Local polarization of Lambda hyperons across RHIC-BES energies:

role of initial condition, baryon chemical potential and other possible corrections

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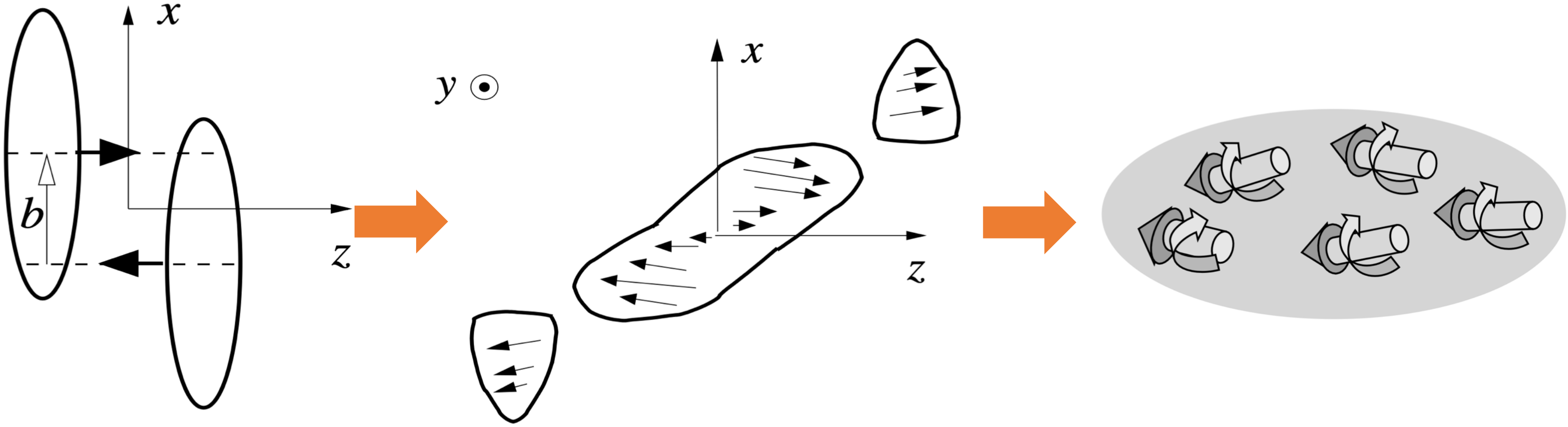
Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions,
UCLA + Online, Dec. 2 – 4, 2022

Outline

- **Introduction on polarization in heavy ion collisions**
- **Global and Local polarization across RHIC-BES energies:**
  Role of initial conditions and baryon chemical potential
- **Helicity polarization across RHIC-BES energies:**
  Probing the thermal or kinetic vorticity from data
- **Corrections from effective interactions to local polarization**
- **Summary and discussion**

Introduction

Global orbital angular momentum in HIC



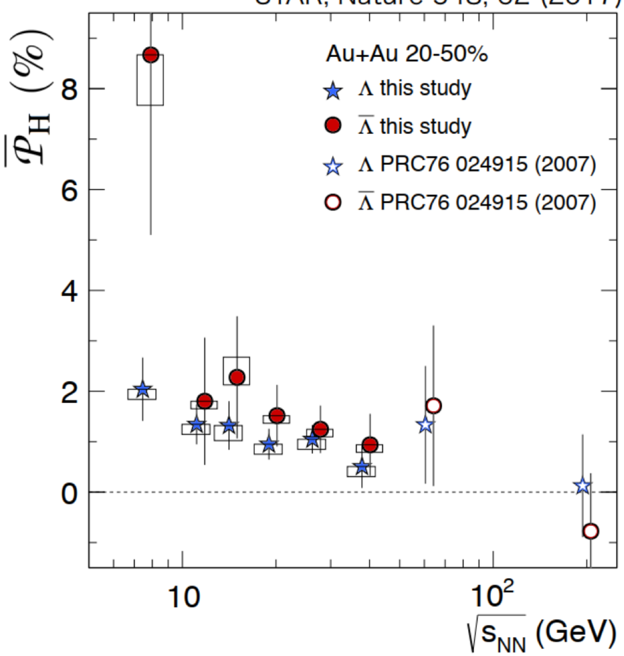
- **Huge global orbital angular momenta are produced in HIC.**
- **Global orbital angular momentum leads to the polarizations of Λ hyperons and vector mesons through spin-orbital coupling.**

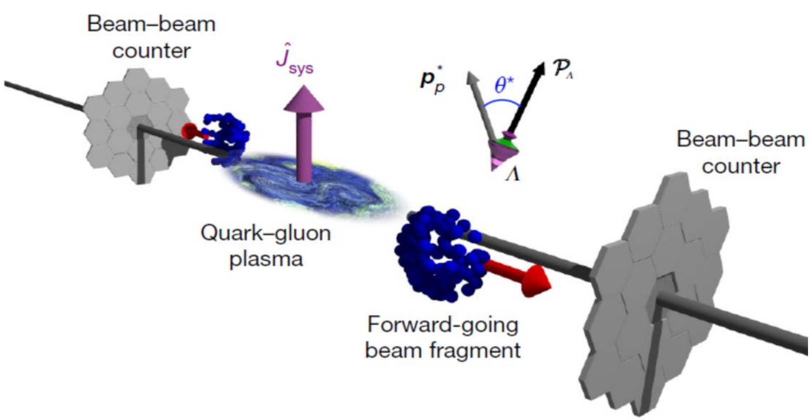
Liang, Wang, PRL (2005); PLB (2005)

Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Global Polarization of Λ and $\overline{\Lambda}$

STAR, Nature 548, 62 (2017)



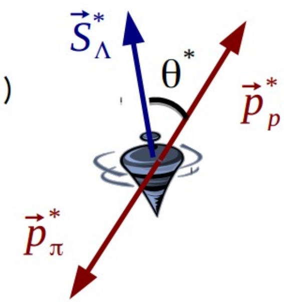


parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}_{\mathbf{\Lambda}} \cdot \mathbf{p}_{\mathbf{p}}^*)$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



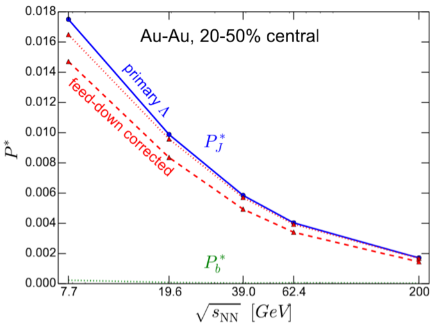
$$\Lambda \rightarrow p + \pi^+$$

(BR: 63.9%, cτ~7.9 cm)

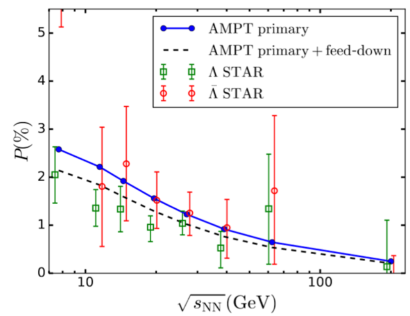
- **ω = (9 ± 1)x10^{21}/s, greater than previously observed in any system.**
- **QGP is most vortical fluid so far.**

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)
…

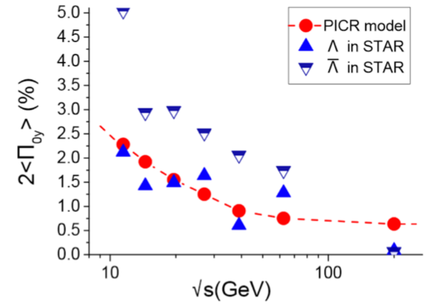
Global Polarization from different models



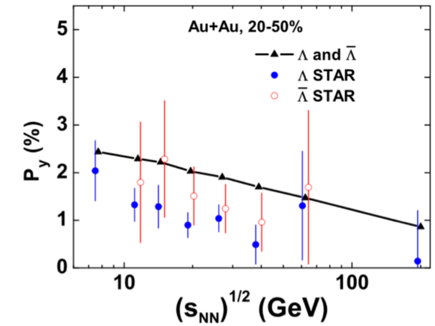
Karpenko, Becattini, EPJC(2017)



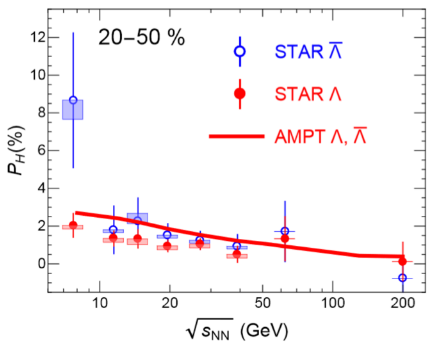
Li, Pang, Wang, Xia PRC(2017)



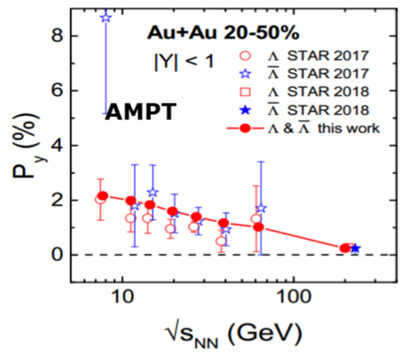
Xie, Wang, Csernai, PRC(2017)



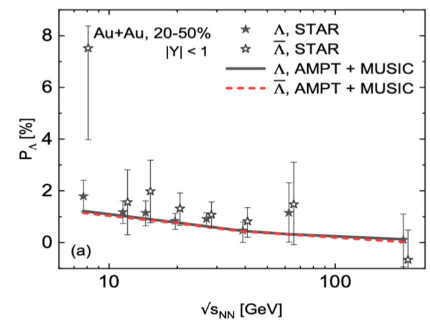
Sun, Ko, PRC(2017)



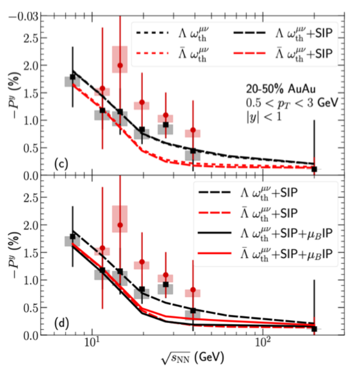
Shi, Li, Liao, PLB(2018)



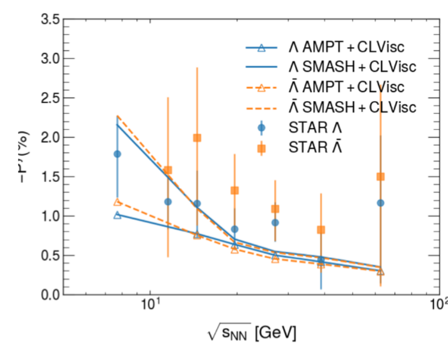
Wei, Deng, Huang, PRC(2019)



Fu, Xu, Huang, Song, PRC (2021)



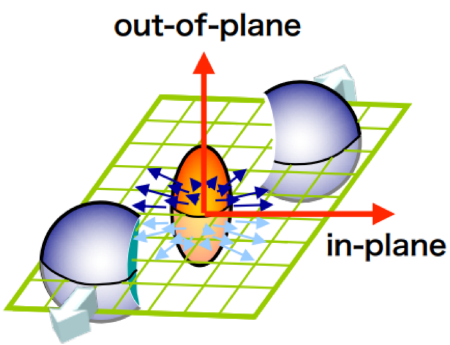
S. Ryu, V. Jupic, C. Shen, PRC (2021)

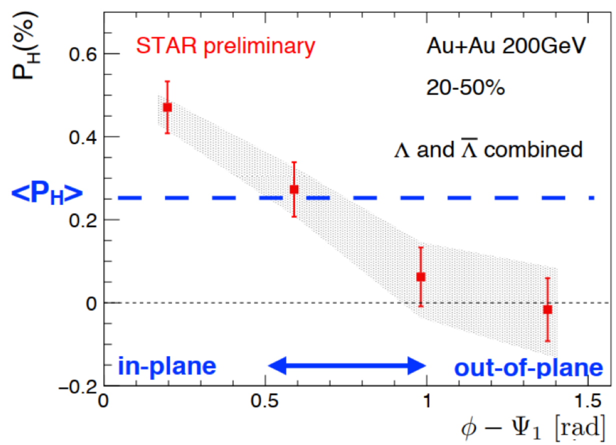


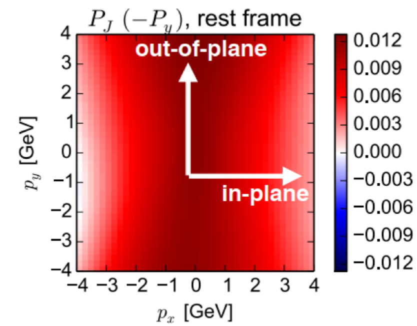
Y.X. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

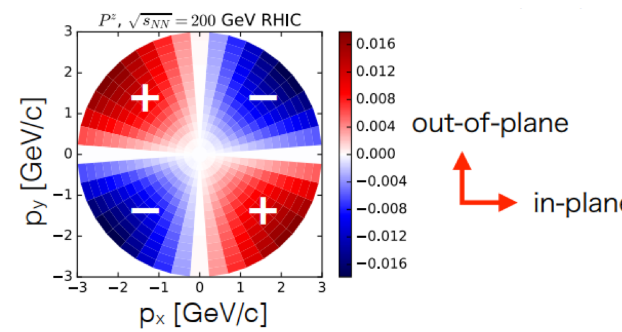
Sign problem in local polarization

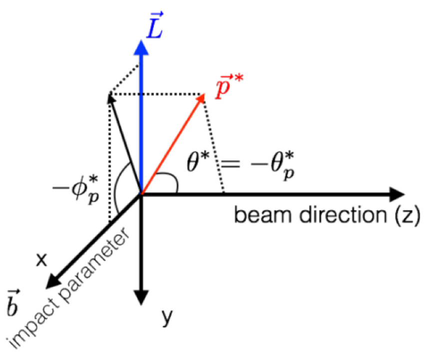
Local polarization cannot be described by the contributions from thermal vorticity only.

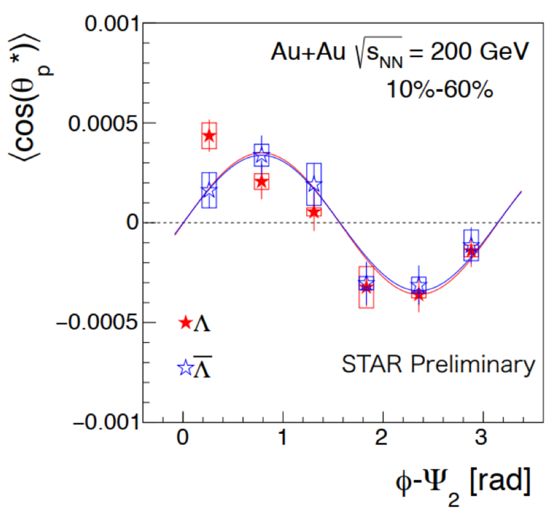




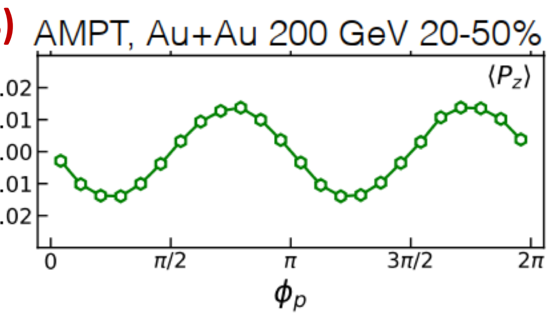








UrQMD : Becattini, Karpenko, PRL (2018)



AMPT: Xia, Li, Tang, Wang, PRC (2018)

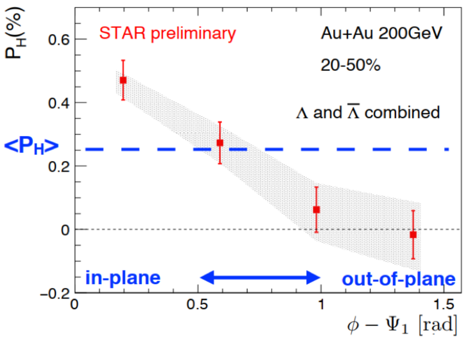
Different approaches

- **Spin hydrodynamics (macroscopic approach)**
  - Florkowski, Friman, Jaiswal, Ryblewski, Speranza (2017-2018);
  - Montenegro, Tinti, Torrieri (2017-2019);
  - Hattori, Hongo, Huang, Matsuo, Taya PLB(2019) ; arXiv: 2201.12390; arXiv: 2205.08051
  - Fukushima, SP, Lecture Note (2020); PLB(2021); Wang, Fang, SP, PRD(2021); Wang, Xie, Fang, SP, PRD (2022)
  - S.Y. Li, M.A Stephanov, H.U Yee, arXiv:2011.12318
  - D. She, A. Huang, D.F. Hou, J.F Liao, arXiv: 2105.04060
- **Quantum kinetic theory with collisions (microscopic approach)**
  - Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD 100, 056018 (2019)
  - Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.
  - Liu, Mameda, Huang, arXiv:2002.03753.
  - Gao, Liang, PRD 2019
  - Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019) ; Z.Y. Wang, arXiv:2205.09334;
  - Li ,Yee, PRD100, 056022 (2019)
  - Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184; Lin, Wang, arXiv:2206.12573
  - Fang, SP, Yang, PRD (2022)
- **Other approaches:**
  - Side-jump effect Liu, Sun, Ko PRL(2020)
  - Mesonic mean-field Csernai, Kapusta, Welle, PRC(2019)
  - Using different vorticity Wu, Pang, Huang, Wang, PRR (2019)
- **Recent reviews:**
  - Gao, Ma, SP, Wang, NST (2020)
  - Gao, Liang, Wang, IJMPA (2021)
  - Hidaka, SP, Yang, Wang, PPNP (2022)

Global and Local polarization across RHIC-BES energies:

Role of initial conditions and baryon chemical potential

Modified Cooper-Frye formula



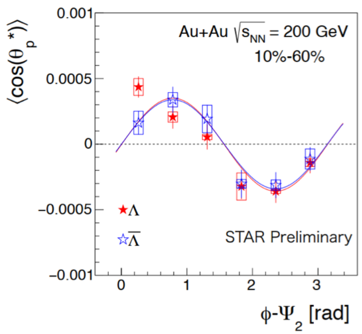
Thermal vorticity $\omega^{\mathrm{th}}_{\rho\sigma} = \frac{1}{2}\left[\partial_\rho\left(\frac{u_\sigma}{T}\right) - \partial_\sigma\left(\frac{u_\rho}{T}\right)\right]$

Distribution function: f_0

$$\mathcal{S}^\mu(p) = \frac{1}{8m_\Lambda}\epsilon^{\mu\nu\rho\sigma}p_\nu\frac{\int d\Sigma_\lambda p^\lambda f_0(1-f_0)\omega^{\mathrm{th}}_{\rho\sigma}}{\int d\Sigma_\lambda p^\lambda f_0}$$

Polarization pseudo vector

Freezeout surface



Karpenko, F. Becattini, EPJC 77 (2017) 213
R.-H. Fang, L.-G. Pang, Q. Wang, X.-N. Wang, PRC94, 024904 (2016)

Polarization and axial current

- **The polarization tensor is connected to the axial current in phase space J_5 by modified Cooper-Frye formula**
  Karpenko, Becattini, EPJC. (2017); Fang, Pang, QW, Wang, PRC (2016)

$$\mathcal{S}^{\mu}(\mathbf{p})=\frac{\int d\Sigma\cdot p\mathcal{J}_5^{\mu}(p,X)}{2m_{\Lambda}\int d\Sigma\cdot\mathcal{N}(p,X)},$$

- **From quantum kinetic theory, we can derive the J_5 in phase space,**

$$\begin{aligned}\mathcal{J}_{\lambda}^{\mu}(p,X) &= 2\pi\mathrm{sign}(u\cdot p)\left\{p^{\mu}+\lambda\frac{\hbar}{2}\delta(p^2)\left[u^{\mu}(p\cdot\omega)-\omega^{\mu}(u\cdot p)\right.\right.\\ &\left.\left.-2S_{(u)}^{\mu\nu}\tilde{E}_{\nu}\right]\partial_{u\cdot p}+\lambda\frac{\hbar}{4}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}\partial_{\nu}^{p}\delta(p^2)\right\}f_{\lambda}^{(0)},\end{aligned}$$

$\lambda=\pm$

+: right

-: left

$$S_{(u)}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}/(2u\cdot p),$$

$$\tilde{E}_{\nu} = E_{\nu}+T\partial_{\nu}\frac{\mu_{\lambda}}{T}+\frac{(u\cdot p)}{T}\partial_{\nu}T-p^{\sigma}[\partial_{<\sigma}u_{\nu>}+\frac{1}{3}\Delta_{\sigma\nu}(\partial\cdot u)+u_{\nu}Du_{\sigma}].$$

$$f_{\lambda}^{(0)}=1/(e^{(u\cdot p-\mu_{\lambda})/T}+1),$$

Y. Hidaka, SP, and D.L. Yang, Phys. Rev. D97, 016004 (2018)

Polarization induced by different sources

- **Axial currents can be decomposed as**

$$\mathcal{J}_5^\mu = \mathcal{J}^\mu_{\text{thermal}} + \mathcal{J}^\mu_{\text{shear}} + \mathcal{J}^\mu_{\text{accT}} + \mathcal{J}^\mu_{\text{chemical}} + \mathcal{J}^\mu_{\text{EB}},$$

where they are related to:

Thermal vorticity

$$\mathcal{J}^\mu_{\text{thermal}} = a\frac{1}{2}\boxed{\epsilon^{\mu\nu\alpha\beta}p_\nu\partial_\alpha\frac{u_\beta}{T}},$$

Shear viscous tensor

$$\mathcal{J}^\mu_{\text{shear}} = -a\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta p^\sigma \boxed{\partial_{<\sigma}u_{\nu>}}$$

Fluid acceleration

$$\mathcal{J}^\mu_{\text{accT}} = -a\frac{1}{2T}\epsilon^{\mu\nu\alpha\beta}p_\nu u_\alpha \boxed{\left(Du_\beta - \frac{1}{T}\partial_\beta T\right)}.$$

Gradient of chemical potential

$$\mathcal{J}^\mu_{\text{chemical}} = a\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta \boxed{\partial_\nu\frac{\mu}{T}},$$

Electromagnetic fields

$$\mathcal{J}^\mu_{\text{EB}} = a\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta \boxed{E_\nu + a\frac{B^\mu}{T}},$$

Y. Hidaka, SP, D.L. Yang, PRD (2018); C. Yi, SP, D.L. Yang, PRC (2021).

Out-of-equilibrium corrections

- **Polarization induced by thermal vorticity, shear viscous tensor and residual part of fluid acceleration at high energy collisions,**

$$\mathcal{S}^{\mu}_{\text{thermal}}(\mathbf{p}) = \frac{\hbar}{8m_{\Lambda}N}\int d\Sigma^{\sigma}p_{\sigma}f_{V}^{(0)}(1-f_{V}^{(0)})\epsilon^{\mu\nu\alpha\beta}p_{\nu}\partial_{\alpha}\frac{u_{\beta}}{T},$$

$$\mathcal{S}^{\mu}_{\text{shear}}(\mathbf{p}) = -\frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_{V}^{(0)}(1-f_{V}^{(0)})\frac{\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}}{(u\cdot p)T}\frac{1}{2}\{p^{\sigma}(\partial_{\sigma}u_{\nu}+\partial_{\nu}u_{\sigma})-Du_{\nu}\}$$

$$\mathcal{S}^{\mu}_{\text{accT}}(\mathbf{p}) = -\frac{\hbar}{8m_{\Lambda}N}\int d\Sigma\cdot pf_{V}^{(0)}(1-f_{V}^{(0)})\frac{1}{T}\epsilon^{\mu\nu\alpha\beta}p_{\nu}u_{\alpha}(Du_{\beta}-\frac{1}{T}\partial_{\beta}T),$$

C. Yi, SP, D.L. Yang, PRC 2021

- **Polarization pseudo vector in the rest frame of Λ can be obtained by Lorentz transformation**

$$\vec{P}^{*}(\mathbf{p}) = \vec{P}(\mathbf{p}) - \frac{\vec{P}(\mathbf{p})\cdot\vec{p}}{p^{0}(p^{0}+m)}\vec{p}, \qquad P^{\mu}(\mathbf{p}) \equiv \frac{1}{s}\mathcal{S}^{\mu}(\mathbf{p}),$$

- **Local polarization is given by the averaging over momentum and rapidity**

$$\langle\vec{P}(\phi_p)\rangle = \frac{\int_{y_{\min}}^{y_{\max}}dy\int_{p_{T\min}}^{p_{T\max}}p_T dp_T[\Phi(\mathbf{p})\vec{P}^{*}(\mathbf{p})]}{\int_{y_{\min}}^{y_{\max}}dy\int_{p_{T\min}}^{p_{T\max}}p_T dp_T\Phi(\mathbf{p})}, \qquad \Phi(\mathbf{p}) = \int d\Sigma^{\mu}p_{\mu}f_{eq}.$$

Shear induced polarization

- Shear induced polarization draws some attentions.
- Shear induced Polarization from massless fermions (Theory):

  Y. Hidaka, SP, and D.L. Yang, PRD97, 016004 (2018);
- Shear induced Polarization from massive fermions:
  - Theory:

    S. Y. F. Liu, Y. Yin, 2103.09200

    F. Becattini, M. Buzzegoli, A. Palermo, 2103.10917
  - Hydrodynamic simulations:

    B. Fu, S. Y. F. Liu, L. Pang, H. Song, Y. Yin, 2103.10403

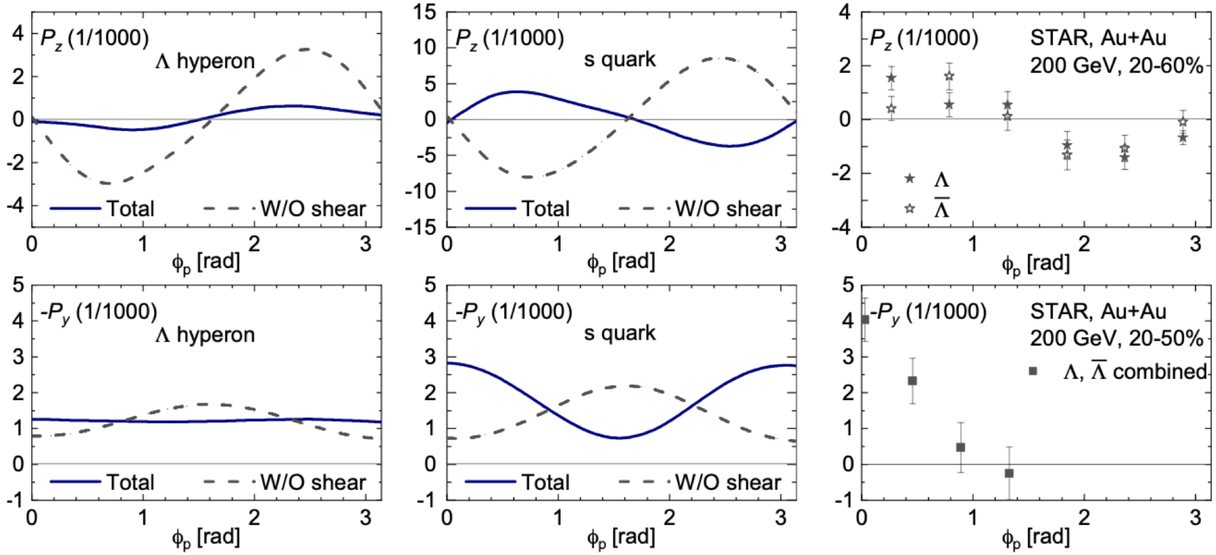
    F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, I. Karpenko, 2103.14621

    C. Yi, SP, D.L. Yang, PRC 2021

    …
- Global polarization induced by shear and gradient of chemical potential

  S. Ryu, V. Jupic, C. Shen, PRC 2021

s quark polarization instead of Λ polarization



Fu, Liu, Pang, Song, Yin, PRL 2021

$$\mathcal{S}^{\mu}_{\rm thermal}(\mathbf{p}) = \frac{\hbar}{8\boxed{m_\Lambda}N}\int d\Sigma^\sigma p_\sigma f_V^{(0)}(1-f_V^{(0)})\epsilon^{\mu\nu\alpha\beta}p_\nu\partial_\alpha\frac{u_\beta}{T},$$

$$\mathcal{S}^{\mu}_{\rm shear}(\mathbf{p}) = -\frac{\hbar}{4\boxed{m_\Lambda}N}\int d\Sigma\cdot p f_V^{(0)}(1-f_V^{(0)})\frac{\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta}{\boxed{(u\cdot p)}T}\frac{1}{2}\{p^\sigma(\partial_\sigma u_\nu+\partial_\nu u_\sigma)-Du_\nu\}$$

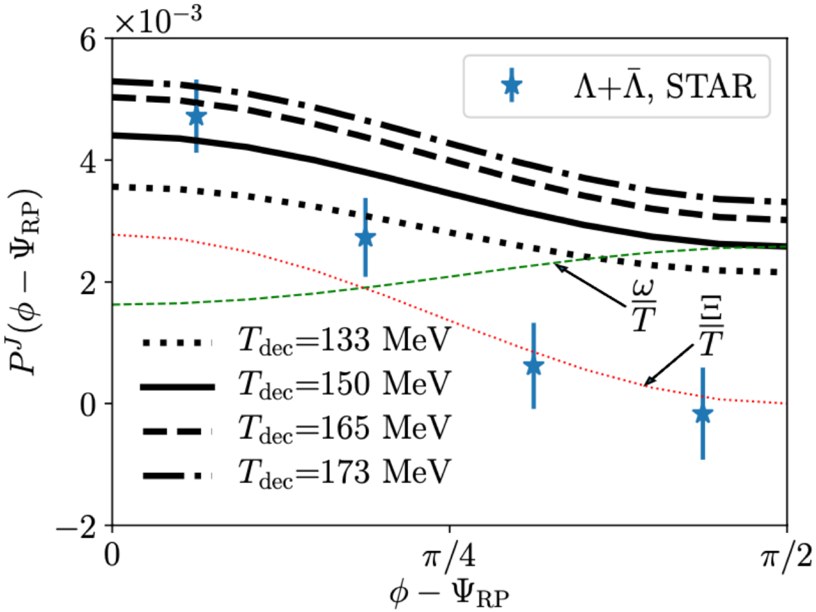
$$(u\cdot p)\sim m$$

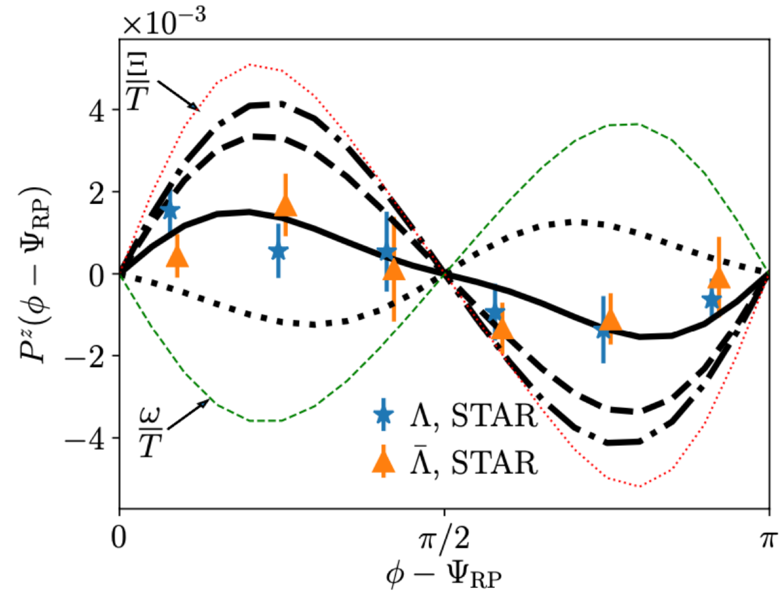
$$m_\Lambda \to m_s \qquad m_s \simeq 0.3\text{GeV}$$

$$m_\Lambda \simeq 1.116\text{GeV}$$

Replacing the mass of Lambda by the mass of s quark, the SIP and total local polarization are enhanced.

Isothermal local equilibrium





$$S^{\mu}_{\mathrm{ILE}}(p) = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p \, n_F (1 - n_F) \left[\omega_{\rho\sigma} + 2\, \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma}\right]}{8 m T_{\mathrm{dec}} \int_\Sigma \mathrm{d}\Sigma \cdot p \, n_F}$$

$$\omega_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho - \partial_\rho u_\sigma\right)$$

All gradient of temperature are neglected.

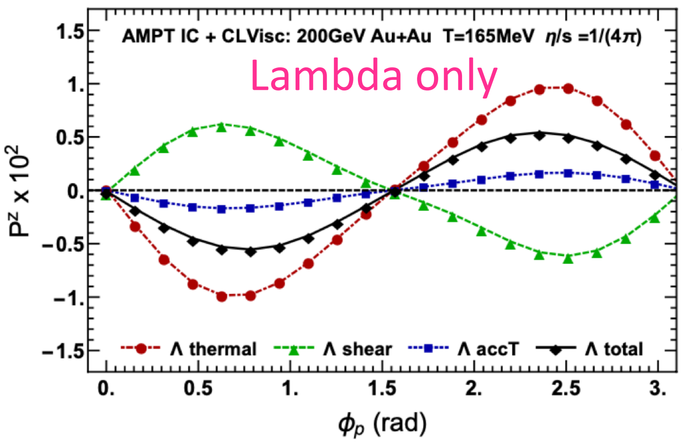
$$\Xi_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho + \partial_\rho u_\sigma\right)$$

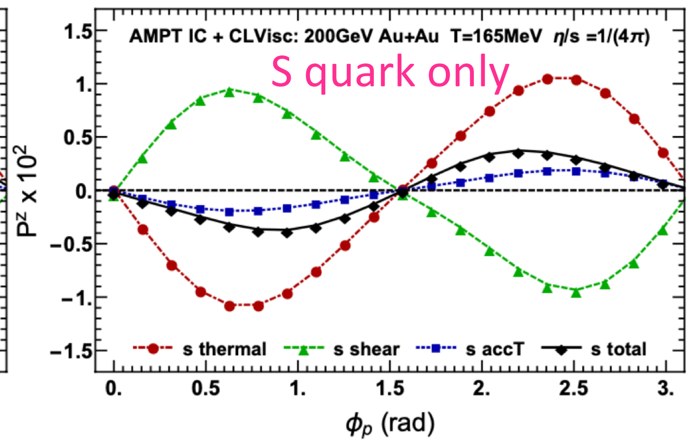
Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, PRL 2021

Local spin polarization induced by shear tensor

(3+1) dimensional viscous hydrodynamic CLVisc with AMPT initial conditions EoS "s95p-pce"

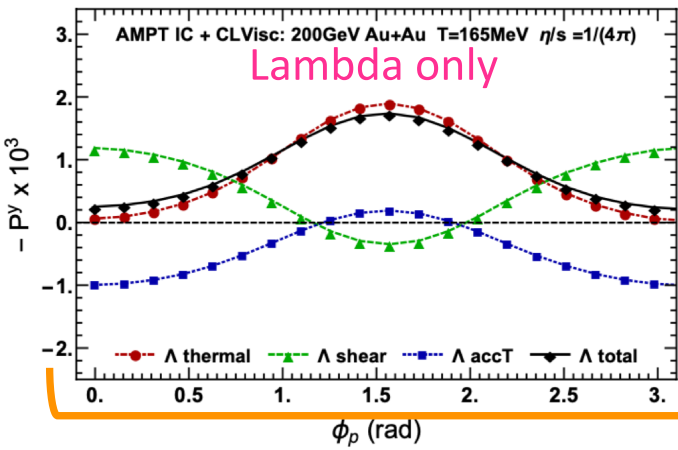
Polarization along beam direction

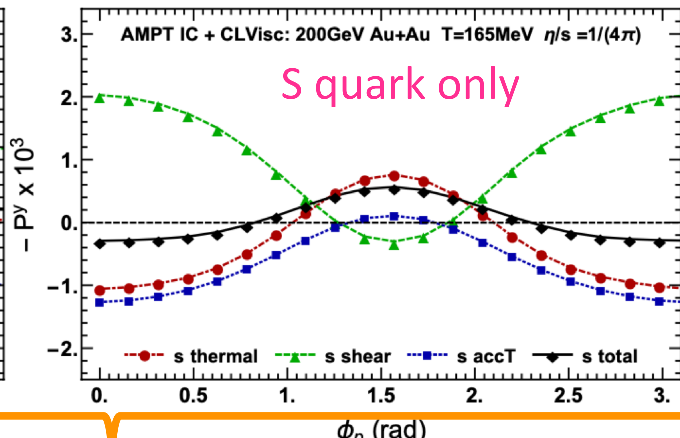




Green lines: Contributions from shear tensor

Black lines: Total local polarization





Polarization along out-of-plane direction

Yi, Pu, Yang, PRC (2021)

Main result for shear induced polarization

We found that

- **Shear induced polarization always give a “correct” sign at 200 GeV.**
- **Total local polarization is sensitive to mass of s quark, EoS, freeze out temperature and eta / s.**

- **The local spin polarization is still an open question. We still need to consider the out-of-equilibrium effects carefully through the spin hydrodynamics and the quantum kinetic theory with collisions.**

Yi, Pu, Yang, PRC (2021)

Polarization induced by gradient of chemical potential

- **"Spin Hall effect" (SHE): polarization induced by the gradient of baryon chemical potential**

$$\mathcal{S}^{\mu}_{\text{chemical}}(\mathbf{p}) = 2\int d\Sigma^{\sigma} F_{\sigma} \frac{1}{(u\cdot p)} \epsilon^{\mu\nu\alpha\beta} p_{\alpha} u_{\beta} \partial_{\nu} \frac{\mu}{T},$$

Also see B. Fu, L. Pang, H. Song, Y. Yin, arXiv:2201.12970.

- **In high energy collisions, the baryon chemical potential is almost vanishing, i.e. SHE is negligible. In low energy collisions, SHE may play a role to the polarization.**

(3+1)-dimensional CLVisc hydrodynamics model

- We study the polarization at RHIC-BES energies via the (3+1)-dimensional CLVisc hydrodynamics model. X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

- Program: Wu, Qin, Pang, Wang, PRC (2022).
- EoS: NEOS-BQS, Monnai, Schenke, Shen, PRC (2019).
- Initial conditions: AMPT and SMASH initial conditions
- We consider the polarization of Lambda and anti-Lambda only + gradient of temperature.

AMPT initial condition (Patron level)

- HIJING model : initial patrons via hard semi-hard scattering and excited strings
- ZPC model : the space-time evolution via elastic scattering

SMASH initial condition (Hadron level)

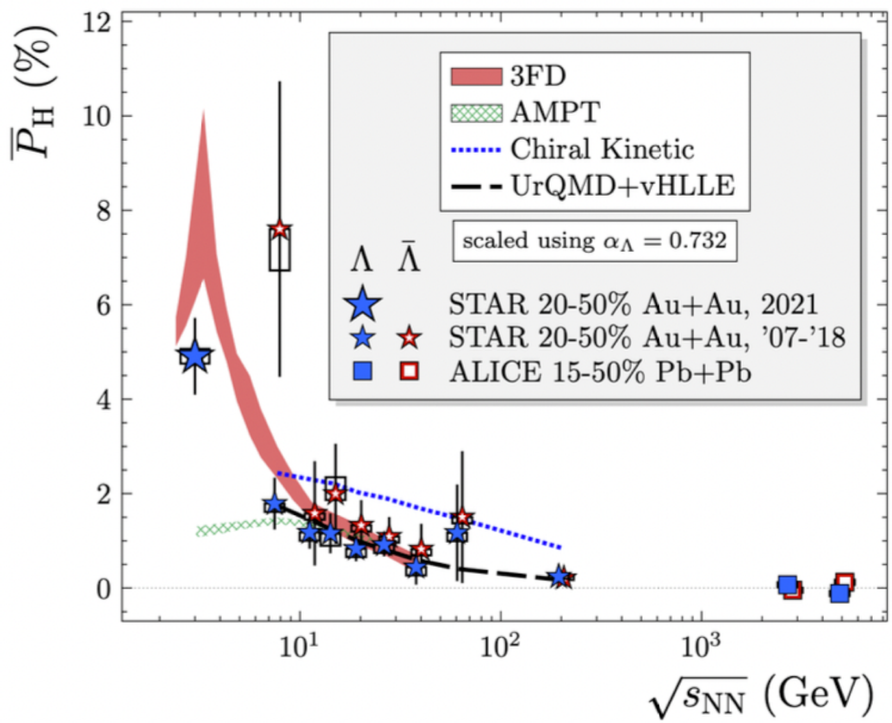
smash

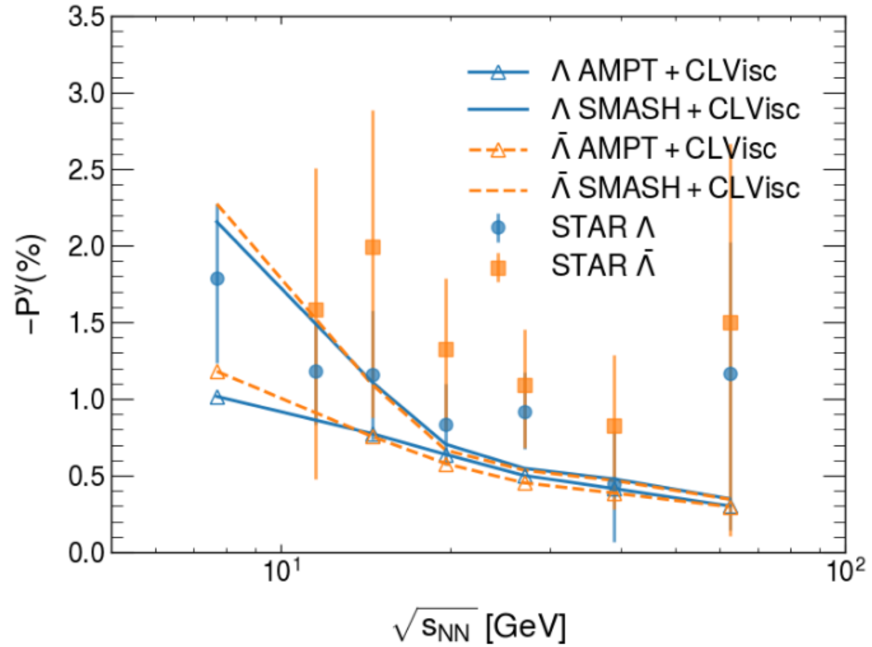
- Effective solutions of $p^\mu \partial_\mu f + m F^\mu \partial_{p_\mu}(f) = C[f]$
- f(t, **x**, **p**) denotes one-particle distribution function
- C[f] includes elastic collisions, resonance formation and decays, string fragmentation
- mesons and baryons up to mass ≈ 2.35 GeV.

At baryon rich region in BES energies, the finite size effect of the initial nucleus in the longitudinal direction may not be negligible. This finite thickness effect has been considered in the SMASH model.

Copy from Xiang-yu Wu's talk

Global polarization





STAR, Phys. Rev. C 104, L061901 (2021)

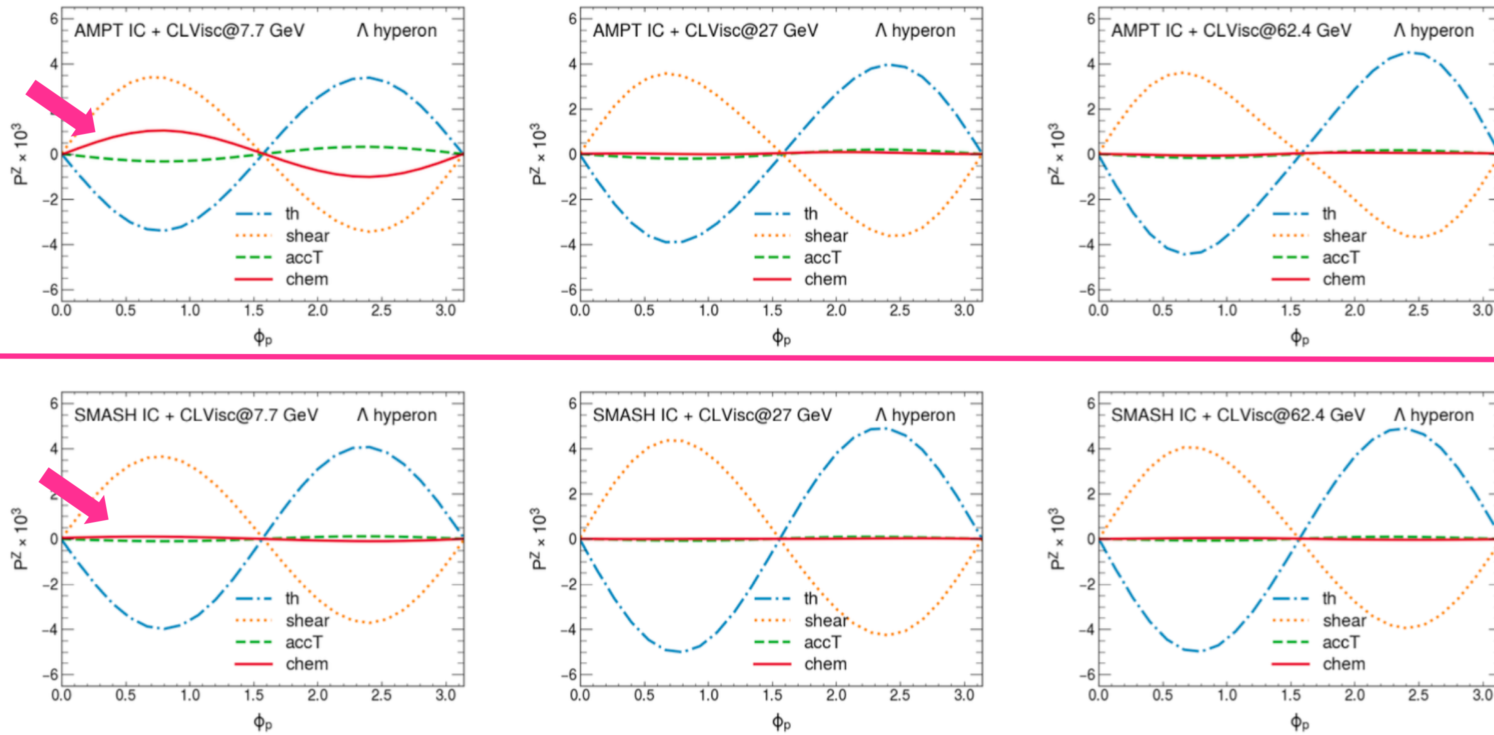
X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

Local polarization alone beam direction (I)

7.7 GeV | 27 GeV | 62.4 GeV

From AMPT Initial condition

From SMASH Initial condition



Red lines: contributions from spin Hall effect

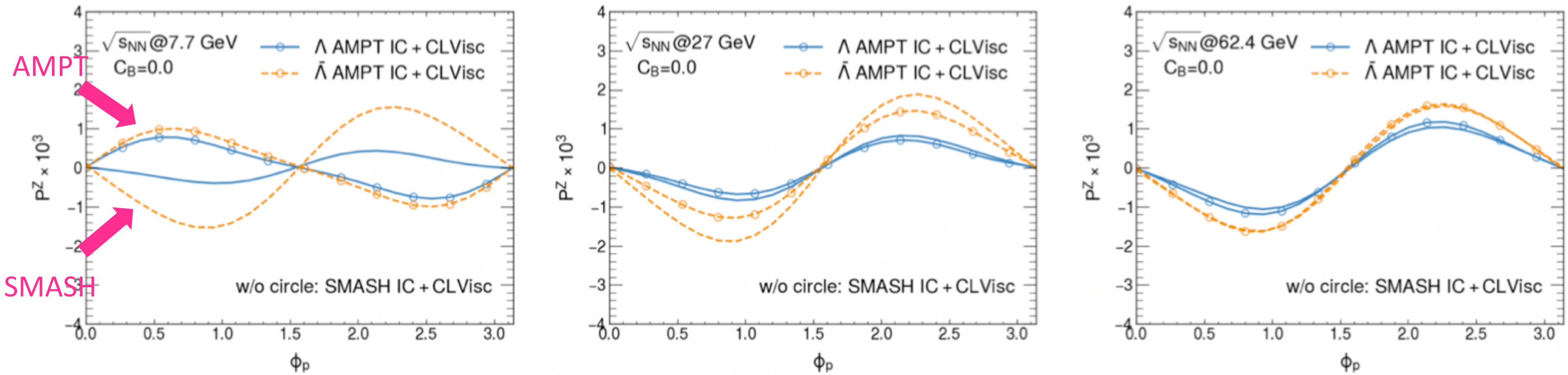
Polarization induced by SHE is almost zero at 27, 62.4GeV and it depends on the initial conditions at 7.7 GeV.

For SMASH, Pz is still almost vanishing at 7.7 GeV.

X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

Total local polarization alone beam direction (II)

7.7 GeV 27 GeV 62.4 GeV



With circle: AMPT; Without circle: SMASH

At 7.7 GeV:

- **The total local polarization from the AMPT initial conditions flip the sign due to SHE**
- **From SMASH, the polarization of anti-Lambda is much larger than the one for Lambda.**
- **A possible way to probe the initial condition?**

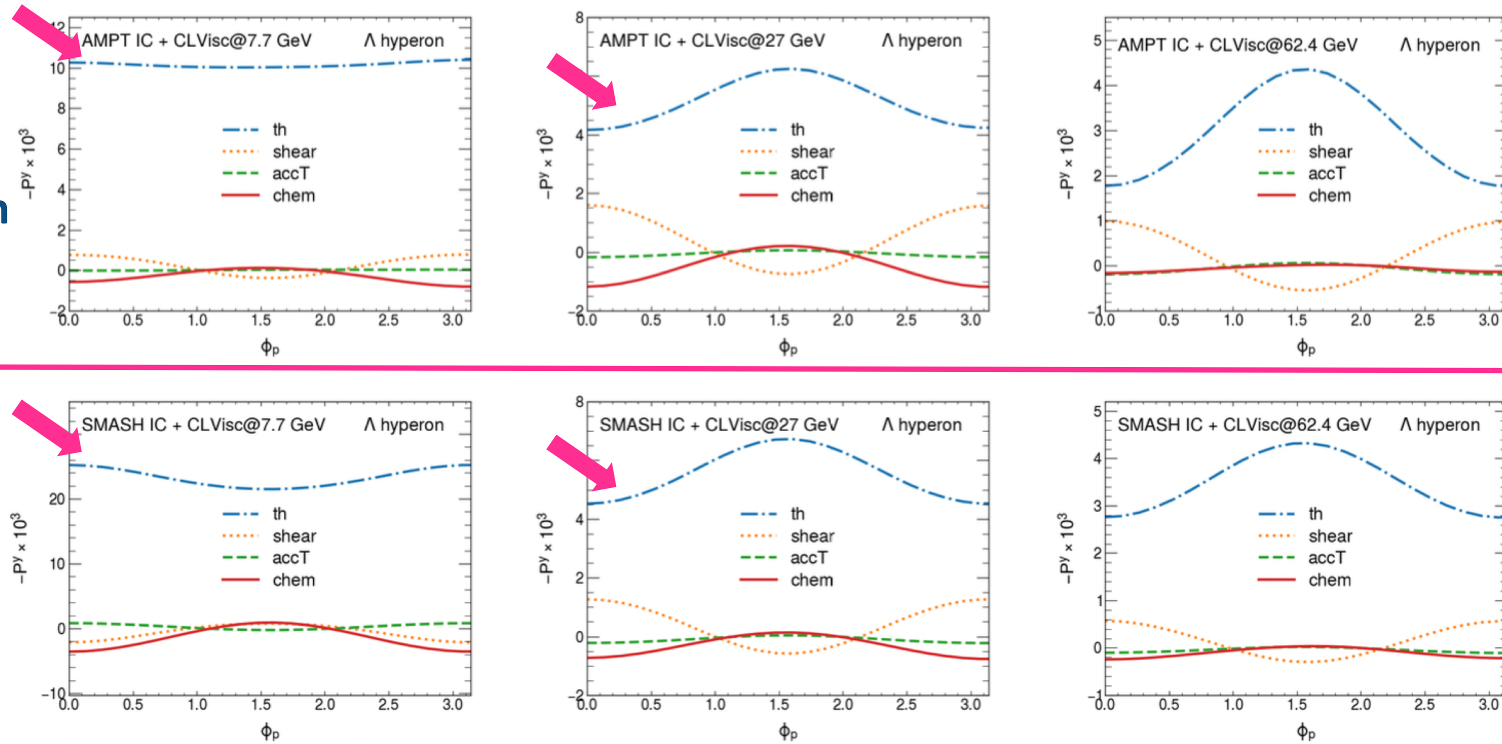
X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

Polarization alone out-of-plane direction (I)

7.7 GeV | 27 GeV | 62.4 GeV

From AMPT Initial condition

From SMASH Initial condition



Blue Dash-dotted lines: contributions from thermal vorticity

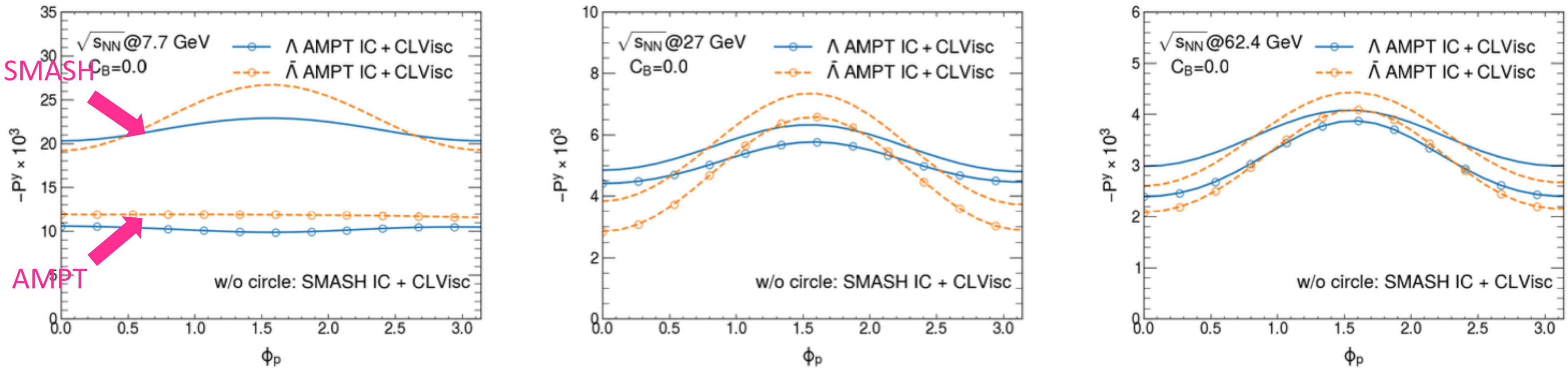
Red lines: contributions from spin Hall effect

- Polarizations induced by thermal vorticity from both AMPT and SMASH at 7.7 GeV are significant difference with those at other collisions energies.

X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

Total local polarization alone out-of-plane direction (II)

7.7 GeV 27 GeV 62.4 GeV



With circle: AMPT; Without circle: SMASH

At 7.7 GeV:

- **From AMPT, the polarization in in-plane is larger than the one in out-of-plane.**

X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

Baryon diffusion dependence (I)

Energy-momentum conservation and net baryon current conservation:

$$\nabla_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu} = eU^\mu U^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$\nabla_\mu J^\mu = 0 \qquad J^\mu = nU^\mu + V^\mu$$

Equation of motion of dissipative current:

$$\Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - \eta\sigma^{\mu\nu}\right) - \frac{4}{3}\pi^{\mu\nu}\theta - \frac{5}{7}\pi^{\alpha\langle}\sigma_\alpha^{\mu\nu\rangle} + \frac{9}{70}\frac{4}{e+P}\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}$$

$$\Delta^{\mu\nu} DV_\mu = -\frac{1}{\tau_V}\left(V^\mu - \kappa_B \nabla^\mu \frac{\mu}{T}\right) - V^\mu\theta - \frac{3}{10}V_\nu\sigma^{\mu\nu}$$

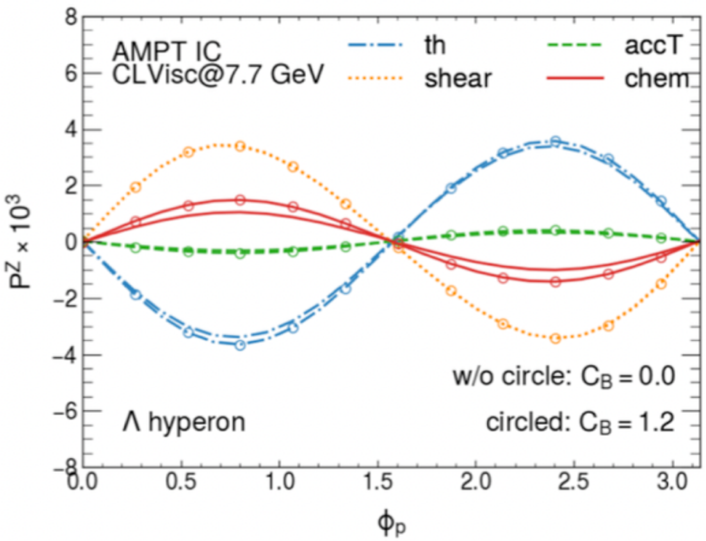
The shear viscosity $\quad \eta = C_\eta \dfrac{e+p}{T}$

The baryon diffusion $\quad \kappa_B = \dfrac{\boxed{C_B}}{T} n \left(\dfrac{1}{3}\cot\left(\dfrac{\mu_B}{T}\right) - \dfrac{nT}{e+P}\right)$

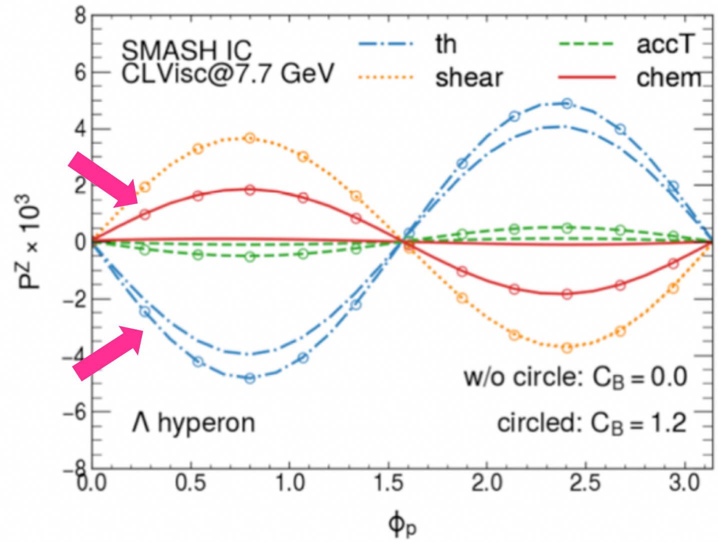
- **We also study the baryon diffusion dependence through changing the parameter C_B. In the figures in previous slides, C_B is set to be zero for simplicity.**

Baryon diffusion dependence (II)

AMPT, 7.7 GeV



SMASH, 7.7 GeV



Blue Dash-dotted lines: contributions from thermal vorticity
Red lines: contributions from spin Hall effect
w.o circle: C_B=0; with circled C_B =1.2

- **Polarization induced by shear tensor and fluid acceleration are insensitive to baryon diffusion.**
- **For SMASH, the polarization induced by SHE and thermal vorticity are enhanced when C_B increases.**

X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

Brief summary for polarization RHIC-BES energies

For AMPT initial condition:

- The polarization induced by SHE gives a sizable contribution and even flip the sign of total local polarization along beam direction at 7.7 GeV.

For SMASH initial condition:

- The polarization induced by SHE gives negligible contribution to the total polarization along beam direction.
- The polarization induced by SHE are also sensitive to the baryon diffusion coefficient C_B.

X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

Two questions (in my opinion)

- For the local polarization, the contributions from different components are mixed.

Can we introduce an observable related to the local polarization from a single source?

- Because the system is not at global equilibrium, we therefore have the contributions from shear tensor and gradient of chemical potential.

Are there any other possible out-of-equilibrium corrections?

Will the new corrections provide a better description for the data?

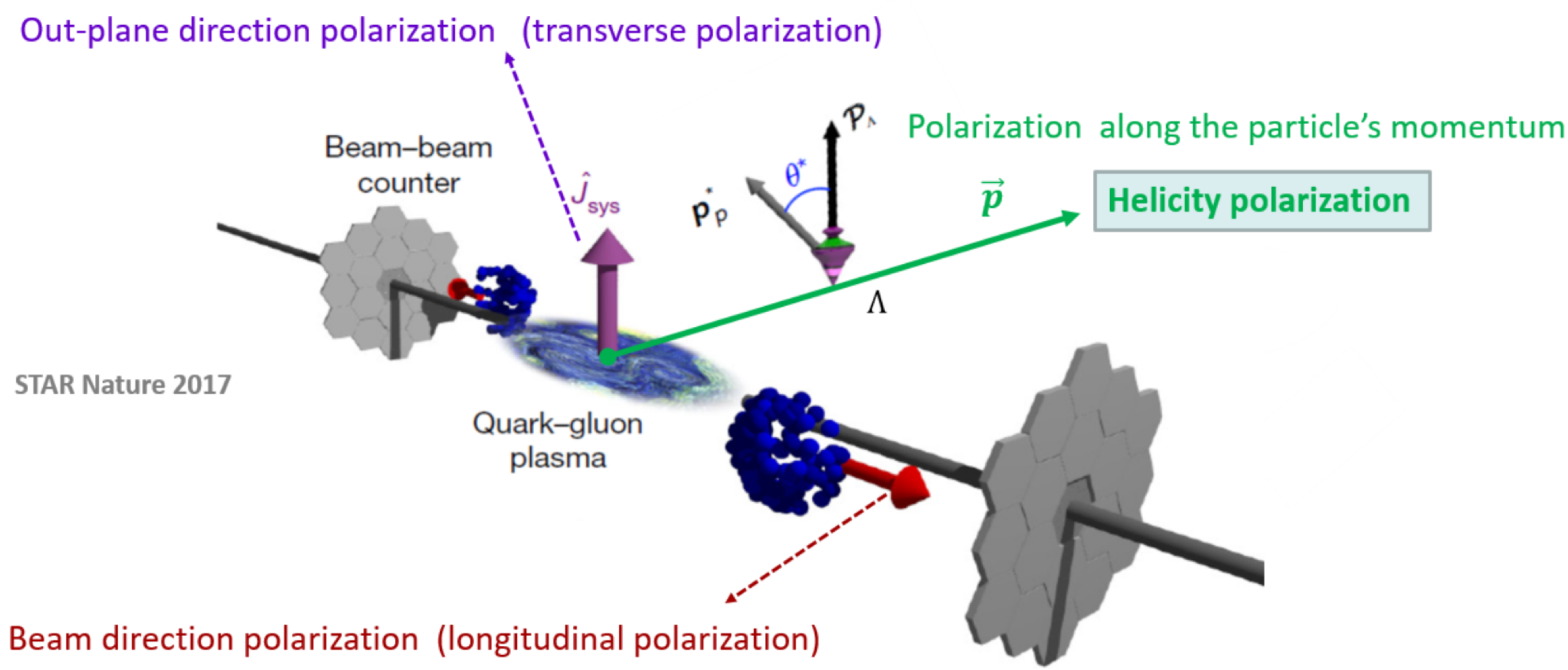
Helicity polarization across RHIC-BES energies:

Probing the thermal or kinetic vorticity from data

Helicity polarization

- The original idea for helicity polarization is proposed by Becattini, Buzzegoli, Palermo, Prokhorov, PLB(2021) and Gao, PRD(2021) to probe the initial chiral chemical potential.
- Helicity instead of spin is widely-used in high energy spin physics. (also see Prof. Qun Wang's talk for the spin alignment of phi meson).

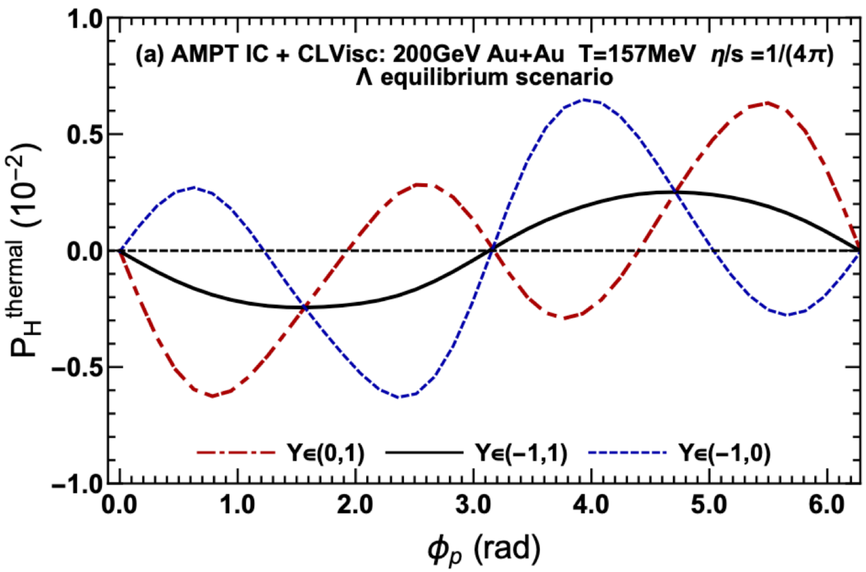
$$S^h = \widehat{\mathbf{p}}\cdot\boldsymbol{S}(\mathbf{p}) = \widehat{p}^x\mathcal{S}^x + \widehat{p}^y\mathcal{S}^y + \widehat{p}^z\mathcal{S}^z,$$



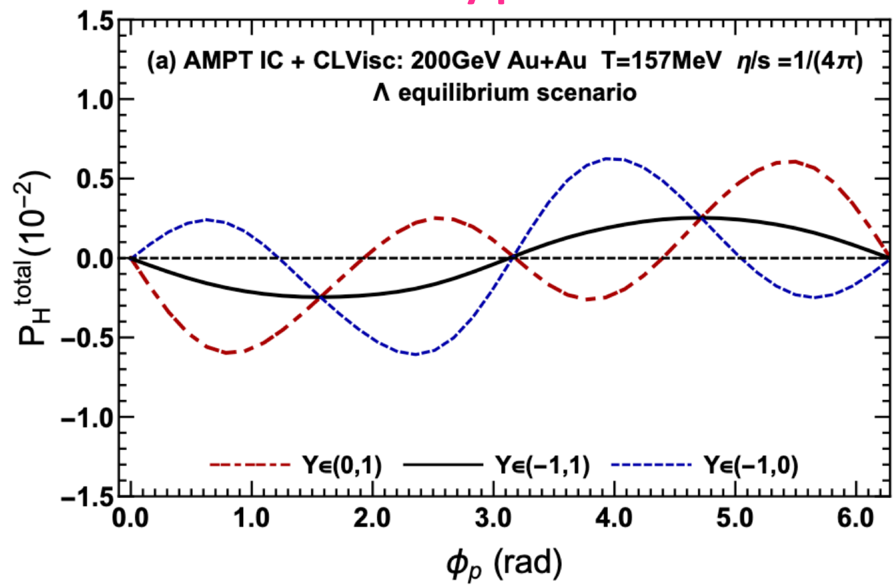
Helicity polarization at 200 GeV

- **Helicity polarization polarization can also induced by thermal, shear and fluid acceleration.**

Contribution from thermal vorticity



Total helicity polarization



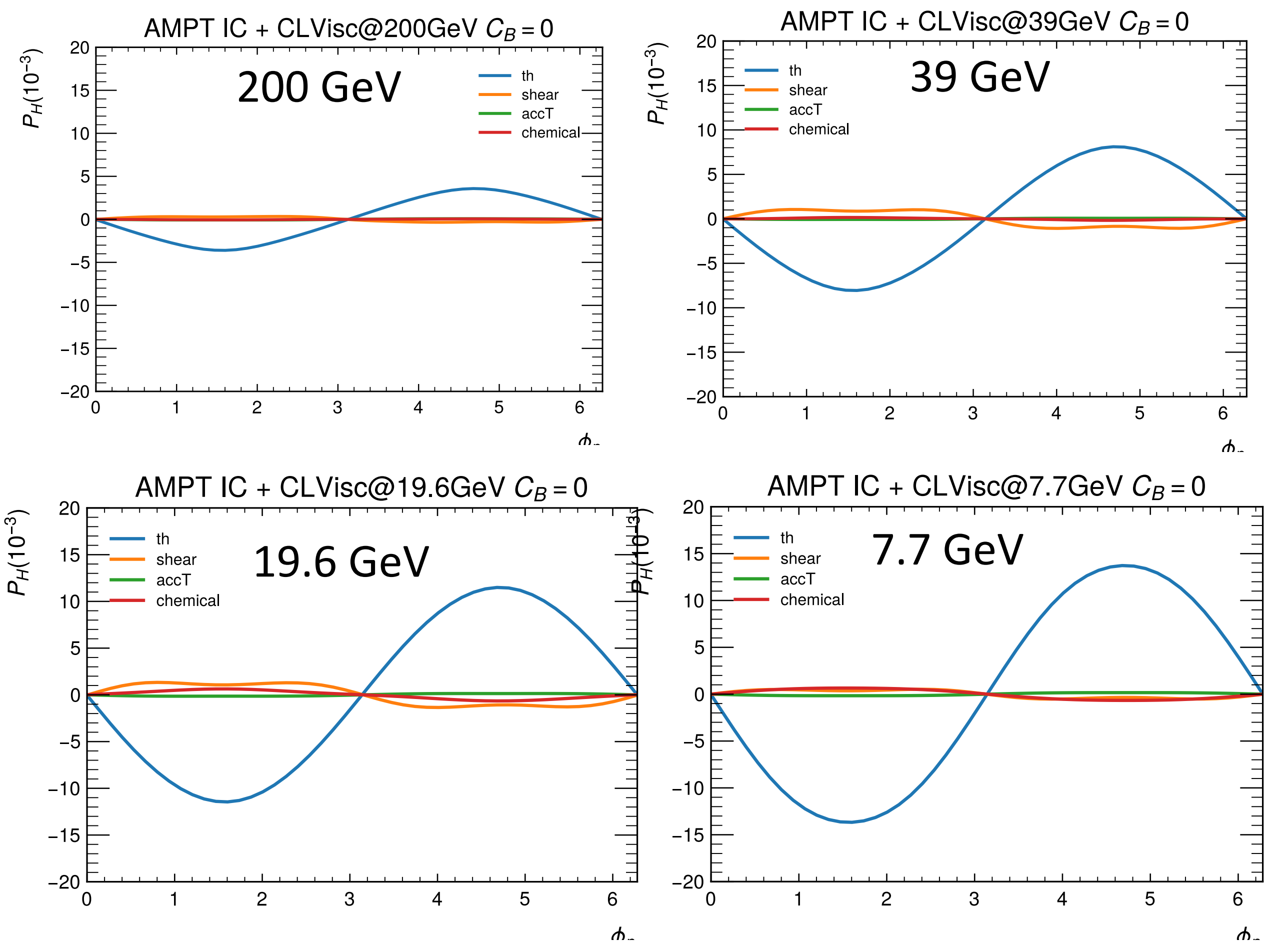
200 GeV, AMPT initial condition, EoS: sp-95pce

- **Our numerical simulation shows that the thermal vorticity dominates over other contributions in helicity polarization. The helicity polarization can be used to detect the vortical structure in the fireball.**

- **Yi, Pu, Gao, Yang, PRC (2022)**

Helicity polarization across RHIC-BES energies

CLVisc hydrodynamics + AMPT initial +EoS: NEOS-BQS + C_B =0



Blue solid lines:

Contribution from thermal vorticity.

Other colors:

Contribution from other components.

- **Helicity polarization induced by thermal vorticity dominates at RHIC-BES energies.**
- **We also check that the above conclusion holds for different EoS, initial conditions and value of C_B.**

Cong Yi, X.Y. Wu, et al. , in preparation

A possible way to probe the local polarization induced by thermal vorticity only?

Corrections from effective interactions to local polarization

Other possible corrections

- **Differences between Λ and anti-Λ**
  - **Strong EB fields: Müller, Schäfer, PRD (2018); Guo, Shi, Feng, Liao PLB (2019); Buzzegoli 2022; Xu, Lin, Huang, Huang, PRD (2022); Peng, Wu, Wang, She, Pu, 2022**
  - **Interplay between chiral and helical vortical effects: Ambrus, Chernodub, EPJC (2022); Chernodub, Ambrus, PRD (2021); Ambrus, JHEP (2020)**
  - **Gradience of baryon chemical potential: Ryu, V. Jupic, and C. Shen PRC (2021); X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022); …**

- **Collisional effects to modified Cooper-Frye formula**

  Fang, SP, Yang, PRD (2022)
  Z.Y. Wang, arXiv:2205.09334
  Lin, Wang, arXiv:2206.12573

- **Corrections from spin potential to modified Cooper-Frye formula**

  Liu, Huang, arXiv: 2109.15301

- **Hadronization**

- **Hadronic interaction after chemical freezeout**

Quantum kinetic theory with collisions

- **Collision term with non-local quantum corrections**

  Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD (2019); PRL (2021)

  Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.

  Liu, Mameda, Huang, arXiv:2002.03753.

  Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019); Wang, Guo, Zhuang, EPJC (2021);Wang, Zhuang, arXiv:2105.00915

  Li ,Yee, PRD100, 056022 (2019)

  Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184

  Fang, SP, Yang, PRD (2022)

  Z.Y. Wang, arXiv:2205.09334; Lin, Wang, arXiv:2206.12573

Recent reviews:

Gao, Ma, SP, Wang, NST (2020)

Gao, Liang, Wang, IJMPA (2021)

Hidaka, SP, Yang, Wang, PPNP (2022)

Theory: Collisions for gauge fields and spin polarization (I)

- **We have derived collision kernel for QED in HTL approximation.**

Eq. for Particle distribution function

$$(p\cdot\partial)f_V^{<}(x,p)=\mathcal{C}_V^{\mathrm{HTL}}[f_V]+\mathcal{O}(\hbar^2),$$

Eq. for Spin distribution function

$$(p\cdot\partial)f_A^{<}(x,p)+\hbar\partial_\mu S_{(u)}^{\mu\nu}\partial_\nu f_V^{<}(x,p)=\mathcal{C}_A^{\mathrm{HTL}}[f_V,f_A]+\mathcal{O}(\hbar^2),$$

- **For the first time, the real QED type collision kernel for axial part:**

$$\begin{aligned}\mathcal{C}_A[f_V,f_A]=&-\frac{e^4\delta(p^2)}{8\pi^2|\boldsymbol{p}|}\ln\frac{T}{m_D}\left\{\frac{2\pi^2}{3\beta^2}|\boldsymbol{p}|F(p)f_A^{<}(p)+\frac{\pi^2}{3\beta^2}|\boldsymbol{p}|^2F(p)[(\hat{p}_\perp\cdot\partial_{p_\perp})-\frac{1}{\beta}(\partial_{p_\perp}\cdot\partial_{p_\perp})]f_A^{<}(p)\right.\\&-\frac{2\pi^2}{3\beta^2}|\boldsymbol{p}|^2f_A^{<}(p)(\hat{p}_\perp\cdot\partial_{p_\perp})f_V^{<}(p)+\hbar F(p)|\boldsymbol{p}|H_{3,\alpha}\partial_{p_\perp}^{\alpha}f_V^{<}(p)\\&-\hbar\frac{\pi^2}{12\beta^2}F(p)|\boldsymbol{p}|\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\nu}u_\beta\partial_{p_\perp,\rho}\partial_\alpha f_V^{<}(p)+\hbar\frac{\pi^2}{6\beta^3}\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\rho}u_\beta\partial_{p_\perp,\nu}\partial_\alpha f_V^{<}(p)\\&+\hbar\frac{\pi^2}{6\beta^2}\epsilon^{\mu\xi\lambda\kappa}p_\lambda u_\kappa\partial_\xi f_V^{<}(p)\partial_{p_\perp,\mu}f_V^{<}(p)\\&\left.-\hbar\frac{\pi^2}{12\beta^3}|\boldsymbol{p}|\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\nu}u_\beta\hat{p}_{\perp,(\gamma}g_{\lambda)\rho}\hat{p}_{\perp,\lambda}\partial_{p_\perp}^{\lambda}\partial_{p_\perp}^{\gamma}\partial_\alpha f_V^{<}(p)\right\}+\mathcal{O}(\hbar^2).\end{aligned}\quad(65)$$

S. Fang, SP, D.L. Yang, PRD (2022), arXiv: 2204.11519

Theory: Collisions for gauge fields and spin polarization (II)

- **We have proved that dynamical spin polarization for a probe is much slower than its thermalization.**

$$\frac{\text{Spin polarization time}}{\text{Thermalization time}} \approx \frac{\Gamma_A(p)}{\Gamma_V(p)} \approx \frac{\hbar H_{3,\alpha}}{T^2|\boldsymbol{p}|} \sim \mathcal{O}\left(\frac{\partial}{|\boldsymbol{p}|}\right),$$

Also see Wang, Guo, Zhuang, EPJC (2021);Wang, Zhuang, arXiv:2105.00915

- **We also derive the Boltzmann equation for spin evolution:**

$$(p\cdot\partial)f_A^<(x,p) + \hbar\partial_\mu S^{\mu\nu}_{(u)}\partial_\nu f^<_{V,leq}(x,p) = \mathcal{C}_A^{\mathrm{HTL}}[f_{V,leq}, f_A] + \mathcal{O}(\hbar^2),$$

$$\begin{aligned}C_A^{\mathrm{HTL}}[f_{V,leq}, f_A] = &-\frac{e^4}{16\pi^3}\frac{\pi^2}{3\beta^2}\ln\frac{T}{m_D}\left\{2\left(f^>_{V,leq}(p) - f^<_{V,leq}(p)\right) + 2|\boldsymbol{p}|\beta f^<_{V,leq}(p)f^>_{V,leq}(p)\right.\\ &\left.+|\boldsymbol{p}|\left[\left(f^>_{V,leq}(p) - f^<_{V,leq}(p)\right)\hat{p}_\perp\cdot\partial_{p_\perp} - \frac{1}{\beta}(\partial_{p_\perp}\cdot\partial_{p_\perp})\right]\right\}f_A^<(p)\\ &+\hbar\frac{e^4}{16\pi^3|\boldsymbol{p}|}\frac{\pi^2}{3\beta^3}\ln\frac{T}{m_D}S^{\alpha\nu}_{(u)}\Omega_{\alpha\nu}f^<_{V,leq}(p)f^>_{V,leq}(p) + \mathcal{O}(\hbar^2),\end{aligned}$$

S. Fang, SP, D.L. Yang, PRD (2022)

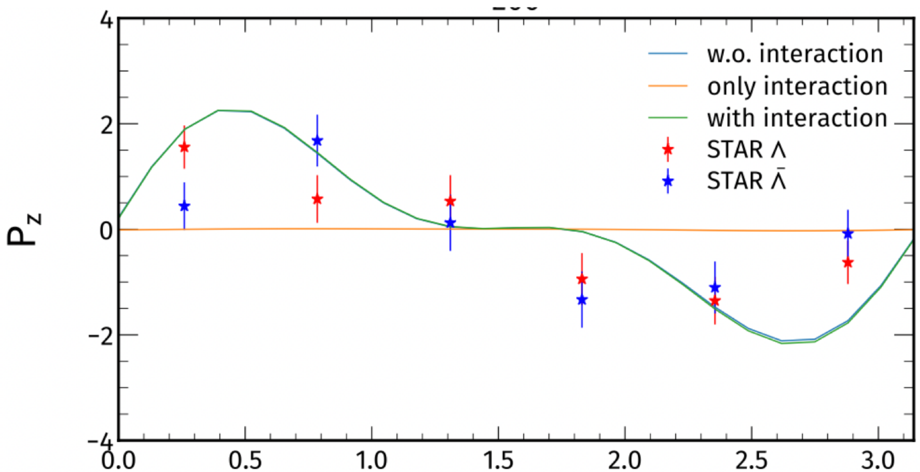
Assumption for simulations

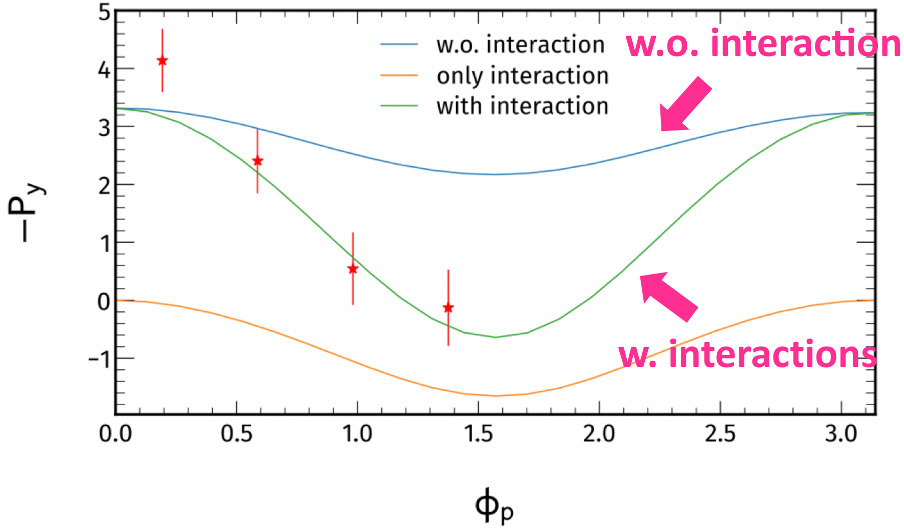
- We assume that the particles are almost at local thermal equilibrium near the freezeout hypersurface.
- We estimate the evolution of spin distribution near the freezeout hypersurface.
- To simplify the calculations, we take some approximations:
  - We only consider qq->qq process, and replace the EM coupling constant e to strong coupling constant g in leading-log order of HTL.
  - We introduce some energy cutoff to simplify the calculations.
  - …

Cong Yi, Shuo Fang, et al., in preparation

Local polarization induced by interactions (Test)

200GeV, CLVisc hydrodynamics + AMPT initial + EoS: NEOS-BQS





- We consider the s quark polarization (s quark scenario):
  - Mass of s quark: 0.4 GeV
- For qq -> qq type effective interactions, we find that the interactions can modify the local polarization.

More systemic studies are needed!

Cong Yi, Shuo Fang, et al., in preparation

Summary

- The local polarization induced by spin Hall effect at low energies are sensitive to initial condition and baryon diffusion.
- For the local polarization, the contributions from different components are mixed.

Can we introduce an observable related to the local polarization from a single source?

Helicity polarization may help us to probe the local polarization from thermal vorticity only.

- Because the system is not at global equilibrium, we therefore have the contributions from shear tensor and gradient of chemical potential.

Are there any other possible out-of-equilibrium corrections?

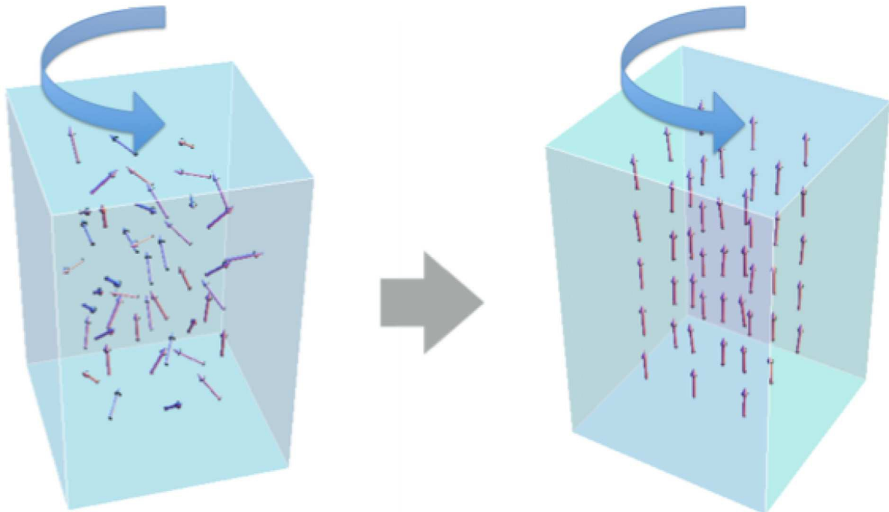
Will the new corrections provide a better description for the data?

Interactions can play a role in local polarization, but the systemic studies are needed.

Thank you for your time!

Backup

Barnet effects and Einstein-de Hass effects



Barnett effect:

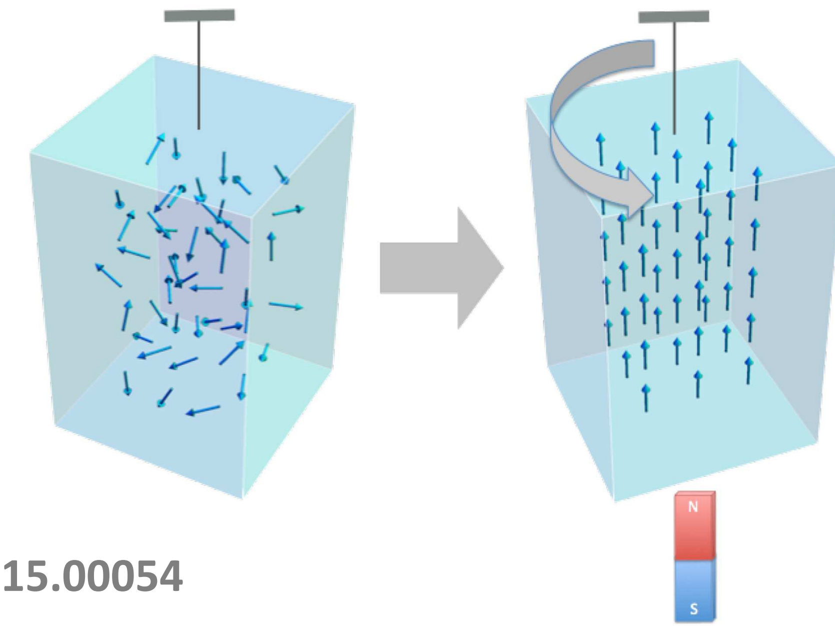
Rotation $\Longrightarrow$ Magnetization

Barnett, Magnetization by rotation, Phys Rev. (1915) 6:239–70.

Einstein-de Haas effect:

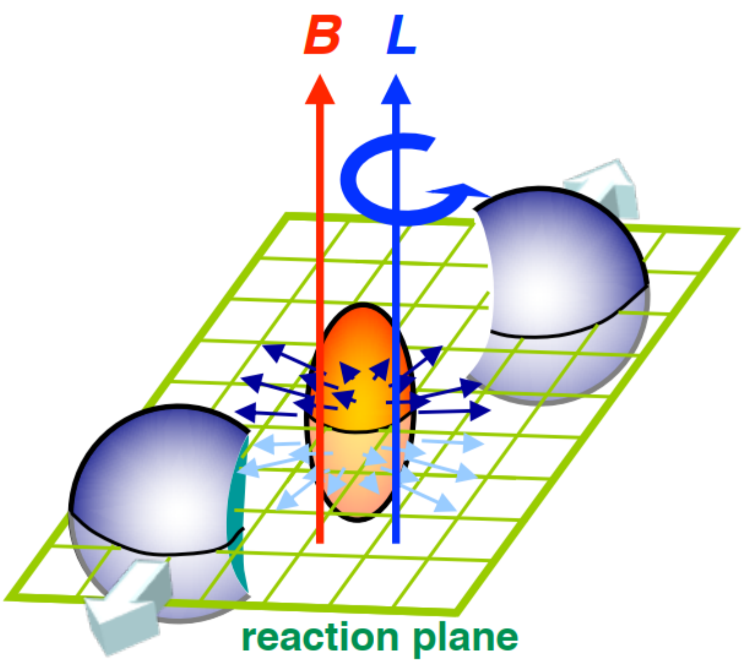
Magnetization $\Longrightarrow$ Rotation

Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents. Verh Dtsch Phys Ges. (1915) 17:152.



Figures: copy from paper doi: 10.3389/fphy.2015.00054

Huge angular momentum



- **Huge global orbital angular momenta are produced**

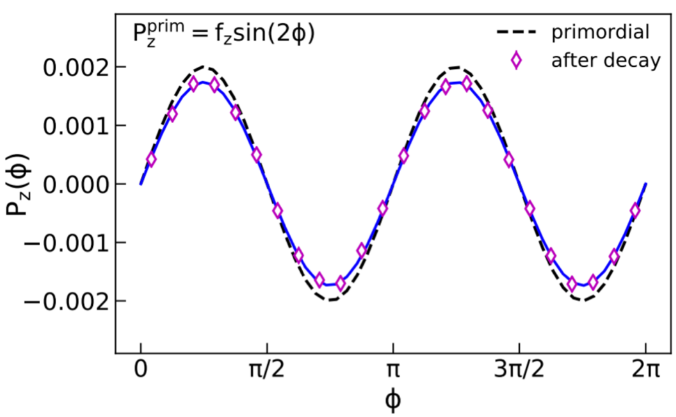
$$L \sim 10^5 \hbar$$

- **How do orbital angular momenta be transferred to the matter created?**

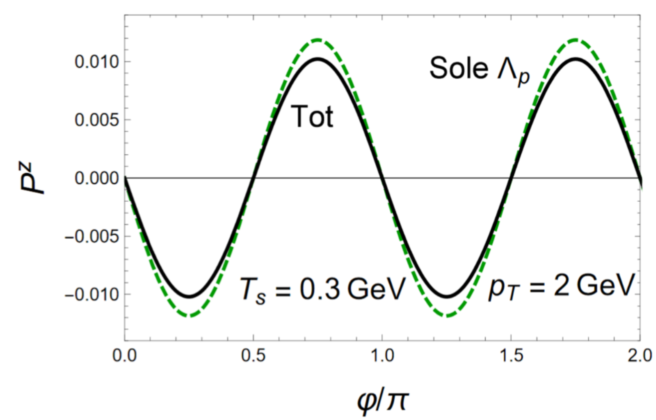
Feed-down effects: NO!

- **Feed-down effects**

  Lambda may come from decays of heavier particles



Xia, Li, Huang, Huang PRC(2019)



Becattini, Cao, Speranza, EPJC(2019)

Collisional kernel

- **An example for collision kernel of NJL type interactions:**

Eq. for Particle distribution function

$$\frac{1}{E_p} p \cdot \partial_x \mathrm{tr}\left[f^{(1)}(x,p)\right] = -\frac{1}{\pi\hbar}\int_0^\infty dp_0\, \mathrm{Im}\mathrm{Tr}\left(I^{(2)}_{\mathrm{coll}}\right) - \frac{1}{2\pi\hbar m}\mathrm{Re}\,\mathrm{Tr}\left(\gamma\cdot\partial_x I^{(1)}_{\mathrm{coll}}\right)$$
$$\equiv \mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right) + \mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll},\,\nabla}\right) + \mathscr{C}_{\mathrm{scalar}}\left(I^{(0)}_{\mathrm{coll,PB}}\right) + \mathscr{C}_{\mathrm{scalar}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right),$$

Eq. for Spin distribution function

$$\frac{1}{E_p} p \cdot \partial_x \mathrm{tr}\left[n_j^{(+)\mu}\tau_j^T f^{(1)}(x,p)\right] = \frac{1}{2\pi\hbar m}\int_0^\infty dp_0\left[\epsilon^{\mu\nu\alpha\beta}p_\nu \mathrm{Im}\mathrm{Tr}\left(\sigma_{\alpha\beta}I^{(2)}_{\mathrm{coll}}\right) + \mathrm{Re}\,\mathrm{Tr}\left(\gamma^5\partial_x^\mu I^{(1)}_{\mathrm{coll}}\right)\right]$$
$$\equiv \mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right) + \mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll},\,\nabla}\right) + \mathscr{C}^\mu_{\mathrm{pol}}\left(I^{(0)}_{\mathrm{coll,PB}}\right) + \mathscr{C}^\mu_{\mathrm{pol}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right).$$

$\mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right)$ $\mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll},\,\nabla}\right)$ $\mathscr{C}_{\mathrm{scalar}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right)$ $\mathscr{C}_{\mathrm{scalar}}\left(I^{(0)}_{\mathrm{coll,PB}}\right)$

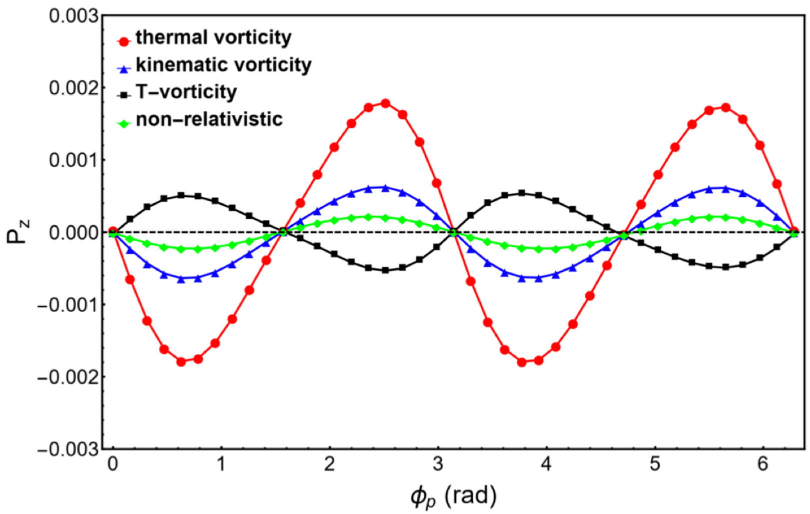
$\mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right)$ $\mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll},\,\nabla}\right)$ $\mathscr{C}^\mu_{\mathrm{pol}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right)$ $\mathscr{C}^\mu_{\mathrm{pol}}\left(I^{(0)}_{\mathrm{coll,PB}}\right)$

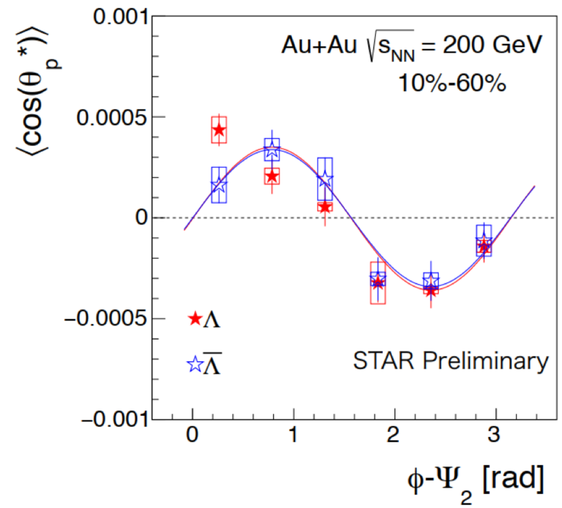
Perturbative Correction to Ordinary terms

Non-local terms related to the space derivatives may be the key to describe the spin-orbital transformation.

Sheng, Weickgenannt, Speranza, Rischke, Wang PRD (2021)

Local polarization from different vorticities





Kinematic vorticity:

$$\omega_{\mu\nu}^{(K)} = -\frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu)$$

T-vorticity:

$$\omega_{\mu\nu}^{(T)} = -\frac{1}{2}[\partial_\mu(Tu_\nu) - \partial_\nu(Tu_\mu)]$$

Non-Relativistic vorticity

$$\omega_{\mu\nu}^{(\mathrm{NR})} = \epsilon_{\nu\mu\rho\eta} u^\rho \omega^\eta$$

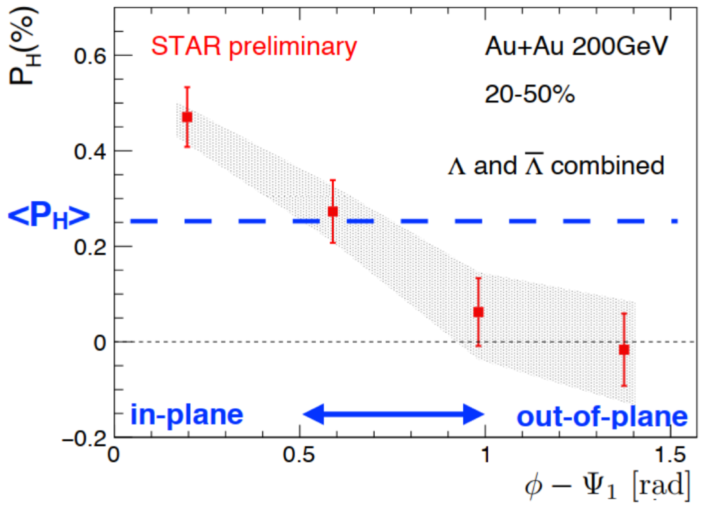
Thermal vorticity:

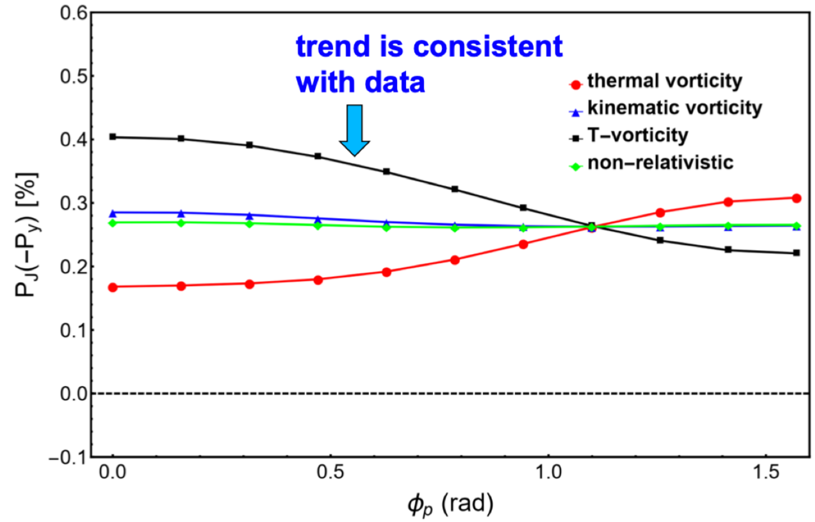
$$\omega_{\rho\sigma}^{\mathrm{th}} = \frac{1}{2}\left[\partial_\rho\left(\frac{u_\sigma}{T}\right) - \partial_\sigma\left(\frac{u_\rho}{T}\right)\right]$$

Wu, Pang, Huang, Wang, PRR 1, 033058(2019)

- Only T-vorticity gives the right trend for both Pz and Py
- Why T-vorticity? Out-of-equilibrium effects?

Local polarization from different vorticities





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Thermal vorticity:

$$\omega_{\rho\sigma}^{\mathrm{th}} = \frac{1}{2}\left[\partial_\rho\left(\frac{u_\sigma}{T}\right) - \partial_\sigma\left(\frac{u_\rho}{T}\right)\right]$$

Wu, Pang, Huang, Wang, PRR 1, 033058(2019)

- Only T-vorticity gives the right trend for both Pz and Py
- Why T-vorticity? Out-of-equilibrium effects?

Chiral kinetic theory (massless fermions)

- **Hamiltonian formulism, effective theory**

  Son, Yamamoto, PRL, (2012); PRD (2013)

- **Path integration**

  Stephanov, Yin, PRL (2012);

  Chen, Son, Stephanov, Yee, Yin, PRL, (2014);

  J.W. Chen, J.Y. Pang, SP, Q. Wang, PRD (2014)

- **Wigner function (Quantum field theory)**
  - hydrodynamics, equilibrium

    Gao, Liang, Pu, Q. Wang, X.N. Wang, PRL 109, 232301 (2012) ; J.W. Chen, SP, Q. Wang, X.N. Wang, PRL (2013);
  - out-of-equilibrium, quantum field theory

    Y. Hidaka, SP, D.L. Yang, PRD(RC) (2017)
  - Other studies

    A.P. Huang, S.Z. Su, Y. Jiang, J.F. Liao, P.F. Zhuang, arXiv:1801.03640

- **World-line formulism**

  N. Muller, R, Venugopalan PRD 2017

Also see recent review:

Gao, Liang, Wang, Int.J.Mod.Phys A 36 (2021), 2130001

Hidaka, SP, D.L. Yang, Q. Wang, arXiv:2201.07644

Wigner function (I)

- **Wigner operator**

$$\hat{W}_{\alpha\beta} = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}_\beta(x_+) U(x_+, x_-) \psi_\alpha(x_-), \quad x = x_+ + x_- \quad y = x_+ - x_-$$

Gauge link $U(x_+, x_-) \equiv e^{-iQ\int_{x_-}^{x_+} dz^\mu A_\mu(z)},$

- **Wigner function:**

$$W(x,p) = \left\langle : \hat{W}(x,p) : \right\rangle$$

W operator in thermal ensemble average and normal ordering of the operators

- **Physical meaning: “density matrix” in quantum field theory**

Vasak, Gyulassy, Elze, Ann. Phys. (N.Y.) 173, 462 (1987);

Elze, Heinz,Phys.Rep.183,81(1989).

Wigner function (II)

- By using the Dirac equation, one can get the master equations for Wigner function

$$\gamma_\mu\left(p^\mu + \frac{i}{2}\nabla^\mu\right)W(x,p) = 0, \qquad \nabla^\mu \equiv \partial_x^\mu - QF^\mu{}_\nu\partial_p^\nu$$

- Matrix decomposition

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^5\gamma^\mu\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right],$$

Charge current

$$\mathscr{V}^\mu = \int\frac{d^4y}{(2\pi)^4}e^{-ip\cdot y} <:\bar{\psi}_\beta\left(x+\frac{1}{2}y\right)\gamma^\mu U\left(x+\frac{1}{2}y, x-\frac{1}{2}y\right)\psi_\alpha\left(x-\frac{1}{2}y\right):>$$

Chiral current

$$\mathscr{A}_\mu = \int\frac{d^4y}{(2\pi)^4}e^{-ip\cdot y} <:\bar{\psi}_\beta\left(x+\frac{1}{2}y\right)\gamma^\mu\gamma^5 U\left(x+\frac{1}{2}y, x-\frac{1}{2}y\right)\psi_\alpha\left(x-\frac{1}{2}y\right):>$$

Solving Wigner function in hbar expansion

- We consider the hbar (gradient) expansion,

$$\mathscr{J}^s_\mu(x,p) = \frac{1}{2}[\mathscr{V}_\mu(x,p) + s\mathscr{A}_\mu(x,p)], \qquad s = \pm$$

$$\mathscr{J}^s_\mu(x,p) = \mathscr{J}^s_{\mu,(0)}(x,p) + \mathscr{J}^s_{\mu,(1)}(x,p) + \cdots ,$$

- Leading order is the classical currents. We introduce the initial distribution function f(x,p) as input.

$$\mathscr{J}^\rho_{(0)s}(x,p) = p^\rho f_s \delta(p^2),$$

- We can then solve the next-to-leading order.

$$\mathscr{J}^\rho_{(1)s}(x,p) = -\frac{s}{2}\tilde{\Omega}^{\rho\lambda} p_\lambda \frac{df_s}{dp_0}\delta(p^2) - \frac{s}{p^2} e\tilde{F}^{\rho\lambda} p_\lambda f_s \delta(p^2).$$

$$\Omega_{\nu\sigma} = \tfrac{1}{2}(\partial_\nu u_\sigma - \partial_\sigma u_\nu), \text{ and } \Omega^{\mu\nu} = -\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\tilde{\Omega}_{\rho\sigma}.$$

Connection to Currents and chiral anomaly

- **Integral over momentum p, we get CME and other quantum transport effects**

$$j_s^\mu = \int d^4p\, \mathscr{J}_s^\mu = n_s u^\mu + \xi_{B,s} B^\mu + \xi_s \omega^\mu,$$

Charge current

$$j^\mu = \sum_{s=\pm} j_s^\mu = n u^\mu + \underline{\xi_B B^\mu} + \xi \omega^\mu,$$

Chiral current

$$j_5^\mu = \sum_{s=\pm} s j_s^\mu = n_5 u^\mu + \xi_{B5} B^\mu + \xi_5 \omega^\mu,$$

$$\xi_B = \frac{e}{2\pi^2}\mu_5, \quad \xi = \frac{1}{\pi^2}\mu\mu_5, \quad \xi_{B5} = \frac{e}{2\pi^2}\mu, \quad \xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}\left(\mu^2 + \mu_5^2\right)$$

- **We also reproduce the chiral anomaly from the kinetic theory.**

$$\partial_\mu j^\mu = 0, \qquad \partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2} E \cdot B.$$

Gao, Liang, Pu, Q. Wang, X.N. Wang, PRL 109, 232301 (2012)

Derivation of chiral kinetic theory

- There is a constrain equation.

$$\nabla^{\mu}\mathscr{J}^{s}_{\mu}(x,p) = 0,$$

- We can insert our results into this equation and get the constraint equation for distribution function.
- Then, we need to integral over p0 to get 3-dim form.

J.W. Chen, SP, Q. Wang, X.N. Wang, PRL (2013);

Y. Hidaka, SP, D.L. Yang, PRD(RC) (2017)

Review: Gao, Liang, Wang, Int.J.Mod.Phys A 36 (2021), 2130001

Hidaka, SP, D.L. Yang, Q. Wang, 2201.07644

Chiral kinetic equation

- **Chiral kinetic theory is a useful tool to study CME.**

$$\sqrt{G}\partial_t f + \sqrt{G}\dot{\mathbf{x}}\cdot\nabla_x f + \sqrt{G}\dot{\mathbf{p}}\cdot\nabla_p f = C[f].$$

- **Particle's effective velocity:**

$$\sqrt{G}\dot{\mathbf{x}} = \frac{\partial\varepsilon}{\partial\mathbf{p}} + \hbar\left(\frac{\partial\varepsilon}{\partial\mathbf{p}}\cdot\boldsymbol{\Omega}\right)\mathbf{B} + \hbar\mathbf{E}\times\boldsymbol{\Omega},$$

- **Effective force:**

$$\sqrt{G}\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial\varepsilon}{\partial\mathbf{p}}\times\mathbf{B} + \hbar(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega},$$

- **Berry curvature**

$$\boldsymbol{\Omega} = \frac{\mathbf{p}}{2|\mathbf{p}|^3}, \qquad \sqrt{G} = 1 + \hbar\mathbf{B}\cdot\boldsymbol{\Omega},$$

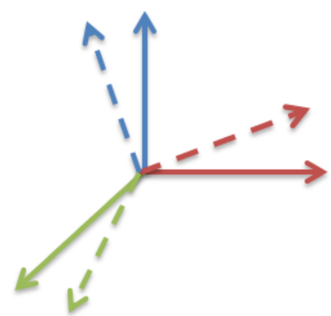
Side-jump effects and Lorentz symmetry

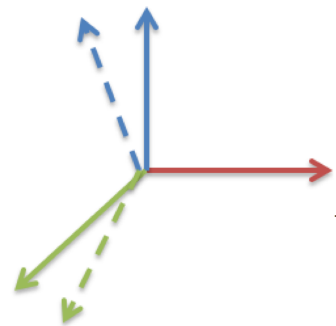
- **The subgroup for Lorentz group for massless fermions and massive fermions are different.**

Massive particles:
(Rest frame)

$$p^\mu = (m, 0, 0, 0)$$

Subgroup: SO(3)





Massless particles:
(No rest frame)

$$p^\mu = (|p_z|, 0, 0, p_z)$$

Subgroup: ISO(2)

- **From quantum field theory, the distribution function is no longer a scalar.**

$$\boxed{f'(x', p', t') = f(x', p', t') + \hbar N^\mu (\partial^x_\mu + F_{\nu\mu}\partial^\nu_p) f,}$$

Infinitesimal Lorentz Transform

$$\delta \mathbf{x} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|},$$

$$\delta \mathbf{p} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|} \times \mathbf{B}$$

Chen, Son, Stephanov, PRL, (2015); Y. Hidaka, SP, D.L. Yang, PRD (2016)