## Polarization: questions and next steps

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## Warne state UNVERTTY

In a spirit of a "retreat" - emphasis not on the "final" results but on current open questions/analyses details
sorry for jumping from one subject to another too often...

Discussions with T. Niida and many others are gratefully acknowledged

- $\Omega$ global polarization, $\gamma_{\Omega}$
- $P_{y} \phi_{H}$ dependence, physics, acceptance effects,
average/global vs $P_{y, 0}$
- $P_{z}$ higher harmonics, hydro, BW, SIP, Cooper-Frye
- $P_{x}$ BW, SIP
- $P_{\phi}$ track reconstruction efficiency
- spin alignment:
physics questions, SA + elliptic flow + acceptance effects


## $\Xi$ and $\Omega$ global polarization

Global Polarization of $\Xi$ and $\Omega$ Hyperons in $\mathbf{A u}+$ Au Collisions at $\sqrt{s_{N N}}=\mathbf{2 0 0} \mathbf{G e V}$

$$
\mathbf{P}_{\Lambda}^{*}=\frac{\left(\alpha_{\Xi}+\mathbf{P}_{\Xi}^{*} \cdot \hat{\boldsymbol{p}}_{\Lambda}^{*}\right) \hat{\boldsymbol{p}}_{\Lambda}^{*}+\beta_{\Xi} \mathbf{P}_{\Xi}^{*} \times \hat{\boldsymbol{p}}_{\Lambda}^{*}+\gamma_{\Xi} \hat{\boldsymbol{p}}_{\Lambda}^{*} \times\left(\mathbf{P}_{\Xi}^{*} \times \hat{\boldsymbol{p}}_{\Lambda}^{*}\right)}{1+\alpha_{\Xi} \mathbf{P}_{\Xi}^{*} \cdot \hat{\boldsymbol{p}}_{\Lambda}^{*}}
$$

$$
a^{2}+\beta^{2}+\gamma^{2}=1
$$

$$
\mathbf{P}_{\Lambda}^{*}=C_{\Omega^{-} \Lambda} \mathbf{P}_{\Omega}^{*}=\frac{1}{5}\left(1+4 \gamma_{\Omega}\right) \mathbf{P}_{\Omega}^{*} .
$$

A way to measure the decay parameter $\gamma_{\Omega}$ !

## $P_{y}(\phi)$ physics

Compare with $\exp$ data: $P_{y}(\phi)$ with $\&$ without SIP


## $P_{y}(\phi)$, parametrization, acceptance effects

$$
P_{H}\left(\phi_{H}-\Psi_{\mathrm{RP}}, p_{t}^{H}, \eta^{H}\right)=P_{0}\left(p_{t}^{H}, \eta^{H}\right)+2 P_{2}\left(p_{t}^{H}, \eta^{H}\right) \cos \left\{2\left[\phi_{H}-\Psi_{\mathrm{RP}}\right]\right\}
$$

$$
\begin{aligned}
\left\langle\sin \left(\Psi_{\mathrm{RP}}-\phi^{*}\right)\right\rangle= & \int \frac{d \Omega^{*}}{4 \pi} \frac{d \phi_{H}}{2 \pi} A\left(\mathbf{p}_{H}, \mathbf{p}^{*}\right) \int_{0}^{2 \pi} \frac{d \Psi_{\mathrm{RP}}}{2 \pi}\left\{1+2 v_{2, H} \cos \left[2\left(\phi_{H}-\Psi_{\mathrm{RP}}\right)\right]\right\} \\
& \sin \left(\Psi_{\mathrm{RP}}-\phi^{*}\right)\left[1+\alpha_{H} P_{H}\left(\mathbf{p}_{H} ; \Psi_{\mathrm{RP}}\right) \sin \theta^{*} \cdot \sin \left(\phi^{*}-\Psi_{\mathrm{RP}}\right)\right]
\end{aligned}
$$

$$
\left\langle\sin \left(\Psi_{\mathrm{RP}}-\phi^{*}\right)\right\rangle=\frac{\alpha_{H} \pi}{8}\left[A_{0}\left(P_{0}+2 P_{2} v_{2}\right)-A_{2}\left(P_{2}+P_{0} v_{2}\right)\right]
$$

$$
\left\langle\sin \left(\Psi_{\mathrm{RP}}-\phi^{*}\right) \cos \left[2\left(\phi_{H}-\phi^{*}\right)\right]\right\rangle=\frac{\alpha_{H} \pi}{8}\left[A_{0}\left(P_{2}+P_{0} v_{2}\right)-\frac{1}{2} A_{2}\left(P_{0}+3 P_{2} v_{2}\right)\right]
$$

$$
A_{0}\left(p_{t}^{H}, \eta^{H}\right)=\frac{4}{\pi} \int \frac{d \Omega^{*}}{4 \pi} \frac{d \phi_{H}}{2 \pi} A\left(\mathbf{p}_{H}, \mathbf{p}^{*}\right) \sin \theta^{*}
$$

$$
A_{2}\left(p_{t}^{H}, \eta^{H}\right)=\frac{4}{\pi} \int \frac{d \Omega^{*}}{4 \pi} \frac{d \phi_{H}}{2 \pi} A\left(\mathbf{p}_{H}, \mathbf{p}^{*}\right) \sin \theta^{*} \cos \left[2\left(\phi_{H}-\phi^{*}\right)\right]
$$

## A better way?

Idea: calculate $\left\langle\cos \left(\Theta^{*}\right)\right\rangle$, where $\Theta^{*}$ is the angle relative to the quantization axis

$$
P_{H}\left(\phi_{H}-\Psi_{\mathrm{RP}}, p_{t}^{H}, \eta^{H}\right)=P_{0}\left(p_{t}^{H}, \eta^{H}\right)+2 P_{2}\left(p_{t}^{H}, \eta^{H}\right) \cos \left\{2\left[\phi_{H}-\Psi_{\mathrm{RP}}\right]\right\}
$$

$$
\left\langle\sin \left(\Psi_{\mathrm{RP}}-\phi^{*}\right) \sin \theta^{*}\right\rangle=\frac{\alpha_{H}}{3}\left[\tilde{A}_{0}\left(P_{0}+2 P_{2} v_{2}\right)-\tilde{A}_{2}\left(P_{2}+P_{0} v_{2}\right)\right]
$$

$$
\left\langle\sin \left(\Psi_{\mathrm{RP}}-\phi^{*}\right) \sin \theta^{*} \cos \left[2\left(\phi_{H}-\phi^{*}\right)\right]\right\rangle=\frac{\alpha_{H}}{3}\left[\tilde{A}_{0}\left(P_{2}+P_{0} v_{2}\right)-\frac{1}{2} \tilde{A}_{2}\left(P_{0}+3 P_{2} v_{2}\right)\right]
$$

$$
\begin{aligned}
\tilde{A}_{0}\left(p_{t}^{H}, \eta^{H}\right) & =\frac{3}{2} \int \frac{d \Omega^{*}}{4 \pi} \frac{d \phi_{H}}{2 \pi} A\left(\mathbf{p}_{H}, \mathbf{p}^{*}\right) \sin ^{2} \theta^{*} . \\
\tilde{A}_{2}\left(p_{t}^{H}, \eta^{H}\right) & =\frac{3}{2} \int \frac{d \Omega^{*}}{4 \pi} \frac{d \phi_{H}}{2 \pi} A\left(\mathbf{p}_{H}, \mathbf{p}^{*}\right) \sin ^{2} \theta^{*} \cos \left[2\left(\phi_{H}-\phi^{*}\right)\right] .
\end{aligned}
$$

## $P_{y}(\phi)$ physics

Compare with $\exp$ data: $P_{y}(\phi)$ with $\&$ without SIP


Global polarization - $P_{y}$

$$
A u+A u, b=7 \mathrm{fm} \quad \text { "np" - \# of nucleon participants }
$$


$y_{z}^{C M}=\ln (n p A / n p B) / 2$

$v_{z}$ is calculated as velocity of the center of mass

$d v_{z}^{*} / d x$




## $P_{y}(\phi)$ physics

Compare with $\exp$ data: $P_{y}(\phi)$ with \& without SIP


## $\mathrm{dv}_{z}^{*} / \mathrm{dx}(\mathrm{npA}+\mathrm{npB})$


$P_{y}(\phi)$ important for:

- check old preliminary
- SIP
- "Blast wave" - see dist. above

It is not clear why hydro without SIP predicts larger polarization "out-of-plane" - which is at odds with expectation from the right plot
$P_{y, 0}$ and $P_{y, c 2}$


$$
P_{z, s n}=\left\langle P_{z} \sin \left[n\left(\phi-\Psi_{n}\right)\right]\right\rangle
$$



## Measurements and models

Polarization of $\Lambda$ and $\bar{\Lambda}$ Hyperons along the Beam Direction in $\mathrm{Pb}-\mathrm{Pb}$ Collisions at $\sqrt{s}_{N N}=5.02 \mathrm{TeV}$



## Relative contributions to polarization

fluid rest frame $u^{\mu}=(1,0,0,0)$
$\omega^{\mu}=(0, \boldsymbol{\omega}):$

$$
\begin{aligned}
& S^{0}(x, p)=\frac{1}{8 m}\left(1-n_{F}\right) \frac{\boldsymbol{\omega} \cdot \mathbf{p}}{T} \\
& \mathbf{S}(x, p)=\frac{1}{8 m}\left(1-n_{F}\right)\left(-\frac{\mathbf{p} \times \boldsymbol{\nabla} T}{T^{2}}+2 \frac{E \boldsymbol{\omega}}{T}+\frac{\mathbf{p} \times \mathbf{A}}{T}\right)
\end{aligned}
$$

$$
\mathbf{S}^{*}=\mathbf{S}-\frac{\mathbf{p} \cdot \mathbf{S}}{E(E+m)} \mathbf{p}
$$

Contributions due to $\nabla T$ and $A$ should be small in nonrelativistic limit!

$$
\begin{gathered}
S_{i}^{(\text {vort })} \approx \frac{E}{8 m T} \epsilon_{i k j} \frac{1}{2}\left(\partial_{k} v_{j}-\partial_{j} v_{k}\right) \\
S_{i}^{(\text {shear })} \approx \frac{1}{4 m T E} \epsilon_{i k j} p_{k} p_{m} \frac{1}{2}\left(\partial_{j} v_{m}+\partial_{m} v_{j}\right)
\end{gathered}
$$

## Similarly for SIP

Contribution from $d v_{z} / d x$ :
$S_{x} \propto p_{x} p_{y} \propto \sin (2 \phi)$
$S_{y} \propto p_{z}^{2}-p_{x}^{2} \propto \sim 1+\cos (2 \phi)$
$S_{z} \propto p_{y} p_{z} \propto \sin (2 \theta) \sin (\phi)$

Momentum in the rest frame of the fluid - averaging over the production volume should further suppress such contributions.

## Vorticity and Polarization in Heavy Ion Collisions: Hydrodynamic Models

## How is it consistent with equation in the previous slide?

Iurii Karpenko



Fig. 26 Contributions to the global (left panel) and quadrupole longitudinal (right panel) components of $\Lambda$ polarization stemming from gradients of temperature (dotted lines), acceleration (dashed lines) and vorticity (dash-dotted lines). Solid lines show the sums of all 3 contributions. The hydrodynamic calculation with vHLLE is performed with an averaged Monte Carlo Glauber IS corresponding to 20-50\% central Au-Au collisions at $\sqrt{S_{\mathrm{NN}}}=200 \mathrm{GeV}$ RHIC energy.

## $\left\langle P_{z} \sin \left[2\left(\phi_{H}-\Psi_{n}\right)\right]\right\rangle$



Using average over Ru+Ru and $\mathrm{Zr}+\mathrm{Zr}$
Assuming the same polarization for $\Lambda$ and $\bar{\Lambda}$

## $\left\langle P_{z} \sin \left[n\left(\phi_{H}-\Psi_{n}\right)\right]\right\rangle$




## Using average over Ru+Ru and $\mathrm{Zr}+\mathrm{Zr}$ <br> Assuming the same polarization for $\Lambda$ and $\bar{\Lambda}$

## $\left\langle P_{z} \sin \left[4\left(\phi_{H}-\Psi_{n}\right)\right]\right\rangle$



Centrality (\%)

## $P_{x}:$ SIP vs vorticity

SIP:

$$
S_{i}^{(\xi)} \approx \frac{1}{4 T} \frac{1}{m E} \epsilon_{i k j} p_{k} p_{m} \frac{1}{2}\left(\partial_{j} u_{m}+\partial_{m} u_{j}\right) \quad \propto \sin \left[2\left(\phi_{h}^{*}-\Psi_{2}\right)\right]
$$

$$
\begin{aligned}
& u_{i} \text { - fluid velocity } \\
& \text { Star denotes the value in the rest frame of fluid element }
\end{aligned}
$$

Vorticity


$$
\begin{aligned}
& S_{i}^{(\omega)} \approx \frac{1}{8 T} \epsilon_{i k j} \frac{1}{2}\left(\partial_{k} u_{j}-\partial_{j} u_{k}\right) \\
& \quad \propto \sin \left[2\left(\phi_{h}-\Psi_{2}\right)\right]
\end{aligned}
$$

Wiil be difficult to separate the two contributions

## The Cooper-Frye prescription

| PH YSICAL REVIEW D | VOLUME 10, NUMBER 1 |
| :---: | :---: |
| Single-particle distribution in the hydrodynamic and statistical thermodynamic models |  |
| of multiparticle production |  |
| Fred Cooper* and Graham Frye |  |
| Belfer Graduate School of Science, Yeshiva University, New York, New York 10033 |  |

In both models, one assumes that the collision process yields a distribution of collective motions. In Hagedorn's approach these collective motions are called fireballs; in Landau's approach the collective motions are that of the hadronic fluid

Milekhin's ${ }^{6}$ version of Landau's model, in which $d N / d^{3} v$ is proportional to the distribution of entropy in the fluid. In a notation explained below [see Eq. (18)], Milekhin's expression is

$$
\begin{equation*}
\frac{d N}{d^{3} v}=\bar{n}(\overrightarrow{\mathrm{v}}) u^{\mu} \frac{\partial \sigma_{\mu}}{\partial^{3} v} \tag{4}
\end{equation*}
$$

Equations (1) and (4) can be combined to give

$$
\begin{equation*}
E \frac{d N}{d^{3} p} \stackrel{?}{=} \int_{\sigma} g(\bar{E}, \bar{T}(\overrightarrow{\mathrm{v}})) \bar{E} u^{\mu} d \sigma_{\mu} \tag{5}
\end{equation*}
$$

Equation (5) yields the correct number of particles, but it is inconsistent with energy conservation [see Eq. (20)], so we are led to consider how one determines $E d N / d^{3} p$ for the simplest system, an expanding ideal gas.
$t$ if we choose $d \sigma_{\mu}=\left(d^{3} x, \overrightarrow{0}\right)$. The invariant singleparticle distribution in momentum space, of those particles on $\sigma$, is

$$
\begin{equation*}
E \frac{d N}{d^{3} p}=\int_{\sigma} f(x, p) p^{\mu} d \sigma_{\mu} \tag{9}
\end{equation*}
$$

Equation (9) is to be compared with Eq. (5) under the assumption that the fluid is locally in thermodynamic equilibrium,

$$
\begin{equation*}
f(x, p)=g(\bar{E}(v(x)), T(x)) \tag{10}
\end{equation*}
$$

The contrast between Eqs. (5) and (9) is that $p^{\mu}$ has been replaced by $\bar{E} u^{\mu}$ in Eq. (5). To choose

Is BW "closer" to Milekhin's prescription?

## $P_{\phi}$ in asymmetric collisions



## Vorticity and particle polarization in heavy ion collisions (exper-

 imental perspective)
## Sergei A. Voloshin ${ }^{1, \star}$

Finally, we mention another very interesting possibility for vorticity studies in asymmetric nuclear collisions such as $\mathrm{Cu}+\mathrm{Au}$. For relatively central collisions, when during the collision a smaller nucleus is fully "absorbed" by the larger one (e.g. such collisions can be selected by requiring no signal in the zero degree calorimeter in the lighter nucleus beam direction), one can easily imagine a configuration with toroidal velocity field, and as a consequence, a vorticity field in the form of a circle. The direction of the polarization in such a case would be given by $\hat{\mathbf{p}_{\mathrm{T}}} \times \hat{\mathbf{z}}$, where $\hat{\mathbf{p}_{\mathrm{T}}}$ and $\hat{\mathbf{z}}$ are the unit vectors along the particle transverse momentum and the (lighter nucleus) beam direction.

One of the analyses, where the results directly depends on the correction: the effect - nonzero results can be faked by "slightly off" acceptance/efficiency correction. In that, it is very different from the global or $P_{z}$ analyses, where "wrong correction", could lead only to a relatively small difference in the magnitude of the effect.


This is one of the reasons for 3 years old Cu-Au analysis being still "in progress"

## Spin alignment and elliptic flow


angular distribution $\propto \sin ^{2} \theta$, where $\theta$ is the angle relative to the spin direction (in the resonance rest frame), and consequently $\propto \cos (2 \phi)$, where the angle $\phi$ is now the azimuthal angle with respect to the reaction plane, and thus would contribute to the elliptic flow (modulo distortions due to transformation from the resonance rest frame). Such an additional contribution could probably explain the very strong elliptic flow observed at RHIC (recall, that in transverse momentum region, $p_{t} \sim 3 \mathrm{GeV} /$ c elliptic flow at RHIC can not be explained by any model [4]).
pion elliptic flow from rho decays


Fig. 1. Azimuthal anisotropy $v_{2}$ of pions from the decay of $\rho$ vec tor mesons that have spin alignment according to Eq. (13) with $\rho_{00}^{0}=1 / 3$ (solid line), 0 (dot-dashed line) and 1 (dashed line).

$$
\frac{d N}{d \cos \theta^{*}} \propto w_{0}\left|Y_{1,0}\right|^{2}+w_{+1}\left|Y_{1,1}\right|^{2}+w_{-1}\left|Y_{1,-1}\right|^{2} \propto w_{0} \cos ^{2} \theta^{*}+\left(w_{+1}+w_{-1}\right) \sin ^{2} \theta^{*} / 2
$$

$$
\mathrm{NSM}: \rho_{00} \approx \frac{1}{3+(\omega / T)^{2}}
$$

$$
\begin{aligned}
\rho_{00}^{\rho(\mathrm{rec})} & =\frac{1-P_{q}^{2}}{3+P_{q}^{2}} \\
\rho_{00}^{V(\mathrm{frag})} & =\frac{1+\beta P_{q}^{2}}{3-\beta P_{q}^{2}}
\end{aligned}
$$

Z.-T. Liang, X.-N. Wang / Physics Letters B 629 (2005) 20-26
$\mathrm{v}_{2}$ of pions from $100 \%$ polarized rho decays is $\sim 20 \%$ !

Strangeness in Quark Matter, Utrecht University, July 10-15,2017 S.A. Volo

Evidence of Spin-Orbital Angular Momentum Interactions in Relativistic
Heavy-Ion Collisions
S. Acharya et al.*
(The ALICE Collaboration)


## Thermal model:

$$
\begin{aligned}
& \rho_{00}=0.15 \Rightarrow w\left(s_{z}=+1\right)=0.82 \\
& w(0)=0.15, w(-1)=0.03
\end{aligned}
$$

## Spin-alignment: STAR

Observation of Global Spin Alignment of $\phi$ and $K^{* 0}$ Vector Mesons in Nuclear Collisions
(STAR Collaboration)


RHIC: Mean field of $\varphi$ meson plays a role?
Does it change from RHIC to LHC?
X. Sheng, L. Oliva, and Q. Wang, PRD101.096005(2020)
X. Sheng, Q.Wang, and X. Wang, PRD102.056013 (2020)

If it is related to the vorticity, it must depend on the direction. In mean field approach (as well as any others) -
what are the predictions for $\rho_{1,1}$ and $\rho_{-1,-1}$ ?

One possibility for noticeable spin alignment might be strong, fluctuating in direction, polarization, e.g vorticity, (the mechanism discussed by B. Mueller).
This possibility might be checked with $\Lambda \Lambda$ correlations

Helicity conservation and heavy resonance decays into vector mesons?

## Spin alignment, elliptic flow, and efficiency

$$
\frac{d N}{d \cos \theta^{*}} \propto\left(1-\rho_{00}\right)+\left(3 \rho_{00}-1\right) \cos ^{2} \theta^{*}
$$



It ,ight be better to present an efficiency (1d) plot vs $\hat{n}_{p}^{*} \cdot \hat{n}_{\Lambda}$

Reconstruction efficiency changes $\sim \mathcal{O}(1)$ with the emission angle relative to the reaction plane


Opacity/width reflects efficiency and/or multiplicity

The efficiency entangles elliptic flow and polarization, neither of them can be measured independently

## Summary

$P_{z}$ measurements surprisingly (or not?) well agree with the BW expectations
It is not clear how/why $\nabla_{\mu} T$ and $A_{\mu}$ and SIP contributions appear to be large/significant A specific, better unique, predictions for SIP, SHE, etc. are needed

Is the "Cooper-Frye" prescription really good for polarization calculations?

Spin alignment: I think a thorough review and understanding of the detector effects are needed

New techniques are being developed,
Many new measurements appear and many will be available soon including higher harmonics, $P_{x}$, differential $P_{y}$

## EXTRA SLIDES

## Spin alignment in vector meson decays

Strong decays of vector mesons in to two (pseudo)scalar particles

| $K^{* 0} \rightarrow \pi+K$ |
| :---: |
| $\phi \rightarrow K^{-}+K^{+}$ |

$$
\frac{d N}{d \cos \theta^{*}} \propto\left(1-\rho_{00}\right)+\left(3 \rho_{00}-1\right) \cos ^{2} \theta^{*}
$$

$$
\rho_{00}=w_{0}-\text { probability for } s_{z}=0
$$

$$
\rho_{00}=\frac{1}{3}-\frac{4}{3}\left\langle\cos \left[2\left(\phi_{p}^{*}-\Psi_{\mathrm{RP}}\right)\right]\right\rangle
$$

$$
\frac{d N}{d \cos \theta^{*}} \propto w_{0}\left|Y_{1,0}\right|^{2}+w_{+1}\left|Y_{1,1}\right|^{2}+w_{-1}\left|Y_{1,-1}\right|^{2} \propto w_{0} \cos ^{2} \theta^{*}+\left(w_{+1}+w_{-1}\right) \sin ^{2} \theta^{*} / 2
$$

$$
V \rightarrow l^{+} l^{-}
$$

$$
W(\theta, \phi) \propto \frac{1}{3+\lambda_{\theta}}\left(1+\lambda_{\theta} \cos ^{2} \theta+\lambda_{\phi} \sin ^{2} \theta \cos 2 \phi+\lambda_{\theta \phi} \sin 2 \theta \cos \phi\right)
$$

$$
\begin{aligned}
& \text { Unlike } K^{0^{*}} \rightarrow K \pi \\
& \text { and } \phi \rightarrow K^{+} K^{-} \text {, the daughters } \\
& \text { in } J / \psi \rightarrow l^{+} l^{-} \text {have spin } 1 / 2 \\
& \hline
\end{aligned}
$$

