TMDs and Precision Theory

TMDs = Transverse Momentum Dependent Distributions

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MIT

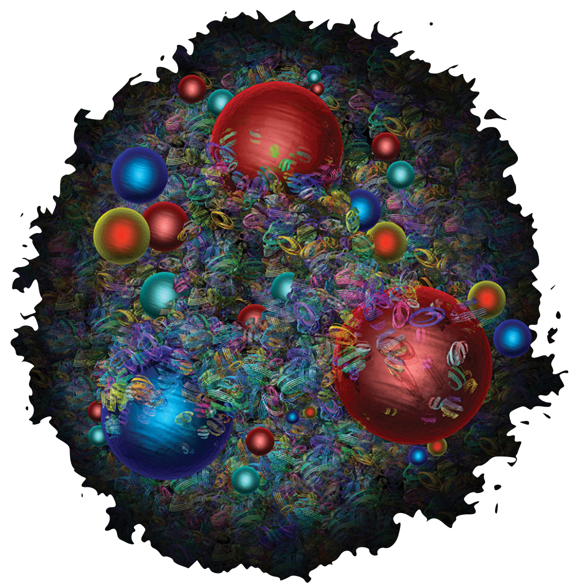
CFNS Workshop on
“Theory for the EIC in the next decade”
MIT, Cambridge
September 20, 2022

Massachusetts Institute of Technology

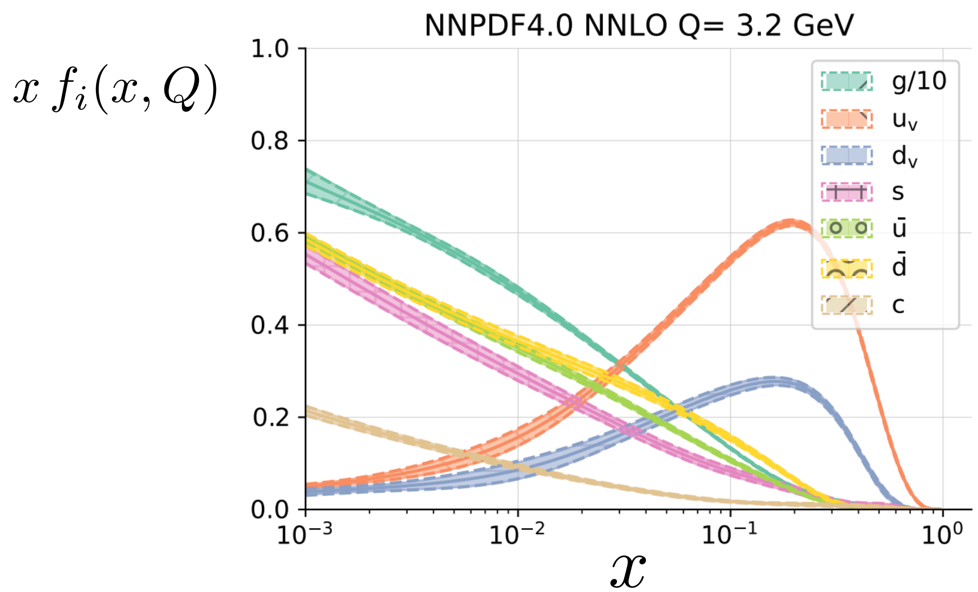
U.S. DEPARTMENT OF ENERGY | Office of Science

TMD Collaboration

Unravelling the Mysteries of Relativistic Hadronic Bound States



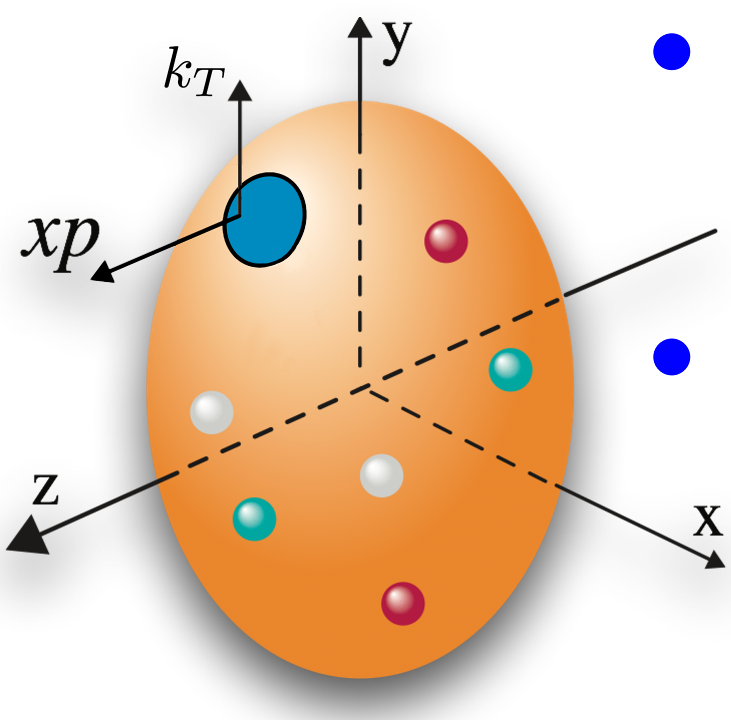
- Parton distributions provide fundamental description



Snapshot of the bound constituents

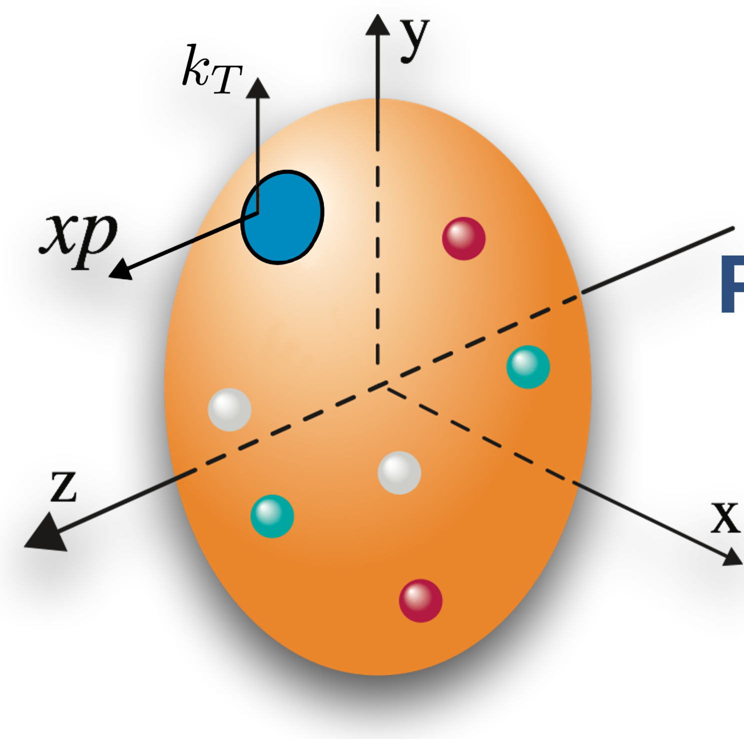
Learn about confined quarks and gluons

- Next frontier includes:
  - more detailed structure (spin, gluons, ion targets, …)
  - precision info on hadronization (frag. functions, …)
    - multi-dimensional distributions (GPDs, TMDs, …)
  - new probes (eg. qgq operators rather than just qq or gg)
    - new phenomena in asymptotic regimes (eg. saturation at small-x)



targets for EIC

TMDs



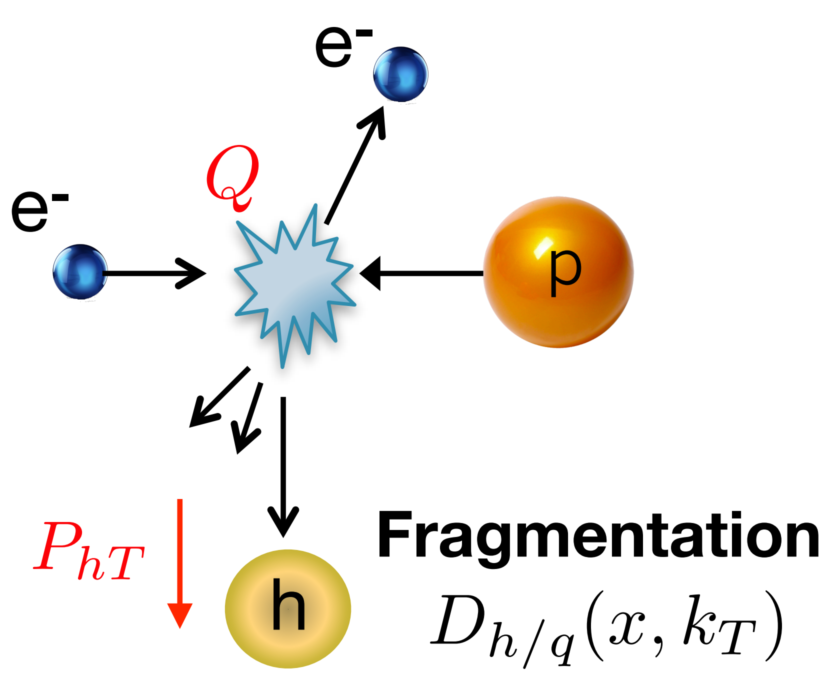
3D momentum distributions

PDF $f_{q/P}(x, \vec{k}_T, \mu, \zeta)$ **of confined partons**

FF $D_{\pi/q}(x, \vec{k}_T, \mu, \zeta)$ **of hadronizing partons**

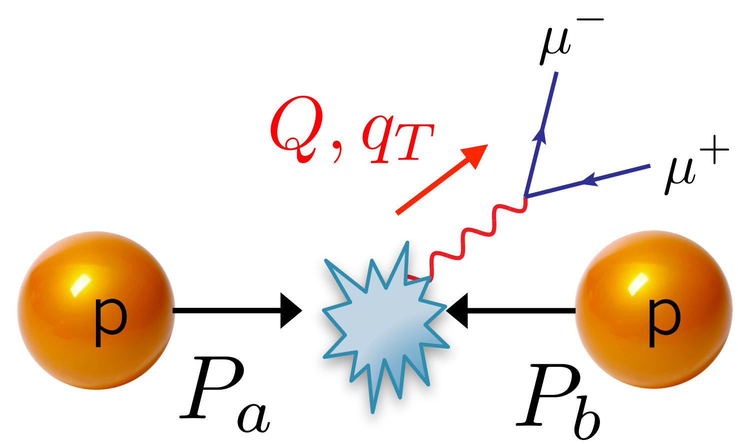
Semi-Inclusive DIS

$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$



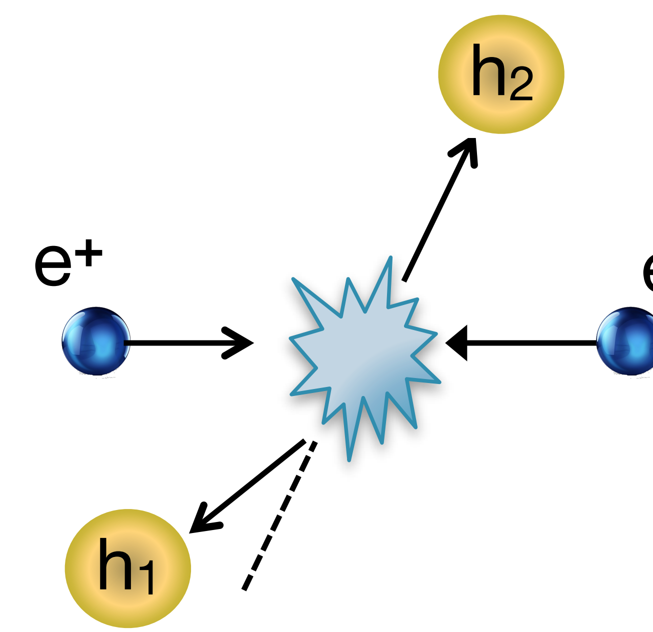
Drell-Yan

$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$



Dihadron in e+e-

$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$



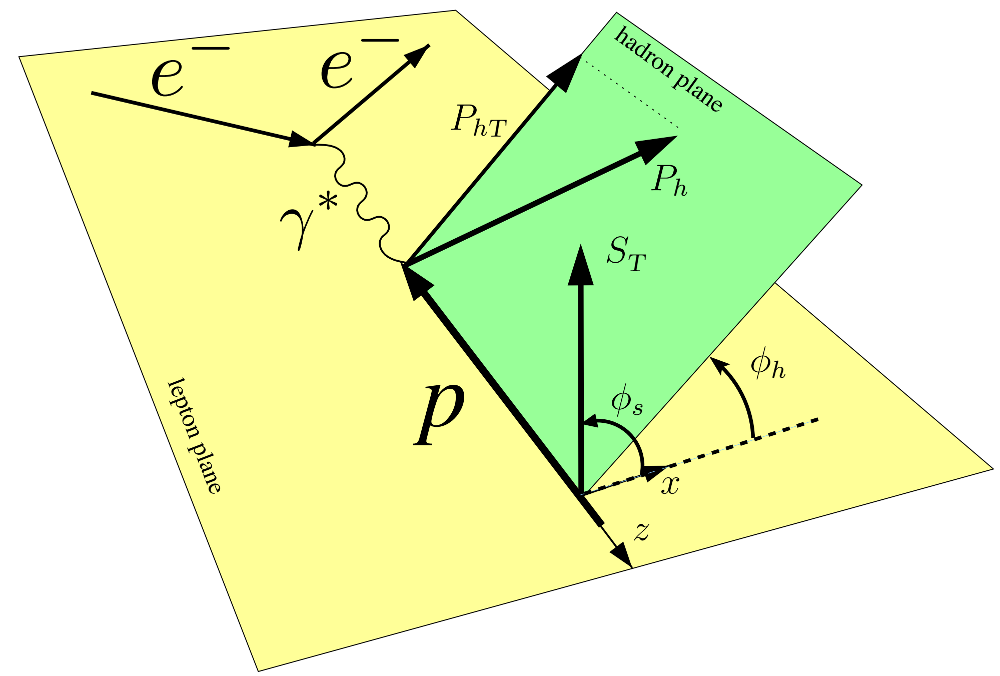
Two Scale probe: $q_T \ll Q$ $\rightarrow$ $\Lambda_{\text{QCD}} \sim q_T$, $\Lambda_{\text{QCD}} \ll q_T$

Observables (leading order in $q_T \ll Q$)

SIDIS with polarized electron & proton: $e^- p \xrightarrow{\gamma^*} e^- h X$

Key target for EIC

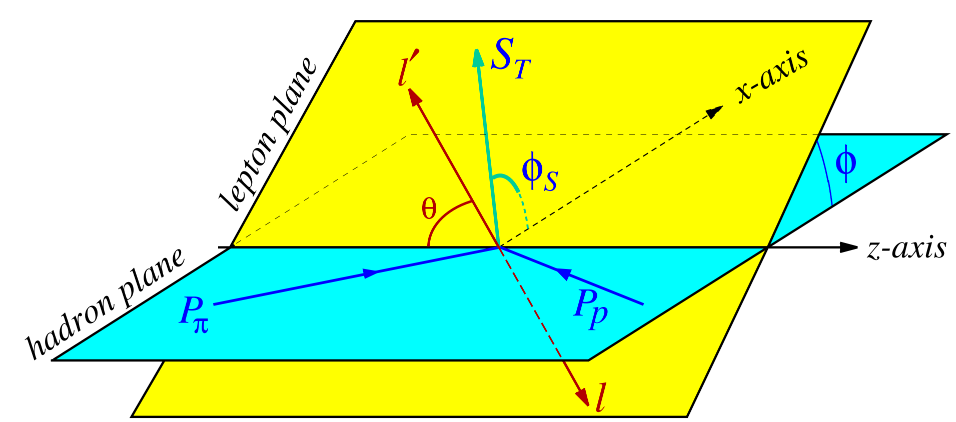
$$\frac{d^6\sigma}{dx_B\, dy\, dz_h\, d\phi_S\, d\phi_h\, dP_{hT}^2} = \frac{\alpha_{em}^2}{x_B\, y\, Q^2}\left(1 - y + \frac{1}{2}y^2\right)\Big[F_{UU,T} + \cos(2\phi_h)\, p_1\, F_{UU}^{\cos(2\phi_h)}$$
$$+ S_L \sin(2\phi_h)\, p_1\, F_{UL}^{\sin(2\phi_h)} + S_L\, \lambda p_2\, F_{LL}$$
$$+ S_T \sin(\phi_h - \phi_S)\, F_{UT,T}^{\sin(\phi_h-\phi_S)}$$
$$+ S_T \sin(\phi_h + \phi_S)\, p_1\, F_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+ \lambda\, S_T \cos(\phi_h - \phi_S)\, p_2\, F_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+ S_T \sin(3\phi_h - \phi_S)\, p_1\, F_{UT}^{\sin(3\phi_h-\phi_S)}\Big]$$



TMDPDFs * TMDFFs: $f_{1,p} D_{1,h},\ h_{1,p}^{\perp} H_{1,h}^{\perp},\ \ldots$

Drell-Yan with pol. proton: $\pi p \xrightarrow{\gamma^*} \ell^+ \ell^- X$

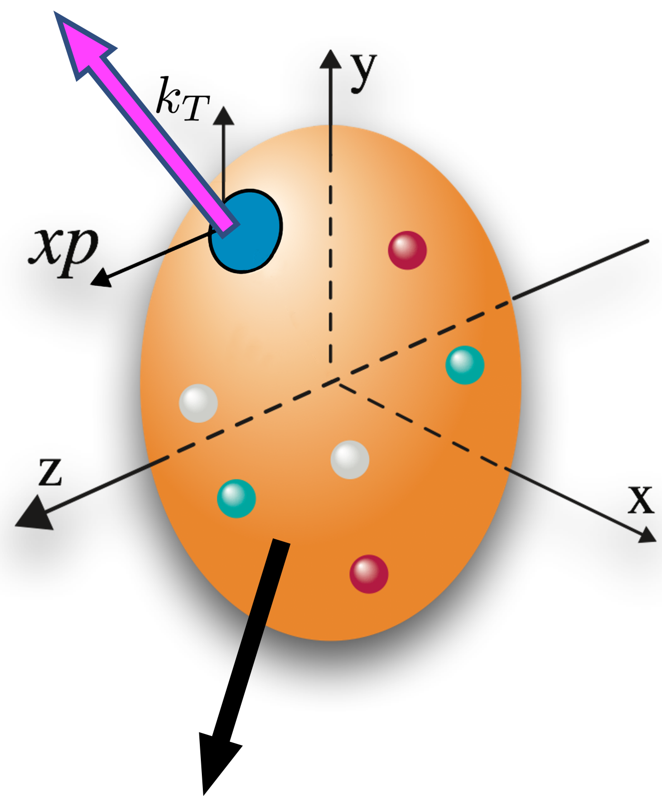
$$\frac{d\sigma}{d^4q\, d\Omega} = \frac{\alpha_{em}^2}{F\, Q^2}\left\{\left[(1 + \cos^2\theta) F_{UU}^1 + \sin^2\theta \cos(2\phi) F_{UU}^{\cos 2\phi}\right] + \ldots\right\}$$



products of TMDPDFs: $f_{1,\pi} f_{1,p},\ h_{1,\pi}^{\perp} h_{1,p}^{\perp},\ \ldots$

TMDs with Polarization

Quark Polarization Γ



Nucleon Polarization

$S_L, \overrightarrow{S}_T$

Leading Quark TMDPDFs

Nucleon Spin   Quark Spin

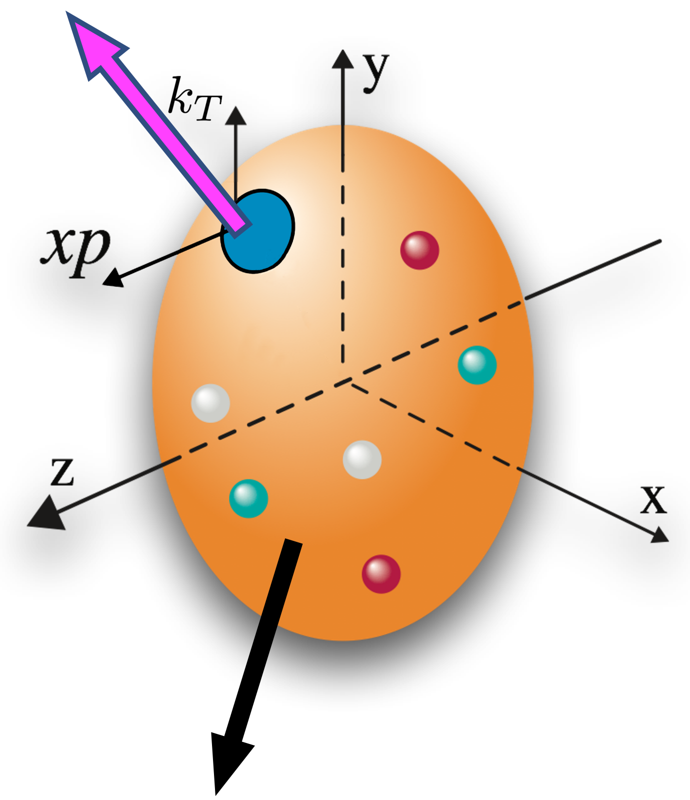
		Quark Polarization		
		Un-Polarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
Nucleon Polarization	**U**	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity $h_{1T}^\perp$ = − Pretzelosity

8 different TMDs which encode spin-momentum correlations

eg. $$f_i^{[\gamma^+]}(x,\vec{k}_T) = f_1(x,k_T) - \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{T\sigma}}{M} f_{1T}^\perp(x,k_T)$$

TMDs with Polarization

Quark Polarization Γ



Nucleon Polarization

$S_L, \overrightarrow{S}_T$

Leading Quark TMDPDFs

Nucleon Spin   Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity $h_{1T}^\perp$ = − Pretzelosity

Also have index i=u,d,g,... just like longitudinal PDFs

Have analogous table for TMD Fragmentation functions.

Explore the 3D Structure of Hadrons with these distributions!

Field Theory

- **Rigorous Factorization Theorems**

Cross Sections ⟷ TMDs

$$\frac{d\sigma}{dQ\,dY\,dq_T^2} = H(Q,\mu) \int d^2\vec{b}_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\, f_q(x_a,\vec{b}_T,\mu,\zeta_a)\, f_q(x_b,\vec{b}_T,\mu,\zeta_b)\Big[1+\mathcal{O}\Big(\frac{q_T^2}{Q^2}\Big)\Big]$$

CSS (Collins, Soper, Sterman)

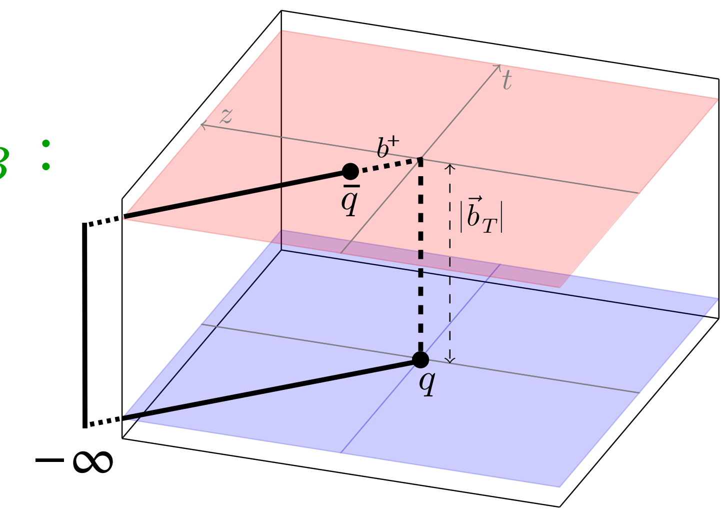
SCET (Soft Collinear Effective Theory)

- **TMD Definitions** (constructions & schemes)

→ full understanding now available

$$f_q(x,\vec{b}_T,\mu,\zeta) \sim Z_{\rm uv}\,\langle p|O_B|p\rangle \Big/ \sqrt{\langle 0|O_S|0\rangle}$$

O_B :



→ tractable methods with **Lattice QCD** (see Yong Zhao's talk)

- **Universality** same TMDs in DY, SIDIS, e^+e^-

<u>but</u> with sign flip for Sivers and Boer-Mulders:

$$f_{1T}^{\perp\,\rm SIDIS} = -f_{1T}^{\perp\,\rm DY}$$
$$h_1^{\perp\,\rm SIDIS} = -h_1^{\perp\,\rm DY}$$

Brodsky-Huang-Schmidt; Collins, …

directly probes final/initial state interactions with "spectator" partons in the proton!

Field Theory

- **Evolution**

Sum large logarithms: $\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$

$$f_q(x, \vec{b}_T, \mu, \zeta) = \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^q(\mu', \zeta_0)\right] \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln \frac{\zeta}{\zeta_0}\right] f_q(x, \vec{b}_T, \mu_0, \zeta_0)$$

CS kernel — Boundary condition

Perturbative γ_i^q : Leading Log (LL) → Next-to-leading log (NLL) → NNLL → N^{3}LL → N^{4}LL

Nonperturbative γ_ζ^q : fit to data using models, or calculate with Lattice QCD

- **Operator Product Expansion and (non)perturbative inputs**

$$f_{i/h}(x, b_T, \mu, \zeta) = f_{i/h}^{\text{pert}}(x, b^*(b_T), \mu, \zeta)\, f_{i/h}^{\text{NP}}(x, b_T)$$

perturbative — nonperturbative (models, lattice)

$b_T^{-1} \sim q_T \gg \Lambda_{\text{QCD}}$ — $b_T^{-1} \sim q_T \sim \Lambda_{\text{QCD}}$

OPE: $$f_{i/h}^{\text{pert}}(x, b_T, \mu, \zeta) = \sum_j \int \frac{dy}{y} C_{ij}(x/y, b_T, \mu, \zeta) f_{j/h}(y, \mu)$$

LO (α_s^0) → **NLO** (α_s) → **NNLO** (α_s^2) → **N^{3}LO** (α_s^3)

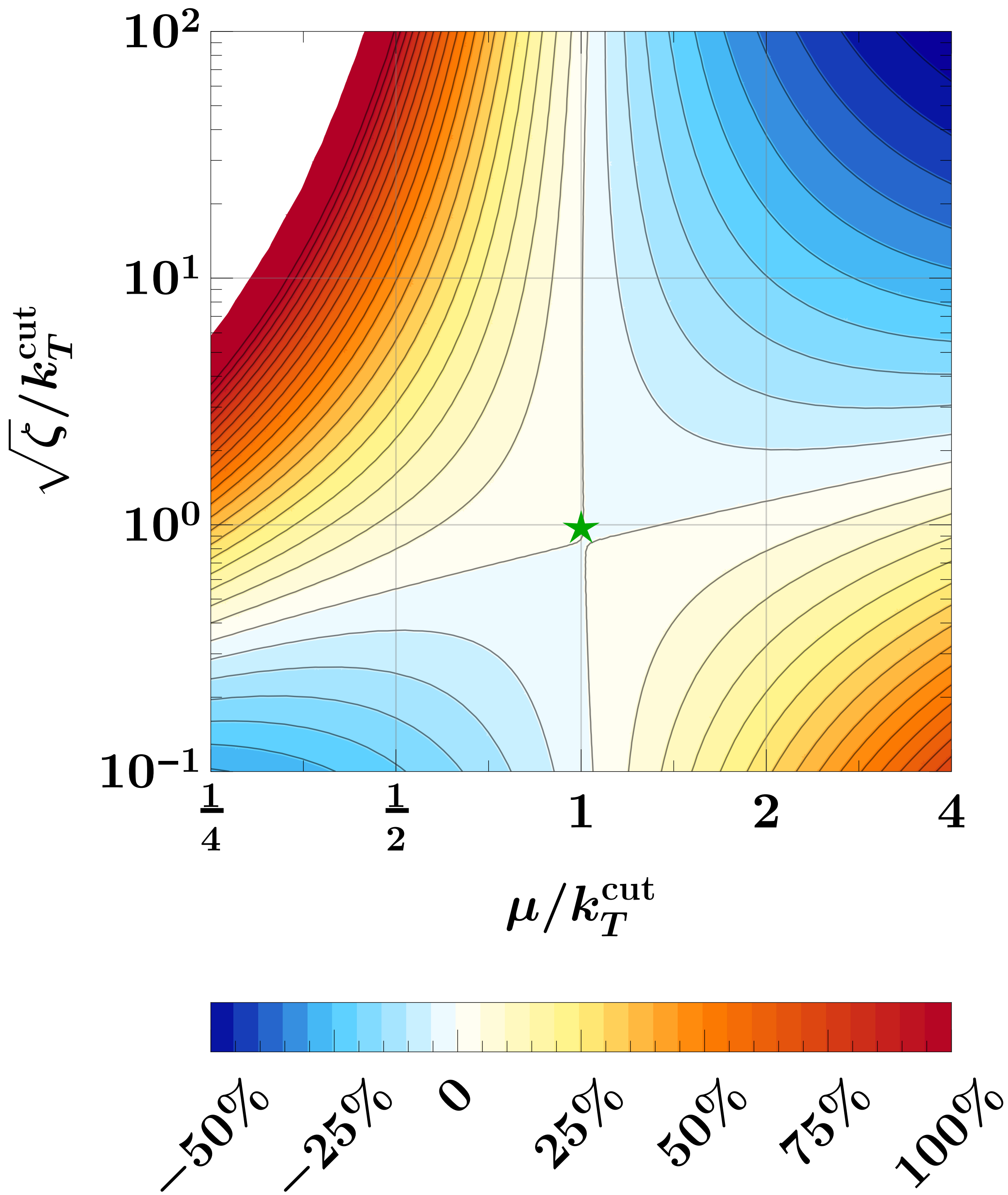
longitudinal PDFs

Example of question we now know how to answer

$$\int^{\mathrm{k_T^{cut}}} d^2k_T \; f(x, k_T, \mu, \zeta) \stackrel{?}{=} f(x, \mu)$$

$$\int^{k_T^{\mathrm{cut}}} \mathrm{d}^2\vec{k}_T \, f_d(x, \vec{k}_T, \mu, \zeta)/f_d(x, \mu) - 1$$

$x = 0.01,\, k_T^{\mathrm{cut}} = 10\,\mathrm{GeV}$

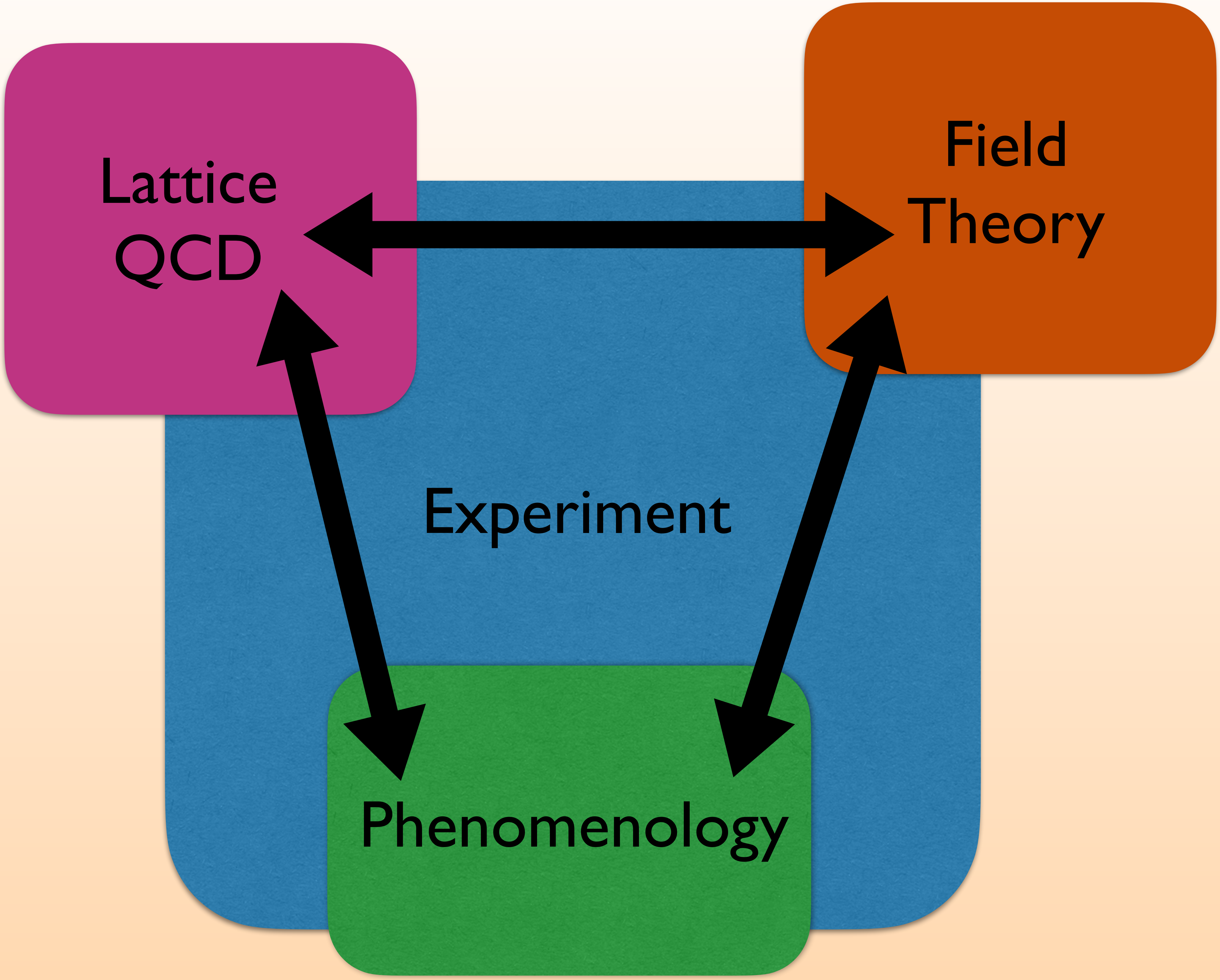


- Intuitive expectation is robust in the vicinity of $\mu = \sqrt{\zeta} = k_T^{\mathrm{cut}}$
- Sizable corrections away from this region
- Uncertainty from $q_T^{\mathrm{cut}} b_T^{\mathrm{cut}} \gg 1$ and nonperturbative corrections are small (at 1% level)
- Perturbative uncertainty at N3LL dominates (at 4% level)

$$\boxed{\int^{k_T^{\mathrm{cut}}} d^2\vec{k}_T \; f^{\mathrm{TMD}}(x, k_T, \mu = k_T^{\mathrm{cut}}, \zeta) = f^{\mathrm{coll}}(x, \mu = k_T^{\mathrm{cut}})}$$

[Ebert, Michel, IS, Sun, 2201.07237]

TMDs



TMD Handbook

August 16, 2022

A modern introduction to the physics of
Transverse Momentum Dependent distributions

12 chapters
470 pages
(soon to be released)

Renaud Boussarie
Matthias Burkardt
Martha Constantinou
William Detmold
Markus Ebert
Michael Engelhardt
Sean Fleming
Leonard Gamberg
Xiangdong Ji
Zhong-Bo Kang
Christopher Lee
Keh-Fei Liu
Simonetta Liuti
Thomas Mehen *
Andreas Metz
John Negele
Daniel Pitonyak
Alexei Prokudin
Jian-Wei Qiu
Abha Rajan
Marc Schlegel
Phiala Shanahan
Peter Schweitzer
Iain W. Stewart *
Andrey Tarasov
Raju Venugopalan
Ivan Vitev
Feng Yuan
Yong Zhao

* - Editors

TMD Collaboration

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Recent Results
and
Future Opportunities

- **Multi-loop results**
- **Global fit ingredients**
- **Nonperturbative modeling**
- **New observables** (q_*, τ_1)
- **New distributions (Subleading power TMDs)**

Multi-loop results

TMD physics with state-of-the-art precision

Accuracy	H, $\mathcal{J}$	$\Gamma_{\rm cusp}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop	–	–	1-loop
NLL	Tree level	2-loop	1-loop	1-loop	2-loop
NLL′	1-loop	2-loop	1-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	2-loop	3-loop
NNLL′	2-loop	3-loop	2-loop	2-loop	3-loop
N^{3}LL	2-loop	4-loop	3-loop	3-loop	4-loop
N^{3}LL′	3-loop	4-loop	3-loop	3-loop	4-loop
N^{4}LL	3-loop	5-loop	4-loop	4-loop	5-loop
N^{4}LL′	4-loop	5-loop	4-loop	4-loop	5-loop

Key new ingredients:

- OPE for TMD PDFs and FFs to 3-loops (all channels)

  Ebert, Mistlberger, Vita (2020)

  Luo, Yang, Zhu, Zhu (2020)

$$f_{i/h}^{\rm pert}(x, b_T, \mu, \zeta) = \sum_j \int \frac{dy}{y} C_{ij}(x/y, b_T, \mu, \zeta) f_{j/h}(y, \mu)$$

- CS kernel to 4-loops

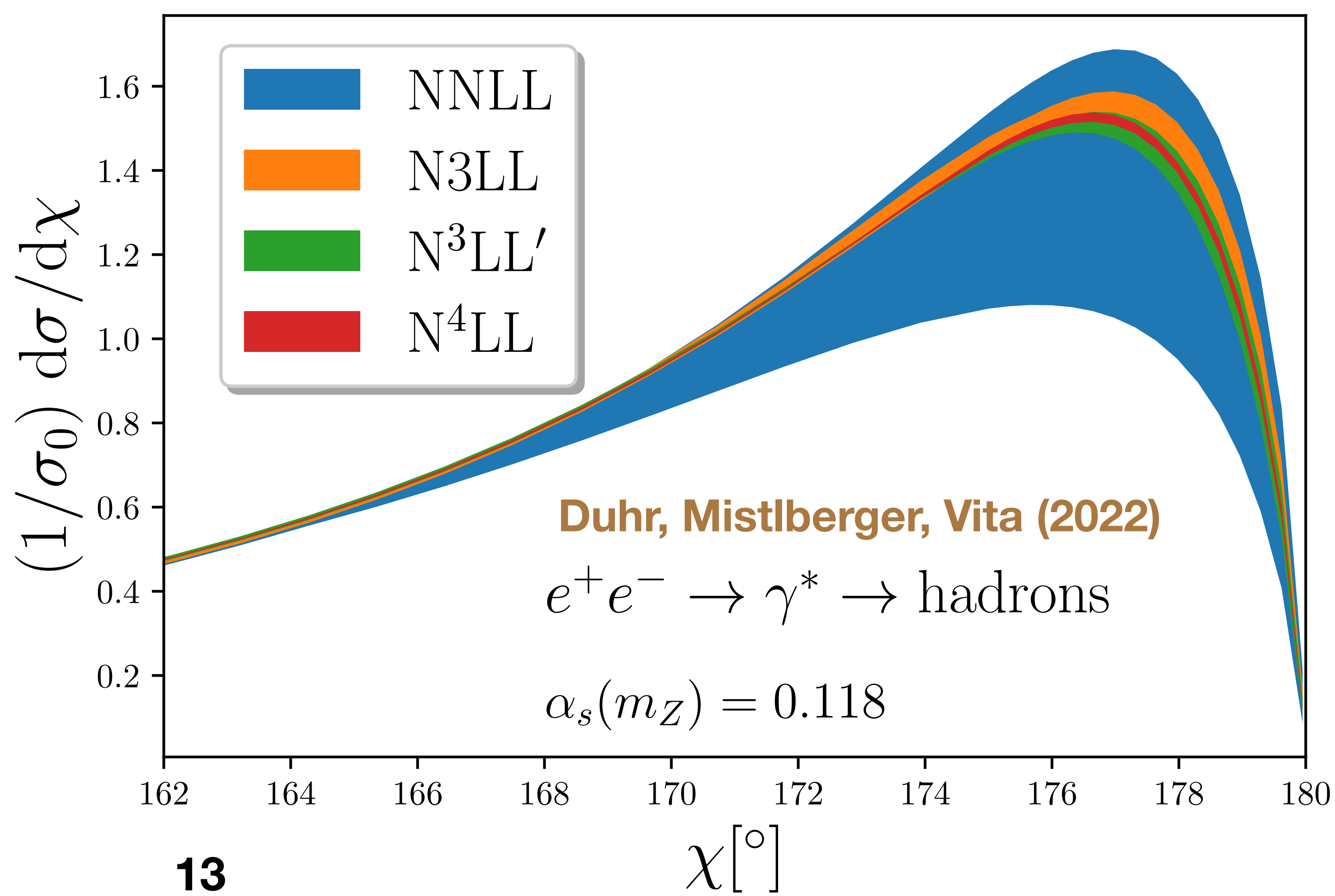
  Duhr, Mistlberger, Vita (2022)

  Moult, Zhu, Zhu (2022)

$$\gamma_\zeta^q[\alpha_s] = \alpha_s\, \gamma_\zeta^{q(1)} + \alpha_s^2\, \gamma_\zeta^{q(2)} + \alpha_s^3\, \gamma_\zeta^{q(3)} + \alpha_s^4\, \gamma_\zeta^{q(4)} + \ldots$$

(3-loop result: Li, Zhu 2016; Vladimirov 2016)

eg. of precision (EEC TMD observable)



Multi-loop results

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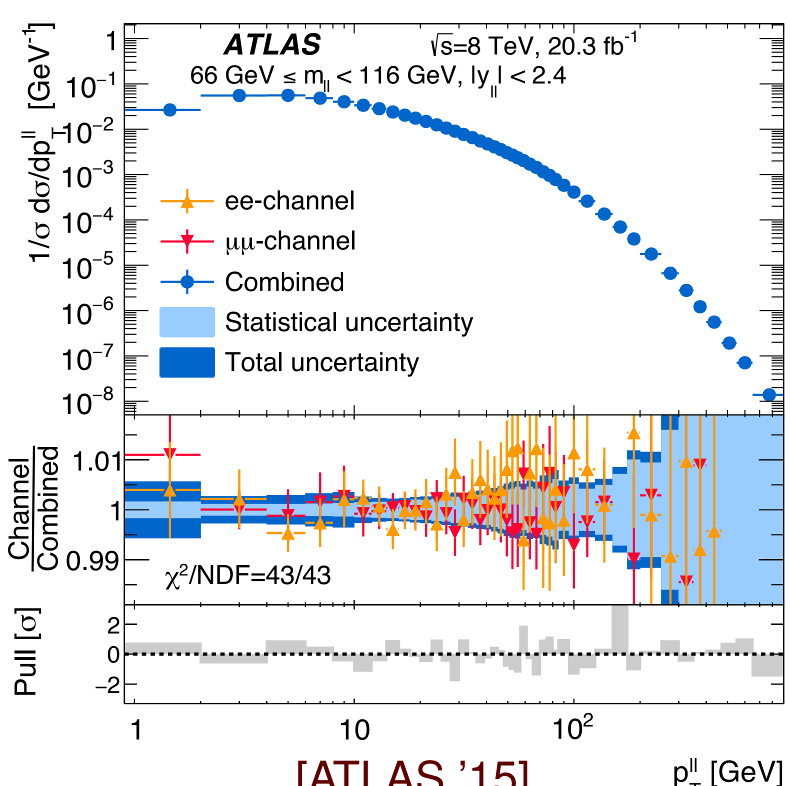
$\mathrm{N^3LL}$	2-loop	4-loop	3-loop	3-loop	4-loop
$\mathrm{N^3LL'}$	3-loop	4-loop	3-loop	3-loop	4-loop
$\mathrm{N^4LL}$	3-loop	5-loop	4-loop	4-loop	5-loop
$\mathrm{N^4LL'}$	4-loop	5-loop	4-loop	4-loop	5-loop

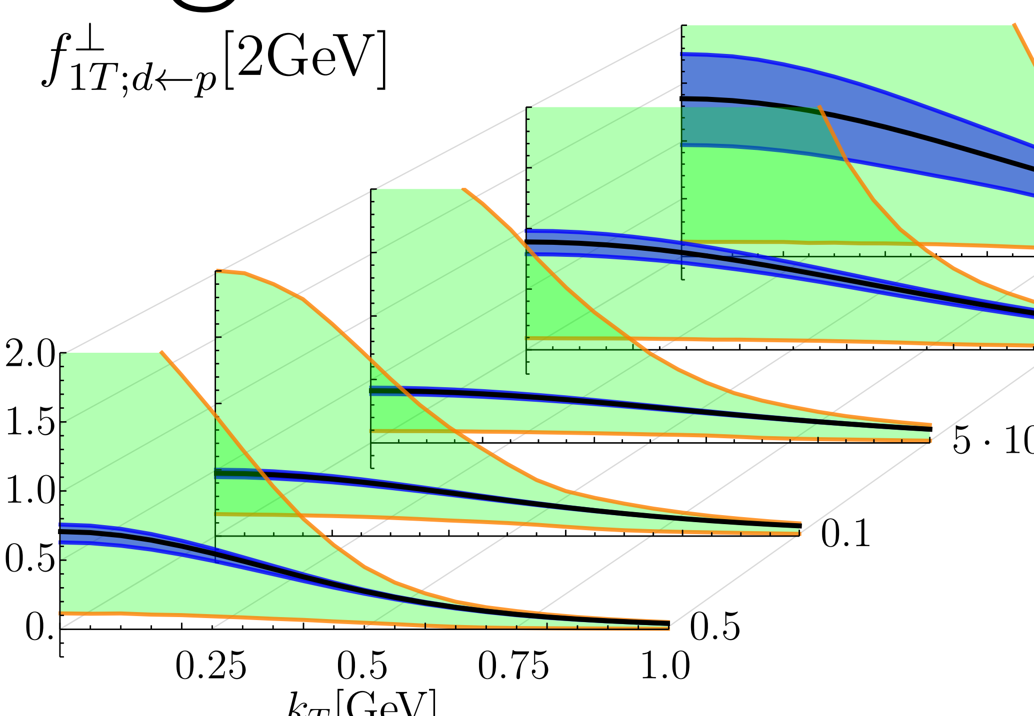
Future Opportunities:

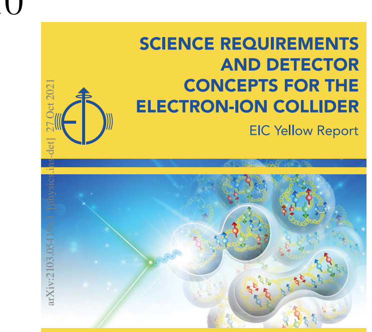
- Make high precision predictions for unpolarized SIDIS and Drell-Yan using these results!
- Need analogous multi-loop calculations for spin dependent TMD PDFs and TMD FFs to fully exploit the EIC program: tools are there to do it!
- Theoretical perturbative uncertainty currently dominate in certain regions: still the case? Reduce to level commensurate with experimental uncertainties?



[ATLAS '15]

Sivers TMD@EIC:





Global fit ingredients

- Current highest precision global fits to SIDIS and Drell-Yan data (eg. N^3LL + NNLO) are often done with constraints

eg. SV19 = Scimemi, Vladimirov (1912.06532)

Pavia19 = Bachetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici (1912.07550)

$q_T/Q < 0.2 - 0.25$ **(TMD region with 4-6% power corrections)**

Only unpolarized data used.

(See talk by Nobuo Sato for more results, JAM, …)

Future Opportunities:

- More work is needed to fully and accurately handle multiple kinematic regions:

  $q_T \ll Q$ and $q_T \sim Q$ and thus <u>fully exploit</u> the available/expected data

- Move towards fits that simultaneously fit unpolarized and spin-dependent data

- Consider also observables involving jets **(see talk by Zhongbo Kang)**

Nonperturbative modeling

so far, fixed functional forms

SV19

TMDPDF: 5
TMDFF: 4
CS kernel: 2

$$f_{NP}(x,b) = \exp\left(-\frac{\lambda_1(1-x)+\lambda_2 x + x(1-x)\lambda_5}{\sqrt{1+\lambda_3 x^{\lambda_4}\boldsymbol{b}^2}}\boldsymbol{b}^2\right)$$

$$D_{NP}(x,b) = \exp\left(-\frac{\eta_1 z+\eta_2(1-z)}{\sqrt{1+\eta_3(\boldsymbol{b}/z)^2}}\frac{\boldsymbol{b}^2}{z^2}\right)\left(1+\eta_4\frac{\boldsymbol{b}^2}{z^2}\right)$$

$$\gamma_\zeta^q(\mu,b) = \gamma_\zeta^{q\,\mathrm{pert}}(\mu,b^*) - \frac{1}{2}c_0 b b^*$$

$$b^*(b) = \frac{b}{\sqrt{1+\boldsymbol{b}^2/B_{\mathrm{NP}}^2}}$$

Pavia19

TMDPDF: 7

CS kernel: 2

$$f_{\mathrm{NP}}(x,b_T) = \left[\frac{1-\lambda}{1+g_1(x)\frac{b_T^2}{4}} + \lambda\exp\left(-g_{1B}(x)\frac{b_T^2}{4}\right)\right]$$

$$g_1(x) = \frac{N_1}{x\sigma}\exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{x}{\alpha}\right)\right]$$

$$g_{1B}(x) = \frac{N_{1B}}{x\sigma_B}\exp\left[-\frac{1}{2\sigma_B^2}\ln^2\left(\frac{x}{\alpha_B}\right)\right]$$

$$\gamma_\zeta^q(\mu,b) = \gamma_\zeta^{q\,\mathrm{pert}}(\mu,b_*) - \frac{1}{2}(g_2 b_T^2 + g_{2B} b_T^4)$$

$$b_*(b_T) = b_{\max}\left(\frac{1-\exp\left(-\frac{b_T^4}{b_{\max}^4}\right)}{1-\exp\left(-\frac{b_T^4}{b_{\min}^4}\right)}\right)^{\frac{1}{4}}$$

(model for b* used to split perturbative & non-perturbative parts)

Global Fits

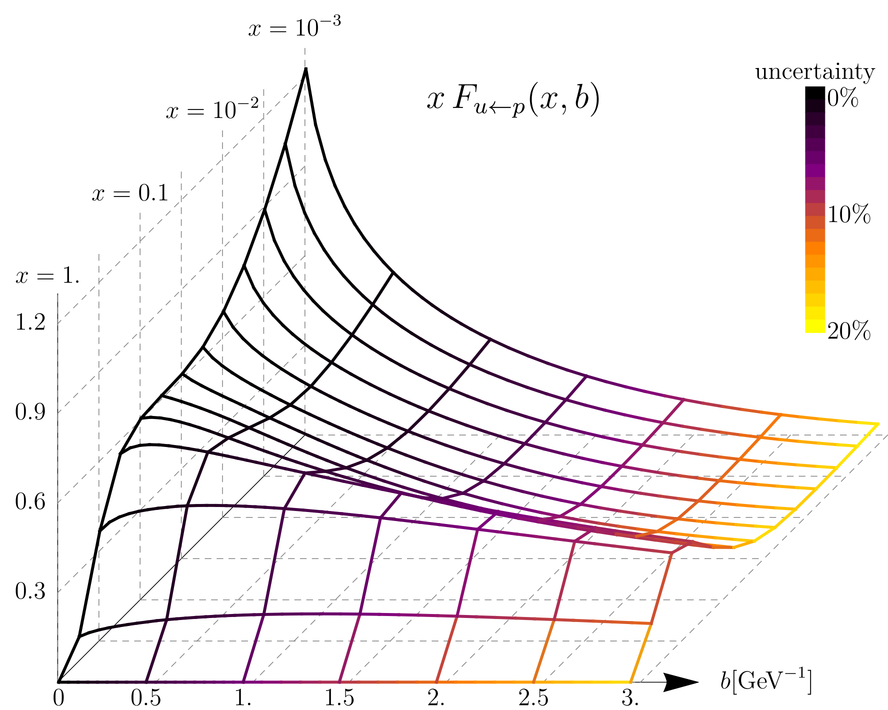
SV19 = Scimemi, Vladimirov (1912.06532)

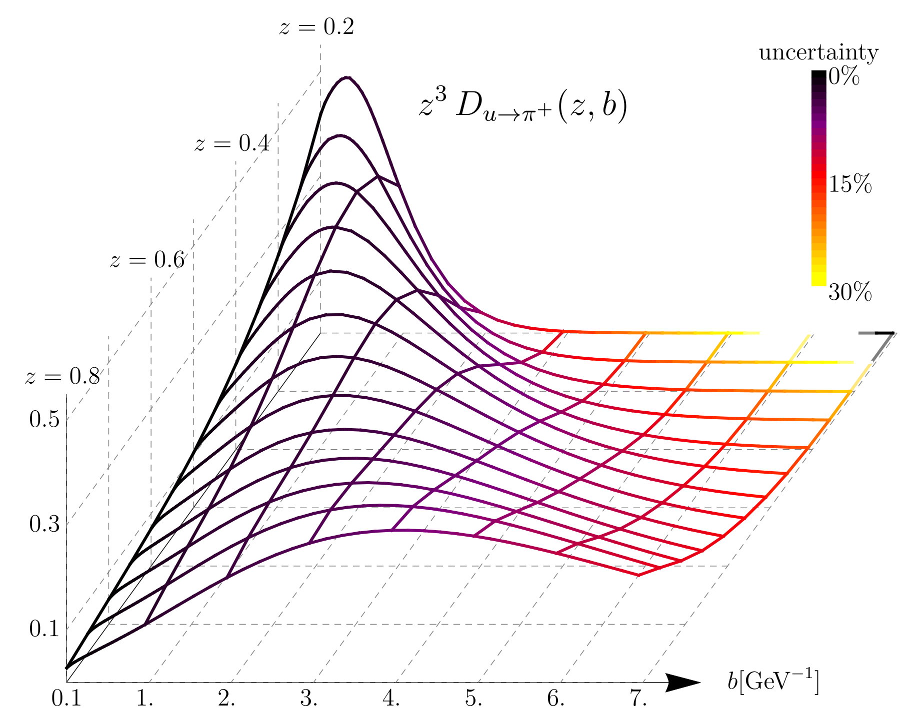
Pavia19 = Bachetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici (1912.07550)

Fit Results:

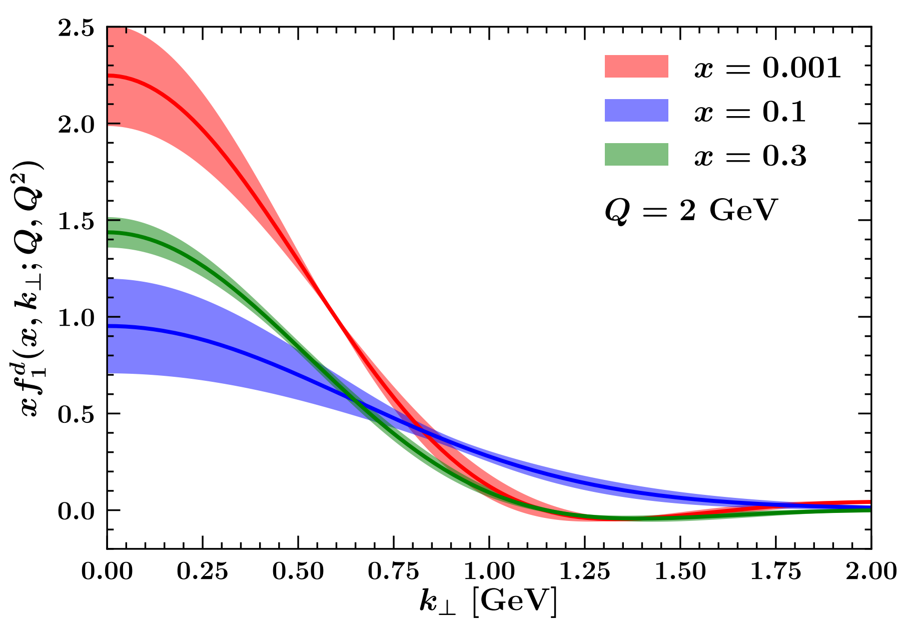
for TMD PDF & TMD FF

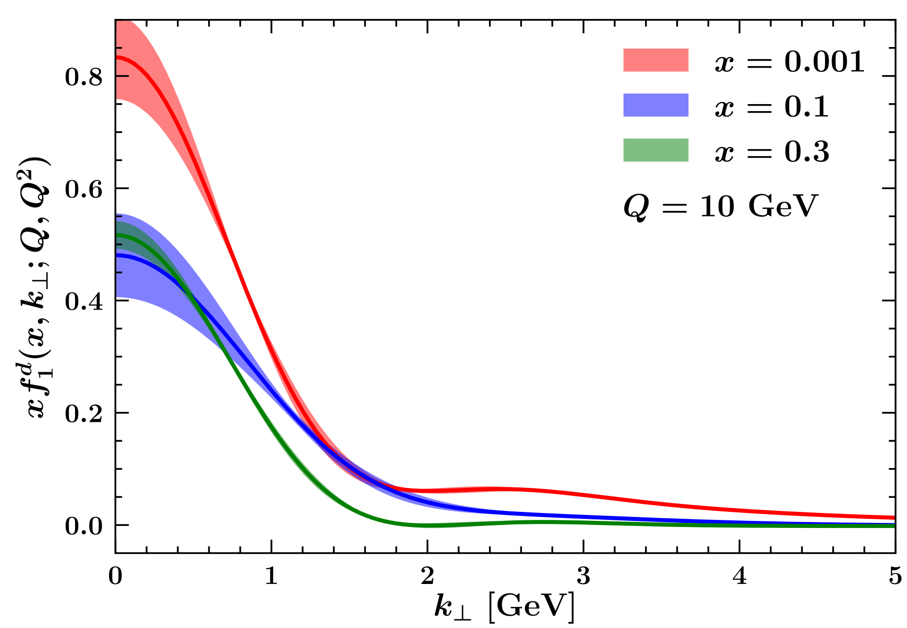
SV19





Pavia19





Precise determinations for a given fit form.

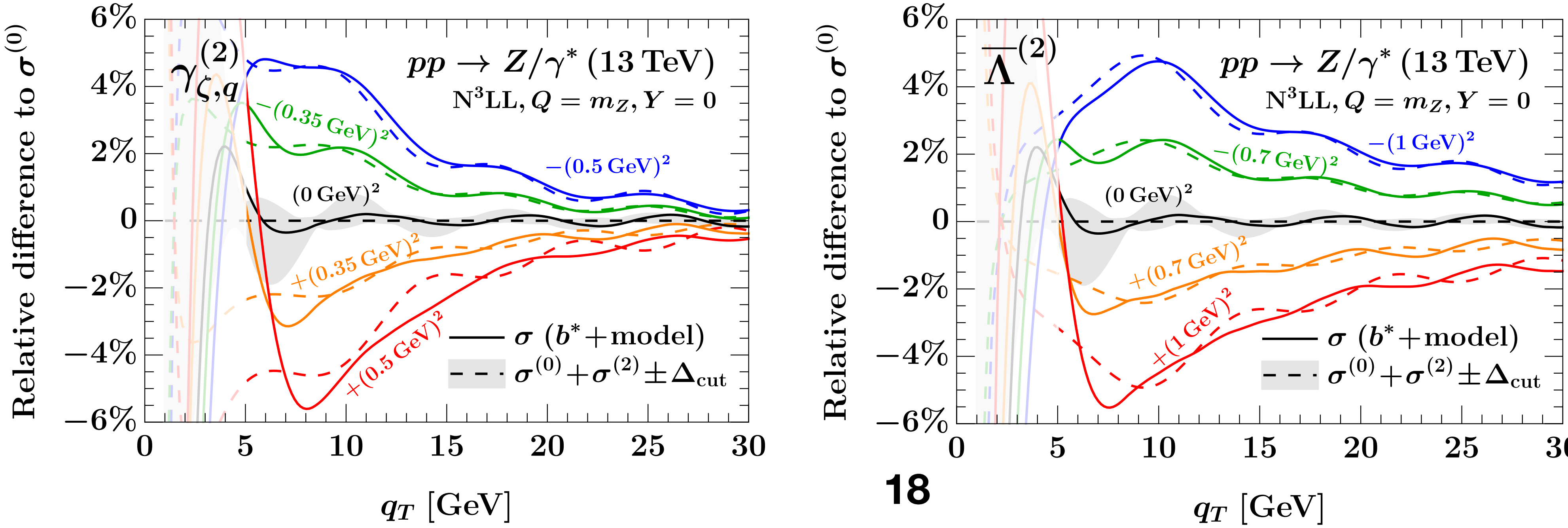
Nonperturbative modeling

Future Opportunities:

- More extensive exploration of dependence on functional forms, Neural Net, …
- Many fits assume flavor universality for intrinsic TMDs (same for up, down, …).
  Need to continue to move away from this assumption.
  Future precision data (EIC) will also play an important role. eg. need to determine antiquark in SIDIS to test SIVERS sign flip [Bury, Prokudin, Vladimirov, 2012.05135]
- Lattice QCD results will become more accurate and need to be used
- Interpretation of non-perturbative model parameters in current fits is also difficult.
  Different meaning for parameters in <u>same</u> functional form, with different b* choice.

  Methods that are model independent (no b*) could be exploited

[Ebert, Michel, IS, Sun, 2201.07237]



New observables

q_* **"A Better Angle on Hadron Transverse Momentum Distributions at the EIC"**

Gao, Michel, Sun, IS (this week!)

$$e^-(\ell) + N(P) \to e^-(\ell') + h(P_h) + X$$

Key challenge for probing hadronization and confinement at EIC is ability to accurately reconstruct final $\vec{\ell}\,'$, and thus $\vec{P}_{hT}$ (defined relative to $\vec{q}$)

eg. $Q = 20\,\mathrm{GeV}, \Delta\ell' = 0.5\,\mathrm{GeV} \implies$ 50% uncertainty on $P_{hT}/z = 1\,\mathrm{GeV}$

This makes it difficult to measure TMDs in the most interesting region!

New observables

Future EIC Opportunity

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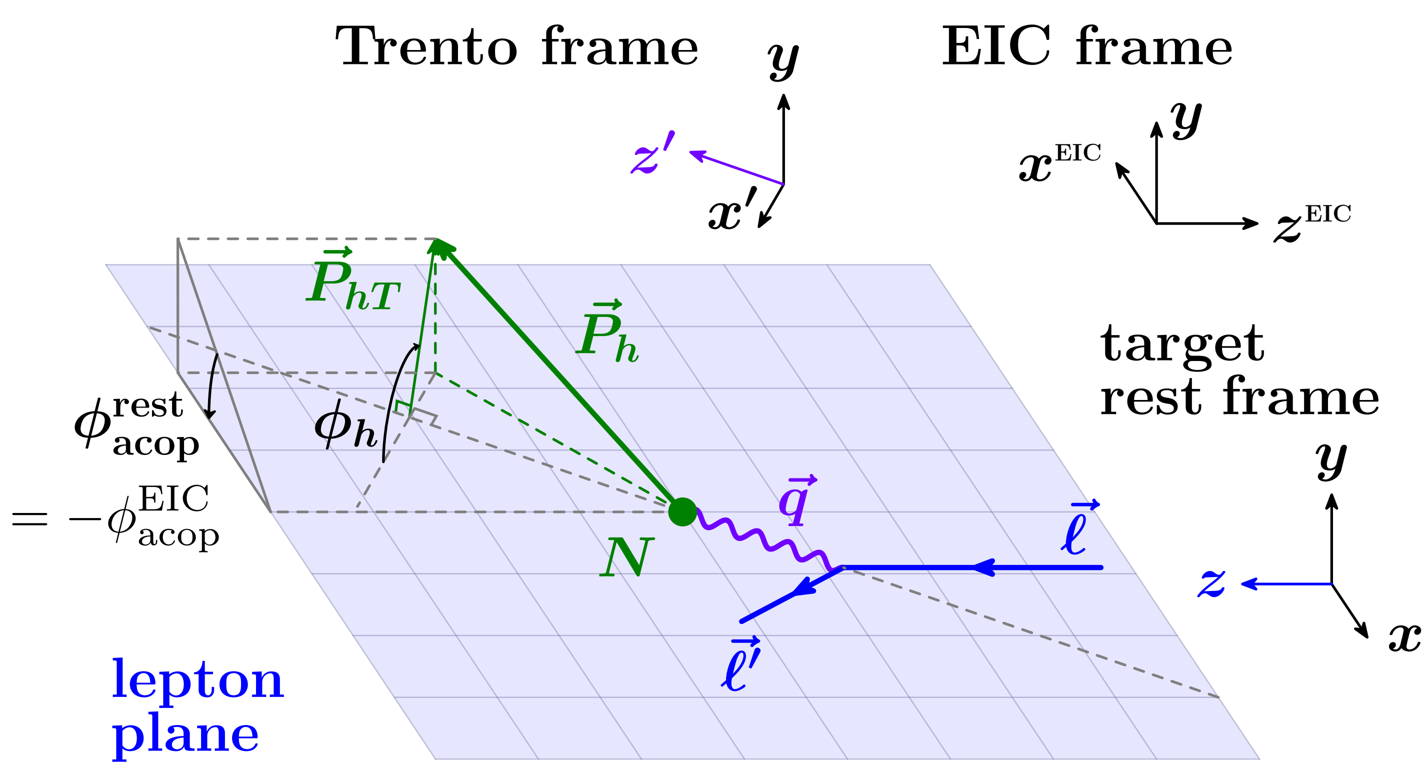
eg. $Q = 20\,\mathrm{GeV}, \Delta\ell' = 0.5\,\mathrm{GeV} \implies$ 50% uncertainty on $P_{hT}/z = 1\,\mathrm{GeV}$

Solve by replacing $\vec{P}_{hT}$ by an angular observable q_*



measure with angles in EIC lab frame:

$$q_* \equiv 2P^0_{\mathrm{EIC}} \frac{e^{\eta_h}}{1 + e^{(\eta_h - \eta_{\ell'})}} \tan\phi^{\mathrm{EIC}}_{\mathrm{acop}}$$

rapidities

$P_{hT} \ll Qz$:

$$q_* = -\sin\phi_h \frac{P_{hT}}{z} + \dots$$

q_* **obeys a standard SIDIS TMD factorization theorem**

measure same TMDs

New observables

Future EIC Opportunity

Gao, Michel, Sun, IS (this week!)

q_* **"A Better Angle on Hadron Transverse Momentum Distributions at the EIC"**

$$q_* \equiv 2P^0_{\mathrm{EIC}} \frac{e^{\eta_h}}{1+e^{(\eta_h-\eta_{\ell'})}} \tan\phi^{\mathrm{EIC}}_{\mathrm{acop}}$$

$$e^-(\ell) + N(P) \to e^-(\ell') + h(P_h) + X$$

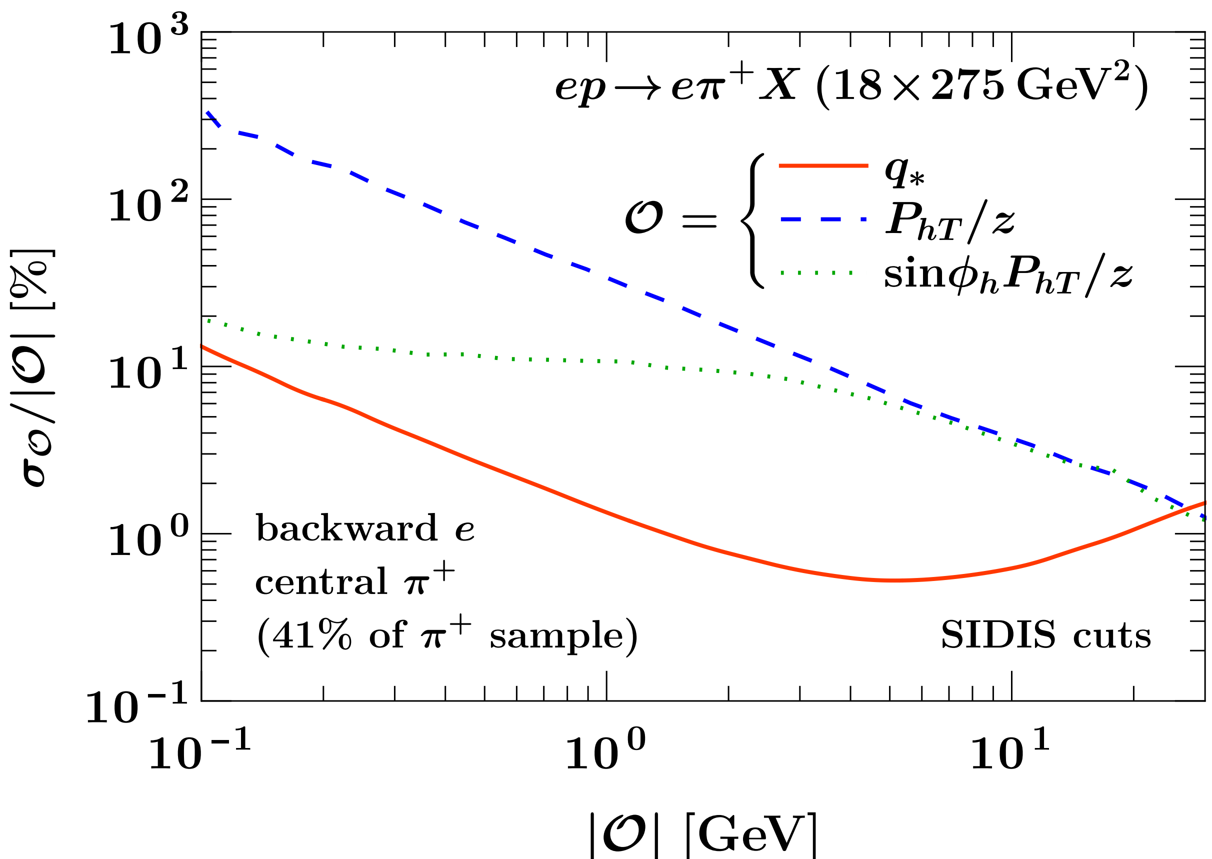
SIDIS

Compare EIC resolution for q_* vs. P_{hT}/z

expected event-level resolution for these TMD observables:

Pythia simulation with EIC cuts

Gaussian smearing of final state e^-, h momenta using resolutions that match the tracking performance given in EIC Yellow report.



q_* **gives an order of magnitude improvement!**

τ_1

Event shapes:

DIS thrust @ N3LL and angularity @ NNLL

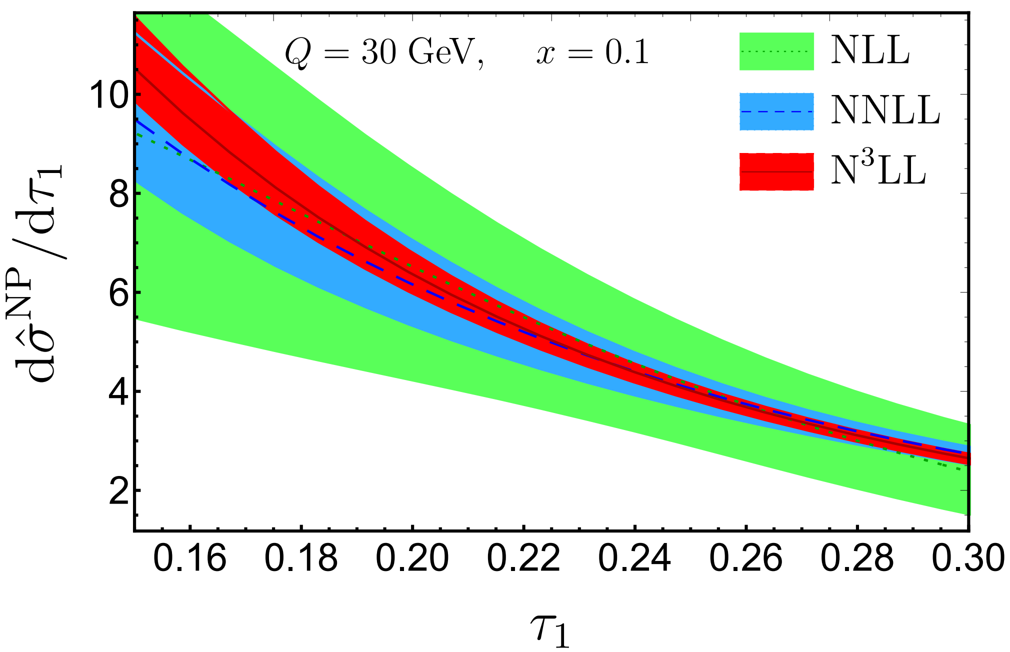
Lee, Kang, IS (2013, 2021); Zhu, Kang, Maji (2021)

Future EIC Opportunity

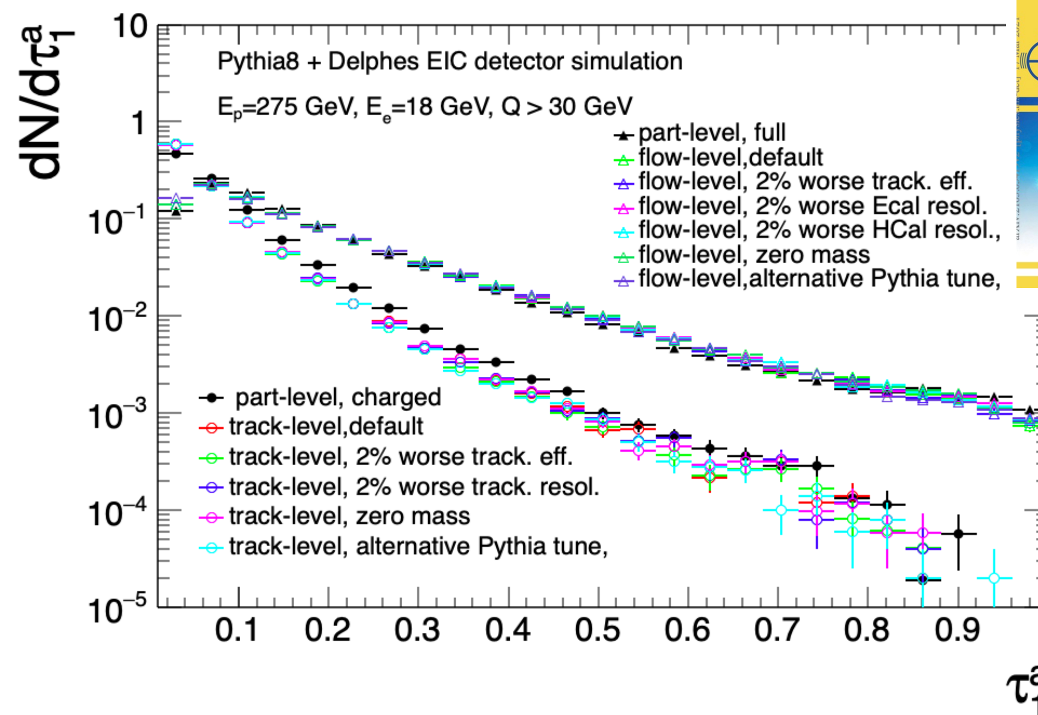
Replace final hadron in SIDIS by a jet

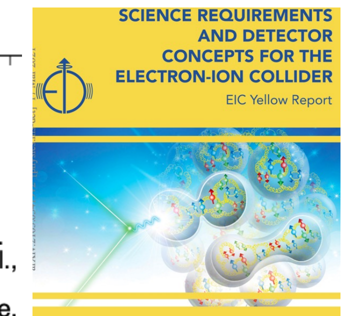
Sensitive to standard PDFs (not TMDs, but some vars do have TMD integral)

Theory (convergence):

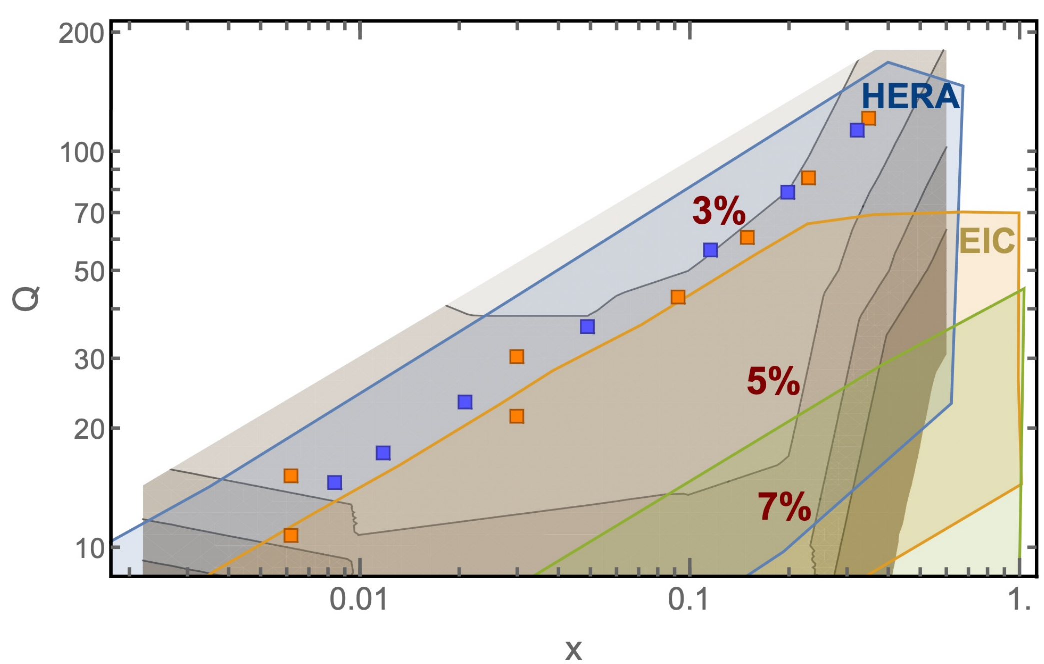


EIC simulation:





Perturbative uncertainty for EIC:



τ_1 **Event shapes:**

DIS thrust @ N3LL and angularity @ NNLL

Lee, Kang, IS (2013, 2021); Zhu, Kang, Maji (2021)

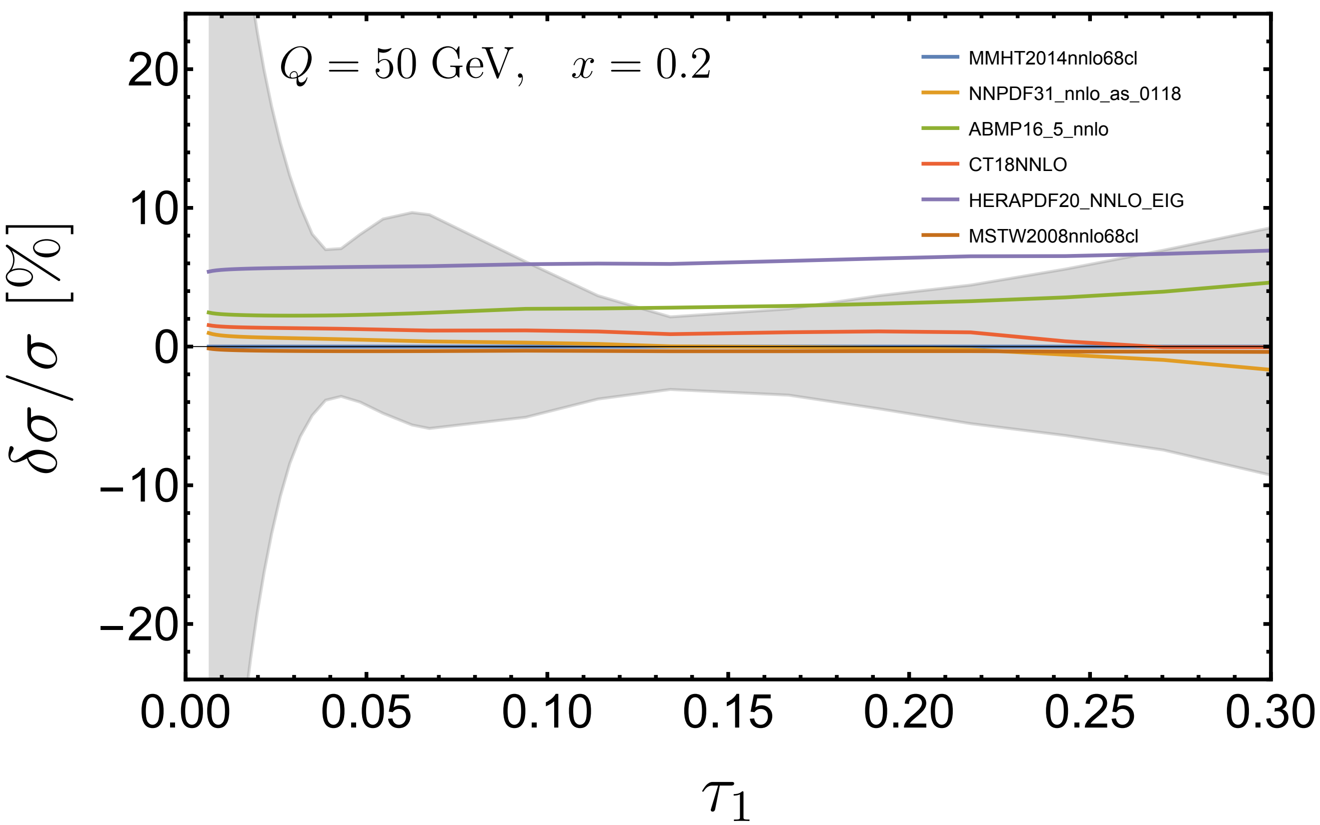
Future EIC Opportunity

Replace final hadron in SIDIS by a jet

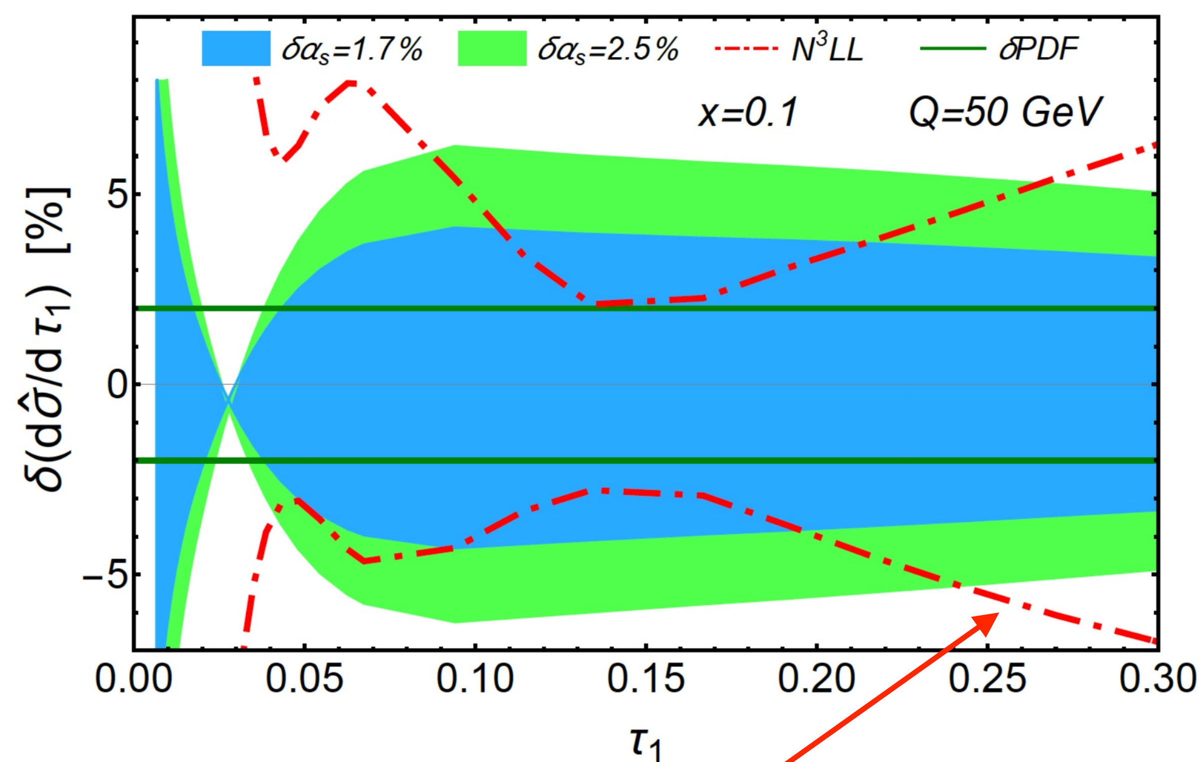
Sensitive to standard PDFs (not TMDs, but some vars do have TMD integral)

Precision predictions for: measuring PDFs and $\alpha_s(m_Z)$ at EIC

Global proton PDF uncertainty ~ pert.uncertainty, hence promising for ion PDFs:



Sensitivity to $\alpha_s(m_Z)$:



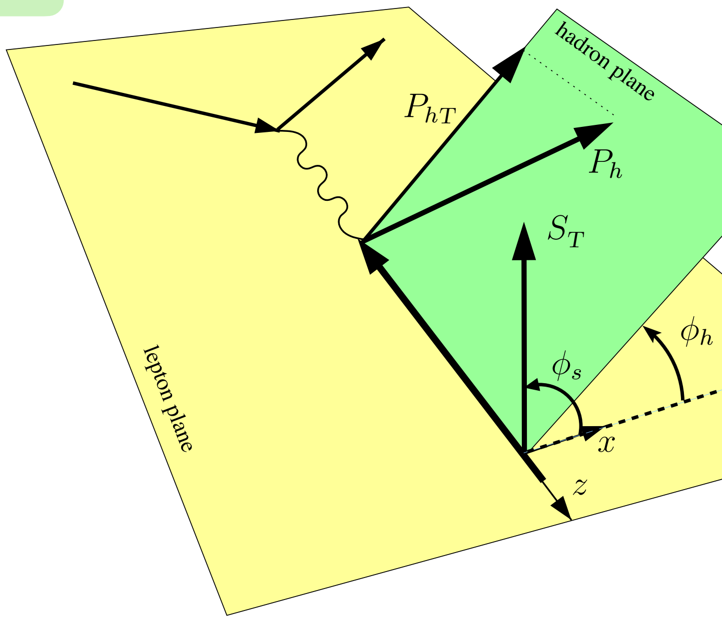
Currently working on including full $\mathcal{O}(\alpha_s^2)$ corrections which will reduce this uncertainty.

Azimuthal Asymmetries in SIDIS at subleading power

- At subleading order in $q_T/Q \ll 1$ there are new TMD probes:
  8 new structure functions in SIDIS

famous Cahn ('78) effect: transverse motion of quarks gives a $\cos(\phi_h)$ asymmetry

$$\frac{d^6\sigma_{\text{subleading}}}{dx\,dy\,dz_h\,d\phi_S\,d\phi_h\,dP_{hT}^2} = \frac{\alpha_{em}^2}{x\,y\,Q^2}\left(1-y+\frac{1}{2}y^2\right)\Big\{\cos(\phi_h)\,p_3\,F_{UU}^{\cos(\phi_h)}$$
$$+\lambda\sin(\phi_h)\,p_4\,F_{LU}^{\sin(\phi_h)} + S_L\sin(\phi_h)\,p_3\,F_{UL}^{\sin(\phi_h)} + \lambda\,S_L\cos(\phi_h)\,p_4\,F_{LL}^{\cos(\phi_h)}$$
$$+S_T\sin(2\phi_h-\phi_S)\,p_3\,F_{UT}^{\sin(2\phi_h-\phi_S)} + S_T\sin(\phi_S)\,p_3\,F_{UT}^{\sin(\phi_S)}$$
$$+\lambda\,S_T\cos(\phi_S)\,p_4\,F_{LT}^{\cos(\phi_S)} + \lambda\,S_T\cos(2\phi_h-\phi_S)\,p_4\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big\}$$

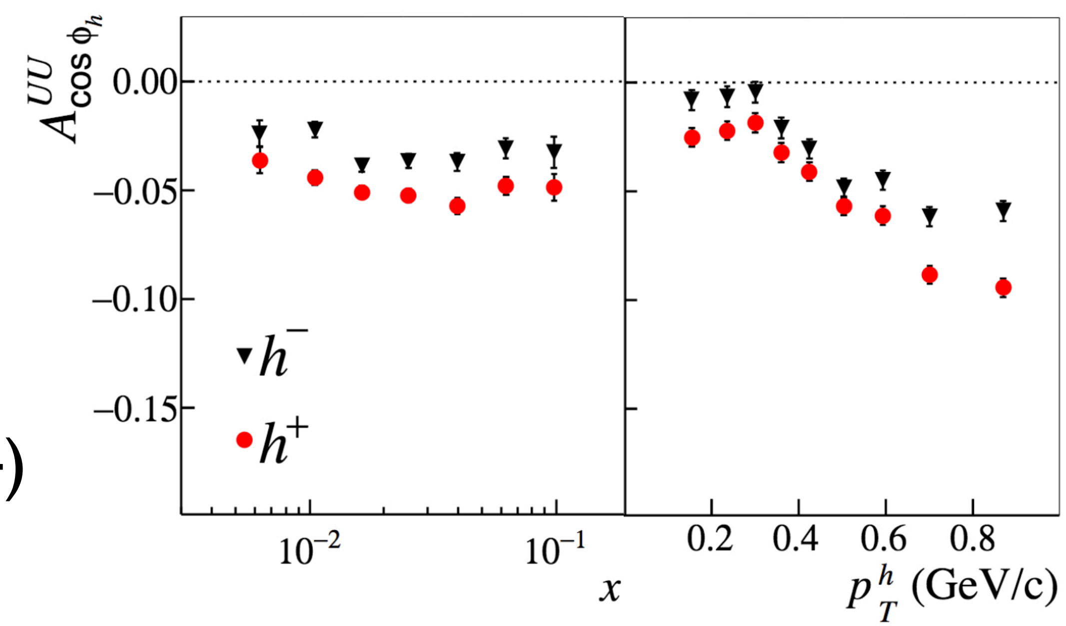


Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel (2006)

- Tree level parton model shows these give access to subleading power TMDs

Mulders, Tangerman (1995); Bachetta et al (2008)(2019)

Compass measurement (2014)



Factorization for Azimuthal Asymmetries in SIDIS at next-to-leading power

- Recently we derived* all orders subleading power factorization theorems for these SIDIS structure functions (using SCET)

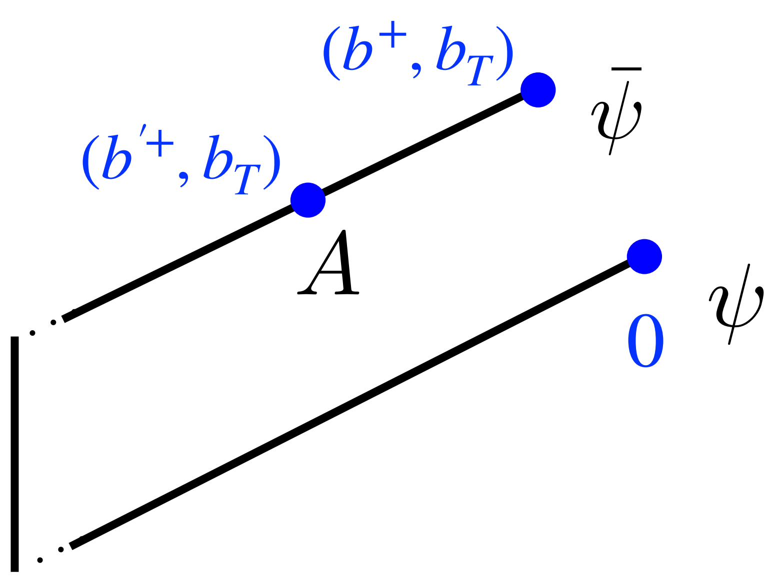
  Ebert, Gao, IS (2112.07680)

eg. Cahn effect

$$
\begin{aligned}
F_{UU}^{\cos(\phi_h)} = \mathcal{F}\Big\{ & -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\,h_1^\perp H_1^\perp\Big] && \text{(Kinematic corrections)} \\
& - \mathcal{H}^{(0)}\Big[\frac{p_{Tx}+k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp\Big] && \text{(From the } \mathcal{P}_\perp \text{ operators)} \\
& + \mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(k_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} p_{Tx}\,\tilde{h}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{px}\,f_1\tilde{D}^\perp + \frac{M_h}{M_N} k_{Tx}\,h_1^\perp \tilde{H}\Big)\Big]\Big\} && \text{(From the } \mathcal{B}_\perp \text{ operators)}
\end{aligned}
$$

- Only new TMDs are quark-gluon-quark (QGQ) correlators

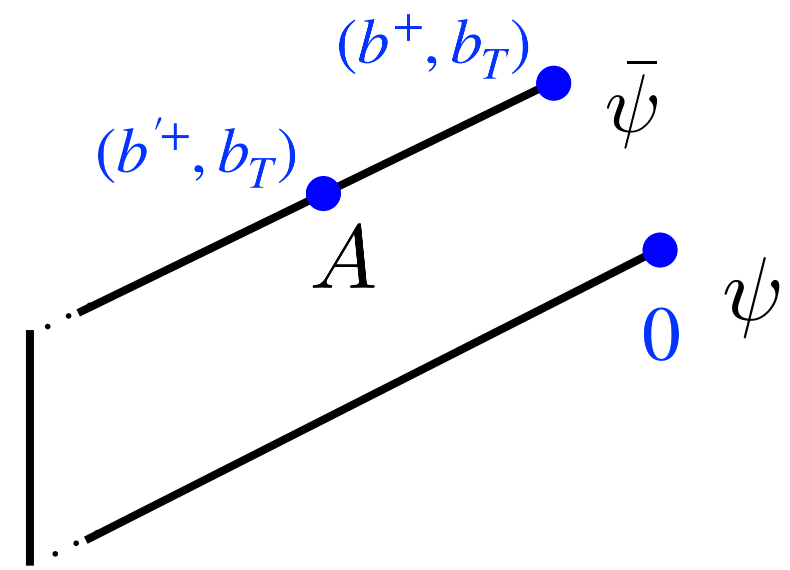
$$\tilde{f}_{\mathrm{QGQ}} \sim \langle p|\bar{\psi} A \psi|p\rangle$$



Factorization for Azimuthal Asymmetries in SIDIS at next-to-leading power

- TMD quark-gluon-quark (QGQ) correlators

$$\tilde{f}_{\rm QGQ} \sim \langle p|\bar{\psi} A \psi|p\rangle$$



16 new PDFs & 4 new FFs for SIDIS

Subleading Quark-Gluon-Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$\tilde{f}^{\perp}, \tilde{g}^{\perp}$	$\tilde{e}\,,\ \tilde{h}$
	L	$\tilde{f}_L^{\perp},\ \tilde{g}_L^{\perp}$	$\tilde{e}_L,\ \tilde{h}_L$
	T	$\tilde{f}_T,\ \tilde{f}_T^{\perp},\ \tilde{g}_T,\ \tilde{g}_T^{\perp}$	$\tilde{e}_T,\ \tilde{e}_T^{\perp},\ \tilde{h}_T,\ \tilde{h}_T^{\perp}$

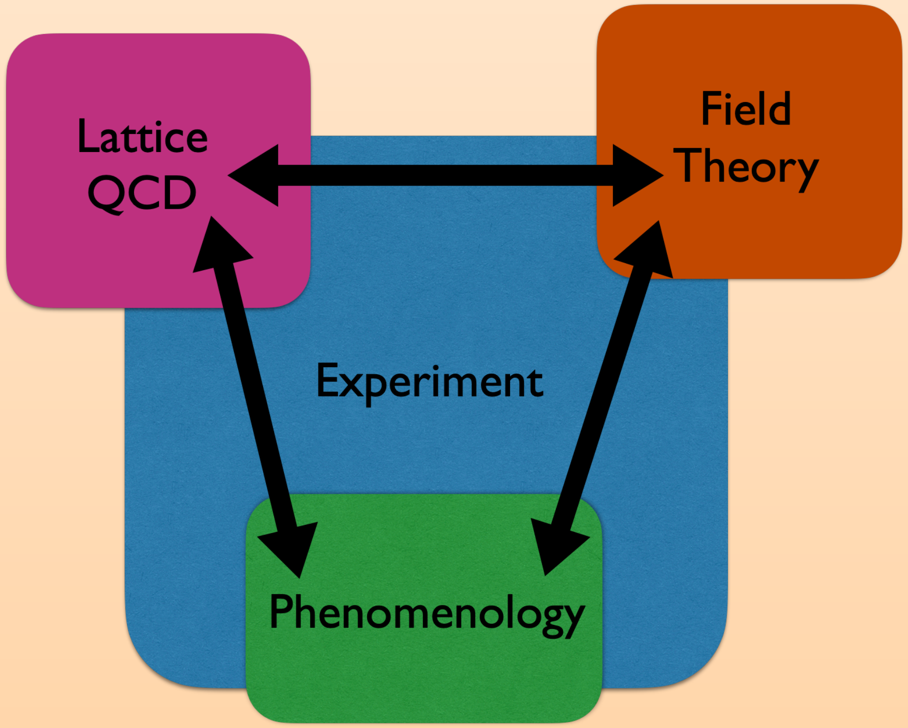
Subleading Quark-Gluon-Quark TMDFFs

	Quark Chirality	
	Chiral Even	Chiral Odd
Unpolarized (or Spin 0) Hadrons	$\tilde{D}^{\perp},\ \tilde{G}^{\perp}$	$\tilde{E}\,,\ \tilde{H}$

additional probes for hadronic structure, now on a firm theoretical footing!

Summary

- TMDs provide novel information about hadrons:
  confinement, hadronization, correlations,
  access to interesting phenomena in hadron structure
- Prospects are bright for precise measurements, with
  high precision theoretical predictions for cross sections
- Opportunities with new TMD experimental probes, eg. $q_*, \ \tau_1$
- Opportunities with new probes of hadron structure (QGQ TMD correlators). Lots to do to study subleading power theoretically.
- Can look forward to global fits of TMDs reaching towards
  the level we have now for PDFs
- Continuing the strong support for EIC theory directions, and the synergy among various communities are both necessary for this program to succeed



Backup

Global Fits

SV19 = Scimemi, Vladimirov (1912.06532)

Pavia19 = Bachetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici (1912.07550)

Fit Results:

SV19

$$\chi^2/N_{pt} = 1.06$$

NP-parameters			
RAD	$B_{\rm NP} = 1.93 \pm 0.22$	$c_0 = (4.27 \pm 1.05) \times 10^{-2}$	
TMDPDF	$\lambda_1 = 0.224 \pm 0.029$	$\lambda_2 = 9.24 \pm 0.46$	$\lambda_3 = 375. \pm 89.$
	$\lambda_4 = 2.15 \pm 0.19$	$\lambda_5 = -4.97 \pm 1.37$	
TMDFF	$\eta_1 = 0.233 \pm 0.018$	$\eta_2 = 0.479 \pm 0.025$	
	$\eta_3 = 0.472 \pm 0.041$	$\eta_4 = 0.511 \pm 0.040$	

Pavia19

$$\chi^2/N_{pt} = 1.02$$

Parameter	Value
g_2	0.036 ± 0.009
N_1	0.625 ± 0.282
α	0.205 ± 0.010
σ	0.370 ± 0.063
λ	0.580 ± 0.092
N_{1B}	0.044 ± 0.012
α_B	0.069 ± 0.009
σ_B	0.356 ± 0.075
g_{2B}	0.012 ± 0.003

Low and High energy data are well described

CS kernel parameters are less sensitive to input PDF set

Universality of CS kernel satisfied by DY vs. SIDIS data

Global Fits

Bury, Prokudin, Vladimirov (2012.05135)

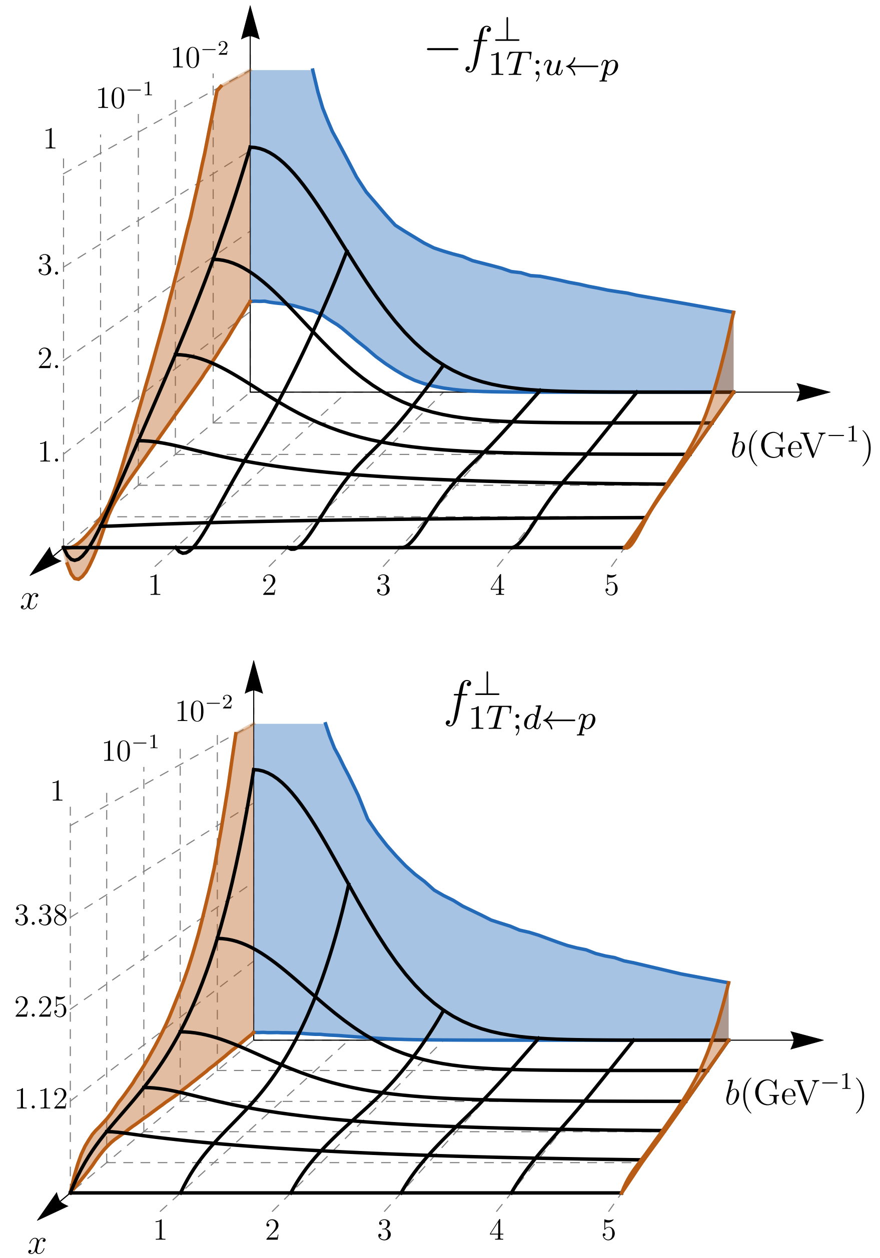
Extraction of Sivers function from global fit to SIDIS, DY, and W/Z data [76 bins: HERMES, COMPASS, Jlab (SIDIS); STAR(W/Z); COMPASS (DY)]

$$f_{1T}^{\perp\,\mathrm{SIDIS}} = -f_{1T}^{\perp\,\mathrm{DY}}$$

N3LL analysis following SV19

Flavor dependent parametrization (no matching)

$$f_{1T;q\leftarrow h}^{\perp}(x,b) = N_q \frac{(1-x)x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q,\epsilon_q)} \exp\left(-\frac{r_0+xr_1}{\sqrt{1+r_2x^2b^2}}b^2\right)$$



Results:

Good global fit: $\chi^2/N_{pt} = 0.88$

Opposite signs for up and down Sivers functions

Data not precise enough to confirm sign flip

$f_{1T}^{\perp\,\mathrm{SIDIS}} = +f_{1T}^{\perp\,\mathrm{DY}}$ **gives** $\chi^2/N_{pt} = 1.0$

Need higher precision data (Jlab, EIC) to pin down antiquark in SIDIS

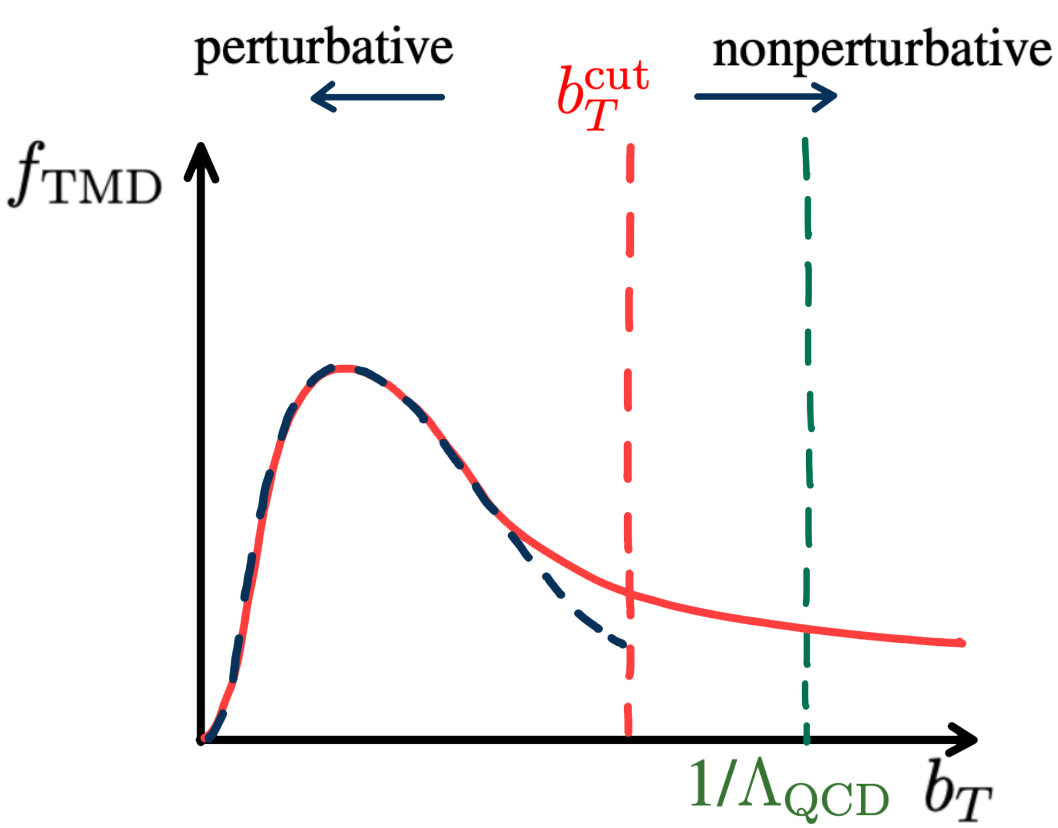
Disentangling Long and Short Distances in Momentum-Space TMDs

Ebert, Michel, IS, Sun (arXiv:2201.07237)

- Use of b* entangles perturbative and nonperturbative TMD components
- Would like to extract nonpert. information without relying on b*

  Intuition: perturbative q_T should be dominated by perturbative b_T

- Recently we developed a method to do this by setting up a systematic expansion



$$\int_0^\infty d^2 b_T \, e^{i\vec{b}_T\cdot\vec{q}_T} f(b_T)$$

$$= \int_0^{b_T^{\rm cut}} d^2 b_T \, e^{i\vec{b}_T\cdot\vec{q}_T} f(b_T) \qquad \text{perturbative } b_T$$

$$+ \int_{b_T^{\rm cut}}^\infty d^2 b_T \, e^{i\vec{b}_T\cdot\vec{q}_T} f(b_T) \qquad \text{syst. expansion in } q_T b_T^{\rm cut} \gg 1$$

Evolution

$$f_q(x, \vec{b}_T, \mu, \zeta)$$

μ = renormalization scale

ζ = Collins-Soper parameter

Sum large logarithms: $\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$

$$\zeta = 2\left(xP^+ e^{-y_n}\right)^2$$

Solution:

$$f_q(x, \vec{b}_T, \mu, \zeta) = \exp\Big[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^q(\mu', \zeta_0)\Big] \exp\Big[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln \frac{\zeta}{\zeta_0}\Big] \; f_q(x, \vec{b}_T, \mu_0, \zeta_0)$$

OPE: expansion in the region $1/b_T, \mu_0, \mu \gg \Lambda_{\rm QCD}$

$$\underbrace{\gamma_{\zeta,i}^{(0)}(b_T, \mu)}_{\text{perturbative}} + \underbrace{b_T^2 \gamma_{\zeta,i}^{(2)}(b_T)}_{\text{nonperturbative}} + \mathcal{O}\left[(\Lambda_{\rm QCD} b_T)^4\right]$$

$$\begin{aligned} f_i(x, b_T, \mu, \zeta) &= f_i^{(0)}(x, b_T, \mu, \zeta) + b_T^2 f_i^{(2)}(x, b_T, \mu, \zeta) + \mathcal{O}\left[(\Lambda_{\rm QCD} b_T)^4\right] \\ &= f_i^{(0)}(x, b_T, \mu, \zeta)\Big(1 + \underbrace{\Lambda_i^{(2)}(x)}_{\text{nonperturbative}}\, b_T^2 + \mathcal{O}\left[(\Lambda_{\rm QCD} b_T)^4\right]\Big) \end{aligned}$$

multiplicative form

$$f_i^{(0)}(x, b_T, \mu, \zeta) = \sum_j \int \frac{dz}{z} \underbrace{C_{ij}\Big(\frac{x}{z}, b_T, \mu, \zeta\Big)}_{\text{perturbative}} \; \underbrace{f_j(z, \mu)}_{\text{nonperturbative PDF}}$$