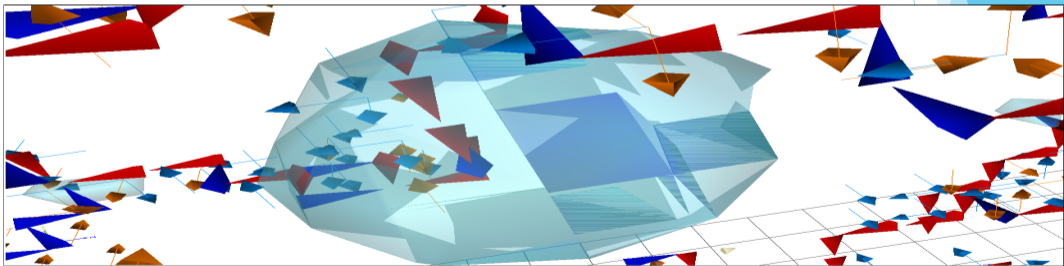


Emergent phenomena from centre vortices

Waseem Kamleh

Collaborators

James Biddle, Derek Leinweber, Adam Virgili



RIKEN/BNL Seminar
Zoom, 27 October 2022

Emergent phenomena

An emergent behaviour or emergent property can appear when a number of simple entities (agents) operate in an environment, forming more complex behaviors as a collective.



Emergent features of QCD

Non-trivial vacuum state (c.f. QED)

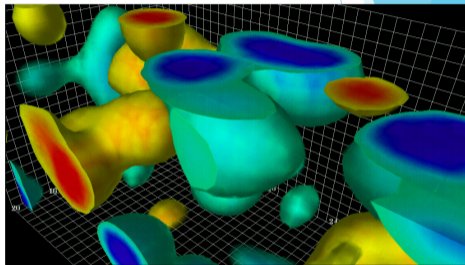
Requires a non-perturbative description.

Two key *emergent* features

Confinement of quarks

Dynamical mass generation

Can centre vortices explain the emergent features of QCD?

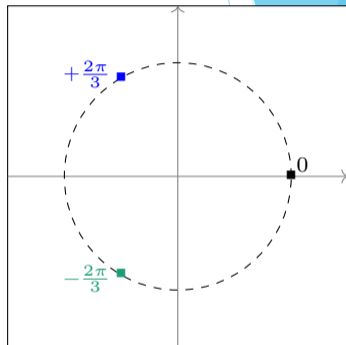


Centre group of SU(3)

Centre elements commute with every group element,

$$z = \exp\left(\frac{2\pi i}{3}m\right)I, \quad m \in \{-1, 0, 1\} \simeq \mathbb{Z}_3$$

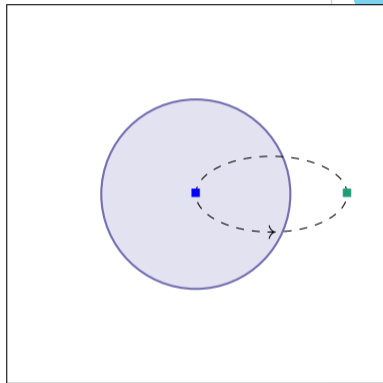
Each of the three centre phases corresponds to a centre element of SU(3)



Centre vortices

A singular gauge transformation, discontinuous on a 3-volume, creates a (thin) *centre vortex* on the 2-dimensional boundary surface.

A Wilson loop $W(C)$ along a curve $C = \partial A$ is topologically linked if the vortex pierces the enclosed area A only once.



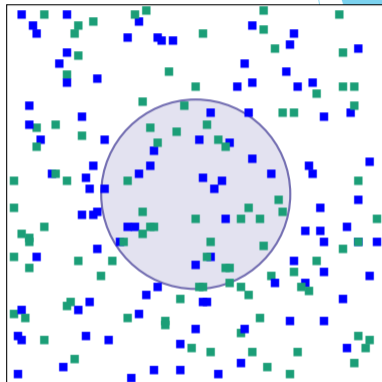
Confinement

The piercing vortex generates a non-trivial centre phase z ,

$$W(C) \rightarrow zW(C)$$

If centre vortices percolate through a volume with density ρ , this gives rise to an area law for the Wilson loop

$$W(C) = e^{-2\rho A}$$



Lattice QCD

Discretise space-time onto 4D hypercube

Gauge field $U_\mu(x) \in SU(3)$ becomes unitary

$32^3 \times 64$ (periodic) lattice volume

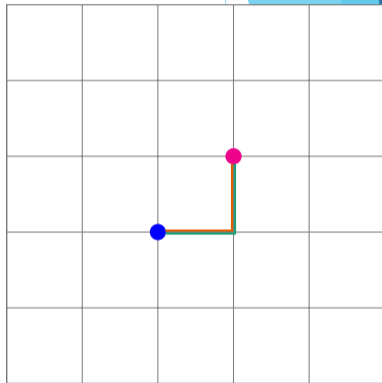
3 ensembles

Pure gauge (PG), $a = 0.100$ fm

Dynamical $m_\pi = 701$ MeV, $a = 0.1022$ fm

Dynamical $m_\pi = 156$ MeV, $a = 0.0933$ fm

S. Aoki *et al* (PACS-CS), Phys. Rev. D **79**, 034503



Identifying centre vortices

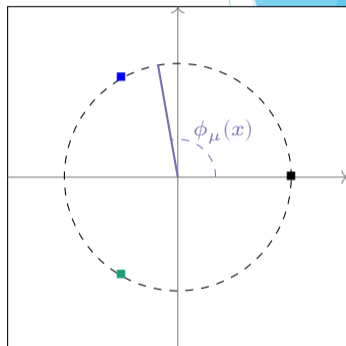
Transform to Maximal Centre Gauge

$$\sum_{x,\mu} \text{Re Tr}[U_\mu^\Omega(x) Z_\mu^\dagger(x)] \rightarrow \text{Max}$$

$\Omega(x)$ maximises overlap with centre elements.

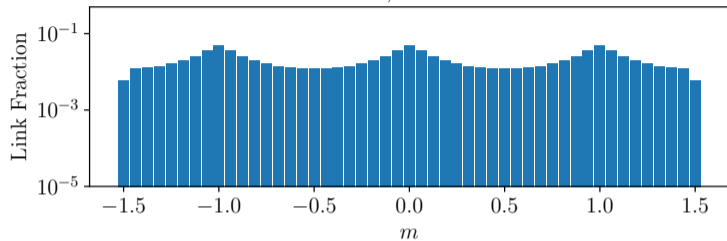
Project onto \mathbb{Z}_3 by choosing closest centre element to the phase of

$$\frac{1}{3} \text{Tr} U_\mu^\Omega(x) = r_\mu(x) \exp(i\phi_\mu(x))$$

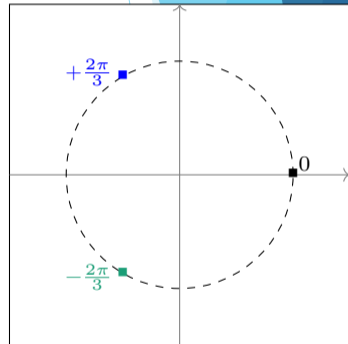
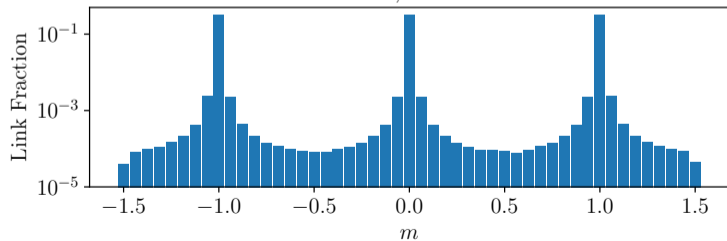


Centre phase distribution

PG, before

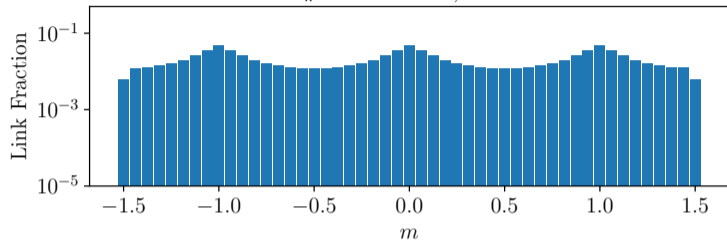


PG, after

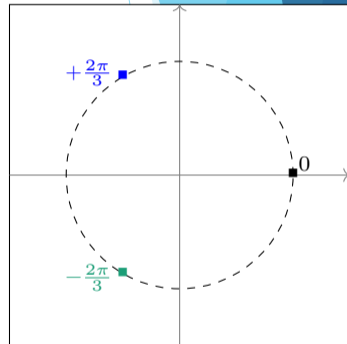
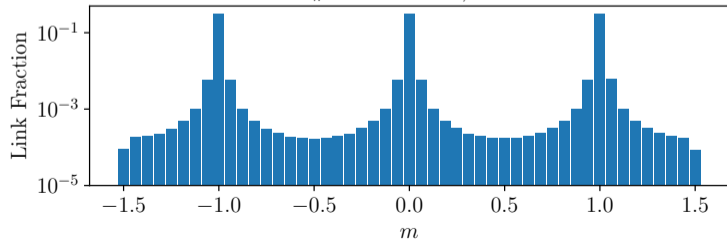


Centre phase distribution

$m_\pi = 156$ MeV, before



$m_\pi = 156$ MeV, after



Identifying centre vortices

The centre vortex field lives on the dual lattice,

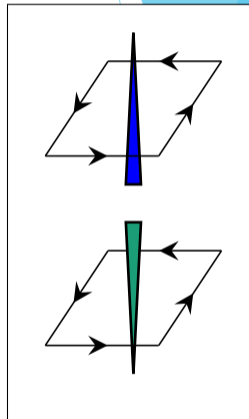
$$\bar{x} = x + \frac{a}{2}(\hat{\mu} + \hat{\nu} - \hat{\lambda} - \hat{\kappa})$$

The centre flux $m_{\kappa\lambda}(\bar{x})$ through an elementary plaquette is

$$P_{\mu\nu}(x) = \exp\left(\frac{\pi i}{3} \epsilon_{\kappa\lambda\mu\nu} m_{\kappa\lambda}(\bar{x})\right)$$

Centre-projected plaquette is pierced by a (P-)vortex if

$$\begin{aligned} P_{\mu\nu}(x) &= Z_{\mu}(x)Z_{\nu}(x + \mu)Z_{\mu}^{\dagger}(x + \nu)Z_{\nu}^{\dagger}(x) \\ &= \exp\left(\frac{\pm 2\pi i}{3}\right) I \end{aligned}$$



Centre vortices on the lattice

Untouched configurations

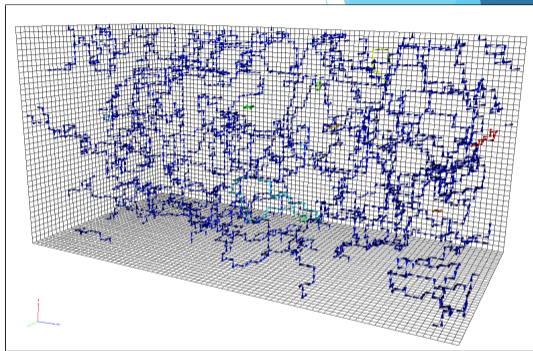
$$U_\mu(x)$$

Vortex-only configurations

$$Z_\mu(x) = \exp\left[\frac{2\pi i}{3}m_\mu(x)\right]I$$

Vortex removed configurations

$$R_\mu(x) = Z_\mu^\dagger(x)U_\mu^\Omega(x)$$



Visualization of center vortex structure

J.C. Biddle, WK, D.B. Leinweber

Phys. Rev. D 102 (2020) 3, 034504

Emergent features of QCD

Can centre vortices explain the emergent features of QCD?

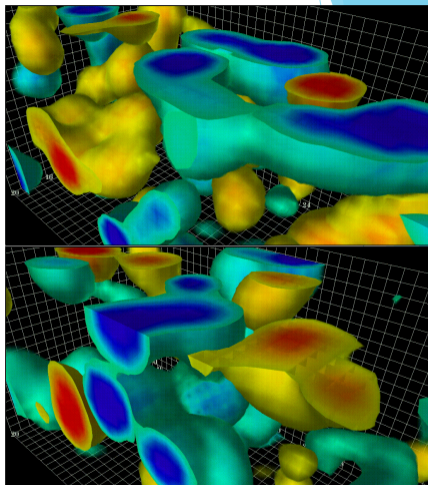
Confinement of quarks

Static quark potential

Gluon propagator

Dynamical mass generation

Quark propagator



Connection between center vortices and instantons through gauge-field smoothing

A. Trewartha, WK, D.B. Leinweber, Phys. Rev. D 92 (2015) 7, 074507

Static quark potential

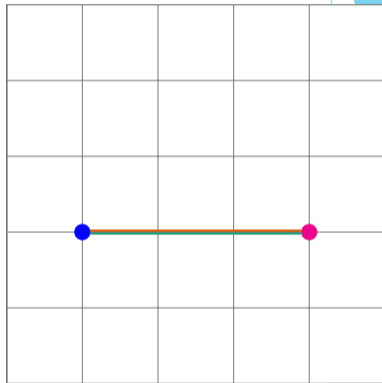
Wilson loop $W(\vec{r}, t)$ measures the potential between two static quarks

Confinement:

→ Wilson loop falls off with asymptotic area law

→ Static quark potential is linear

$$V(r) \sim \sigma r \leftrightarrow W(\vec{r}, t) \sim e^{-\sigma A}$$



Static quark potential

Untouched (UT) field: Cornell

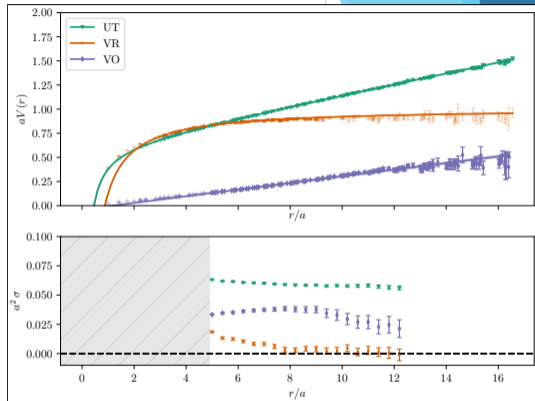
$$V(r) = V_0 - \alpha/r + \sigma r$$

Vortex-only (VO) field: Linear

$$V(r) = V_0 + \sigma r$$

Vortex removed (VR) field: Coulomb

$$V(r) = V_0 - \alpha/r$$



Pure gauge

Static quark potential from centre vortices in the presence of dynamical fermions

J.C. Biddle, WK, D.B. Leinweber, arXiv:2206.00844

Static quark potential

Untouched (UT) field: Cornell

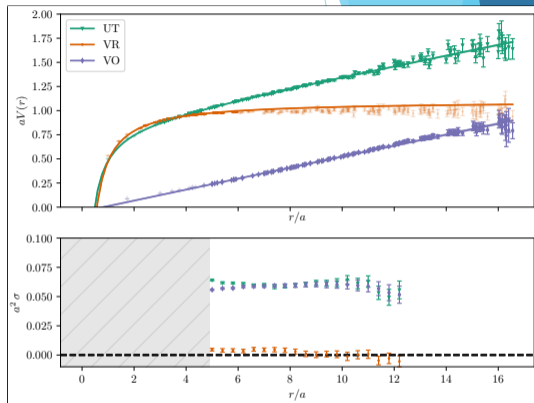
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$$V(r) = V_0 + \sigma r$$

Vortex removed (VR) field: Coulomb

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$$m_\pi = 701 \text{ MeV}$$

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Static quark potential

Untouched (UT) field: Cornell

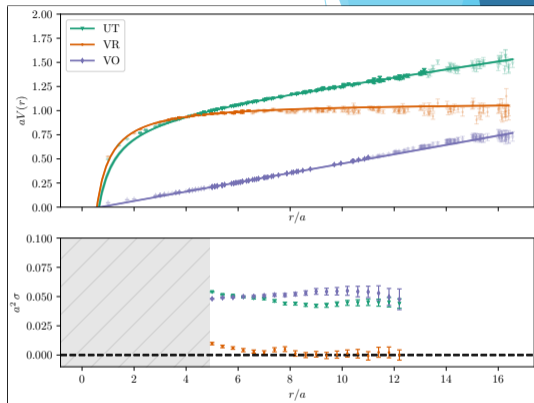
$$V(r) = V_0 - \alpha/r + \sigma r$$

Vortex-only (VO) field: Linear

$$V(r) = V_0 + \sigma r$$

Vortex removed (VR) field: Coulomb

$$V(r) = V_0 - \alpha/r$$



$m_\pi = 156$ MeV

Static quark potential from centre vortices in the presence of dynamical fermions

J.C. Biddle, WK, D.B. Leinweber, arXiv:2206.00844

Anti-screened Coulomb potential

Fit the **vortex removed** potential with an anti-screening (AS) functional

$$V_{\text{as}}(r) = V_0 - \frac{\alpha}{1 - e^{-\rho r}}$$

At large r , the effective coupling increases to create a constant potential

$$V_{\text{as}}(r) \rightarrow V_0 - \alpha$$

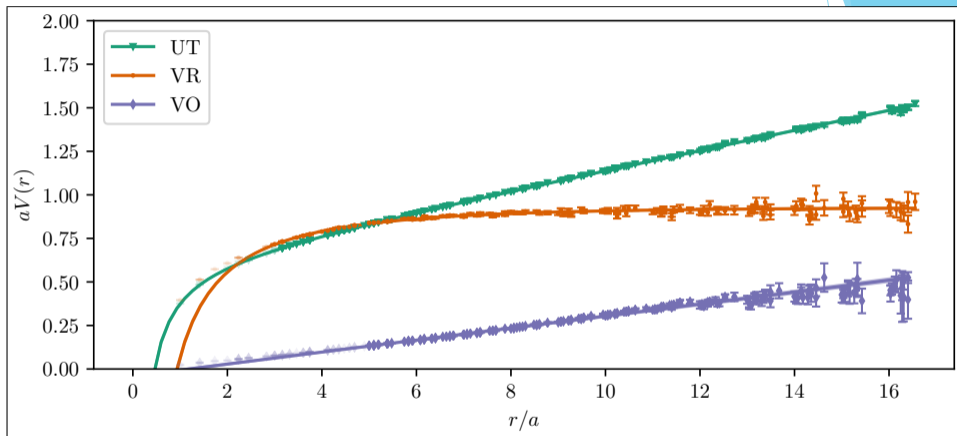
At small r , $\tilde{\alpha} = \alpha/\rho$ becomes the effective Coulomb coefficient with

$$V_{\text{as}}(r) \rightarrow V_0 - \tilde{\alpha}/r$$

Fit the **untouched** potential by fixing ρ and adding the linear string tension,

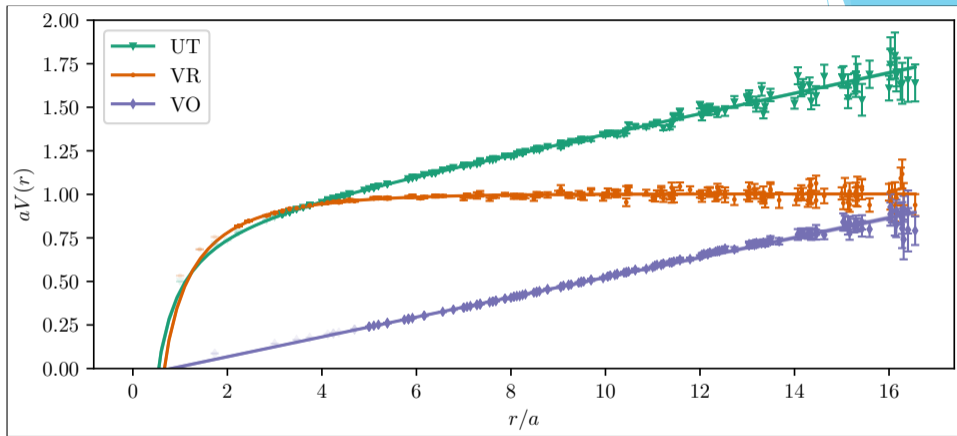
$$V(r) = V_0 - \frac{\alpha}{1 - e^{-\rho r}} + \sigma r$$

Static quark potential – anti-screened Coulomb potential



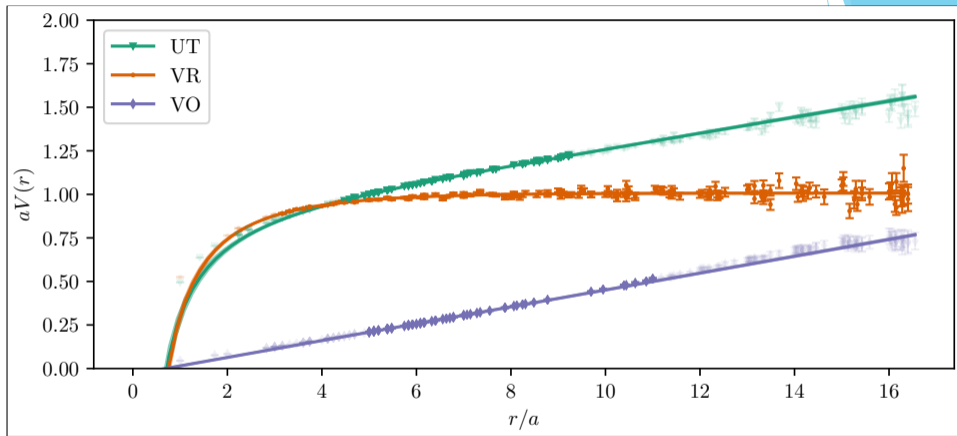
Pure gauge

Static quark potential – anti-screened Coulomb potential



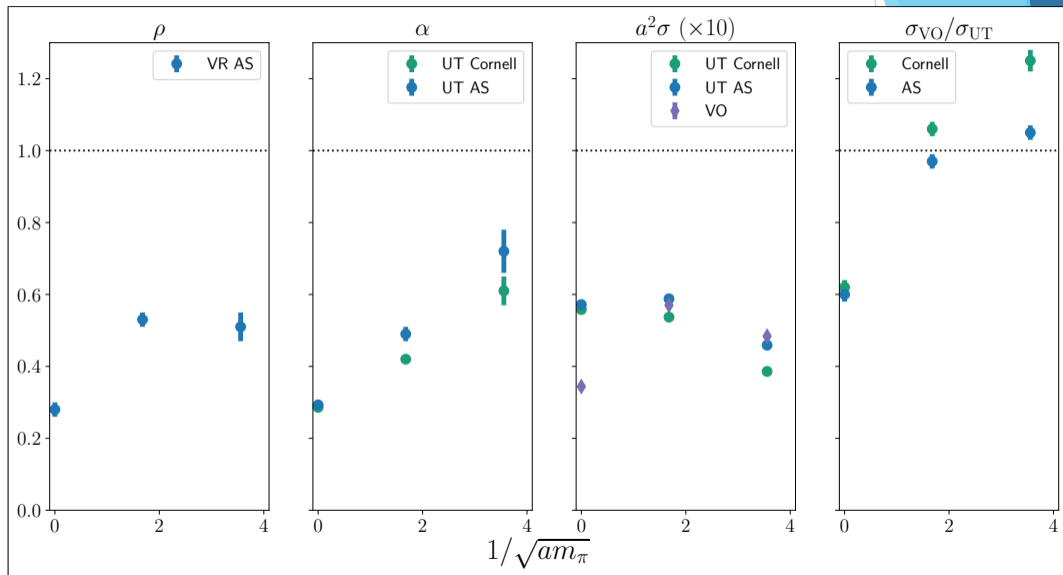
$$m_\pi = 701 \text{ MeV}$$

Static quark potential – anti-screened Coulomb potential



$m_\pi = 156$ MeV

Static quark potential – fit parameters



Gluon propagator

The nonperturbative scalar gluon propagator in momentum space is

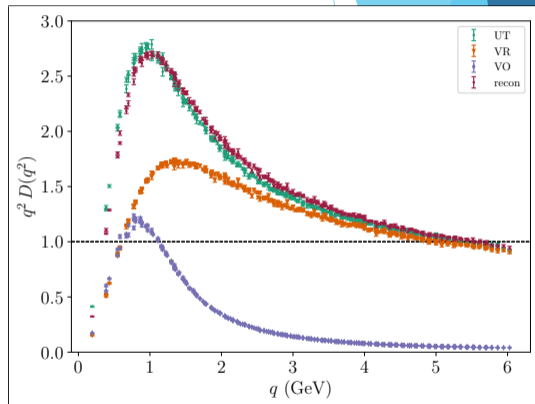
$$D(q^2) \equiv \frac{Z(q^2)}{q^2} \rightarrow \frac{1}{q^2}$$

Consider the renormalisation function

$$Z(q^2) = q^2 D(q^2)$$

Renormalise $Z(q^2) = 1$ at $q = 5.5$ GeV

Calculated in Landau gauge



Gluon propagator – pure gauge

Vortex removal (VR)

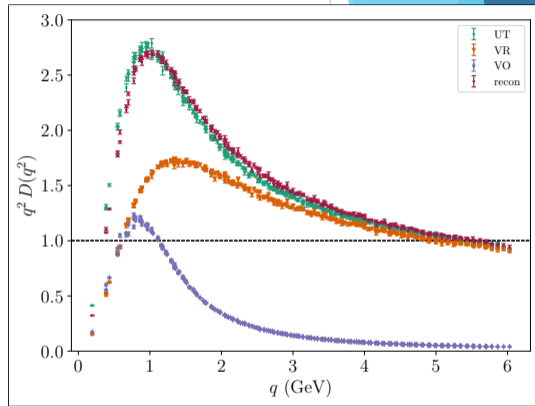
- Suppresses infrared enhancement
- Preserves perturbative UV behaviour

Vortex-only (VO) configurations

- Capture the long-distance physics

Reconstruction of the propagator

- Linear combination of the vortex-modified fields
- Near perfect agreement



Gluon propagator on a center-vortex background

J.C. Biddle, WK, D.B. Leinweber, Phys. Rev. D 98 (2018) 9, 094504

Gluon propagator – dynamical ($m_\pi = 156$ MeV)

Dynamical fermions (UT)

Suppress the overall infrared strength

Vortex removal (VR)

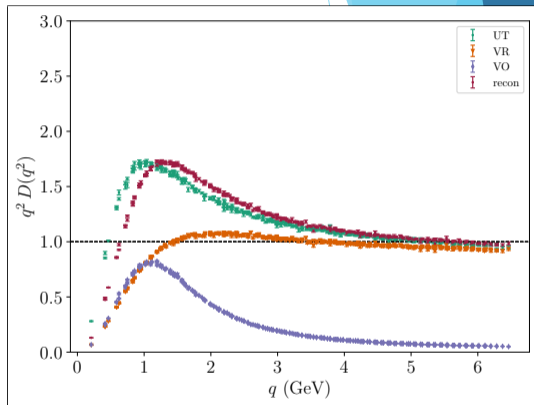
Almost eliminates infrared enhancement

Vortex-only (VO) configurations

Captures the long-distance physics

Reconstruction

Less perfect agreement



Impact of dynamical fermions on the center vortex gluon propagator

J.C. Biddle, WK, D.B. Leinweber, Phys. Rev. D 106 (2022) 1, 014506

Spectral positivity

For an arbitrary two-point function $D(x - y)$ the scalar propagator has spectral representation

$$D(p^2) = \int_0^\infty dm^2 \frac{\rho(m^2)}{p^2 + m^2}$$

To represent correlations between physical particles, need $\rho(m^2) \geq 0$

To check this we can calculate the Euclidean correlator,

$$C(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dp_0 D(p_0^2) e^{-ip_0 t} = \int_0^\infty dm e^{-mt} \rho(m^2)$$

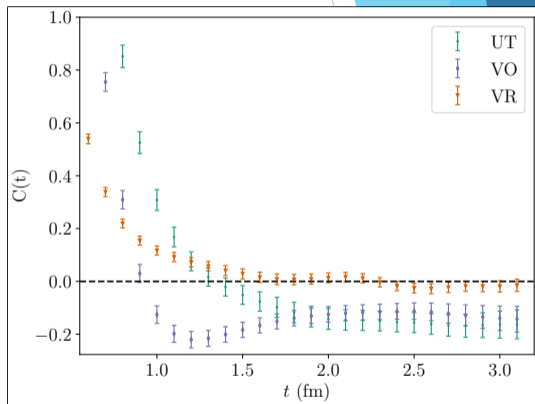
If $C(t) < 0$ for any t , then positivity is violated and gluons are confined

Euclidean correlator – pure gauge

Untouched (UT) exhibits positivity violation, as is necessary in the confining phase.

Vortex-only (VO) captures the essence of gluon confinement.

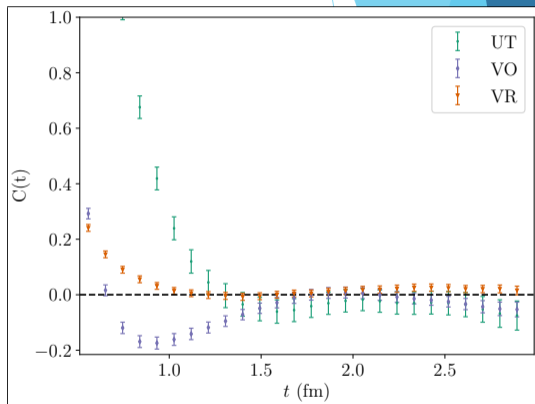
Vortex-removed (VR) exhibits hints of positivity violation at large t .



Euclidean correlator – dynamical $m_\pi = 156$ MeV

Untouched and vortex only are similar to the pure gauge sector.

Vortex-removed (VR) shows no positivity violation, indicating deconfinement of gluons.



Overlap quark propagator - pure gauge

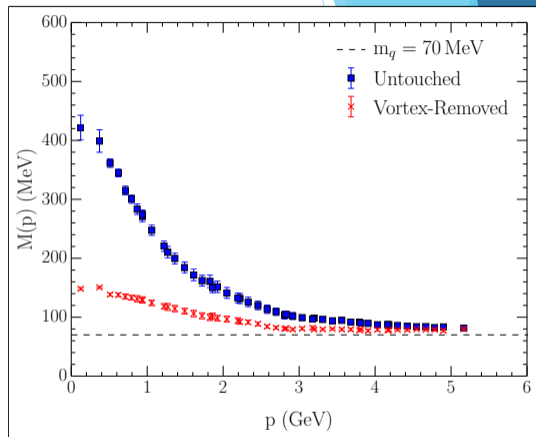
$$S(p) = \frac{Z(p)}{i\not{p} + M(p)}$$

$M(p)$ is the mass function

$Z(p)$ is the renormalisation function

Calculated in Landau gauge

Overlap fermion valence quarks



Evidence that centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory

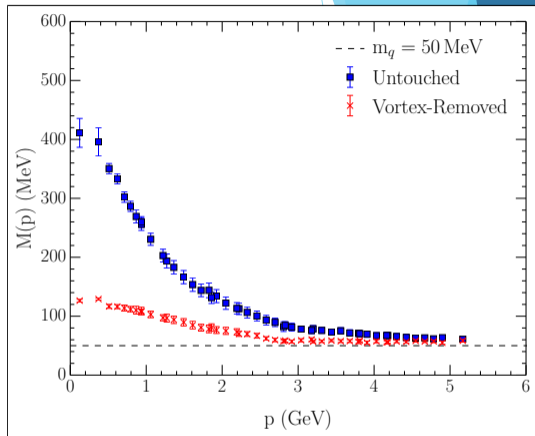
A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

Overlap quark propagator - pure gauge

$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}$$

Infrared behaviour of $M(p)$

Dynamical mass generation



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Overlap quark propagator - pure gauge

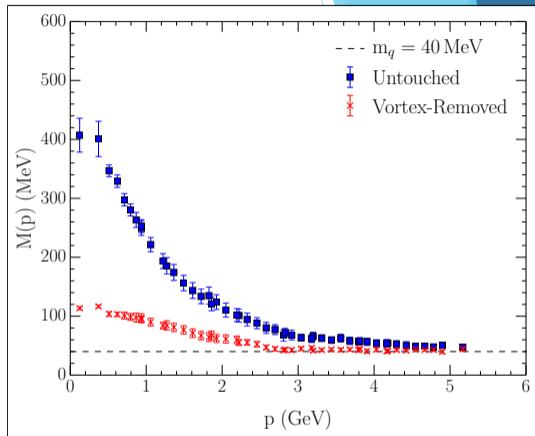
$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}$$

Infrared behaviour of $M(p)$

Dynamical mass generation

→ Increases as m_q decreases

→ Dynamical chiral symmetry breaking
($D\chi$ SB)



Evidence that centre vortices underpin dynamical chiral symmetry breaking in $SU(3)$ gauge theory

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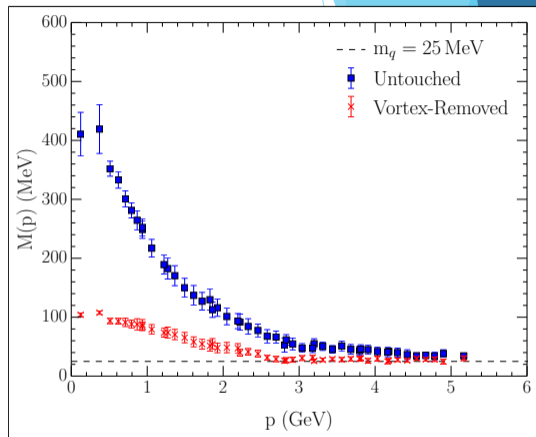
Overlap quark propagator - pure gauge

$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}$$

Infrared behaviour of $M(p)$

Dynamical mass generation

- Increases as m_q decreases
- Dynamical chiral symmetry breaking ($D\chi SB$)
- Reduced by vortex removal



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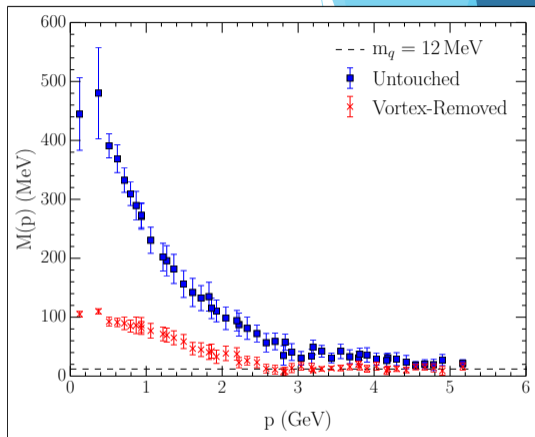
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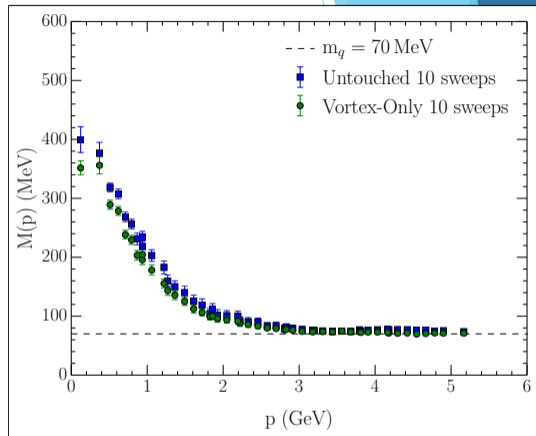
Overlap quark propagator - pure gauge

Vortex only field is very rough

Overlap has smoothness condition

→ 10 sweeps of cooling

→ Smooth untouched to compare



Evidence that centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory

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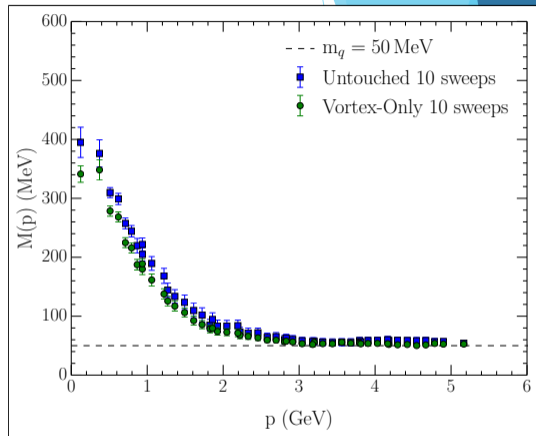
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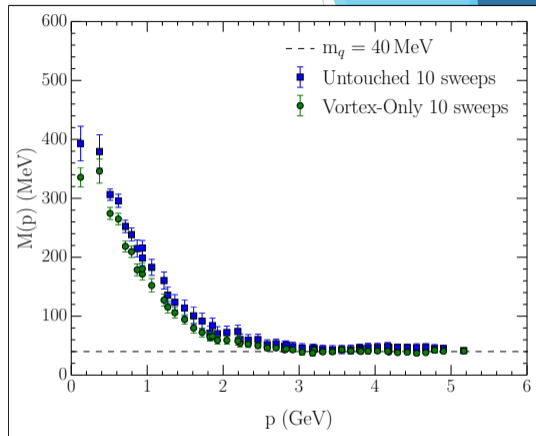
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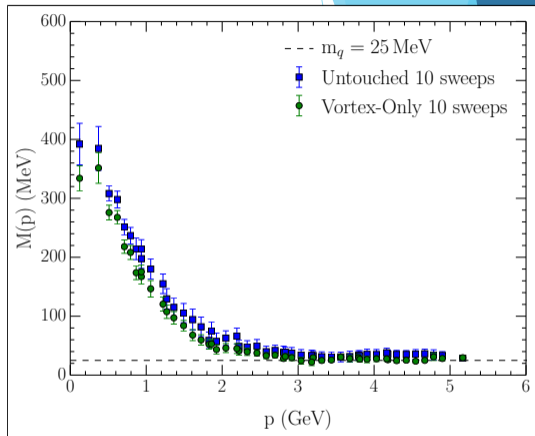
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Dynamical mass generation completely reproduced from smoothed centre vortices



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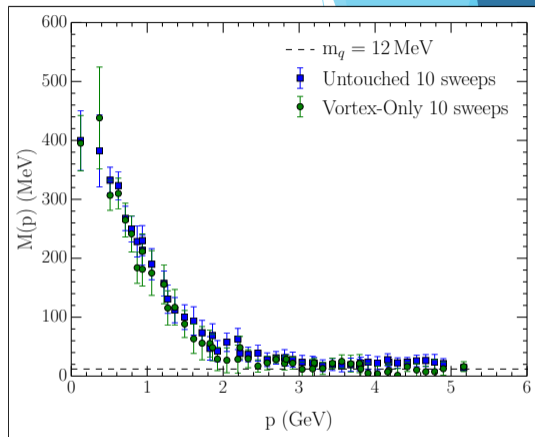
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Centrifuge preconditioned smoothing

Work with centre phase field

$$e^{i\lambda_\mu(x)} I \rightarrow [\lambda_\mu(x), \lambda_\mu(x), \lambda_\mu(x)]$$

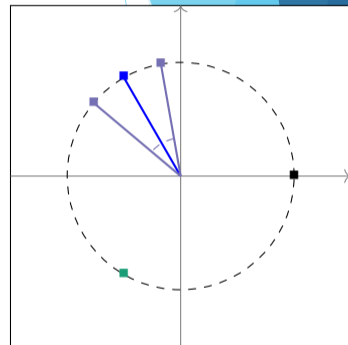
Define the staple phase

$$\sigma_\mu(x) = \frac{1}{6} \sum_{\nu \neq \mu} \left(\begin{array}{c} \uparrow \rightarrow \downarrow \\ \cdot \quad \cdot \end{array} + \begin{array}{c} \cdot \quad \cdot \\ \downarrow \rightarrow \uparrow \end{array} \right)$$

Select a pair of indices randomly for each link,

$$\lambda_\mu(x) \rightarrow (1 \mp \omega) \lambda_\mu(x) \pm \omega \sigma_\mu(x)$$

Update corresponds to a phase rotation by $\mp\omega(\lambda - \sigma)$



Smoothing algorithms for projected center-vortex gauge fields

A. Virgili, WK, D.B. Leinweber, Phys.Rev.D 106 (2022) 1, 014505

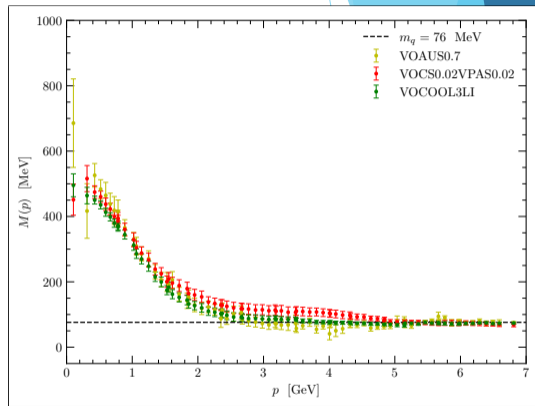
Overlap quark propagator - dynamical $m_\pi = 156$ MeV

Centrifuge preconditioning $\omega = 0.02$

Annealed smoothing, $\alpha = 0.7$

Vortex preserving, $\alpha = 0.02$

10 sweeps of cooling for comparison



PRELIMINARY

Overlap quark propagator - dynamical $m_\pi = 156$ MeV

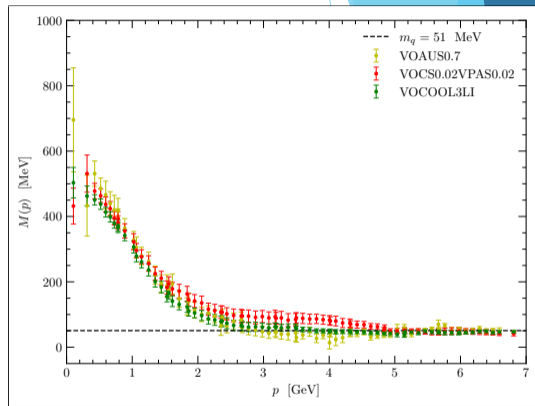
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Dynamical mass generation from
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PRELIMINARY

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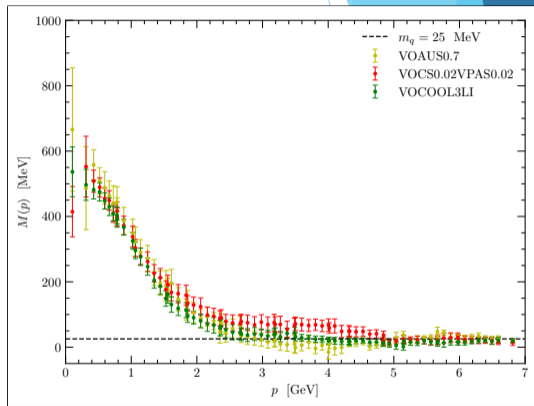
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PRELIMINARY

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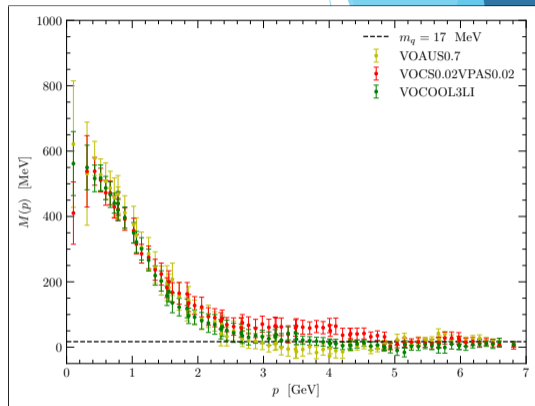
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PRELIMINARY

Overlap quark propagator - dynamical $m_\pi = 156$ MeV

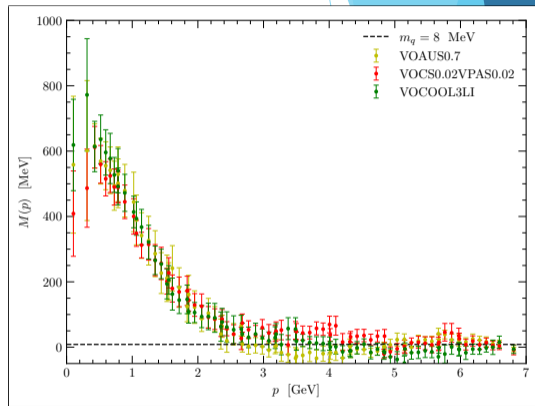
Centrifuge preconditioning $\omega = 0.02$

Annealed smoothing, $\alpha = 0.7$

Vortex preserving, $\alpha = 0.02$

10 sweeps of cooling for comparison

Dynamical mass generation from
smoothed centre vortices



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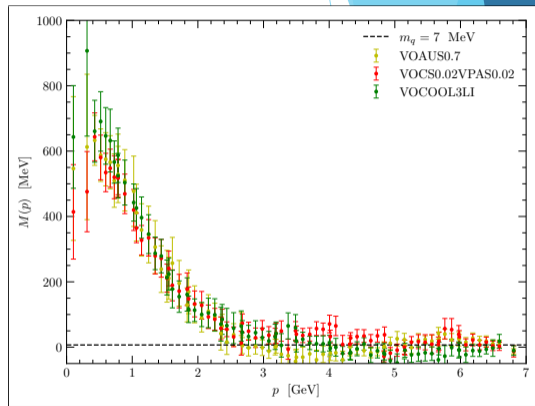
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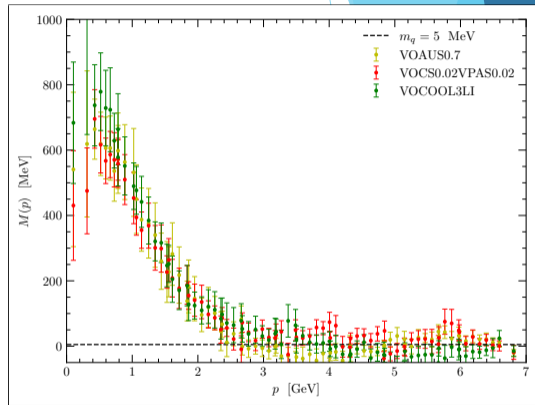
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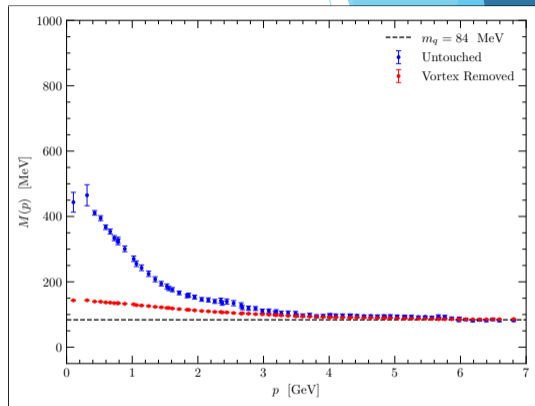
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Dynamical mass generation

→ Increases as m_q decreases

Vortex removed dynamical mass

→ Significantly reduced at large m_q



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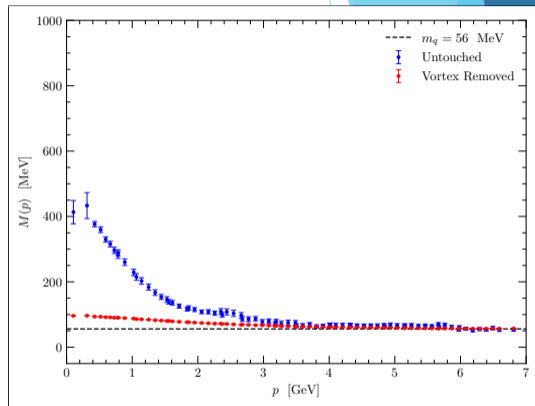
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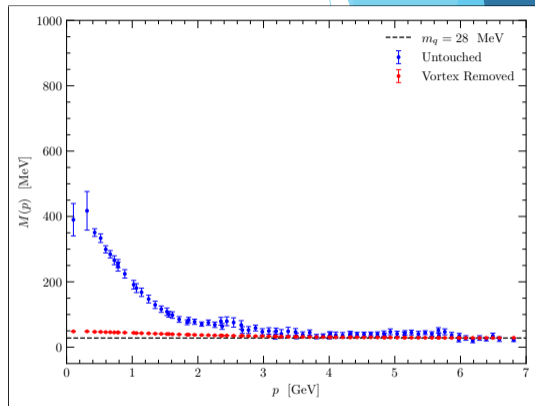
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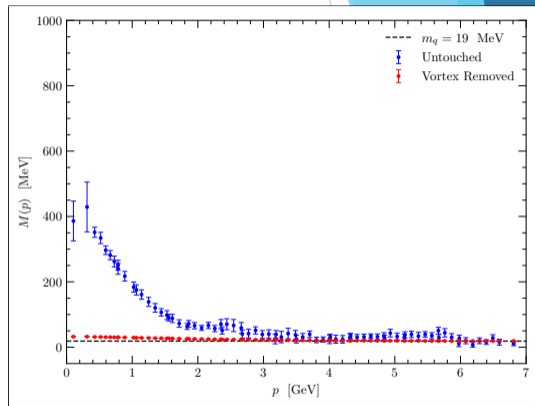
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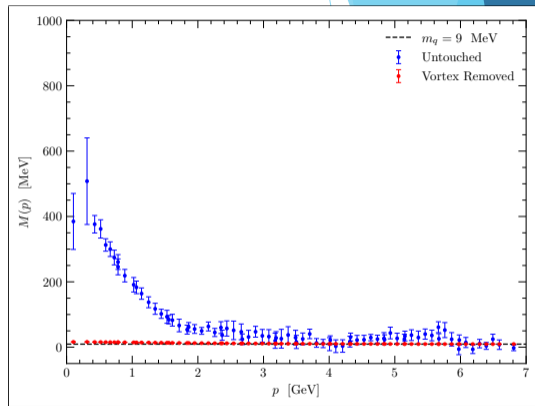
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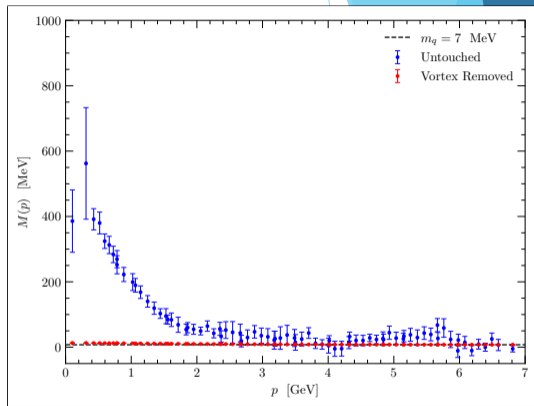
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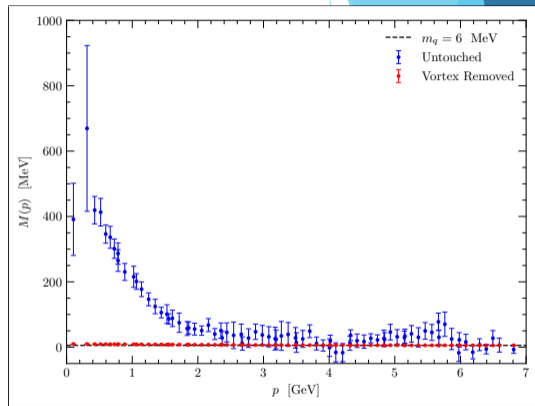
Dynamical mass generation

→ Increases as m_q decreases

Vortex removed dynamical mass

→ Significantly reduced at large m_q

→ Completely eliminated at small m_q



PRELIMINARY

Scorecard: Emergent phenomena from centre vortices

Static quark potential;

Vortex removal removes confining linear potential.

Pure gauge vortex-only field recreates $\sim 2/3$ of string tension.

Dynamical vortex-only field recreates \sim full string tension.

Gluon propagator;

Vortex-only field is confining (violates spectral positivity).

Pure gauge vortex removal shows possible positivity violation.

Dynamical vortex removal causes gluons to be deconfined.

Overlap quark propagator;

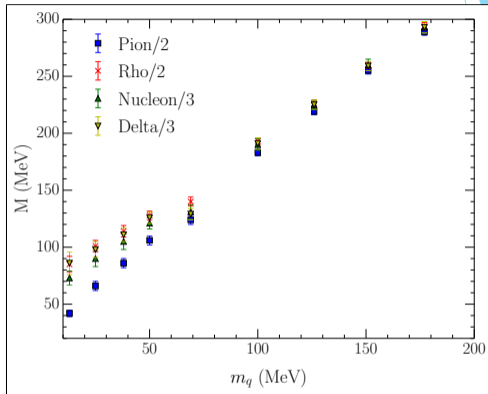
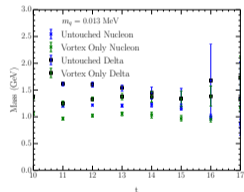
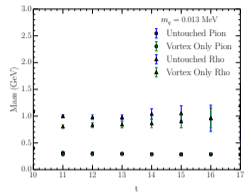
Pure gauge vortex removal diminishes dynamical mass generation.

Dynamical vortex removal eliminates dynamical mass generation.

Pure gauge **smoothed*** vortex-only field reproduces dynamical mass.

Dynamical **smoothed*** vortex-only field generates excess dynamical mass.

Bonus points: Hadron spectrum

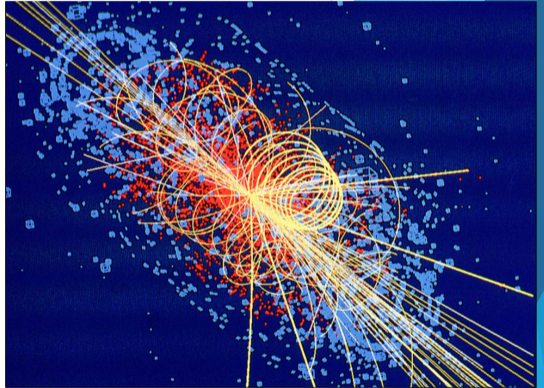


Centre vortex removal restores chiral symmetry

A. Trewartha, WK, D.B. Leinweber, J. Phys. G 44 (2017) 12, 125002

Origin of mass

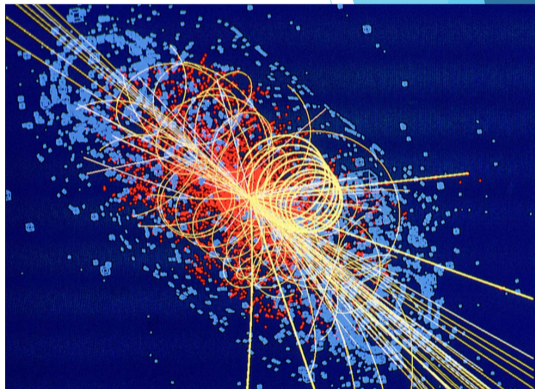
$$3 + 3 + 5 = 938 \text{ MeV?}$$



Origin of mass

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Higgs contributes $\sim 1\%$ of your mass

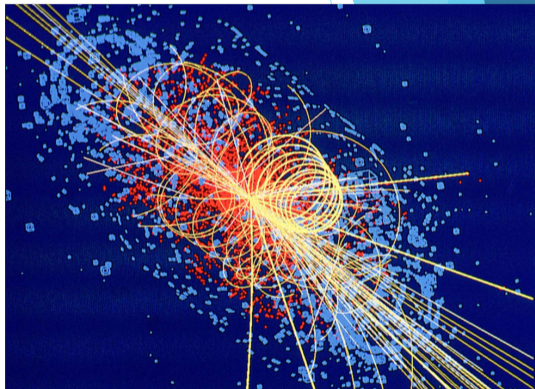


Origin of mass

$$3 + 3 + 5 = 938 \text{ MeV?}$$

Higgs contributes $\sim 1\%$ of your mass

QCD generates the other $\sim 99\%$



Summary

It would be tempting to abolish the $SU(3)$ color theory for hadrons altogether, replacing it by a $Z(3)$ theory on a Euclidean lattice and taking the continuum limit close to the critical point. This is presumably not justifiable.

“On the phase transition towards permanent quark confinement”

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G. 't Hooft, 1978

Centre vortices appear to contain all information necessary to reproduce confinement and $D\chi$ SB in $SU(3)$ gauge theory with dynamical fermions.