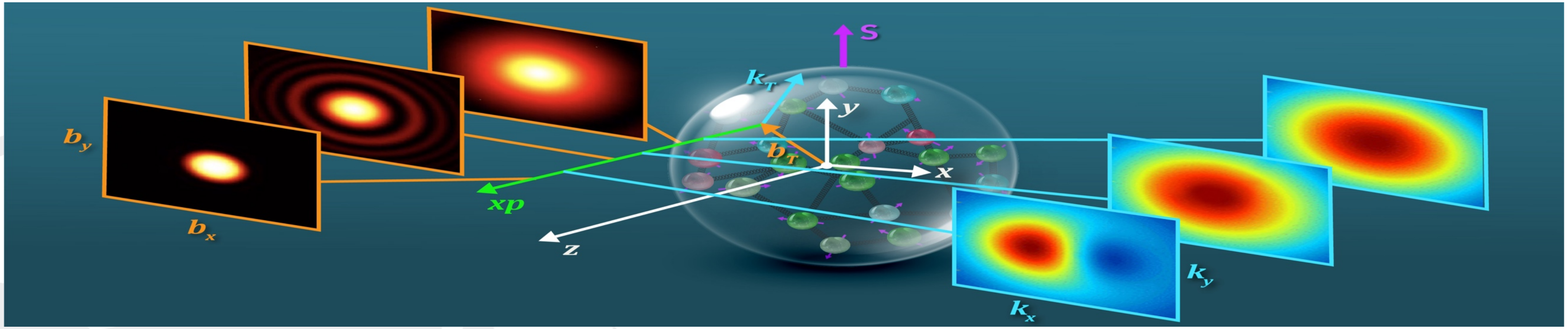
Center for Frontiers in Nuclear Science

Workshop: Theory for EIC in the next decade

September 20 - 22, 2022, MIT, Maclaurin Buildings, Building 4

Massachusetts Institute of Technology

Exploring the x-dependence of GPDs at EIC



3D Tomographic images of hadrons in slides of parton momentum fraction x

Jianwei Qiu
Jefferson Lab, Theory Center

Jefferson Lab

TMD Collaboration

U.S. DEPARTMENT OF ENERGY | Office of Science

JSA

QCD Landscape of Nucleons and Nuclei

Color Confinement

Asymptotic freedom

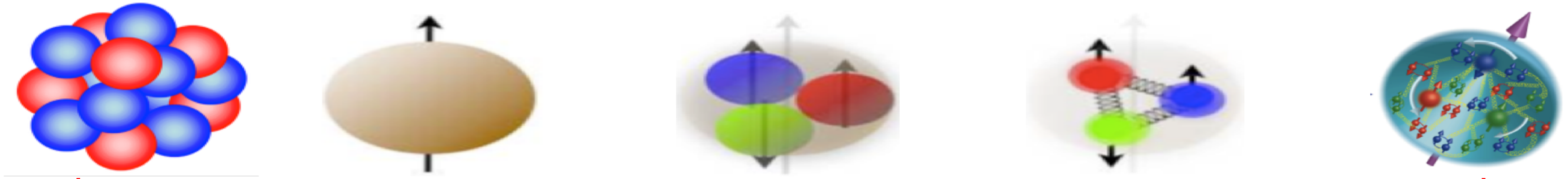
Q (GeV)

20 MeV (10 fm)

200 MeV (1 fm)

2 GeV (1/10 fm)

Probing scale



QCD at the Fermi Scale: Femto-science (0.1-10 fm)

- *The most interesting, rich, and complex regime of the theory!*
- *All emergent phenomena depend on the scale at which we probe them!*

Asymptotic regime PQCD works beautifully!

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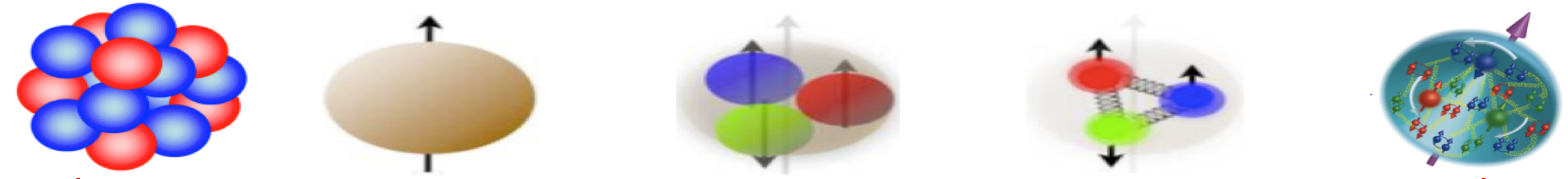

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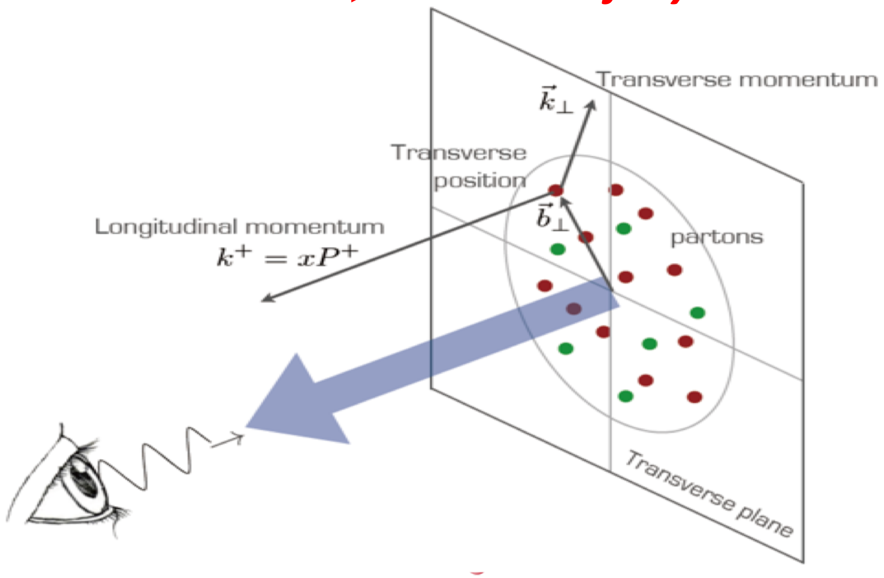
❑ **Need new observables with two distinctive: scales:**

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

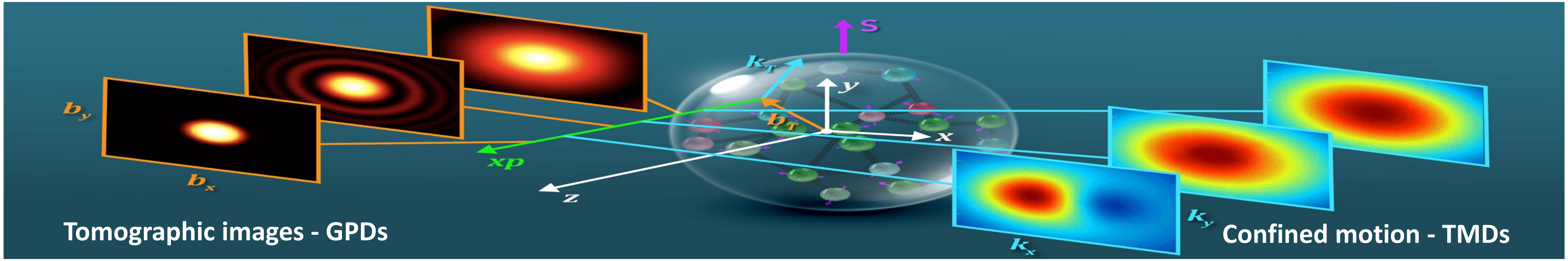
- **Hard scale:** Q_1 **to localize the probe to see the particle nature of quarks/gluons**
- **"Soft" scale:** Q_2 **could be more sensitive to the hadron structure ~ 1/fm**



"See" Internal Structure of Hadron without seeing quarks/gluons?

❑ **3D hadron structure:**

NO quarks and gluons can be seen in isolation!



SciDAC5 – QuantOm Collab.

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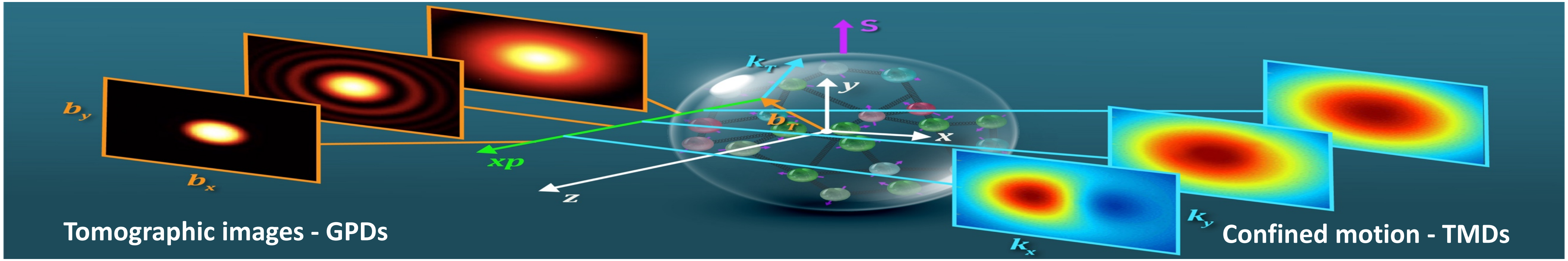


❑ QCD factorization – Matching hadrons to partons:

SciDAC5 – QuantOm Collab.

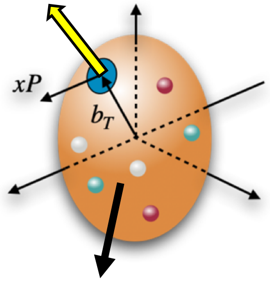
Lattice QCD data

Experimental data

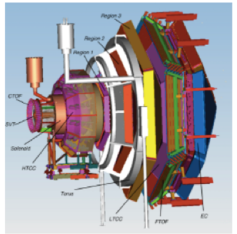
Factorization Theory

$$= \int \cdots \int \quad \boxed{\text{PQCD Calculation}} \quad \otimes \quad \boxed{\text{Quantum Correlation Functions Unknown}}$$

Quark Polarization



Nucleon Polarization



Jefferson Lab

KEK

BROOKHAVEN NATIONAL LABORATORY

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Fermilab

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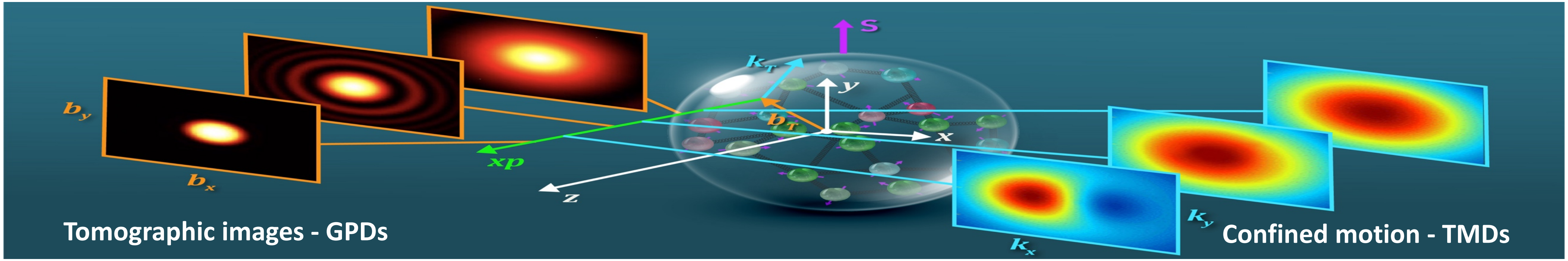

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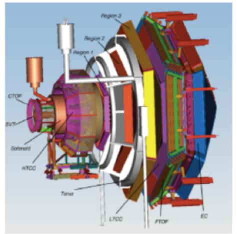

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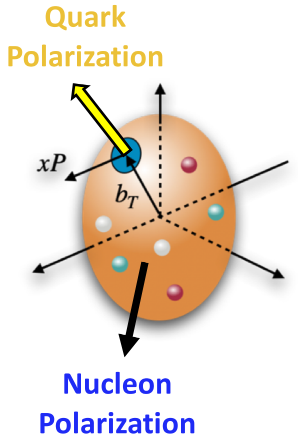
Factorization Theory

$$= \int \cdots \int \boxed{\text{PQCD Calculation}} \otimes \boxed{\text{Quantum Correlation Functions Unknown}}$$

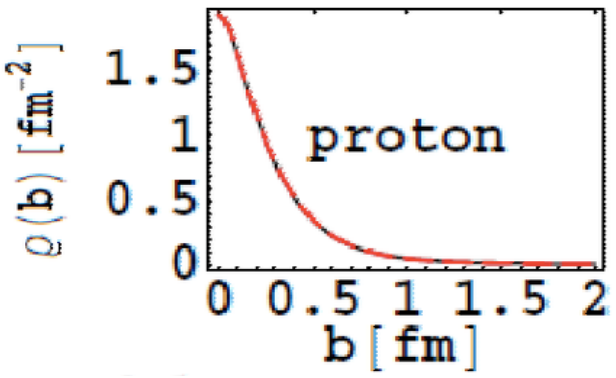
Bayesian inference
Theory/Phenomenology
AI/ML, Computation

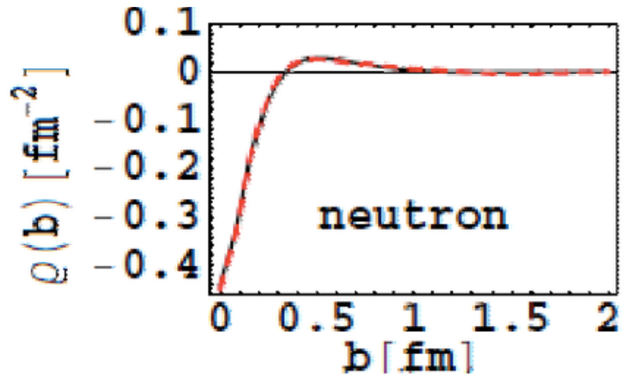
Extract QCFs

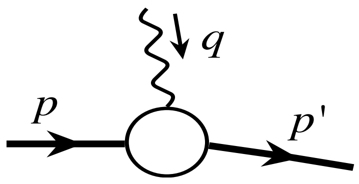


How to Explore Internal Structure of Hadron without Breaking it?

- Form factors: Elastic electric form factor → Charge distributions

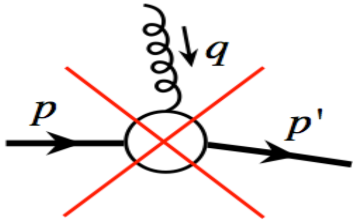






Proton "Radius"
in EM charge distribution

- But, there is NO elastic "color" form factor!



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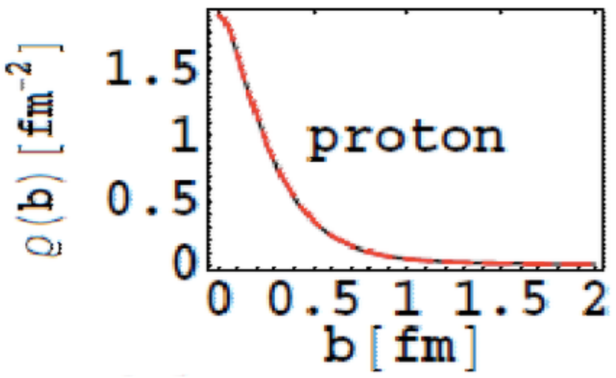

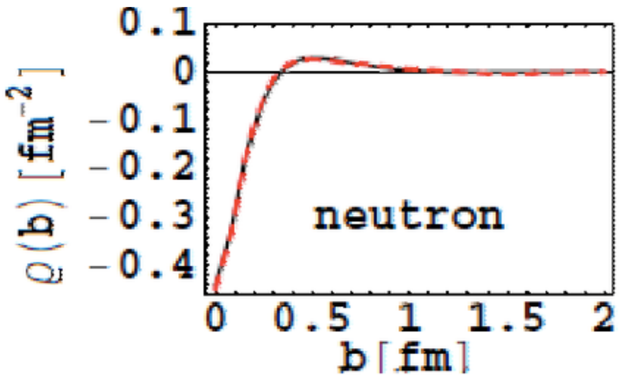

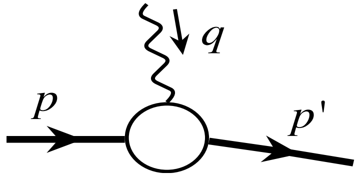

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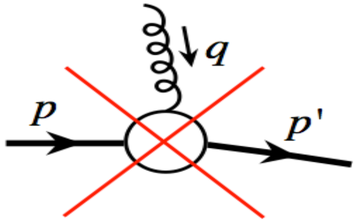

No Proton "Radius"
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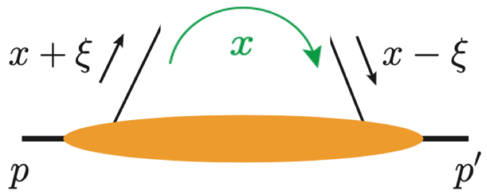
- **3D hadron tomography:**

Generalized "form factor" for quark and parton "density" distribution

Generalized PDF (GPD) – without breaking the proton

$$F_{q/h}(x, \xi, t) \qquad \text{skewness} \qquad \xi = \frac{(P-P')^+}{(P+P')^+}$$



Spatial distribution of quark/gluon density, quark/gluon correlations, …

Generalized Parton Distribution (GPD)

Definition:

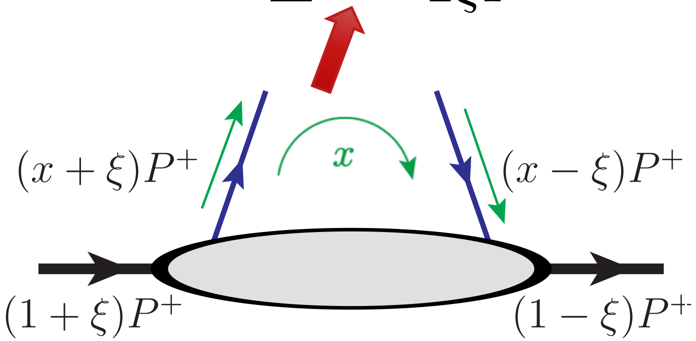
$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right].$$

$$\Delta^+ = 2\xi P^+$$



$$P^+ = \frac{p^+ + p'^+}{2}$$

$$\Delta = p - p'$$

Similar definition for gluon GPDs

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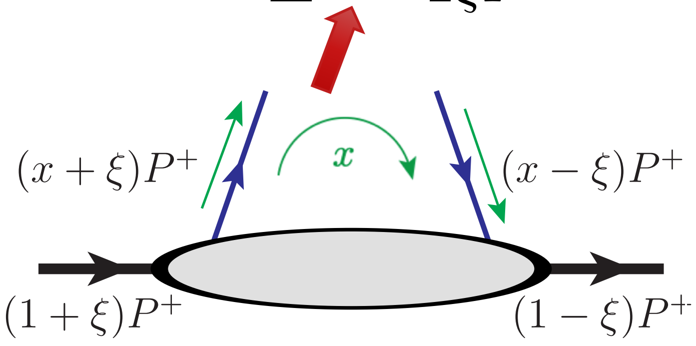


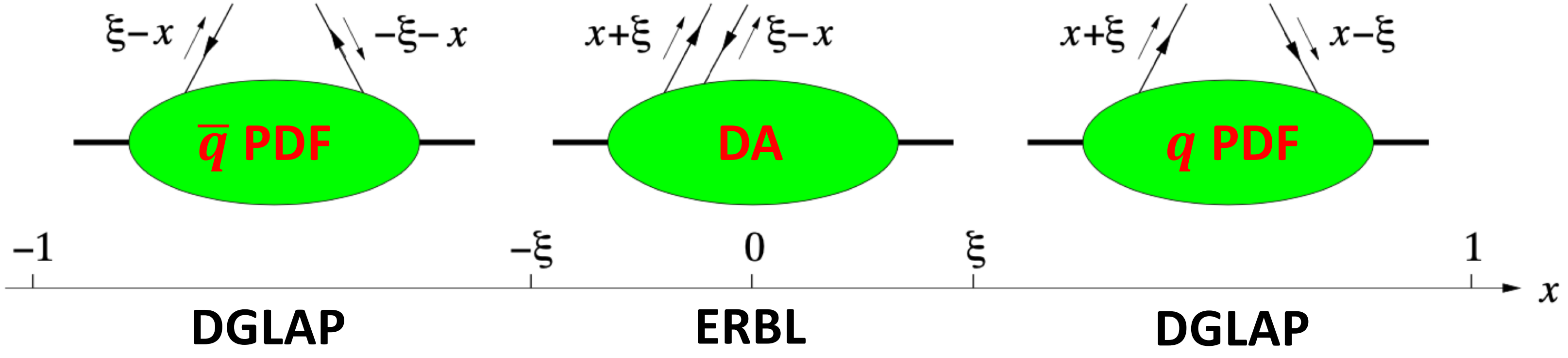
$$P^+ = \frac{p^+ + p'^+}{2}$$

$$\Delta = p - p'$$

Similar definition for gluon GPDs

- **Combine *PDF* and *Distribution Amplitude (DA):***

Forward limit $\xi = t = 0$ **:** $\quad H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x)$



Properties of GPDs

- “Mass” – QCD energy-momentum tensor:

Ji, PRL78, 1997

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \text{with} \quad T_q^{\mu\nu} = \bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \qquad T_g^{\mu\nu} = G^{\mu\alpha} G_\alpha{}^\nu + \frac{1}{4} g^{\mu\nu} G^{\alpha\beta} G_{\alpha\beta}$$

- Form factors:

$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = A_{q,g}(t) \bar{u} P^{(\mu}\gamma^{\nu)} u + B_{q,g}(t) \bar{u} \frac{P^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha}{2m} u + C_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{m} \bar{u} u + \bar{C}_{q,g}(t) m g^{\mu\nu} \bar{u} u$$

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- "Spin" – Light-cone helicity operator:

$$J^3 = \int \mathrm{d}x^- \, \mathrm{d}^2\mathbf{x} \, M^{+12}(x) \quad \text{with} \quad M^{\alpha\mu\nu} = T^{\alpha\nu}x^\mu - T^{\alpha\mu}x^\nu$$

- Connection to the proton spin:

$$\langle J_q^3 \rangle = \tfrac{1}{2}[A_q(0) + B_q(0)] \,, \qquad \langle J_g^3 \rangle = \tfrac{1}{2}[A_g(0) + B_g(0)]$$

$$A_q(t) + B_q(t) = \int_{-1}^{1} \mathrm{d}x\, x[H_q(x,\xi,t) + E_q(x,\xi,t)] \qquad A_g(t) + B_g(t) = \int_0^1 \mathrm{d}x [H_g(x,\xi,t) + E_g(x,\xi,t)]$$

Need to know the x-dependence of GPDs to construct the proper moments!

Properties of GPDs

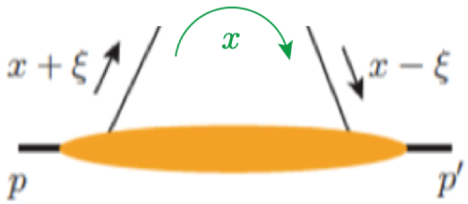
- **Impact parameter dependent parton density distribution:**

M. Burkdart, PRD 2000

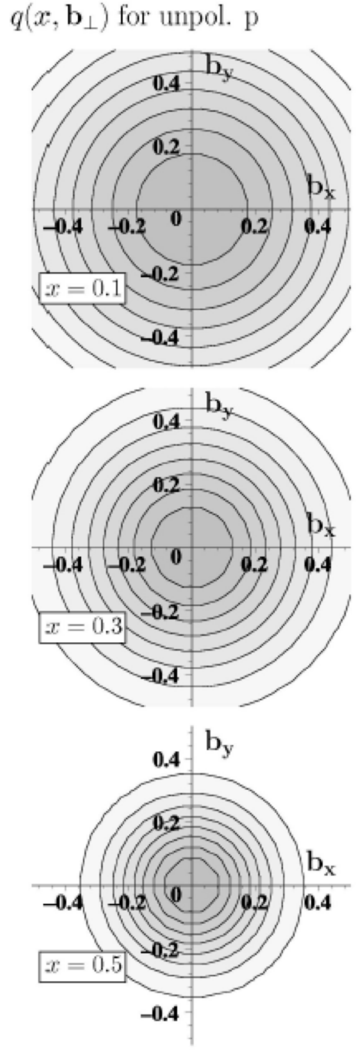
$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$

Quark density in $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$

Unpolarized proton



- x = momentum flow between the pair
- $b_\perp$ = conjugate to the diffracted momentum
- Small x: large "meson cloud"
- Large x: compact "valence core"
- $b_\perp \to 0$ = $x \to 1$ narrow distribution



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❑ Impact parameter dependent parton density distribution:

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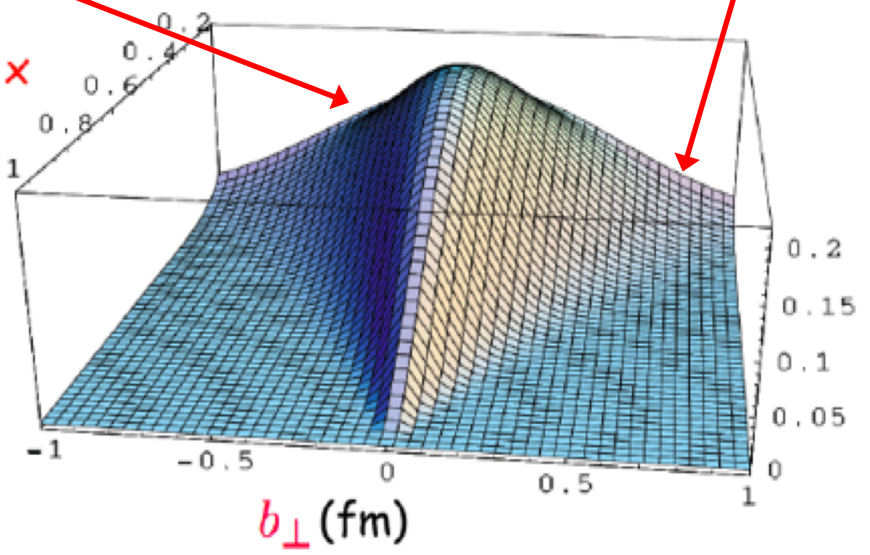
How fast does glue density fall?

3D image

How far does glue density spread?



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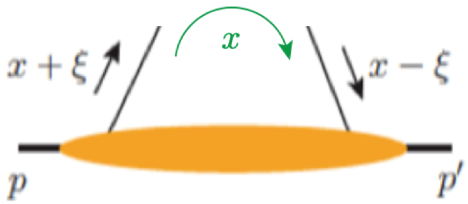

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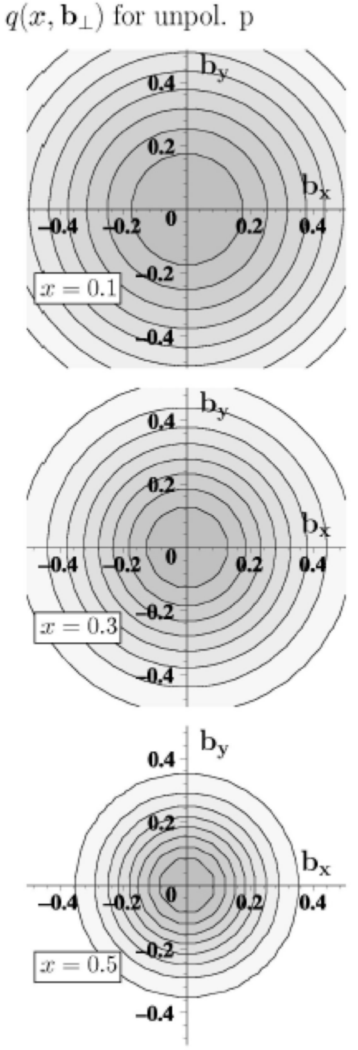

➡ **Proton radii of quark and gluon spatial density distribution,** $r_q(x)$ **&** $r_g(x)$

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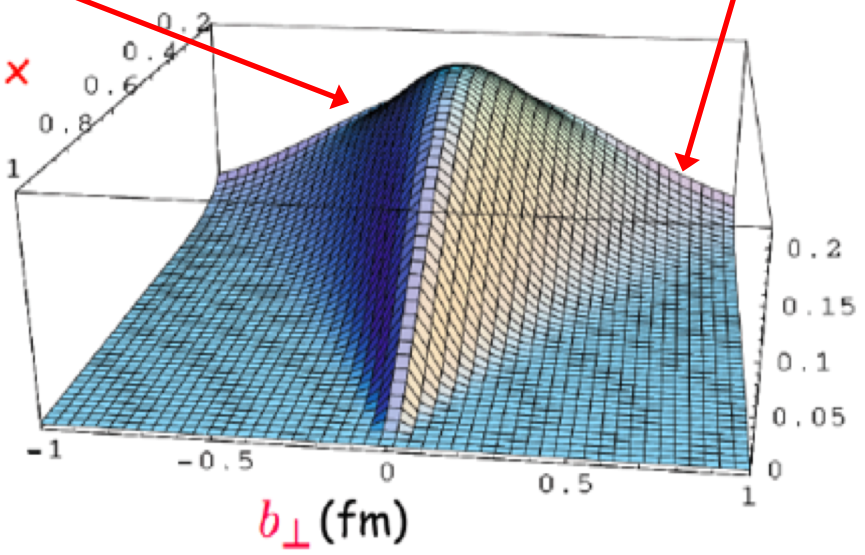

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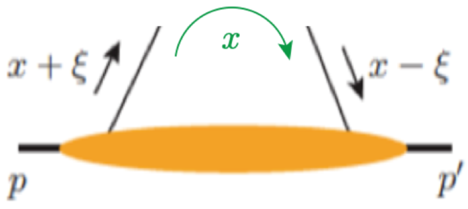
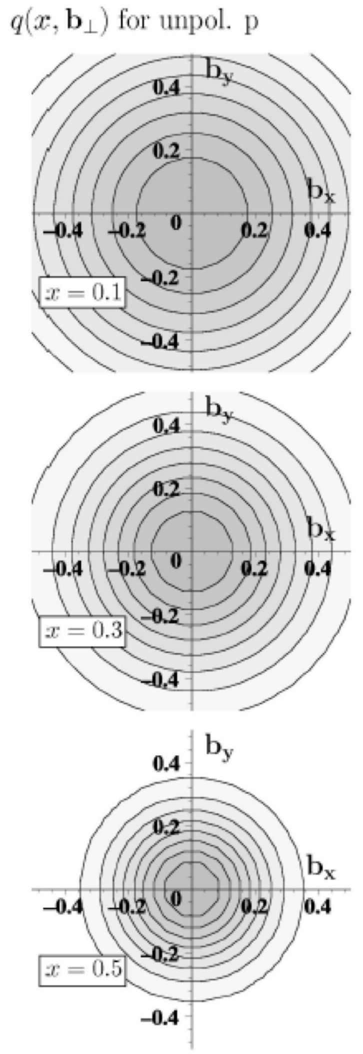


- **Should** $r_q(x)$ **>** $r_g(x)$**, or vice versa?**
- **Could** $r_g(x)$ **saturates as** $x \to 0$
- **How do they compare with known radius (EM charge radius, mass radius, …)?**
- **Tomographic images in slides of different x value!**



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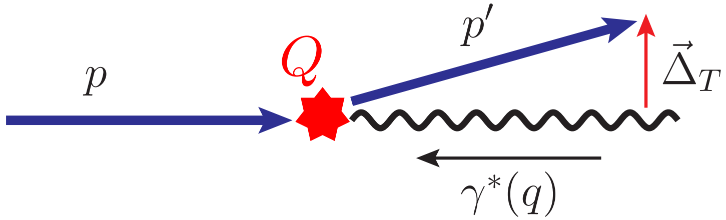
Exclusive Diffractive Process for Extracting GPDs at the EIC

- Hit the proton hard without breaking it $\Longrightarrow$ Diffractive scattering to keep proton intact



HERA discovery:

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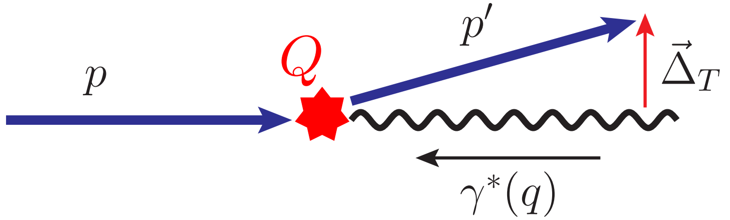

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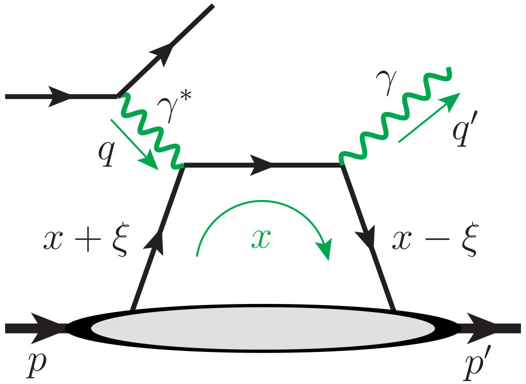
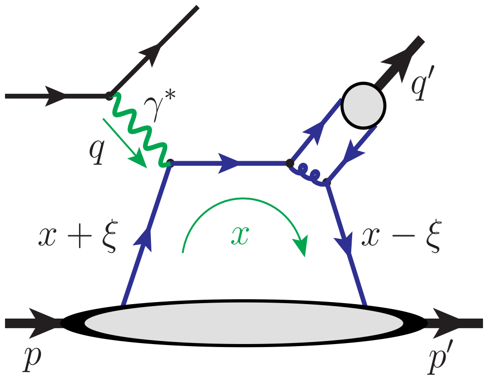
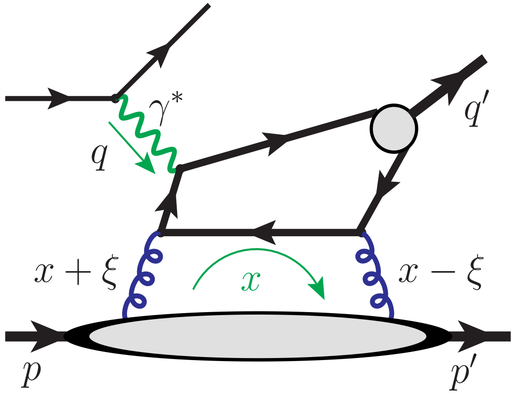
- **Known exclusive processes for extracting GPDs in lepton-hadron collisions:**



DVCS: Q^2 >> |t|



DVMP



DVQP

+ DDVCS, …

Feature: Two-scale observables $\quad Q^2 \gg |t| \qquad t = (p-p')^2$

- **Hard scale Q: allows pQCD, factorization**
- **Low scale t: probes non-pert. hadron structure**

Factorization

GPDs: $f_{i/h}(x, \xi, t; \mu)$

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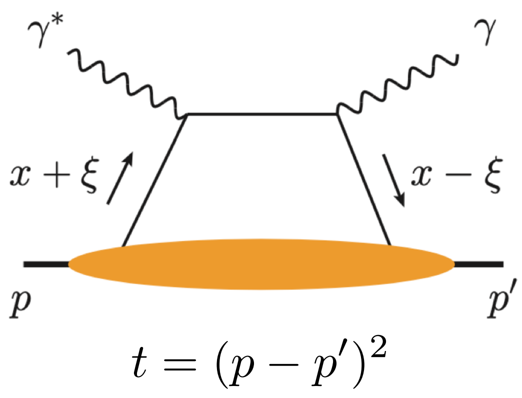
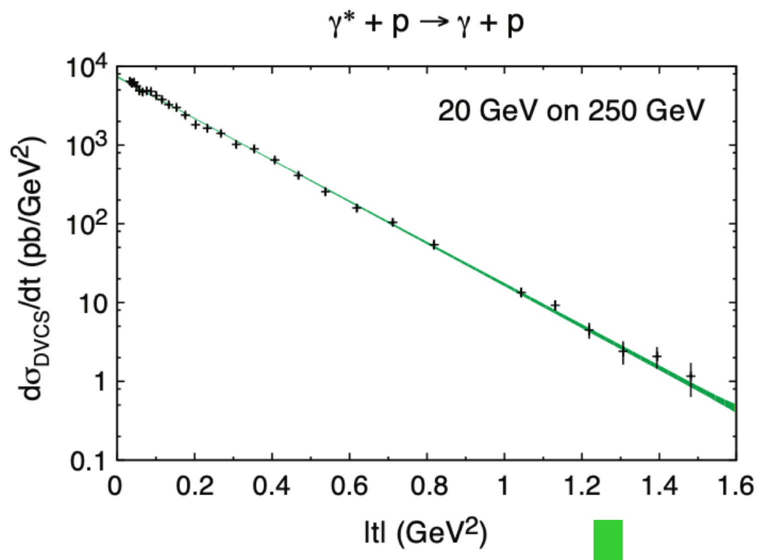
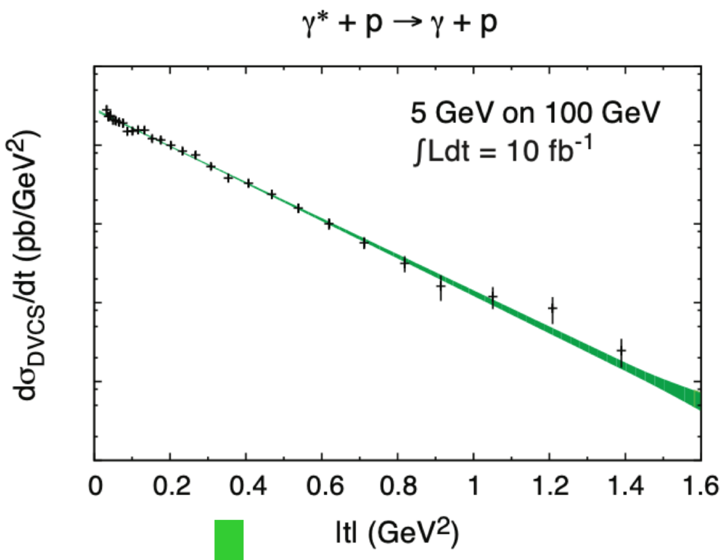
DVCS at a Future EIC (White Paper)

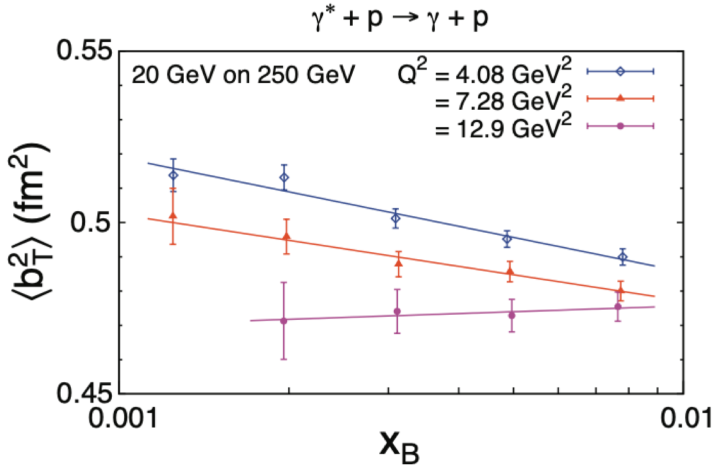
❑ Cross Sections:

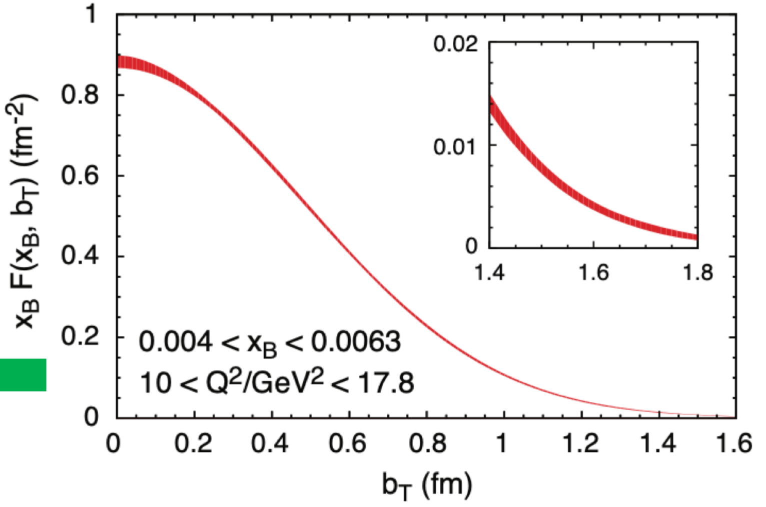


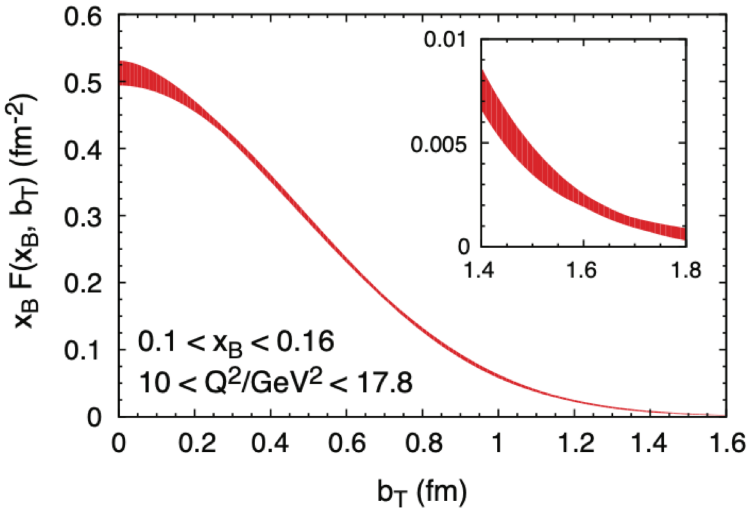




❑ Spatial distributions:



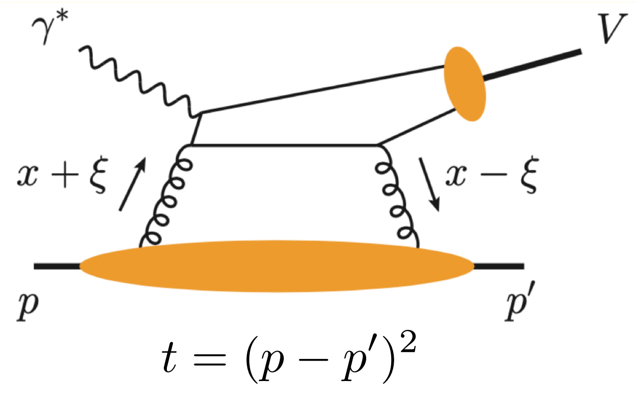


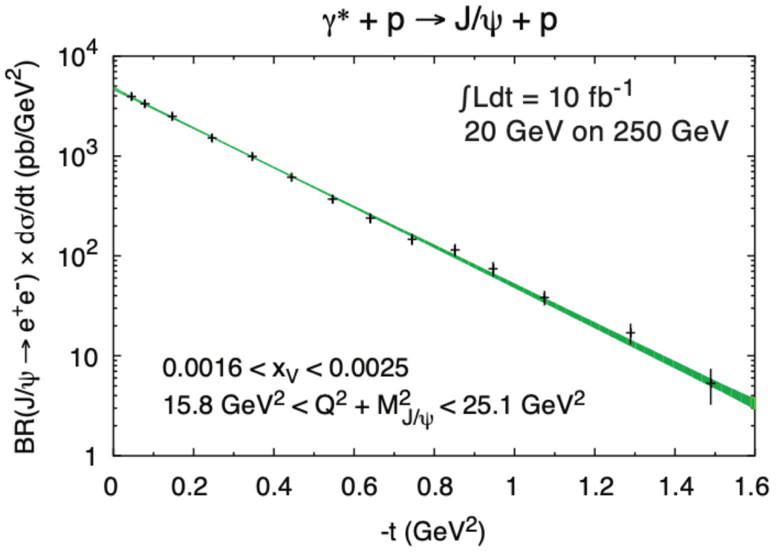


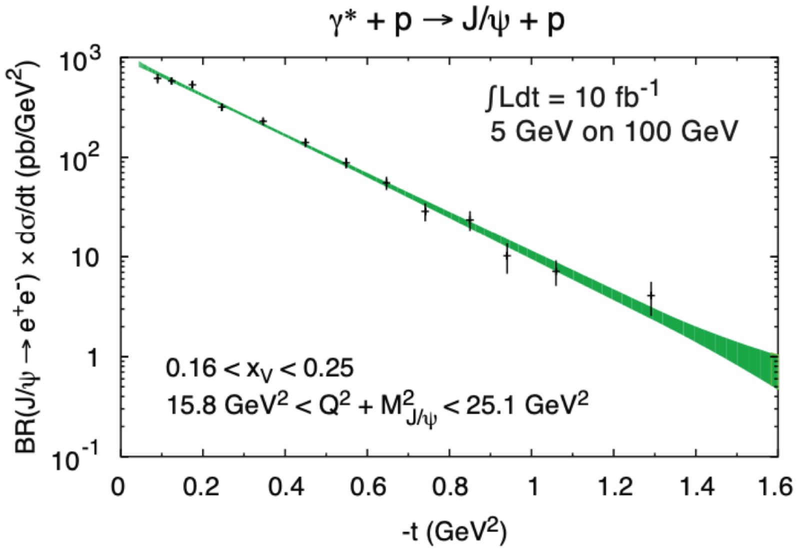
Effective "proton radius" in terms of quarks as a function of x_B

Imaging the Gluon at the EIC (White Paper)

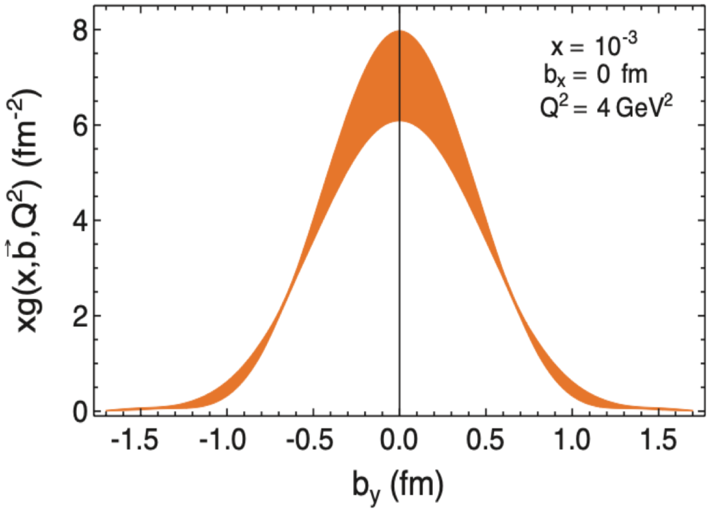
- Exclusive vector meson production:

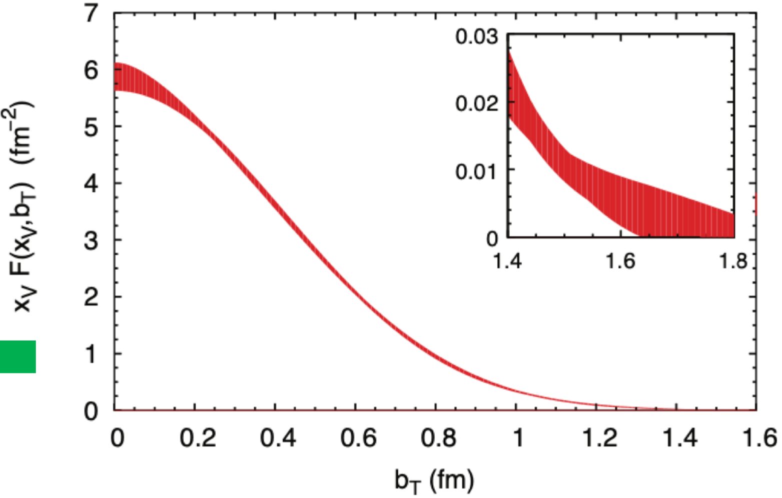


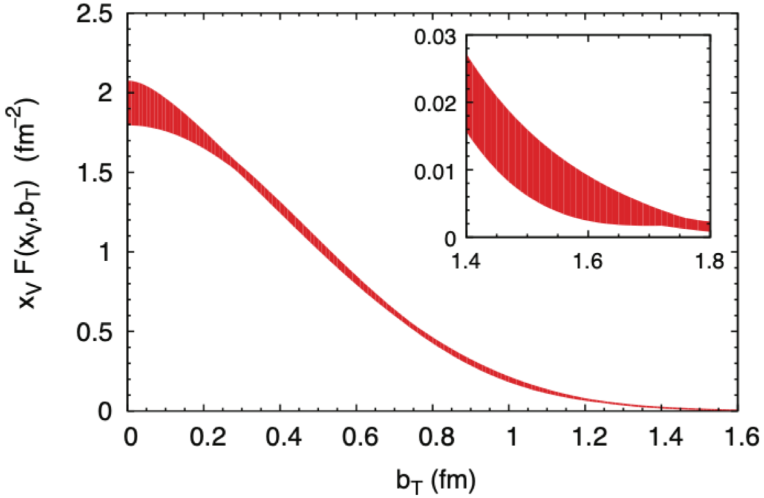




- Spatial distributions:





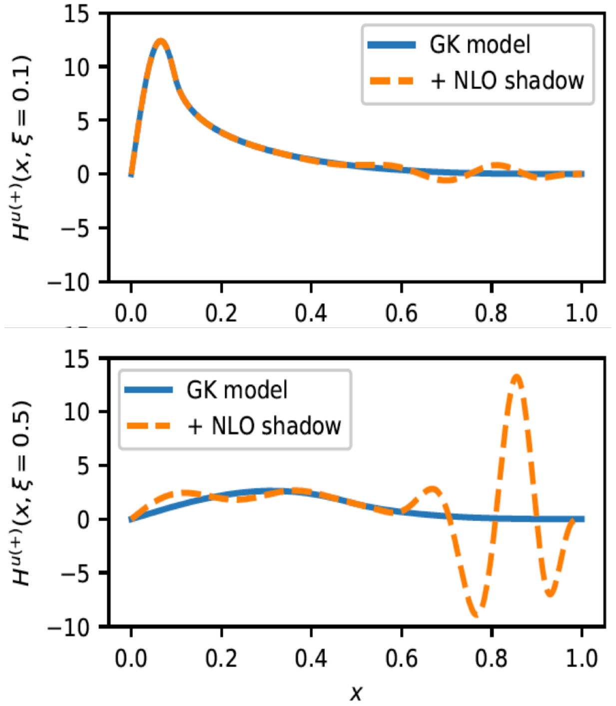


The b_T space density for gluons

Effective "proton radius" in terms of gluons

It is Difficult to Extract the x-dependence of GPD – Why?

❑ "Shadow GPDs"



Blue and dashed
Fit the same CFFs !

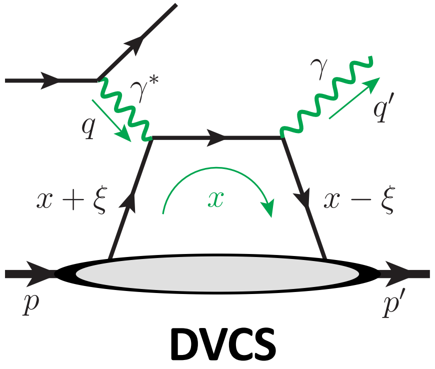
PRD103 (2021) 114019

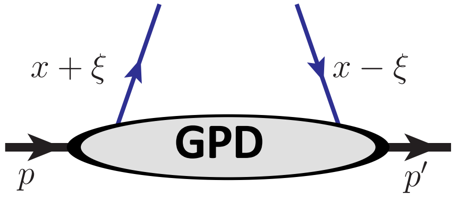
❑ Amplitude nature: exclusive processes

$x \sim$ **loop momentum**

$$\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x F(x, \xi, t) \cdot C(x, \xi; Q/\mu)$$

never pin down to some x





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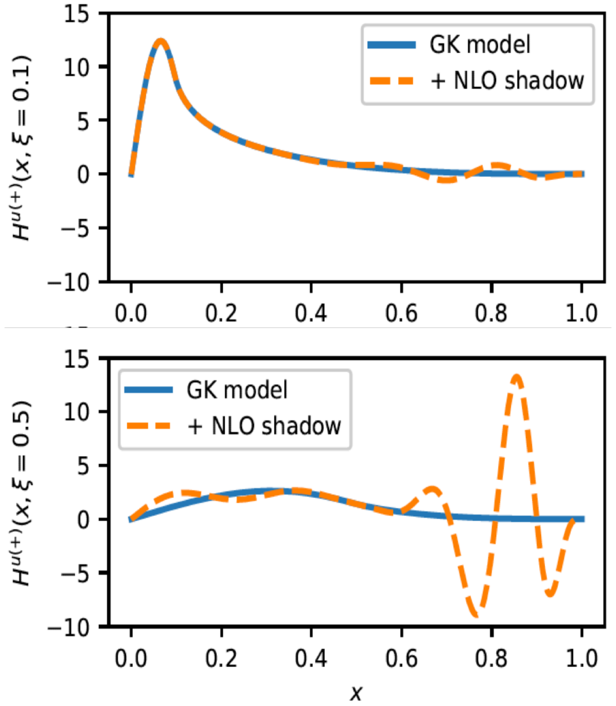

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$$\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x F(x, \xi, t) \cdot C(x, \xi; Q/\mu)$$

never pin down to some x

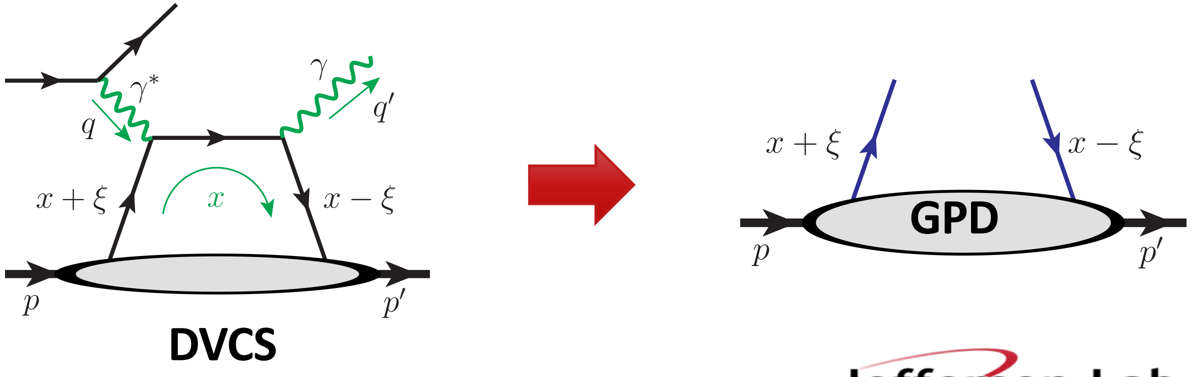
❑ Sensitivity to x comes from $C(x, \xi; Q/\mu)$

At LO, DVCS hard coefficient factorizes

$$C(x, \xi; Q/\mu) = C_Q(Q/\mu) \cdot C_x(x, \xi) \propto \frac{1}{x - \xi + i\varepsilon} \cdots$$

$$\Rightarrow \quad i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \frac{F(x, \xi, t)}{x - \xi + i\varepsilon} \equiv \text{“}F_0(\xi, t)\text{”}$$

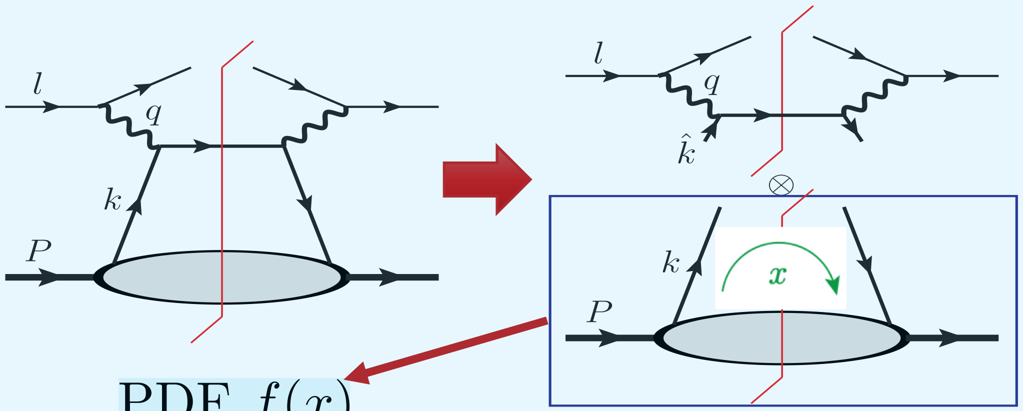
- also true for most other processes
- x-dependence is only constrained by a "moment"
- easy to fit to the data



Jefferson Lab

Inclusive Process vs. Exclusive Process

❑ Deeply Inelastic Scattering (DIS):



PDF $f(x)$

$x = k^+/P^+$

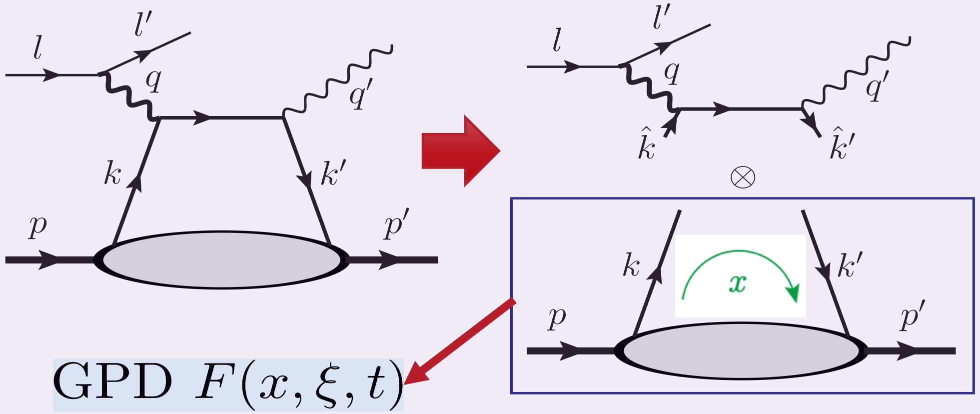
Cross section: **Cut diagrams**

$$\sigma_{\rm DIS} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\, \hat{\sigma}(x/x_B)$$

- **PDF ~ probability**
- **At LO:** $x = x_B$ $\quad \hat{\sigma}^{(\rm LO)}(x/x_B) \propto \delta(x - x_B)$
- **Beyond LO:** $x \in [x_B, 1]$

x-dependence: **Part of measurement**

❑ Deeply Virtual Compton Scattering (DVCS)



GPD $F(x,\xi,t)$

$$x = \frac{(k+k')^+}{(p+p')^+}$$

$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

$$t = (p-p')^2$$

Amplitude: **Uncut diagrams**

$$\mathcal{M}_{\rm DVCS}(\xi,t) \simeq \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t)\, \hat{\mathcal{M}}(x,\xi)$$

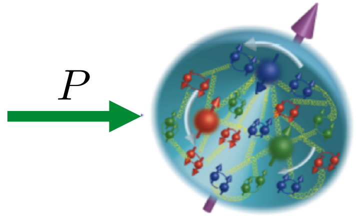
- **GPD ~ amplitude**
- $\boldsymbol{k^+ = (x+\xi)\, P^+}$ **is loop momentum**
- **At any order:** $x \in [-1, 1]$

x-dependence: **Hard to measure**

Single-Diffractive Hard Exclusive Processes (SDHEP)

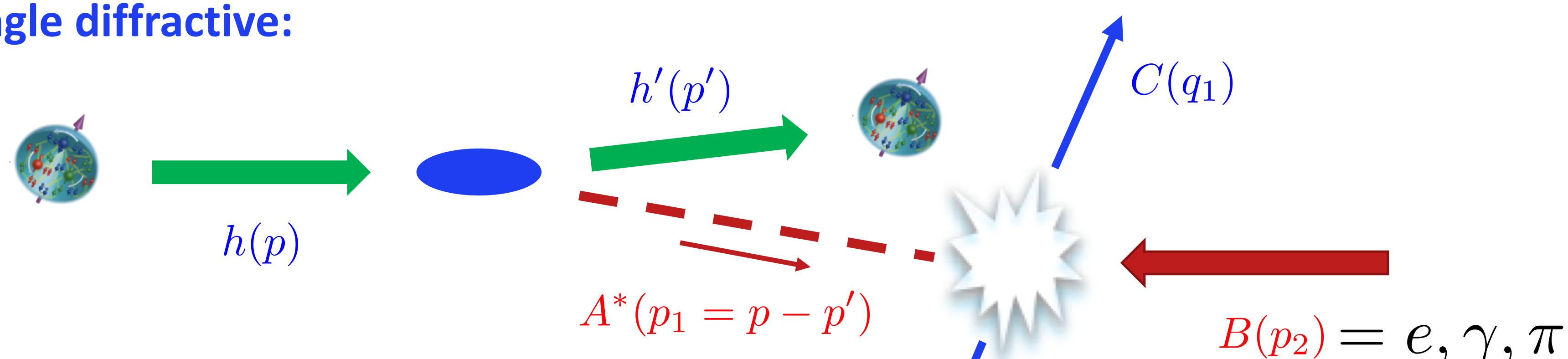
- **Two-stage diffractive $2 \to 3$ hard exclusive processes:**

Qiu & Yu, JHEP 08 (2022) 103



Probing its structure without breaking it!

  - **Single diffractive:**



$$h(p) \to h'(p') + A^*(p_1 = p - p')$$

$$t = (p - p')^2 \equiv p_1^2$$

  - **Hard probe:** $2 \to 2$ **high** q_T **exclusive process**

$$A^*(p_1) + B(p_2) \to C(q_1) + D(q_2)$$

$$(p - p') \cdot n \gg \sqrt{|t|} \quad \Longleftrightarrow \quad |q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$$

Jefferson Lab

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Qiu & Yu, JHEP 08 (2022) 103

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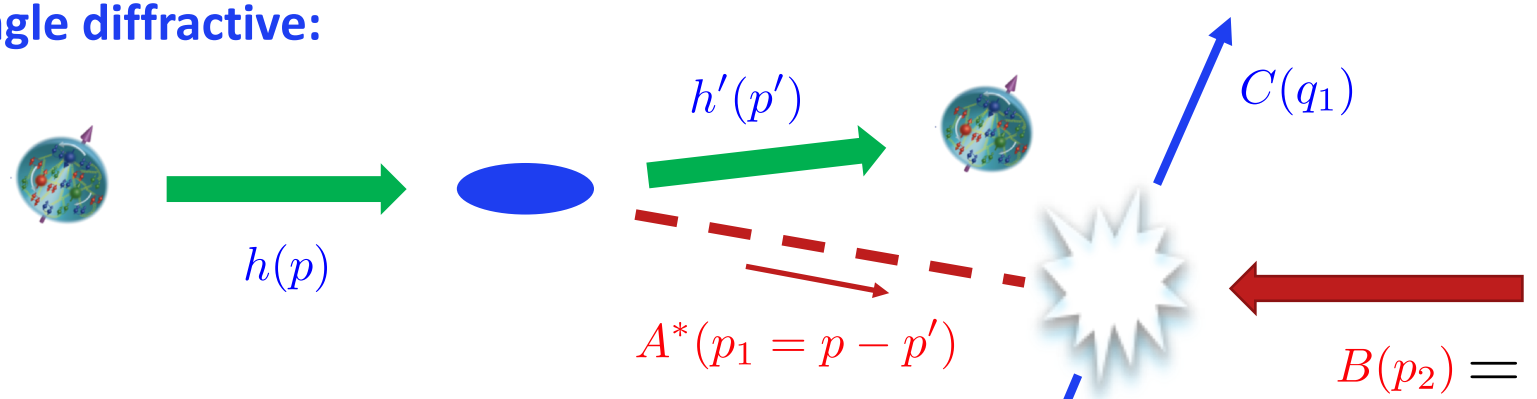

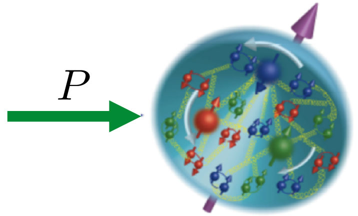

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- **The single diffractive $2 \to 3$ exclusive hard processes:**

$$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$$

- **Necessary condition for QCD factorization:**

$$|q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$$

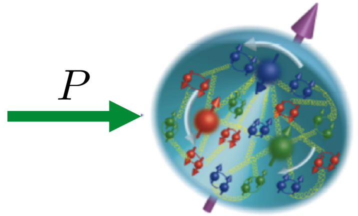
The state $A^*(p_1)$ lives much longer than $2 \to 2$ hard exclusive collision!

Not necessarily sufficient!

Single-Diffractive Hard Exclusive Processes (SDHEP)

- **Two-stage diffractive $2 \to 3$ hard exclusive processes:**

Qiu & Yu, JHEP 08 (2022) 103



Probing its structure without breaking it!

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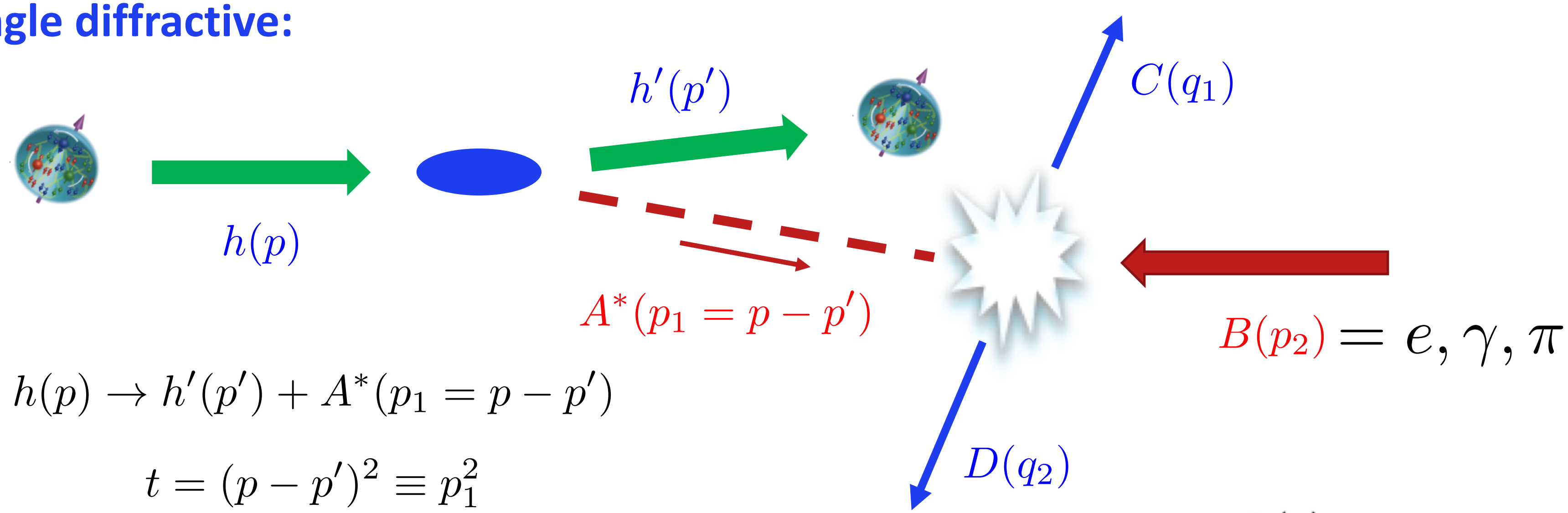

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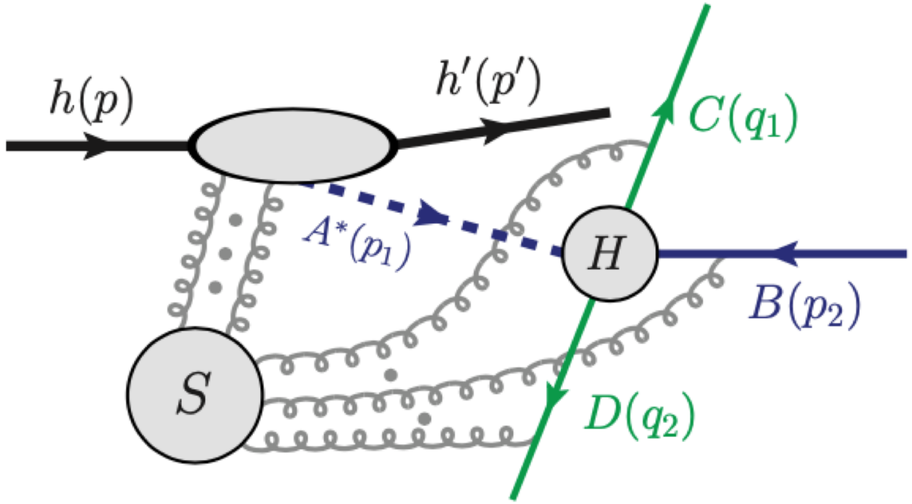


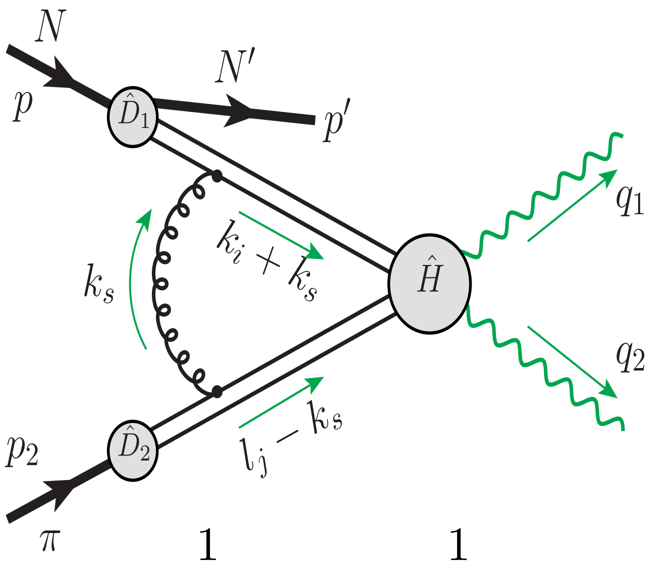
Challenge for QCD Factorization of SDHEP

❑ **Example:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$ $\qquad \lambda \sim m_\pi / Q, \qquad Q \sim q_T$

Gluons in the Glauber region: $k_s = (\lambda^2, \lambda^2, \lambda)\,Q$

Transverse component contribute to the leading region!

ERBL region **(Efremov, Radyushkin, Brodsky, Lepage)**



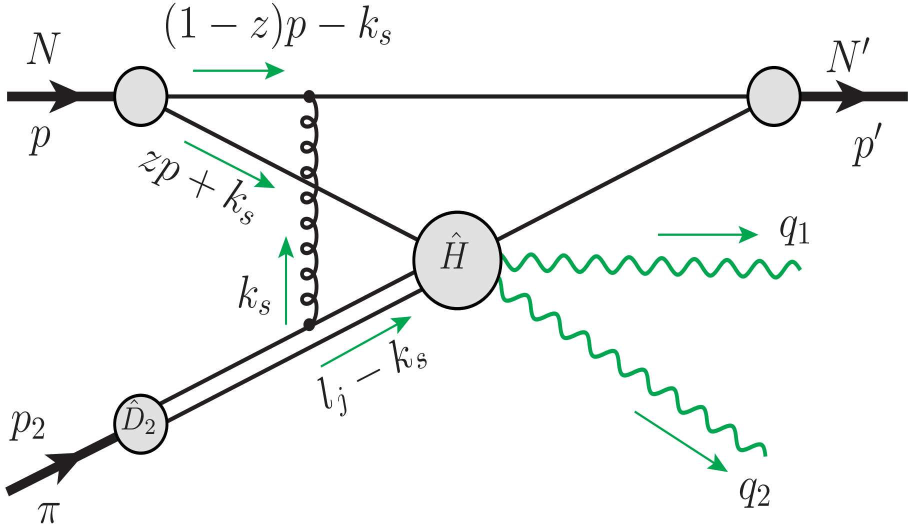
$$\frac{1}{k_s^2 + i\varepsilon} \to \frac{1}{-\mathbf{k}_s^2 + i\varepsilon}$$

$$\frac{1}{(k_i + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

$$\frac{1}{(l_j - k_s)^2 + i\varepsilon} \to \frac{1}{-k_s^+ + i\varepsilon}$$

No pinch!

DGLAP region



$$\frac{1}{((1-z)p - k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- - i\varepsilon}$$

$$\frac{1}{(zp + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

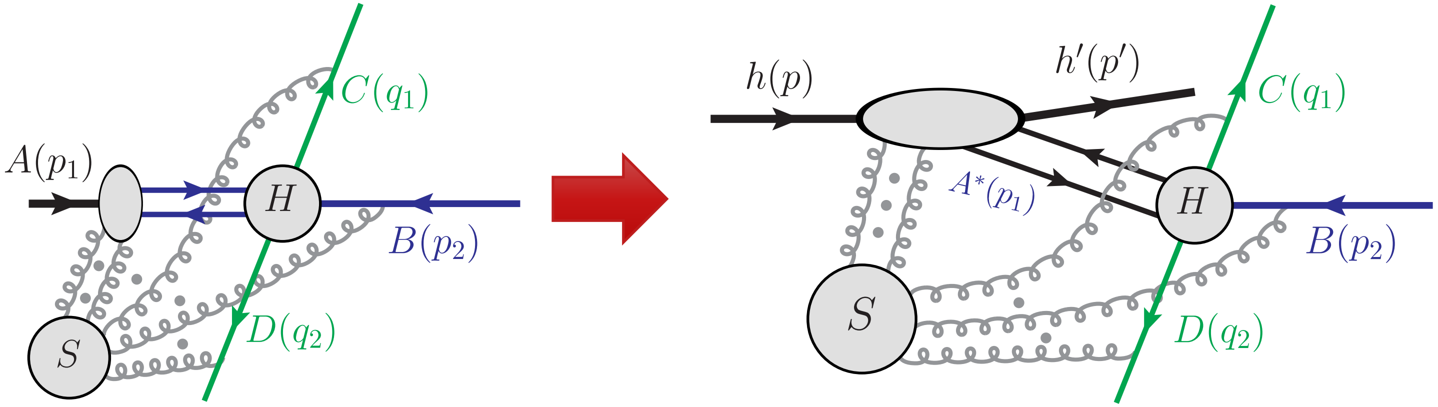
Pinched!

Same conclusion if k_s flows Through N'!

Factorization in the Two-Stage Paradigm

Qiu & Yu, JHEP 08 (2022) 103

Factorization for 2-parton channel factorization:



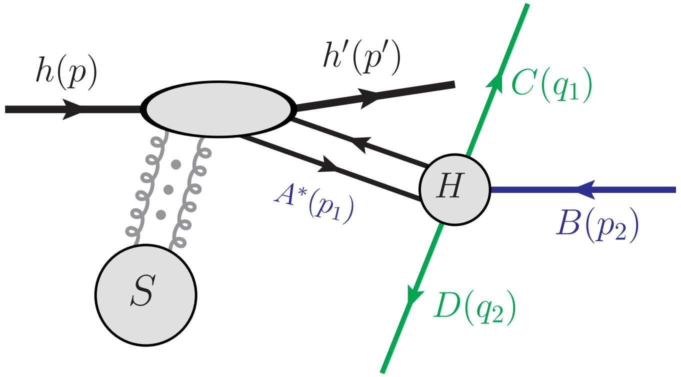
Only complication:

$\boldsymbol{k_s^-}$ **is pinched in Glauber region for DGLAP region.**

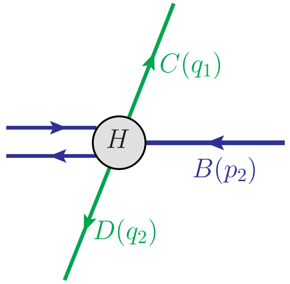
$$k_s^+ \mapsto k_s^+ \pm i\mathcal{O}(Q)$$

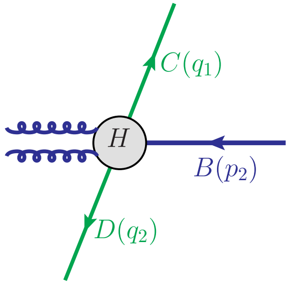
Glauber → ***h*-collinear region**

Soft gluons cancel for the meson-initialized process if *C* and *D* are mesons:



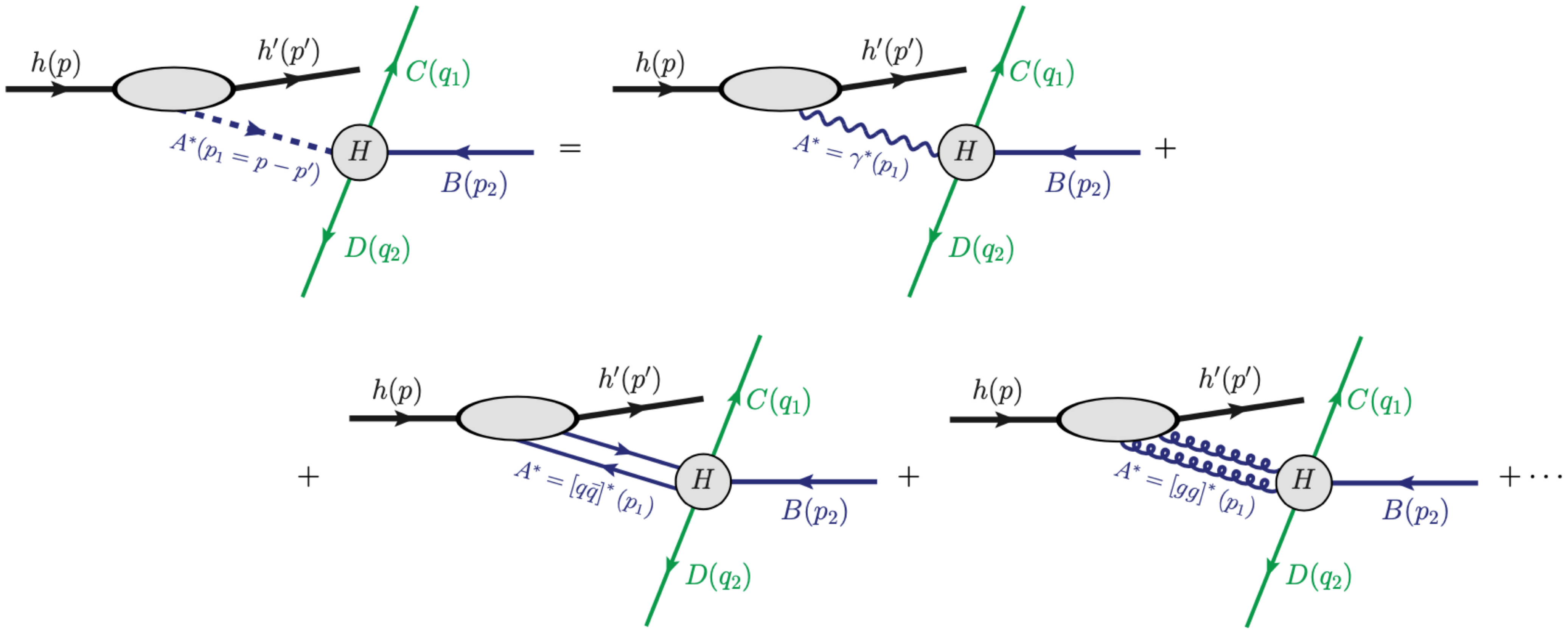
Soft gluons are no longer pinched and can be deformed into *h*-collinear region

GPD $\otimes$ 

or 

Single-Diffractive Hard Exclusive Processes (SDHEP)

- **Two-stage diffractive $2 \to 3$ hard exclusive processes:**



The exchanged state *A*(p-p')* is a sum of all possible partonic states, $\sum_{n=1,2,..}$, allowed by

- **Quantum numbers of *h(p) – h'(p')***
- **Symmetry of producing non-vanishing *H***

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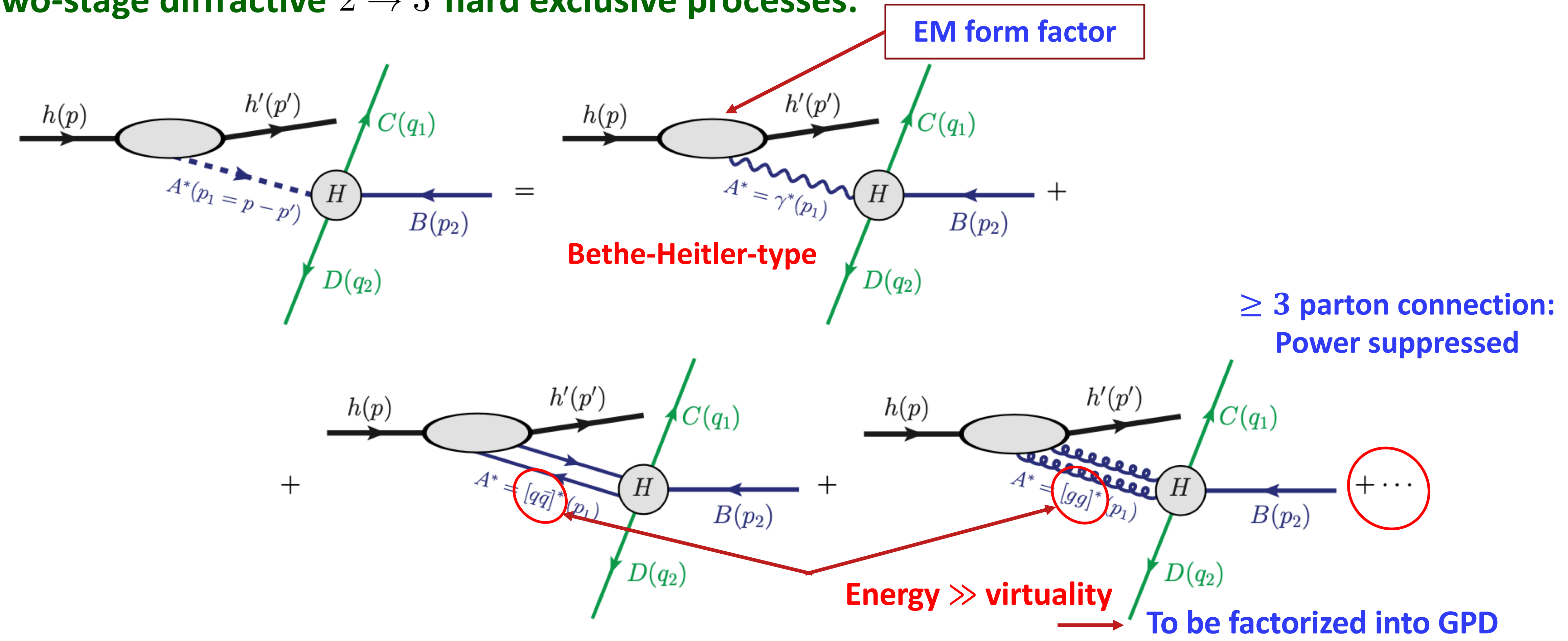

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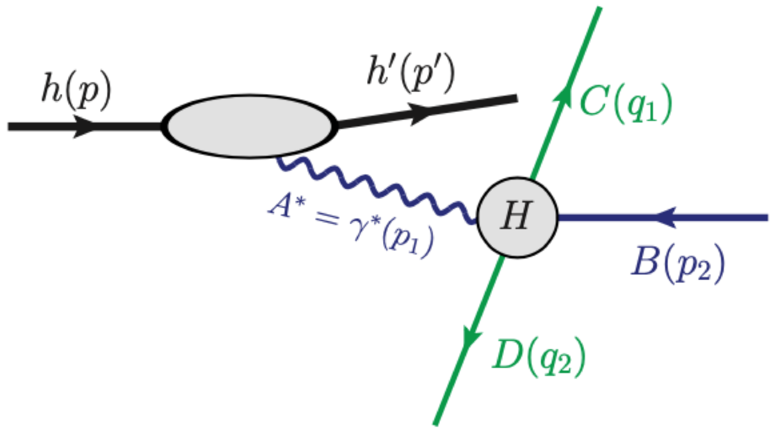
General Discussion on n=1 state: γ^*

❑ Exchange of a virtual photon:

$$\mathcal{M}^{(1)} = \frac{ie^2}{t}\langle h'(p')|J^\mu(0)|h(p)\rangle \langle C(q_1)D(q_2)|J_\mu(0)|B(p_2)\rangle$$

$$\equiv \frac{ie^2}{t} F^\mu(p,p')\,\mathcal{H}_\mu(p_1,p_2,q_1,q_2)$$

$$J^\mu = \sum_{i\in q} Q_i \bar{\psi}_i \gamma^\mu \psi_i$$



$$F^\mu(p,p') = \langle h'(p')|J^\mu(0)|h(p)\rangle$$

$$= F_1^h(t)\,\bar{u}(p')\gamma^\mu u(p) + F_2^h(t)\,\bar{u}(p')\frac{i\sigma^{\mu\nu}p_{1\nu}}{2m_h}u(p)$$

Forbidden for $p \to n$ (or $n \to p$) transition GPDs
Or not allowed by H

Has a leading component , $F^+ \propto \mathcal{O}(Q)$, as h-h' fast along "+"

$$F^+\mathcal{H}^- = \frac{1}{p_1^+}F^+\left(p_1^+\mathcal{H}^-\right) = \frac{1}{p_1^+}F^+\left(p_1\cdot\mathcal{H} + \boldsymbol{p}_{1\perp}\cdot\boldsymbol{\mathcal{H}}_\perp - p_1^-\mathcal{H}^+\right) \sim \mathcal{O}(\sqrt{|t|})$$

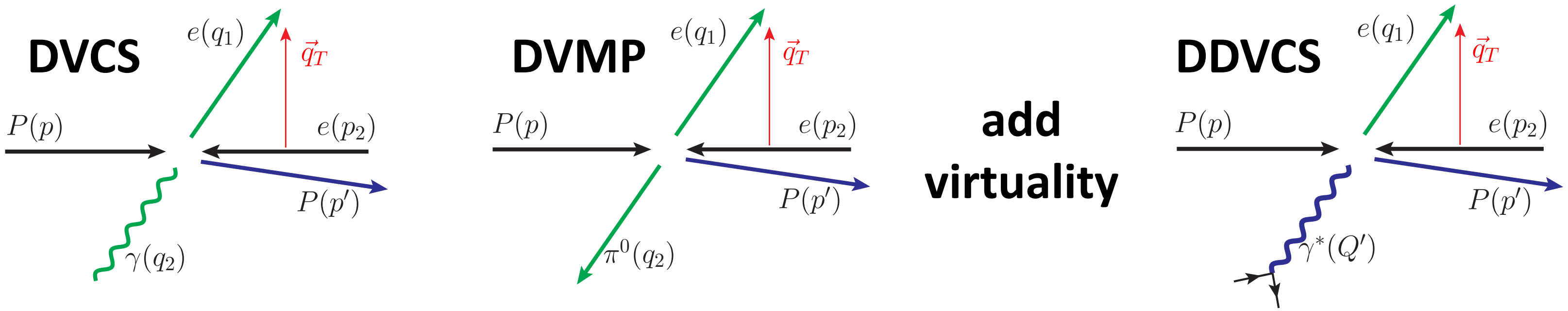
Leading power of $F\cdot\mathcal{H}$

➡ $\mathcal{M}^{(1)} \sim \mathcal{O}(1/\sqrt{|t|})$ **Higher power than n=2 contribution, but, higher power in power of α_{EM}**

$\mathcal{M}^{(2)} \sim \mathcal{O}(1/Q)$ ➡ $\mathcal{M}^{(1)}/\mathcal{M}^{(2)} \sim \mathcal{O}(Q/\sqrt{|t|})$

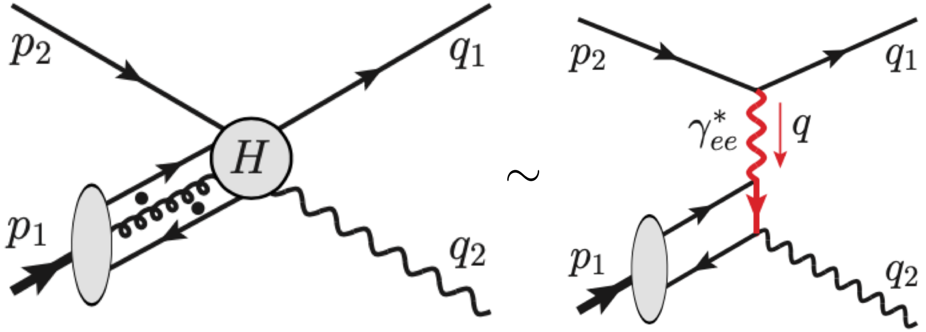
If we neglect contribution from $n\geq 3$, $\mathcal{M}^{(1+2)}_{\mathrm{SDHEP}} \sim$ is up to corrections at $\mathcal{O}(\sqrt{|t|}/Q^2)$

SDHEP with a Lepton Beam – JLab, EIC



- **DVCS:** $h(p) = \text{Proton}(p),\ h'(p') = \text{Proton}(p'),\ B(p_2) = \text{electron}(p_2),\ C(q_1) = \text{electron}(q_1),\ D(q_2) = \text{photon}(q_2)$

Stage-1:



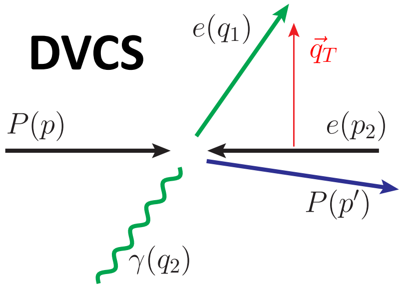
Stage-1: $2 \to 2$ exclusive process

$$\mathcal{M}_{Me\to e\gamma} = \sum_i \int_0^1 \mathrm{d}z\, D_i(z)\, C_{ie\to e\gamma}(z, q_T)$$

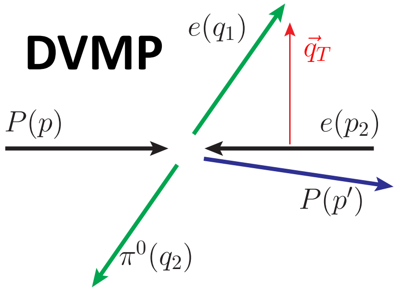
SDHEP with a Lepton Beam – JLab, EIC

(JLab, EIC, …)

DVCS

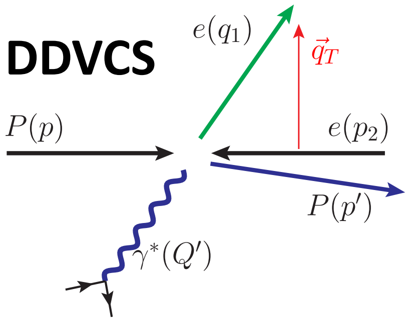


DVMP



add virtuality

DDVCS



DHQP…

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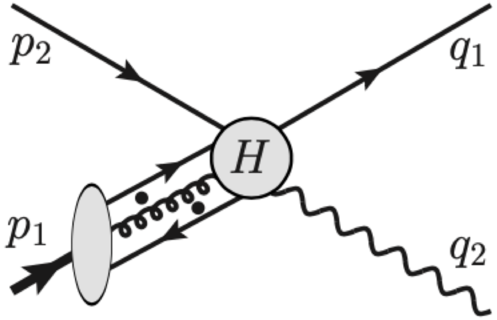
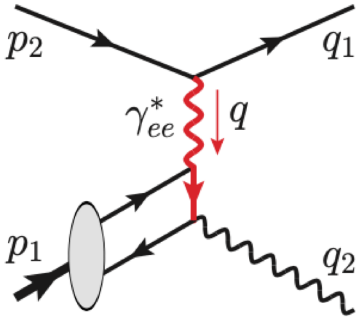
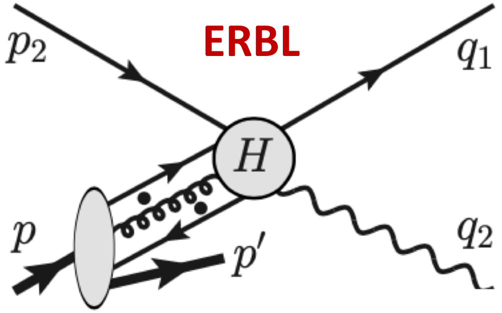


$\sim$

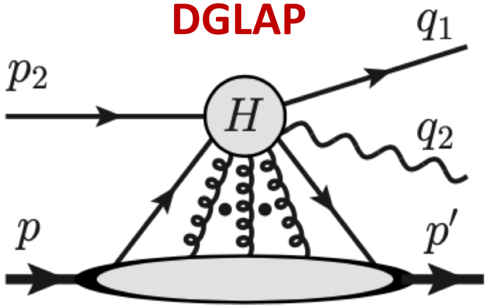


Stage-2:

ERBL



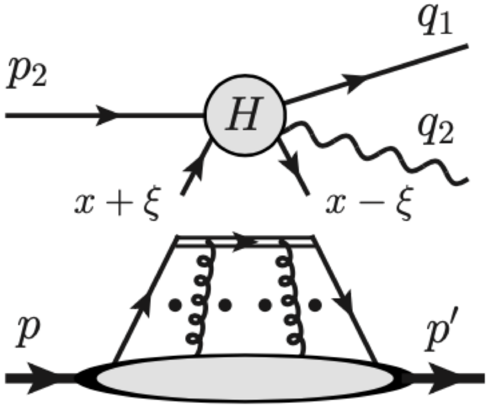
DGLAP



Stage-1: $2 \to 2$ exclusive process

$$\mathcal{M}_{Me\to e\gamma} = \sum_i \int_0^1 \mathrm{d}z\, D_i(z)\, C_{ie\to e\gamma}(z, q_T)$$

$$\mathcal{M}^{(2)}_{he\to h'e\gamma} = \sum_i \int_{-1}^1 \mathrm{d}x\, F_i^h(x,\xi,t)\, C_{ie\to e\gamma}(x,\xi,q_T),$$



$$x = \frac{(k+k')^+}{(p+p')^+}$$

$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

SDHEP with a Lepton Beam – JLab, EIC

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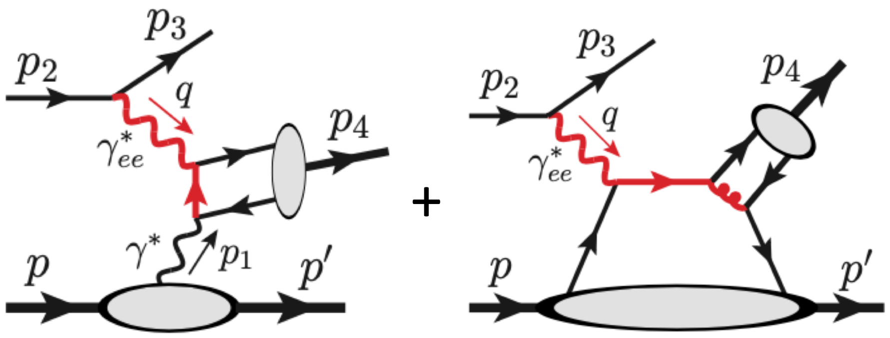
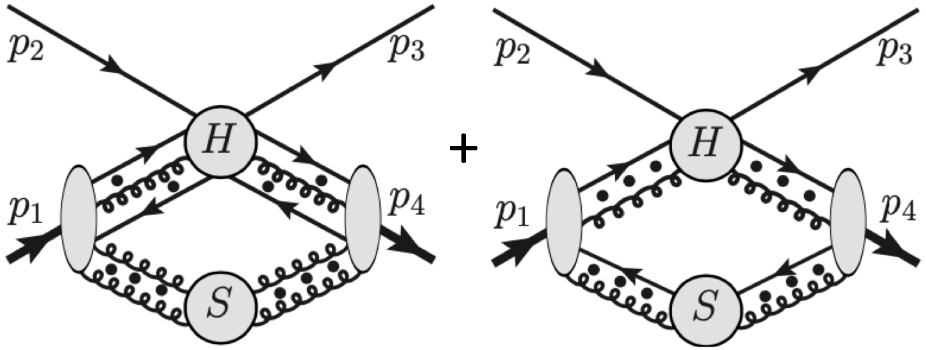
  - **Leading order diagram:**

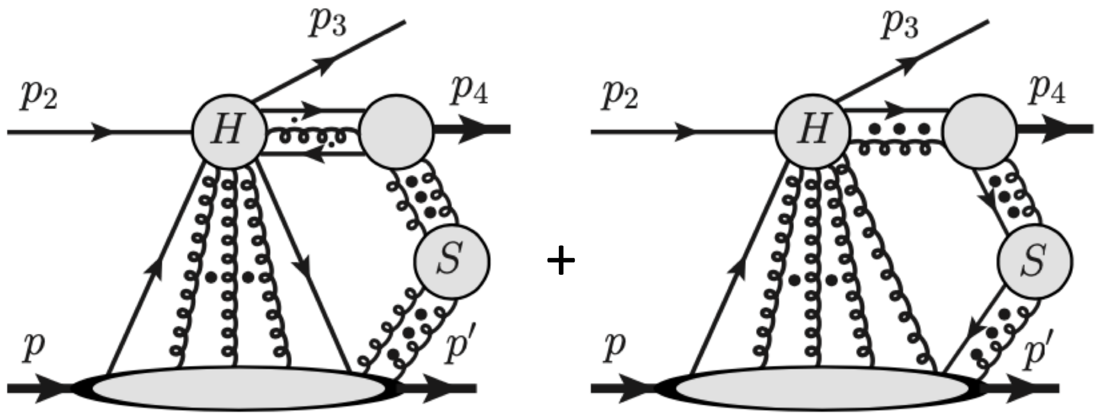


  - **Leading pinch surface:**

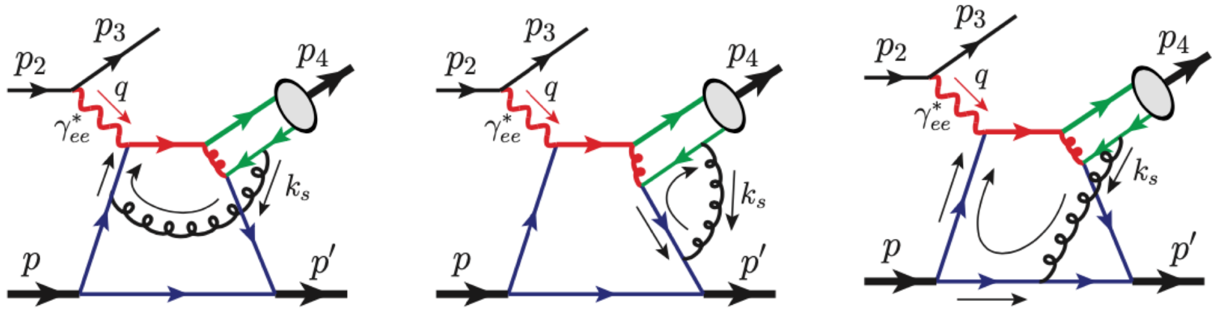


$$\mathcal{M}_{M_A e \to e M_D} = \sum_{i,j} \int_0^1 \mathrm{d}z_1\, \mathrm{d}z_2\, D_{i/A}(z_A) \times$$
$$\times\, C_{ie\to ej}(z_A, z_D, q_T)\, D_{j/D}(z_D)$$

  - **SDHEP – Leading region:**



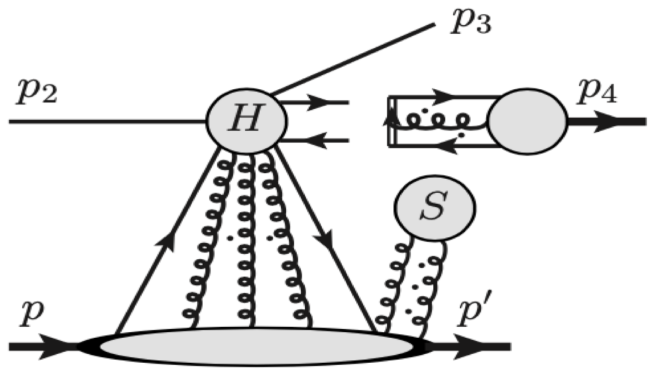
  - **SDHEP – sample diagrams:**

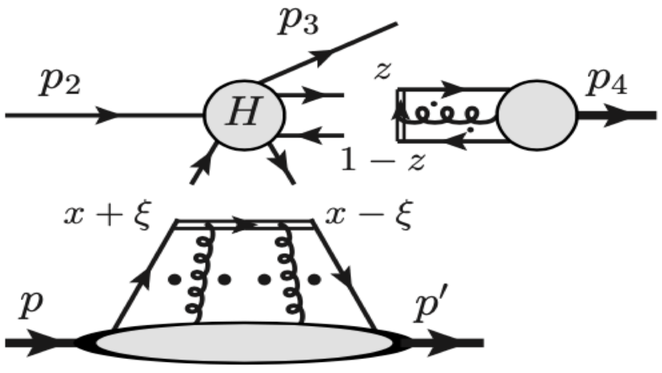


SDHEP with a Lepton Beam – JLab, EIC

❑ **DVMP:** $h(p) = \text{Proton}(p),\ h'(p') = \text{Proton}(p'),\ B(p_2) = \text{electron}(p_2),\ C(q_1) = \text{electron}(p_3),\ D(q_2) = \text{Meson}(p_4)$

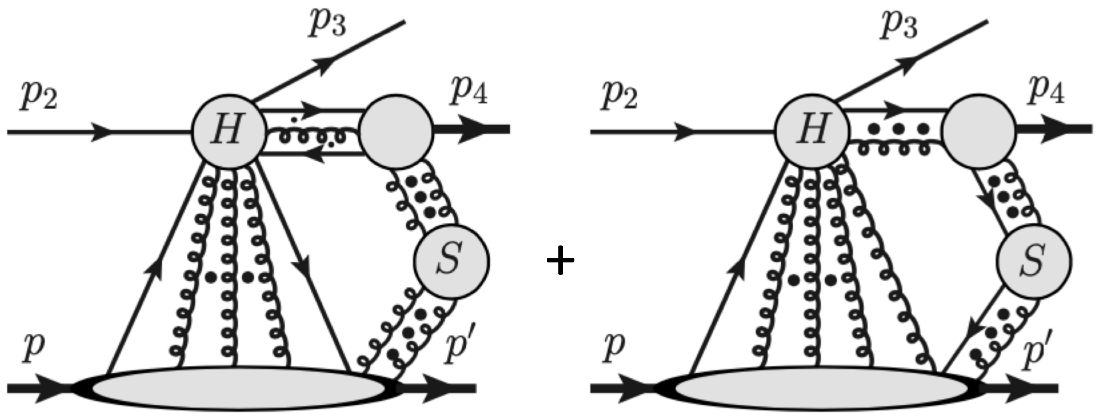
- **Factorization:**





$$\mathcal{M}_{he\to h'eM} = \sum_{i,j} \int_{-1}^{1} \mathrm{d}x \int_{0}^{1} \mathrm{d}z$$
$$\times\, F_i^{hh'}(x,\xi,t)\, C_{ie\to ej}(x,\xi;z_D;q_T)\, D_{j/D}(z_D)$$

- **SDHEP – Leading region:**



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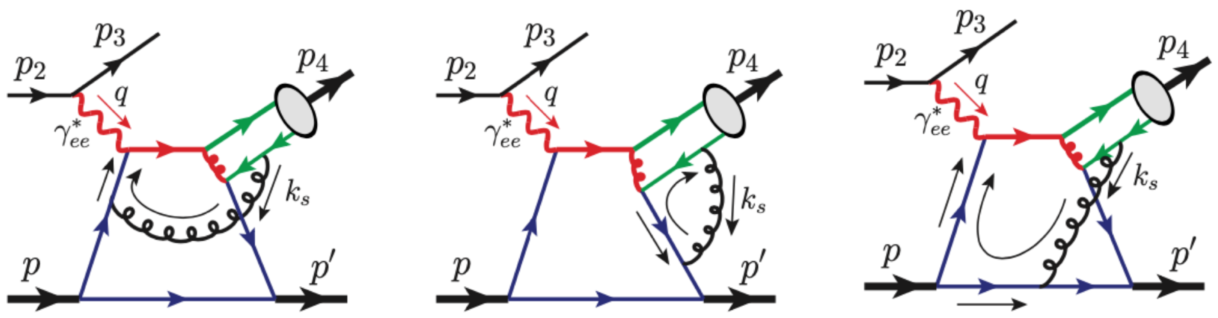
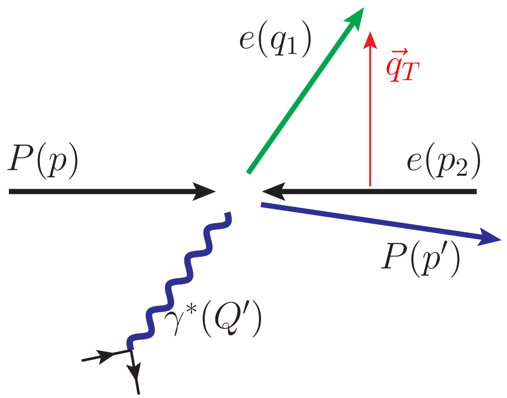


Factorization is valid, but,
not sensitive on x-dependence

SDHEP with a Lepton Beam – JLab, EIC

- **DDVCS:**

$$h(p) = \mathrm{Proton}(p),\ h'(p') = \mathrm{Proton}(p'),\ B(p_2) = \mathrm{electron}(p_2)$$
$$C(q_1) = \mathrm{electron}(q_1),\ D(q_2) = \gamma^*[\to e^+e^-](q')$$



Factorization: Can be factorized in the same way as DVCS!

- **Heavy meson production (high q_T):**

Factorization could be valid if $p_T >> m_Q$, not sure for other regions.

SDHEP with a Lepton Beam – JLab, EIC

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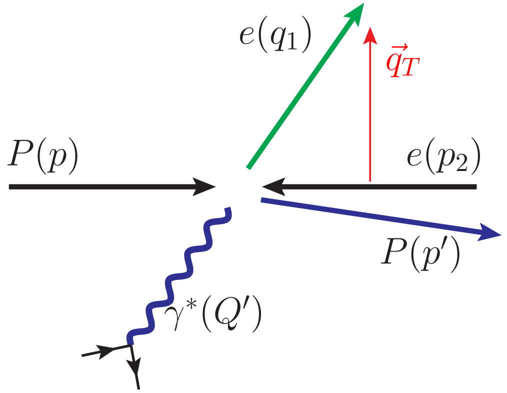


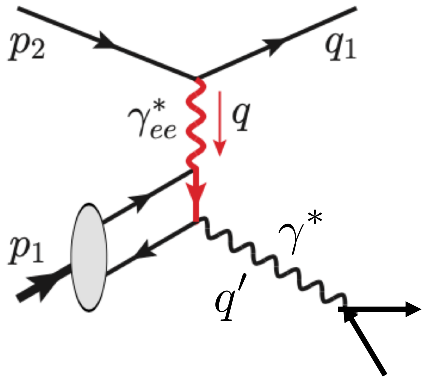
Factorization: Can be factorized in the same way as DVCS!

- **Heavy meson production (high q_T):**

Factorization could be valid if p_T >> m_Q, not sure for other regions.

- **The x-dependence on GPDs:**

As explained earlier, the type of DVCS and DVMP processes only sensitive to: $\int_{-1}^{1} dx \frac{F(x,\xi,t)}{x-\xi \pm i\varepsilon}$



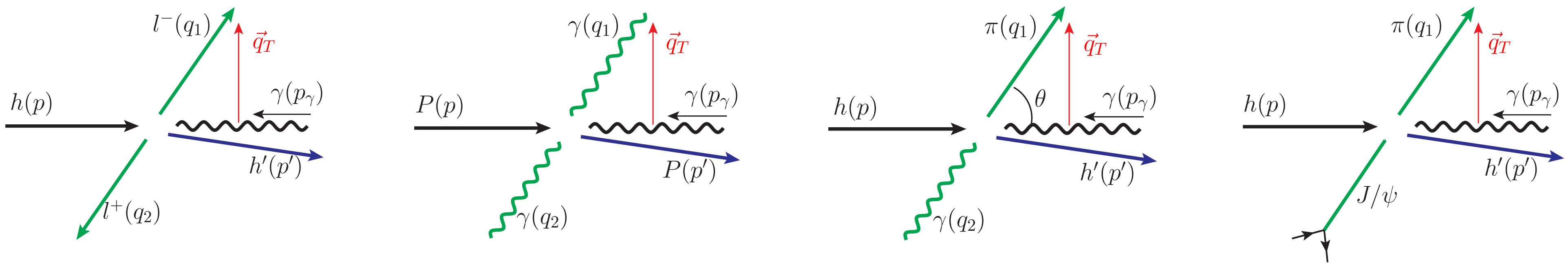
For DDVCS: **Transverse momentum flow from the final-state lepton and the virtual photon is sensitive to the virtuality of the dilepton**

$$Q'^2 \equiv q'^2 = \left(\frac{2\xi}{x_B(1+\xi)} - 1\right)$$

Direct sensitive to external variable, x_B, directly sensitive to q_T

Experimental challenge to distinguish the scattered lepton ! ***But, needs luminosity!!!***

SDHEP with a Photon Beam – JLab, EIC



(JLab Hall-D, …)

- **Dilepton & Diphoton production:**

A. Pedrak, et al. Phys.Rev.D96 (2017)074008, …

Both n=1 and n=2 should contribution, and factorizable

- **Real photon + meson pair production:**

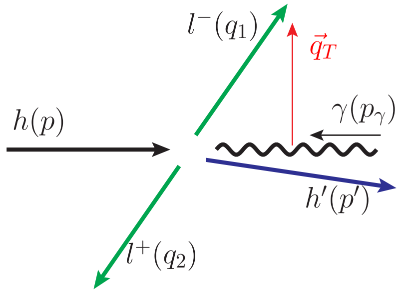
G. Duplancic, et al. JHEP 11 (2018) 179, …

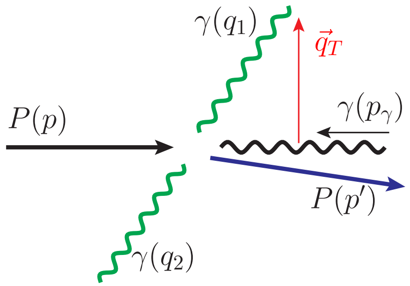
The n=1 channel is forbidden for a charge meson: $\pi^{\pm}$, or transversely polarized vector meson, ρ_T ,

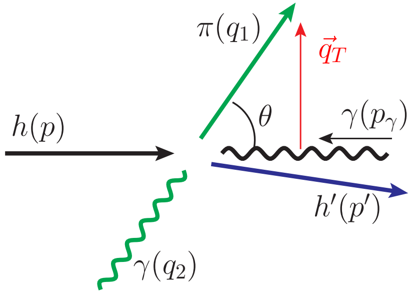
but, allowed for the production of a longitudinally polarized vector meson like ρ_L .

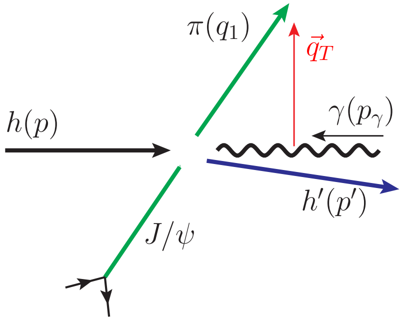
Factorization arguments are the same as that for DVMP.

SDHEP with a Photon Beam – JLab, EIC









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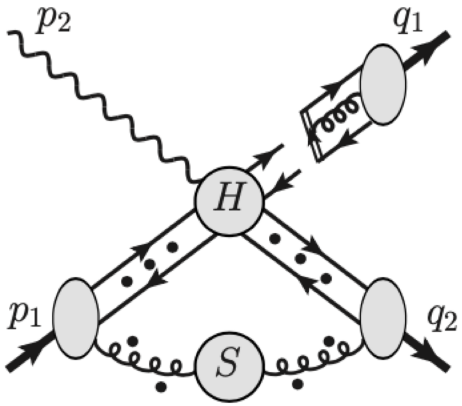
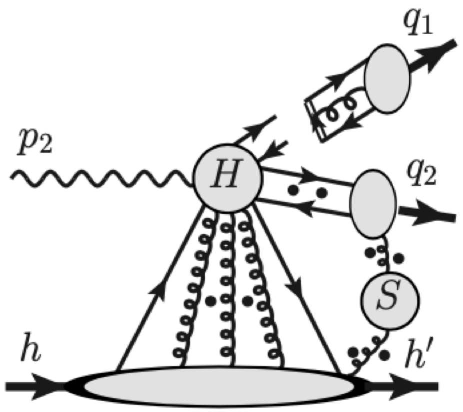
  Factorization arguments are the same as that for DVMP.

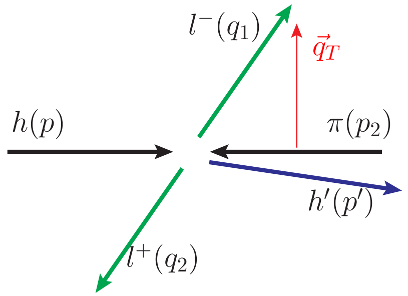
- **Light meson pair production – New:**

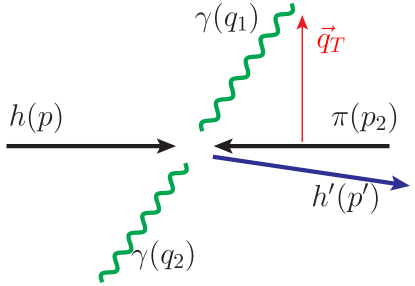
$$\mathcal{M}_{h\gamma\to h' M_C M_D} = \sum_{i,j,k} \int_{-1}^{1} \mathrm{d}x \int_{0}^{1} \mathrm{d}z_C\, \mathrm{d}z_D\; F_i^{hh'}(x,\xi,t) \\ \times C_{i\gamma\to jk}(x,\xi; z_C, z_D; q_T)\, D_{j/C}(z_C)\, D_{k/D}(z_D)$$





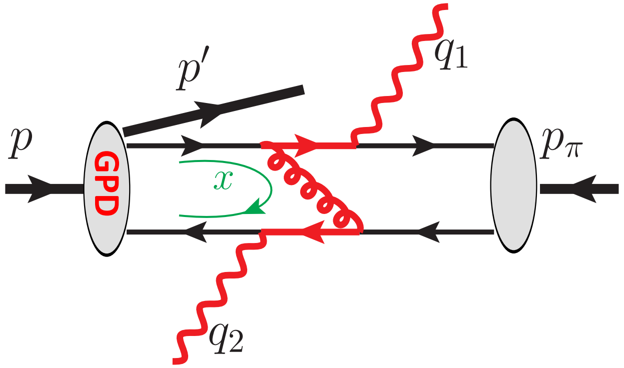
SDHEP with a Light Meson Beam – J-PARC, Amber





+ photon-meson pair, meson-meson pair

(Complementary to JLab/EIC)



❑ Two-photon production: $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$

- **Kinematical observables:**

t, ξ, q_T

- $t = (p - p')^2$
- $\xi = (p^+ - p'^+)/(p^+ + p'^+)$

Hard scale: $q_T \gg \Lambda_{\rm QCD}$

Soft scale: $t \sim \Lambda^2_{\rm QCD}$

- **Factorization:**

[suppressing DA factor]

$$\mathcal{M}(t, \xi, q_T) = \int_{-1}^{1} \mathrm{d}x\, F(x, \xi, t; \mu) \cdot C(x, \xi; q_T/\mu) + \mathcal{O}(\Lambda_{\rm QCD}/q_T)$$

$$\Longrightarrow \quad \frac{\mathrm{d}\sigma}{\mathrm{d}t\; \mathrm{d}\xi\; \mathrm{d}q_T} \sim |\mathcal{M}(t, \xi, q_T)|^2$$

$x \leftrightarrow q_T$

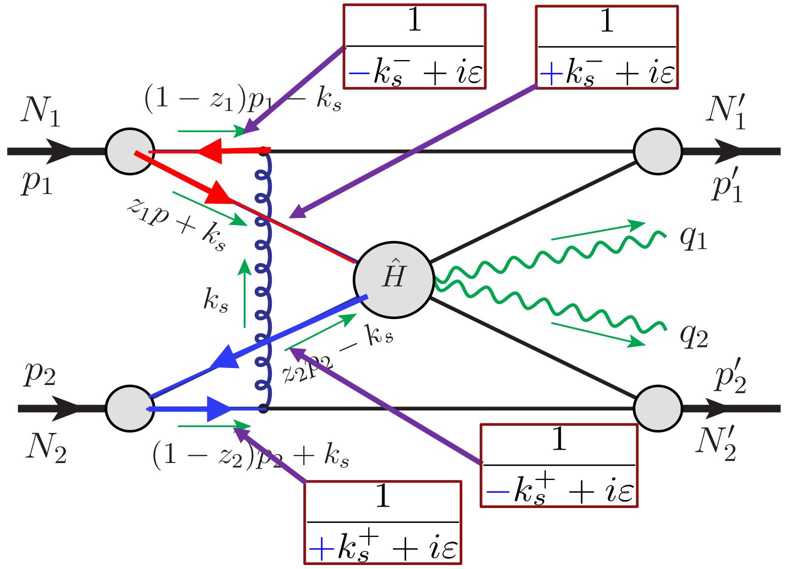
q_T distribution is "conjugate" to x distribution

Jefferson Lab

Discussion & more Opportunities

- **Why single-diffractive?**

  We need diffractive process to keep proton intact.
  But, double diffractive process is not factorizable!



Discussion & more Opportunities

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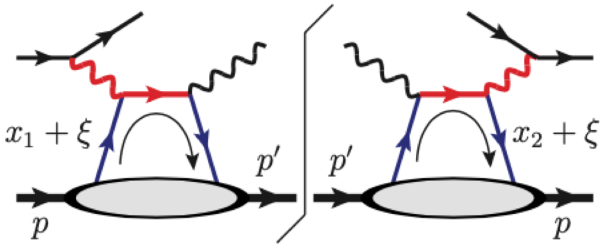
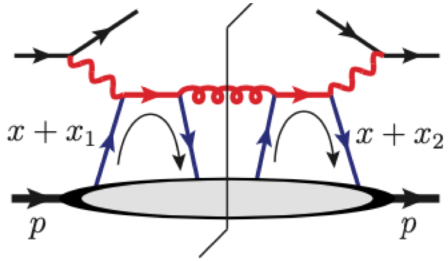
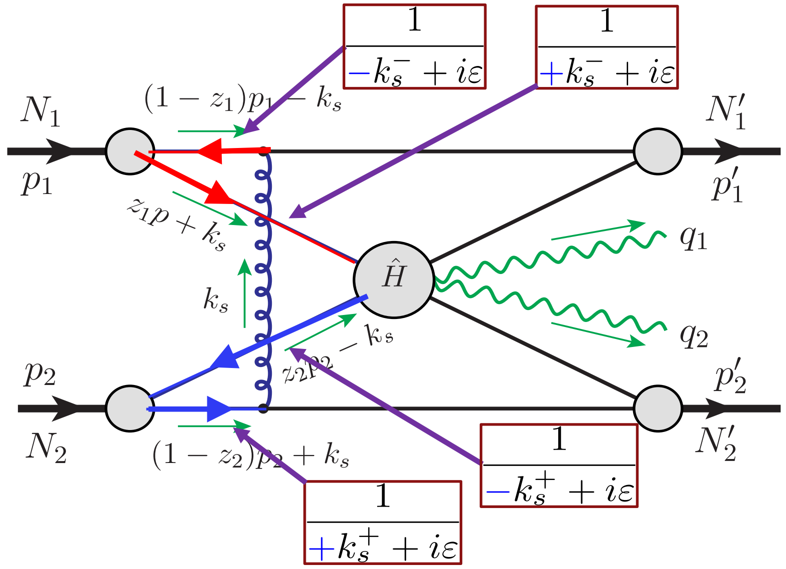
- **Connection to high-twist inclusive production:**

DVCS



Twist-4 DIS





Not sensitive to the loop momentum fraction: x_i

Discussion & more Opportunities

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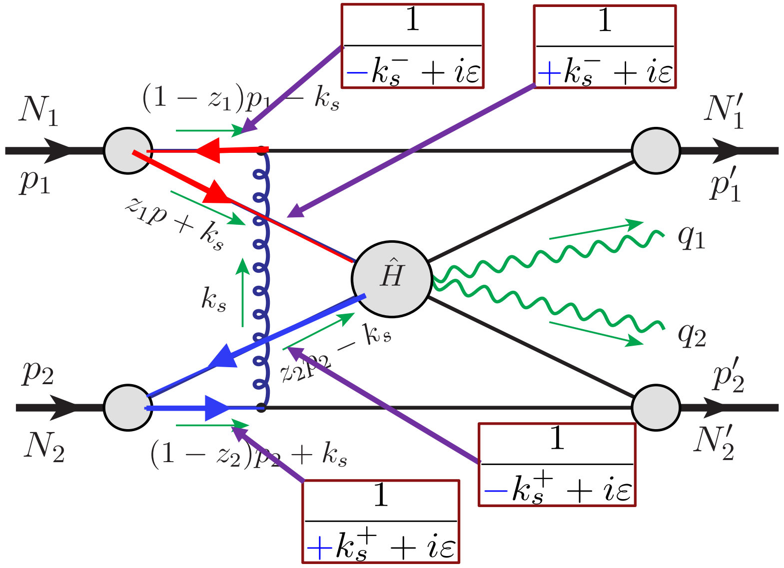

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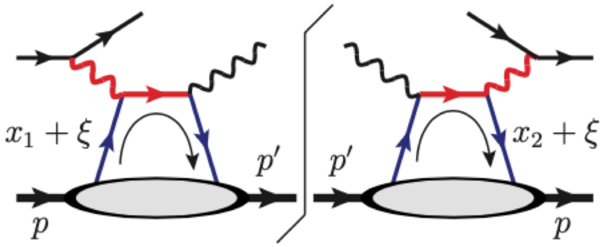

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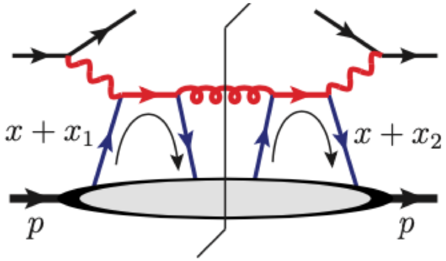
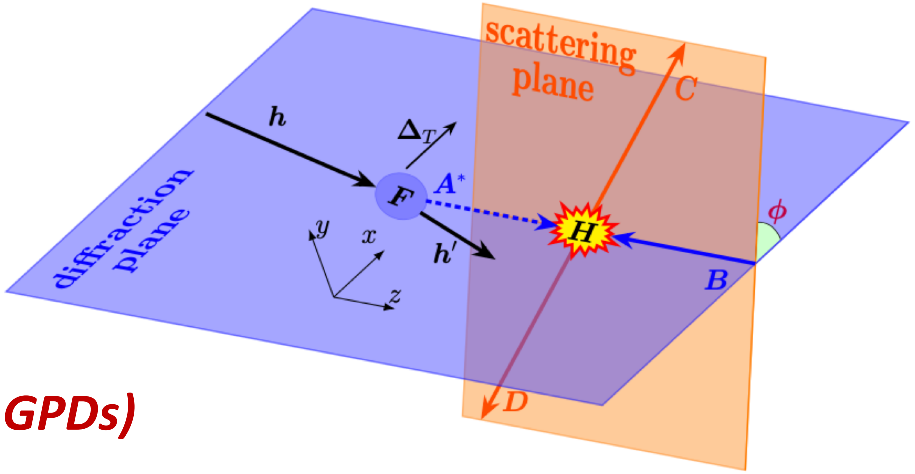


❑ More opportunities:

- Diffractive plane
- Exclusive hard scattering plane
- Angular modulation between the two planes

➡ *Selection from different exchange state A* (or different GPDs)*



Summary and Outlook

- **GPDs are fundamental parton correlation functions:**
  - Carry rich information on emergent hadron properties (mass, spin, …) from QCD/parton dynamics
  - Are responsible for the tomographic images of confined quarks and gluons inside a bound hadron
  - Provide the much needed hints on how confined quarks/gluons respond to the probing scale, …

  Extracting their x-dependence from experimental observable(s) is non-trivial, but, full of opportunities, …

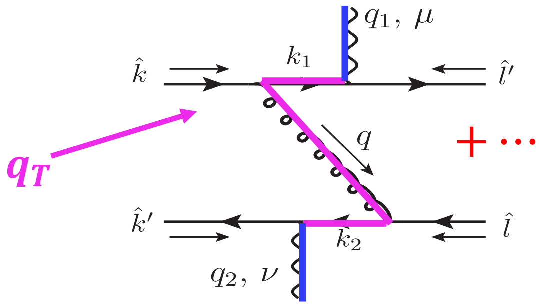
- **Introduced the single diffractive $2 \to 3$ hard exclusive processes (SDHEP) for extracting GPDs, …**
  - Explored both necessary and sufficient conditions for the leading power QCD factorization
  - Covered all existing/known processes for extracting GPDs, plus ideas for new observables, …
  - Introduced a path forward to identify new SDHEPs that could be sensitive to x-dependence of GPDs
  - Angular modulation between diffractive plane and hard scattering plane could provide unique opportunity to separate various GPDs

  Following the pioneering work on GPDs over 25 years ago, we now have renewed opportunities for exploring the physics of GPDs and the confined phenomena of QCD. Such effort in both theory And experiment should be strongly supported!

Thanks!

Meson-initiated Single Diffractive Hard Processes

❑ Hard part for A-type:



New

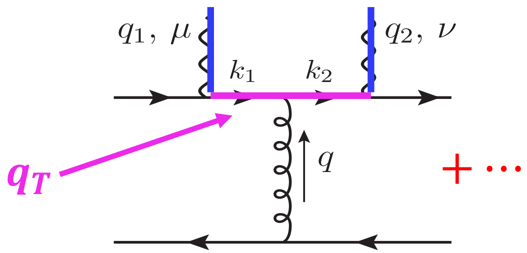
$\kappa = 4\, q_T^2/\hat{s}$

- **Gluon propagator** $q^2 = -\frac{\hat{s}}{4}\left[\left(2z_1 - 1 - \sqrt{1-\kappa}\right)\left(2z_2 - 1 - \sqrt{1-\kappa}\right) + \kappa\right]$

$$\Longrightarrow \quad \mathcal{M} \propto \int_0^1 \mathrm{d}z_1\, \mathrm{d}z_2\, \frac{\phi(z_1)\phi(z_2)}{(1-z_1)\,(1-z_2)\,\left[\left(2z_1 - 1 - \sqrt{1-\kappa}\right)\left(2z_2 - 1 - \sqrt{1-\kappa}\right) + \kappa\right]}$$

- **Change q_T changes the z_1-z_2 integral.**
- **$\mathrm{d}\sigma/\mathrm{d}q_T^2$ provides sensitivity to the DA's functional form of z.**

❑ Hard part for B-type:



Like "time-like" form factor

- **Gluon propagator** $q^2 = z_2(1-z_1)\hat{s}$

$$\Longrightarrow \quad \mathcal{M} \propto \int_0^1 \mathrm{d}z_1\, \mathrm{d}z_1\, \frac{\phi(z_1)\phi(z_2)}{z_1\,(1-z_1)\, z_2\,(1-z_2)} \sim \left[\int_0^1 \mathrm{d}z\, \frac{\phi(z)}{z\,(1-z)}\right]^2$$

- ***Not* sensitive to DA functional form.**
- **Relies on $\phi(z) = 0$ at end points.**
- **Sudakov resummation could suppress the end-point sensitivity.**

Li, Sterman, 1992

Numerical results

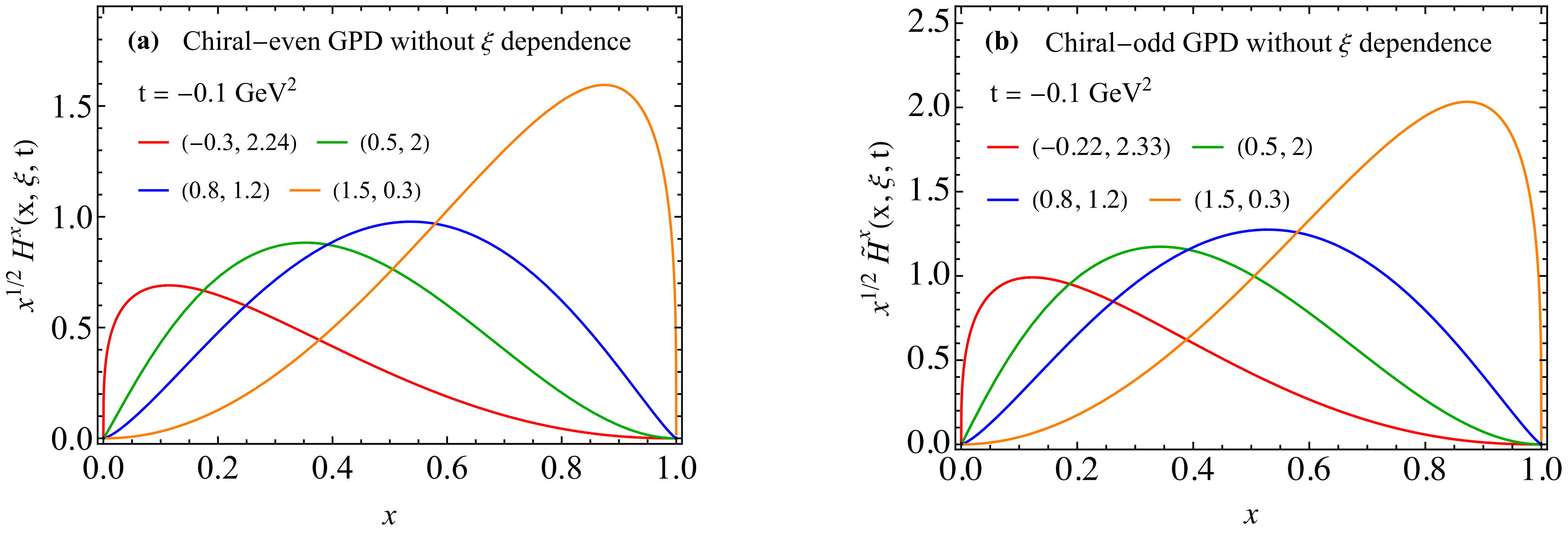
❑ GPD models – simplified GK model:

Goloskokov, Kroll
hep-ph/0501242
arXiv: 0708.3569
arXiv: 0906.0460

$$H_{pn}(x,\xi,t) = \theta(x)\, x^{-0.9\,(t/\mathrm{GeV}^2)} \frac{x^\rho (1-x)^\tau}{B(1+\rho, 1+\tau)}$$

$$\widetilde{H}_{pn}(x,\xi,t) = \theta(x)\, x^{-0.45\,(t/\mathrm{GeV}^2)} \frac{1.267\, x^\rho (1-x)^\tau}{B(1+\rho, 1+\tau)}$$

- **Neglect $E, \widetilde{E}$. Neglect evolution effect.**
- **Tune (ρ, τ) to control x shape.**
- **Fix DA: $D(z) = N\, z^{0.63}(1-z)^{0.63}$**



Numerical results

$$\frac{d\sigma}{dt\,d\xi\;dq_T} \sim |H(x,\xi,t)|^2$$

Relative q_T shape

$$\frac{\sigma_{\text{tot}}^{-1} d\sigma/dq_T}{\text{some shape func}}$$

$$\sigma_{\text{tot}} = \int_{1\ \text{GeV}}^{\sqrt{\hat{s}}/2} dq_T \frac{d\sigma}{dt d\xi dq_T}$$