Pure Quark and Gluon Jet Observables

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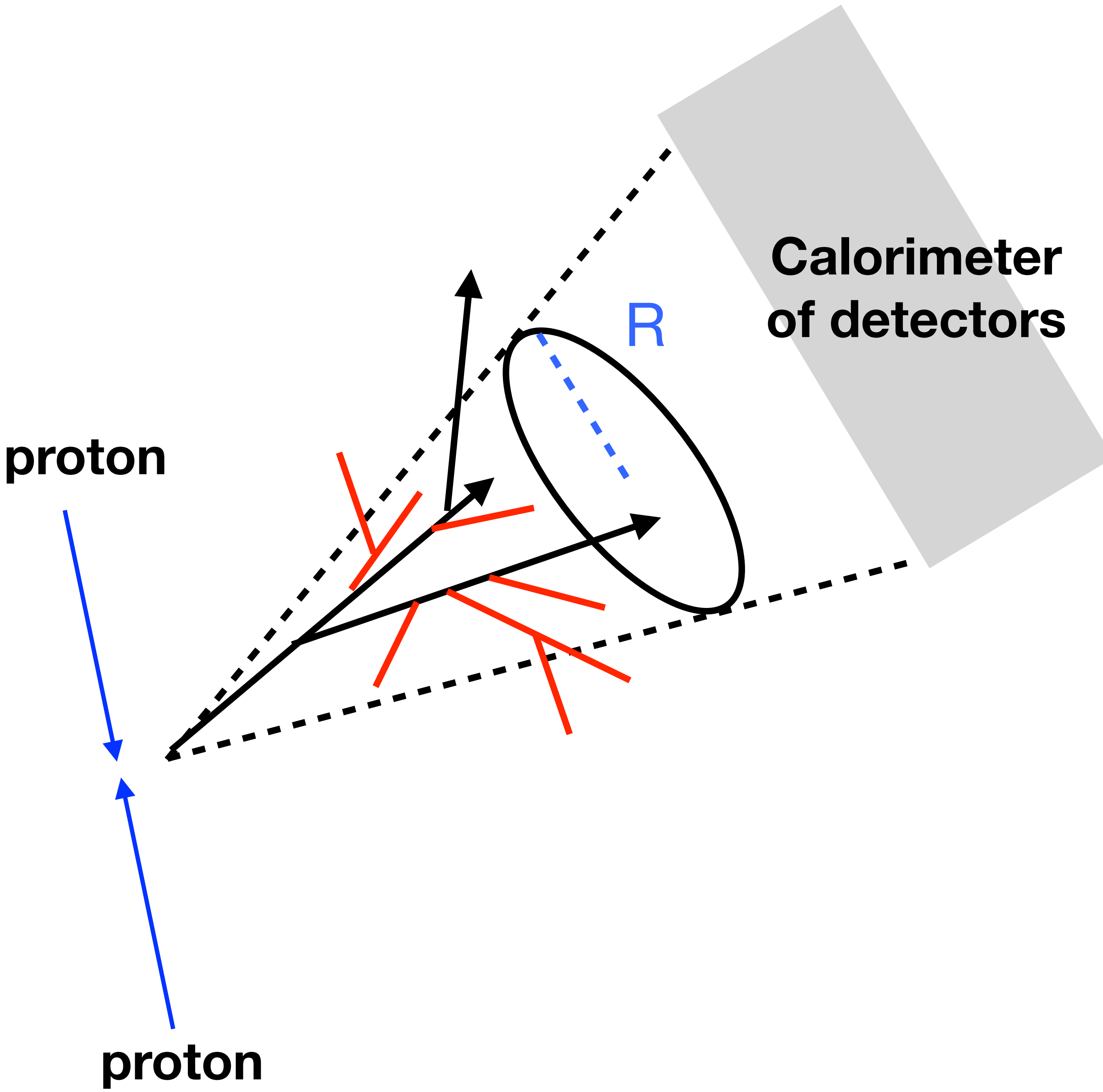
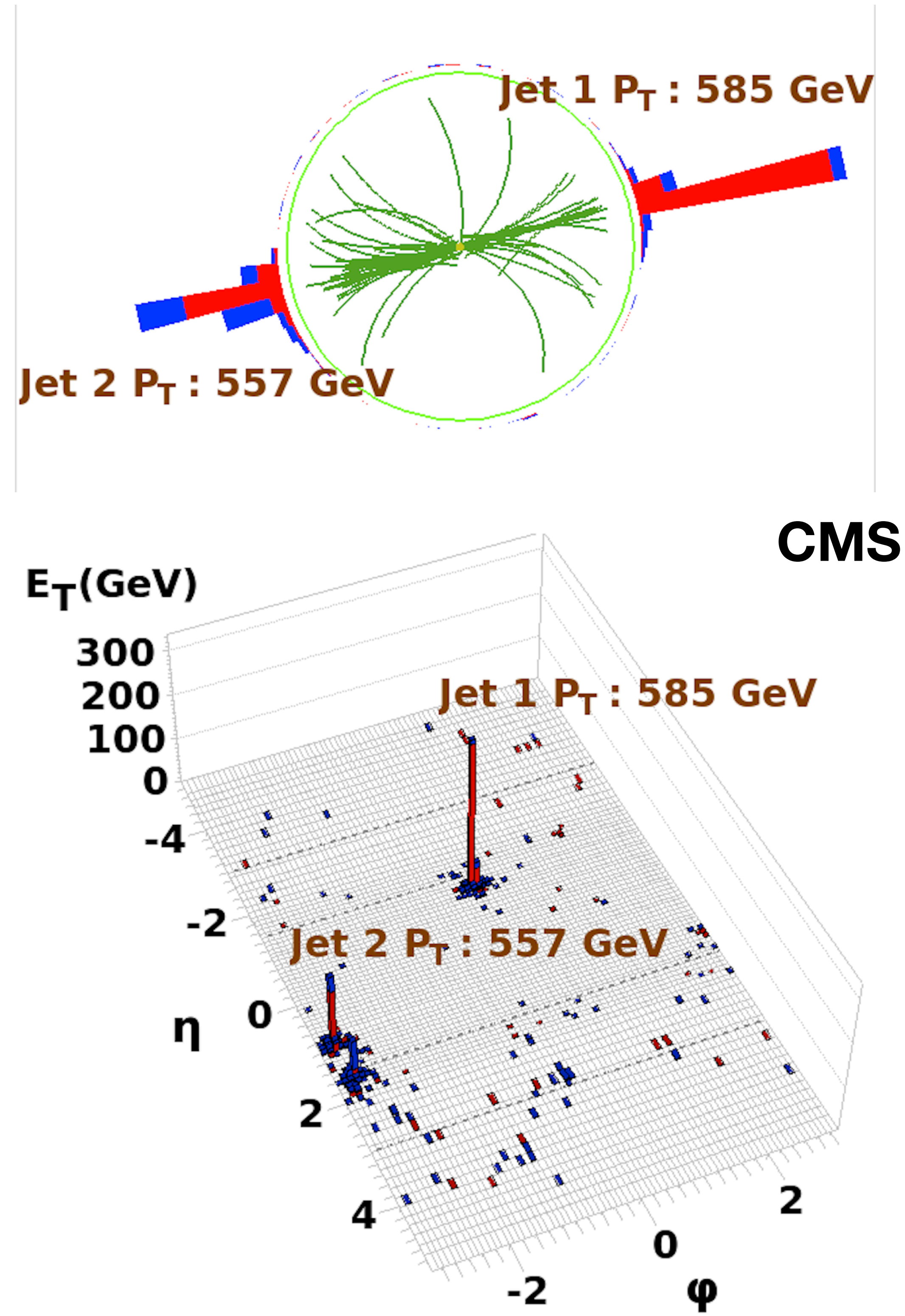
Jets in High Energy Collisions

- **Jets**: collimated spray of particles + finding algorithm (e.g. anti-kT)

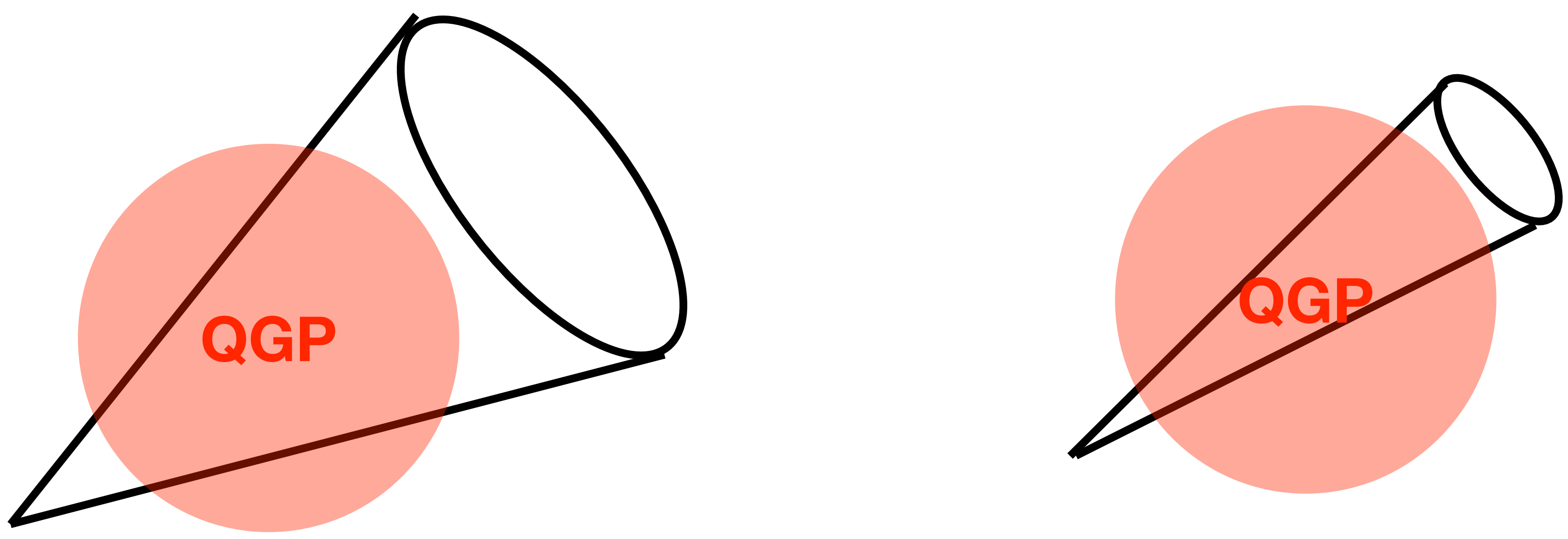




Jets in Heavy Ion Collisions

- **Jets in quark-gluon plasma (QGP)**

Fat jets lose more energy than narrow jets



- **Gluon-initiated jets are fatter in general than quark-initiated jets**

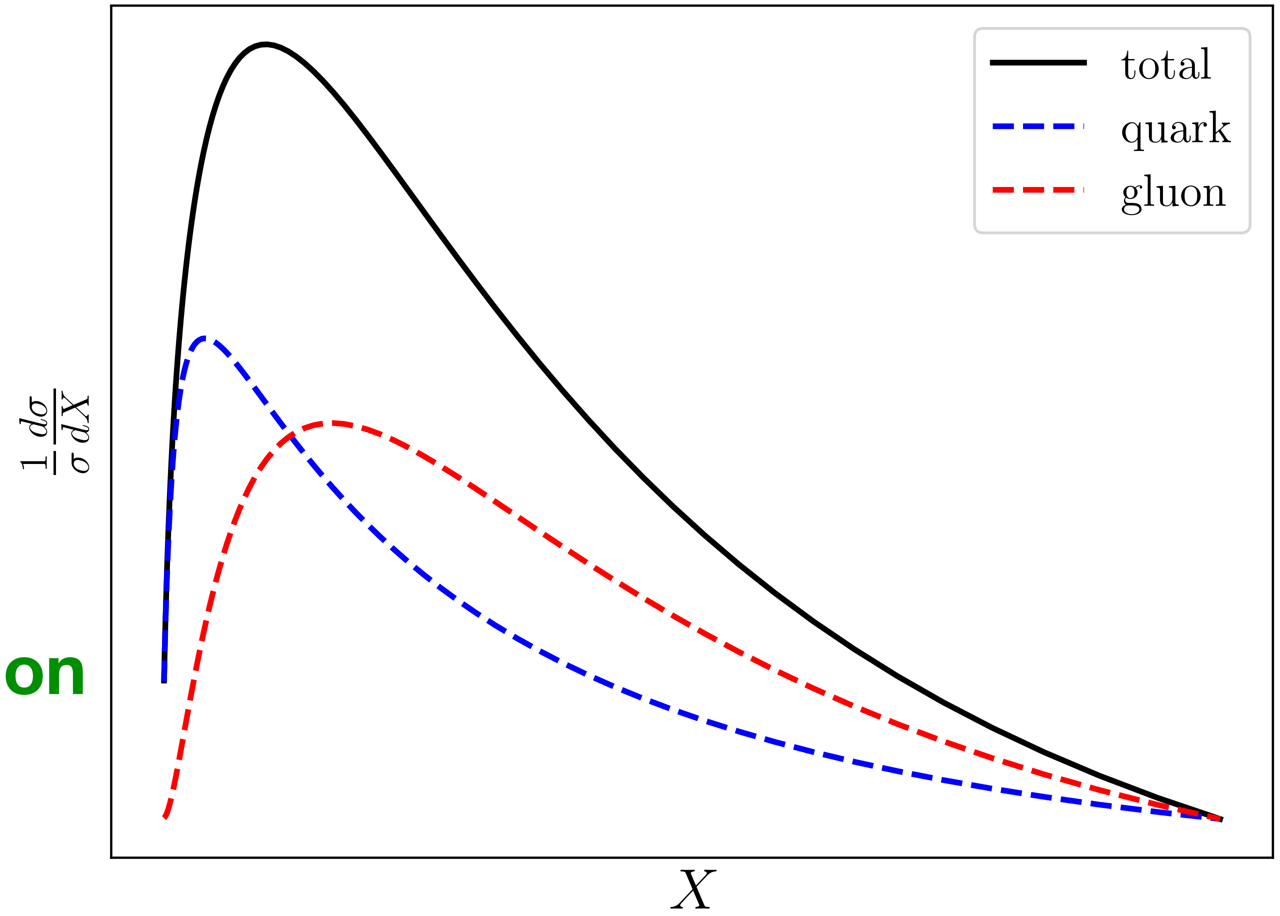
If we can separate gluon jets from quark jets in the data sample, we can better use jets to probe QGP

Disentangling Quark- and Gluon-Initiated Jets

- **Jet observables contain quark & gluon contributions**

$$X = f_q X_q + f_g X_g$$

Only know total distribution from measurements, want to know individual distribution



- **Motivations of quark/gluon discrimination**
  - Better understand QCD jets
  - Improve probes of QGP in heavy ion collisions
  - Constrain parton shower generators
  - Increase sensitivity in BSM physics searches
- **Can we extract quark & gluon fractions (and distributions)?**

Disentangling Quark- and Gluon-Initiated Jets

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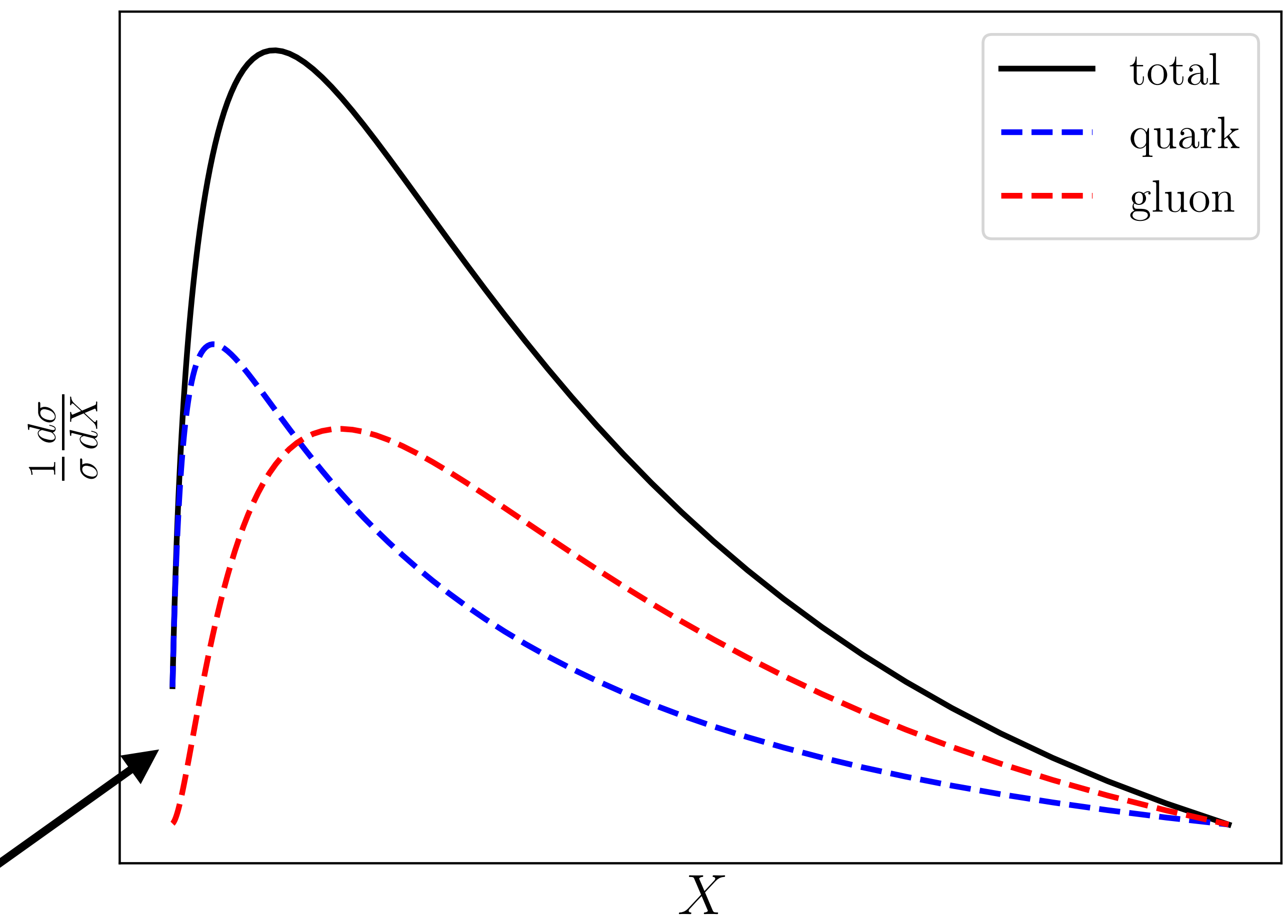


- **Strategy: different Sudakov factors near tail on left**

$$d\sigma_q/dX \sim \exp(-\#\, C_F \log^2)$$

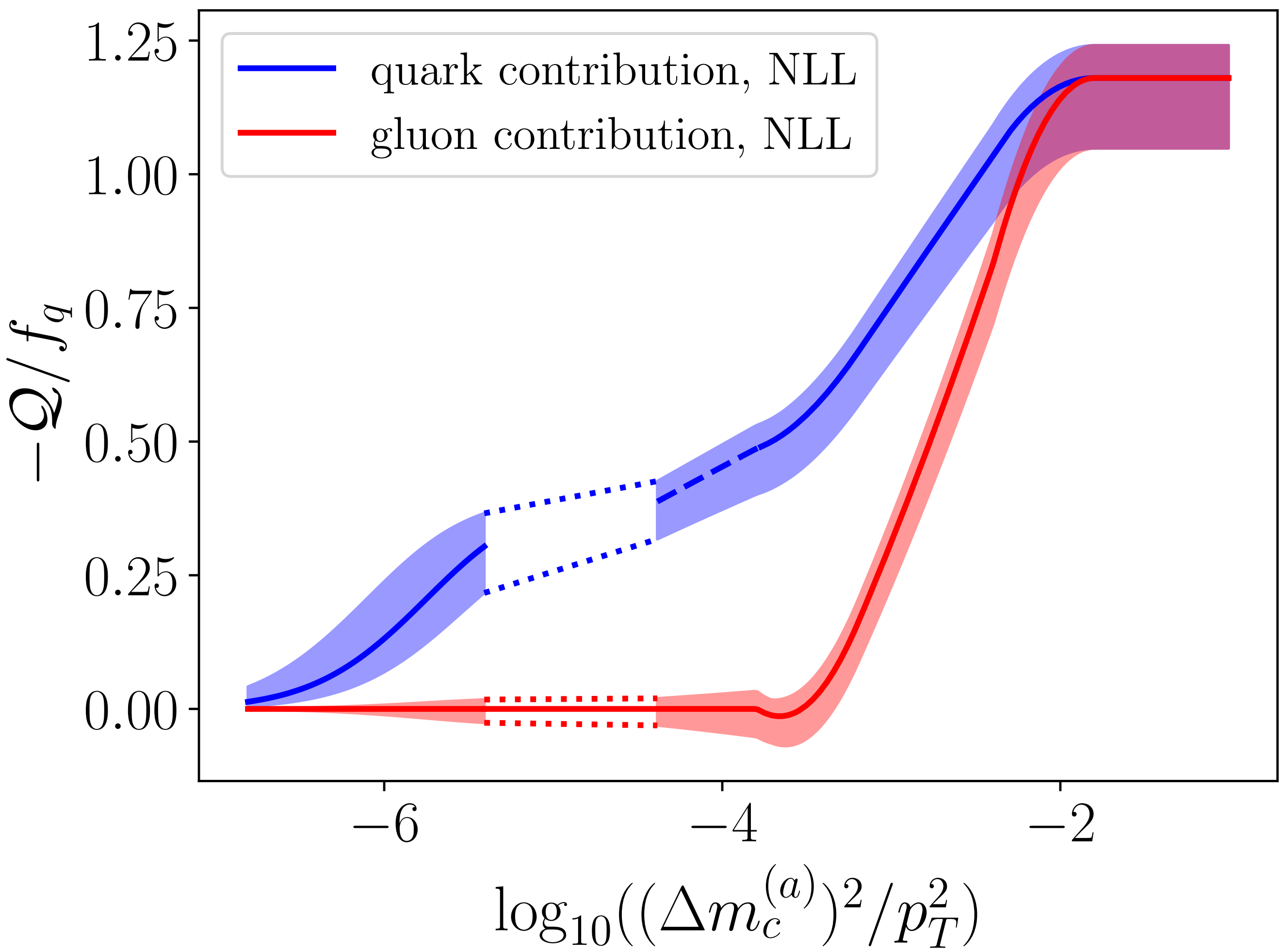
$$d\sigma_g/dX \sim \exp(-\#\, C_A \log^2)$$

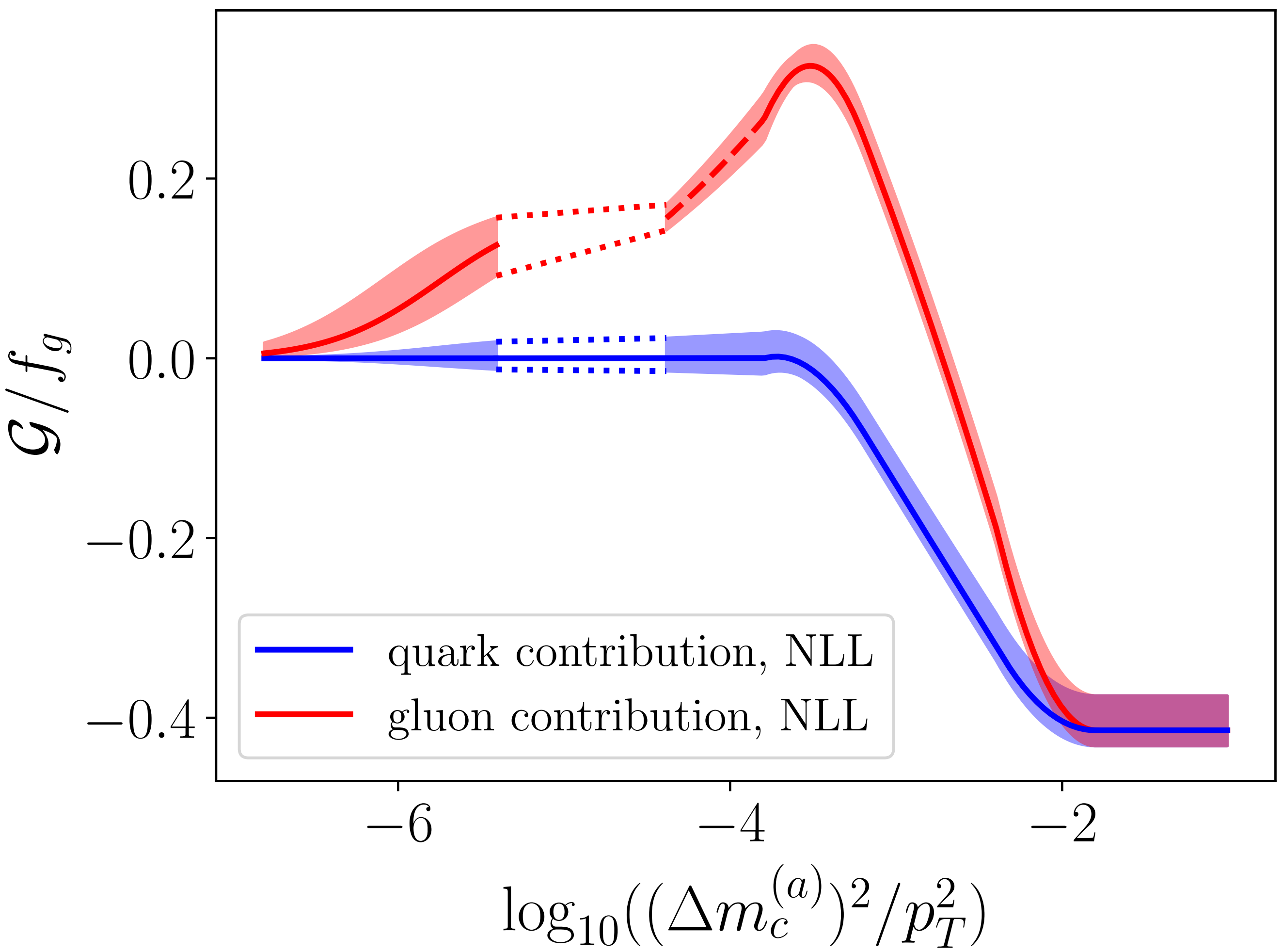
$$C_j = \begin{cases} C_F = \frac{N_c^2-1}{2N_c} & \text{quark} \\ C_A = N_c & \text{gluon} \end{cases}$$

Tail region large experimental uncertainty

Not 100% efficiency

Pure Quark and Gluon Jet Observables





Observable only has quark or gluon contribution in wide kinematic region !

Contents

- Collinear drop grooming technique
  - Jet mass without grooming
  - Jet mass with grooming: soft drop and collinear drop
  - Relevant modes and factorization
- Cumulative jet mass in perturbative and nonperturbative regimes
- Construction of pure quark/gluon observables: NLL v.s. Monte Carlo
- Conclusions

I. Jet Mass and Grooming

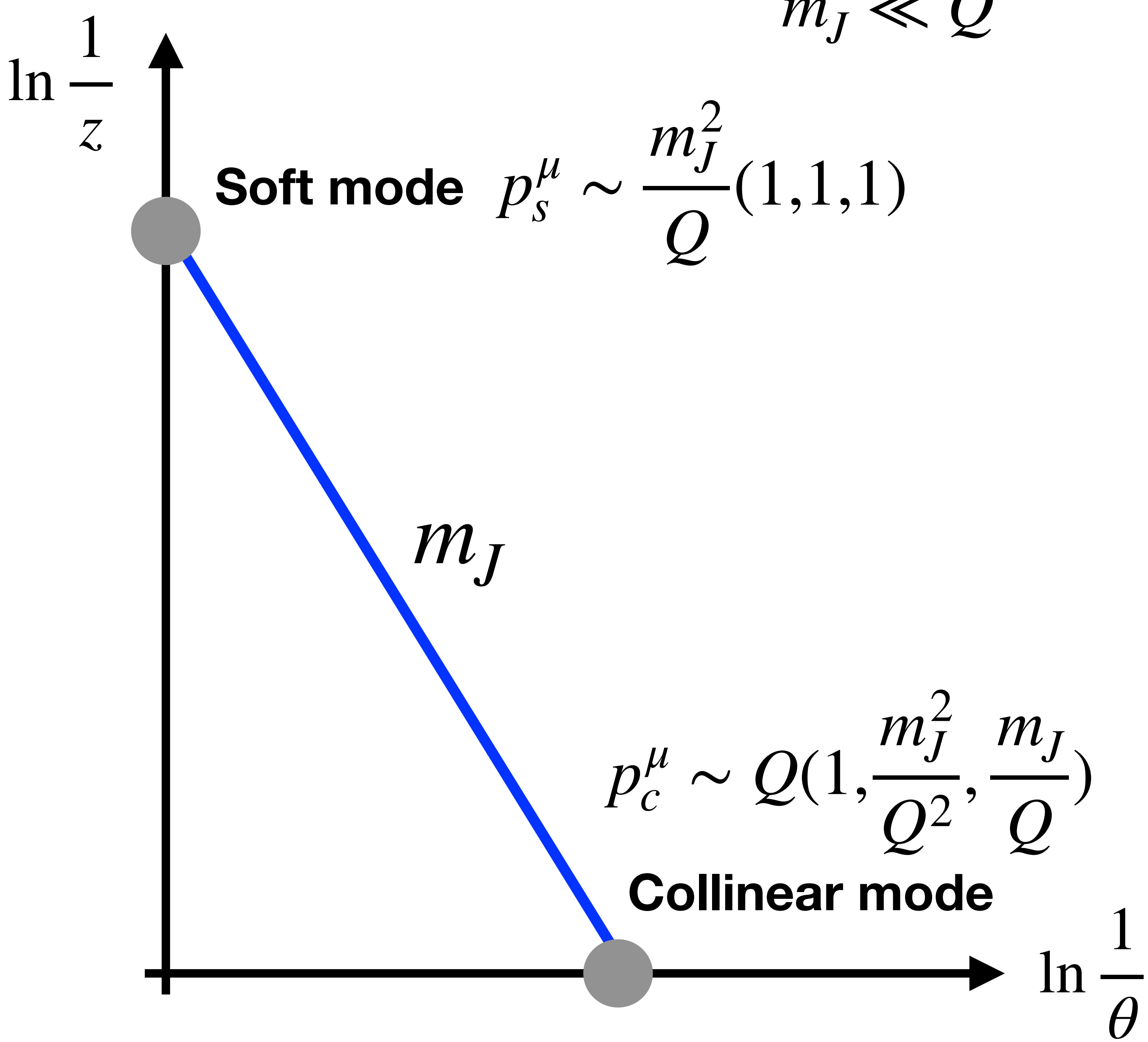
Jet Mass without Grooming

- **Jet mass**

$$m_J^2 = \Big(\sum_{i\in J} p_i^\mu\Big)^2 = P_J^-\Big(\sum_{i\in J} p_i^+\Big) \qquad p_i^\mu \sim z P_J^-(1, \theta^2, \theta)$$

$$Q \equiv P_J^- \qquad m_J^2 \sim Q^2 z \theta^2$$

$$m_J \ll Q$$



$$m_J^2 \sim Q(p_c^+ + p_s^+)$$

↓ Convolution

$$\frac{d\sigma}{dm_J} \sim H \times J \otimes S$$

H: hard function

J: jet function for collinear mode

S: soft function for soft mode

Jet Grooming: Soft Drop

- **Soft Drop w/ parameters** $(z_{\rm cut}, \beta)$

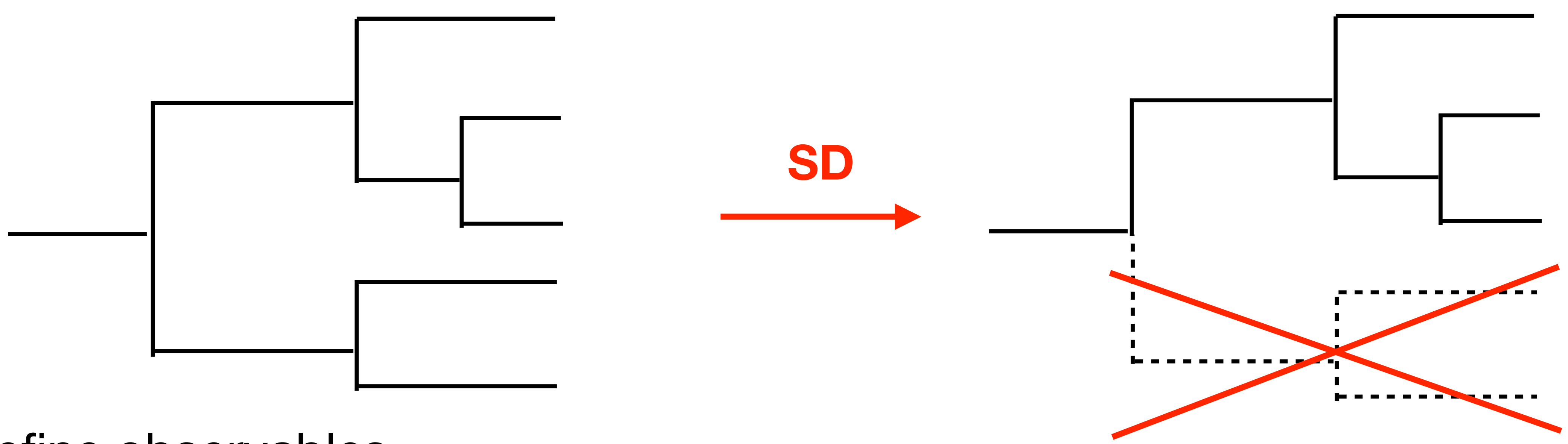
M.Dasgupta,A.Fregoso,S.Marzani G.P. Salam arXiv:1307.0007

A.J.Larkoski,S.Marzani,G.Soyez J.Thaler arXiv:1402.2657

  - Start with jet defined by anti-kT with radius R
  - Re-cluster the jet in Cambridge-Aachen algorithm: first combine pairs w/ smallest

$$\Delta R_{ij} = \frac{2p_i \cdot p_j}{p_{Ti}\, p_{Tj}} = (\theta_i - \theta_j)\cosh y_J \approx \theta_{ij} \cosh \eta_J$$

  - Obtain a tree, consistent with LL branching history: large angle radiated first
  - Keep removing the softer branch until $\dfrac{\min(p_{Ti}, p_{Tj})}{p_{Ti} + p_{Tj}} > z_{\rm cut}\left(\dfrac{\Delta R_{ij}}{R_0}\right)^{\beta} \approx \tilde{z}_{\rm cut}\, \theta_{ij}^{\beta}$



  - Define observables

Jet Mass in Soft Drop

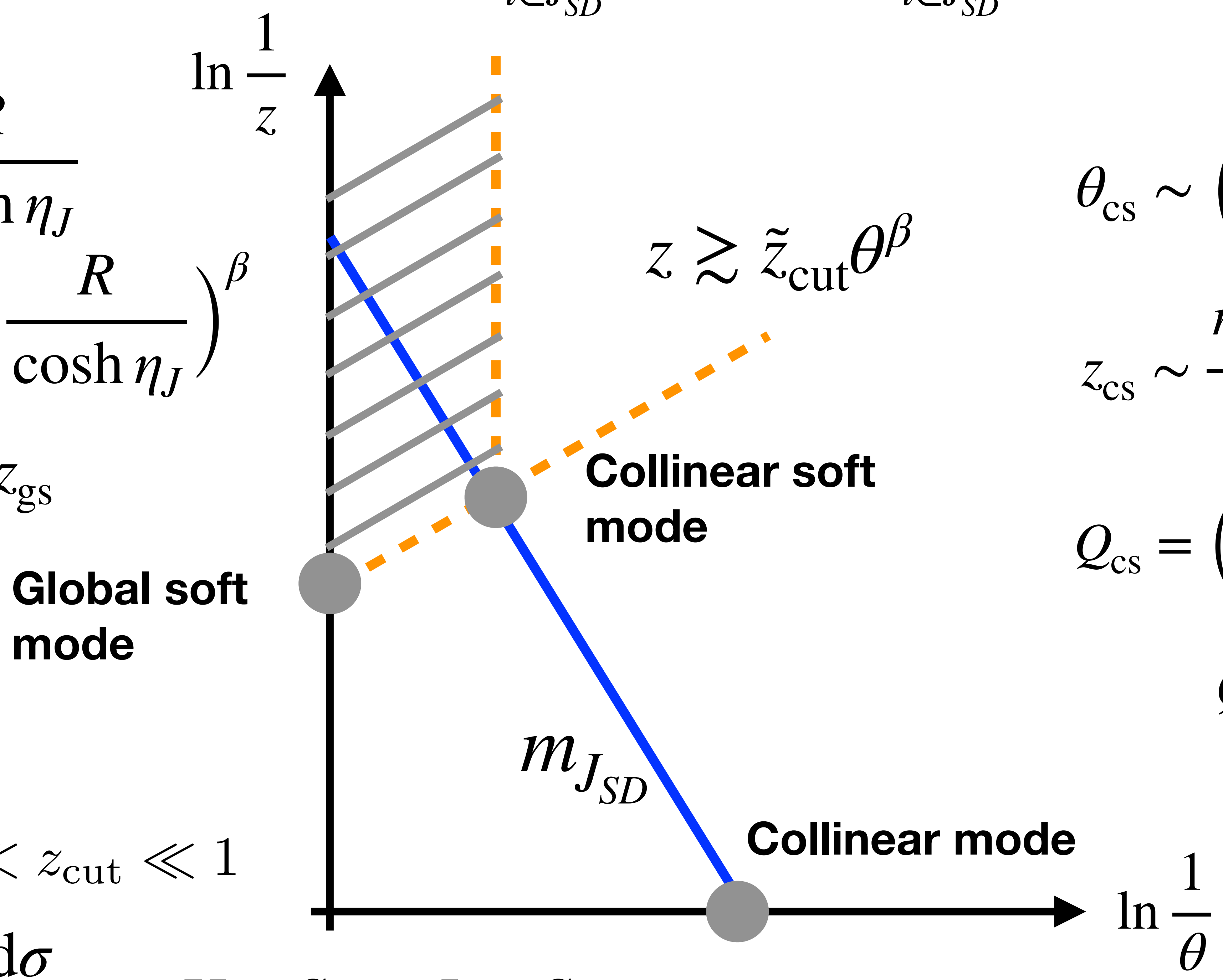
- **Jet mass in SD:** $m_{J_{SD}}^2 = \Big(\sum_{i \in J_{SD}} p_i^\mu\Big)^2 = P_J^- \Big(\sum_{i \in J_{SD}} p_i^+\Big)$

To contribute, must pass SD criterion

$$\theta_{\rm gs} = \frac{R}{\cosh \eta_J}$$

$$z_{\rm gs} = \tilde{z}_{\rm cut}\left(\frac{R}{\cosh \eta_J}\right)^\beta$$

$$Q_{\rm gs} = p_T R z_{\rm gs}$$



$$\theta_{\rm cs} \sim \left(\frac{m_{J_{SD}}^2}{Q Q_{\rm cut}}\right)^{\frac{1}{2+\beta}}$$

$$z_{\rm cs} \sim \frac{m_{J_{SD}}^2}{Q^2}\frac{1}{\theta_{\rm cs}^2}$$

$$Q_{\rm cs} = \left(\frac{m_{J_{\rm SD}}^2}{Q}\right)^{\frac{1+\beta}{2+\beta}} Q_{\rm cut}^{\frac{1}{2+\beta}}$$

$$Q_{\rm cut} = 2^\beta Q \tilde{z}_{\rm cut}$$

$$\frac{m_{J_{SD}}^2}{(p_T R)^2} \ll z_{\rm cut} \ll 1$$

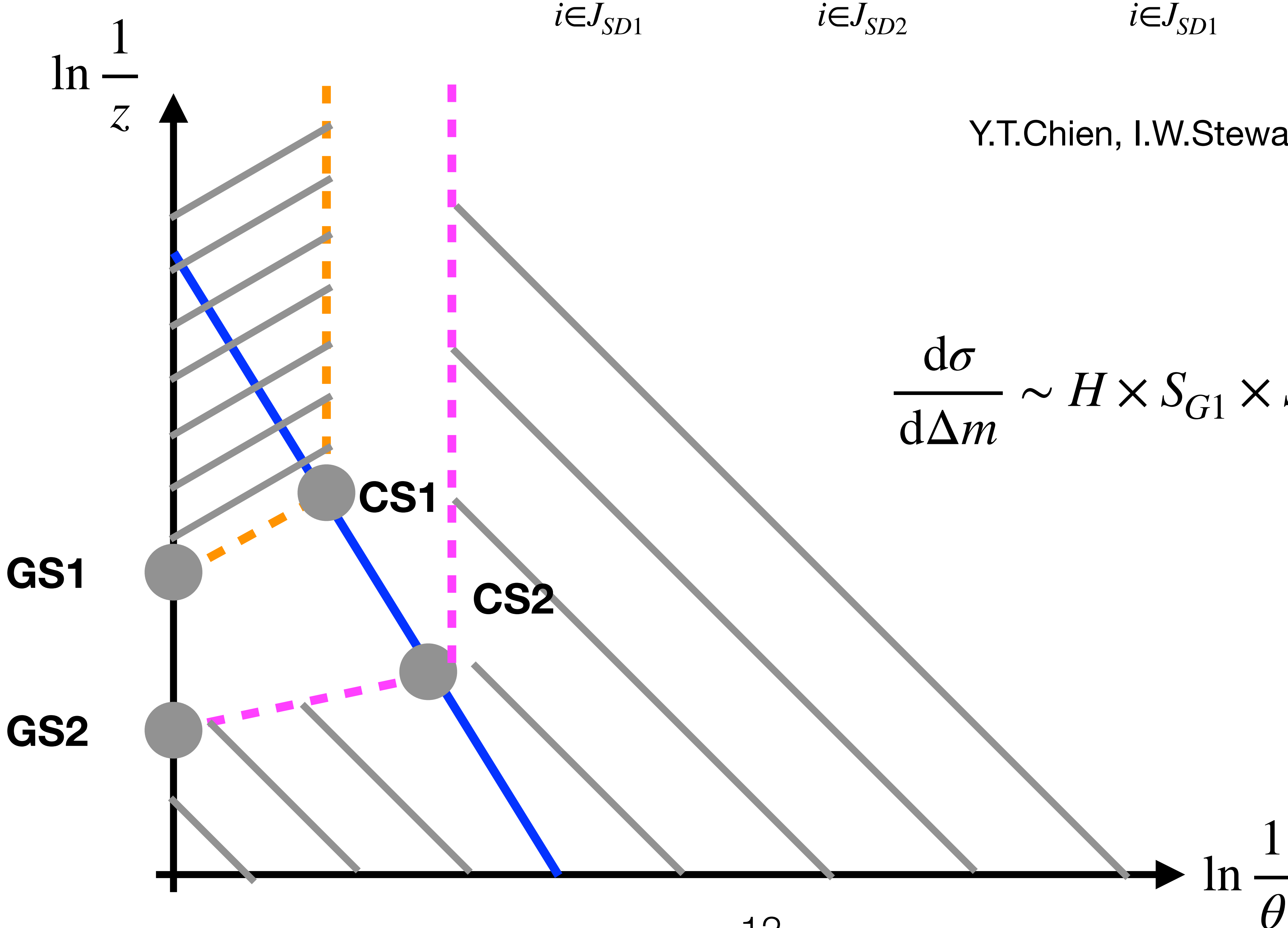
$$\frac{d\sigma}{dm_{J_{SD}}} \sim H \times S_G \times J \otimes S_C$$

C.Frye,A.J.Larkoski,M.D.Schwartz, K.Yan arXiv:1603.09338

Jet Mass in Collinear Drop

- **Jet mass in CD: CD defined from two SD's, second one more aggressive**

$$\Delta m^2 = m^2_{J_{SD1}} - m^2_{J_{SD2}} = \Big(\sum_{i\in J_{SD1}} p_i^\mu\Big)^2 - \Big(\sum_{i\in J_{SD2}} p_i^\mu\Big)^2 = Q\Big(\sum_{i\in J_{SD1}} p_i^+ - \sum_{i\in J_{SD2}} p_i^+\Big)$$



Y.T.Chien, I.W.Stewart, arXiv:1907.11107

$$\frac{d\sigma}{d\Delta m} \sim H \times S_{G1} \times S_{G2} \times S_{C1} \otimes S_{C2}$$

Differential Jet Mass Distribution

- **Differential cross section**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Delta m^2} = \sum_{j=q,g} N_j^{\mathrm{CD}}(p_T, \eta_J, R, \tilde{z}_{\mathrm{cut}\,i}, \beta_i, \mu)\, P_j^{\mathrm{CD}}(\Delta m^2, Q, \tilde{z}_{\mathrm{cut}\,i}, \beta_i, \mu)$$

Determine normalization
Independent of Δm
Dependent on jet kinematics and R

Determine the shape
Dependent on Δm

- **Normalization**

$$N_j^{\mathrm{CD}}(p_T, \eta_J, R, \tilde{z}_{\mathrm{cut}\,i}, \beta_i, \mu) = H_j(p_T, \eta_J, R) \otimes_\Omega S_{G_j}(Q_{\mathrm{gs}1}, R, \beta_1, \mu) \otimes_\Omega S_{\overline{G}_j}(Q_{\mathrm{gs}2}, R, \beta_2, \mu)$$

Hard function only depends
on jet kinematics and R

Global soft function depends
on CD parameters

$$H_j(p_T, \eta_J, R) = \sum_{a,b} f_a \otimes f_b \otimes H_{abj}(p_T, \eta_J, R)$$

Differential Jet Mass Distribution

- **Differential cross section**

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Determine normalization
Independent of Δm
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- **Normalization**

$$N_j^{\mathrm{CD}}(p_T, \eta_J, R, \tilde{z}_{\mathrm{cut}\,i}, \beta_i, \mu) = H_j(p_T, \eta_J, R) \otimes_\Omega S_{G_j}(Q_{\mathrm{gs}1}, R, \beta_1, \mu) \otimes_\Omega S_{\overline{G}_j}(Q_{\mathrm{gs}2}, R, \beta_2, \mu)$$

- **Shape**

$$\hat{P}_j^{\mathrm{CD}}(\Delta m^2, Q, \tilde{z}_{\mathrm{cut}\,i}, \beta_i, \mu) = Q_{\mathrm{cut}1}^{\frac{1}{1+\beta_1}} Q_{\mathrm{cut}2}^{\frac{1}{1+\beta_2}} \int \mathrm{d}\ell_1^+ \mathrm{d}\ell_2^+ \delta\big(\Delta m^2 - Q\ell_1^+ - Q\ell_2^+\big) \times \hat{S}_{C_j}\big(\ell_1^+ Q_{\mathrm{cut}1}^{\frac{1}{1+\beta_1}}, \beta_1, \mu\big) \hat{D}_{C_j}\big(\ell_2^+ Q_{\mathrm{cut}2}^{\frac{1}{1+\beta_2}}, \beta_2, \mu\big)$$

Convolution of two collinear-soft functions

Global Soft Functions

- **One loop in dim. reg.**

$$S_{G_j}(Q_{\rm gs1},\beta_1,\epsilon)=1+\frac{4g^2C_j\mu^{2\epsilon}e^{\epsilon\gamma_E}}{(4\pi)^\epsilon}\int\frac{{\rm d}^dk}{(2\pi)^d}\frac{1}{k^+k^-}2\pi\delta^+(k^2)\,\overline{\Theta}^{\rm (gs)}_{\rm SD1}\,\Theta_{\rm alg}$$

$$S_{\overline{G}_j}(Q_{\rm gs2},\beta_2,\epsilon)=1+\frac{4g^2C_j\mu^{2\epsilon}e^{\epsilon\gamma_E}}{(4\pi)^\epsilon}\int\frac{{\rm d}^dk}{(2\pi)^d}\frac{1}{k^+k^-}2\pi\delta^+(k^2)\Big(-\overline{\Theta}^{\rm (gs)}_{\rm SD2}\Big)\Theta_{\rm alg}$$

$$\overline{\Theta}^{\rm (gs)}_{{\rm SD}\,i}=\theta\bigg(Q\tilde{z}_{{\rm cut}\,i}\Big(\frac{2k^+}{k^-}\Big)^{\frac{\beta_i}{2}}-k^+-k^-\bigg)\qquad\qquad\Theta_{\rm alg}=\theta\bigg(R^2-4\cosh^2\eta_J\,\frac{k^+}{k^-}\bigg)$$

Failing soft drop **Inside jet**

- **Renormalization in MSbar**

$$S^{\rm ren}_{G_j}(Q_{\rm gs1},\beta_1,\mu)=1+\frac{\alpha_s(\mu)C_j}{\pi(1+\beta_1)}\bigg(\ln^2\frac{\mu}{Q_{\rm gs1}}-\frac{\pi^2}{24}\bigg)$$

$$S^{\rm ren}_{\overline{G}_j}(Q_{\rm gs2},\beta_2,\mu)=1-\frac{\alpha_s(\mu)C_j}{\pi(1+\beta_2)}\bigg(\ln^2\frac{\mu}{Q_{\rm gs2}}-\frac{\pi^2}{24}\bigg)$$

Global Soft Functions

- **One loop in dim. reg.**

$$S_{G_j}(Q_{\text{gs}1}, \beta_1, \epsilon) = 1 + \frac{4g^2 C_j \mu^{2\epsilon} e^{\epsilon\gamma_E}}{(4\pi)^\epsilon} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{k^+ k^-} 2\pi\delta^+(k^2)\, \overline{\Theta}^{(\text{gs})}_{\text{SD}1}\, \Theta_{\text{alg}}$$

$$S_{\overline{G}_j}(Q_{\text{gs}2}, \beta_2, \epsilon) = 1 + \frac{4g^2 C_j \mu^{2\epsilon} e^{\epsilon\gamma_E}}{(4\pi)^\epsilon} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{k^+ k^-} 2\pi\delta^+(k^2)\big(- \overline{\Theta}^{(\text{gs})}_{\text{SD}2}\big)\Theta_{\text{alg}}$$

- **RGE of global soft functions**

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \ln S^{\text{ren}}_{G_j}(Q_{\text{gs}1}, \beta_1, \mu) = \frac{2C_j}{1+\beta_1}\Gamma_{\text{cusp}} \ln\frac{\mu}{Q_{\text{gs}1}} + \gamma_{S_{G_j}}$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \ln S^{\text{ren}}_{\overline{G}_j}(Q_{\text{gs}2}, \beta_2, \mu) = -\frac{2C_j}{1+\beta_2}\Gamma_{\text{cusp}} \ln\frac{\mu}{Q_{\text{gs}2}} - \gamma_{\overline{S}_{G_j}}$$

Collinear Soft Functions

- **One loop in dim. reg.**

$$\hat{S}_{C_j}(\ell_1^+,\beta_1,\mu):\quad \frac{4g^2C_j\mu^{2\epsilon}e^{\epsilon\gamma_E}}{(4\pi)^\epsilon}\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{2\pi\delta^+(k^2)}{k^+k^-}\Big(\delta(k^+-\ell_1^+)-\delta(\ell_1^+)\Big)\Theta_{\mathrm{SD1}}^{(\mathrm{cs})}$$

$$\hat{D}_{C_j}(\ell_2^+,\beta_2,\mu):\quad \frac{4g^2C_j\mu^{2\epsilon}e^{\epsilon\gamma_E}}{(4\pi)^\epsilon}\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{2\pi\delta^+(k^2)}{k^+k^-}\Big(\delta(k^+-\ell_2^+)-\delta(\ell_2^+)\Big)\Big(-\Theta_{\mathrm{SD2}}^{(\mathrm{cs})}\Big)$$

- **Renormalization in MSbar in Laplace space**

$$\tilde{f}(y)=\int_0^\infty \mathrm{d}\Delta m^2\,\exp\big(-ye^{-\gamma_E}\Delta m^2\big)f(\Delta m^2)$$

$$\tilde{\hat{S}}_{C_j}\Big(yQQ_{\mathrm{cut1}}^{\frac{-1}{1+\beta_1}},\beta_1,\mu\Big)=1+\frac{\alpha_sC_j}{\pi}\frac{2+\beta_1}{1+\beta_1}\left(-\ln^2\frac{\mu\,y^{\frac{1+\beta_1}{2+\beta_1}}Q^{\frac{1+\beta_1}{2+\beta_1}}}{Q_{\mathrm{cut1}}^{\frac{1}{2+\beta_1}}}+\frac{\pi^2}{24}\right)$$

$$\tilde{\hat{D}}_{C_j}\Big(yQQ_{\mathrm{cut2}}^{\frac{-1}{1+\beta_2}},\beta_2,\mu\Big)=1-\frac{\alpha_sC_j}{\pi}\frac{2+\beta_2}{1+\beta_2}\left(-\ln^2\frac{\mu\,y^{\frac{1+\beta_2}{2+\beta_2}}Q^{\frac{1+\beta_2}{2+\beta_2}}}{Q_{\mathrm{cut2}}^{\frac{1}{2+\beta_2}}}+\frac{\pi^2}{24}\right)$$

Collinear Soft Functions

- **One loop in dim. reg.**

$$\hat{S}_{C_j}(\ell_1^+, \beta_1, \mu): \quad \frac{4g^2 C_j \mu^{2\epsilon} e^{\epsilon\gamma_E}}{(4\pi)^\epsilon} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{2\pi\delta^+(k^2)}{k^+ k^-} \Big(\delta(k^+ - \ell_1^+) - \delta(\ell_1^+)\Big) \Theta_{\mathrm{SD1}}^{(\mathrm{cs})}$$

$$\hat{D}_{C_j}(\ell_2^+, \beta_2, \mu): \quad \frac{4g^2 C_j \mu^{2\epsilon} e^{\epsilon\gamma_E}}{(4\pi)^\epsilon} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{2\pi\delta^+(k^2)}{k^+ k^-} \Big(\delta(k^+ - \ell_2^+) - \delta(\ell_2^+)\Big) \big(- \Theta_{\mathrm{SD2}}^{(\mathrm{cs})}\big)$$

- **RGE of collinear soft functions**

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \ln \tilde{\hat{S}}_{C_j}\Big(yQQ_{\mathrm{cut1}}^{\frac{-1}{1+\beta_1}}, \beta_1, \mu\Big) = 2C_j \Gamma_{\mathrm{cusp}}[\alpha_s] \ln \frac{Q_{\mathrm{cut1}}^{\frac{1}{1+\beta_1}}}{\mu^{\frac{2+\beta_1}{1+\beta_1}} Qy} + \gamma_{S_{C_j}}[\alpha_s]$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \ln \tilde{\hat{D}}_{C_j}\Big(yQQ_{\mathrm{cut2}}^{\frac{-1}{1+\beta_2}}, \beta_2, \mu\Big) = -2C_j \Gamma_{\mathrm{cusp}}[\alpha_s] \ln \frac{Q_{\mathrm{cut2}}^{\frac{1}{1+\beta_2}}}{\mu^{\frac{2+\beta_2}{1+\beta_2}} Qy} + \gamma_{D_{C_j}}[\alpha_s]$$

Cumulative Jet Mass in Collinear Drop

- **Cumulative jet mass distribution**

$$\Sigma(\Delta m_c^2) = \frac{1}{\sigma}\int_0^{\Delta m_c^2} \mathrm{d}\Delta m^2 \, \frac{\mathrm{d}\sigma}{\mathrm{d}\Delta m^2}$$

- **In perturbative region**

$$\hat{\Sigma}(\Delta m_c) = \sum_{j=q,g} f_j \, \hat{\Sigma}_j(\Delta m_c)$$

- **At NLL** $f_j = H_j(p_T, \eta_J, R)/\sigma_0$

Y.T.Chien, I.W.Stewart, 1907.11107

$$\hat{\Sigma}_j^{\rm NLL} = \exp\left[\frac{2C_j}{1+\beta_1}K(\mu_{\rm gs1},\mu) - \frac{2C_j}{1+\beta_2}K(\mu_{\rm gs2},\mu) - 2C_j\frac{2+\beta_1}{1+\beta_1}K(\mu_{\rm cs1},\mu) + 2C_j\frac{2+\beta_2}{1+\beta_2}K(\mu_{\rm cs2},\mu)\right]$$

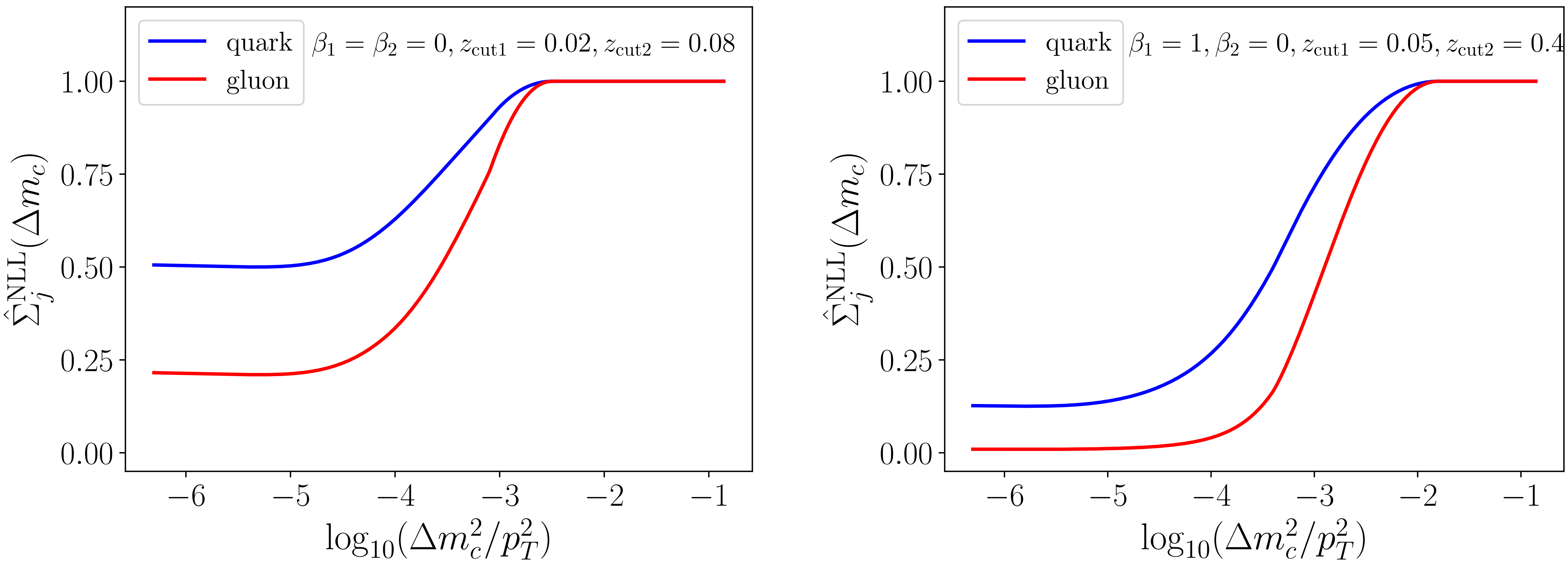
$$\times \left(\frac{\mu_{\rm gs1}}{Q_{\rm gs1}}\right)^{\frac{2C_j}{1+\beta_1}\omega(\mu_{\rm gs1},\mu)} \left(\frac{\mu_{\rm gs2}}{Q_{\rm gs2}}\right)^{\frac{-2C_j}{1+\beta_2}\omega(\mu_{\rm gs2},\mu)} \left(\frac{Q_{\rm cut1}^{\frac{1}{1+\beta_1}}}{Q\mu_{\rm cs1}^{\frac{2+\beta_1}{1+\beta_1}}}\right)^{2C_j\omega(\mu_{\rm cs1},\mu)} \left(\frac{Q_{\rm cut2}^{\frac{1}{1+\beta_2}}}{Q\mu_{\rm cs2}^{\frac{2+\beta_2}{1+\beta_2}}}\right)^{-2C_j\omega(\mu_{\rm cs2},\mu)} \left.\frac{(e^{-\gamma_E}\Delta m_c^2)^\eta}{\Gamma(1+\eta)}\right|_{\eta=2C_j\omega(\mu_{\rm cs1},\mu_{\rm cs2})}$$

Sudakov factors with both negative and positive signs

$$K(\mu_1,\mu_2) = \int_{\alpha_s(\mu_1)}^{\alpha_s(\mu_2)} d\alpha \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\mu_1)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')} \qquad \omega(\mu_1,\mu_2) = \int_{\alpha_s(\mu_1)}^{\alpha_s(\mu_2)} d\alpha \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}$$

Cumulative Jet Mass in CD: Perturbative

$p_T = 800$ GeV $\quad R = 0.2$



Finite constant as $\Delta m_c \to 0$ in CD; In SD, go to zero

A significant fraction of events have the two SD jet masses equal, the constant gives this fraction

The constant depends on quark/gluon, CD parameters —> exploited later

Cumulative Jet Mass in CD: Nonperturbative

- **Solution to RGE of collinear-soft function and natural scale**

$$\hat{\tilde{S}}^{\text{ren}}_{C_j}\left(yQQ_{\text{cut}1}^{\frac{-1}{1+\beta_1}},\beta_1,\mu\right)=\hat{\tilde{S}}^{\text{ren}}_{C_j}\left(yQQ_{\text{cut}1}^{\frac{-1}{1+\beta_1}},\beta_1,\mu_1\right)\exp\left(-2C_j\frac{2+\beta_1}{1+\beta_1}K(\mu_1,\mu)+\omega_{S_{C_j}}(\mu_1,\mu)\right)\left(\frac{Q_{\text{cut}1}^{\frac{1}{1+\beta_1}}}{yQ\mu_1^{\frac{2+\beta_1}{1+\beta_1}}}\right)^{2C_j\omega(\mu_1,\mu)}$$

How to pick up the scale μ_1 for boundary condition?

Use perturbative calculation

$$\hat{\tilde{S}}^{\text{ren}}_{C_j}\left(yQQ_{\text{cut}a}^{\frac{-1}{1+\beta_1}},\beta_1,\mu_1\right)=1+\frac{\alpha_s C_j}{\pi}\frac{2+\beta_1}{1+\beta_1}\left(-\ln^2\frac{\mu_1 y^{\frac{1+\beta_1}{2+\beta_1}}Q^{\frac{1+\beta_1}{2+\beta_1}}}{Q_{\text{cut}1}^{\frac{1}{2+\beta_1}}}+\frac{\pi^2}{24}\right)$$

Choose $\mu_1\sim\left(\frac{\Delta m^2}{Q}\right)^{\frac{1+\beta_1}{2+\beta_1}}Q_{\text{cut}1}^{\frac{1}{2+\beta_1}}$ to minimize the log in boundary term

- **In nonperturbative regime, have to stop running at** $\mu_1=\Lambda_{\text{CS1}}>1\ \text{GeV}$

As Δm^2 decreases $\left(\frac{\Delta m^2}{Q}\right)^{\frac{1+\beta_1}{2+\beta_1}}Q_{\text{cut}1}^{\frac{1}{2+\beta_1}}\sim\Lambda_{\text{QCD}}$

Nonperturbative corrections important in this region

Nonperturbative Correction via Shape Function

- **Convoluting perturbative CS function with nonperturbative shape function**

$$S_{C_j}\big(\ell_1^+ Q_{\rm cut1}^{\frac{1}{1+\beta_1}}, \beta_1, \mu\big) = \int_0^{+\infty} \mathrm{d}k_1\, \hat{S}_{C_j}\big(\ell_1^+ Q_{\rm cut1}^{\frac{1}{1+\beta_1}} - k_1^{\frac{2+\beta_1}{1+\beta_1}}, \beta_1, \mu\big) F_1^j(k_1, \beta_1)$$

Independent of $z_{\rm cut1}$

A.H.Hoang, S.Mantry, A.Pathak, I.W.Stewart, arXiv:1906.11843

- **In Laplace space:**

$$\widetilde{S}_{C_j}\big(yQQ_{\rm cut1}^{\frac{-1}{1+\beta_1}}, \beta_1, \mu\big) = \widetilde{\hat{S}}_{C_j}\big(yQQ_{\rm cut1}^{\frac{-1}{1+\beta_1}}, \beta_1, \mu\big)\, \widetilde{F}_1^j\big(yQQ_{\rm cut1}^{\frac{-1}{1+\beta_1}}, \beta_1\big)$$

$$= \widetilde{\hat{S}}_{C_j}\big(yQQ_{\rm cut1}^{\frac{-1}{1+\beta_1}}, \beta_1, \Lambda_{\rm cs1}\big)\, \widetilde{F}_1^j\big(yQQ_{\rm cut1}^{\frac{-1}{1+\beta_1}}, \beta_1\big) \exp\Big(-2C_j \frac{2+\beta_1}{1+\beta_1} K(\Lambda_{\rm cs1}, \mu) + \omega_{S_{C_j}}(\Lambda_{\rm cs1}, \mu)\Big) \left(\frac{Q_{\rm cut1}^{\frac{1}{1+\beta_1}}}{yQ\Lambda_{\rm cs1}^{\frac{2+\beta_1}{1+\beta_1}}}\right)^{2C_j\omega(\Lambda_{\rm cs1},\mu)}$$

Same procedure can be done for the other collinear-soft function

Can define shape function in MSbar scheme at $\mu = \Lambda_{{\rm CS}\,i}$

$$\widetilde{F}_i^{(\overline{\rm MS})j}\big(yQQ_{{\rm cut}\,i}^{\frac{-1}{1+\beta_i}}, \beta_i, \Lambda_{{\rm cs}\,i}\big) = \widetilde{F}_i^j\big(yQQ_{{\rm cut}\,i}^{\frac{-1}{1+\beta_i}}, \beta_i\big)\, \widetilde{\hat{S}}_{C_j}\big(yQQ_{{\rm cut}\,i}^{\frac{-1}{1+\beta_i}}, \beta_i, \Lambda_{{\rm cs}\,i}\big)$$

$\Lambda_{{\rm CS}\,i}$ **dependence canceled between perturbative evolution and shape function**

Cumulative Jet Mass in Nonperturbative Regime

$$\Sigma^{\mathrm{NLL}}(\Delta m_c) = \sum_{j=q,g} f_j\, \hat{\Sigma}_j^{\mathrm{NLL}}\, \mathscr{F}_j(\Delta m_c)$$

$$\hat{\Sigma}_j^{\mathrm{NLL}} = \exp\left[\frac{2C_j}{1+\beta_1}K(\mu_{\mathrm{gs1}},\mu) - \frac{2C_j}{1+\beta_2}K(\mu_{\mathrm{gs2}},\mu) + \frac{2C_j(\beta_1-\beta_2)}{(1+\beta_1)(1+\beta_2)}K(\Lambda_{\mathrm{cs}},\mu)\right]$$
$$\times\left(\frac{\mu_{\mathrm{gs1}}}{Q_{\mathrm{gs1}}}\right)^{\frac{2C_j}{1+\beta_1}\omega(\mu_{\mathrm{gs1}},\mu)}\left(\frac{\mu_{\mathrm{gs2}}}{Q_{\mathrm{gs2}}}\right)^{\frac{-2C_j}{1+\beta_2}\omega(\mu_{\mathrm{gs2}},\mu)}\left(\frac{Q_{\mathrm{cut1}}^{\frac{1}{1+\beta_1}}}{\Lambda_{\mathrm{cs}}^{\frac{1}{1+\beta_1}}}\frac{\Lambda_{\mathrm{cs}}^{\frac{1}{1+\beta_2}}}{Q_{\mathrm{cut2}}^{\frac{1}{1+\beta_2}}}\right)^{2C_j\omega(\Lambda_{\mathrm{cs}},\mu)}$$

$$\mathscr{F}_j(\Delta m_c) = \int \mathrm{d}k_1\mathrm{d}k_2\, F_1^j(k_1,\beta_1)F_2^j(k_2,\beta_2)\,\Theta\left(\Delta m_c^2 - QQ_{\mathrm{cut1}}^{\frac{-1}{1+\beta_1}}k_1^{\frac{2+\beta_1}{1+\beta_1}} - QQ_{\mathrm{cut2}}^{\frac{-1}{1+\beta_2}}k_2^{\frac{2+\beta_2}{1+\beta_2}}\right)$$

We choose $\Lambda_{\mathrm{CS1}} = \Lambda_{\mathrm{CS2}} \sim 2\ \mathrm{GeV}$

Difference choices —> different shape functions, final results should be same

II. Construction of Pure Quark and Gluon Observables

Construction of Pure Quark/Gluon Observables

- **General strategy**

Take cumulative jet mass in CD $\Sigma(\Delta m_c) = \sum_{j=q,g} f_j \hat{\Sigma}_j \mathcal{F}_j(\Delta m_c)$

Form linear combination from two sets of CD parameters: $z_{\text{cut}\,i}^{(a)},\ z_{\text{cut}\,i}^{(b)},\ i = 1,2$

$$\mathcal{Q} = \Sigma(\Delta m_c^{(b)}, p_T, \eta_J, R, z_{\text{cut}\,i}^{(b)}, \beta_i) - \xi_g \Sigma(\Delta m_c^{(a)}, p_T, \eta_J, R, z_{\text{cut}\,i}^{(a)}, \beta_i)$$

$$\mathcal{G} = \Sigma(\Delta m_c^{(b)}, p_T, \eta_J, R, z_{\text{cut}\,i}^{(b)}, \beta_i) - \xi_q \Sigma(\Delta m_c^{(a)}, p_T, \eta_J, R, z_{\text{cut}\,i}^{(a)}, \beta_i)$$

Find value of ξ_g such that gluon contribution to $\mathcal{Q}$ vanishes

$$\hat{\Sigma}_g^{(b)} \mathcal{F}_g^{(b)} - \xi_g \hat{\Sigma}_g^{(a)} \mathcal{F}_g^{(a)} = 0$$

Find value of ξ_q such that quark contribution to $\mathcal{G}$ vanishes

$$\hat{\Sigma}_q^{(b)} \mathcal{F}_q^{(b)} - \xi_q \hat{\Sigma}_q^{(a)} \mathcal{F}_q^{(a)} = 0$$

Key: values of $\xi_q,\ \xi_g$ independent of $\Delta m_c^{(a)}$ and $\Delta m_c^{(b)}$

Rebinning Jet Masses

- **Make arguments of shape function the same in (a) and (b)**

$$(Q^{(a)}_{\mathrm{cut}1})^{\frac{1}{1+\beta_1}}(\Delta m_c^{(a)})^2 = (Q^{(b)}_{\mathrm{cut}1})^{\frac{1}{1+\beta_1}}(\Delta m_c^{(b)})^2$$

$$(Q^{(a)}_{\mathrm{cut}2})^{\frac{1}{1+\beta_2}}(\Delta m_c^{(a)})^2 = (Q^{(b)}_{\mathrm{cut}2})^{\frac{1}{1+\beta_2}}(\Delta m_c^{(b)})^2$$

- **Solved by constraints**

$$(\Delta m_c^{(b)})^2 = (\Delta m_c^{(a)})^2 \left(\frac{z^{(a)}_{\mathrm{cut}1}}{z^{(b)}_{\mathrm{cut}1}}\right)^{\frac{1}{1+\beta_1}} \qquad z^{(b)}_{\mathrm{cut}2} = z^{(a)}_{\mathrm{cut}2}\left(\frac{z^{(b)}_{\mathrm{cut}1}}{z^{(a)}_{\mathrm{cut}1}}\right)^{\frac{1+\beta_2}{1+\beta_1}}$$

- **Shape functions become common factor** $\mathscr{F}_j^{(a)} = \mathscr{F}_j^{(b)} \equiv \mathscr{F}_j$

$$\mathscr{Q} = \sum_{j=q,g} f_j \mathscr{F}_j \left(\hat{\Sigma}_j^{(b)} - \xi_g \hat{\Sigma}_j^{(a)}\right) \qquad \mathscr{G} = \sum_{j=q,g} f_j \mathscr{F}_j \left(\hat{\Sigma}_j^{(b)} - \xi_q \hat{\Sigma}_j^{(a)}\right)$$

$$\xi_g = \frac{\hat{\Sigma}_g^{(b)}}{\hat{\Sigma}_g^{(a)}}$$

Coefficients defined purely perturbatively

Also works in perturbative regime

$$\xi_q = \frac{\hat{\Sigma}_q^{(b)}}{\hat{\Sigma}_q^{(a)}}$$

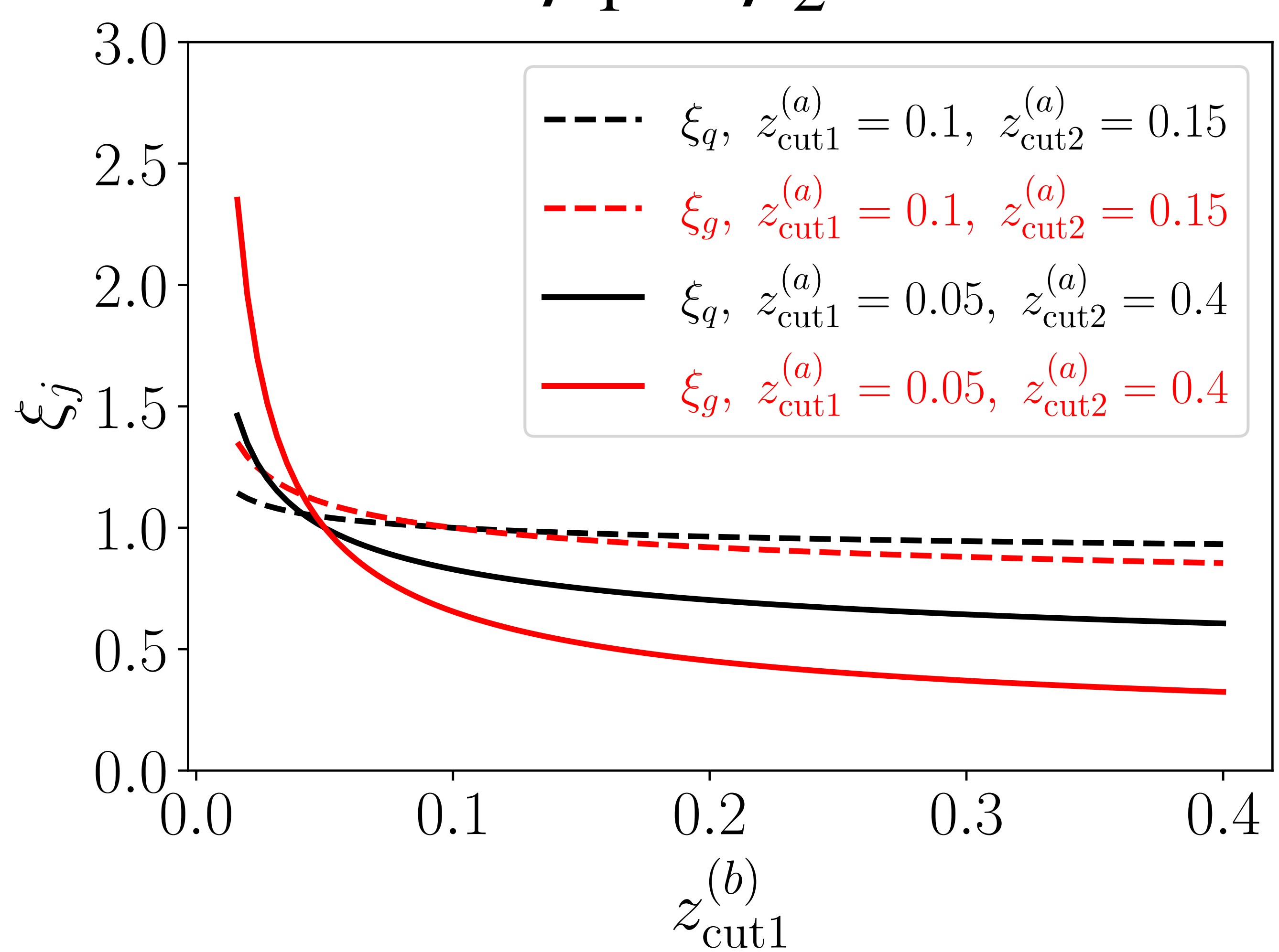
Perturbative Determination of ξ_j

- **At NLL**

$$\xi_j = \exp\left[\frac{2C_j}{1+\beta_1}K\left(\mu_{\rm gs1}^{(b)},\mu_{\rm gs1}^{(a)}\right) - \frac{2C_j}{1+\beta_2}K\left(\mu_{\rm gs2}^{(b)},\mu_{\rm gs2}^{(a)}\right) + \frac{2C_j}{1+\beta_1}\omega\left(\mu_{\rm gs1}^{(a)},\mu_{\rm gs2}^{(a)}\right)\ln\frac{z_{\rm cut\,1}^{(a)}}{z_{\rm cut\,1}^{(b)}}\right]$$
$$\times\left(\frac{\mu_{\rm gs1}^{(b)}}{Q_{\rm gs1}^{(b)}}\right)^{\frac{2C_j}{1+\beta_1}\omega\left(\mu_{\rm gs1}^{(b)},\mu_{\rm gs1}^{(a)}\right)}\left(\frac{\mu_{\rm gs2}^{(b)}}{Q_{\rm gs2}^{(b)}}\right)^{\frac{-2C_j}{1+\beta_2}\omega\left(\mu_{\rm gs2}^{(b)},\mu_{\rm gs2}^{(a)}\right)}$$

Stronger discrimination power requires largely separated ξ_j , and thus largely separeted $z_{{\rm cut}\,i}$

$\beta_1 = \beta_2 = 0$



Optimize Parameter Choice

- **Have constructed a class of pure quark and gluon observables, want to optimize parameters to maximize disentangling power**

  - GS scales perturbative: $Q_{\mathrm{gs}\,i}^{(a/b)} = p_T R\, z_{\mathrm{cut}\,i}^{(a/b)} (R/R_0)^{\beta_i} \gg \Lambda_{\mathrm{QCD}}$

  - Largely separated $z_{\mathrm{cut1}}^{(a)}$ and $z_{\mathrm{cut2}}^{(a)}$, largely separated $z_{\mathrm{cut1}}^{(a)}$ and $z_{\mathrm{cut1}}^{(b)}$

  - Remove contamination of external soft radiation (e.g. ISR) and underlying events (MPI): (1) small jet radius R; (2) $z_{\mathrm{cut1}}^{(a/b)} \gtrsim 0.15$ for $R \sim 1$

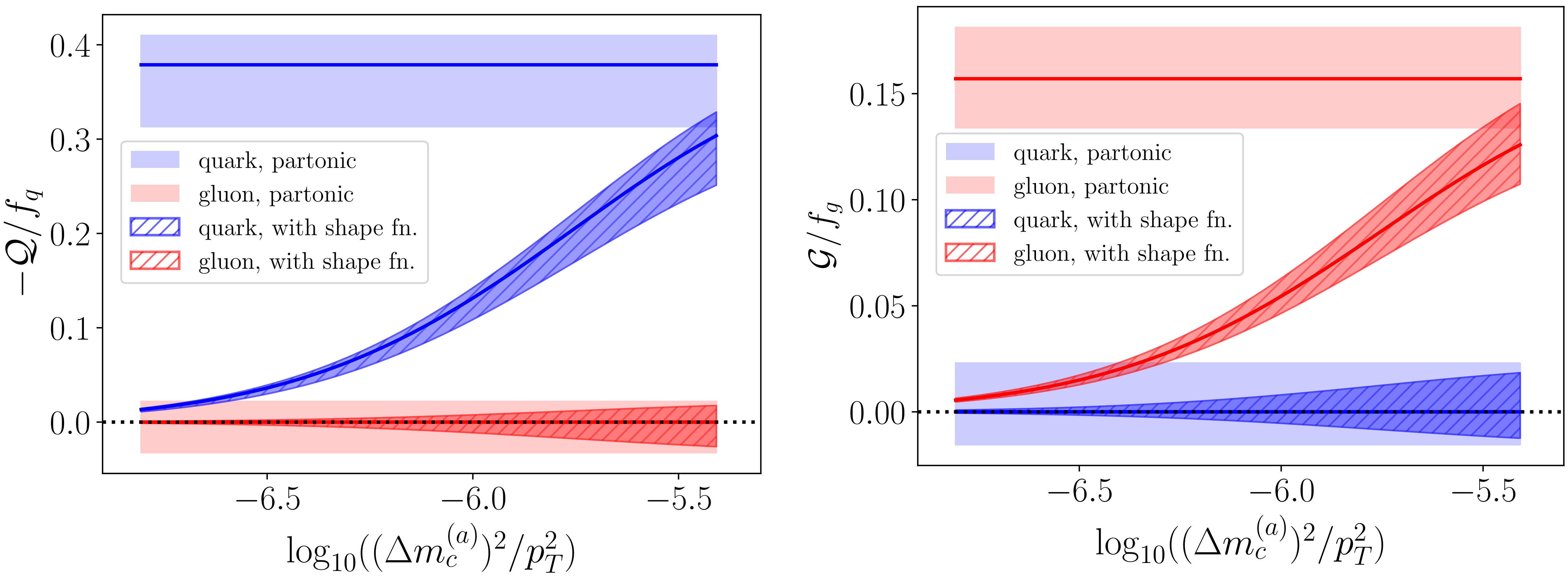
  - Factorization formula: $z_{\mathrm{cut}\,i} \ll 1$

It turns out small jet radius works

Pure Quark and Gluon Observables in Nonperturbative Regime

$$p_T = 800 \text{ GeV}, \ \eta_J = 0, \ R = 0.2$$

$$\beta_1 = \beta_2 = 0, \ z_{\text{cut1}}^{(a)} = 0.1, \ z_{\text{cut2}}^{(a)} = 0.4, \ z_{\text{cut1}}^{(b)} = 0.02, \ z_{\text{cut2}}^{(b)} = 0.08$$

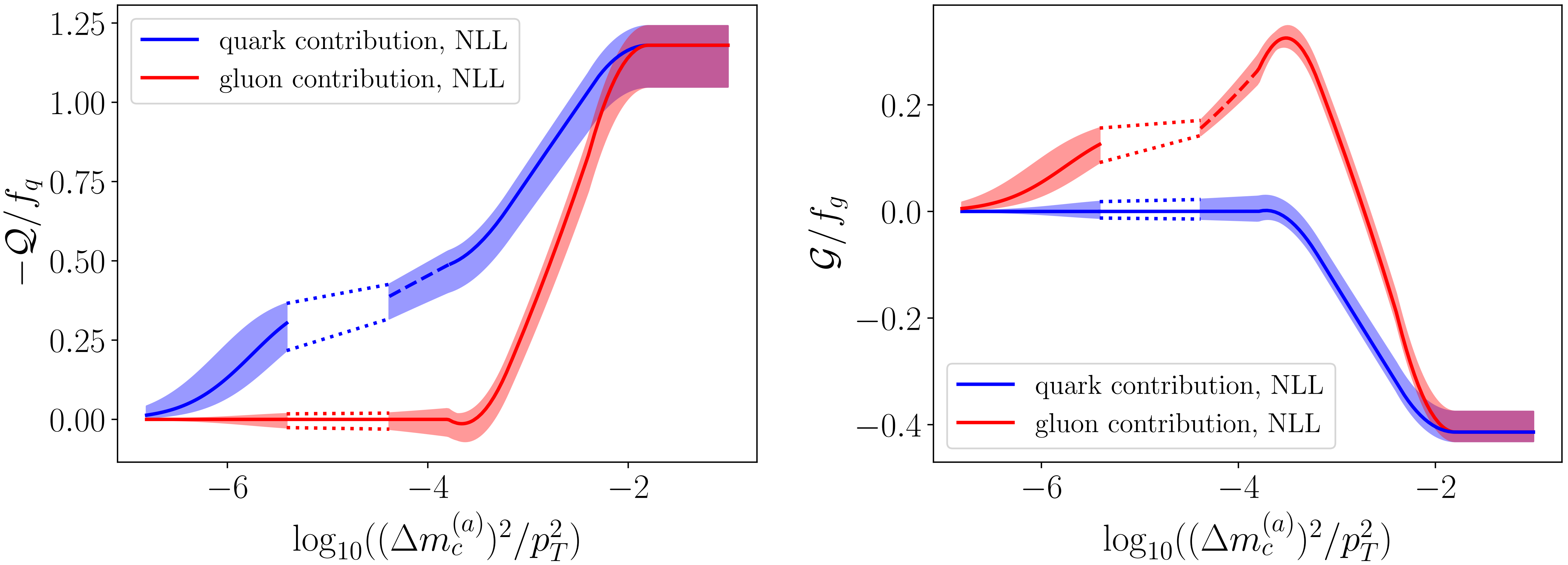


Shape function makes distributions vanishing in small mass limit

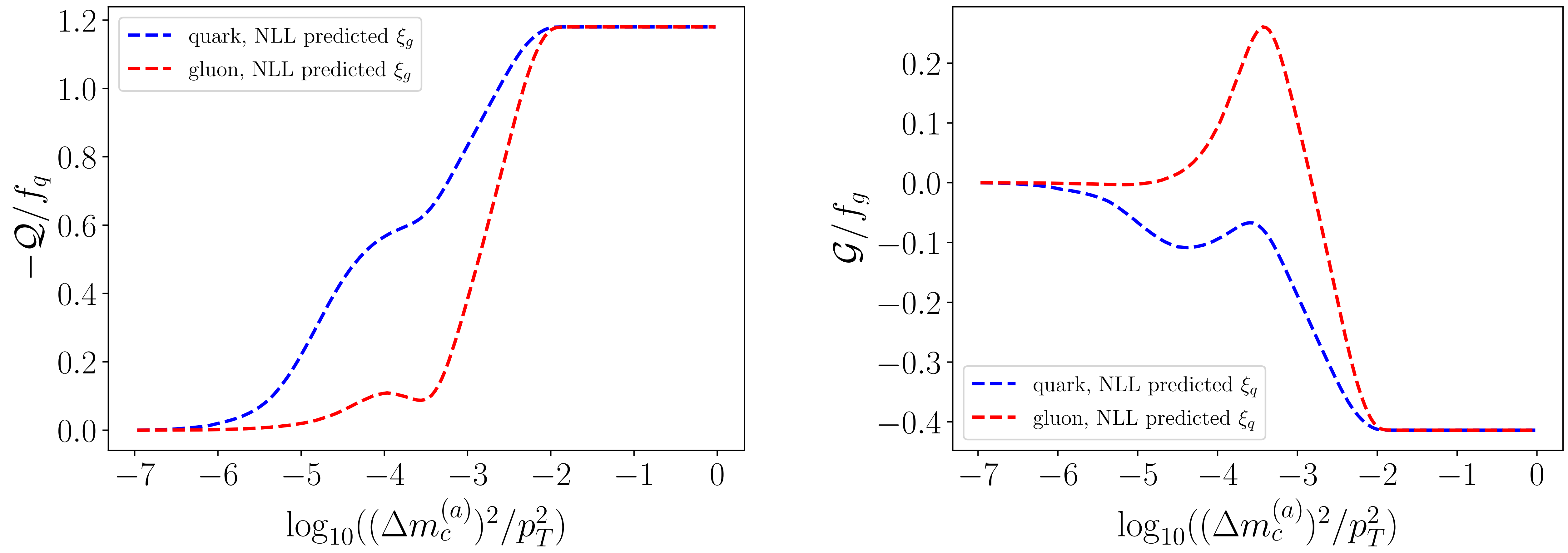
Pure Quark and Gluon Observables in Full Regime

$$p_T = 800 \text{ GeV}, \; \eta_J = 0, \; R = 0.2$$

$$\beta_1 = \beta_2 = 0, \; z_{\text{cut1}}^{(a)} = 0.1, \; z_{\text{cut2}}^{(a)} = 0.4, \; z_{\text{cut1}}^{(b)} = 0.02, \; z_{\text{cut2}}^{(b)} = 0.08$$

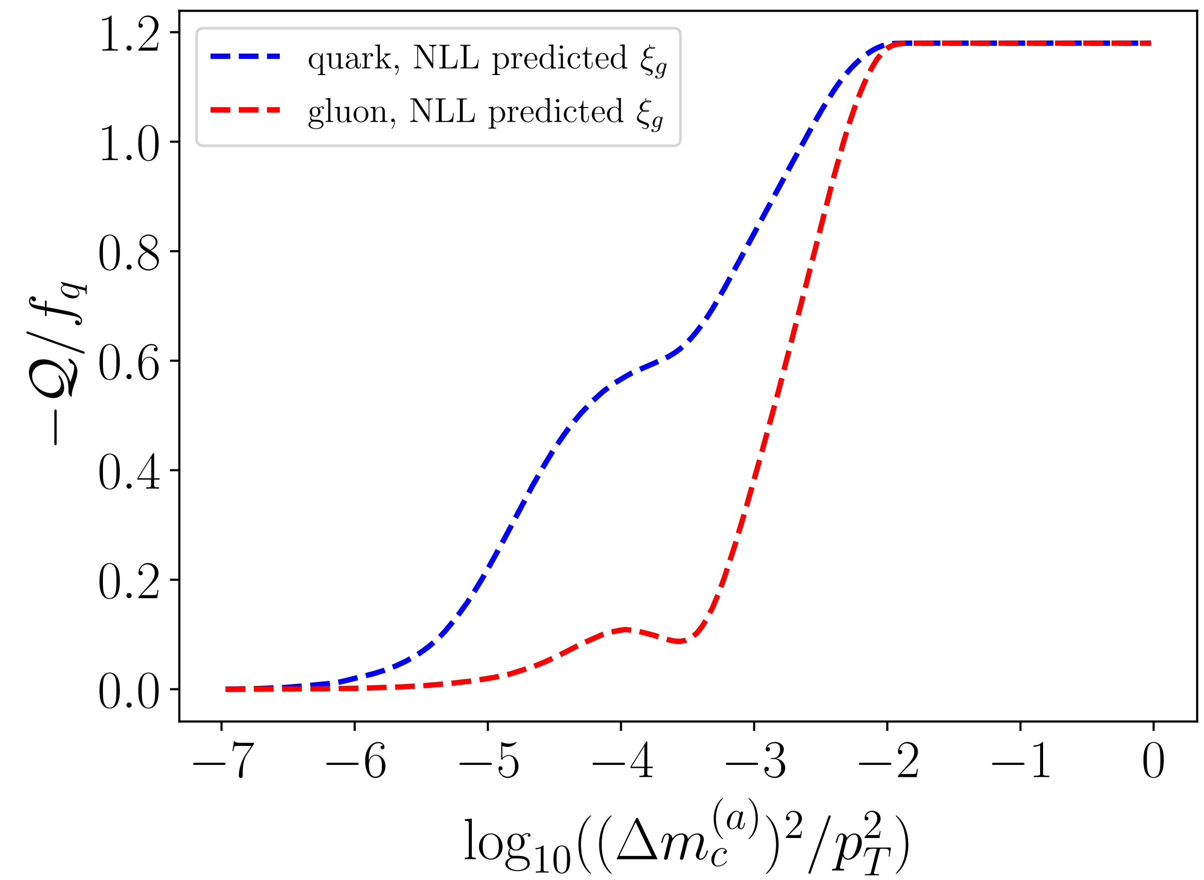


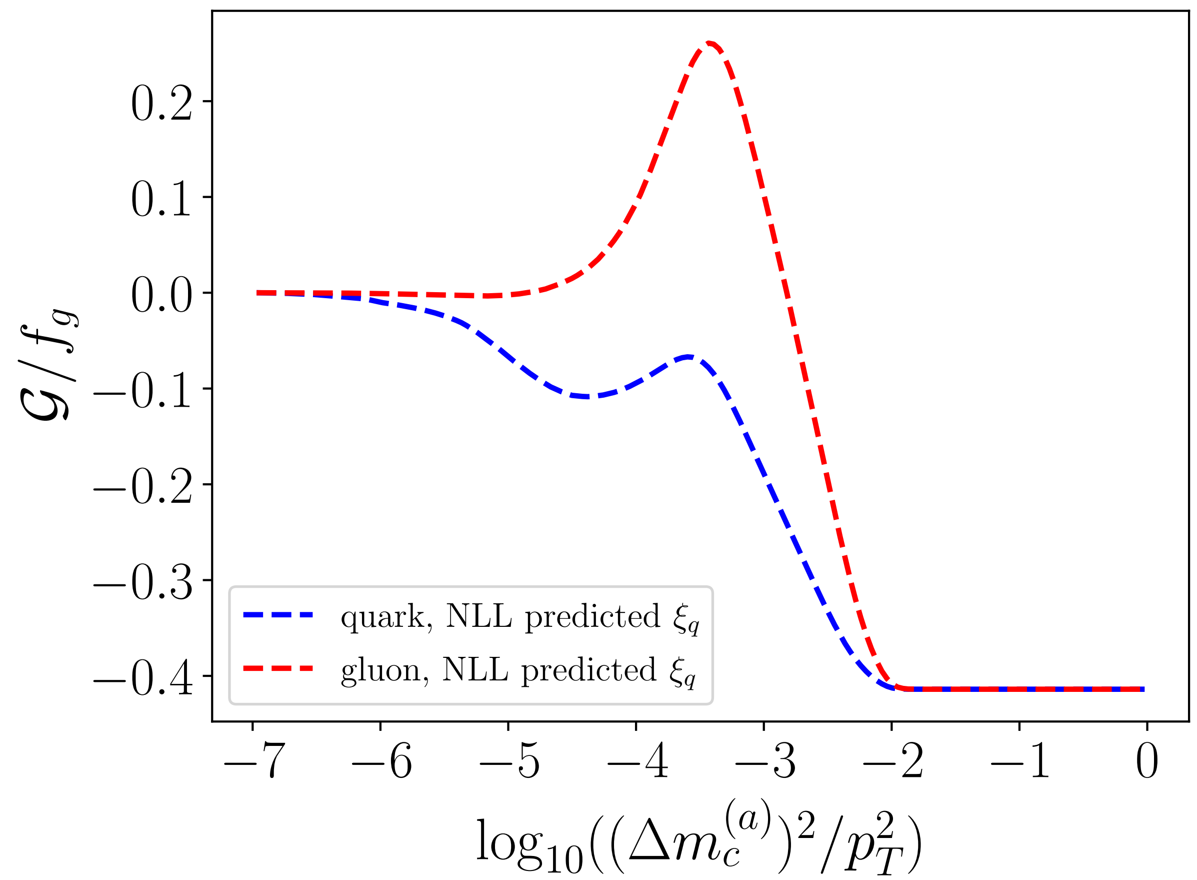
Monte Carlo Studies w/o ISR and MPI

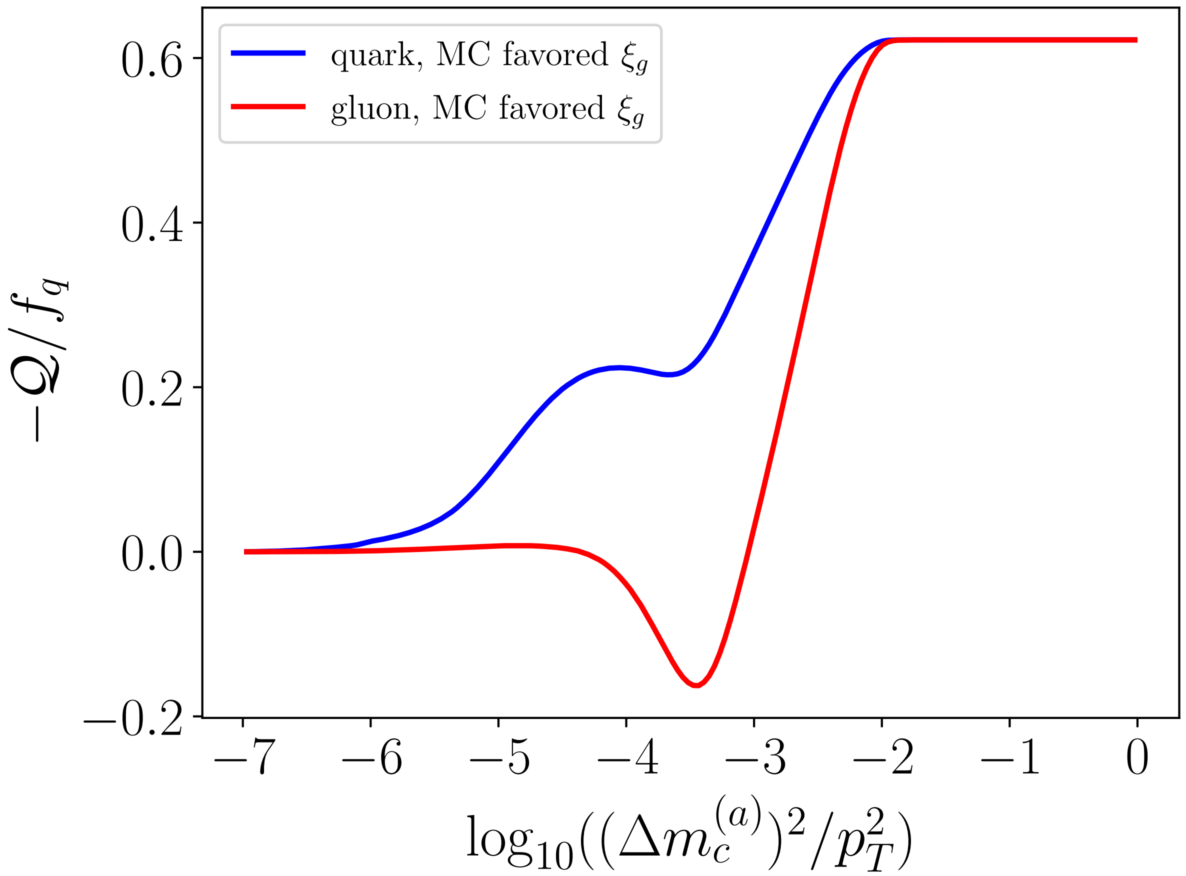


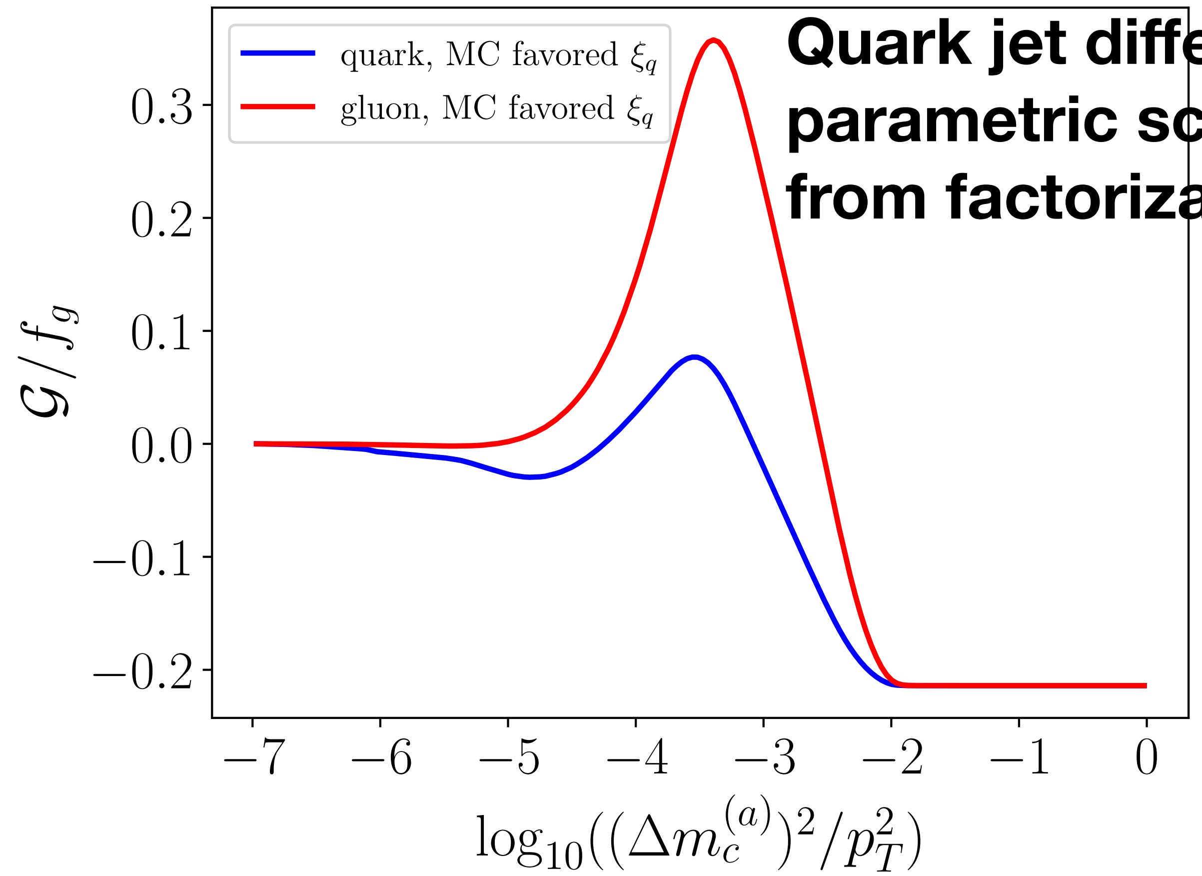
Monte Carlos may not be tuned well to describe soft radiations:
(1) determination of linear combination coefficients
(2) shape of spectrum

Monte Carlo Studies w/o ISR and MPI



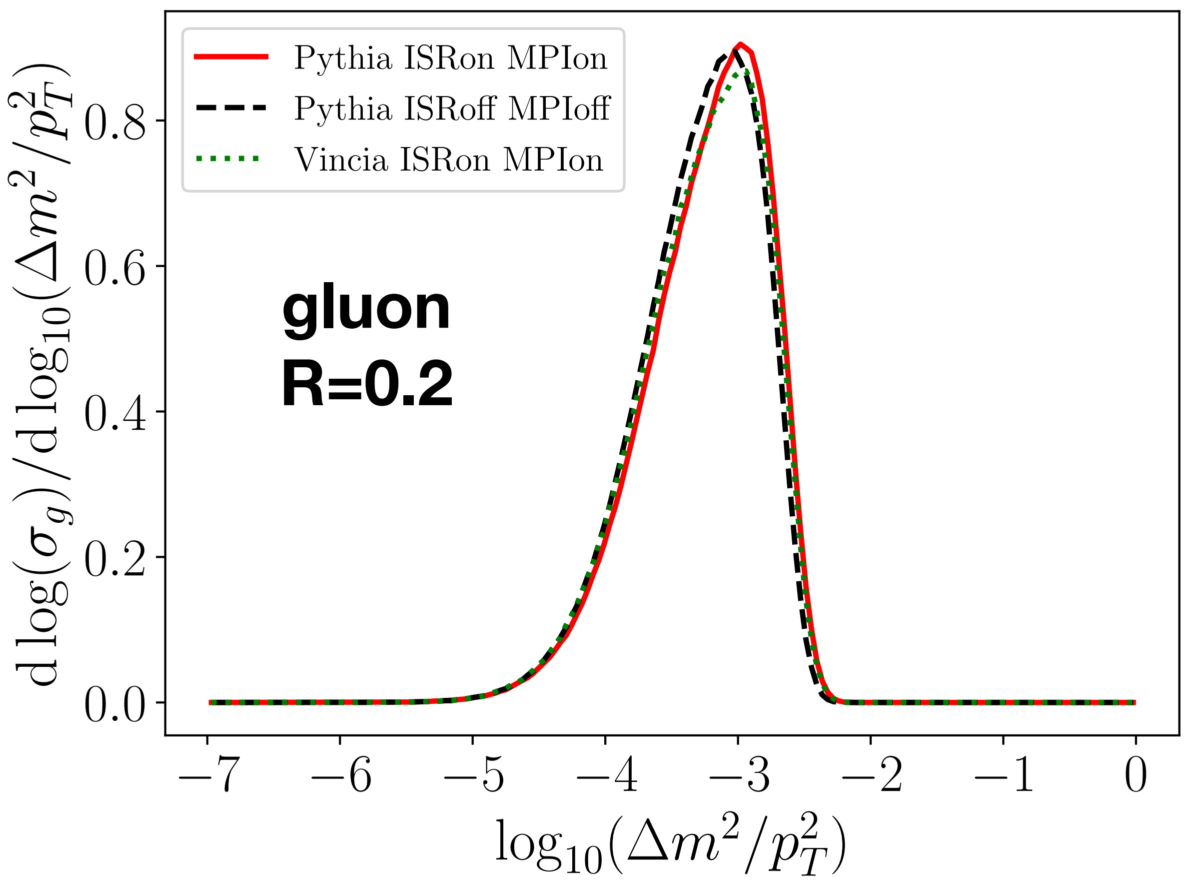


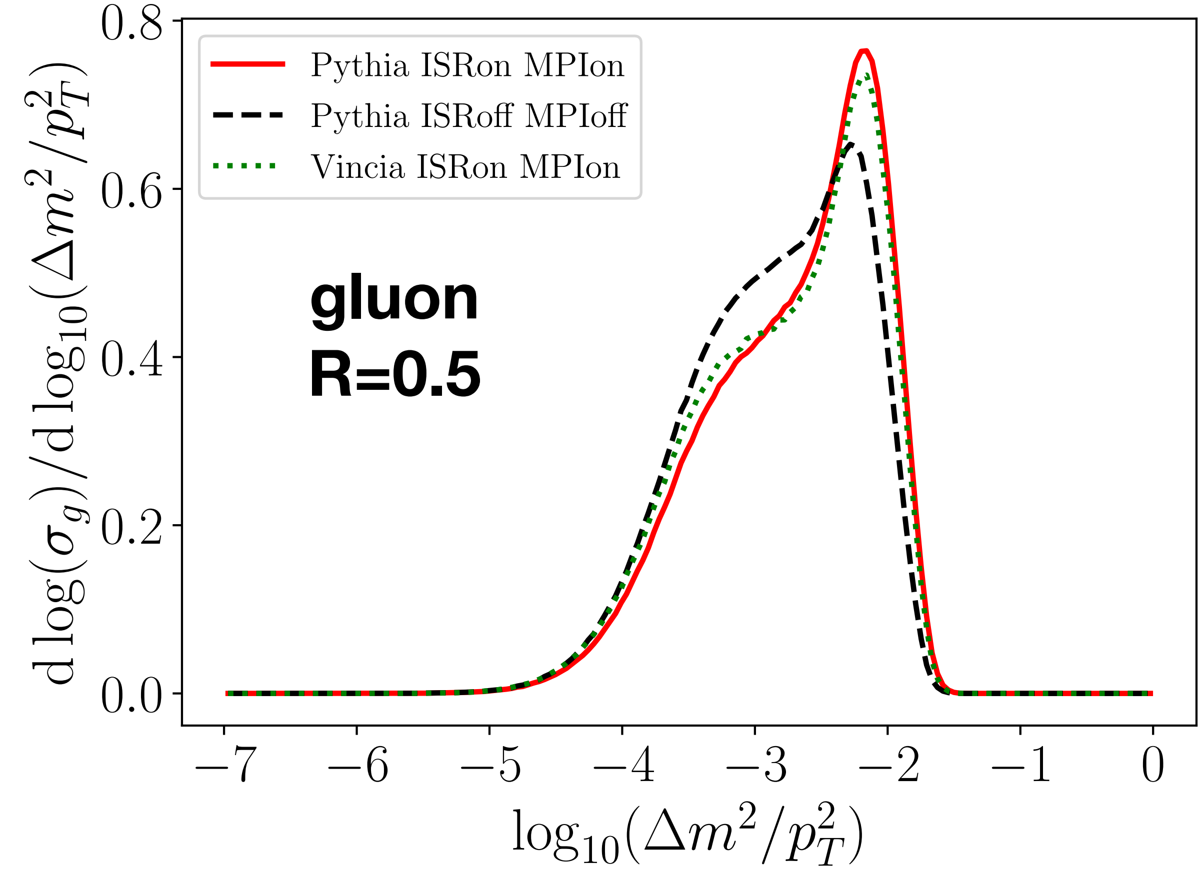


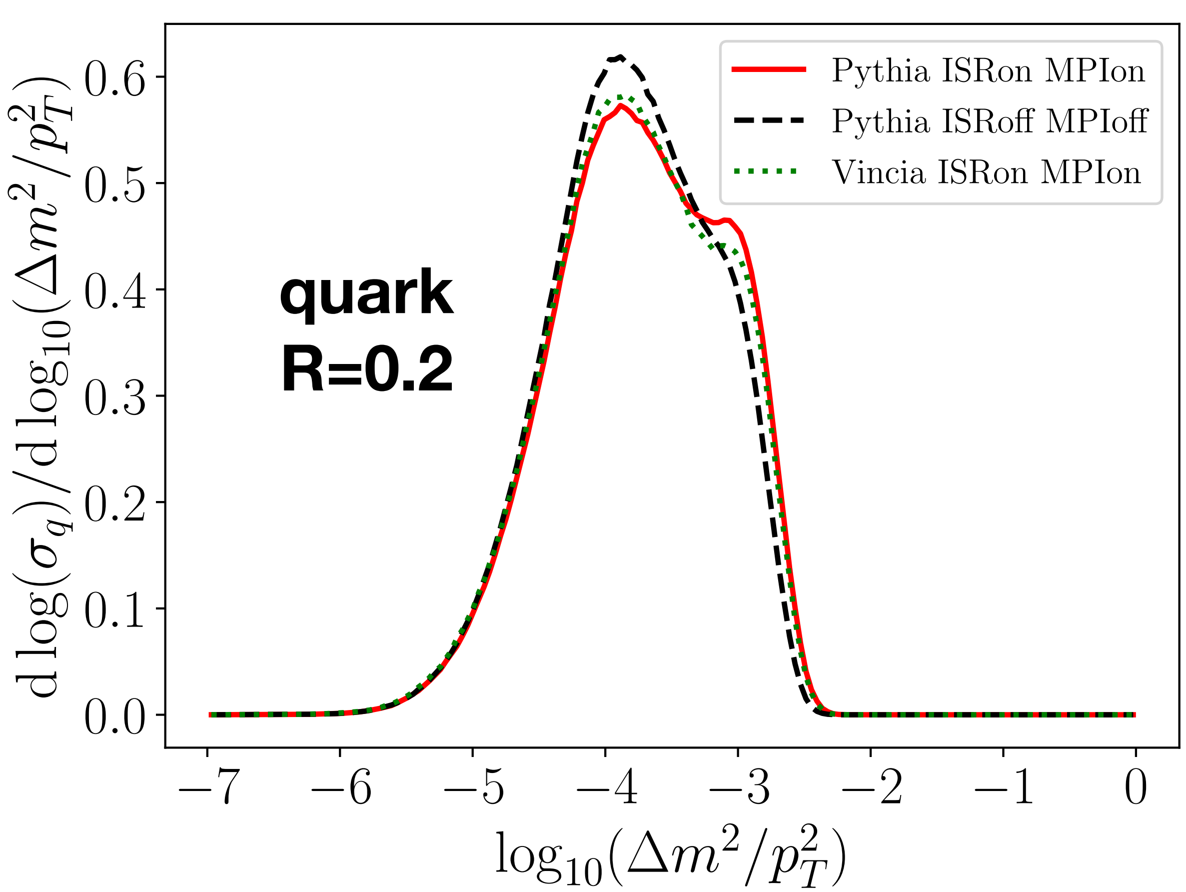


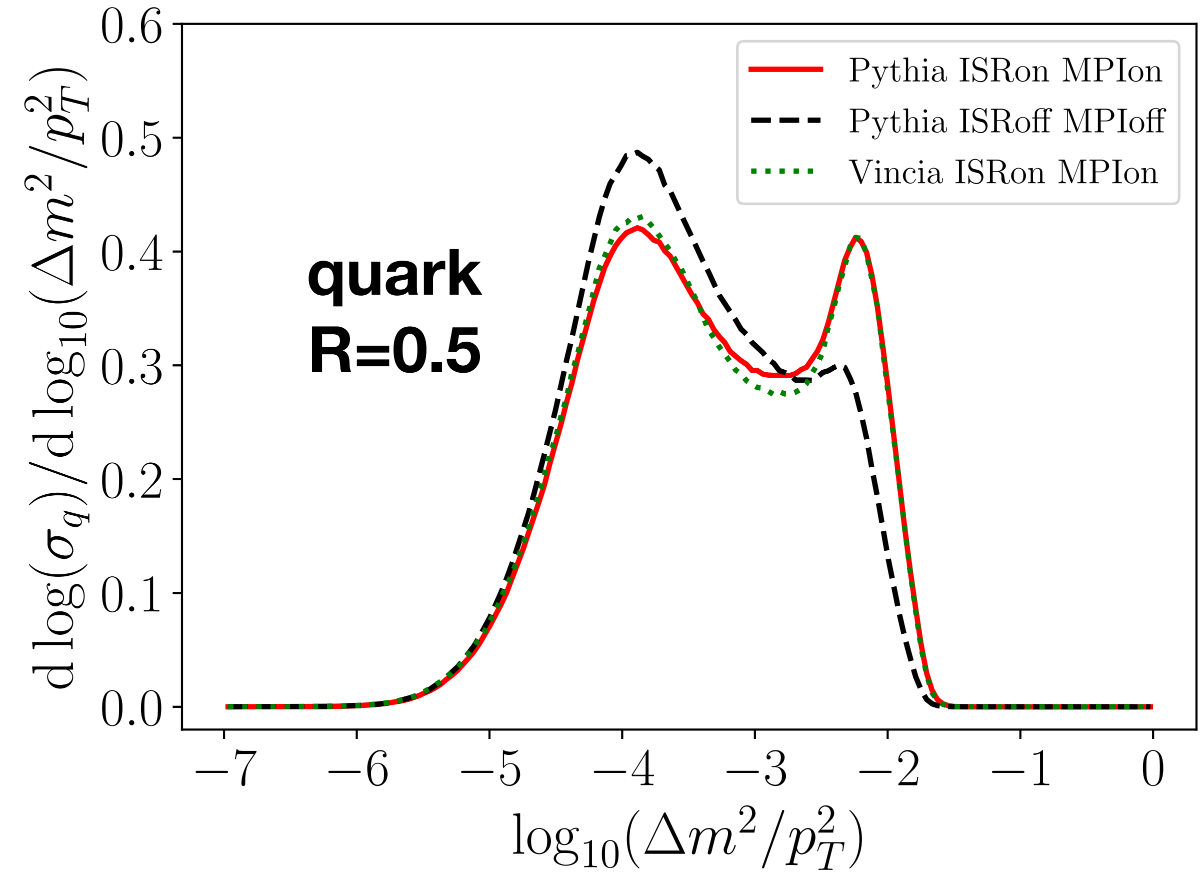
Quark jet different parametric scaling from factorization

Impact of ISR and MPI

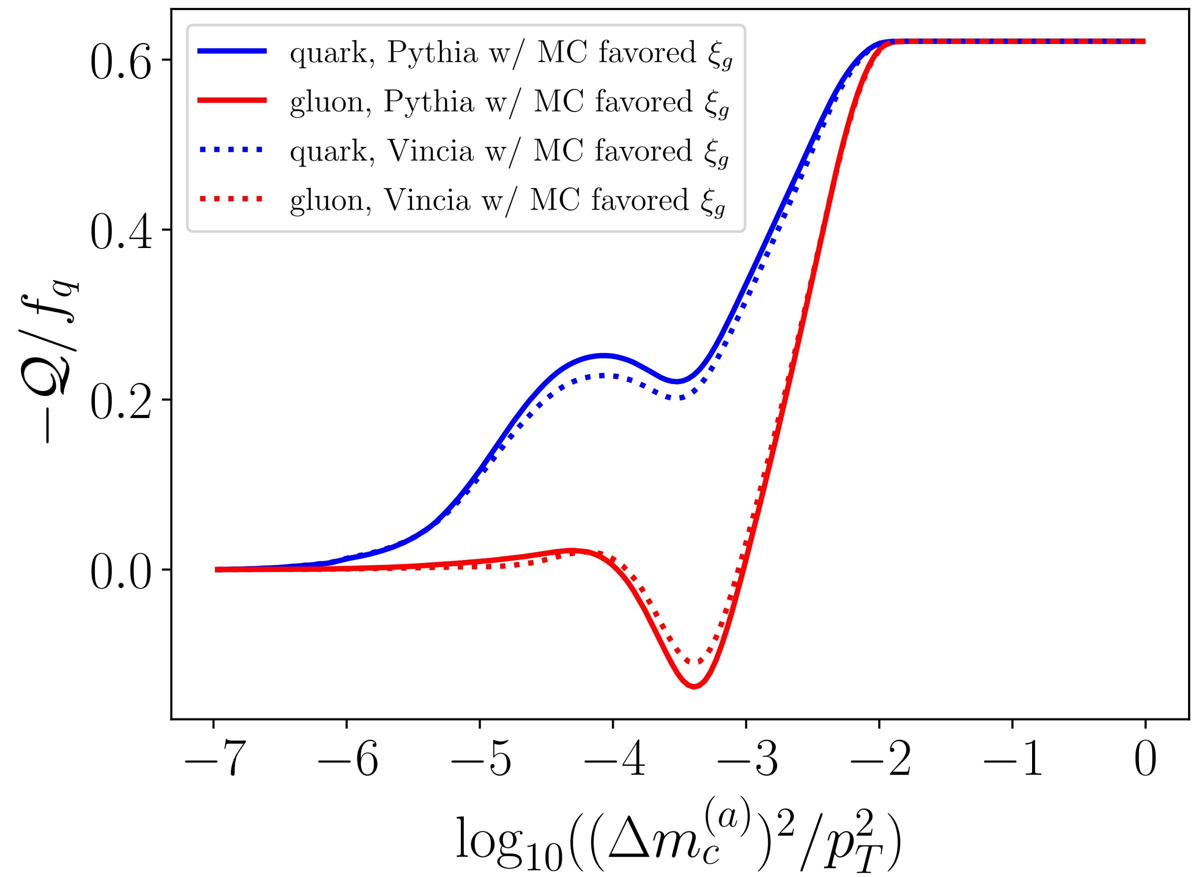


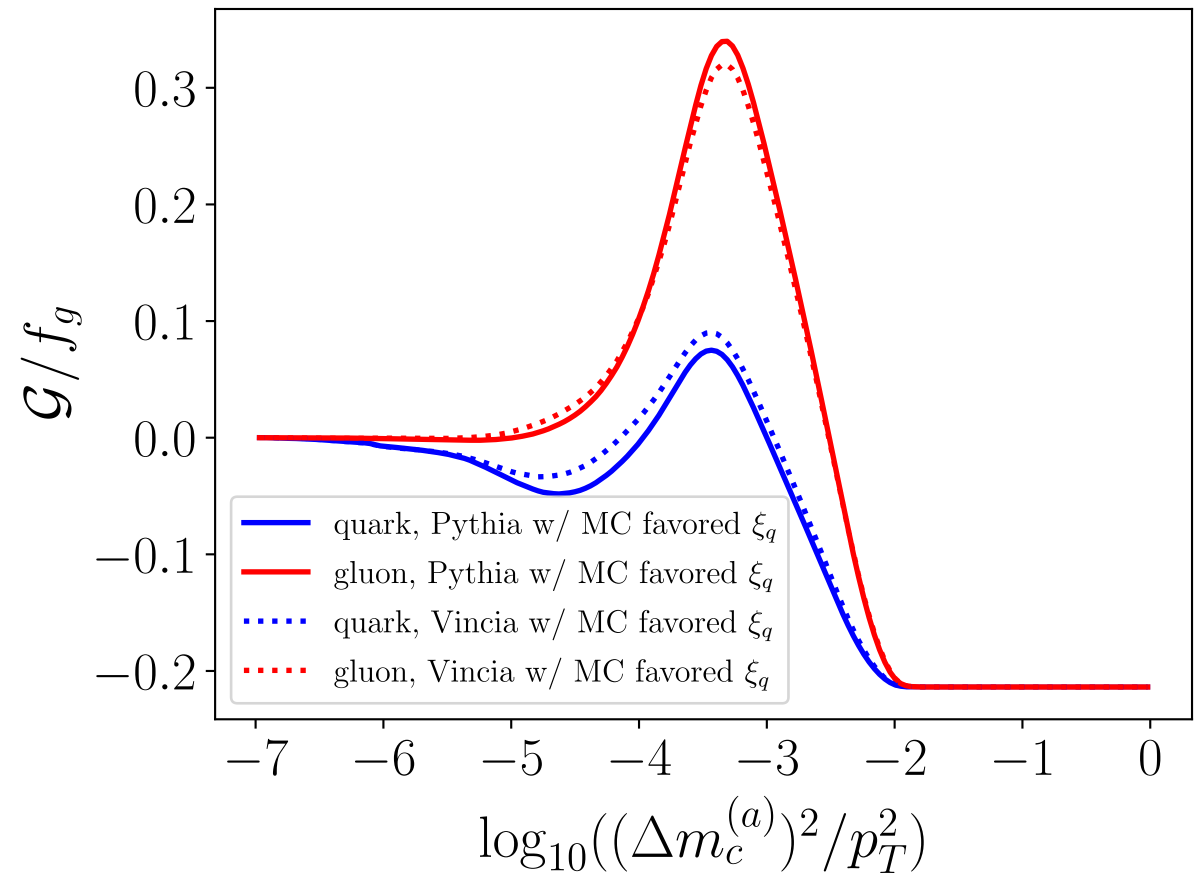






Monte Carlo Studies w ISR and MPI





Most contamination removed, construction with same linear combination coefficients works for pure quark observable

Pure gluon observable does not work, not a problem of ISR/MPI
Mass dependence of quark jet spectrum in Monte Carlo different from factorization

Conclusions

- Construct pure quark/gluon observables with collinear drop
    - Linear combination of cumulative jet mass observables with different CD parameters $z_{\mathrm{cut}\,i}$
    - Rescaling jet mass —> robust against nonperturbative correction
- Monte Carlo studies
    - Small jet radius removes ISR/MPI effects
    - Pure quark observable works; gluon observable suffers from different parametric scaling behavior of quark jets

Backup: Model of Shape Function

- **Models of shape function**

$$F_i^j(k_i, \beta_i) = \frac{1}{\Lambda}\left(\sum_{n=0}^{\infty} c_n^j(\beta_i) f_n(x, p)\right)^2 \qquad x = \frac{k_i}{\Lambda} \qquad \Lambda \sim \Lambda_{\mathrm{QCD}}$$

Basis function

$$f_n(x,p) = \sqrt{(2n+1)Y(x,p)}\,P_n\big(y(x)\big)$$

$$y(x,p) = -1 + 2\int_0^x \mathrm{d}x'\, Y(x',p)$$

$$Y(x,p) = \frac{(p+1)^{p+1}}{\Gamma(p+1)} x^p e^{-(p+1)x}$$

Legendre polynomial

Most importantly $x \to 0 \quad F_i^j(x) \sim x^p$

Z.Ligeti, I.W.Stewart, F.J.Tackmann arXiv:0807.1926