Probing the Hydrodynamics of Strongly Coupled QFTs

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arXiv:2206.04765 with Romatschke

arXiv:2104.02024 with Cohen, Lamm, Yamauchi

arXiv:2111.08158

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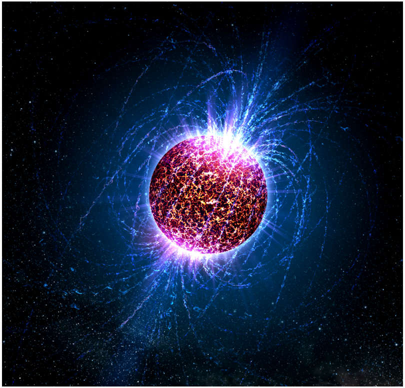
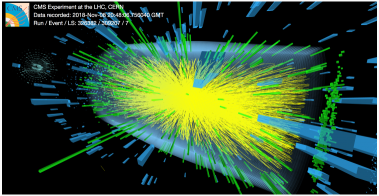
The universe has not yet equilibrated

Heat death of the universe is not expected for $\sim 10^{100}$ more years.





For now: need nonperturbative methods for nonequilibrium physics.

Hydrodynamics

Finite-temperature quantum matter (hard to simulate!), when "zoomed out", is described by *classical* hydrodynamics ("easy").

Navier-Stokes:

$$\rho \frac{\mathrm{d}u_i}{\mathrm{d}t} + \partial_i p = \eta \left(\frac{1}{3} \partial_i \partial_j u_j + \partial_j^2 u_i \right) + \zeta \cdots$$

More systematic (and relativistic): $\partial_\mu T^{\mu\nu} = 0$, expand $T^{\mu\nu}$ in ∇:

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} \underbrace{- 2\eta \nabla^{\langle \mu} u^{\nu \rangle} - \zeta \Delta^{\mu\nu} \nabla^{\perp}_{\lambda} u^{\lambda}}_{T^{\mu\nu}_{(1)}} + \underbrace{\kappa \left[R^{\langle ij \rangle} - 2 R^{t \langle ij \rangle t} \right] + \cdots}_{T^{\mu\nu}_{(2)}} + \cdots$$

Gradient expansion: long distances, long times

Transport coefficients: LECs through which quantum effects can appear

Sound waves

$$\int dx\, \sin kx \,\langle T^{00}(x,t)\, T^{00}(0,0)\rangle \sim \exp\left[i c_s k t - \left(\frac{\zeta + \frac{2(d-1)}{d}\eta}{\epsilon + P} \right) k^2 t \right]$$

Shear waves

$$\int dx\, \sin kx \,\langle T^{01}(x,t)\, T^{01}(0,0)\rangle \sim e^{-\frac{\eta}{\epsilon+P} k^2 t}$$

"Shear channel"

$$\langle T^{12}(\omega,k)\, T^{12}(\omega,k)\rangle = P - i\eta\omega + O(\omega^2) + O(k)$$

Air

Paradox: $\eta_{\mathrm{atm}} \sim 10^{-5}\,\frac{\mathrm{kg}}{\mathrm{m\,s}}$ results in a sound attenuation time on the order of days, but everyday experience says it should be seconds.

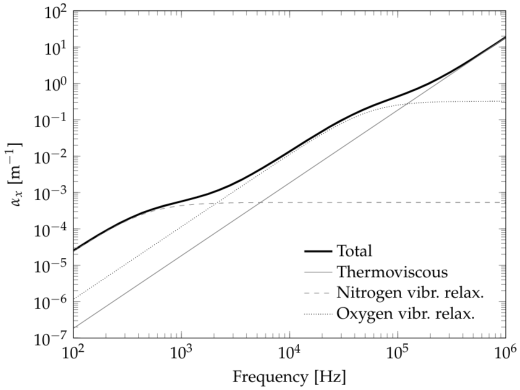


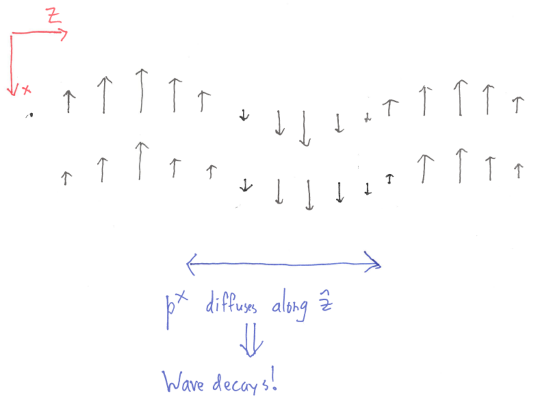
Figure 2.7: Absorption coefficient and its composition for air at 293.15 K, atmospheric pressure and 70 % relative humidity. Rotational relaxation for nitrogen and oxygen is represented as bulk viscosity.

(Figure from E. M. Viggen's thesis.)

Shear viscosity

Two equivalent phrasings of what η represents:

Transport of x-momentum along $\hat{z}$ OR decay of shear waves



Shear waves in a free theory

Impart some momentum at $x = 0$. After time t, what is the amplitude of the shear wave?

$$C(t) \sim \int d^3v \underbrace{\rho(v)}_{\sim e^{-v^2}} \cos(k v_z t) \sim e^{-t^2}$$

This decay is *gaussian*, faster than any exponential. $\eta_{\mathrm{free}} = \infty$.

Physically: the transport of momentum (carried by individual particles) is entirely unobstructed.

Another example: in a rigid body, $\eta = \infty$. (Proof: pick up a pen.)

A nontrivial lower bound on transport?

In a free theory, particles are free, transport momentum efficiently. Sound waves decay super-exponentially.

In a rigid body, particles can't move at all. Phonons are free, and don't decay, so they transport momentum efficiently.

In a rigid body, phonons are efficient transporters *because* particles can't move.

Blocking one mechanism of momentum transport opens up another one (by allowing sound waves to propagate without decaying).

A more careful version of this argument is in Kovtun, Moore, Romatschke, "The Stickiness of Sound".

Status of the KSS conjecture

KSS Conjecture: $\frac{\eta}{s} \geq \frac{1}{4\pi}$ in "all" theories.

Counterexample from T. Cohen (`arXiv:0702136`) involves a large number of weakly interacting massive species.

In this family of models $\frac{\eta}{s}$ is made arbitrarily small, but shear waves still decay quickly.

Moral: bound the decay constant, not $\frac{\eta}{s}$!

See e.g. `arXiv:2111.08158` (Lawrence) and `arXiv:2005.06482` (Baggioli and Li)

Large-N_f expansions

Nonrelativistic fermions in three dimensions:

$$S = \int_0^\beta d\tau \int d^3\mathbf{x} \left[\sum_f \psi^\dagger_{s,f}(\mathbf{x}) \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) \psi_{s,f}(\mathbf{x}) + \sum_{f,f'} \frac{4\pi a_s}{mN} \psi^\dagger_{\uparrow,f}(\mathbf{x}) \psi^\dagger_{\downarrow,f'}(\mathbf{x}) \psi_{\downarrow,f}(\mathbf{x}) \psi_{\uparrow,f'}(\mathbf{x}) \right]$$

(The limit $a_s \to -\infty$ is the unitary Fermi gas.)

To expand in powers of N^{-1}, introduce an auxiliary field to make N a parameter.

$$S_{\text{eff}} = -N \left[\log\det \left(\partial_\tau - \sigma_z \frac{\nabla^2}{2m} - \sigma_z \mu + i\zeta^* \sigma_- - i\zeta\sigma_+ \right) - \int \frac{m\zeta\zeta^*}{4\pi a_s} \right]$$

Evaluate the path integral $Z = \int \mathcal{D}\zeta e^{-S_{\text{eff}}}$ with a saddle-point expansion.

Thermodynamic transport

Not all LECs in the hydrodynamic expansion are specific to out-of-equilibrium physics.

- Pressure
- Gravitational wave-to-matter coupling (κ)

These appear when the spacetime metric undergoes a time-independent perturbation.

Equivalently, these are detectable from fluctuations in thermodynamic equilibrium.

Gravitational wave-to-matter coupling of the unitary Fermi gas

$$T^{ij} \supset \kappa \left[R^{\langle ij \rangle} - 2R^{t\langle ij \rangle t} \right]$$

Thermodynamic transport can be seen from *Euclidean* correlators:

$$\kappa = \frac{\partial^2}{\partial k^2} \langle T^{12} T^{12} \rangle (\omega = 0, k) \Big|_{k=0}$$

For nonrelativistic fermions, the stress-energy tensor is:

$$T^{12} = \frac{1}{4m} \left[\partial_1 \Psi^\dagger \sigma_z \partial_2 \Psi + \partial_2 \Psi^\dagger \sigma_z \partial_1 \Psi - \partial_1 \partial_2 \Psi^\dagger \sigma_z \Psi - \Psi^\dagger \sigma_z \partial_1 \partial_2 \Psi \right] - \frac{is}{4m} \partial_k \Sigma_k$$

where $\Sigma_3 = \Psi \sigma_x (\partial_1 - i\partial_2) \Psi + \Psi^\dagger \sigma_x (\partial_1 + i\partial_2) \Psi^\dagger$

Evaluated at the saddle point:

$$C(k) = -\frac{2}{m^2} \int \frac{d^4 p}{(2\pi)^4} \frac{\mathbf{p}_1^2 \mathbf{p}_2^2 \left[(\epsilon_{\mathbf{p}} - \mu)(\epsilon_{\mathbf{k}+\mathbf{p}} - \mu) - \omega^2 - \Delta^2 \right]}{\left[(\epsilon_{\mathbf{p}} - \mu)^2 + \omega^2 + \Delta^2 \right] \left[(\epsilon_{\mathbf{p}+\mathbf{k}} - \mu)^2 + \omega^2 + \Delta^2 \right]} + \frac{s^2 \mathbf{k}^2}{2m^2} \int \frac{d^4 p}{(2\pi)^4} \mathbf{p}_1^2 \mathrm{tr}[\sigma_x G(\omega, \mathbf{p}) \sigma_x G(-\omega, -\mathbf{p})]$$

Integrating:

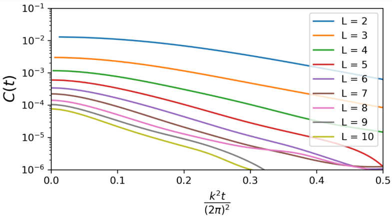
$$\lim_{a_s \to -\infty} = \frac{(2m\mu)^{\frac{3}{2}}}{3\pi^2 m} \frac{1}{\xi^{\frac{3}{2}}} \frac{N}{12} = \frac{n}{12m}$$

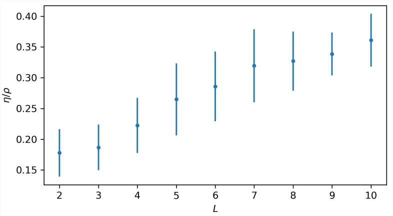
Viscosity from molecular dynamics

From an equilibrated MD simulation, compute a time-series:

$$f(T) = \sum_n p_x(T) \sin k_z x_z(T)$$

Plot the autocorrelation and fit the decay to $f(T) \sim e^{-\frac{\eta k^2}{\rho} t}$:





(Inter-particle potential $V(r) \sim e^{-2r}$)

Viscosity from a quantum computer

The best model of a cat is another cat. (*Norbert Wiener*)

Quantum simulations are *conceptually* simple.

- Physical Hilbert space $\mathcal{H}_P$; qubit Hilbert space $\mathcal{H}_C \approx \mathbb{C}^{2Q}$
- Define an injective map $\mathcal{H}_P \to \mathcal{H}_C$
- Decompose operators H and $\mathcal{O}$ in 1- and 2-Pauli terms
- Time-evolve, possibly with $H = H(t)$, and then measure!

Either measure T^{01} repeatedly and look at autocorrelation (best on large systems), or explicitly measure $\langle T^{01} T^{01} \rangle$ via linear response (better on small systems).

See `arXiv:2104.02024` (Cohen, Lamm, Lawrence, Yamauchi) for details.

Real-time path integrals

Let's derive a (lattice) path integral for time-separated correlators.

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle \propto \mathrm{Tr}(e^{-\delta H})^{\beta/\delta}(e^{iH\delta})^{t/\delta}\mathcal{O}(e^{-iH\delta})^{t/\delta}\mathcal{O}$$

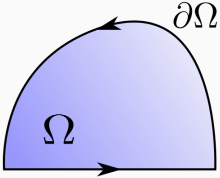
$$\propto \int \mathcal{D}\phi\, \overbrace{\langle\phi|e^{-\delta H}|\phi'\rangle\cdots\underbrace{\langle\phi'|e^{iHt}|\phi''\rangle\cdots\langle\phi''|e^{-iHt}|\phi'''\rangle}_{\text{Complex!}}}^{p(\phi)\in\mathbb{C}}\mathcal{O}(\phi'')\mathcal{O}(\phi''')$$

The "probability distribution" is complex!

Moreover, if you replace the complex $p(\phi)$ with $|p(\phi)|$, almost all of the weight cancels. ($\int |p| \gg \int p$)

(One can also study real-time correlators by analytic continuation. I have nothing to say about this here.)

Cauchy's Theorem

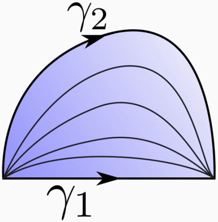


For any holomorphic function $f(z)$:

$$0 = \int_{\partial\Omega} f \, \mathrm{d}z$$

$$\int_{\gamma_1} f \, \mathrm{d}z = \int_{\gamma_2} f \, \mathrm{d}z$$

If we can continuously deform γ_1 to γ_2, "tracing out" Ω.



Physics is unchanged, but fluctuations can be removed ($\int |p(\phi)|$ changes)

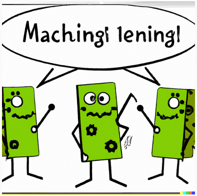
Stokes' Theorem

$$\int \mathcal{D}\phi \, e^{-S(\phi)} = \int e^{-S(\phi)} + \nabla \cdot v(\phi)$$

This is a strict generalization of the Cauchy's theorem based methods.

Any sign problem can be removed by some appropriate v.

The vector field v is responsible for representing the "pure fluctuation" part of the integrand.



The transverse Ising model

$$H = -\mu \sum_r \sigma_x(r) - \sum_{\langle rr' \rangle} \sigma_z(r)\sigma_z(r')$$

In one dimension, this is (dual to via Jordan-Wigner) a theory of free fermions. Tuning $\mu \to \mu_c$ makes the fermions massless. This is a free CFT.

In two dimensions, this theory is interacting. Tuning to $\mu_c \approx 3.044$ yields the $O(1)$ (Ising) CFT; also accessible from scalar field theory.

Thermodynamics and RG flows around the $O(N)$ models are very well understood (in $2+1$ dimensions):

- Bootstrap
- Monte carlo
- ϵ expansion

Simulating the transverse Ising model

Ising spin $\leftrightarrow$ qubit

On a universal quantum computer, time-evolve via Suzuki-Trotter:

$$e^{-i(A+B)\epsilon} \approx e^{-iA\epsilon}e^{-iB\epsilon}$$

TIM is the first model with nontrivial hydrodynamics likely to be accessed.

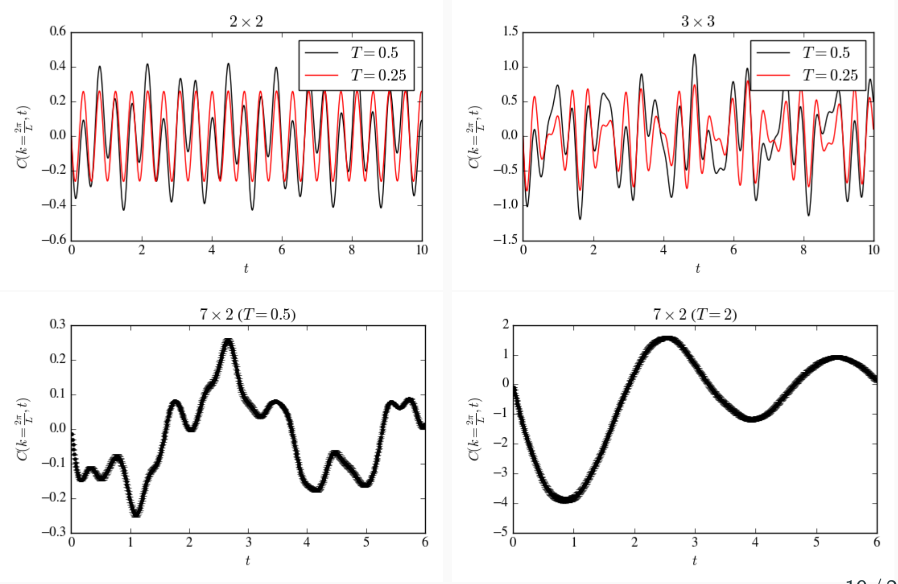
Exponential-cost classical algorithm

Start with a Haar-random $|\Psi\rangle$.
Apply $e^{-\frac{\beta}{2}H}$ to obtain a thermal state. (Agrees with canonical ensemble in the large-volume limit.)
Apply operator $\mathcal{O}$.
Apply $(1 - i\epsilon H)^{\frac{t}{\epsilon}}$ to time-evolve.
Apply $\mathcal{O}$ again to get expectation value.

In both cases, evaluate $\langle T^{00}(t,x)T^{00}(0,0)\rangle$ and observe the decay.

Sound attenuation in the transverse Ising model

In progress!



Large-N calculation in the $O(N)$ CFT

From arXiv:2104.06435 (Romatschke):

$$\frac{1}{N}\left(\frac{\eta T}{\epsilon + P}\right)_{O(N)} = 0.42(1) + O(N^{-1})$$

Recklessly extrapolating gives an Ising shear viscosity of

$$4\pi\frac{\eta T}{\epsilon + P} = 5.28(12).$$

Do other methods agree?

The Shape of Things To Come

- Most analytic methods are limited to narrow regimes. (Weak coupling; large N; holographic theories.)
- Scalable quantum computers will come online *eventually*; will they come before efficient classical methods are available?
- Machine learning for sign problems
- Bootstrap methods

How slow can transport be?