Francesco Becattini,
University of Florence and INFN

Recent developments of spin polarization phenomenology in relativistic heavy ion collisions

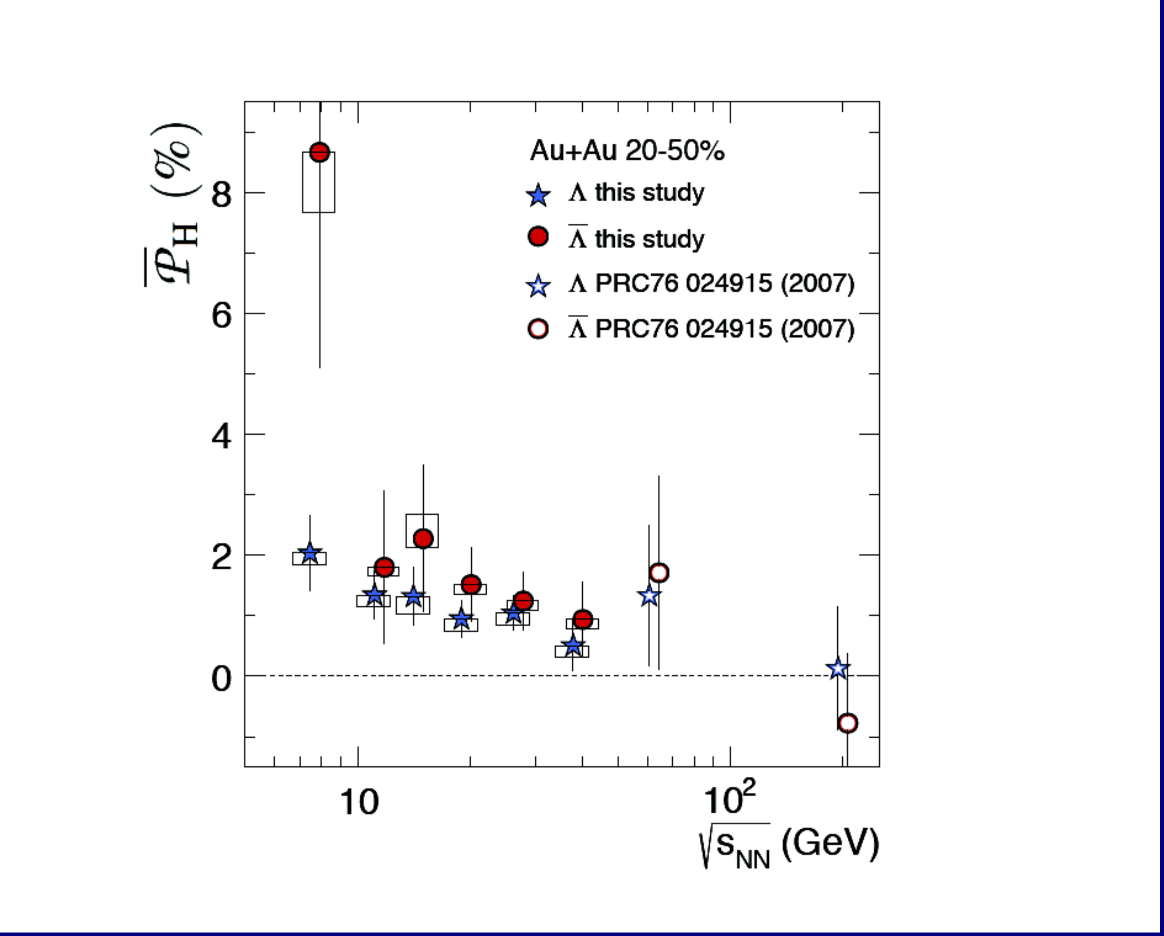
OUTLINE

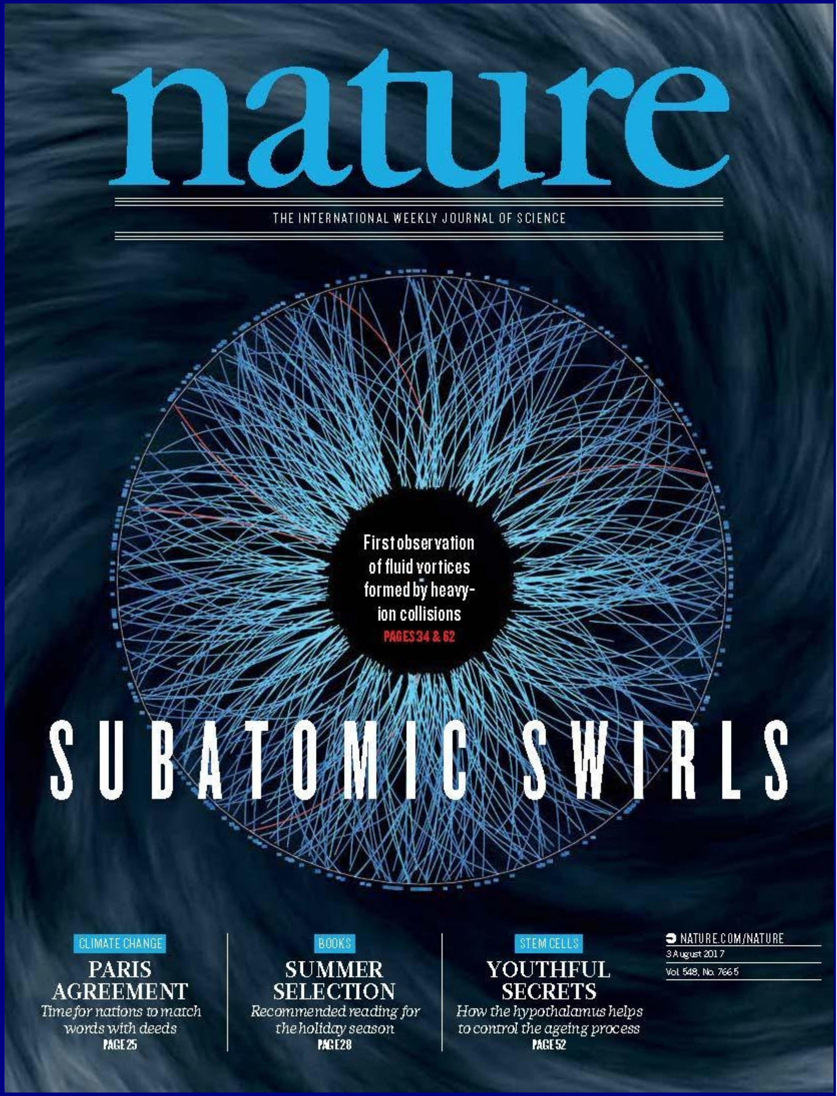
- Introduction and local polarization puzzles
- Local thermodynamic equilibrium and its expansion
- Spin-thermal shear coupling and isothermal local equilibrium
- Conclusions

Chirality retreat UCLA Dec 2-4 2022

Introduction

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017

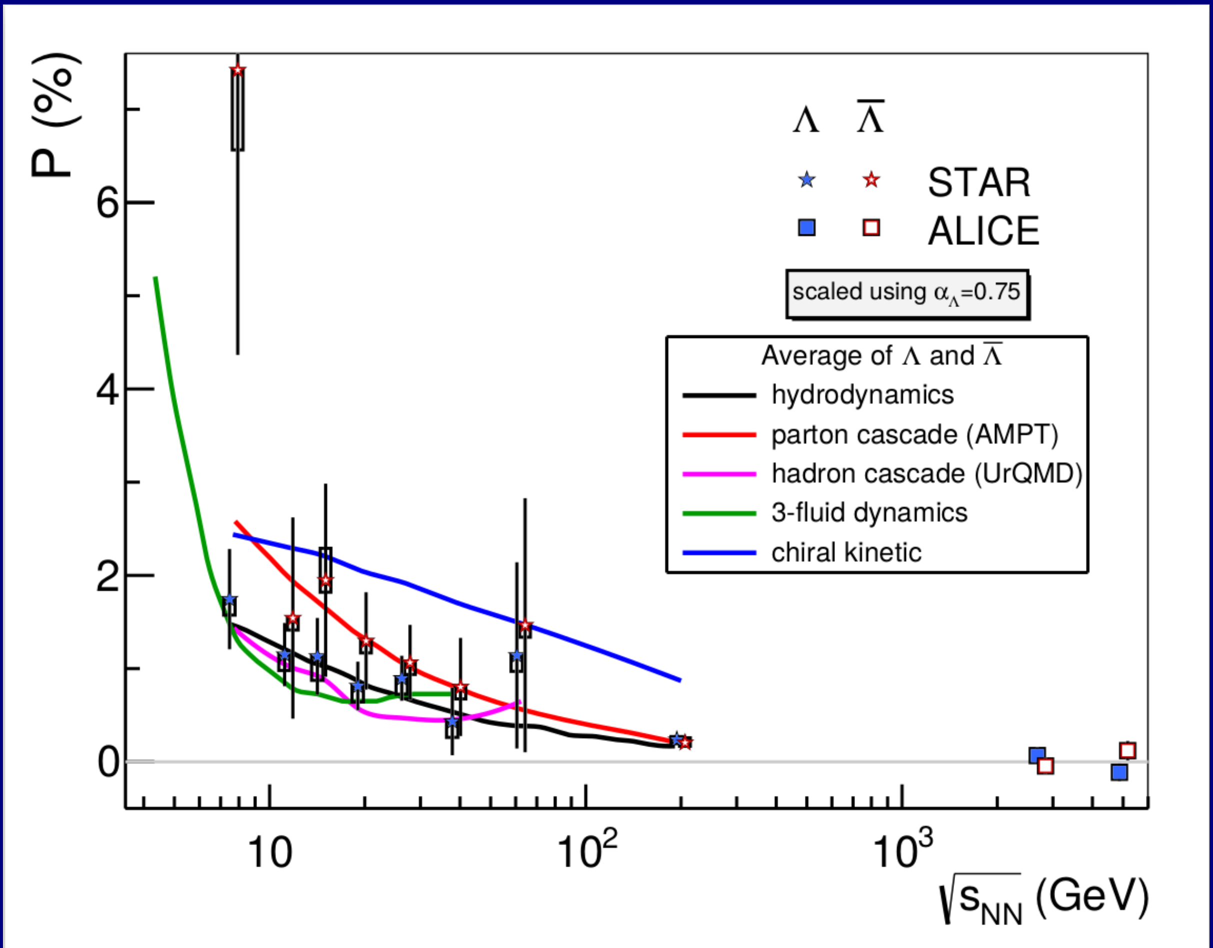




Particle and antiparticle have the same polarization sign.
This shows that the phenomenon cannot be driven
by a mean field (such as EM) whose coupling is *C-odd.*
Definitely favours the thermodynamic (equipartition) interpretation

Comparison with the data (date Jan 2020)

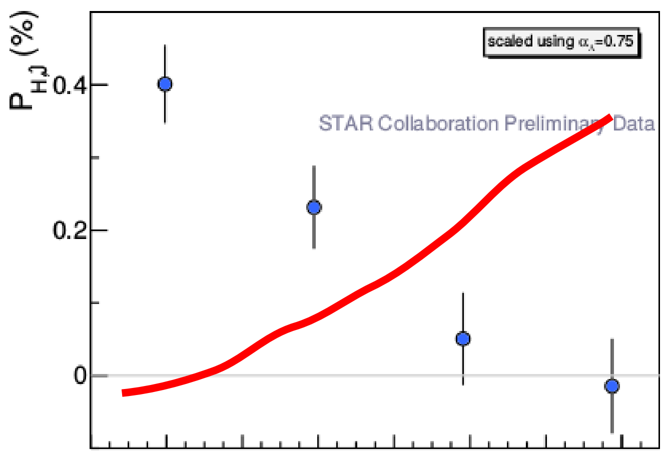
F. B., M. Lisa, Polarization and vorticity in the QGP, Ann. Rev. Part, Nucl. Sc. 70, 395 (2020)



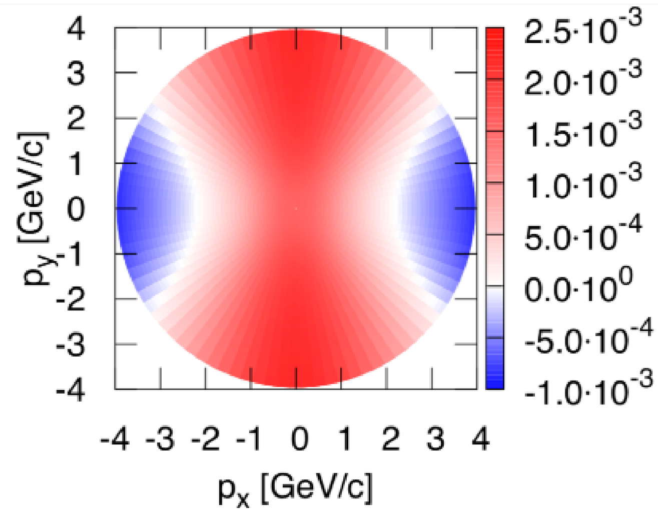
$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \; n_F}$$

Different models of the collision, same formula for polarization

Puzzles: momentum dependence of polarization (until march 2021)

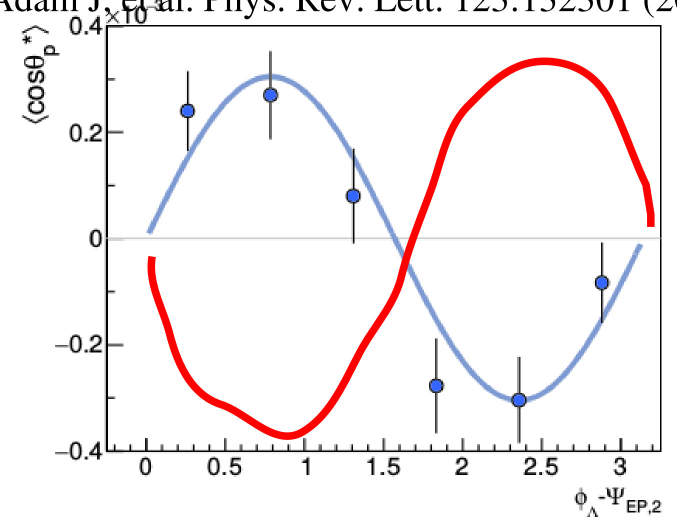


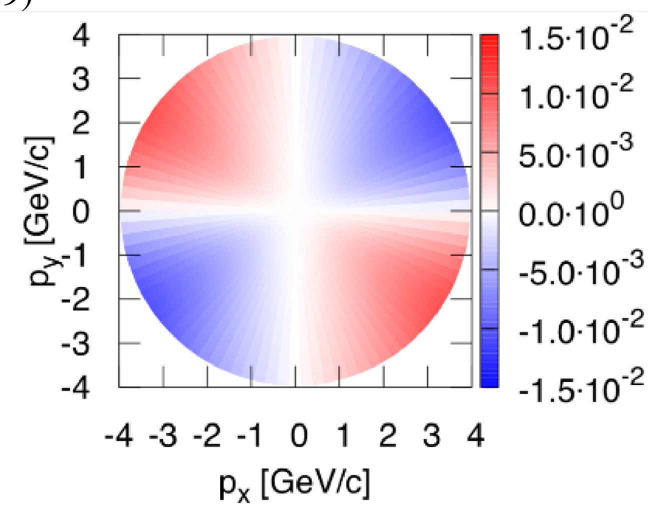
Niida T. Nucl. Phys. A982:511 (2019)



Theory prediction

Adam J, et al. Phys. Rev. Lett. 123:132301 (2019)





Not the effect of decays:

X. L. Xia, H. Li, X.G. Huang and H. Z. Huang, Phys. Rev. C 100 (2019), 014913

F. B., G. Cao and E. Speranza, Eur. Phys. J. C 79 (2019) 741

A brief theory summary

F. Becattini, Lecture Notes in Physics 987, 15 (2021) arXiv:2004.04050

Spin polarization vector for spin ½ particles:

$$S^\mu(p) = \frac{1}{2} \frac{\int \mathrm{d}\Sigma \cdot p \; \mathrm{tr}_4(\gamma^\mu \gamma^5 W_+(x,p))}{\int \mathrm{d}\Sigma \cdot p \; \mathrm{tr}_4 W_+(x,p)}$$

Wigner function is an expectation value of an integrated two-point function of the Dirac field *(see e.g. A. Palermo's talk)*

$$W(x,k) = \mathrm{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

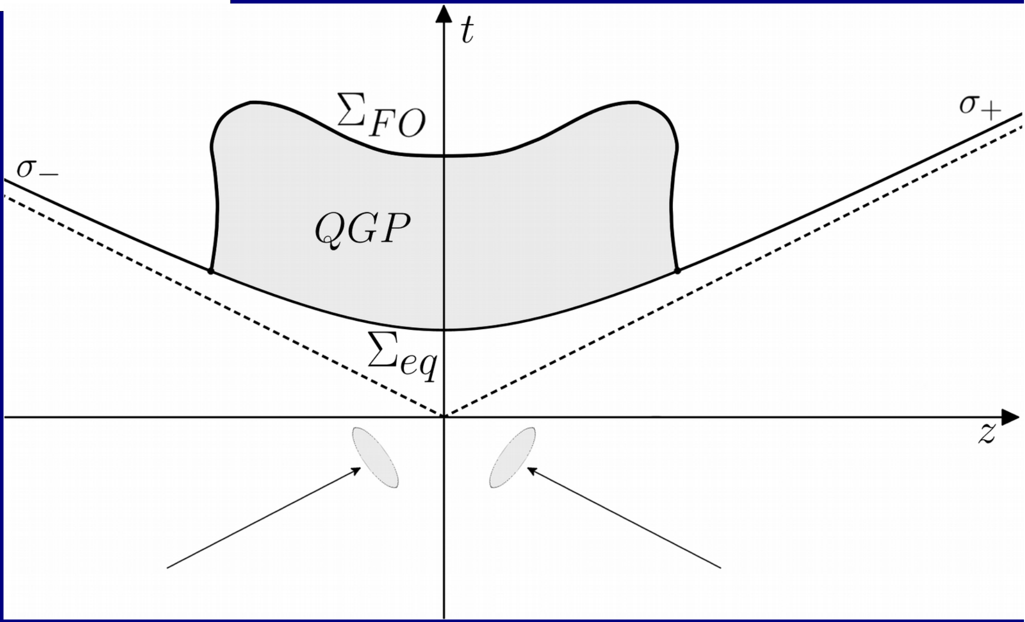
One needs to know the statistical operator to calculate mean values

$$\langle \widehat{X} \rangle = \mathrm{tr}(\widehat{\rho}\widehat{X})$$

Local thermodynamic equilibrium approximation

$$\widehat{\rho} \simeq \widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma(\tau)} {\rm d}\Sigma_\mu \, \widehat{T}_B^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu\right]$$
$$= \frac{1}{Z}\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \, \widehat{T}_B^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu\right]$$

Corresponds to the ideal fluid approximation.
Neglecting dissipative term in the exponent of the density operator



$$W(x,k) \simeq W(x,k)_{\rm LE} = {\rm Tr}(\widehat{\rho}_{\rm LE}\widehat{W}(x,k))$$

Mean value of a local operator: Taylor expansion

$$W(x,k)_{\rm LE} = \frac{1}{Z}{\rm Tr}\left(\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y) \widehat{T}_B^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\widehat{j}^\mu(y)\right] \widehat{W}(x,k)\right)$$

Expand the β and ζ fields from the point *x* where the Wigner operator is to be evaluated

$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \ldots$$

$$\int_\Sigma {\rm d}\Sigma_\mu T_B^{\mu\nu}(y)\beta_\nu(x) = \beta_\nu(x)\int_\Sigma {\rm d}\Sigma_\mu T_B^{\mu\nu}(y) = \beta_\nu(x)\widehat{P}^\nu$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp\left[-\beta_\mu(x)\widehat{P}^\mu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}_x^{\mu\nu} - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}_x^{\mu\nu} + \ldots\right]$$

$$\widehat{J}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu\widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu\widehat{T}_B^{\lambda\mu}(y)$$

neglected until 2021

$$\widehat{\rho}_{\mathrm{LE}} \simeq \frac{1}{Z} \exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$$

Thermal vorticity
Adimensional in natural units

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu)$$

Thermal shear
Adimensional in natural units

At global equilibrium the thermal shear vanishes because the four-temperature fulfills the Killing equation

Surprise: thermal shear does contribute!

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau\, n_F}$$

$$n_F = (\mathrm{e}^{\beta\cdot p-\xi}+1)^{-1}$$

It is a NON-dissipative effect!

$$\boxed{S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\; n_F(1-n_F)\hat{t}_\nu\xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\; n_F},}$$

F. B., M. Buzzegoli, A. Palermo, Phys. Lett. B 820 (2021) 136519

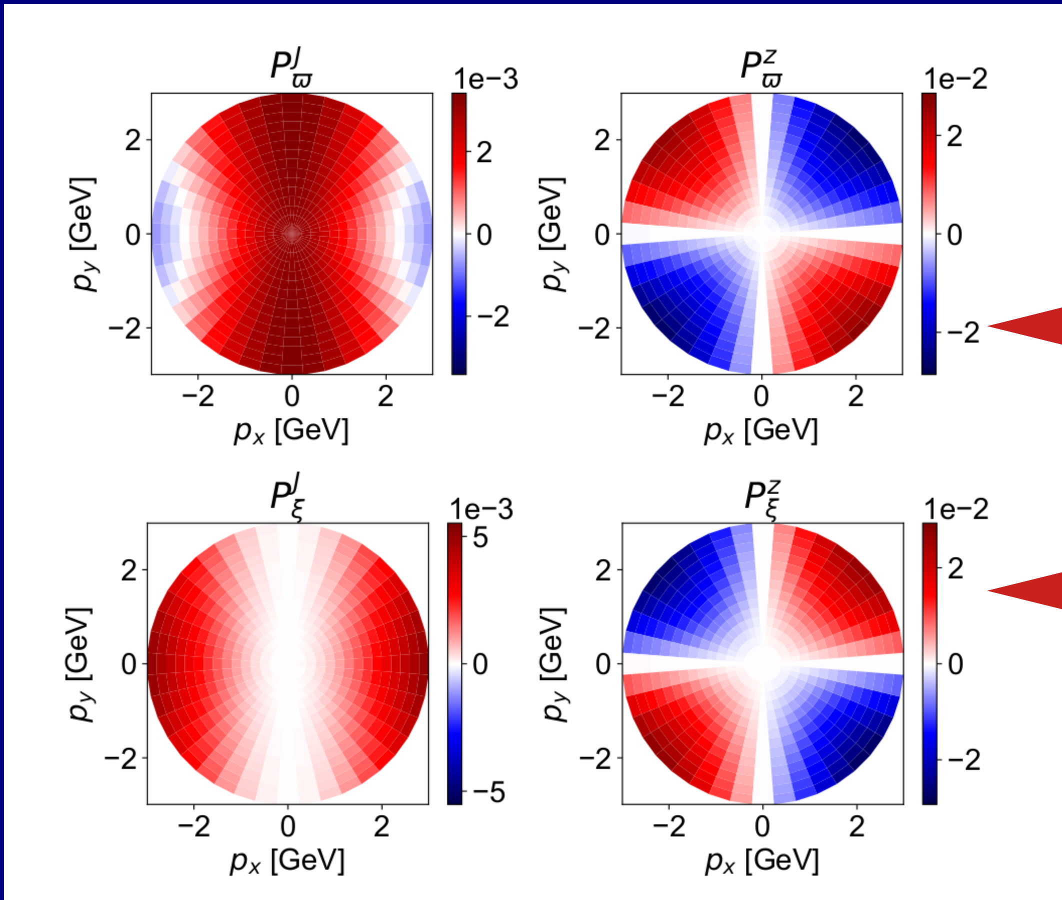
Same (though not precisely the same) formula obtained by Liu and Yin with a different method:

S. Liu, Y. Yin, JHEP 07 (2021) 188

The additional local equilibrium term has been confirmed in more analyses:

C. Yi, S. Pu, D. L. Yang, Phys.Rev.C 104 (2021) 6, 064901
Y. C. Liu, X. G. Huang, arXiv 2109.15301, Sci.China Phys.Mech.Astron. 65 (2022) 7, 272011

Hydro calculations

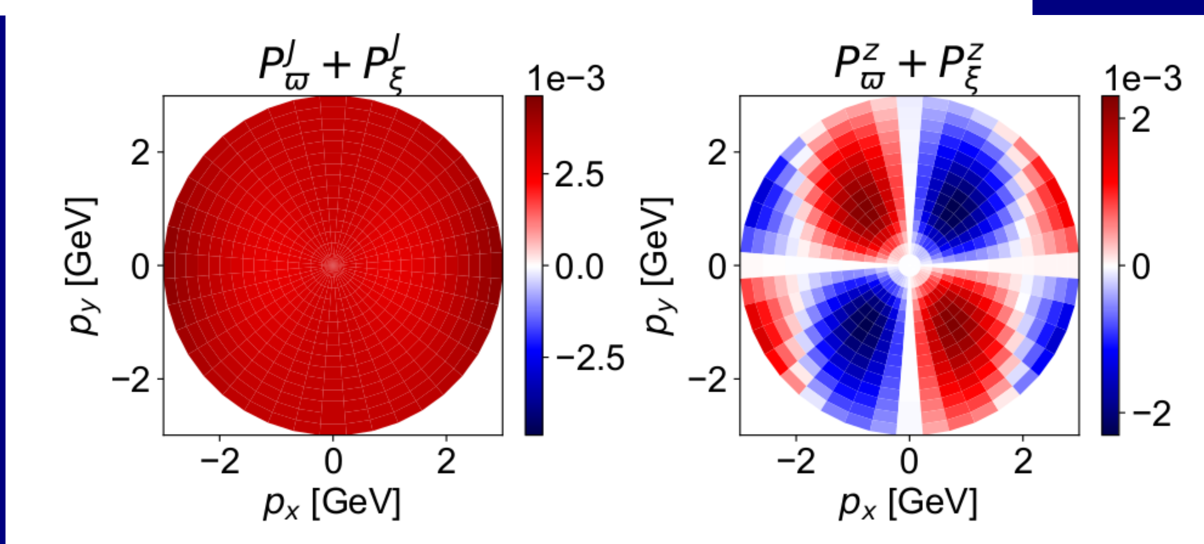


Based on the hydrodynamic code VHLLE (author I. Karpenko) tuned to reproduce Au-Au momentum spectra at RHIC top energy.
Similar output with ECHO-QGP (main author G. Inghirami).

Thermal vorticity

Thermal shear

Right pattern!

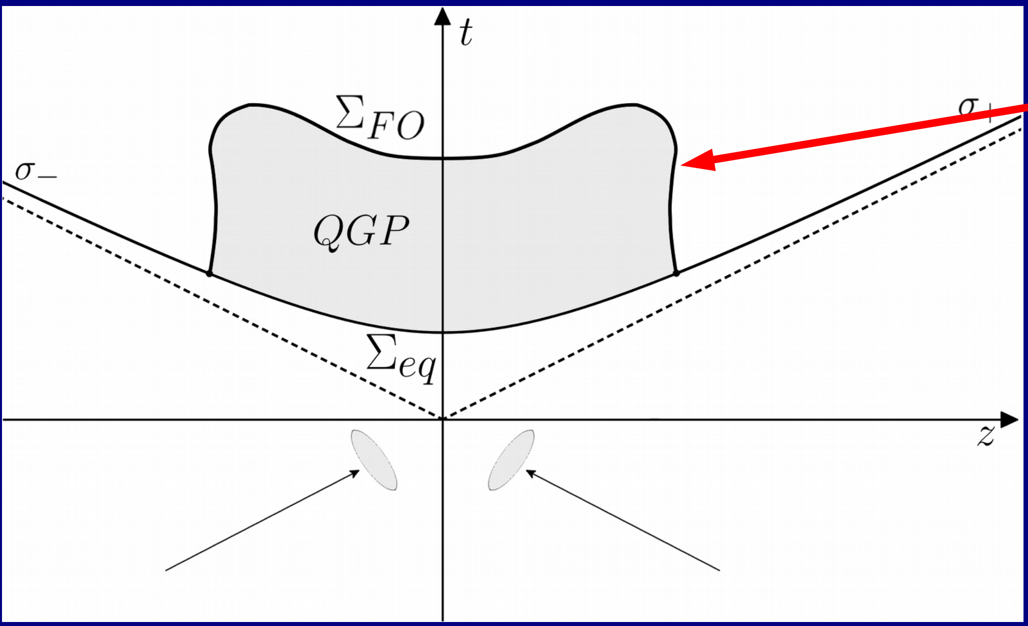


SUMMING UP

Not sufficient to restore the agreement between data and model

Calculations fully consistent with:
B. Fu, S. Liu, L. Pang, H. Song and Y. Yin, Phys. Rev. Lett. 127 (2021) 14, 142301

Isothermal hadronization at very high energy



At high energy, Σ_{FO} expected to be T= constant!

$$\beta^\mu = (1/T) u^\mu$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \; \widehat{T}^{\mu\nu} \beta_\nu \right] = \frac{1}{Z} \exp\left[- \frac{1}{T} \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \; \widehat{T}^{\mu\nu} u_\nu \right]$$

NOW u (and just u) can be expanded!

$$u_\nu(y) = u_\nu(x) + \partial_\lambda u_\nu(x)(y-x)^\lambda + \ldots$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x) \widehat{P}^\mu - \frac{1}{2T}(\partial_\mu u_\nu(x) - \partial_\nu u_\mu(x)) \widehat{J}^{\mu\nu}_x - \frac{1}{2T}(\partial_\mu u_\nu(x) + \partial_\nu u_\mu(x)) \widehat{Q}^{\mu\nu}_x + \ldots]$$

Spin mean vector at leading order with isothermal local equilibrium (ILE)

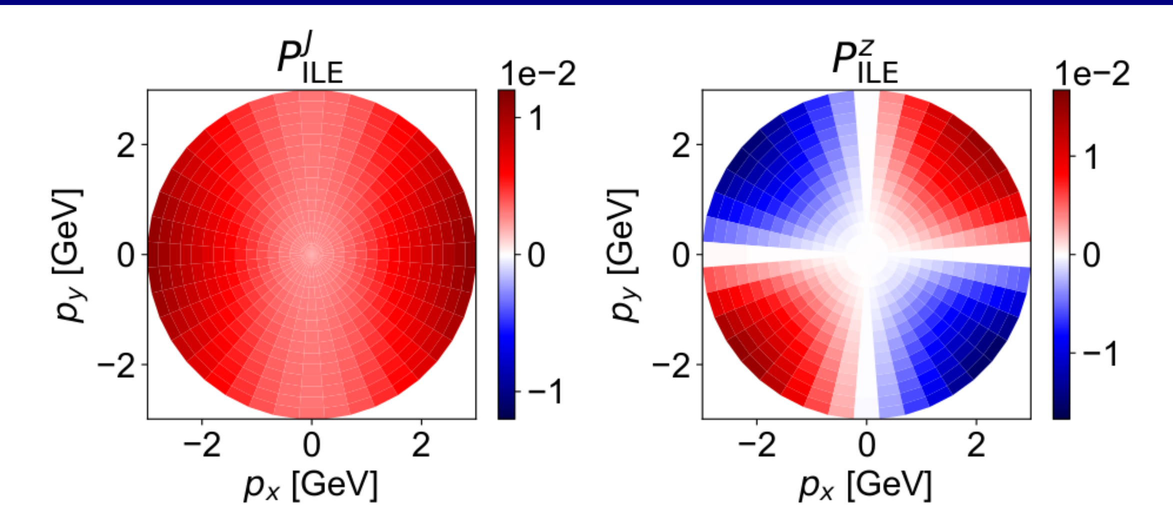
Readily found by replacing the gradients of β with those of u

$$S^{\mu}_{\mathrm{ILE}}(p) = \quad (\\ - \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F (1-n_F) \left[\omega_{\rho\sigma} + 2\, \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma}\right]}{8 m T_{\mathrm{dec}} \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

$$\omega_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho - \partial_\rho u_\sigma\right)$$

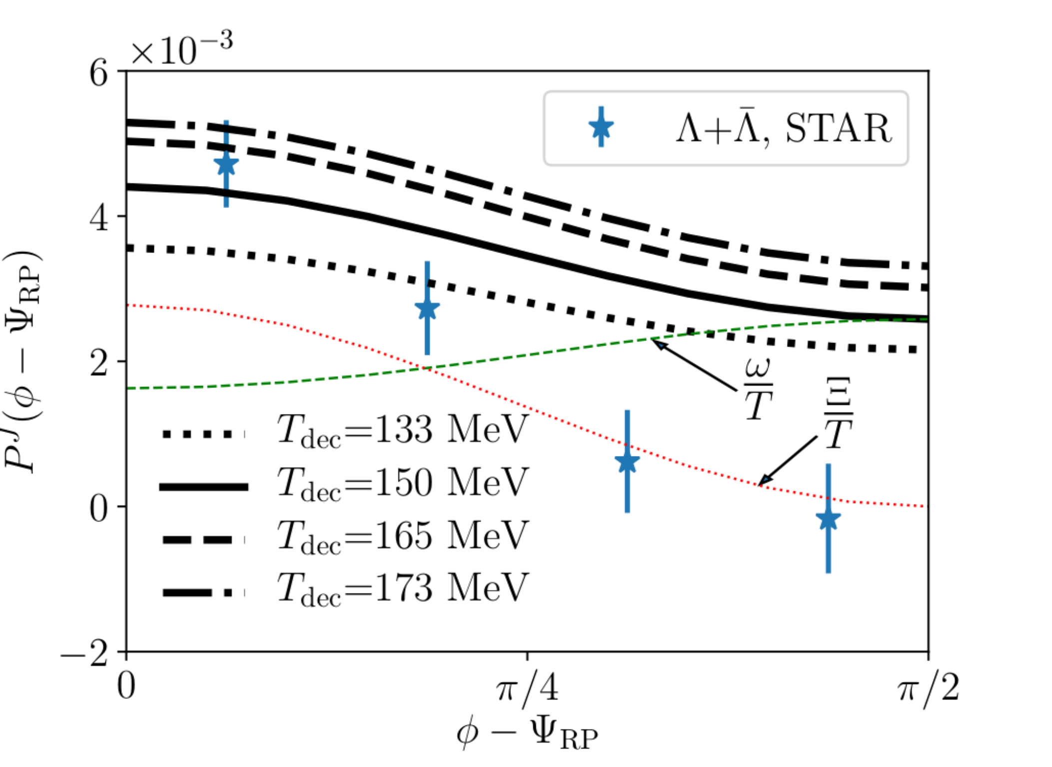
$$\Xi_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho + \partial_\rho u_\sigma\right)$$

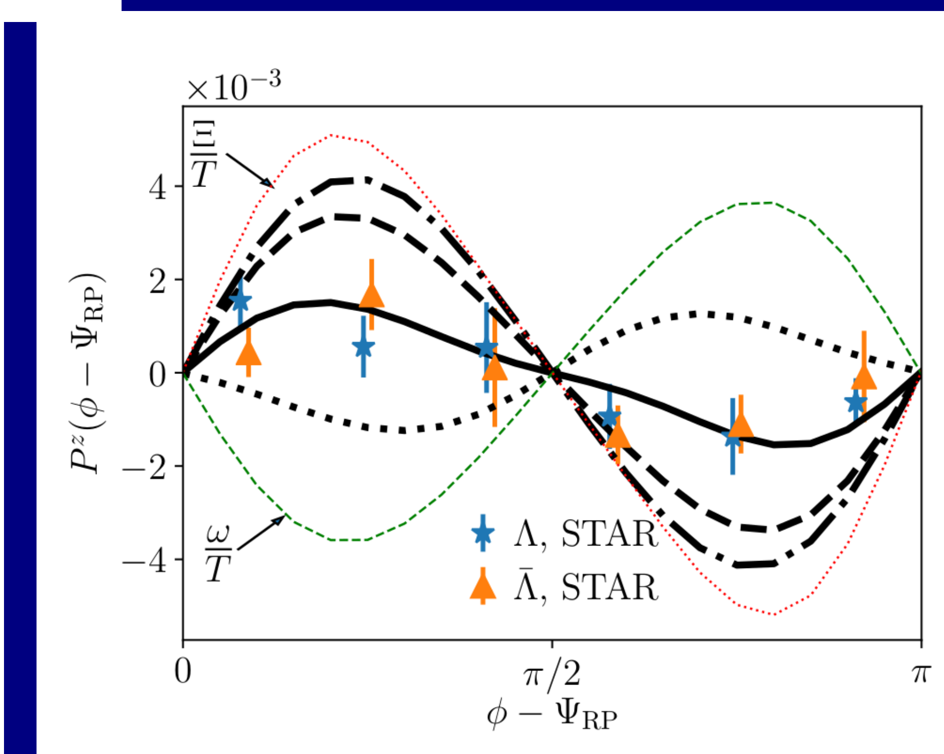
Isothermal local equilibrium: results



Apply the new formula (for primary hadrons)

$$S^\mu_{\rm ILE}(p) = - \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F) \left[\omega_{\rho\sigma} + 2\,\hat{t}_\rho \frac{p^\lambda}{\varepsilon}\Xi_{\lambda\sigma}\right]}{8 m T_{\rm dec} \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$





Recent study of Λ polarization with shear contribution

S. Alzhrani, S. Ryu, C. Shen, Phys.Rev.C 106 (2022) 1, 014905,

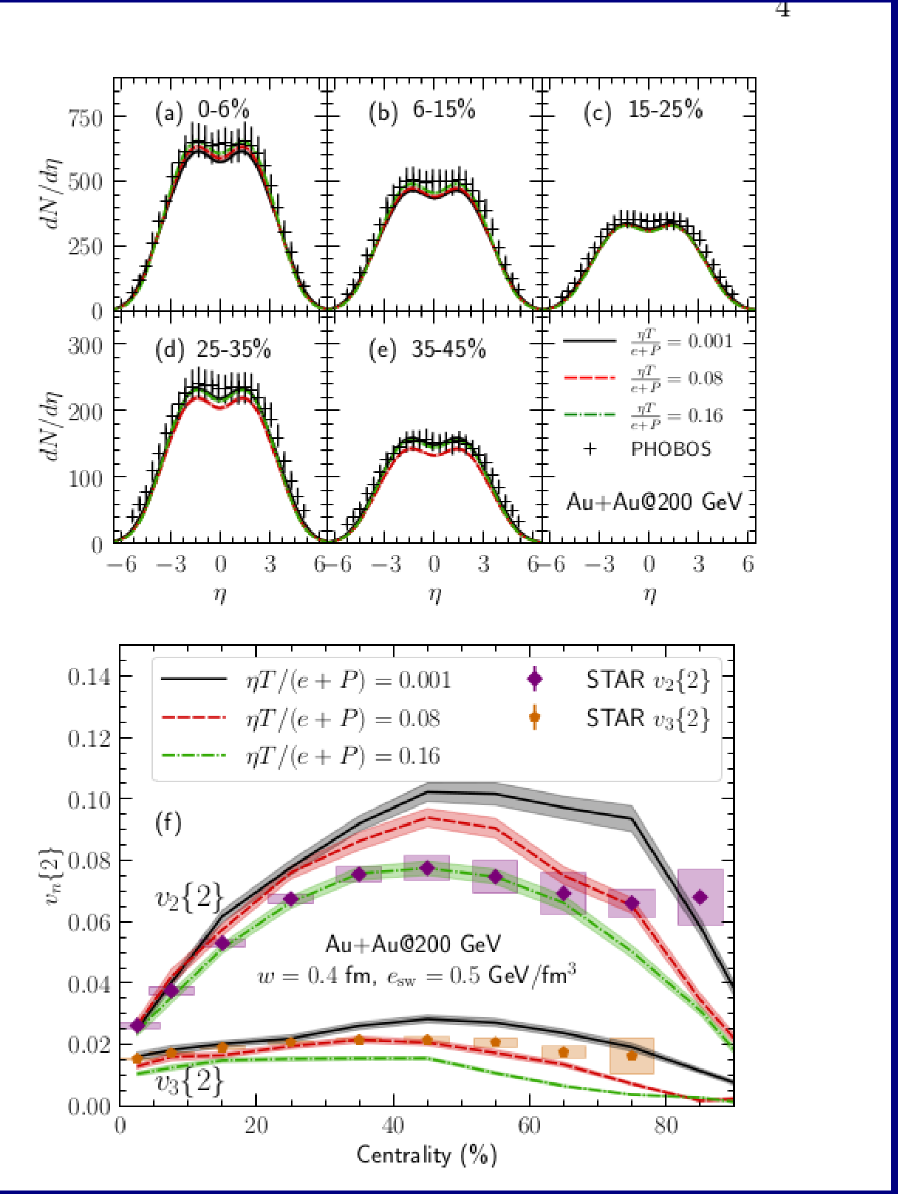
A model study at 200 GeV

3+1 D viscous hydro with specific initial conditions designed in:

C. Shen and S.Alzhrani,
Phys. Rev. C 102, 014909 (2020),
arXiv:2003.05852 [nucl-th].

S. Ryu, V. Jupic, C. Shen,
Phys. Rev. C 104, 054908 (2021)

Parameter	Description	Value
w [fm]	initial hot spot width	0.4, 0.8, 1.2
η_0	space-time rapidity plateau size	2.5
σ_η	space-time rapidity fall off width	0.5
f	initial longitudinal flow fraction	0.15
τ_0 [fm/c]	hydrodynamics starting time	1
$\eta T/(e+P)$	specific shear viscosity	0, 0.08, 0.16
$e_{\rm sw}$ [GeV/fm^3]	particlization energy density	0.25, 0.5



Polarization induced by thermal shear

There are two differences in the thermal shear-spin formula

$$\mathcal{A}^\mu_{\rm BBP} = -\varepsilon^{\mu\rho\sigma\tau}\left(\frac{1}{2}\omega_{\rho\sigma}p_\tau + \frac{1}{E}\hat{t}_\rho\xi_{\sigma\lambda}p^\lambda p_\tau\right)$$

F. B., M. Buzzegoli, A. Palermo,
Phys. Lett. B 820 (2021) 136519

$$S^\mu(p^\alpha) = \frac{1}{4m}\frac{\int d\Sigma\cdot p\, n_0(1-n_0)\mathcal{A}^\mu}{\int d\Sigma\cdot p\, n_0},$$

S. Liu, Y. Yin, JHEP 07 (2021) 188

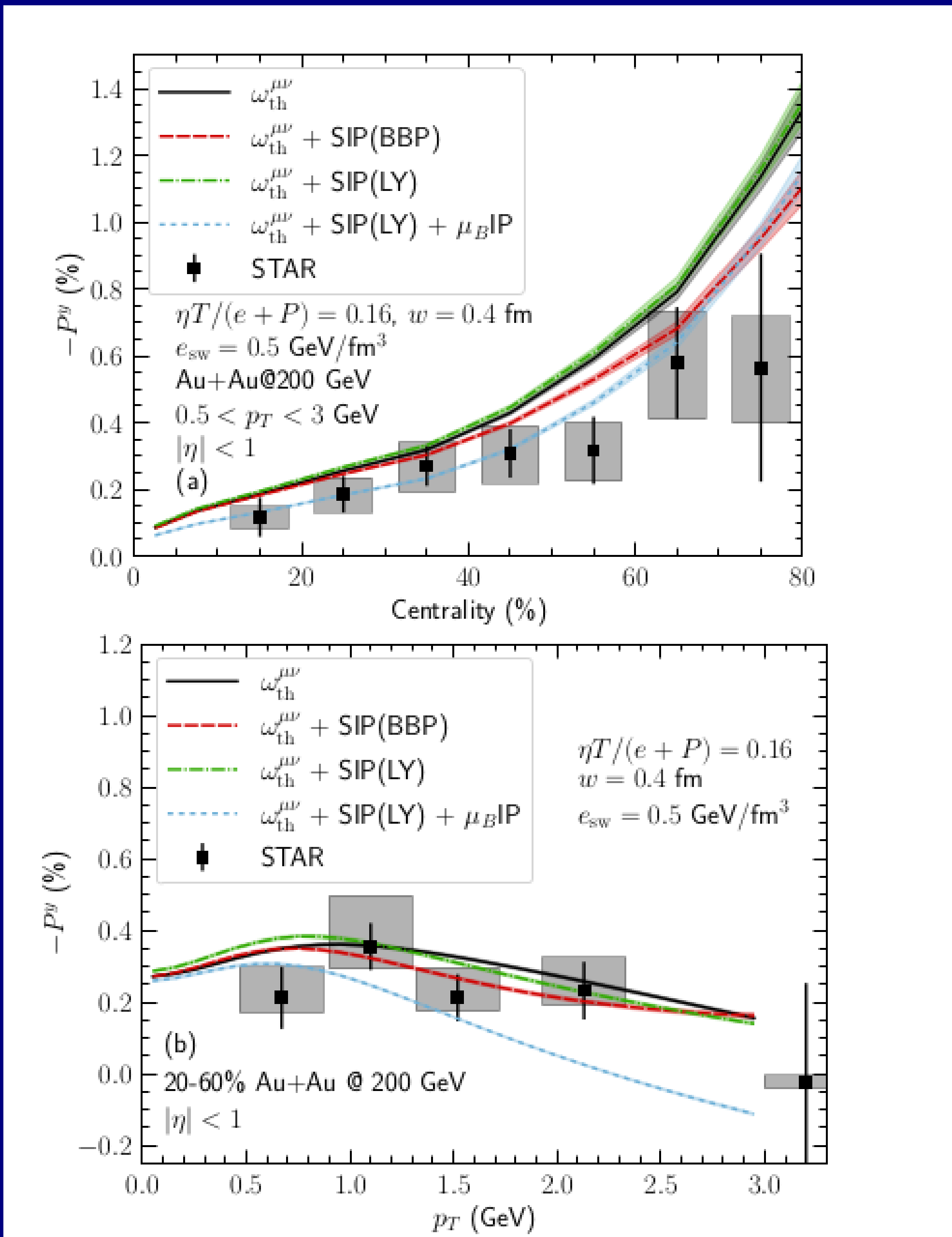
$$\mathcal{A}^\mu_{\rm LY} = -\varepsilon^{\mu\rho\sigma\tau}\left[\frac{1}{2}\omega_{\rho\sigma}p_\tau + \frac{1}{E}u_\rho\xi_{\sigma\lambda}p^\lambda_\perp p_\tau + \frac{b_i}{\beta E}u_\rho p^\perp_\sigma \partial^\perp_\tau(\beta\mu_B)\right].$$

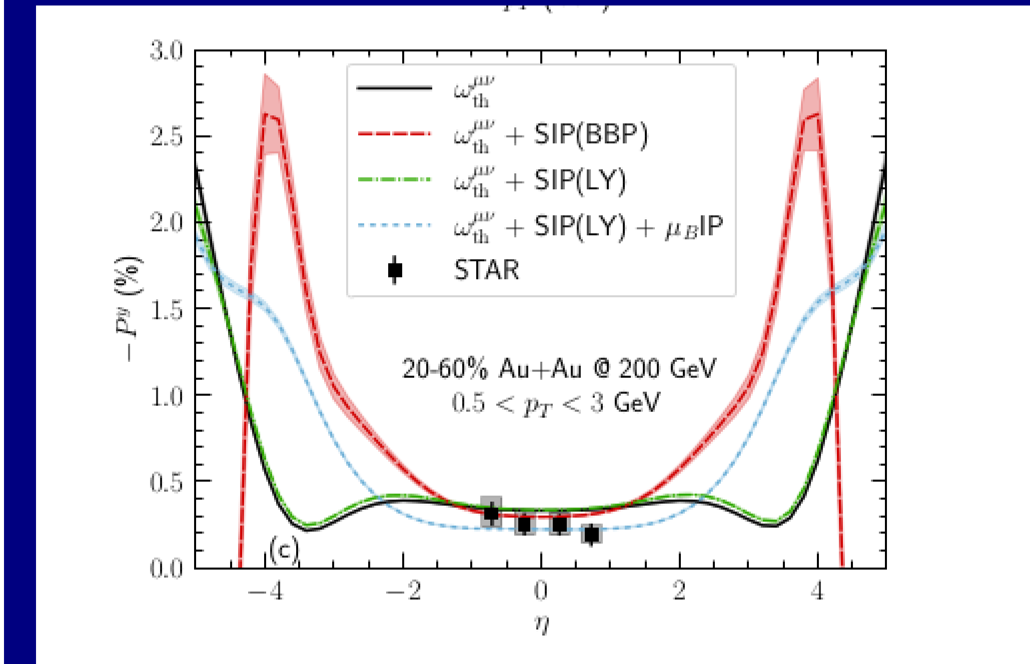
$$p^\lambda_\perp = p^\lambda - (u\cdot p)u^\lambda.$$

$$\xi_{\sigma\rho} = \frac{1}{2}\partial_\sigma\left(\frac{1}{T}\right)u_\rho + \frac{1}{2}\partial_\rho\left(\frac{1}{T}\right)u_\sigma + \frac{1}{2T}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \frac{1}{T}\sigma_{\rho\sigma} + \frac{1}{3T}\theta\Delta_{\rho\sigma}$$

In the LY formula, both the thermal gradient
and acceleration terms are killed by the four-velocity u replacing t and the p⊥ replacing p

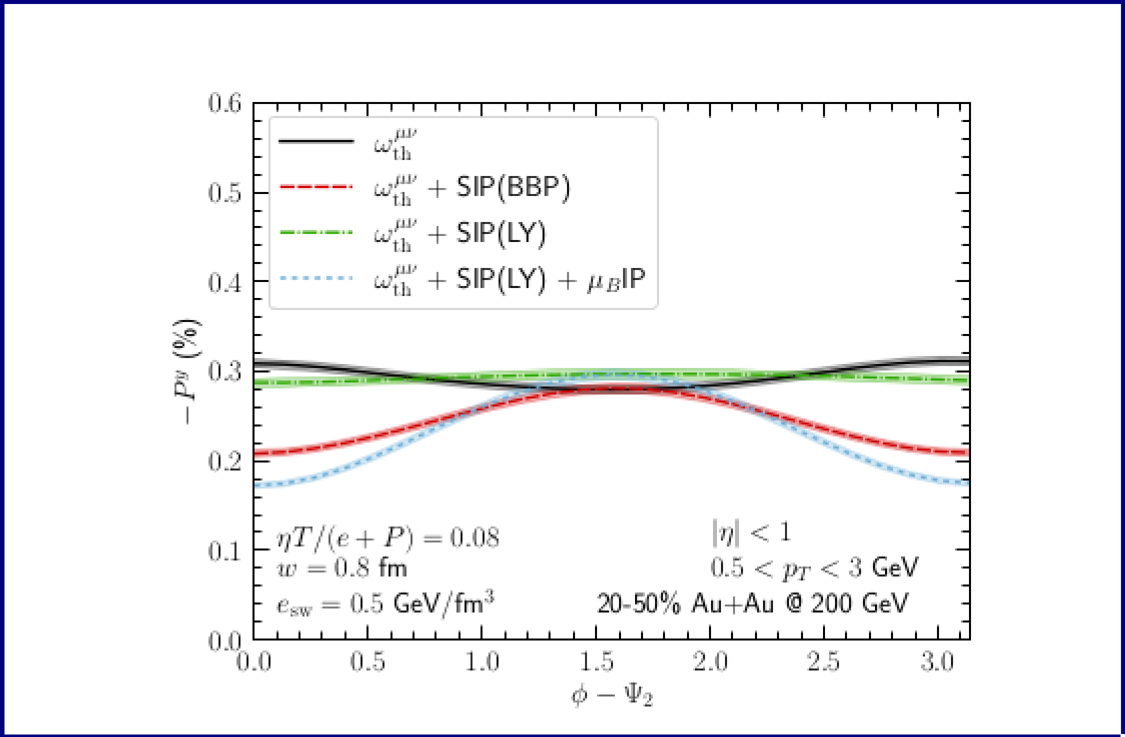
Results – Global polarization



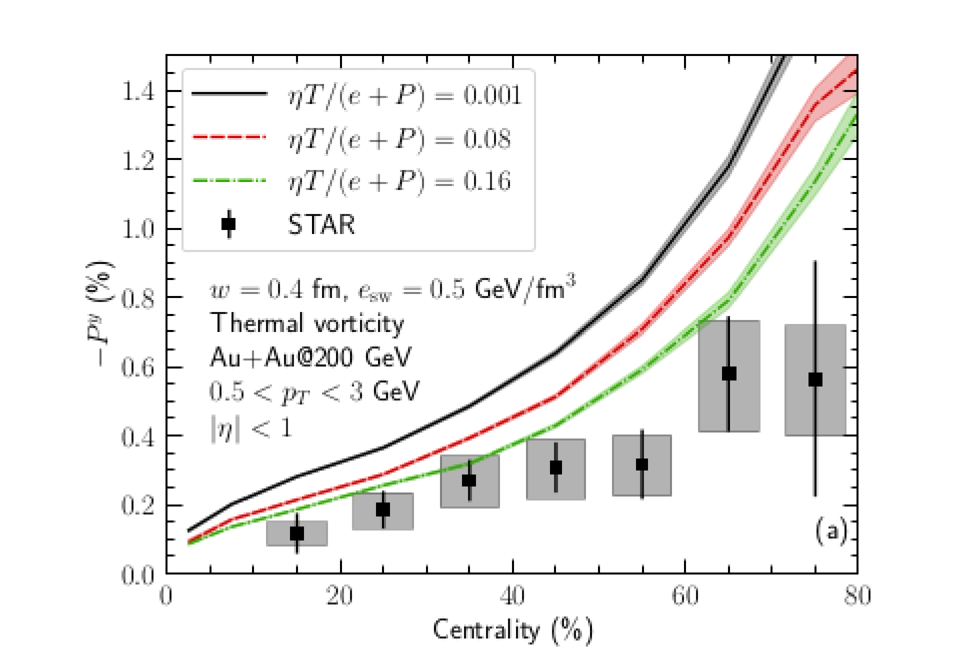


NOTE: BBP results include the contrinution of thermal gradients; the authors did not use the isothermal freeze-out hypothesis

Results – Polarization along J

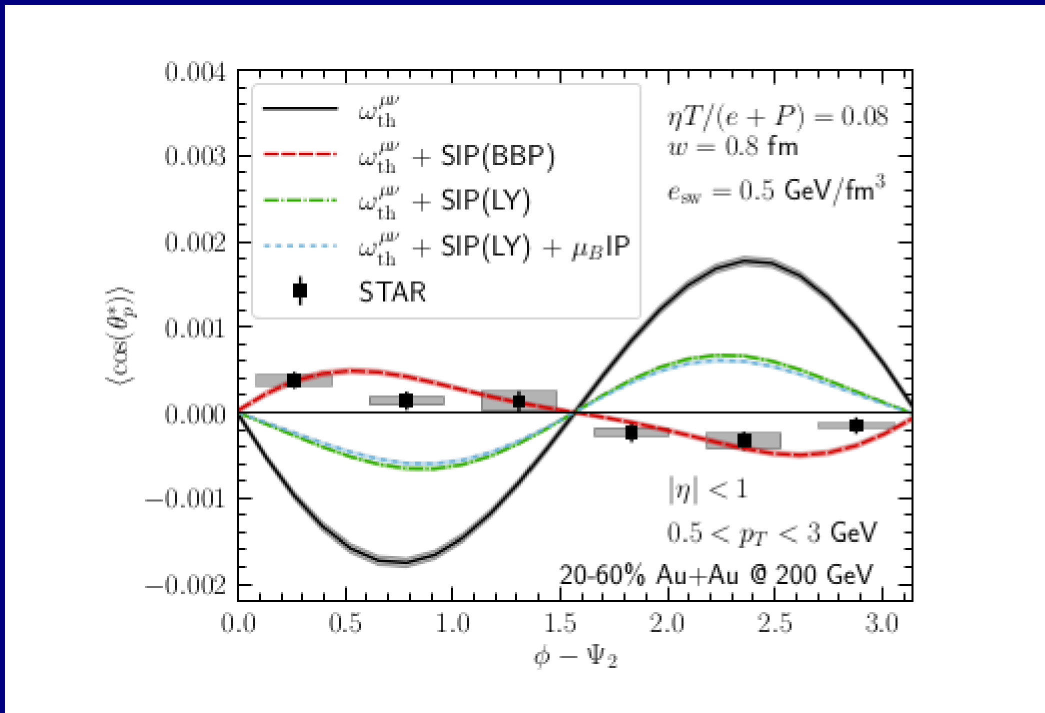


Dependence on the azimuthal angle of the Λ



Dependence of the integrated polarization on centrality with varying viscosity

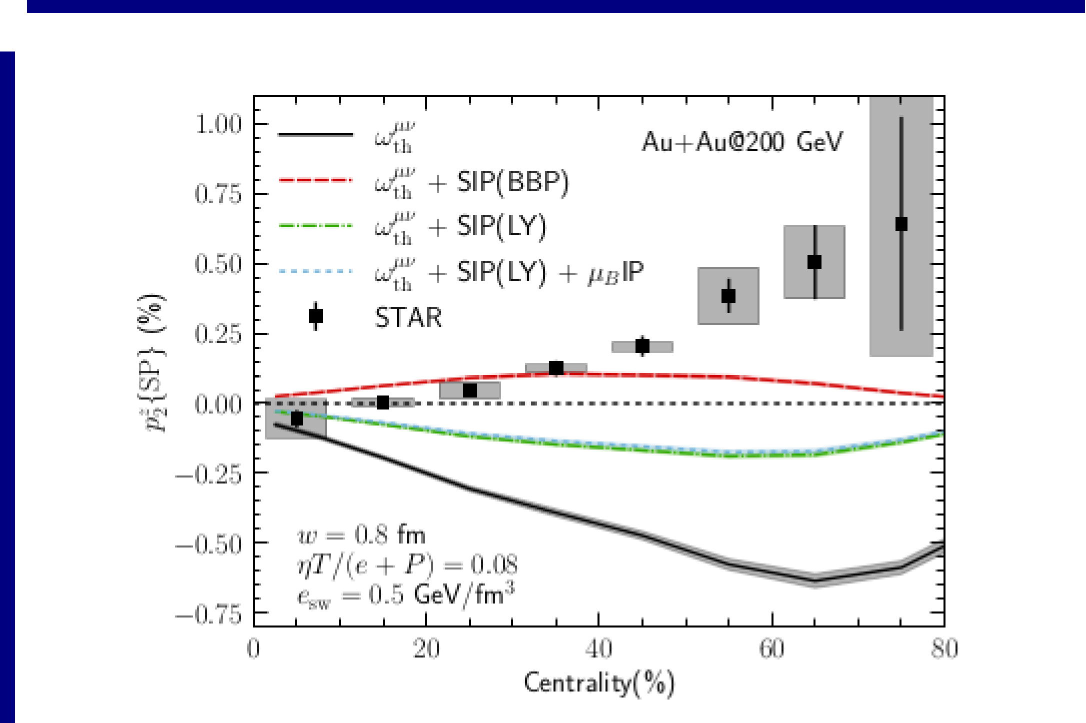
Results – Longitudinal polarization



Dependence of the longitudinal component of the spin polarization vector on the azimuthal angle of the Λ

Different predictions of VHLLE and ECHO-QGP (see previous slides)

Dependence of the dominant Fourier component (sin 2ϕ) on centrality



Summary and outlook

- Spin polarization at local equilibrium induced by thermal shear can solve the puzzles of the hydrodynamic predictions

- It is a new non-dissipative term, not dependent on unknown dynamical transport coefficient. What about its non-relativistic limit?

- Two different formulations of this effect leading to different results

- Difference in the predictions of hydro codes: sensitivity to initial conditions?

MUCH WORK TO BE DONE!

Polarization has a great potential to pin down the initial conditions and the QGP evolution which is yet unexploited to a large extent

Application to relativistic heavy ion collisions

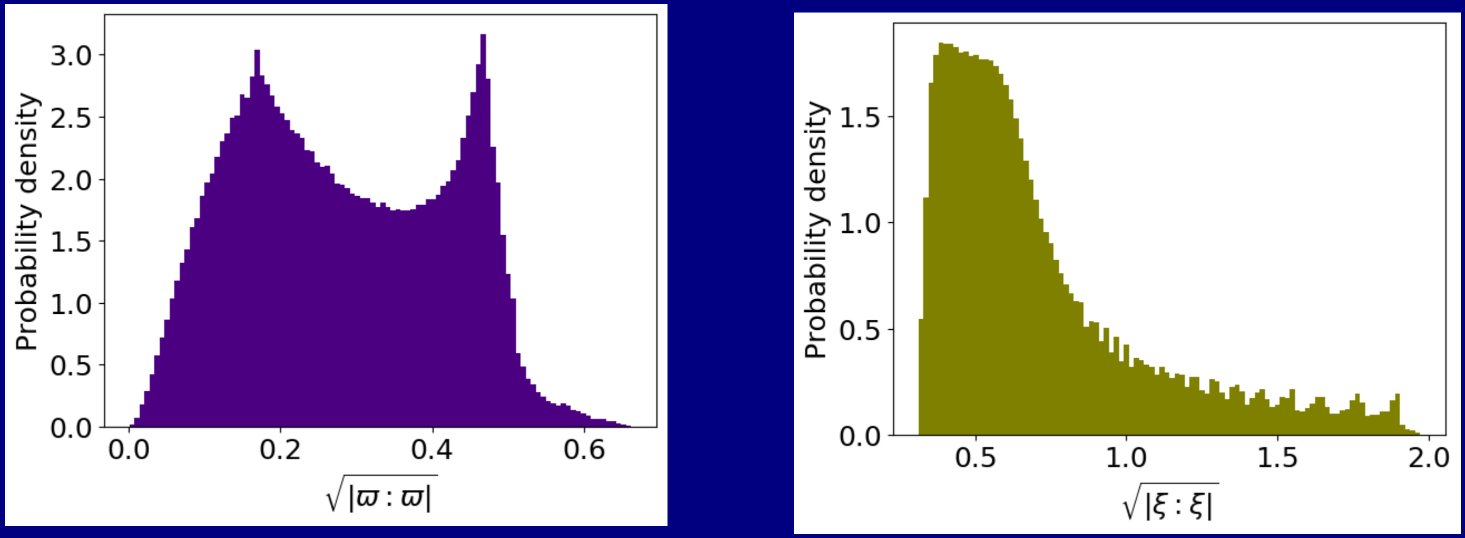
F. B., M. Buzzegoli, A. Palermo, G. Inghirami and I. Karpenko, arXiv:2103.14621, maybe appearing in PRL soon

$$S^\mu = S^\mu_\varpi + S^\mu_\xi$$

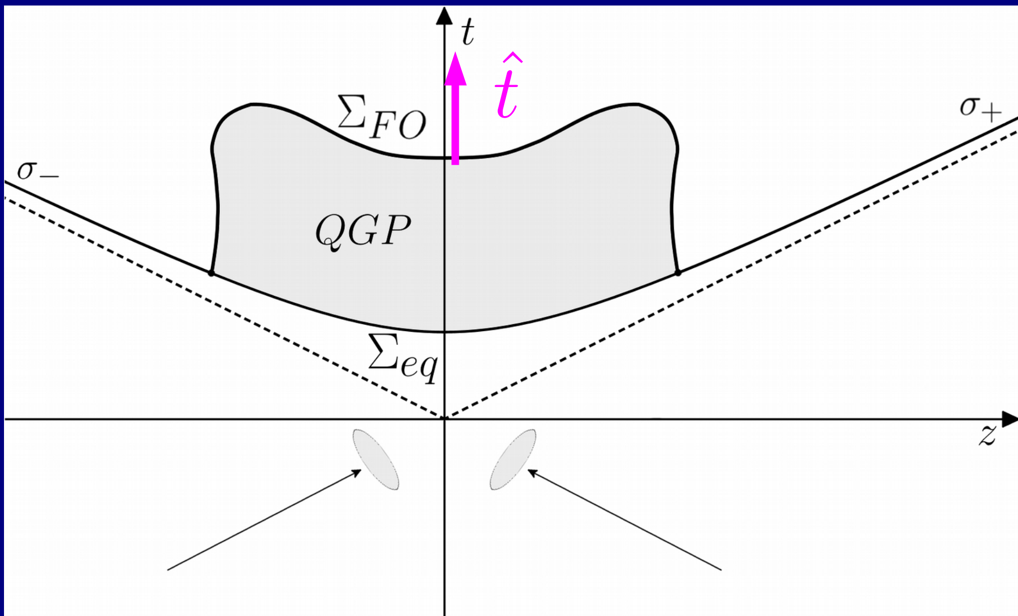
$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\rho\sigma\tau} \frac{p_\tau p^\lambda}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\hat{t}_\rho \xi_{\sigma\lambda}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

Is linear response theory adequate?



Why a dependence on Σ ?



The thermal shear term depends on the correlator:

$$\langle \widehat{Q}_x^{\mu\nu} \widehat{W}(x,p) \rangle$$

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int_{\Sigma_{FO}} \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

The divergence of the integrand of $J^{\mu\nu}$ vanishes, therefore it does not depend on the integration hypersurface (it is a constant of motion) and J is thus a tensor operator:

$$\widehat{\Lambda} \widehat{J}_x^{\mu\nu} \widehat{\Lambda}^{-1} = \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{J}_x^{\alpha\beta}$$

The divergence of the integrand of $Q^{\mu\nu}$ does not vanish, therefore it does depend on the integration hypersurface and Q is NOT a tensor operator

$$\widehat{\Lambda} \widehat{Q}^{\mu\nu} \widehat{\Lambda}^{-1} \neq \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{Q}^{\alpha\beta}$$

What is this new term?

It is a quantum, *non-dissipative*, correction to local equilibrium

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\; n_F(1-n_F)\hat{t}_\nu \xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\; n_F},$$

Does it have a non-relativistic limit?

$$\xi_{\sigma\rho} = \frac{1}{2}\partial_\sigma\left(\frac{1}{T}\right)u_\rho + \frac{1}{2}\partial_\rho\left(\frac{1}{T}\right)u_\sigma + \frac{1}{2T}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \frac{1}{T}\sigma_{\rho\sigma} + \frac{1}{3T}\theta\Delta_{\rho\sigma}$$

$$\sigma_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\theta$$

A is the acceleration field

All terms are relativistic (they vanish in the infinite *c* limit) EXCEPT grad T terms, which give rise to:

$$\mathbf{S}_\xi = \frac{1}{8}\mathbf{v}\times\frac{\int \mathrm{d}^3\mathrm{x}\; n_F(1-n_F)\nabla\left(\frac{1}{T}\right)}{\int \mathrm{d}^3\mathrm{x}\; n_F}$$

There is an equal contribution in the NR limit from thermal vorticity

Isothermal local equilibrium

The most appropriate setting for relativistic heavy ion collisions at very high energy!

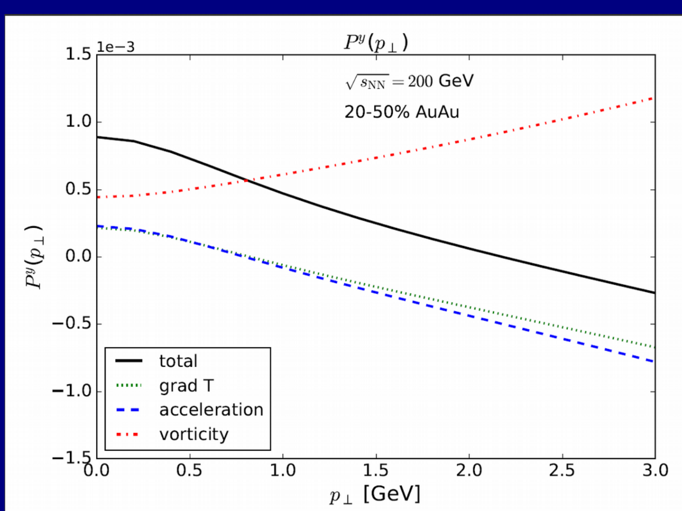
Both thermal shear and thermal vorticity include temperature gradients

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$$

$$\beta^\mu = (1/T)u^\mu$$

Thermal gradients do contribute to the polarization



$$\mathbf{S}^* \propto \frac{\hbar}{KT^2}\mathbf{u} \times \nabla T + \frac{\hbar}{KT}(\boldsymbol{\omega} - \boldsymbol{\omega} \cdot \mathbf{v}\mathbf{u}/c^2) + \frac{\hbar}{KT}\mathbf{A} \times \mathbf{u}/c^2$$

I. Karpenko, Nucl. Phys. A 982 (2019), 519-522, talk given at QM 2018

Is it the best thing to do?

The formulae of the spin vector are based on a Taylor expansion of the density operator

$$W(x,k)_{\rm LE} = \frac{1}{Z}{\rm Tr}\left(\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y)\widehat{T}^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\widehat{j}^\mu(y)\right]\widehat{W}(x,k)\right)$$

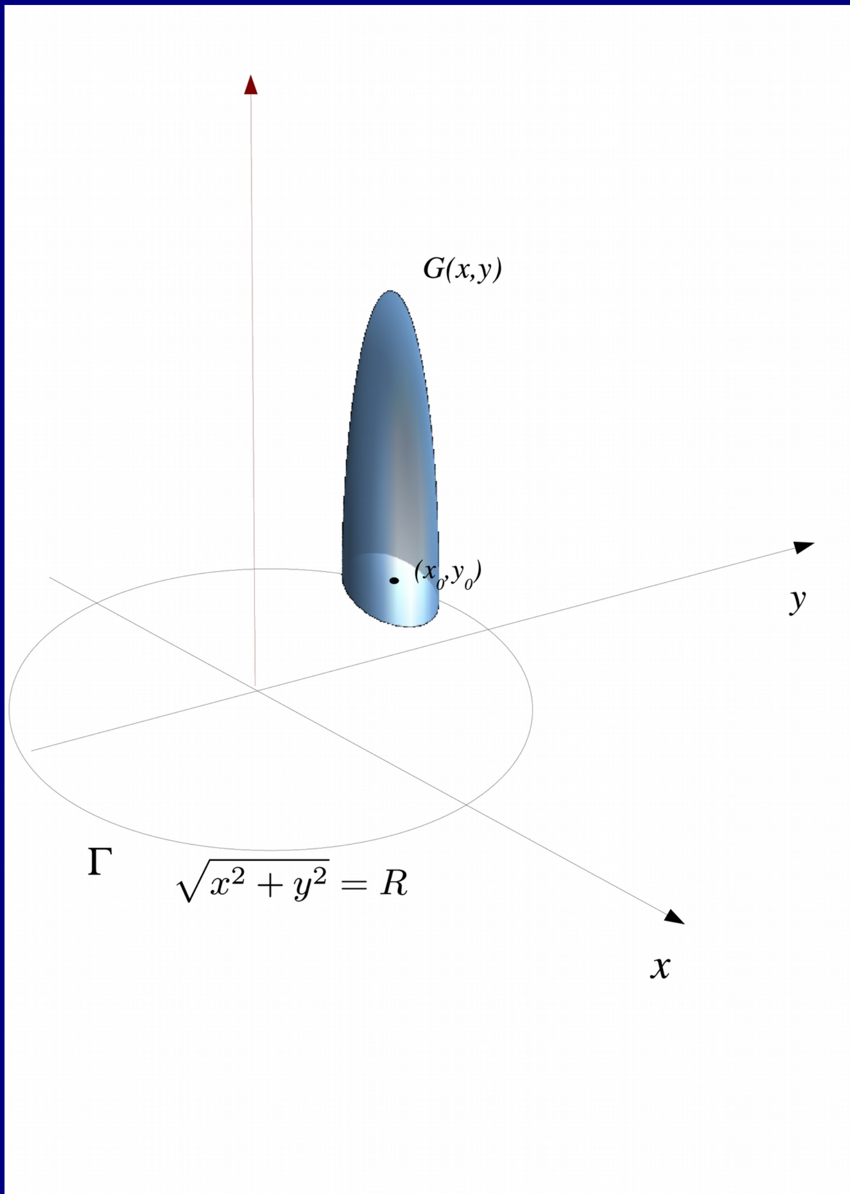
$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \ldots$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp[-\beta_\mu(x)\widehat{P}^\mu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}^{\mu\nu}_x + \ldots]$$

This is generally correct, but it is an approximation after all.

Can we find a better approximation for a special case?

Why the thermal gradients matter: a simple example



Task: approximate the integral

$$W = \int_\Gamma \mathrm{e}^{\sqrt{x^2+y^2}} G(x,y) ds$$

where *G(x,y)* is a peaked function around the point (x_0,y_0) on the circle.

Since G is peaked, one can Taylor expand the exponent about (x_0,y_0)

$$\begin{aligned} W &\simeq \mathrm{e}^{\sqrt{x_0^2+y_0^2}} \int_\Gamma \mathrm{e}^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s \\ &= \mathrm{e}^R \int_\Gamma \mathrm{e}^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s \\ &= \mathrm{e}^R \int_\Gamma \mathrm{e}^{\nabla r|_{(x_0,y_0)}\cdot(\mathbf{x}-\mathbf{x}_0)} G(x,y)\mathrm{d}s \end{aligned}$$

But it is just pointless if we integrate over the circle!

$$W = \mathrm{e}^R \int_\Gamma G(x,y)\mathrm{d}s$$

In the previous example, the Taylor expansion at first order introduces an undesired term:

$$\mathrm{W} = \mathrm{e}^{R} \int_{\Gamma} G(x,y)\mathrm{d}s \qquad\qquad W \simeq \mathrm{e}^{R} \int_{\Gamma} \mathrm{e}^{\nabla r|_{(x_0,y_0)} \cdot (\mathbf{x}-\mathbf{x}_0)} G(x,y)\mathrm{d}s$$

exact — With gradient of r expansion

which is proportional to the gradient of the constant quantity on the circle, perpendicular to the integration line. This term does not vanish in the integration!