A Possibility of Twisted Gauge Backgrounds and Chirality Production in the Heavy-Ion Collision

Kenji Fukushima

The University of Tokyo

— Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions —

What I want to discuss…

Is there any way to make use of not only spin but…

Orbital Angular Momentum (OAM)

In QFT calculations coupled to vorticity $\omega \neq 0$,
OAM plays an intriguing role…

Even in the context of CME, OAM gives a nontrivial relation.

Some analogy to the paraxial photon vortices
(Discussions started, see **Ivanov-Korchagin-Pimikov-Zhang PRL2020, PRD2020 Zou-Zhang-Silenko 2021**, etc…)

Various Good By-Products

Chirality, Vorticity, Magnetic Field

So far, only vorticity has been confirmed experimentally.

Chirality and magnetic field have given a big impact to the QCD studies in a wider context.

Any such impact from vorticity — yes, it has been found!

Various Good By-Products

Chiral Magnetic Effect → Phase Diagram of *B*-QCD

Magnetic Field (strong) — Lattice QCD OK!

Axial Chemical Potential / Chirality — Lattice feasible

Inverse Magnetic Catalysis

Polarization → Phase Diagram of ω-QCD

Vorticity / Angular Velocity (small) — Lattice partially

Spin / Orbital Angular Momenta — No reliable lattice…

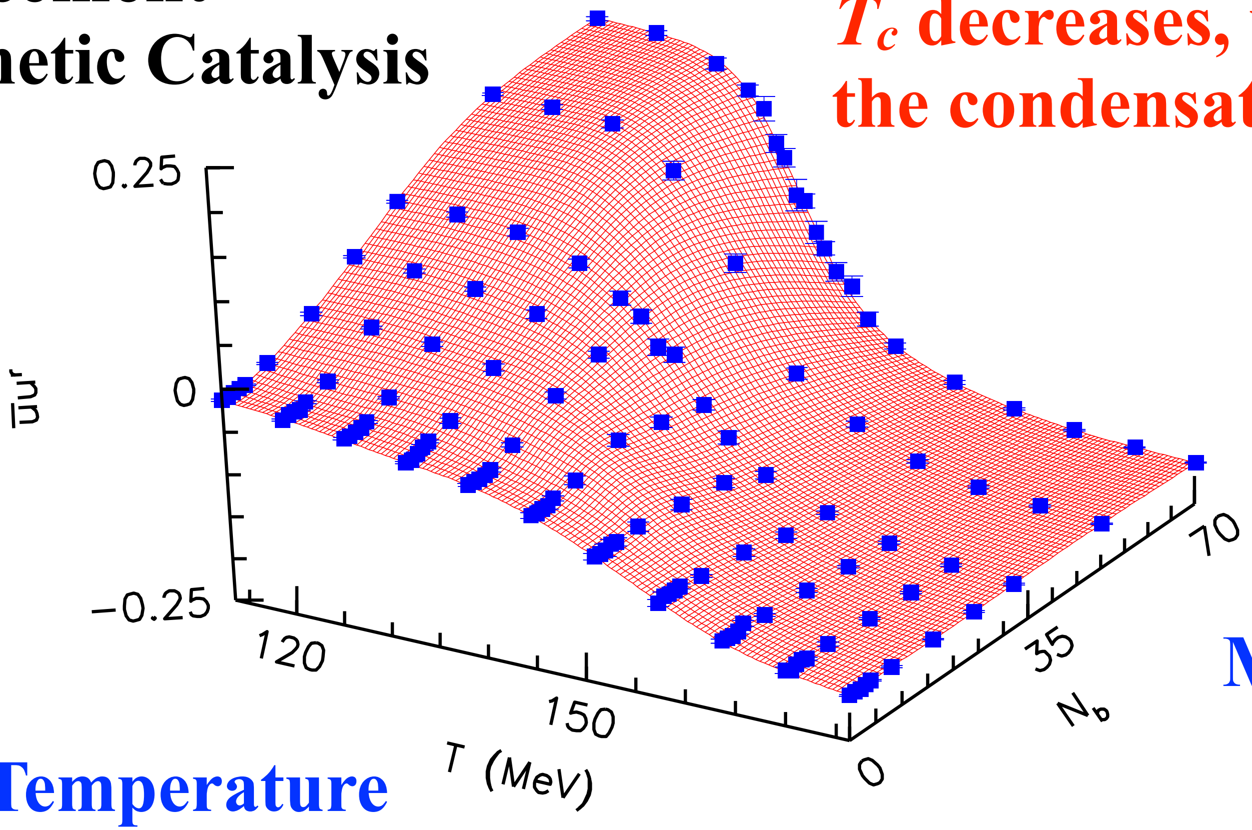
Controversy !

Various Good By-Products

Magnetic field killed many finite-density models

Enhancement
~ Magnetic Catalysis

T_c decreases, while the condensate increases



Temperature

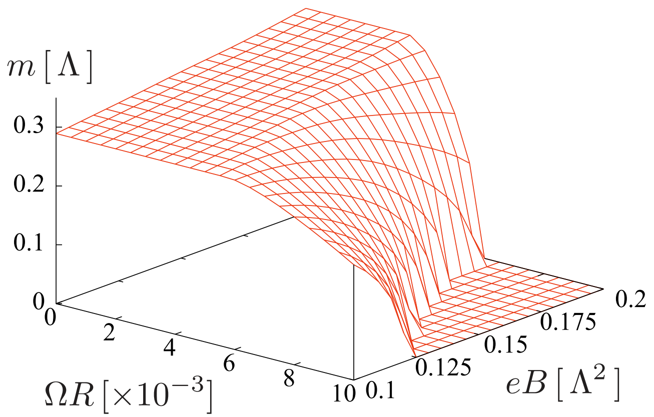
Magnetic Field

Bali *et al*. JHEP (2011)

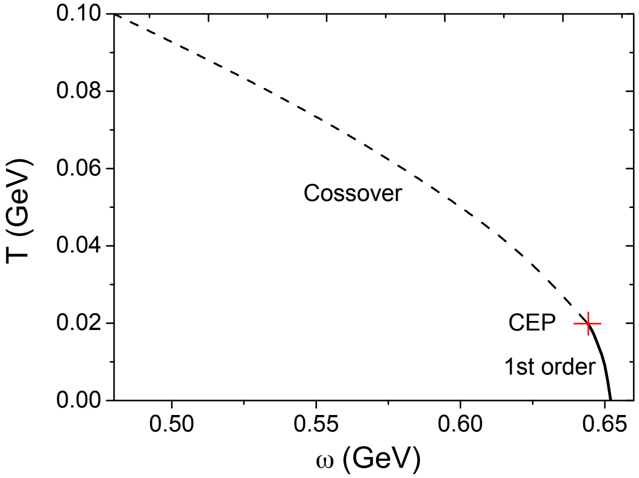
Various Good By-Products

Vorticity triggered controversies — more theory needed

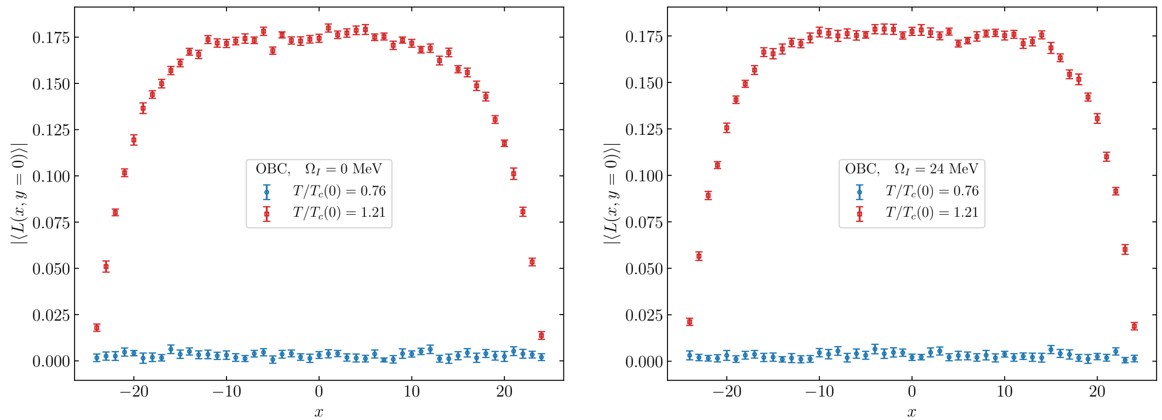
Chen-Fukushima-Huang-Mameda PRD (2015)



Jiang-Liao PRL (2016)



Lattice simulation by Braguta *et al*. PRD (2021)



Deconfinement order parameter
without *imaginary* rotation (left)
and with *imaginary* rotation (right)

$$T_c(\Omega_I^2) \to T_c(\omega^2 = -\Omega_I^2)$$

Various Good By-Products

Vorticity triggered controversies

Jinfeng and I would say rotation favors deconfinement, while Victor says opposite.

Revisited this problem using a HRG (thermal) model:

$$p_i^{\pm} = \pm \frac{T}{8\pi^2} \sum_{\ell=-\infty}^{\infty} \int dk_r^2 \int dk_z \sum_{\nu=\ell}^{\ell+2S_i} J_\nu^2(\underline{k_r r}) \ \log\{1 \pm \exp[-(\varepsilon_{\ell,i} - \mu_i)/T]\}$$

Inhomogeneous

$$\varepsilon_{\ell,i} = \sqrt{k_r^2 + k_z^2 + m_i^2} - (\ell + S_i)\omega$$

$\boldsymbol{J \cdot \omega}$

Theoretically speaking, an equivalent setup to get the global polarization for equilibrated matter.

OAM is discarded by looking at $r \simeq 0$ only.

Even if ω is small, large r leads to large OAM (restricted by causality).

Various Good By-Products

Vorticity triggered controversies

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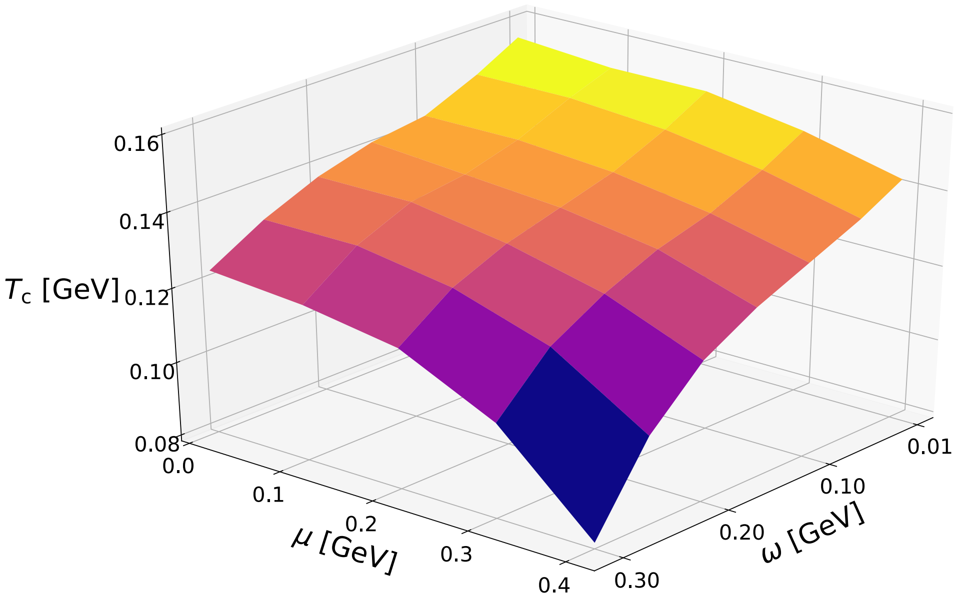
Revisited this problem using a HRG (thermal) model:

Fujimoto-Fukushima-Hidaka PLB (2021)



Also estimated the moment of inertia (dominated by OAM)

$$dI(r) \sim \sigma T^4 r^2 dV$$

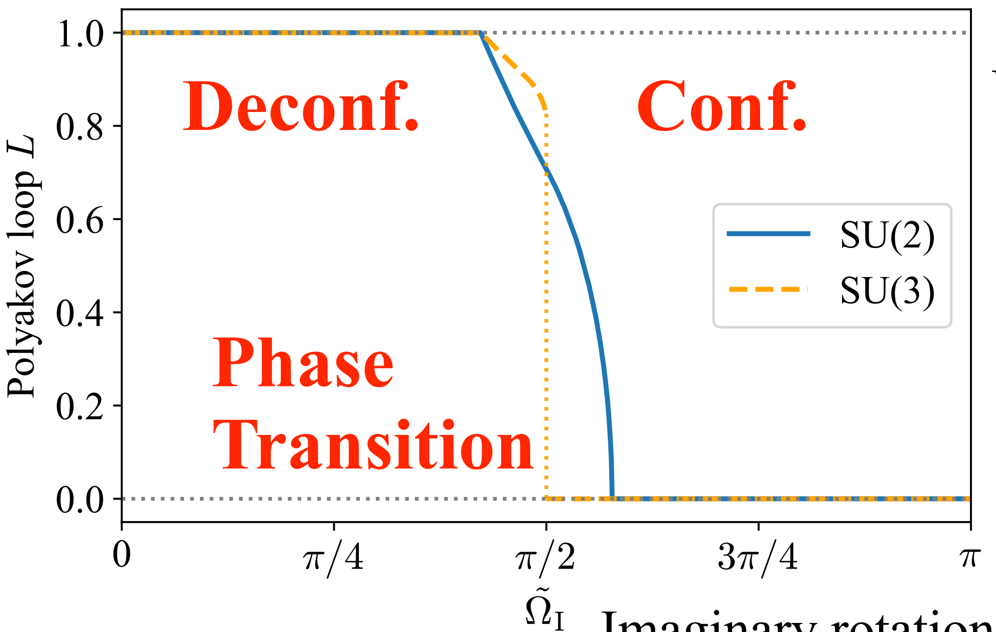
Coefficient given by the enthalpy density (~ mass density).
In principle, a measurable observable

Various Good By-Products

Another QCD-based calculation

Chen-Fukushima-Shimada PRL Editor's Suggestion (2022)

Deconfinement order parameter is reliably calculable at high enough temperature as a function of imaginary rotation.



Imaginary rotation as large as T

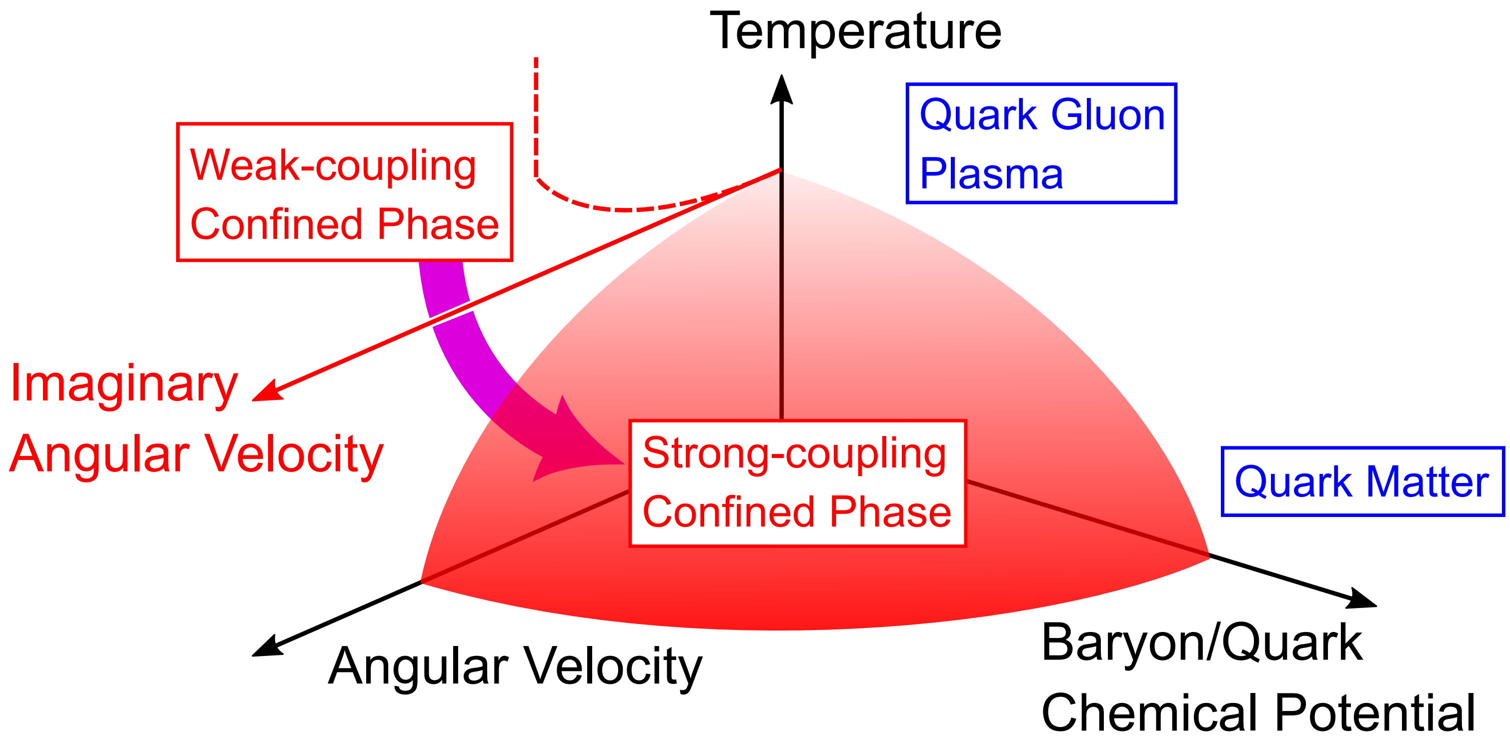
$$V(\boldsymbol{\phi};\tilde{\Omega}_\mathrm{I}) = -\frac{2T^4}{\pi^2}\sum_{\boldsymbol{\alpha}}\sum_{l=1}^{\infty}\frac{\cos(l\boldsymbol{\phi}\cdot\boldsymbol{\alpha})\cos\left(l\tilde{\Omega}_\mathrm{I}\right)}{\left\{l^2+2\tilde{r}^2\left[1-\cos\left(l\tilde{\Omega}_\mathrm{I}\right)\right]\right\}^2}$$

Very singular expression
Analytic continuation questioned
(Analytic distillation won't work)

Various Good By-Products

Another QCD-based calculation

Chen-Fukushima-Shimada PRL Editor's Suggestion (2022)



Imaginary chemical potential doesn't realize this due to the RW PT.

Various Good By-Products

Another QCD-based calculation

Chen-Fukushima-Shimada PRL Editor's Suggestion (2022)

$$V(\phi;\tilde{\Omega}_{\mathrm{I}}) = -\frac{2T^4}{\pi^2}\sum_{\boldsymbol{\alpha}}\sum_{l=1}^{\infty}\frac{\cos(l\boldsymbol{\phi}\cdot\boldsymbol{\alpha})\cos\left(l\tilde{\Omega}_{\mathrm{I}}\right)}{\left\{l^2+2\tilde{r}^2\left[1-\cos\left(l\tilde{\Omega}_{\mathrm{I}}\right)\right]\right\}^2}$$

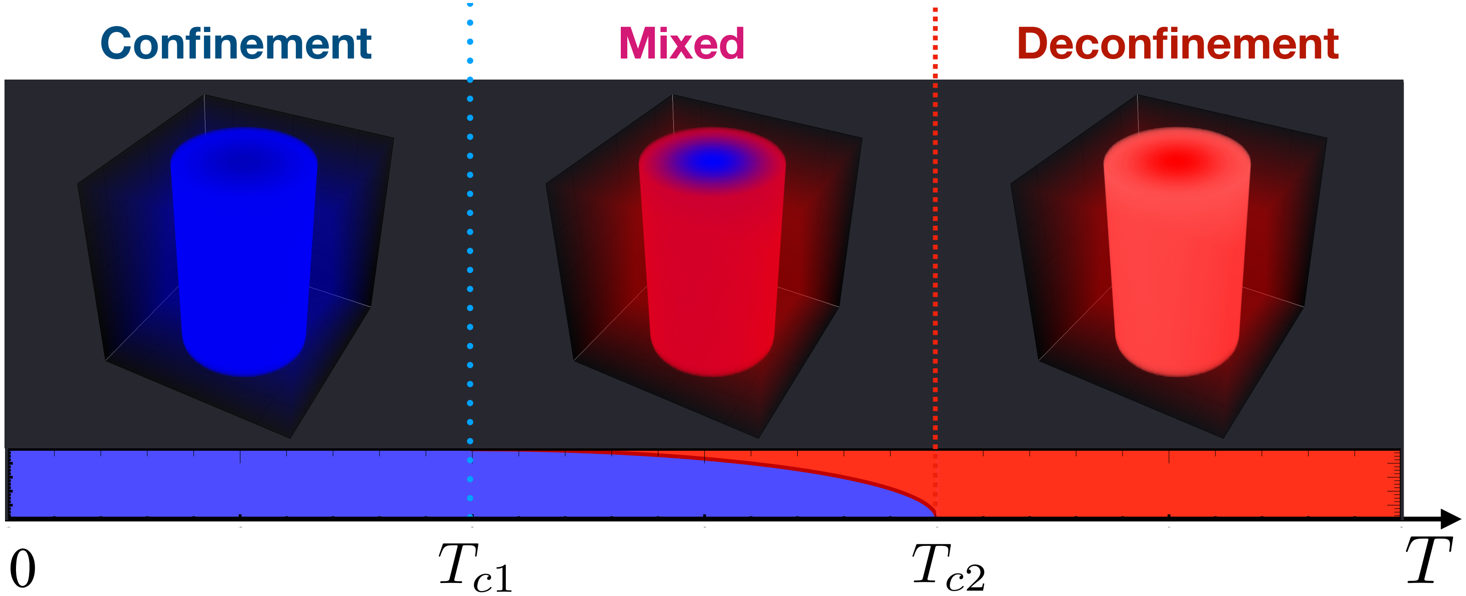
Perturbative expression implies that the analytic continuation is quite problematic (at finite T — Euclidean — and real rotation ω — Minkowskian — simultaneously). Not surprising because of the causality bound.

Lattice results may be dominantly affected by the boundary condition needed for the causality.
Global equilibrium with ω may be sensitive to the boundary?

Various Good By-Products

Related works

Chernodub PRD (2021), (2022)



$$T(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} = T_0$$

Landau-Lifshitz gives the full explanation for this formula.

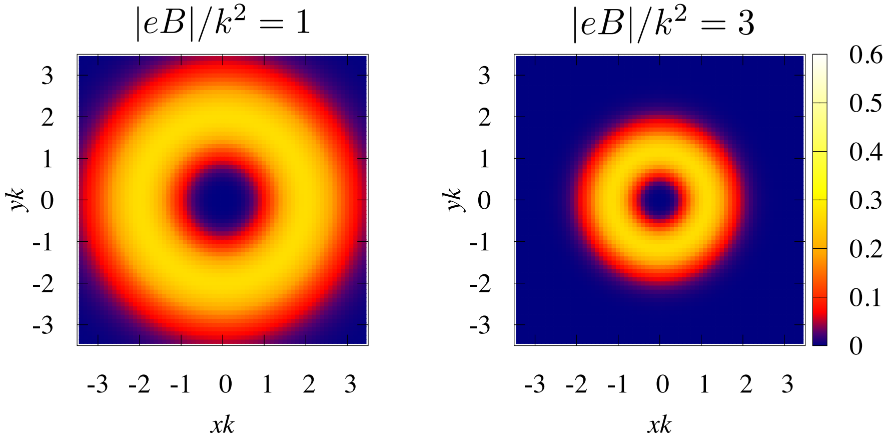
Tedious QFT calcs with l

QFT calculation with rotation in general

This kind of calculation involves complete bases with quantum number *l* of orbital angular momentum.

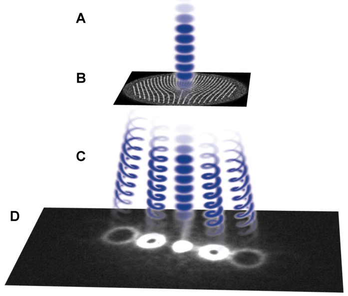
It is also possible to derive the CME formula using the rotating bases.

[Profile of twisted fermionic bases]



Fukushima-Shimazaki-Wang PRD (2020)

Motivated by electron vortex beams:



McMorran *et al*. Science (2011)

Tedious QFT calcs with l

CME — OAM mode-by-mode

Fukushima-Shimazaki-Wang PRD (2020)

For one particle states the mode-by-mode relations read:

$$j^{z(\uparrow)}_{n,l,k} = \rho^{(\uparrow)}_{5\,n,l,k} \qquad j^{z(\downarrow)}_{n,l,k} = -\rho^{(\downarrow)}_{5\,n,l,k}$$

$$\bar{j}^{z(\uparrow)}_{n,l,k} = -\bar{\rho}^{(\uparrow)}_{5\,n,l,k} \qquad \bar{j}^{z(\downarrow)}_{n,l,k} = \bar{\rho}^{(\downarrow)}_{5\,n,l,k}$$

Ordinary CME is not directly related to density.
The relation between j_5 and ρ (not ρ_5) is much more complicated (cf. the CSE formula).
If spin is asymmetric and a finite *l* is favored, then…

Anyway, we need chirality…

Please do not forget:

CME needs not only *B* but also chirality!

$$\frac{dn_5}{dt} + \boldsymbol{\nabla}\cdot\boldsymbol{j}_5 = \frac{q^2}{2\pi^2}\boldsymbol{E}\cdot\boldsymbol{B}$$

Gauge backgrounds are the source for the chirality (and the axial current - spin)

$$\boldsymbol{E}\cdot\boldsymbol{B} = \partial_\mu K^\mu$$

CME/CSE Optical Spin… Why?

$$= \frac{d}{dt}(\boldsymbol{A}\cdot\boldsymbol{B}) + \boldsymbol{\nabla}\cdot(A_0\boldsymbol{B} + \boldsymbol{E}\times\boldsymbol{A})$$

Anyway, we need chirality…

Chiral anomaly looks like a continuity equation with respect to the spin part only.
Nobody knows why it should be so…

Jaffe-Manohar NPB (1990)

constructs the rotation generators. The spin contribution to $\boldsymbol{J}_{(g)}$ is $\int d^3x\,(\boldsymbol{A}\times\boldsymbol{E})$. The analogous integral over $\epsilon^{\mu\nu\lambda\sigma}K_\sigma$ contains an additional term: $\int d^3x\,A^0\boldsymbol{B}$. In $A^0=0$ gauge the two expressions agree. The same is true for the analogous

……

(iii) What is the relation between the gluon spin and the anomalous current

$$K_\sigma=\epsilon_{\sigma\alpha\beta\gamma}\,\mathrm{Tr}\,A^\alpha\left(F^{\beta\gamma}-\tfrac{2}{3}A^\beta A^\gamma\right). \tag{6.1}$$

Anyway, we need chirality…

I still do not understand why it should so, but this is what it is, and this is why only the spin part appears in anomaly induced phenomena.

Is there any other simple gauge configuration that can induce chirality?

LPV from Standing Waves

Electromagnetic plane waves with parallel electric and magnetic fields E||H in free space

Koichi Shimoda, Toshio Kawai, and Kiyoji Uehara
Department of Physics, Keio University, 3-14-1 Hiyoshi, Kohokuku, Yokohama 223, Japan

(Received 31 October 1988; accepted for publication 7 June 1989)

Maxwell's equations for electromagnetic waves in free space, with parallel electric and magnetic fields, are solved under the condition that the fields are functions of z and t, independent of either x or y. A number of interesting examples of plane waves obtained from the general solution are shown. Their properties and the vanishing Poynting vector are discussed.

Usually, the electromagnetic waves (as solutions of the free/source-less Maxwell equations) have propagating E and B perpendicular to each other. However, even plane wave solutions can have nonzero $E \cdot B$ as they are.

LPV from Standing Waves

They found a family of solutions:

$$E_x = \cos(F+G)\cos(F-G)$$

$$E_y = \cos(F+G)\sin(F-G)$$

$$H_x = \sin(F+G)\cos(F-G)$$

$$H_y = \sin(F+G)\sin(F-G)$$

F and *G* are arbitrary functions of $z+t$ and $z-t$

Standing Wave

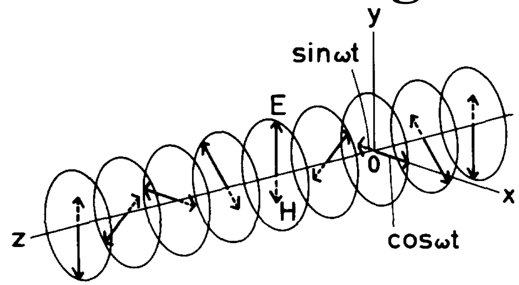
These are not general solutions, and other types of solutions do exist.

Parity-odd domains (local parity violation) may occur locally but its spatial average is zero not to break parity.

LPV from Standing Waves

A particularly interesting example:

Helical Standing Wave



$$F = \frac{k}{2}(z+t) \qquad G = -\frac{k}{2}(z-t)$$

An example of the vector potential (giving *E* and *B*)

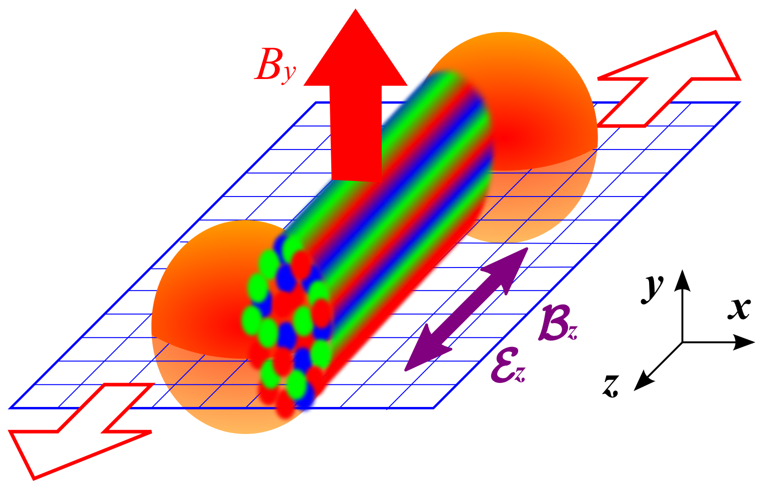
$$A_0 = 0 \qquad \boldsymbol{A} = \frac{a}{\omega}(-\cos\omega z \sin\omega t, \cos\omega z \cos\omega t, 0)$$

$$\boxed{\boldsymbol{E}\cdot\boldsymbol{B} = \frac{a^2}{2}\sin 2\omega z} \qquad \boxed{(\boldsymbol{E}^2 - \boldsymbol{B}^2) = a^2\cos 2\omega z}$$

Twisted-modes: Evtuhov-Siegman (1965) / Chu-Ohkawa (1982)

LPV from Standing Waves

Usually, in the heavy ion collision, the chirality source is assumed to be the Glasma initial condition, that appears as a result of non-linear self-interacting nature of gluons.



But, even the linearized theory (like the Maxwell eqs.) can easily accommodate solutions with LPV.
Helical gluonic background in non-central geometry?

LPV from Standing Waves

A more nontrivial feature:

$$K^\mu = \frac{e^2}{4\pi^2}\varepsilon^{\mu\nu\rho\sigma}A_\nu\partial_\rho A_\sigma$$

$$K^0 = 0 \qquad K^z = \frac{e^2}{4\pi^2}\frac{a^2}{\omega}\cos^2\omega z$$

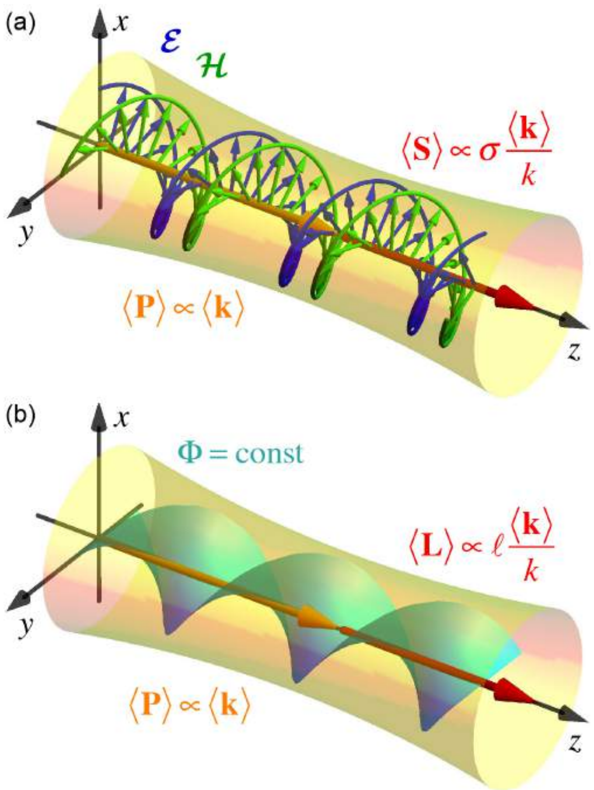
Only the spatial component

Usually, for parallel *E* and *B*, only the magnetic helicity (charge) is nonzero, which induces the chirality, and its coupling to *B* causes the CME current.
With this helical standing wave, only the current appears which may directly contribute to the current.

Such a possibility of gluonic background is not yet studied enough.
MV model needs to be upgraded (like helicity-MV model).

Further Interesting Possibility

Bliokh-Nori Phys.Rept. (2015)



Helical gauge fields have been intensively investigated.

Polarization

Decomposition is well defined in the "paraxial limit".

Photon vortex beam
with finite orbital angular mom.

This is an ideal testing ground for our idea of CME!
We are working on this system (in progress).

Summary

- **Rotation as well as magnetic field has a big impact to a wide context of QCD physics**
  - □ For theoretical calculation the imaginary vorticity (angular velocity) must be analytically continued.
  - □ Imaginary vorticity itself is an interesting device.
- **Mode-by-mode calculation is unavoidable**
  - □ Angular momentum decomposition simplifies CME.
  - □ Easier to see a finite mode CME?
- **Helical gauge backgrounds generated everywhere**
  - □ Even Maxwell eqs. can accommodate such solutions.
  - □ CME from helical standing gluonic backgrounds?