton i converted to tracks, i.e. $p^\mu_{\text{trk}} = x_i p^\mu_i$
- Jet shape measured on track is then

$$\delta(e - \hat{e}(\{p^\mu_i \in X_J\})) \to \delta(e - \hat{e}(\{x_i p^\mu_i \in X_J\}))$$

which requires simultaneous knowledge of all the tracks in jet.

- On the other hand, energy correlators on tracks only require a simple replacement:

$$\mathcal{E}(\hat{n}) \to \mathcal{E}_R(\hat{n}) = T_i(1,\mu)\mathcal{E}(\hat{n}) \qquad T_i(1,\mu) = \int dx\, x\, T_i(x,\mu)$$

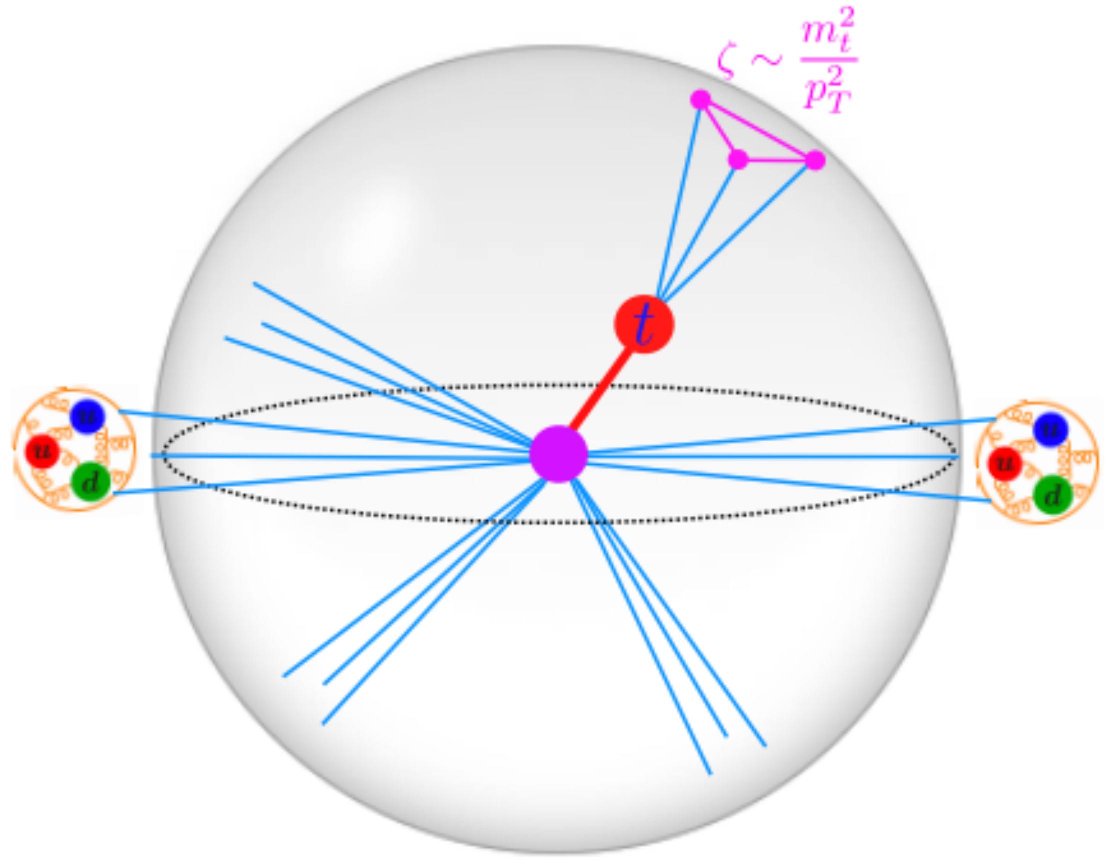
- This simple structure allowed energy correlators on track to be computed at the same accuracy as the state of the art calculation perturbative calculation.

Energy correlators as jet substructure

- Probes fixed scale

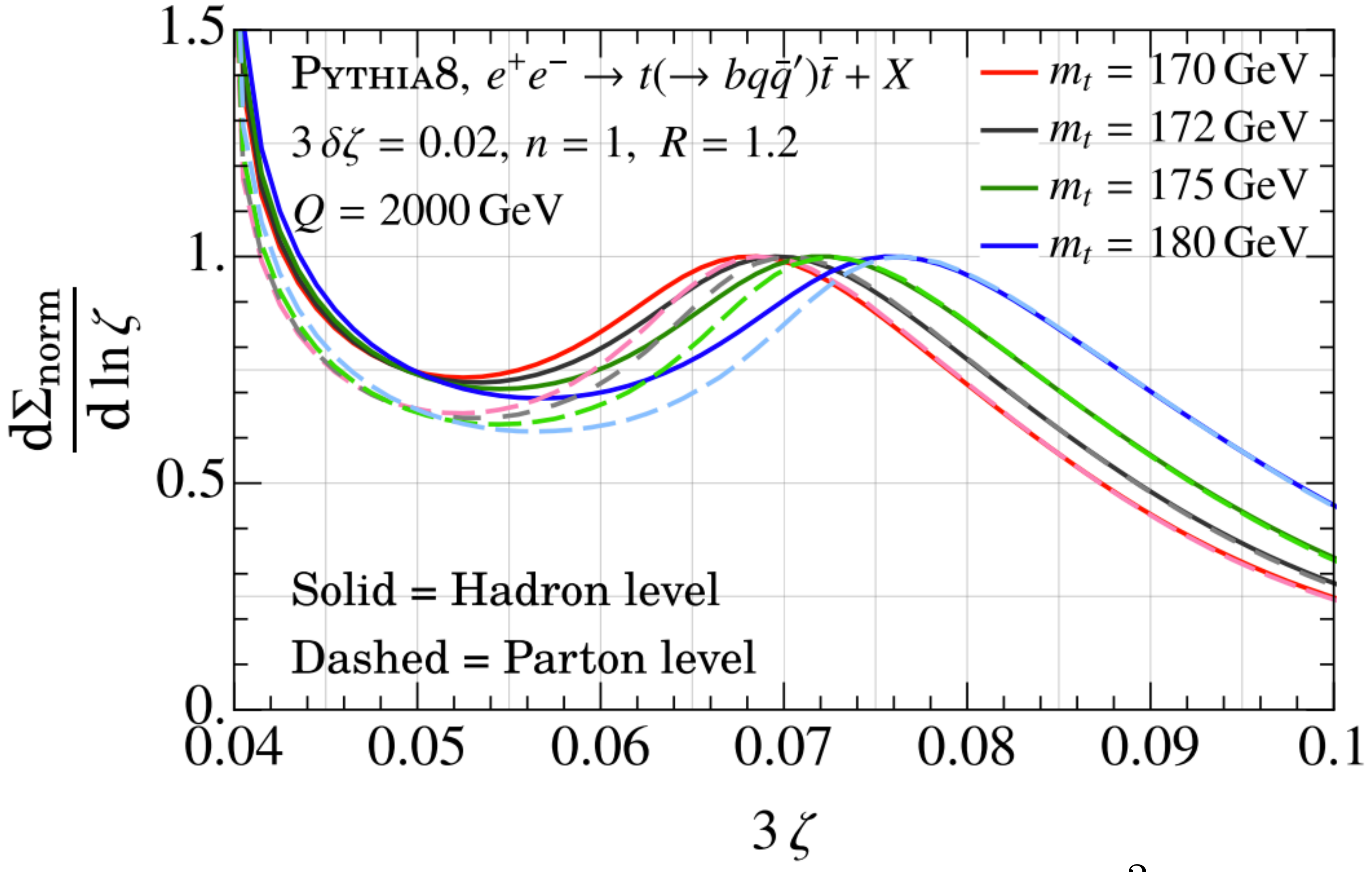
$$\mu \sim 2p_{J,T}\sqrt{z} \sim p_{J,T}R_L$$

scale knob



Example: Top mass

Holguin, Moult, Pathak, Procura, `22



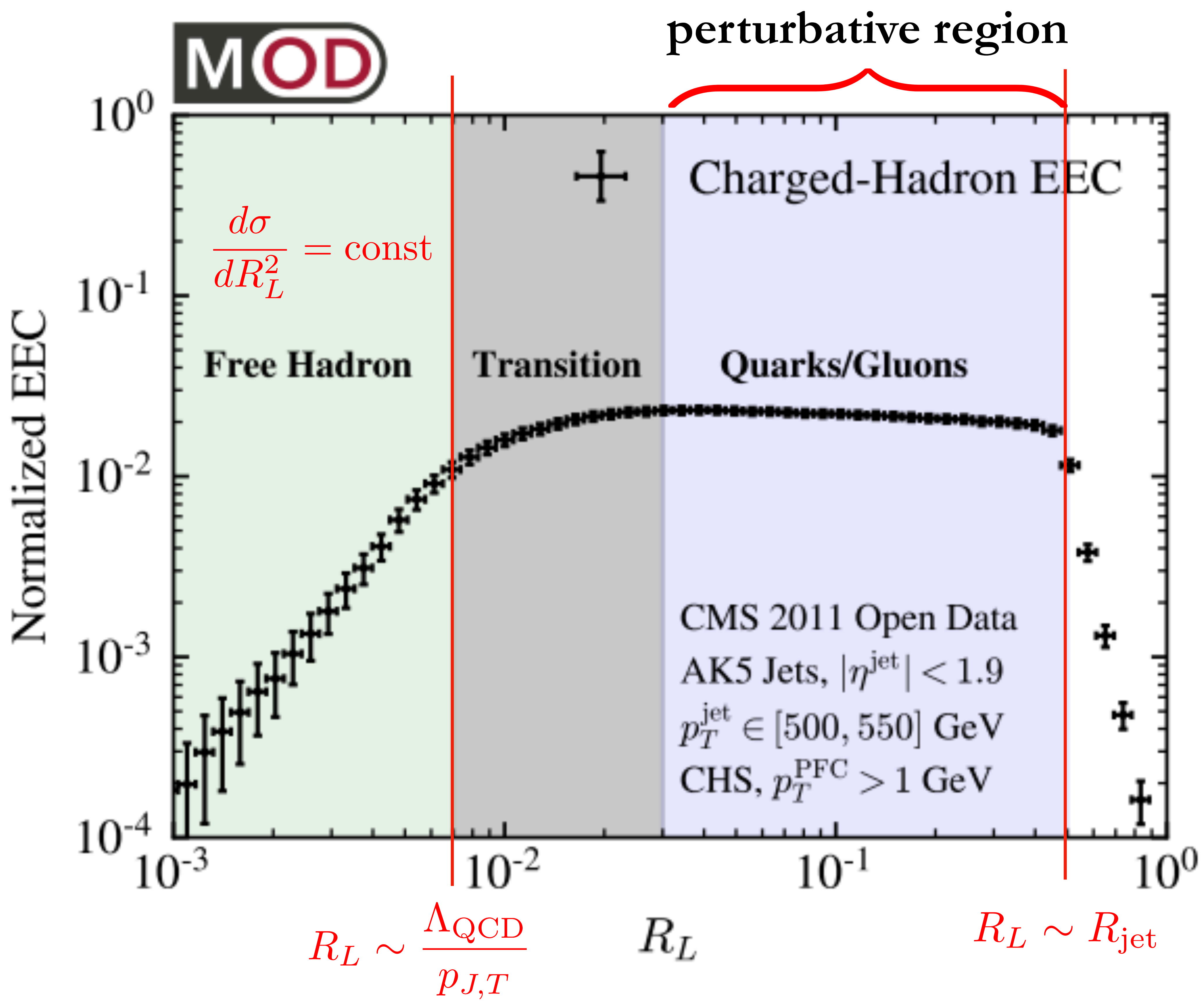
Equilateral triangle, so $\zeta \sim \frac{R_L^2}{4}$

- One sees a clear top mass sensitive peak around $3\zeta \sim \frac{3m_t^2}{Q^2}$

Energy correlators as jet substructure

- Probes fixed scale

$$\mu \sim 2p_{J,T}\sqrt{z} \sim p_{J,T}R_L$$

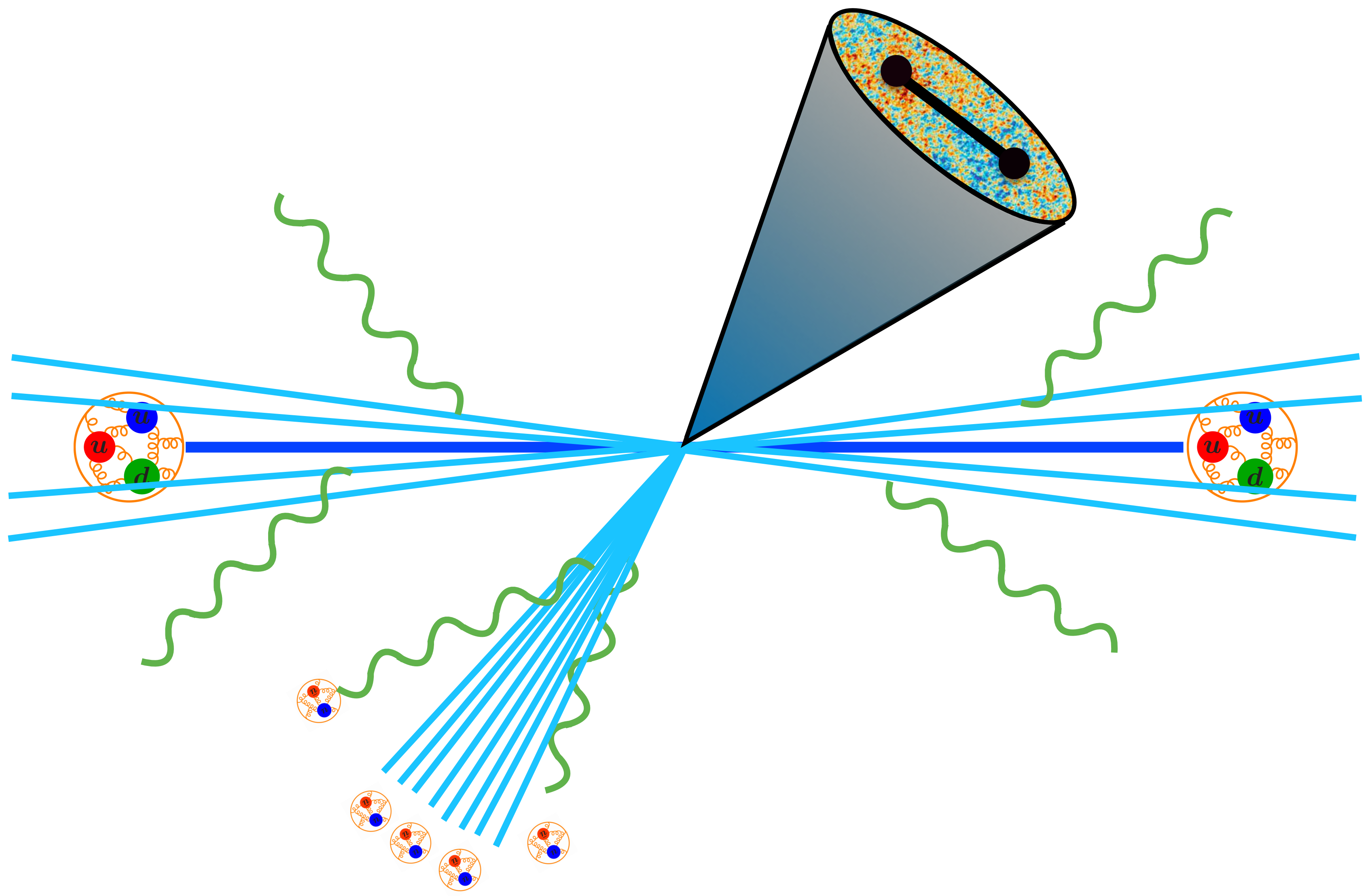


Komiske, Moult, Thaler, Zhu, `22

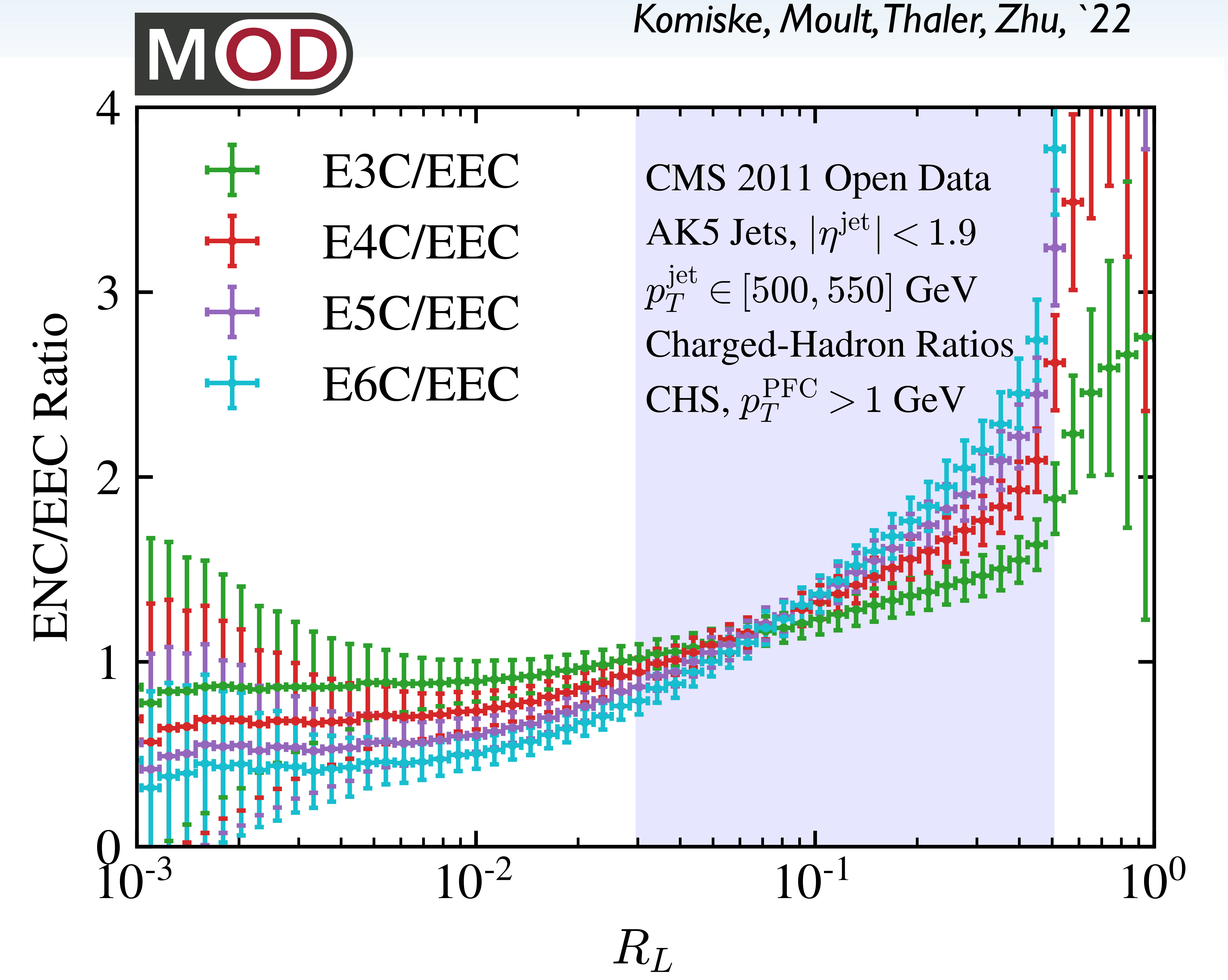
Example: Transition between hadronic and perturbative regions

- Unlike in jet shape, which can have multiple scales at a fixed value of the observable, one really probes fixed angular and associated energy scale.
- This makes the transition between different regions clear.
- Free hadron region is consistent with the freely propagating uniformly distributed hadrons.
- Now, let's look at the perturbative region.

Energy correlators as jet substructure



Komiske, Moult, Thaler, Zhu, `22



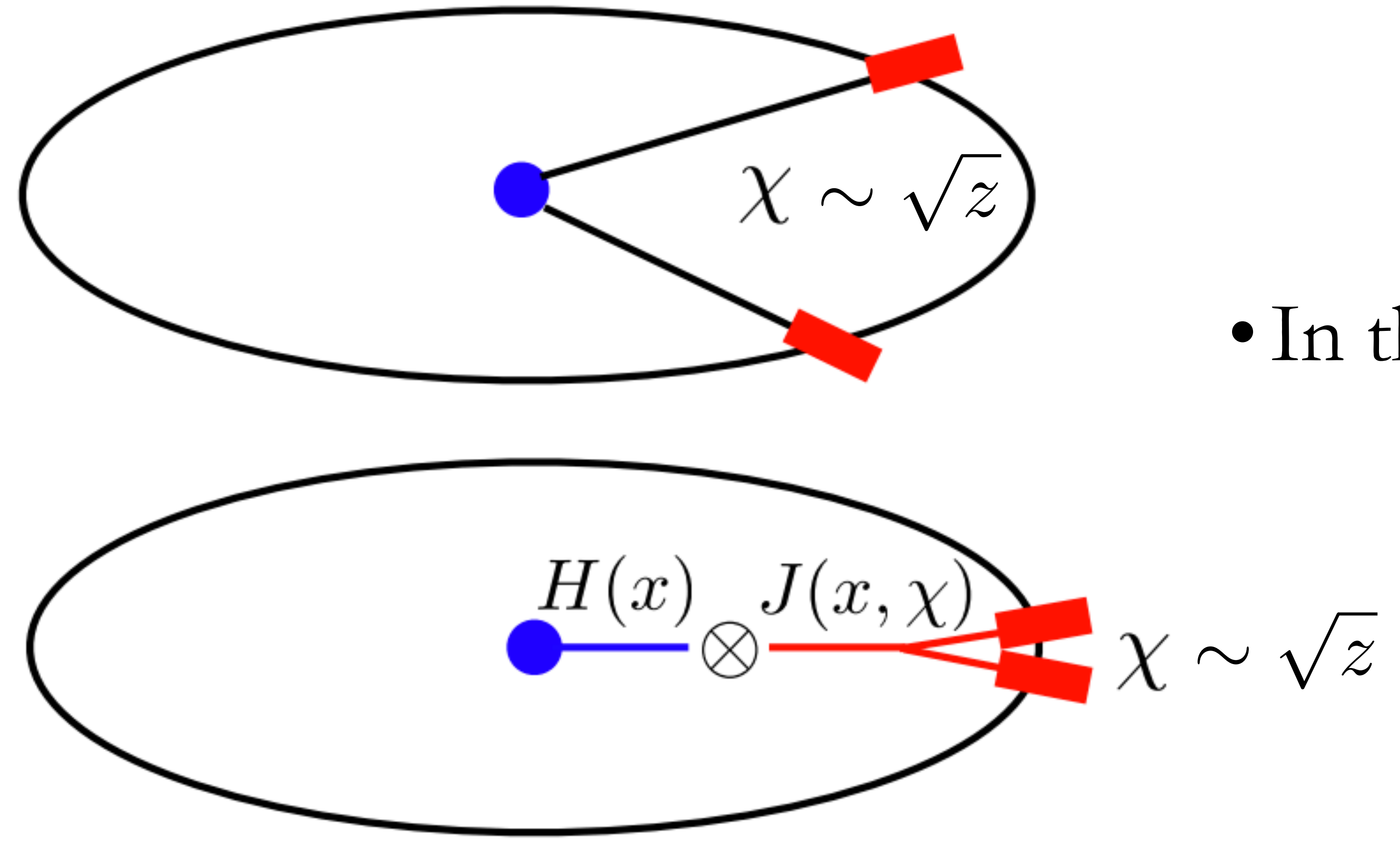
- Want to be able to extend the formalism to study energy correlators as jet substructure at the LHC!

Energy correlators at e^+e^-

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

For convenience, cumulant: $\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) \equiv \frac{1}{\sigma_0}\int_0^z dz' \frac{d\sigma}{dz}\left(z', \ln\frac{Q^2}{\mu^2}, \mu\right)$

$$[\ln^j z/z]_+ \to 1/(j+1) \times \ln^{j+1} z \quad \text{and} \quad \delta(z) \to 1$$



Dixon, Moult, Zhu, `19

- In the collinear limit, $z \to 1$ (i.e. $\chi_{ij}^2 \to 0$), factorizes as (using SCET)

$$\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) = \int_0^1 dx\, x^2 \vec{J}\left(\ln\frac{z x^2 Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

$\mu_{\mathrm{EEC}} \sim \sqrt{z}\, Q$ $\quad$ $\mu_H \sim Q$

Hard function (source)

$$\vec{J} = \{J_q, J_g\}$$

EEC Jet function

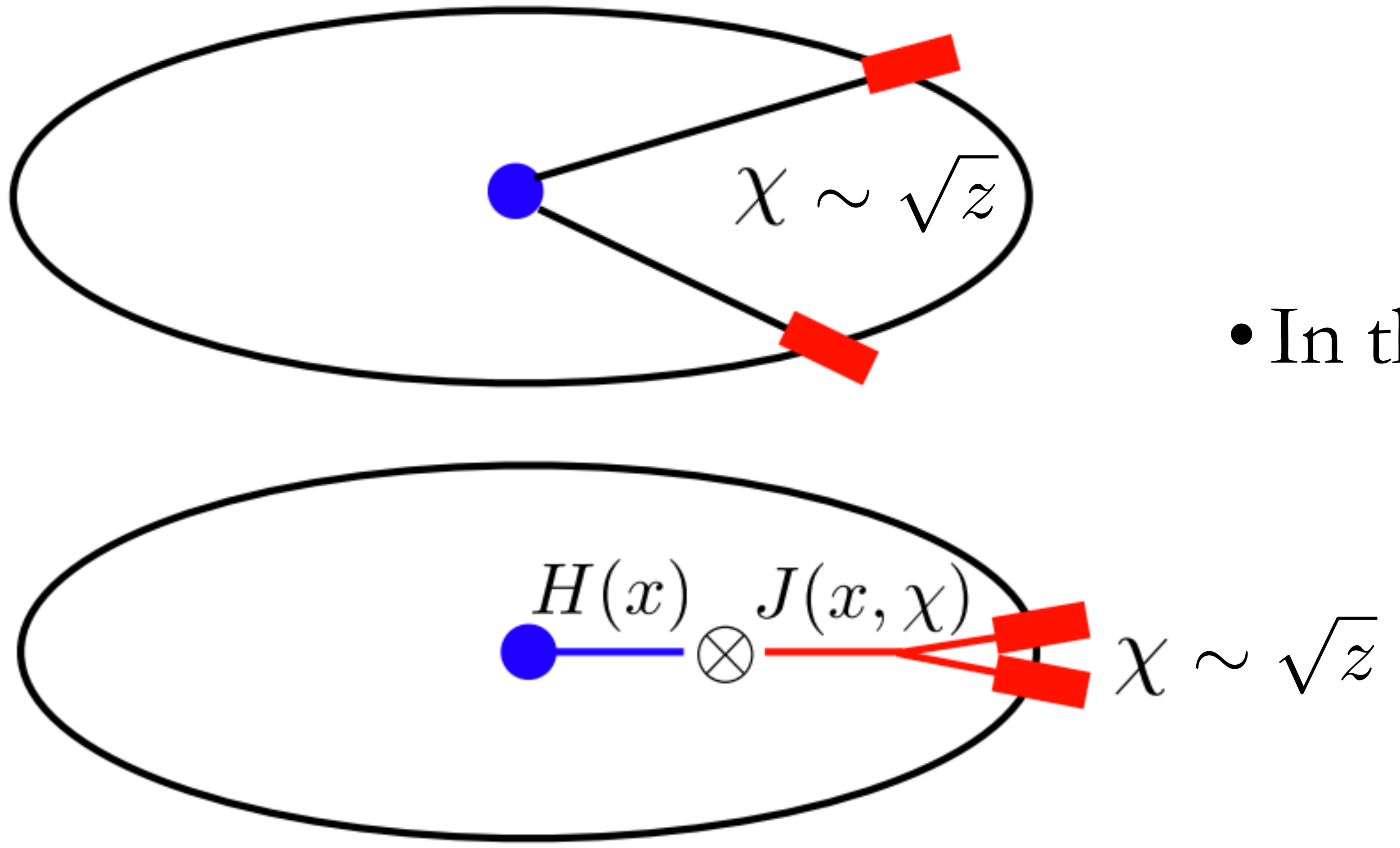
$$J_q(z) = \sum_X \sum_{i,j\in X} \langle 0 | \bar{\chi}_n | X \rangle \frac{E_i E_j}{(Q/2)^2} \Theta(\theta_{ij} < \chi) \langle X | \chi_n | 0 \rangle$$

Energy correlators at e^+e^-

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1 - \cos\chi_{ij}}{2}\right)$$

For convenience, cumulant: $\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) \equiv \frac{1}{\sigma_0}\int_0^z dz' \frac{d\sigma}{dz}\left(z', \ln\frac{Q^2}{\mu^2}, \mu\right)$

$$[\ln^j z/z]_+ \to 1/(j+1) \times \ln^{j+1} z \quad \text{and} \quad \delta(z) \to 1$$



Dixon, Moult, Zhu, `19

- In the collinear limit, $z \to 1$ (i.e. $\chi_{ij}^2 \to 0$), factorizes as (using SCET)

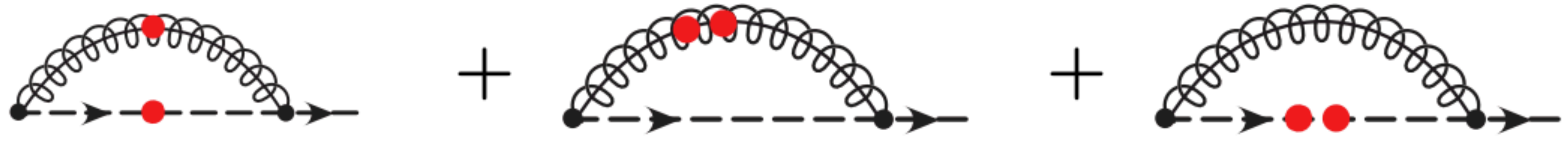
$$\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) = \int_0^1 dx\, x^2 \vec{J}\left(\ln\frac{z x^2 Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

$\mu_{\rm EEC} \sim \sqrt{z}\, Q$ — EEC Jet function, $\vec{J} = \{J_q, J_g\}$

$\mu_H \sim Q$ — Hard function (source)

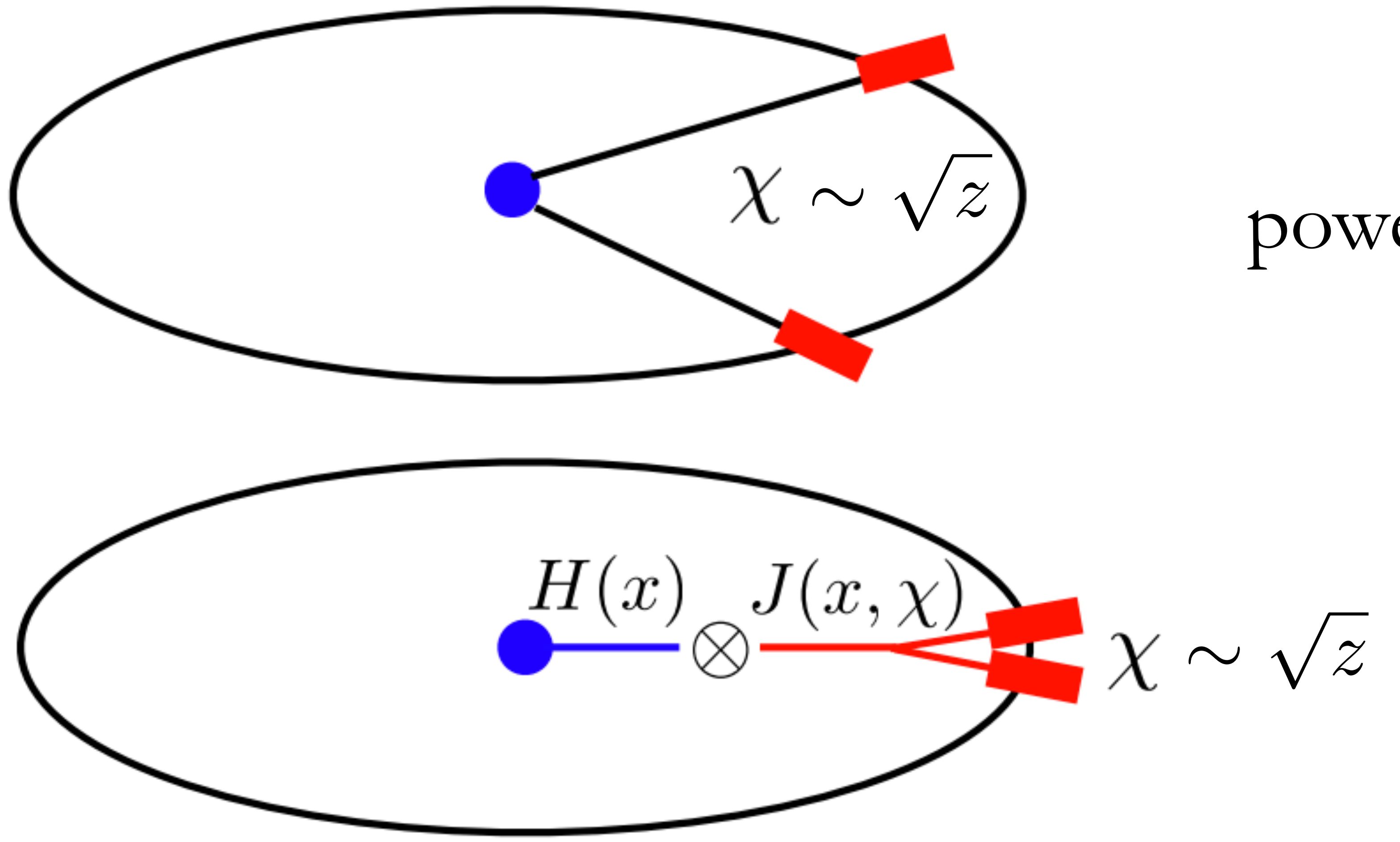
$$\frac{E_i E_j}{Q^2} \sim x^2 x_i x_j$$

$\vec{J}$ at **NLO**



Energy correlators at e^+e^-

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$



Dixon, Moult, Zhu, `19

- In CFTs,

$$\Sigma(z) = \frac{1}{2} C(\alpha_s)\, z^{\gamma_J^{\mathcal{N}=4}(\alpha_s)} \quad \Longleftrightarrow \quad \mathcal{E}(\hat{n}_1)\,\mathcal{E}(\hat{n}_2) = \theta^{\gamma_i} \sum \mathbb{O}_i(\hat{n}_1)$$

power-law behavior with scaling from twist-2 spin-3 anomalous dimension, related to OPE.

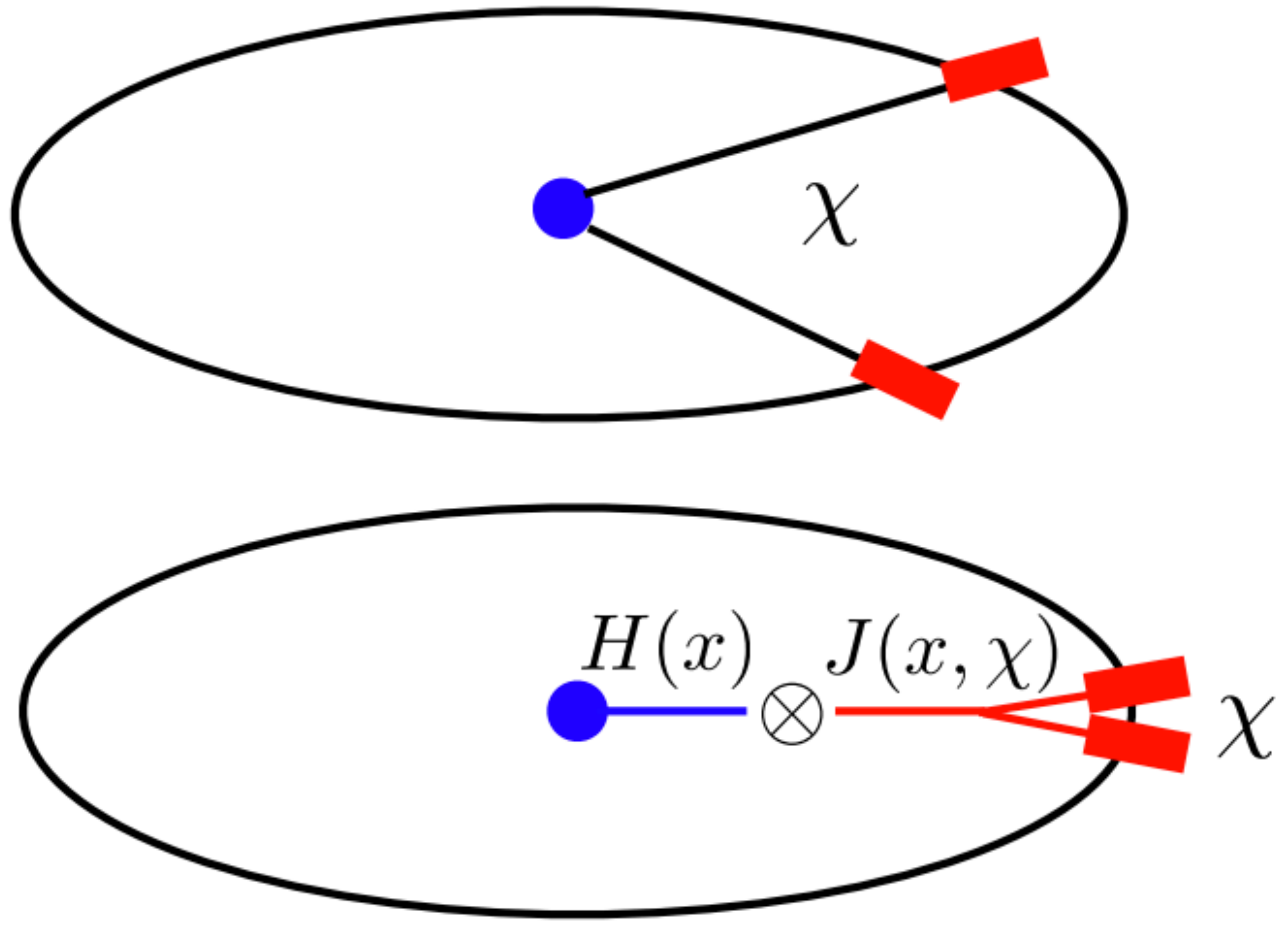
$$\gamma(3) > 0 \implies z\frac{d\sigma}{dz}\Big|_{z\to 0} = 0$$

can be computed using OPE alone!

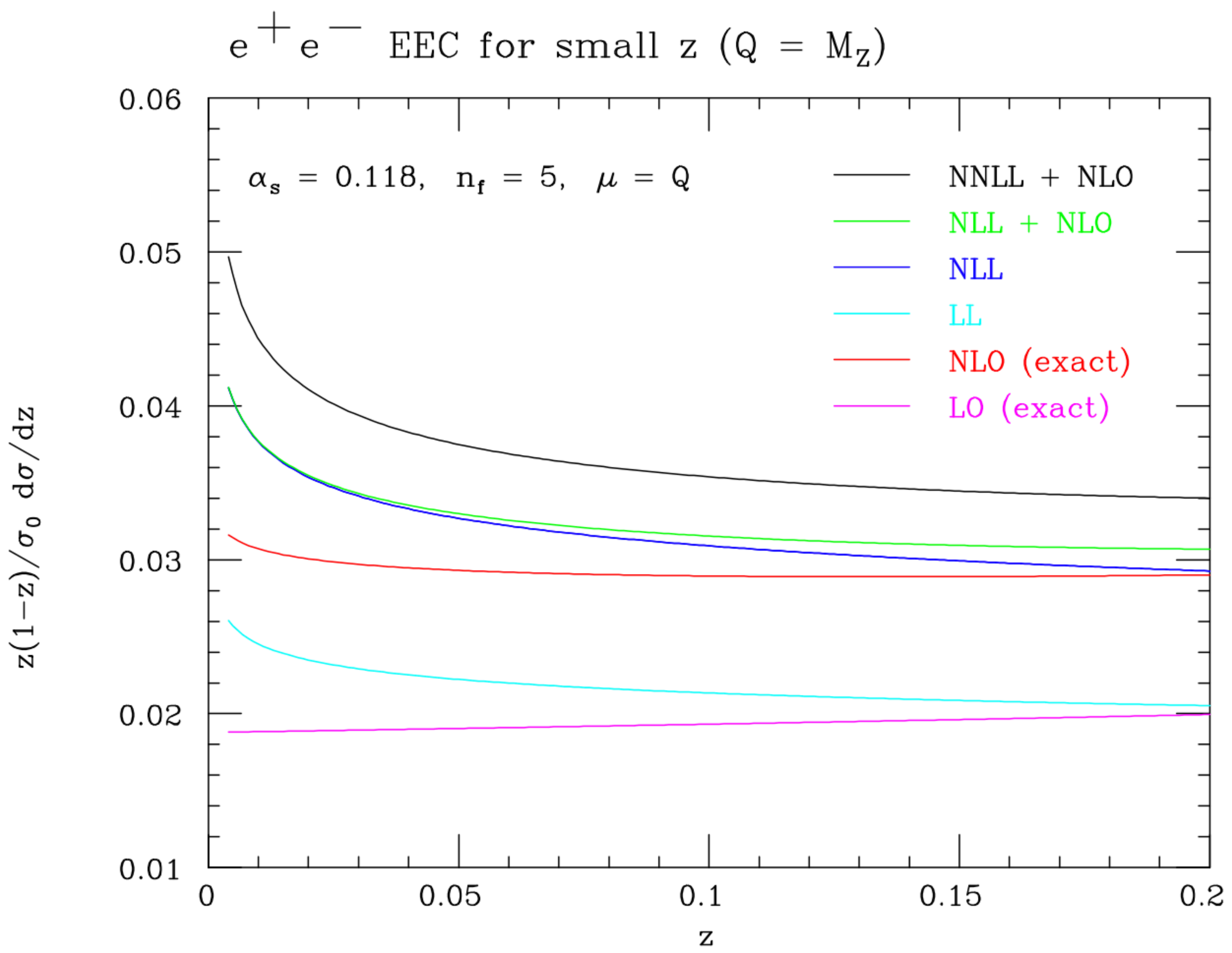
Energy correlators at e^+e^-

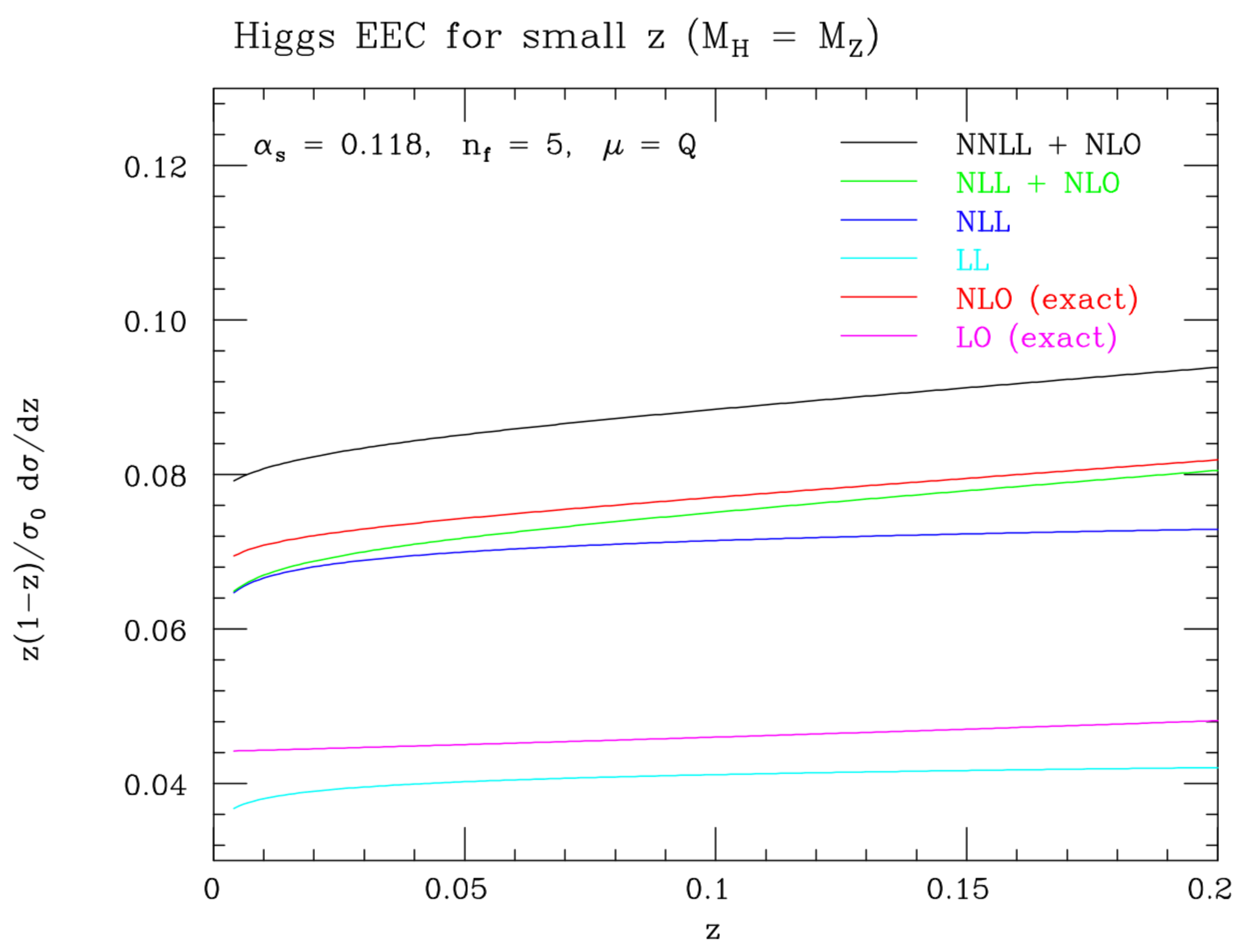
$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

- In non-CFTs (like QCD), there is competition between beta functions and twist-2 spin-3 anomalous dimension.



Dixon, Moult, Zhu, `19





- Higher scale would give larger window of region where the contribution from the twist-two anomalous dimension dominates over that of beta function, giving phenomenological connection to Light-ray OPE and other CFT techniques
- Higher energy provides more particles in jet, allowing us to study higher-point correlators
- Smaller NP corrections

$\Longrightarrow$ **Jets at the LHC!**

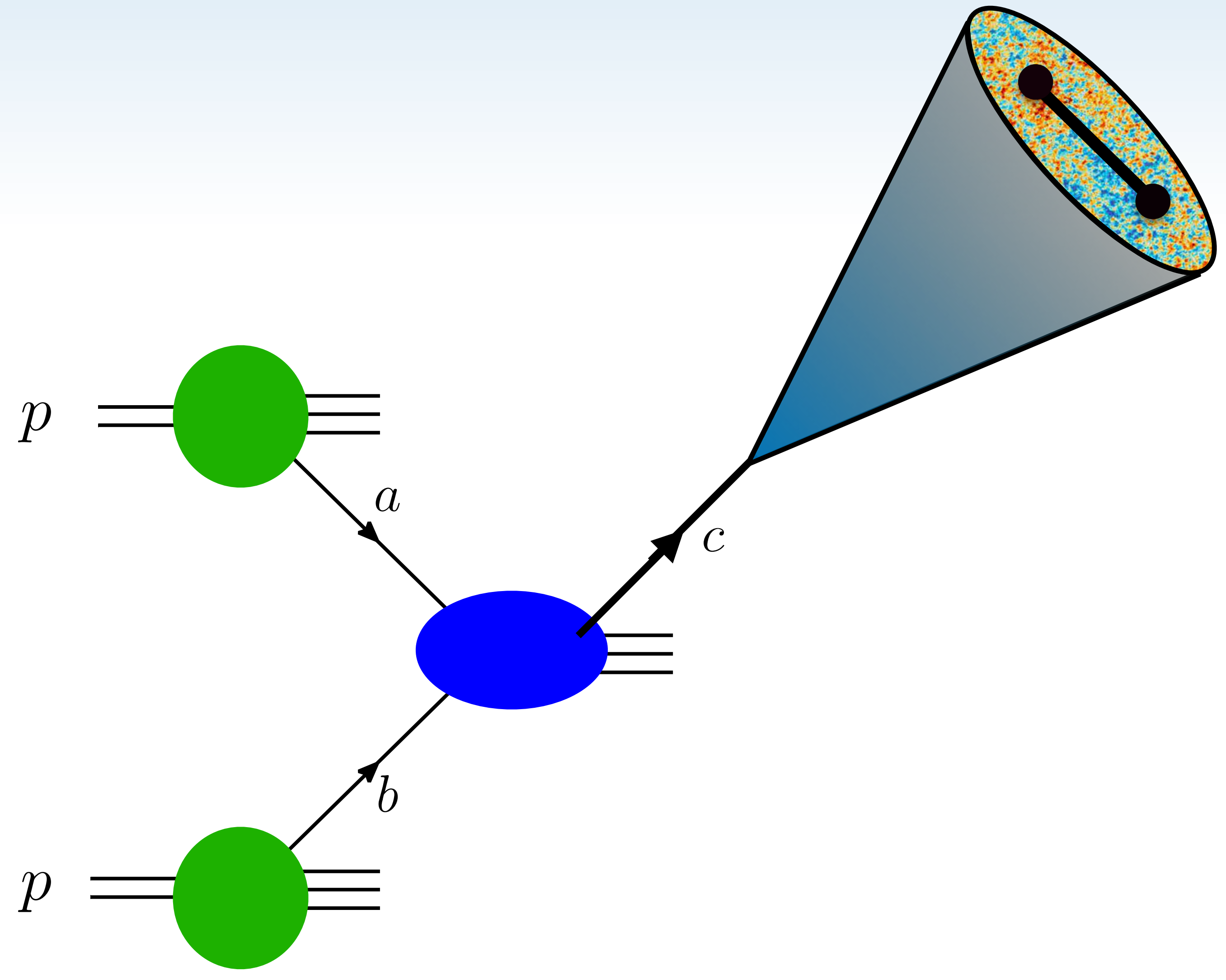
The jet fragmentation function and energy correlators

Factorization

$$\frac{d\sigma^{pp\to \mathrm{jet(ENC)}X}}{dp_T d\eta d\{\zeta\}} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \mathcal{G}_c(\{\zeta\})$$

Λ_{QCD} $\quad p_T \quad p_T R$

$p_T\sqrt{\zeta}$



where $\{\zeta\}$ stands for the collection of angles in N-point correlators

$$\mathcal{G}_c(z,\{\zeta\},p_T R,\mu) = \sum_j \int_0^1 dx\, x^N \mathcal{J}_{ij}(z,x,p_T R,\mu)\, J_{\mathrm{EEC}}(\{\zeta\},x,\mu)$$

- J_{EEC} is the same EEC jet function as e^+e^- case (can use track or other cases too)
- Energy correlators are expectation values on a state $|\Psi\rangle$
  In e^+e^-, the state is created by a local operator.

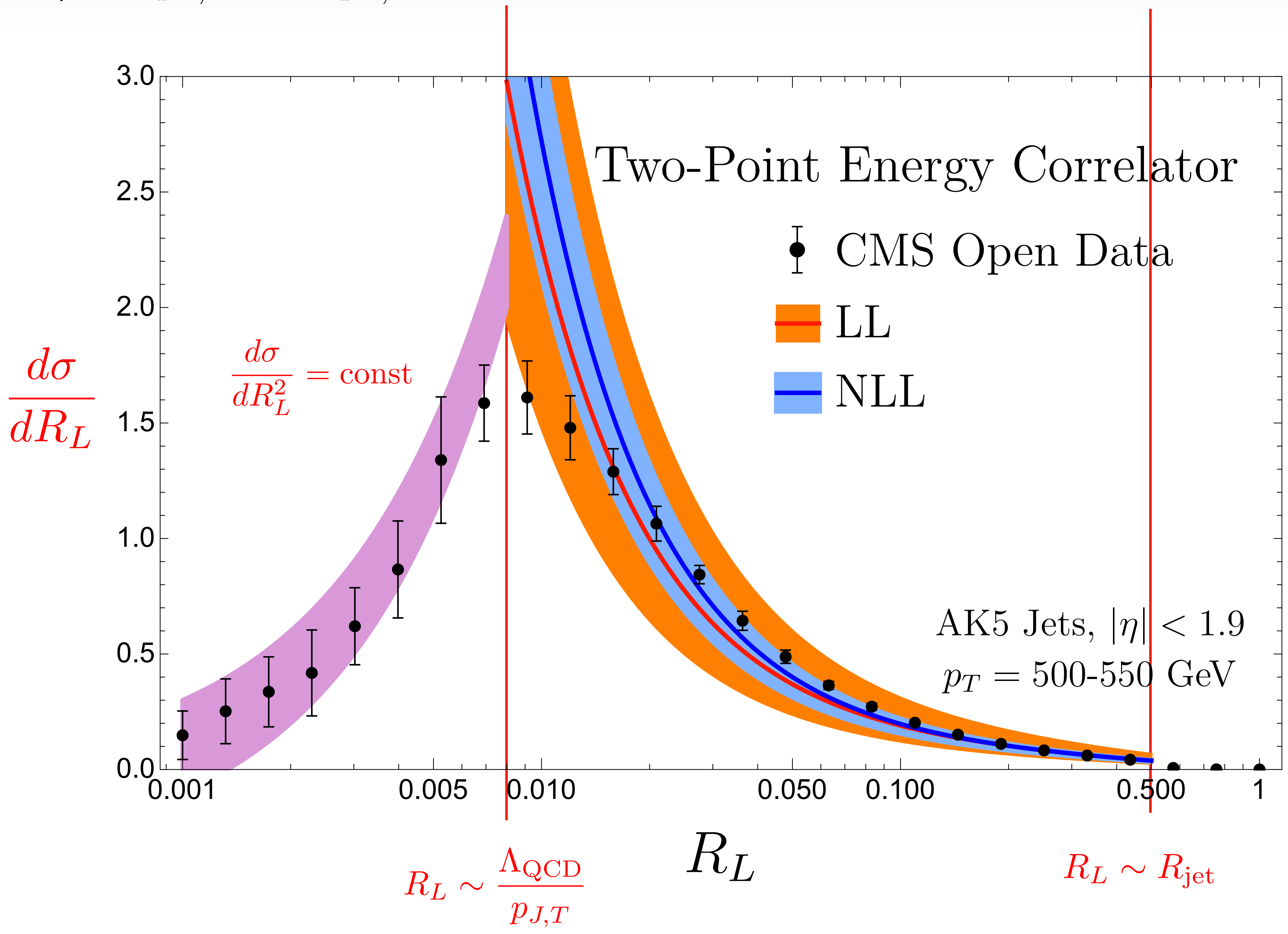
$$\frac{d\sigma}{d\{\zeta\}} \sim \langle \Psi | \mathcal{E}(\hat{n}_1)\cdots\mathcal{E}(\hat{n}_N)|\Psi\rangle$$

- As discussed, $\mathcal{G}_c$, describes how jet algorithms are used to "create" the state $|\Psi\rangle$ in which energy correlators are measured.
- More formally, $|\Psi\rangle = \sum_{\delta,j} c_{\delta,j}|\Psi_{\delta,j}\rangle$ where δ, j are the quantum numbers of the celestial sphere.

Chen, Moult, Sandor, Zhu `22
Chang, Simmons-Duffin `22

2-Point Energy correlators at the LHC

$\mu \sim 2 p_{J,T} \sqrt{z} \sim p_{J,T} R_L$



- One can see clear transition between the perturbative and hadronization regions.
- Perturbative region agrees well with the data without any soft drop grooming, trimming, pruning, etc.
- At very small angle, the result is consistent with uniformly distributed freely propagating hadrons.

KL, Meçaj, Moult, `22

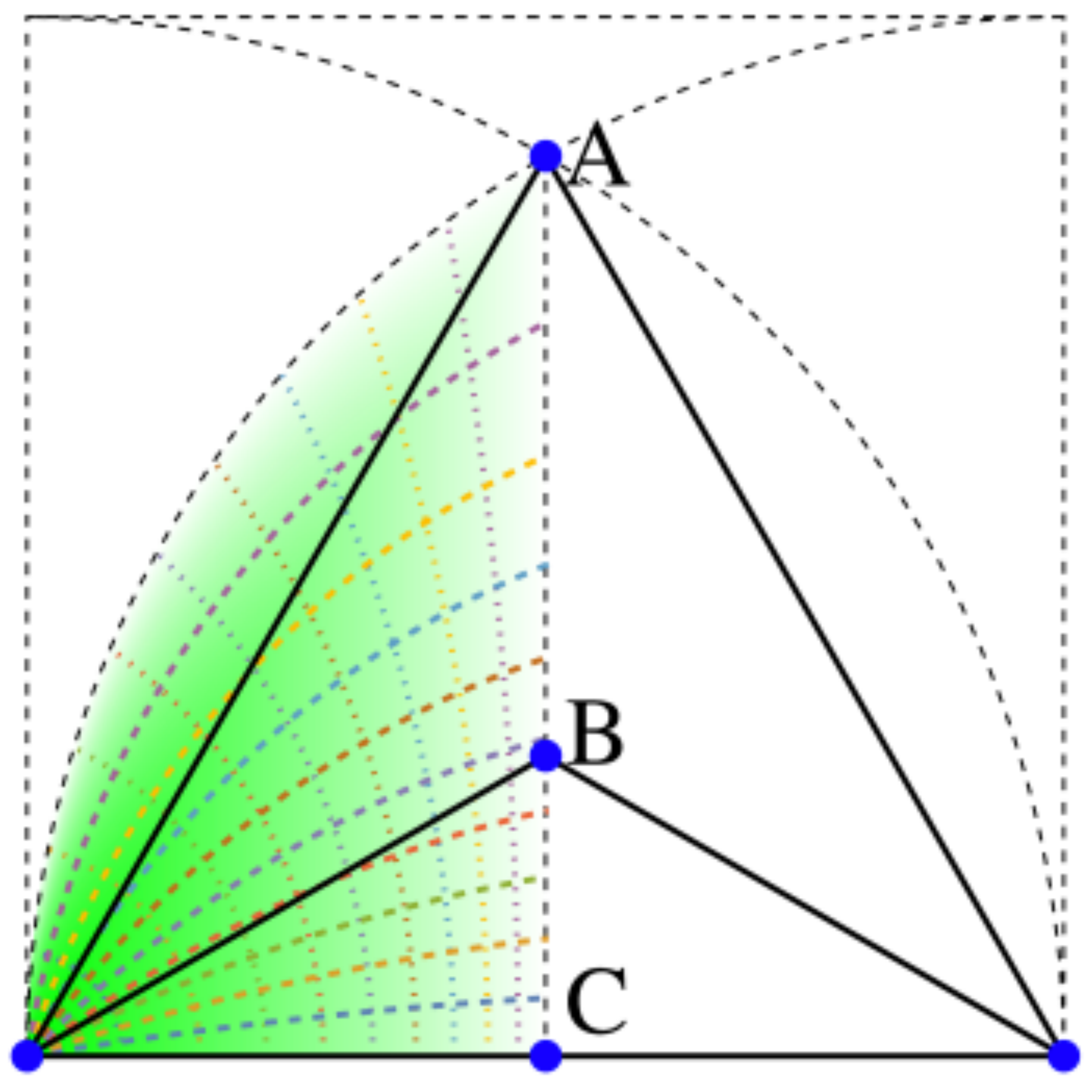
Projected Energy correlators at the LHC

$\mu \sim 2p_{J,T}\sqrt{z} \sim p_{J,T}R_L$

$$J_{\mathrm{EEC}}^{N-\mathrm{proj}}(R_L, x, \mu) = \int d\{\zeta\}\ \delta\left(R_L - \max[\{\zeta\}]\right)\ J_{\mathrm{EEC}}^{N}(\{\zeta\}, x, \mu)$$

- Integrate over all shapes with fixed largest angle, R_L

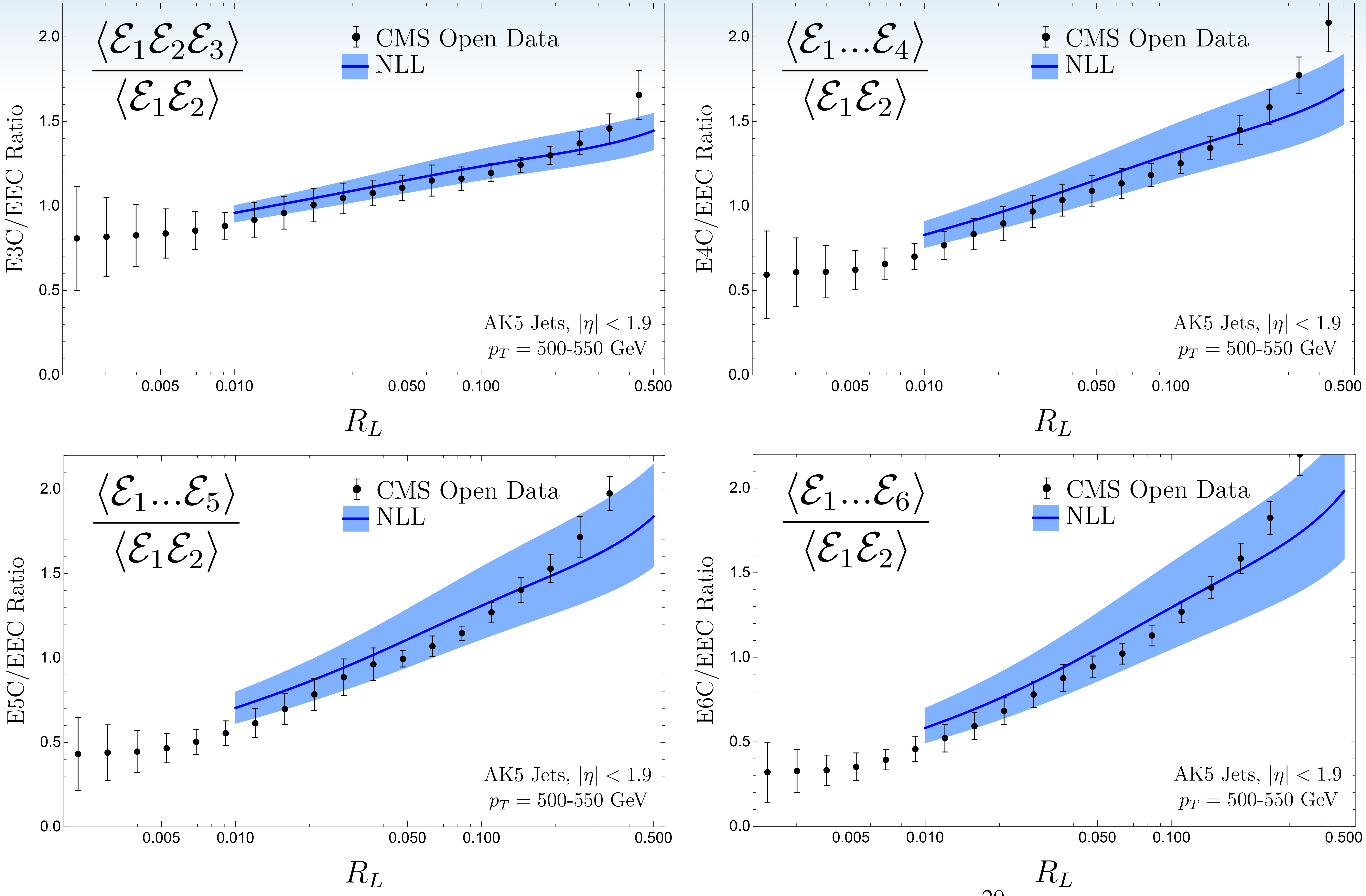
- Related to the OPE limit of the N-point correlators, scales as twist-2 spin-(N+1) anomalous dimension in the conformal limit.



Space of 3-point correlator

KL, Meçaj, Moult, `22

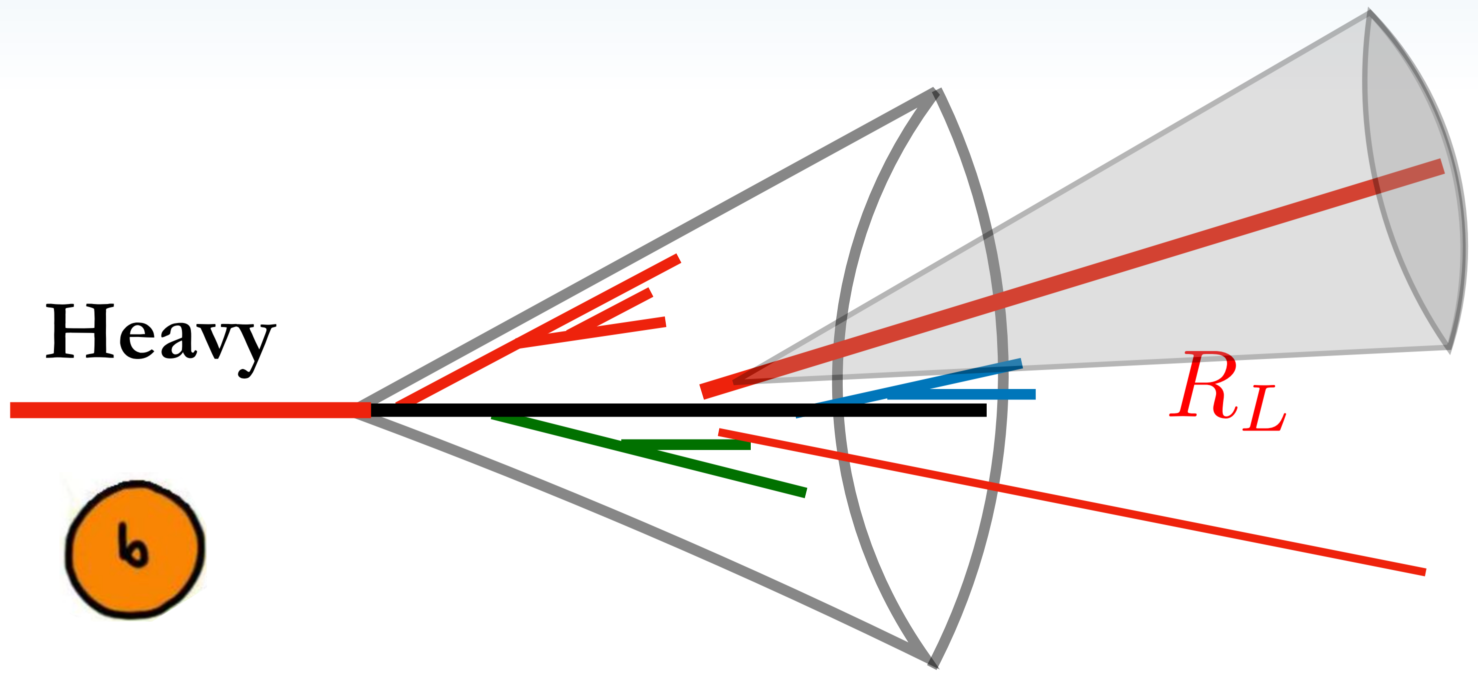
Projected Energy correlators at the LHC



- Slope increases with N as predicted by the light-ray OPEs
- Non-perturbative effects expected to cancel in ratio
- Already at competing order of accuracy as the state-of-the-art calculation of other jet substructure
- Precision calculations of α_s

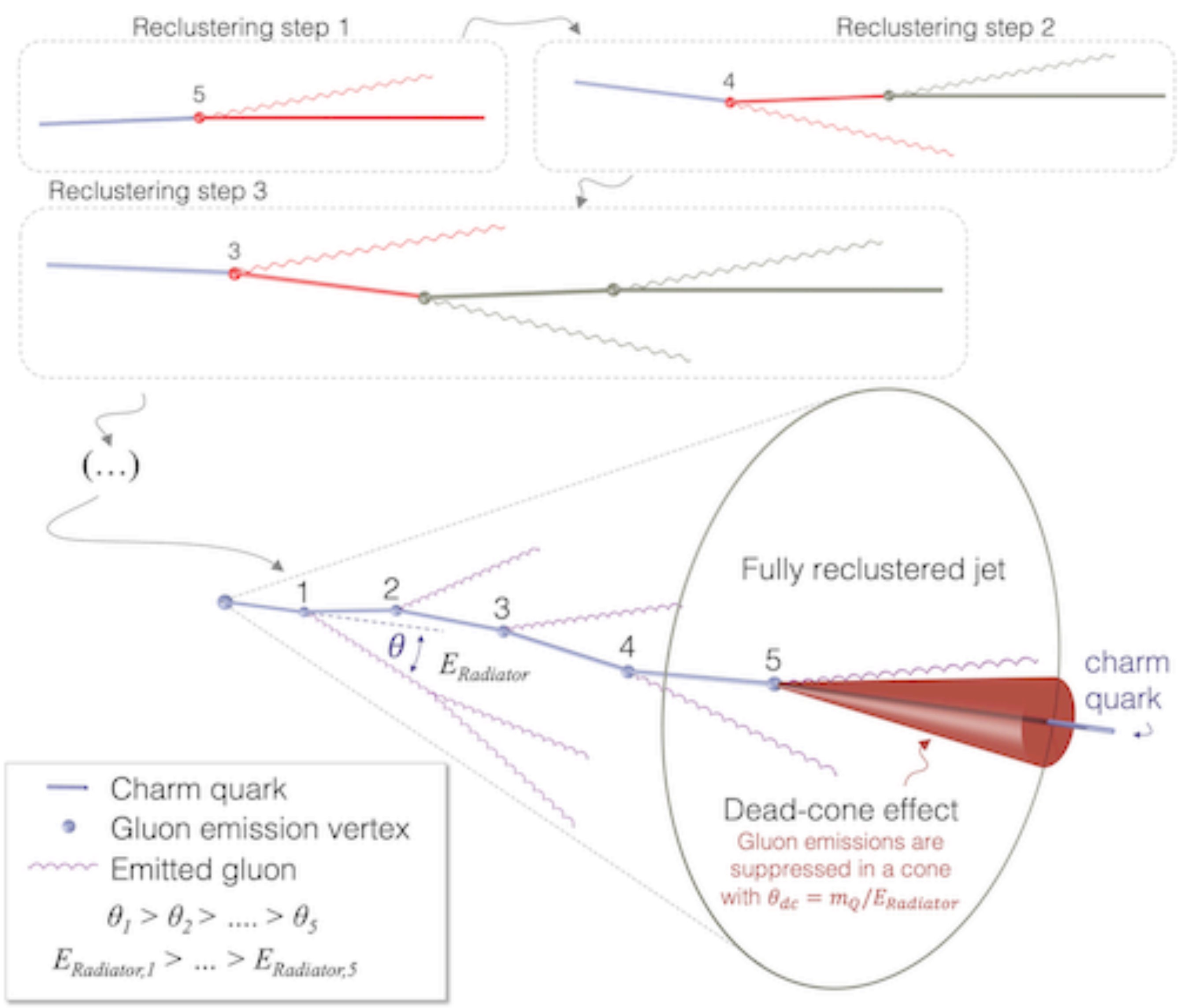
KL, Meçaj, Moult, `22

beautiful and *charming* energy correlators



- What happens if we consider energy correlators between heavy meson and other particles in a heavy jet?
- Heavy quark suppresses gluon emission around the angular region $\theta < \dfrac{M}{p_T}$

dead-cone



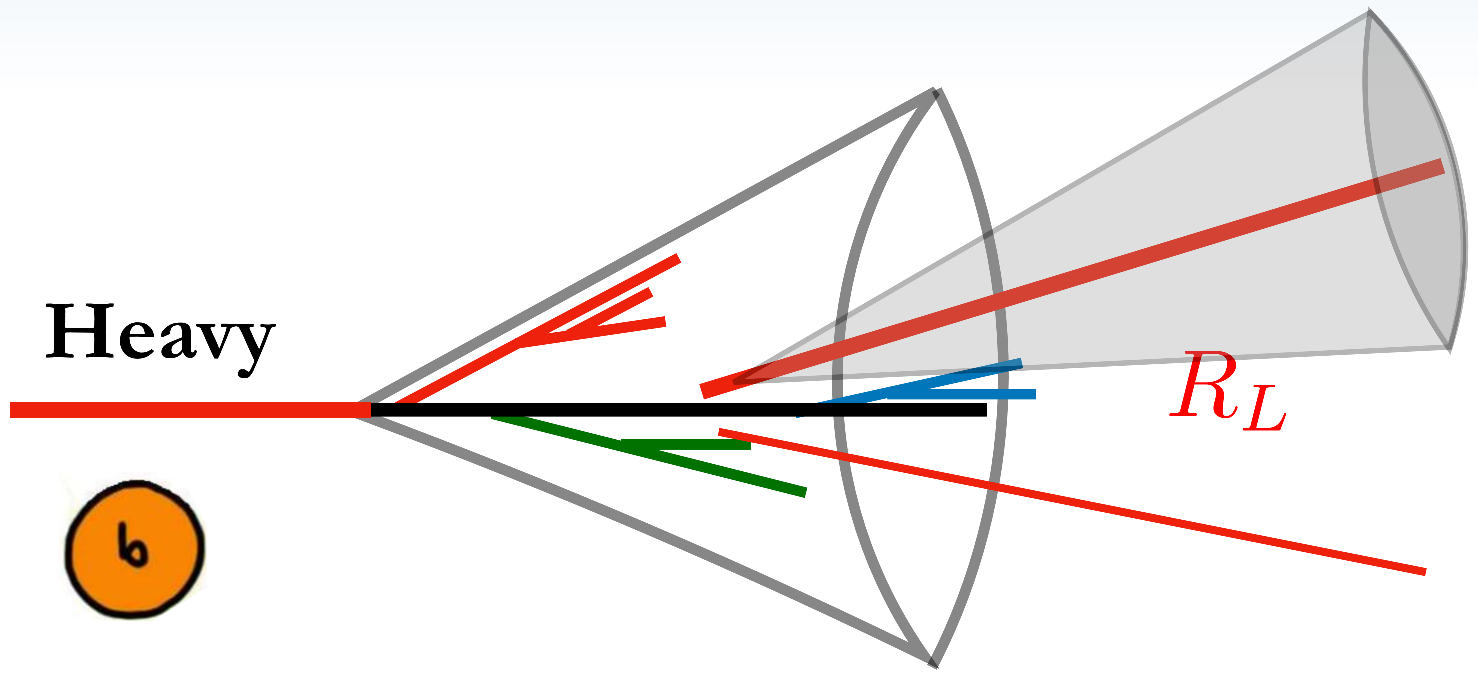
- Recently, ALICE collaboration made a direct observation of the dead-cone effect

ALICE Collaboration `22 (Nature)

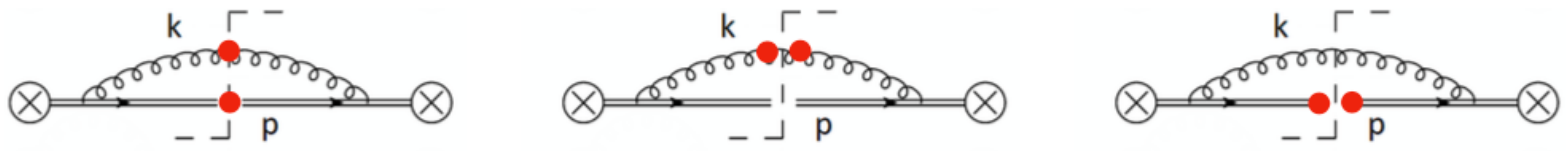
- Sophisticated reclustering techniques.
  Can we observe the dead-cone effect statistically using energy correlators?

KL, Meçaj, Moult, In Progress

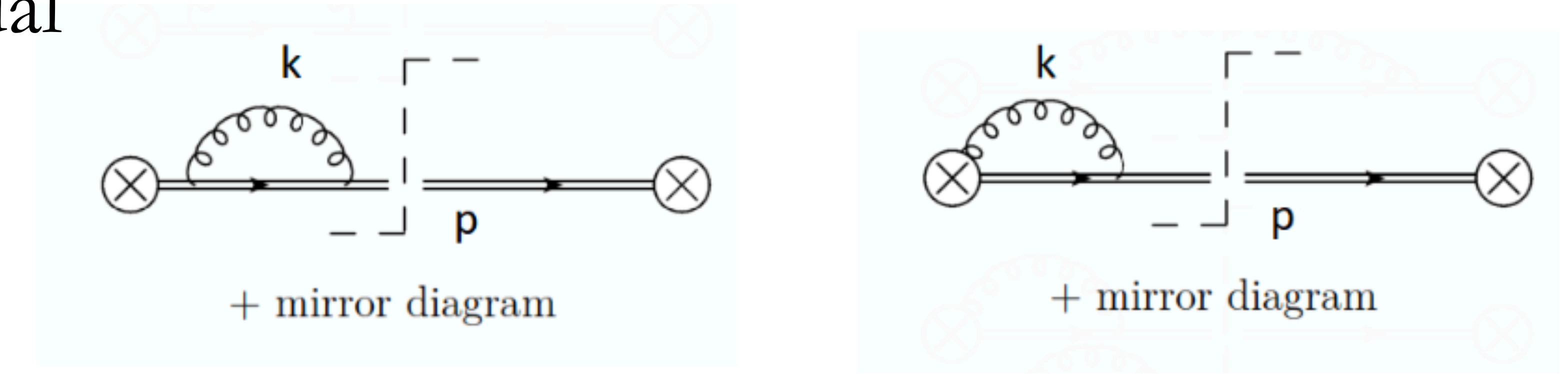
beautiful and *charming* energy correlators



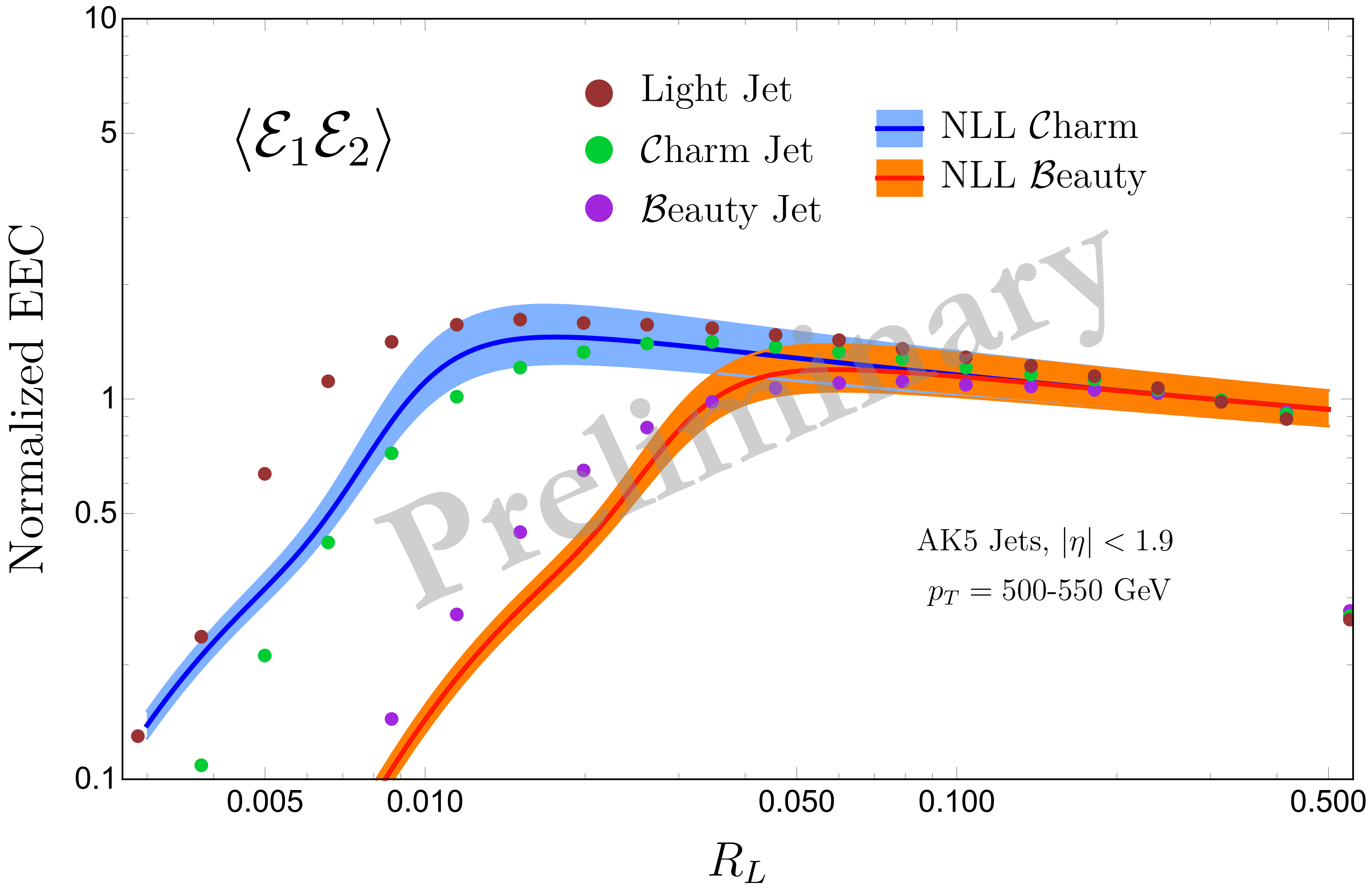
Real



Virtual



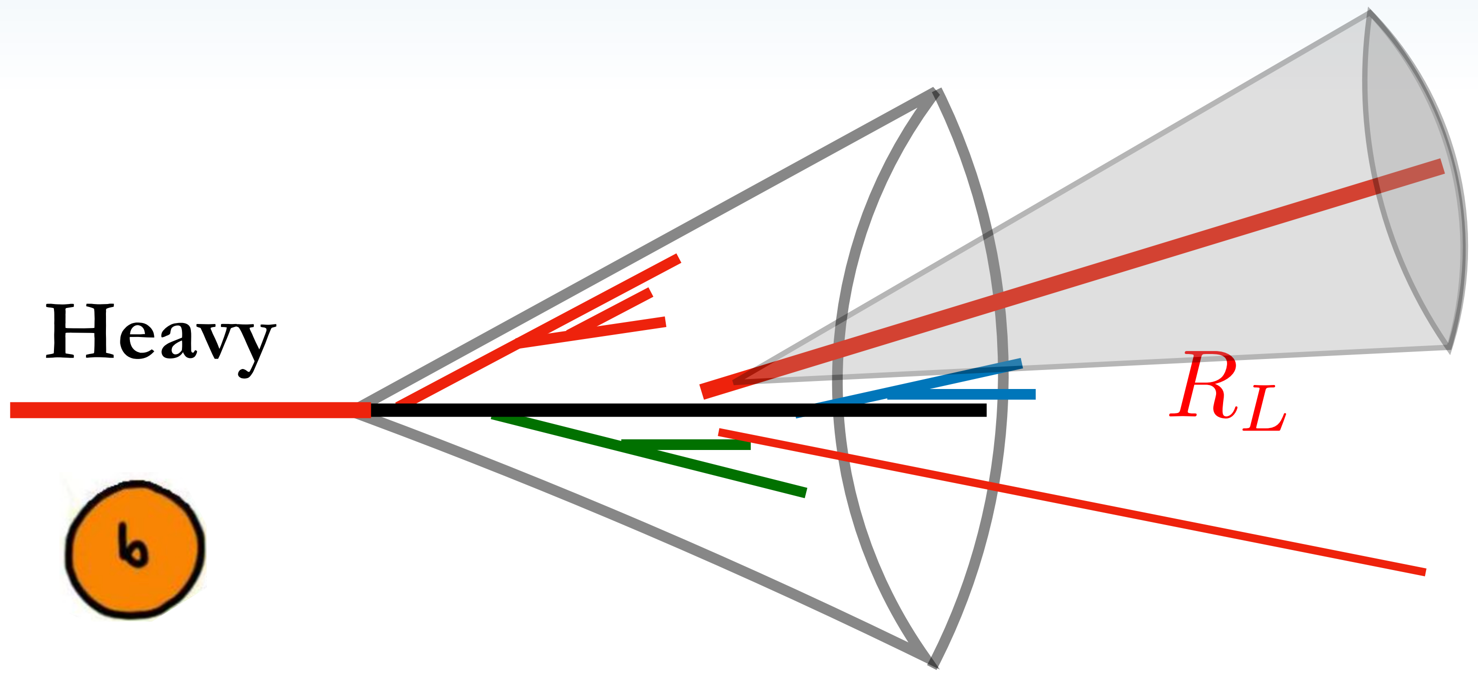
- What happens if we consider energy correlators between heavy meson and other particles in a heavy jet?

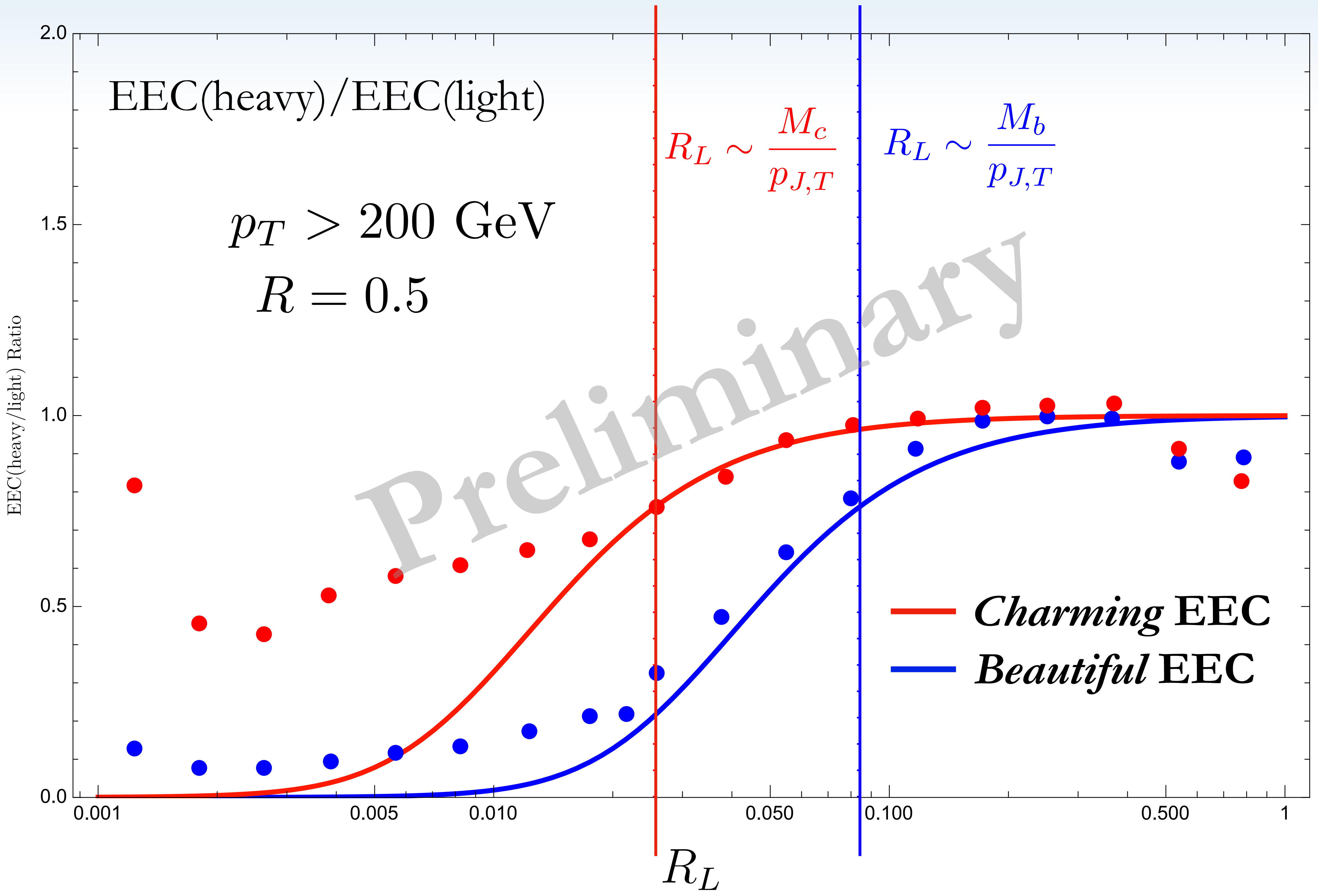


- UV poles match the light jet case as expected
- As the virtuality approaches heavy quark IR scale, we see a turning over.

KL, Meçaj, Moult, In Progress

beautiful and *charming* energy correlators

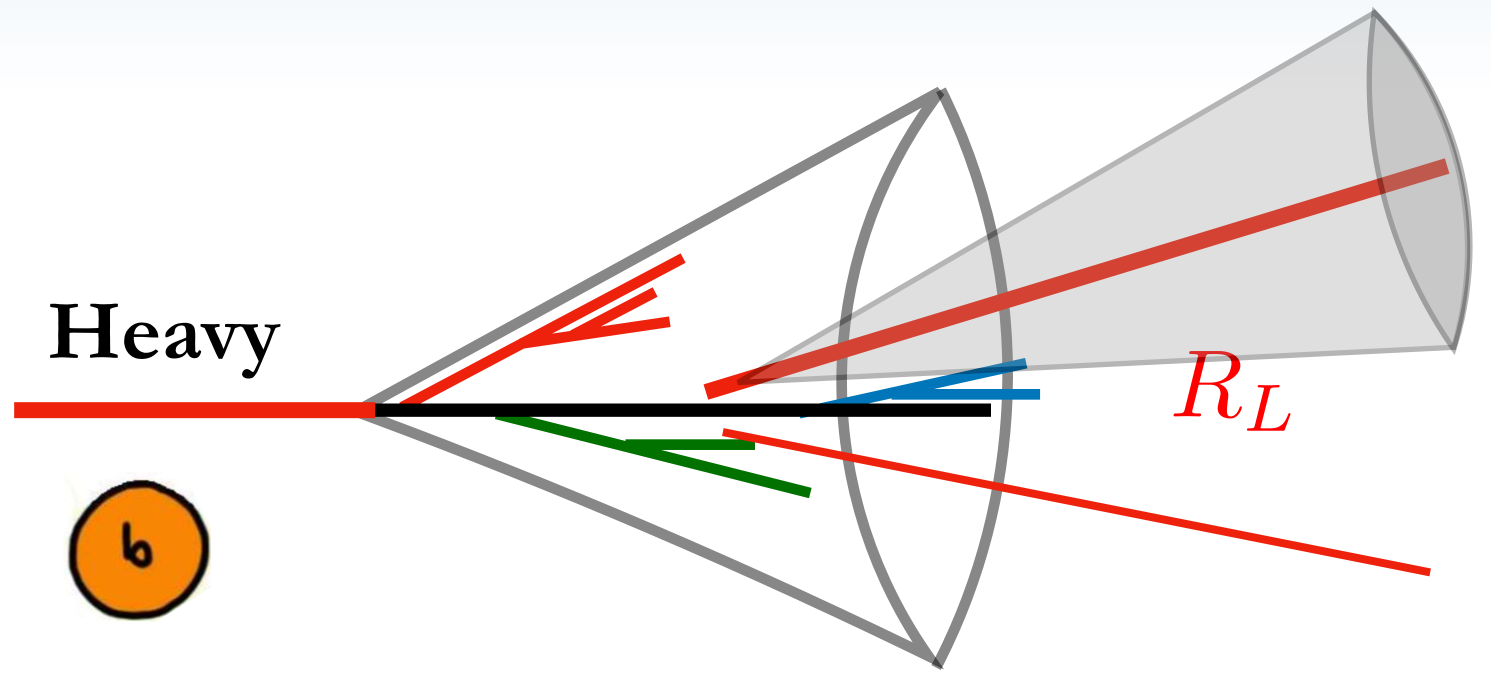


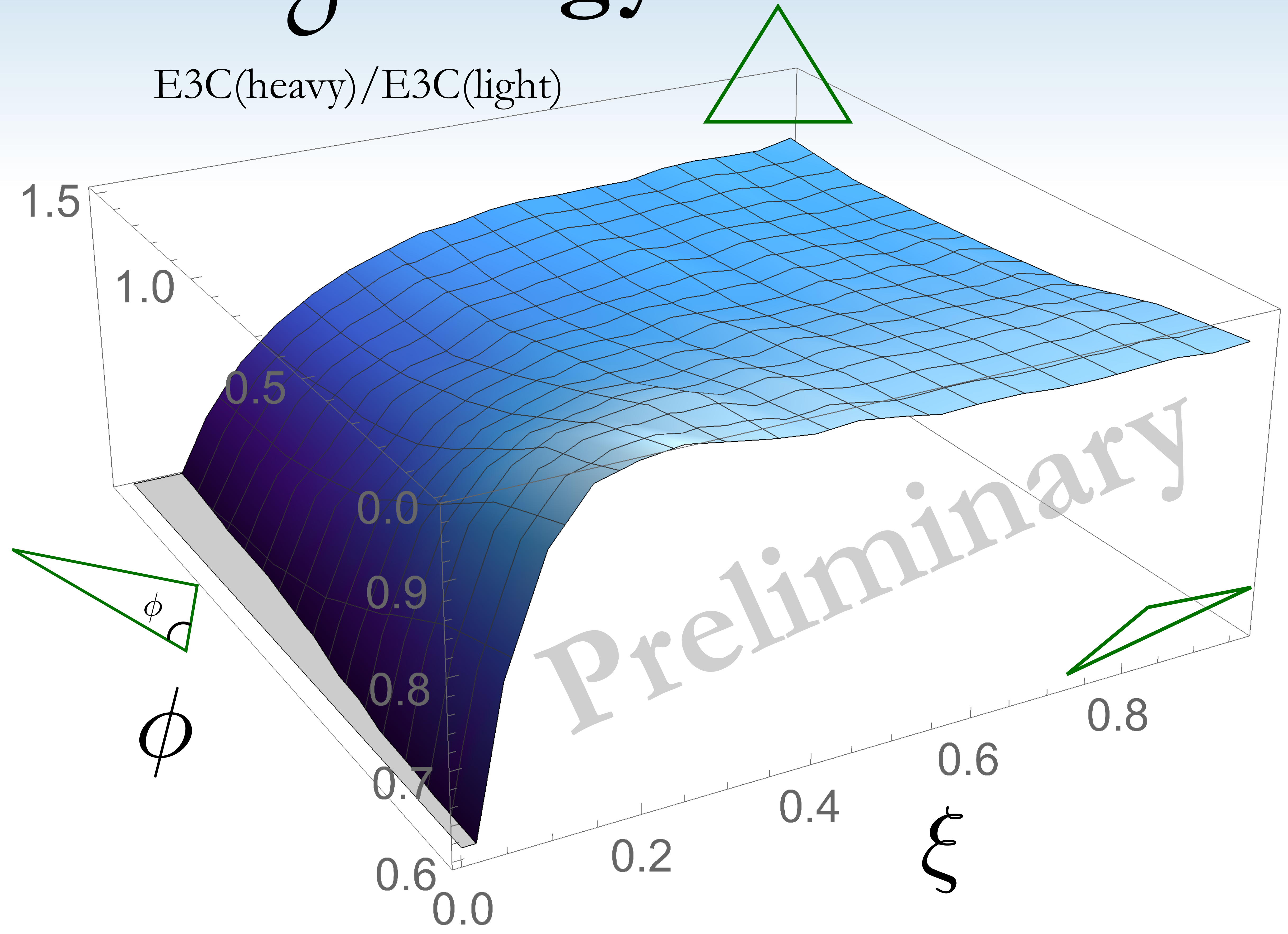


- One observes clear turning around heavy quark scale (both from Pythia and the fixed order calculation).
- Suppression at small angle can be interpreted as a direct signature of the dead-cone

KL, Meçaj, Moult, In Progress

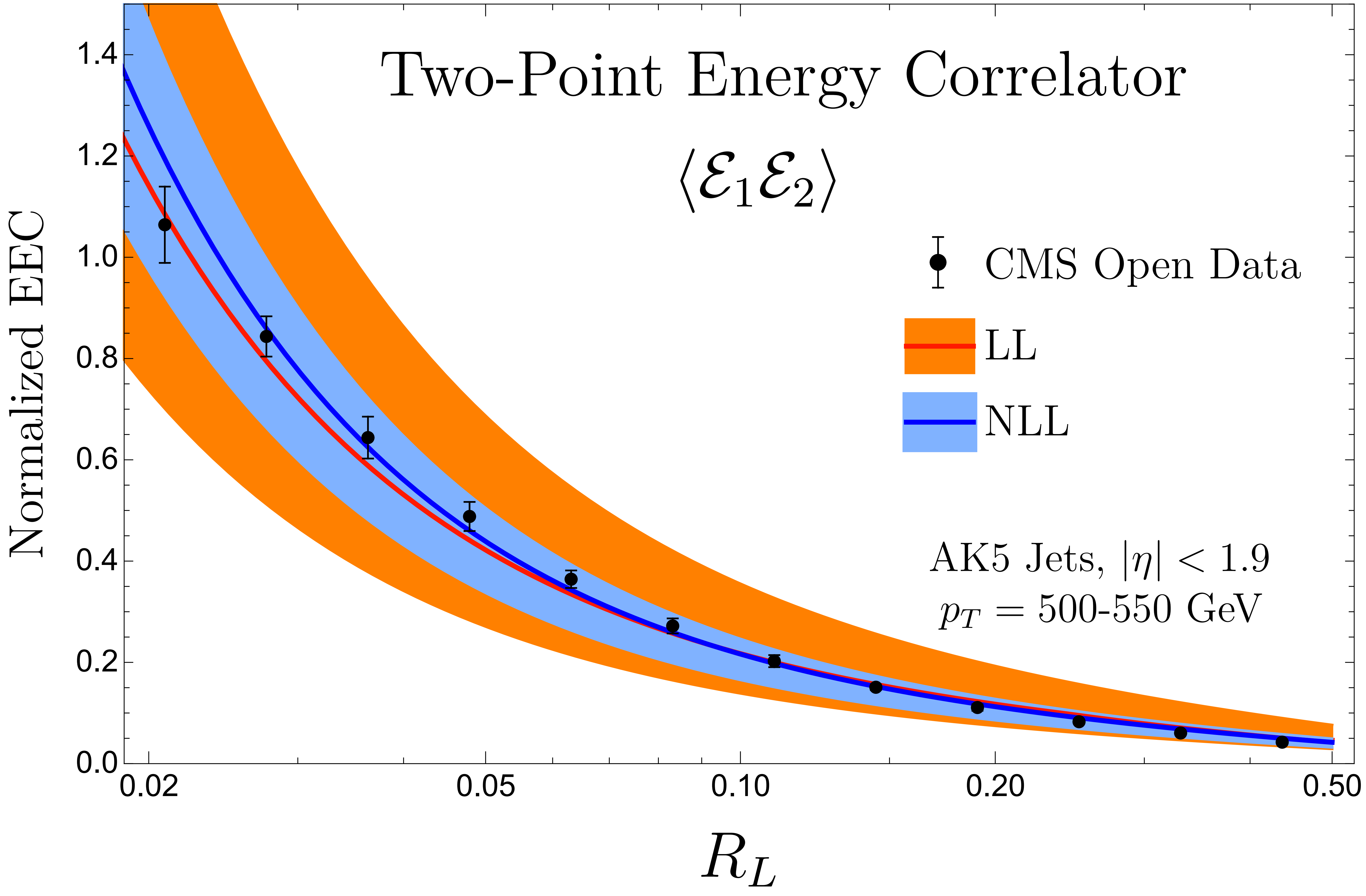
beautiful and *charming* energy correlators

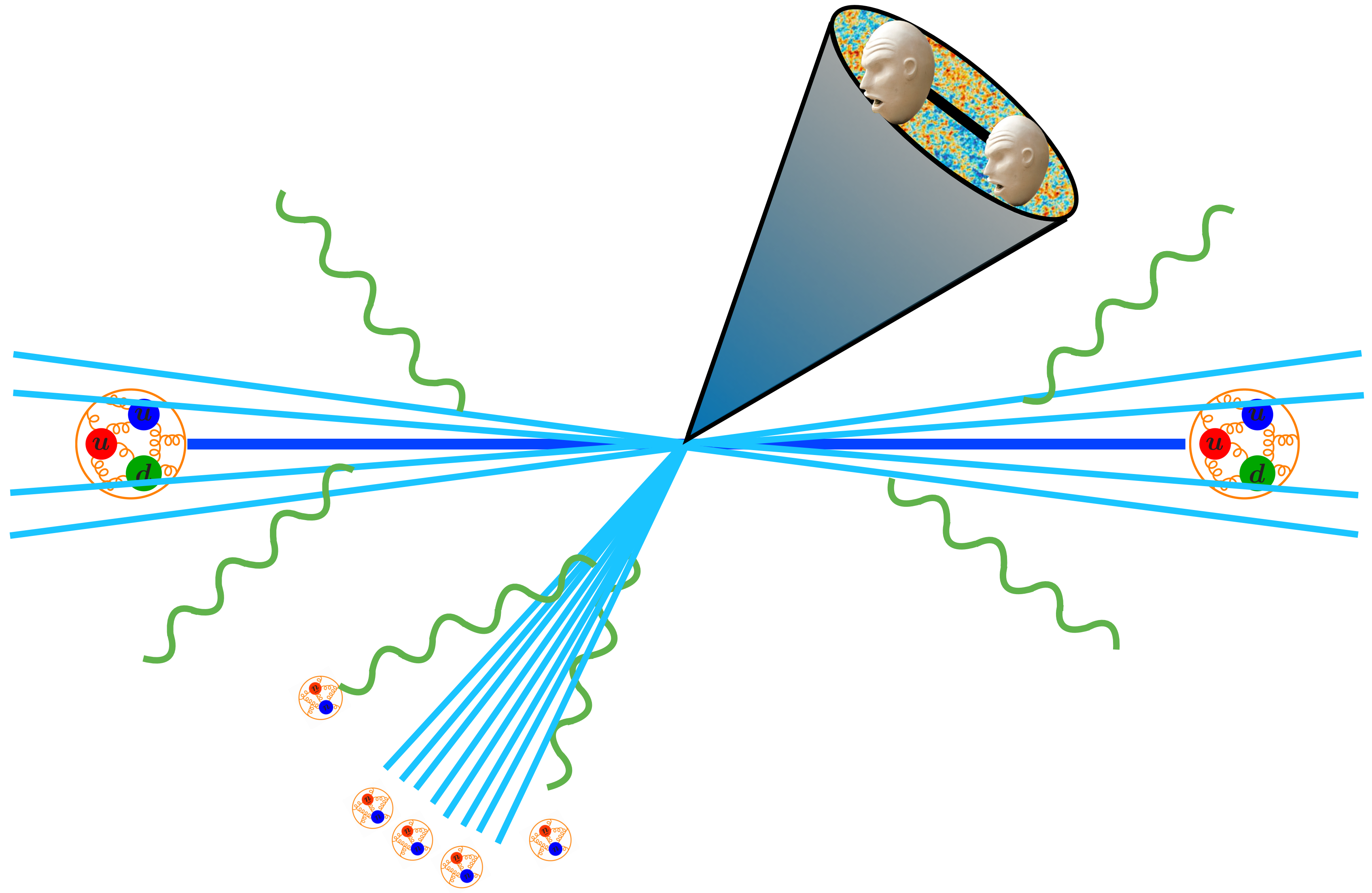




- One observes clear turning around heavy quark scale (both from Pythia and the fixed order calculation).
- Suppression at small angle can be interpreted as a direct signature of the dead-cone

Venturing into precision calculations





Outlook

Czakon, Generet, Mitov, Poncelet `21

Partial results computed

NNLO semi-inclusive hard function ▲

$$\frac{d\sigma^{pp\to \mathrm{jet}(\mathrm{N-proj})X}}{dp_T d\eta dR_L} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^{c} \otimes \mathcal{G}_c^{\mathrm{N-proj}}(R_L)$$

NNLO PDFs ✓

NNPDFs, CTEQ, …

$$\mathcal{G}_c^{\mathrm{N-proj}}(z, R_L, p_T R, \mu) = \sum_j \int_0^1 dx x^N \mathcal{J}_{ij}(z, x, p_T R, \mu)\, J_{\mathrm{EEC}}^{\mathrm{N-proj}}(R_L, x, \mu)$$

Matching coefficients ▲

Partial results

KL, Liu, Moult, In progress

Projected ENC jet function ✓✓

Available even for the track case!

Chen, Moult, Zhang, Zhu, `20

Li, Moult, van Velzen, Waalewijn, Zhu, `21

Jaarsma, Li, Moult, Waalewijn, Zhu, `22

- Unprecedented precision calculation of jet substructure on the horizon!

Outlook

Czakon, Generet, Mitov, Poncelet `21

Partial results computed

NNLO semi-inclusive hard function

$$\frac{d\sigma^{pp\to \mathrm{jet}(\mathrm{N}-\mathrm{proj})X}}{dp_T\, d\eta\, dR_L} = \sum f \otimes f \otimes H^c \otimes \mathcal{G}^{\mathrm{N}-\mathrm{proj}}_c(R_L \ldots)$$

$$\mathcal{G}_c^{\mathrm{N}-\mathrm{proj}}(z, R_L, p_T R, \mu) = \ldots$$

Energy correlators at the **EIC**?

- Universal hadronization effects at the perturbative region,
- Polarized beams,
- Tracking information,
- Heavy flavor jets, etc etc

ion calculation of

e horizon!

n, Moult, Zhang, Zhu, `20

elzen, Waalewijn, Zhu, `21

Moult, Waalewijn, Zhu, `22