TMD Collaboration

UCLA Mani L. Bhaumik Institute for Theoretical Physics

# The Gluon Sivers Asymmetry in Dijet Production at the EIC


John Terry

University of California, Los Angeles → Los Alamos National Lab


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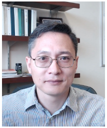
M. Alrashed

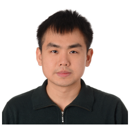
M. Echevarria

Z.B. Kang



J. Reiten

D.Y. Shao



# Overview (How can we use heavy flavor jets to probe the proton spin structure?)

## The Sivers asymmetry before the EIC

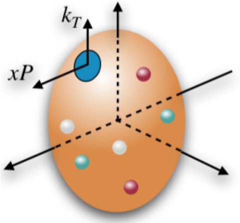
- What is the Sivers function and why is it important?
- What can current data tell us about this Sivers function?

## The gluon Sivers asymmetry at the EIC

- What unique perspective can the EIC offer us in terms of understanding the gluon Sivers function?
- Which processes are optimal for measuring the gluon Sivers function at the EIC?
- How can we generate a factorization and resummation formalism for generating predictions at the EIC.

# The Sivers asymmetry before the EIC

# The Sivers function




*Decomposition of the quark correlation function*

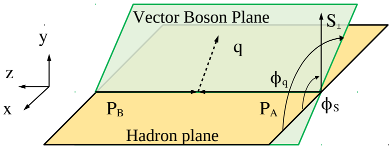
*TMD PDFs*

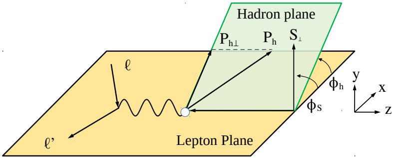
Quark Pol

| Proton Pol: $\Phi$ | U | L | T |
|---|---|---|---|
| U | $f$ | | $h_1^\perp$ |
| L | | $g_1$ | $h_{1L}^\perp$ |
| T | $f_{1T}^\perp$ | $g_{1T}$ | $h_1 \; h_{1T}^\perp$ |

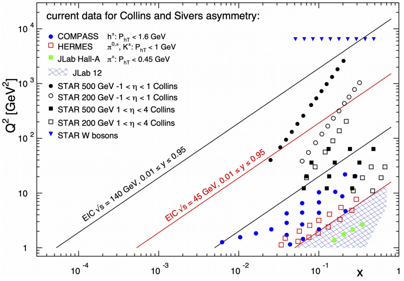
$$\Phi_{jj'}(x,\boldsymbol{k}_T,\boldsymbol{S};\mu,\nu) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi} \left\langle P,\boldsymbol{S} \left| \bar{\psi}_{j'}(\xi)\, \mathcal{U}^{\bar{n}}_{(\xi^-,-\infty;\boldsymbol{\xi}_T)}\, \mathcal{U}^{T}_{(\boldsymbol{\xi}_T,\boldsymbol{0};-\infty)}\, \mathcal{U}^{\bar{n}}_{(-\infty,0,\boldsymbol{0}_T)}\, \psi_j(0) \right| P,\boldsymbol{S} \right\rangle \Big|_{\xi^+=0}$$

$$= \left[ f(x,\boldsymbol{k}_T,\boldsymbol{S};\mu,\nu) - \frac{\epsilon^T_{\mu\nu} k_T^\mu S_T^\nu}{M} f_{1T}^\perp(x,\boldsymbol{k}_T,\boldsymbol{S};\mu,\nu) \right] \frac{\not{n}_{j'j}}{2} + \ldots$$



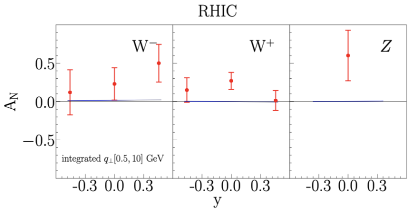



# Our recent global extraction of the Sivers function




| Collab | Ref | Process | $Q_{\rm avg}$ | $N_{\rm data}$ | $\chi^2/N_{\rm data}$ |
|---|---|---|---|---|---|
| COMPASS | [44] | $ld \to lK^0X$ | 2.52 | 7 | 0.770 |
| | | $ld \to lK^-X$ | 2.80 | 11 | 1.325 |
| | | $ld \to lK^+X$ | 1.73 | 13 | 0.749 |
| | | $ld \to l\pi^-X$ | 2.50 | 11 | 0.719 |
| | | $ld \to l\pi^+X$ | 1.69 | 12 | 0.578 |
| | [43] | $lp \to lh^-X$ | 4.02 | 31 | 1.055 |
| | | $lp \to lh^+X$ | 3.93 | 34 | 0.898 |
| | [46] | $\pi^-p \to \gamma^*X$ | 5.34 | 15 | 0.658 |
| HERMES | [41] | $lp \to lK^-X$ | 1.70 | 14 | 0.376 |
| | | $lp \to lK^+X$ | 1.73 | 14 | 1.339 |
| | | $lp \to l\pi^0X$ | 1.76 | 13 | 0.997 |
| | | $lp \to l(\pi^+ - \pi^-)X$ | 1.73 | 15 | 1.252 |
| | | $lp \to l\pi^-X$ | 1.67 | 14 | 1.498 |
| | | $lp \to l\pi^+X$ | 1.69 | 14 | 1.697 |
| JLAB | [45] | $lN \to l\pi^+X$ | 1.41 | 4 | 0.508 |
| | | $lN \to l\pi^-X$ | 1.69 | 4 | 1.048 |
| RHIC | [47] | $pp \to W^+X$ | $M_W$ | 8 | 2.189 |
| | | $pp \to W^-X$ | $M_W$ | 8 | 1.684 |
| | | $pp \to Z^0X$ | $M_Z$ | 1 | 3.270 |
| Total | | | | 226 | 0.989 |

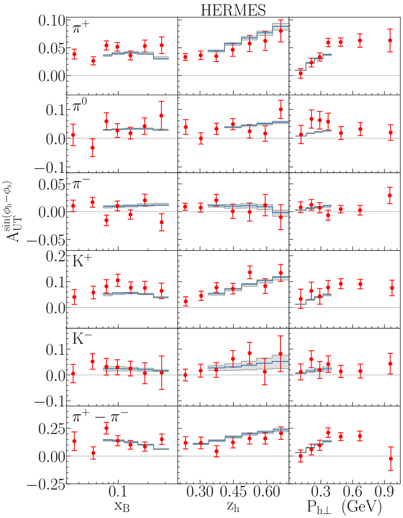
*Old RHIC analysis is inconsistent with our result*

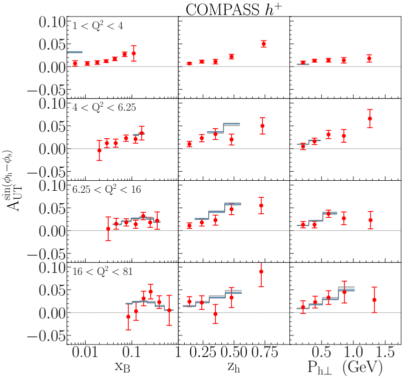



$\chi^2/d.o.f. \sim 1.0$, *12 parameters and 226 data points [2]. Issues with the RHIC data.*

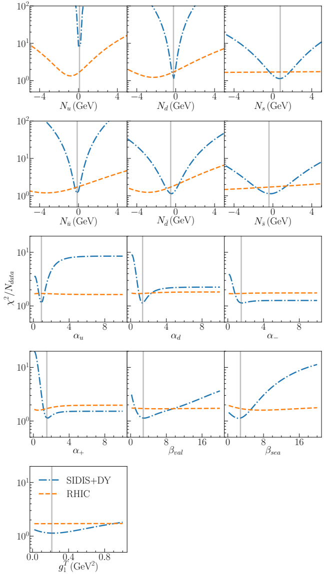
[2] Echevarria, Kang, JT 19

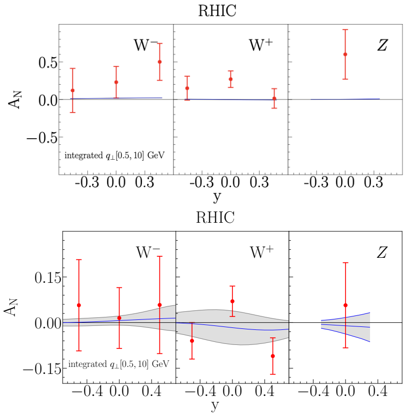
# Our recent global extraction of the Sivers function continued



# Re-analysis of the RHIC data is completely consistent with our result

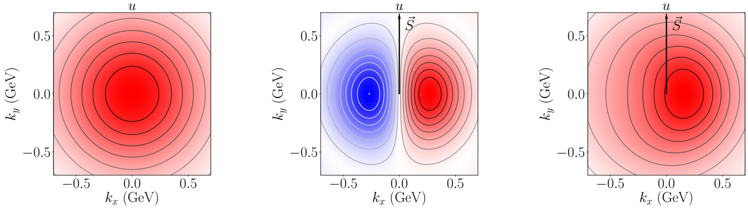


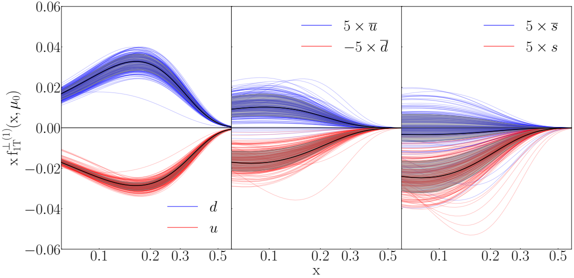




*Re-analysis of the RHIC data is completely consistent with our result.*

# Results for the extraction of the Sivers function $x = 0.2, Q = 2$ GeV

$f_{u/p}(x, k_T)$ $\qquad -\frac{\varepsilon^T_{\mu\nu} k_T^\mu S_T^\nu}{M} f^\perp_{1T,u/p}(x, k_T)$ $\qquad \Phi_{u/p}(x, k_T)$



## The gluon Sivers asymmetry at the EIC

# Process dependence for the gluon Sivers function

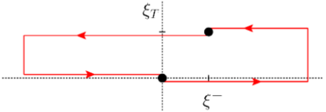
*Two types of gluon Sivers functions*. See [3] for instance.

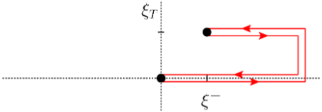
Dipole (d-type)

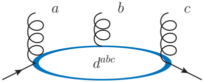
Measurable in $pp^{\uparrow} \to \gamma$ Jet ($qg \to \gamma q$)

Weizsäcker-Williams (f-type)

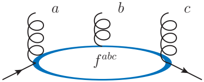
Measurable in $ep^{\uparrow} \to Q\bar{Q}$ ($\gamma g \to Q\bar{Q}$)



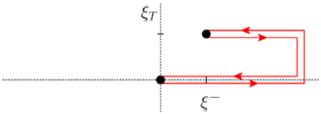



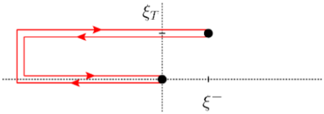

$\neq$




Asymptotically different as small $x$ [4].

*Modified universality for the gluon Sivers* [5]




$$f_{1T}^{\perp[ep^{\uparrow} \to e' Q\bar{Q}X]}(x, p_{\perp}) = -f_{1T}^{\perp[pp^{\uparrow} \to \gamma\gamma X]}(x, p_{\perp})$$


[3] Dominguez, Xiao, Yuan 11; Buffing, Mukherjee, Mulders 13

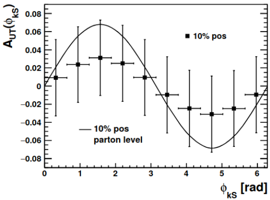
[4] Boer, Echevarria, Mulders, Zhou 15

[5] Boer, Mulders, Pisano, Zhou 16

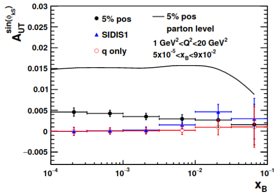
# Gluon Sivers Function at the EIC

Feasibility of measuring the gluon Sivers asymmetry was examined in [6] using a Pythia reweighting analysis $\mathcal{L}_{int} = 10\, fb^{-1}$.

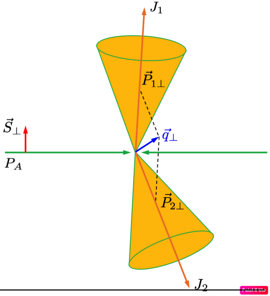
**Open charm ($D \to K\pi$ pair)**

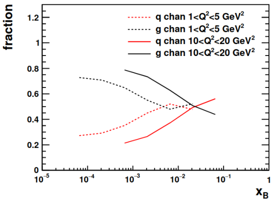



**Di-hadron production**




**Dijet ($\gamma q \to qg$, $\gamma q \to qg$)**

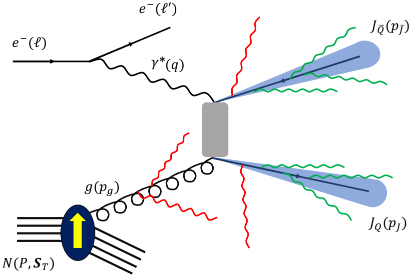




[6] Zheng, Aschenauer, Lee, Xiao, Yin 18

# Imbalance for heavy flavor dijet production [7]




*Mode Analysis*

$(-, +, \perp)$

- Hard: $p_h \sim p_{jT}(1, 1, 1)\,,$
- Jet$_Q$: $p_{J\,Q} \sim p_{jT}(R^2, 1, R)\,,$
- Collinear: $p_c \sim p_{jT}(\lambda^2, 1, \lambda)\,,$
- Soft: $p_s \sim p_{jT}(\lambda, \lambda, \lambda)\,,$
- C-Soft$_Q$: $p_{cs\,Q} \sim q_\perp(R^2, 1, R)\,,$

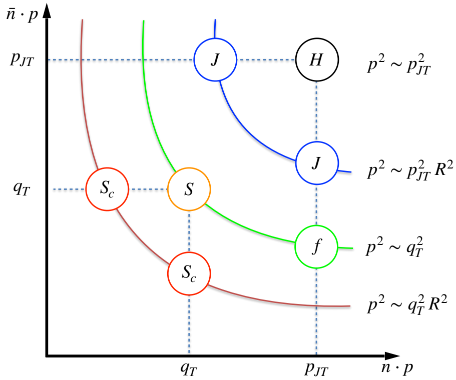
*Also need to consider the heavy-flavor mass of the quarks*

We assume the scale heirarchy

$q_T R \ll q_T \lesssim m_Q \lesssim p_T R \ll p_T$

[7] Kang, Reiten, Shao, JT 20

# Factorization for HF Dijet Production



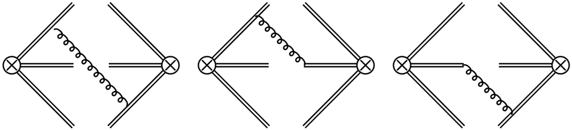

$$\frac{d\sigma^{UU}}{dQ^2 dy d^2\boldsymbol{p}_T dy_J d^2\boldsymbol{q}_T} = H(Q, y, p_T, y_J, \mu) \int \frac{d^2 b}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T} S(\boldsymbol{b}, \mu, \nu)\, f_{g/N}\left(x, b, \mu, \zeta/\nu^2\right)$$
$$\times J_Q(p_T R, m_Q, \mu)\, S_Q^c(\boldsymbol{b}, R, m_Q, \mu)\, J_{\bar{Q}}(p_T R, m_Q, \mu)\, S_{\bar{Q}}^c(\boldsymbol{b}, R, m_Q, \mu)\,,$$

$H$ and $S$ are the same as light flavor dijet production since $m_Q \ll p_T$.

The functions $S_Q^c$ and $J_Q$ are sensitive to the mass.

# Global soft function

*NLO expression can be obtained by explicitly calculating the graphs [8]*



$$S^{\rm NLO}(\boldsymbol{b},\mu,\nu) = -\frac{C_A}{2}\,\mathcal{I}_{BJ} - \frac{C_A}{2}\,\mathcal{I}_{B\bar{J}} + \left(\frac{C_A}{2} - C_F\right)\,\mathcal{I}_{J\bar{J}}$$

$$\mathcal{I}_{ij} = \frac{\alpha_s \mu^{2\epsilon}\pi^{\epsilon}e^{\epsilon\gamma_E}}{\pi^2}\int d^dk\,\delta^+(k^2)e^{-i\,k\cdot b}\frac{n_i\cdot n_j}{(n_i\cdot k)(n_j\cdot k)}\frac{\nu^\eta}{|k^+ - k^-|^\eta}$$

*Can obtain the soft anomalous dimensions*

$$\Gamma^s(\alpha_s) = 2C_F\,\gamma^{\rm cusp}(\alpha_s)\ln\frac{\mu^2}{\mu_b^2} - C_A\,\gamma^{\rm cusp}(\alpha_s)\ln\frac{\nu^2}{\mu^2} + \gamma^s(\alpha_s)$$

$$\gamma_\nu^s(\alpha_s) = -\frac{\alpha_s}{\pi}C_A\ln\frac{\mu^2}{\mu_b^2} + \mathcal{O}(\alpha_s^2)\,,$$

$$\gamma_0^s = 8C_F\ln\left(4c_{bJ}^2\right) + 4(C_A - 2C_F)\ln\frac{\hat{s}}{p_T^2} + 4\,C_A\ln\frac{\hat{s}}{Q^2}$$

$$\frac{\partial}{\partial ln\mu}S(\boldsymbol{b},\mu,\nu) = \Gamma_S^\mu(\mu)\,S(\boldsymbol{b},\mu,\nu)$$

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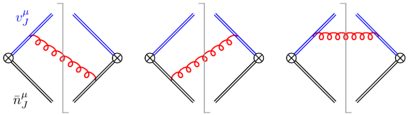
[8] Hornig, Kang, Makris, Mehen 17; del Castillo, Echevarria, Makris, Scimemi 20

# Collinear-soft function

*Blue lines represent heavy Wilson lines*




$$v_J^\mu = \frac{\omega_J}{m_Q}\frac{n_J^\mu}{2} + \frac{m_Q}{\omega_J}\frac{\bar{n}_J^\mu}{2}\,, \qquad \text{with } v_J^2 = 1$$

*Recall the power counting of the collinear-soft modes* $p_{\rm cs} \sim q_\perp(1, R^2, R)$.

$$w_{\alpha\beta} = \frac{\alpha_s \mu^{2\epsilon}\pi^\epsilon e^{\epsilon\gamma_E}}{2\pi^2}\int d^dk\,\delta^+(k^2)e^{-i\,\bar{n}_J\cdot k\,n_J\cdot b/2}\frac{\alpha\cdot\beta}{(\alpha\cdot k)\,(\beta\cdot k)}\,\theta\left[\frac{n_J\cdot k}{\bar{n}_J\cdot k} - \left(\frac{R}{2\cosh y_J}\right)^2\right]$$

*We derive the collinear soft anomalous dimensions*

$$\Gamma^{cs_Q}(\alpha_s) = C_F\gamma^{\rm cusp}(\alpha_s)\ln\frac{R^2\mu_b^2}{\mu^2} + \gamma^{cs_Q}(\alpha_s)$$

$$\gamma_0^{cs_Q} = -4C_F\left[2\ln\left(-2ic_{bJ}\right) - \frac{m_Q^2}{m_Q^2+p_T^2R^2} - \ln\frac{m_Q^2+p_T^2R^2}{p_T^2R^2}\right]$$

$c_{bJ} = \cos\left(\phi_b - \phi_J\right)$ *gives rise to azimuthal asymmetries [9]. We average over the* $\phi_b$ *angle to generate the numerics.*

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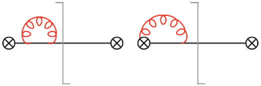
[9] Hatta, Xiao, Yuan, Zhou 20

# Mass dependent jet function

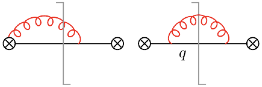
*Virtual Graphs*



Does not depend on the jet radius

$$J_{\mathcal{Q}}^{\mathrm{NLO,V}} = \frac{\alpha_s}{4\pi} C_F \left[ \frac{2}{\epsilon^2} + \frac{1}{\epsilon}\left(1 + 2\ln\frac{\mu^2}{m_{\mathcal{Q}}^2}\right) + \left(1 + \ln\frac{\mu^2}{m_{\mathcal{Q}}^2}\right)\ln\frac{\mu^2}{m_{\mathcal{Q}}^2} + 4 + \frac{\pi^2}{6} \right]$$

*Real Graphs*



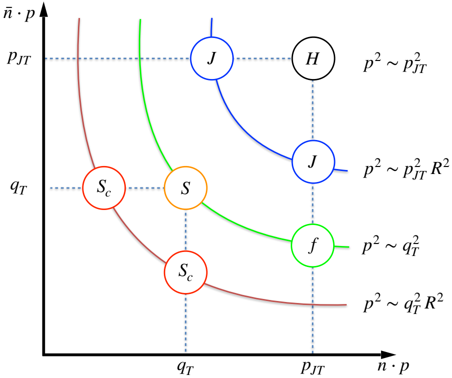

$$\Theta_{\text{anti-}k_T} = \theta\left[ \left( \frac{q^-(\omega_J - q^-)}{\omega_J} \right)^2 \left( \frac{R}{2\cosh y_J} \right)^2 - \boldsymbol{q}_\perp^2 \right]$$

$$J_{\mathcal{Q}}^{\mathrm{NLO,R}}(p_T R, m_{\mathcal{Q}}, \epsilon) = \frac{\alpha_s C_F e^{\epsilon\gamma_E}\mu^{2\epsilon}}{2\pi\Gamma(1-\epsilon)} \int \frac{dq^-}{\omega_J}\frac{d\boldsymbol{q}_\perp^2}{\boldsymbol{q}_\perp^{2\epsilon}} \left[ \frac{q^-}{\omega_J - q^-}\frac{2\boldsymbol{q}_\perp^2\omega_J^4}{[\boldsymbol{q}_\perp^2\omega_J^2 + m_{\mathcal{Q}}^2(\omega_J - q^-)^2]^2} \right.$$

$$\left. + (1-\epsilon)\frac{\omega_J(\omega_J - q^-)}{\boldsymbol{q}_\perp^2\omega_J^2 + m_{\mathcal{Q}}^2(\omega_J - q^-)^2} \right] \theta(\omega_J - q^-)\Theta_{\text{anti-}k_T}$$

$$= \frac{\alpha_s}{4\pi} C_F \left[ -2\ln\left( \frac{m_{\mathcal{Q}}^2 + p_T^2 R^2}{m_{\mathcal{Q}}^2} \right) + 2 - \frac{2m_{\mathcal{Q}}^2}{m_{\mathcal{Q}}^2 + p_T^2 R^2} \right] \frac{1}{\epsilon} + J_{\mathcal{Q}}^{\mathrm{R,fin}},$$

# Resummation of large logarithms




*Perturbative evolution of the cross section: account for broadening of distributions from perturbative radiation*

$$S_{\text{pert}} = \exp\left[\int_{\mu_{\text{TMD}}}^{\mu_h} \frac{d\mu}{\mu}\left(\Gamma^s + \Gamma^{fg}\right) + 2\int_{\mu_j}^{\mu_h} \frac{d\mu}{\mu}\Gamma^j \mathcal{Q} + \int_{\mu_{cs}}^{\mu_h} \frac{d\mu}{\mu}\left(\bar{\Gamma}^{cs}\mathcal{Q} + \bar{\Gamma}^{cs}\bar{\mathcal{Q}}\right)\right]$$

# Unpolarized Phenomenology at NLL

*Mass dependence enters in the anomalous dimensions*

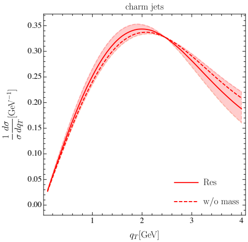
$$\mu_j = p_T R, \quad \mu_{cs} = \mu_{b_*} R\,.$$

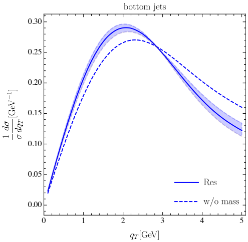
*Non-perturbative parameterization [10]*

$$S_{\rm NP}\left(b, Q_0, n\cdot p_g\right) = g_1 b^2 + \frac{g_2}{2}\frac{C_A}{C_F}\ln\frac{n\cdot p_g}{Q_0}\ln\frac{b}{b_*}\,.$$

$$\text{charm}: \; 5\,\text{GeV} < p_T < 10\,\text{GeV}, \qquad \text{bottom}: \qquad 10\,\text{GeV} < p_T < 15\,\text{GeV},$$

$$|y_J| < 4.5, \text{ see [11]}$$

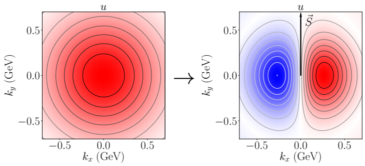



[10] Sun, Isaacson, Yuan, Yuan 14
[11] Aschenauer, Baker et al 14

# Spin Dependent Resummation

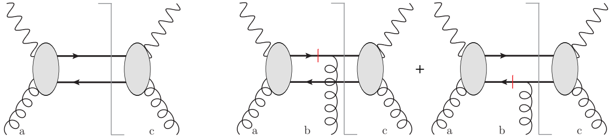
*Replace TMDPDF with the gluon Sivers function*




$$f\left(x, k_T, \mu, \zeta\right) \to \frac{1}{M} \varepsilon_{\alpha\beta}\, S_T^\alpha\, k_T^\beta\, f_{1T}^{\perp, f}\left(x, k_T, \mu, \zeta\right).$$

$$\Gamma^{f\perp} = \Gamma^f \qquad \gamma_\nu^{f\perp} = \gamma_\nu^f$$

*Need to consider additional gauge link attachment for the spin dependent cross section*




$$= C_u\, \delta^{ac}\, h(Q, y, p_T, y_J, \mu_h) \qquad = \left(C_1^\perp + C_2^\perp\right)\left(-i f^{abc}\right) h(Q, y, p_T, y_J, \mu_h)$$

$$C_u = C_1^\perp + C_2^\perp \to \Gamma^{h\perp} = \Gamma^h$$

# Phenomenology

*Non-perturbative collinear parameterization*

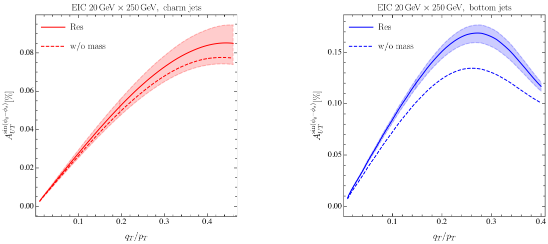
$$f_{1T,g/N}^{\perp f}(x,\mu) = N_g \frac{4\rho\sqrt{2e\rho(1-\rho)g_1}}{M_{\text{proton}}} x^{\alpha_g}(1-x)^{\beta_g} \frac{(\alpha_g+\beta_g)^{\alpha_g+\beta_g}}{\alpha_g^{\alpha_g}\beta_g^{\beta_g}} f_{g/N}(x,\mu)\,,$$

*Parameters from* [12]

$$N_g = 0.65, \quad \alpha_g = 2.8, \quad \beta_g = 2.8, \quad \rho = 0.5, \quad M_{\text{proton}} = 1\,\text{GeV}\,,$$

*For the TMD*

$$S_{\text{NP}}^{\perp}(b, Q_0, n\cdot p_g) = g_1\rho\, b^2 + \frac{g_2}{2}\frac{C_A}{C_F}\ln\frac{n\cdot p_g}{Q_0}\ln\frac{b}{b_*}\,,$$




[12] D'Alesio, Murgia, Pisano 15

# Summary and Outlook

- The non-perturbative structure of the quark Sivers function has been largely explored by experimental data in standard processes.
- Using SCET, we have generated the factorization and resummation formalism for heavy-flavor dijet production at the EIC to probe the gluon Sivers function.
- Once data is available, we can perform extractions of the non-perturbative physics for the gluon-Sivers function from heavy-flavor dijet data.

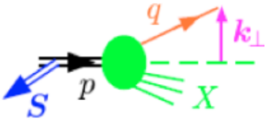
Thank you to the audience and the organizers!

*TMD PDFs*

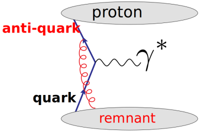
Quark Pol

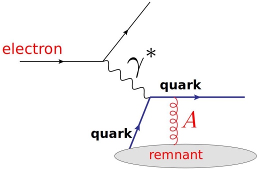
Proton Pol

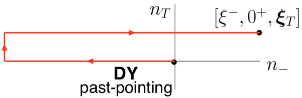
| $\Phi$ | U | L | T |
|---|---|---|---|
| U | $f$ | | $h_1^\perp$ |
| L | | $g_1$ | $h_{1L}^\perp$ |
| T | $f_{1T}^\perp$ | $g_{1T}$ | $h_1$ $h_{1T}^\perp$ |

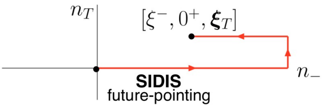


# Signs change of the Sivers function



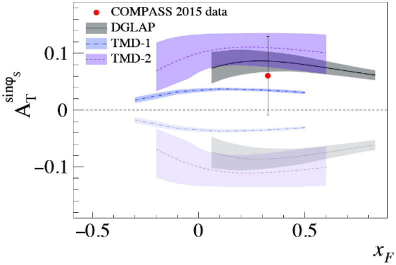




Repulsion between anti-quark and remnant in di-hadron collisions. Attraction between quark and remnant in SIDIS. [1]

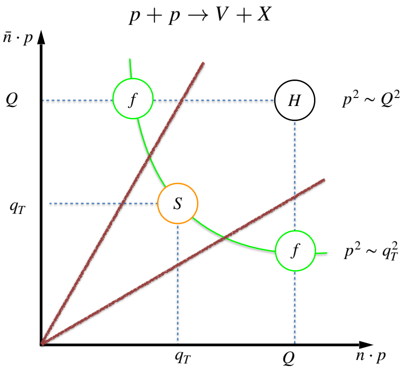
$$f_{1T,q/p}^{\perp,\mathrm{DY}} = -f_{1T,q/p}^{\perp,\mathrm{SIDIS}}$$




[1] Collins 02; Boer-Mulders-Pijlman 03; Collins-Metz 04; Kang-Qiu 09

# Factorization Theorems for standard processes

*Visual representation of the factorization*



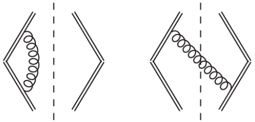

*Mode Analysis*

$(-,+,\perp)$

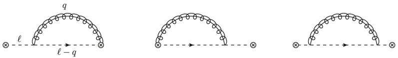
- Hard: $p_h \sim Q(1,1,1)\,,$
- Collinear: $p_c \sim Q(\lambda^2,1,\lambda)\,,$
- Anti-Collinear: $p_{\bar{c}} \sim Q(1,\lambda^2,\lambda)\,,$
- Soft: $p_s \sim Q(\lambda,\lambda,\lambda)\,.$

$$\lambda = qT/Q$$

Soft



Collinear




*Factorization Theorem*

$$\frac{d\sigma}{d\mathcal{PS}d^2q_\perp} = \sigma_0\, H(Q,\mu) \int \prod_{i=1,3} dk_i f_1(x_1,k_{1\perp},\mu,\nu)\, f_2(x_2,k_{2\perp},\mu,\nu)\, S(k_{3\perp},\mu,\nu)$$

# Renormalization Group Consistency

*Hard anomalous dimension: Can obtain from general structure*

$$\Gamma^h(\alpha_s) = C_A \gamma^{\text{cusp}}(\alpha_s) \ln\left(\frac{\hat{u}\,\hat{t}}{\hat{s}\,\mu^2}\right) - 2C_F \gamma^{\text{cusp}}(\alpha_s) \ln\left(\frac{\mu^2}{\hat{s}}\right) + 4\gamma^q(\alpha_s) + 2\gamma^g(\alpha_s)$$

*TMD anomalous dimension: Obtainable through explicit one loop calculation*

$$\Gamma^{f_g}(\alpha_s) = C_A \gamma^{\text{cusp}}(\alpha_s) \ln \frac{\nu^2}{(n \cdot p_g)^2} - 2\gamma^{f_g}(\alpha_s)\,, \quad \gamma_\nu^{f_g}(\alpha_s) = \frac{\alpha_s}{\pi} C_A \ln \frac{\mu^2}{\mu_b^2} + \mathcal{O}(\alpha_s^2)$$

*Cross section should not depend on choice of $\mu$, $\nu$.*

$$\frac{d\sigma^{UU}}{dQ^2 dy d^2\boldsymbol{p}_T dy_J d^2\boldsymbol{q}_T} = H(Q, y, p_T, y_J, \mu) \int \frac{d^2 b}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T} S(\boldsymbol{b}, \mu, \nu)\, f_{g/N}\left(x, b, \mu, \zeta/\nu^2\right)$$
$$\times\, J_{\mathcal{Q}}(p_T R, m_{\mathcal{Q}}, \mu)\, S^c_{\mathcal{Q}}(\boldsymbol{b}, R, m_{\mathcal{Q}}, \mu)\, J_{\bar{\mathcal{Q}}}(p_T R, m_{\mathcal{Q}}, \mu)\, S^c_{\bar{\mathcal{Q}}}(\boldsymbol{b}, R, m_{\mathcal{Q}}, \mu)\,,$$

*Definition of anomalous dimensions*

$$\frac{\partial}{\partial \ln \mu} F = \Gamma^\mu F \qquad \frac{\partial}{\partial \ln \nu} F = \gamma^\nu F$$

*RG consistency requires*

$$\Gamma^h + \Gamma^s + \Gamma^{f_g} + 2\Gamma^{j_{\mathcal{Q}}} + \Gamma^{cs_{\mathcal{Q}}} + \Gamma^{cs_{\bar{\mathcal{Q}}}} = 0 \qquad \gamma^{f_g}_{\nu,0} + \gamma^s_{\nu,0} = 0$$