# *Anomalous Chiral Phenomena*
## Challenges and Perspectives for the Future

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Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions
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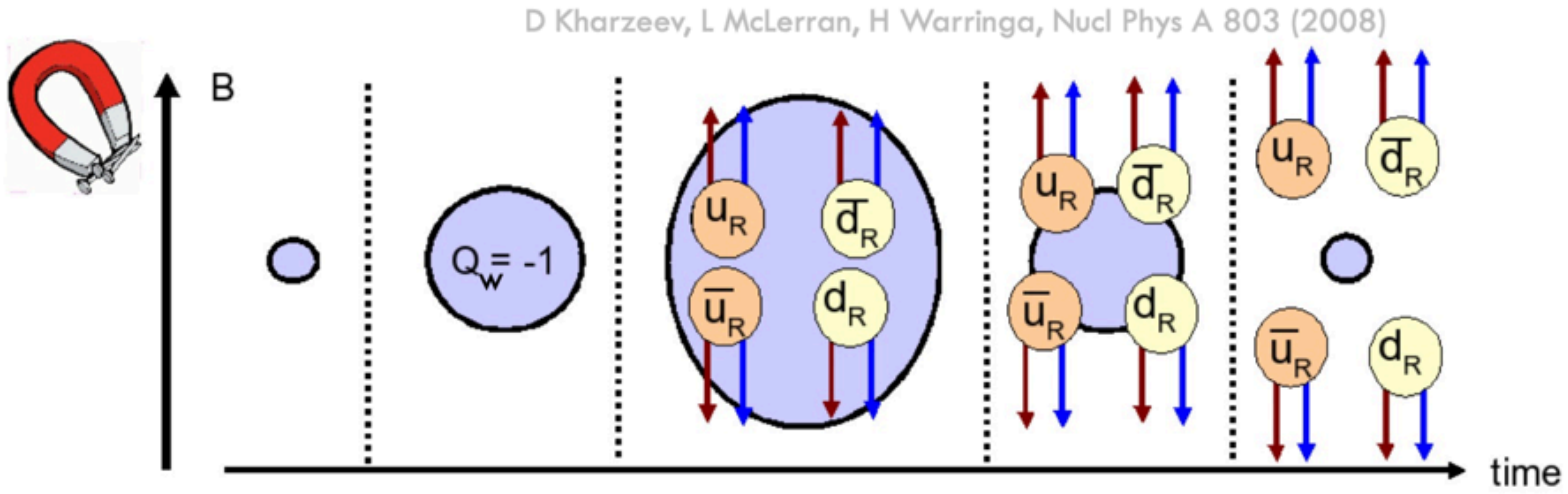
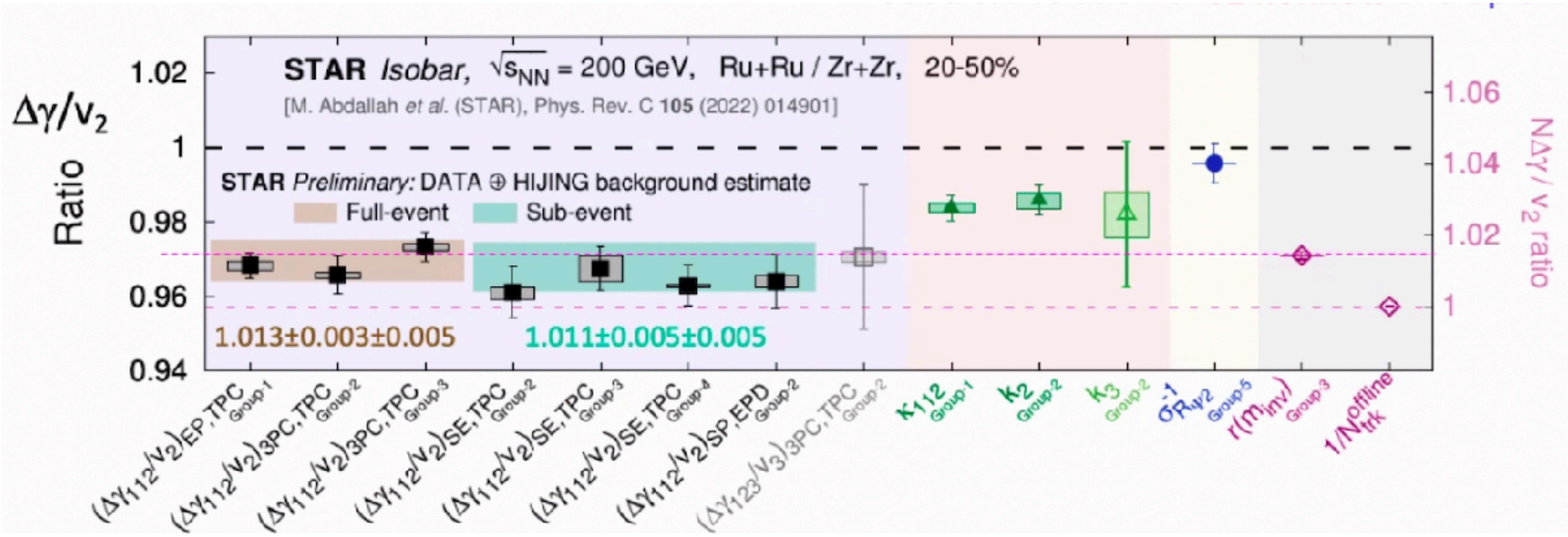
# Anomalous chiral phenomena

- Anomalous chiral current phenomena in the presence of strong magnetic fields, such as the CME and CMW, are an integral property of QCD. They are expected to manifest in certain event-by-event fluctuations under conditions of chiral symmetry restoration.




- The isobar comparison run provided the most stringent test yet for anomalous chiral current fluctuations in heavy ion collisions.



# A very large investment

- The STAR data from the Zr - Ru isobar comparison run constitute a very large investment in time, effort, and money:
  - One full year of RHIC operations (~$200M)
  - Development of the capability to switch between nuclei at every store
  - Dedicated ion source development
  - Dedicated four-months running of the new stable isotope separation facility at ORNL to produce enriched $^{96}$Ru, plus multi-year R&D
  - Multi-months stable operation of STAR
- A conservative estimate of the total cost of the experiment is $300M.
- This is approximately the cost of a dedicated 1-ton neutrino less double beta decay experiment
- What will the heavy ion theory community do with the data?

# Charge separation

- Searches for other manifestations of AChP in the isobar comparison run data will continue, as will searches in other data.
- But it's time for theorists to seriously explore what the new experimental limits tell us.
- Purpose of this talk: How can this be done?

Anomalous current: $\mathbf{J} = \sum_f \frac{(Q_f e)^2}{2\pi^2} \mu_5 \mathbf{B} \equiv C\mu_5 \mathbf{B}$ with $\mu_5 = \frac{3n_5}{T^2}$ ($n_5$ = axial number density)

In QCD, $n_5$ is determined by the winding number density of the gluon field: $n_5 = -\int dt \frac{g^2}{8\pi^2} \mathbf{E}^a \cdot \mathbf{B}^a$

Current conservation $\partial\rho/\partial t = -\nabla \cdot \mathbf{J}$ tells us that the separated charge $\Delta Q$ is

$$\Delta Q = \frac{3Cg^2}{8\pi^2 T^2} \int d^3x \int dt\, \mathbf{B} \cdot \nabla \int dt'\, \mathbf{E}^a \cdot \mathbf{B}^a$$

A rigorous description including charge transport is provided by anomalous hydrodynamics.

# Analysis of CME Limits - General considerations

$$\Delta Q = \frac{3Cg^2}{8\pi^2 T^2} \int d^3x \int dt\, \mathbf{B} \cdot \nabla \int dt'\, \mathbf{E}^a \cdot \mathbf{B}^a$$

$\Delta Q$ is determined by two factors: $\int dt\, \mathbf{B}$, and the space-time dependent fluctuations of $\mathbf{E}^a \cdot \mathbf{B}^a$.

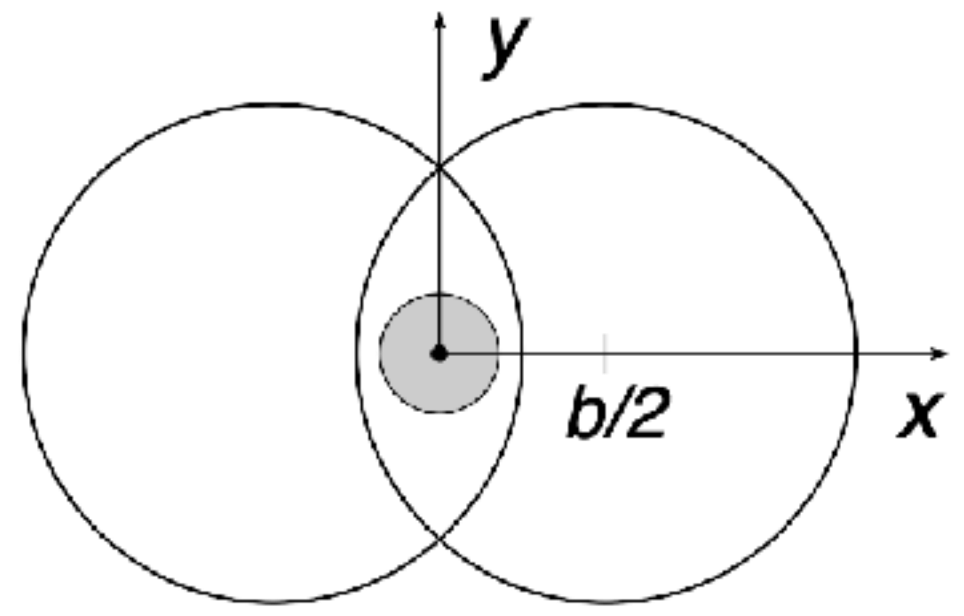
If one of these factors is under theoretical control, the data can provide constraints on the other.

- Magnetic field factor $\int dt\, \mathbf{B}$: Depends on the transport of incident charge distribution and on the response of the QGP medium to time- and space-dependent magnetic fields. The relevant time scale for the variation of this factor, apart from the original impact phase, is driven by the time scale of the QGP response. It should be possible to get a +- 50% estimate of this factor.
- Chern-Simons number density $\mathbf{E}^a \cdot \mathbf{B}^a$: Since this is a nonperturbative quantity and we consider an out-of-equilibrium scenario where lattice gauge theory only gives rough guidance, it is probably more difficult to obtain reliable predictions.
- This suggests a strategy where the magnetic field factor should be considered as controllable and the experimental bound informs our knowledge of the winding number factor.

# Analysis of CME Limits - Magnetic field 1

The contributions to the magnetic field integral: $\int dt\,\mathbf{B}$, take several different forms:

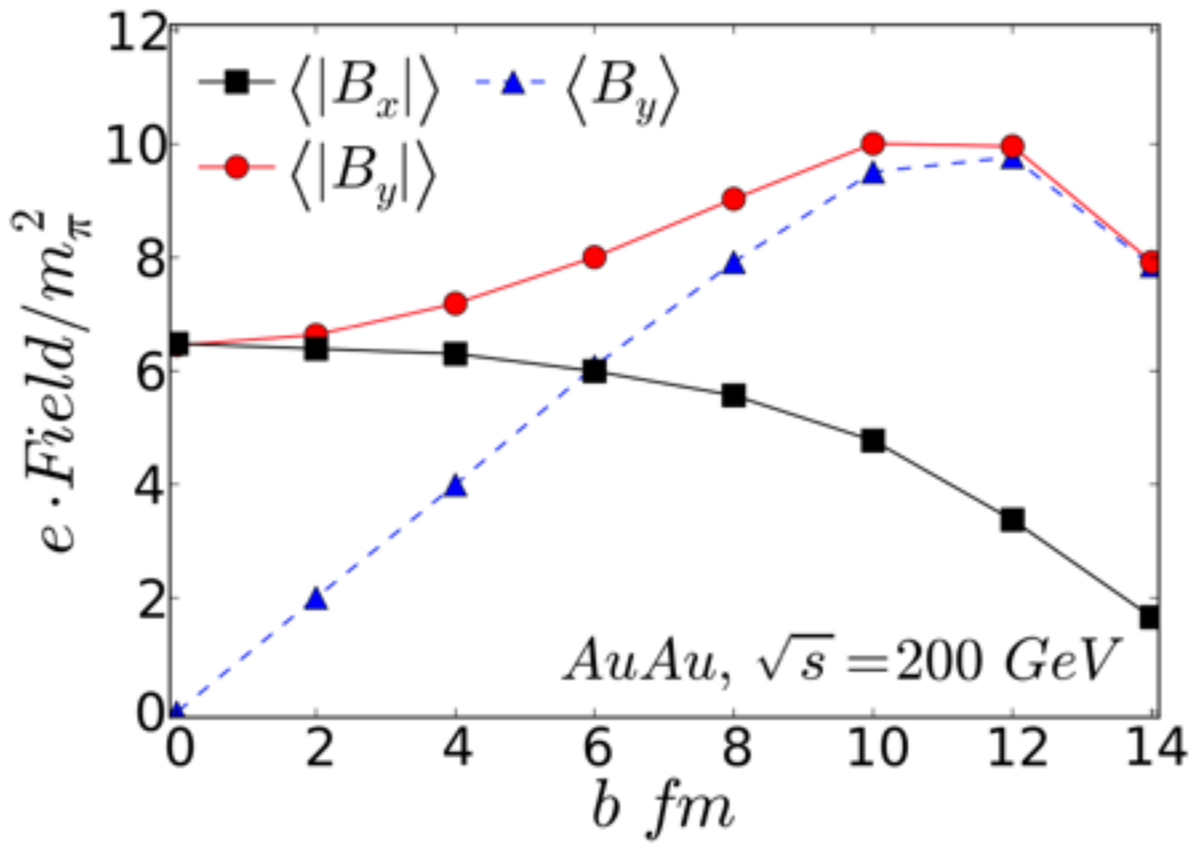
1. The Lorentz contracted colliding nuclei before the collision - time scale $R/\gamma$
2. The spectators and valence quarks in the fragmentation regions - time scale $R/\cosh y_{frag}$
3. The equilibrating glasma - time scales $1/Q_s$, $1/(eQ_s)$
4. The expanding QGP - time scales $1/T$, $1/(eT)$, $R$




A slight compilation arises from the granularity of the nucleon distribution in single events [Bzdak & Skokov, PLB 710 (2012) 171]

The rapidity distribution of charge after the collision can be obtained from the net proton distribution measured by BRAHMS at 200 GeV.

The *response* of the glasma/QGP is of order $eQ_s$, $eT$, and can be estimated using perturbative and lattice techniques. The long-term response is governed by the electric conductivity of the QGP [Tuchin, PRC 91 (2015)]



# Analysis of CME Limits - Magnetic field 2

Medium response is perturbative in the EM field, but partly nonperturbative in the medium dynamics.

In covariant gauge, the EM field is given by $A^\mu = -\frac{1}{k^2}\left(j^\mu_{\rm ext} + j^\mu_{\rm ind}\right)$ with $j^\mu_{\rm ind} = \Pi^\mu_\nu j^\nu$.

For the *B*-field, the relevant response function is $\Pi_\perp(\omega,\vec{k}) = i\sigma(\omega,\vec{k})$.

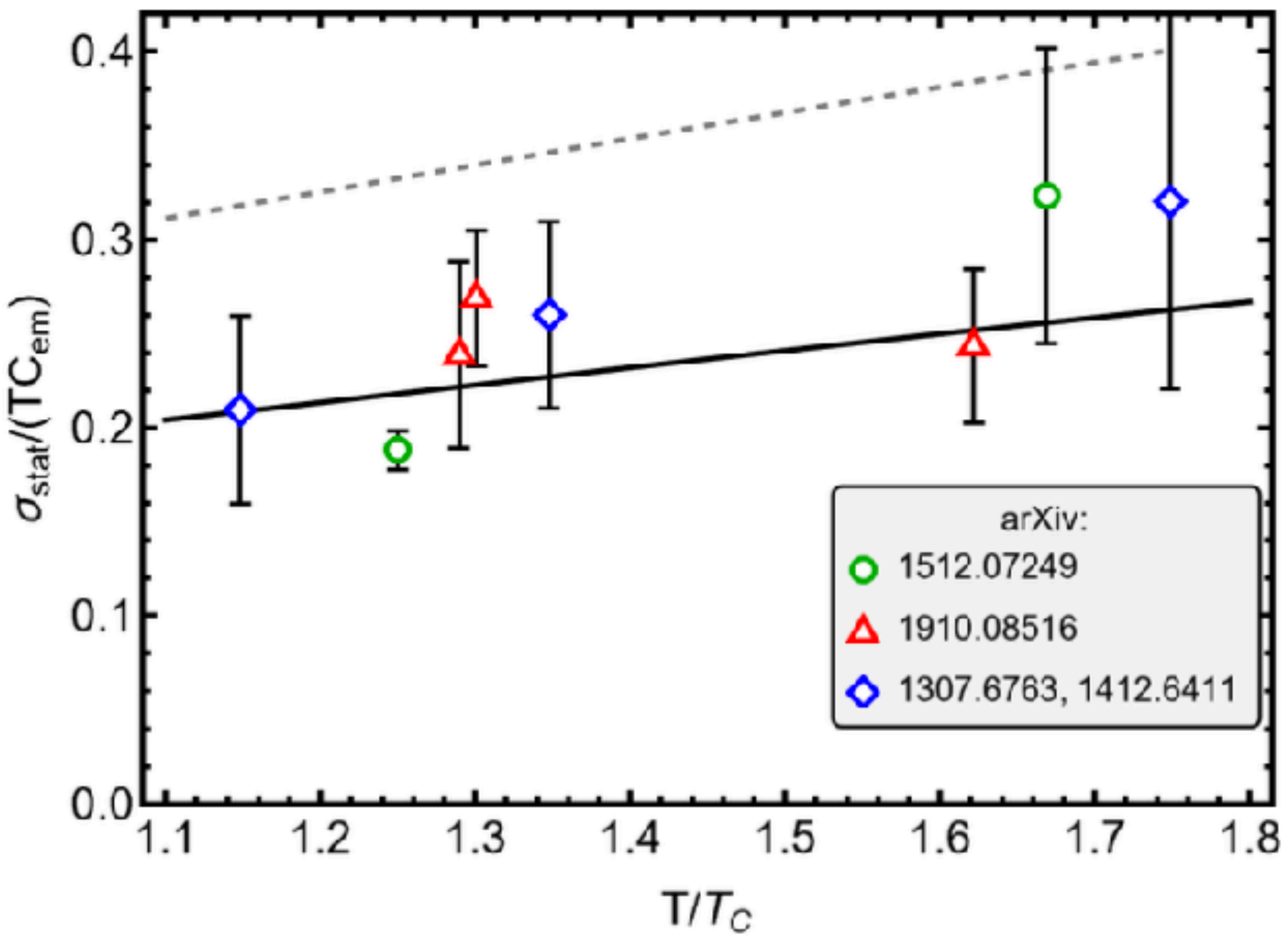
In an equilibrium QGP, the response can be calculated perturbatively for large $\omega, \vec{k}$, and on the lattice for small $\omega, \vec{k}$.

The dominant response comes from the light-cone. At weak coupling [Grayson *et al*., PRD 106 (2022) 014011]:

$$\sigma_\perp\left(\omega = |k|\right) = i\frac{m_{\rm D}^2}{4\omega}\left(\frac{\xi\ln\xi}{\omega^2\tau_c^2} + i\frac{\xi+1}{\omega\tau_c}\right)$$

with $\xi = 1 - 2i\omega\tau_c$, where $\tau_c$ is the gluon mean free time.

Lattice results for $\sigma_{\rm stat} = \sigma_\perp(0,\vec{0})$



# Analysis of CME Limits - Magnetic field 3

Induced magnetic field in QGP [Tuchin, PRC 93 (2016) 014905]:

$$|B_{\rm ind}(t)| \approx \frac{Ze\beta b\sigma}{8\pi t^2} \rightarrow \int dt\,|B_{\rm ind}| \approx \frac{Ze\beta b\sigma}{8\pi\tau_i}$$

Vacuum field contribution: $\int dt\,|B_{\rm vac}| \approx \frac{Ze\beta b}{4\pi R^2}$

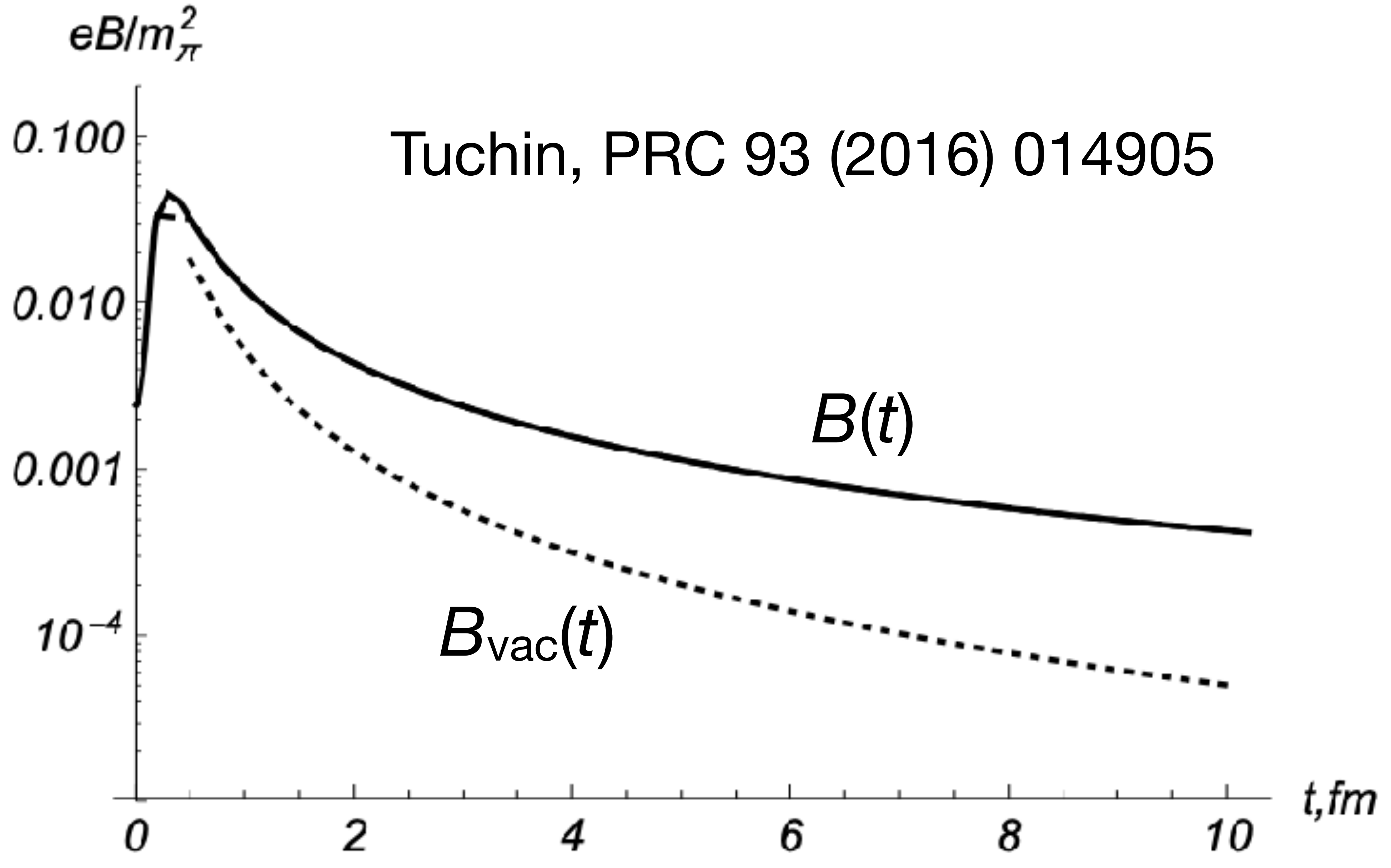
For $\sigma \approx 5\ \mathrm{MeV}$, $\tau_i = 0.5\ \mathrm{fm}/c$ both contributions are approximately equal.

For $\tau > \tau_i$ the medium contribution dominates.

The *B*-field falls off as $B(t) \approx -Zevb\sigma_0/(8\pi t^2)$.

| Time Scale | Formula | Time (fm/c) |
|---|---|---|
| Collision Time | $t_{\rm coll} = 2R/\gamma$ | 0.086[a] |
| Relaxation Time | $\tau_{\rm rel} = 1/\kappa$ | 0.36 |
| Freeze-out time | $t_f$ | 5[b] |
| Decay Time | $t_\sigma = 1/\sigma_0 = \kappa/\omega_p^2$ | 59[c] |

Relevant time scales for *B*-field dynamics.
[Grayson *et al*. PRD 106 (2022) 014011]

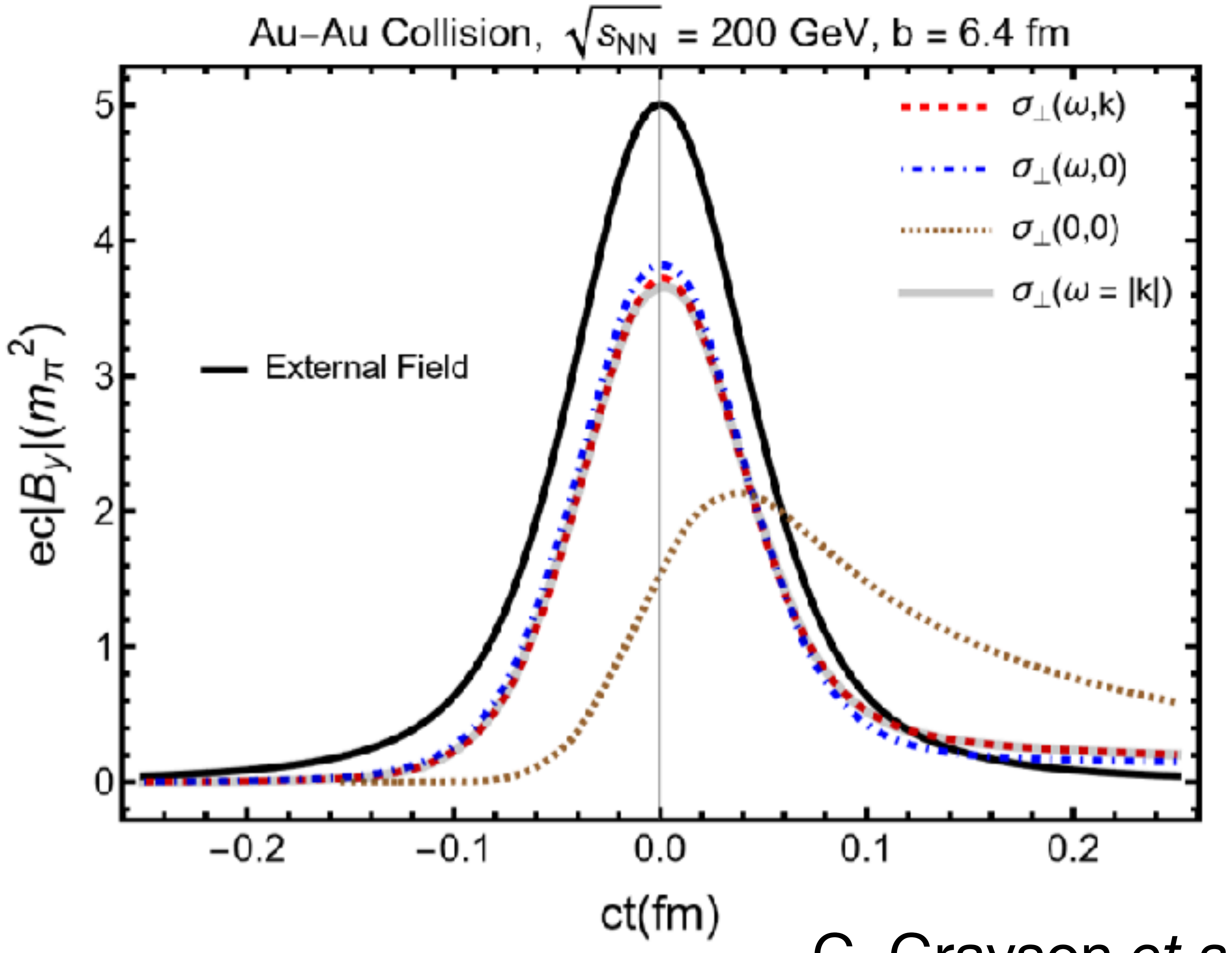


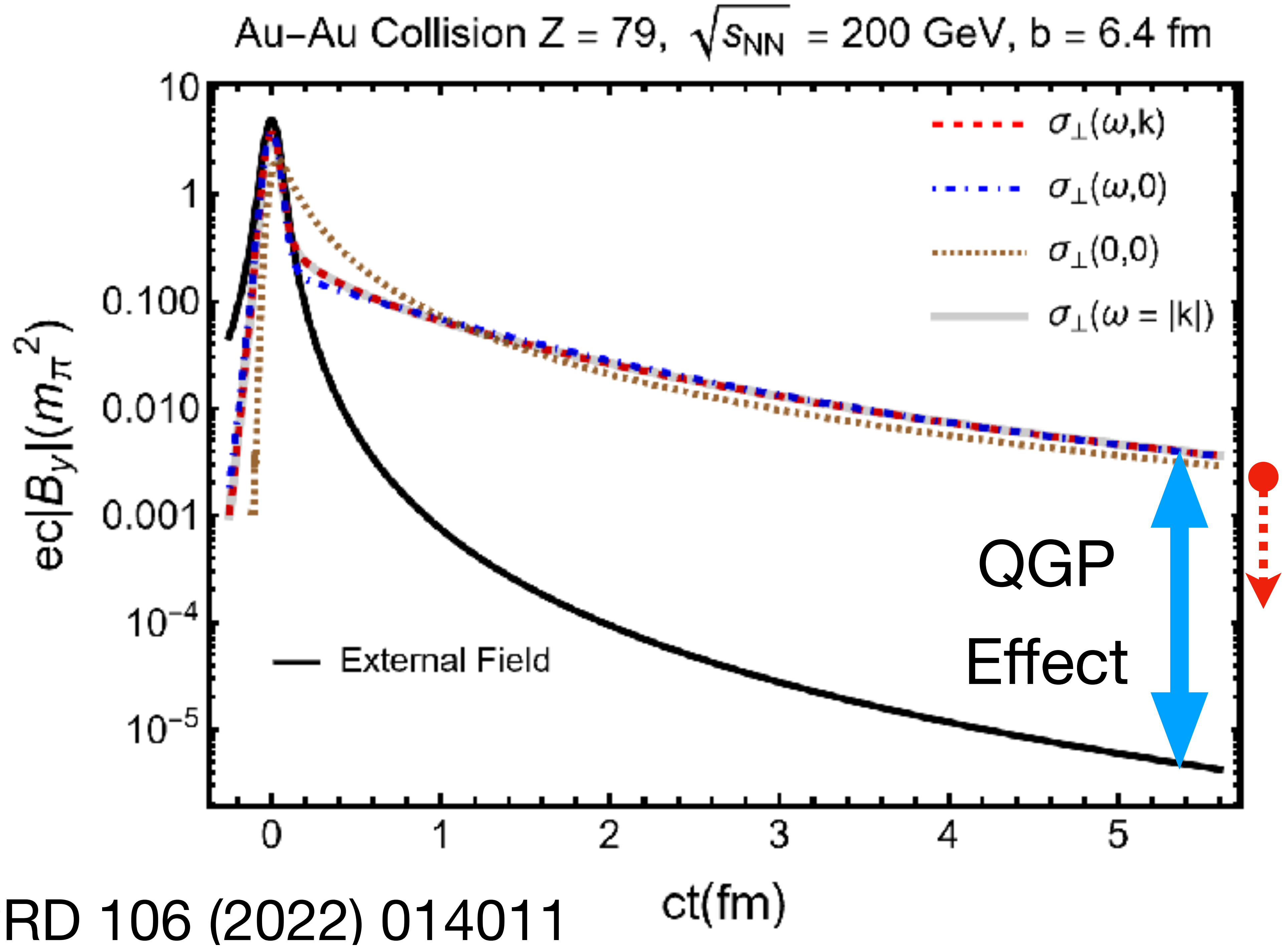
# Analysis of CME Limits - Magnetic field 4

Grayson *et al.* (2022) performed QGP response calculation with full $(\omega, \vec{k})$ dependence of $\Pi_\perp$.

Simplifying (unrealistic) assumptions: Infinite, static, homogeneous QGP.

Other response calculations: Gürsoy, Kharzeev, Rajagopal, PRC 89 (2014) 054905; Inghirami *et al.*, EPJC 80 (2020) 293; Yan & Huang, arXiv: 2104.00831; Wang *et al.,* PRC 105 (2022) L041901.




Bound from $P_\Lambda \approx P_{\bar{\Lambda}}$

BM & A. Schäfer, PRD 98 (2018) 071902

C. Grayson *et al*. PRD 106 (2022) 014011

# Analysis of CME Limits - Magnetic field 5

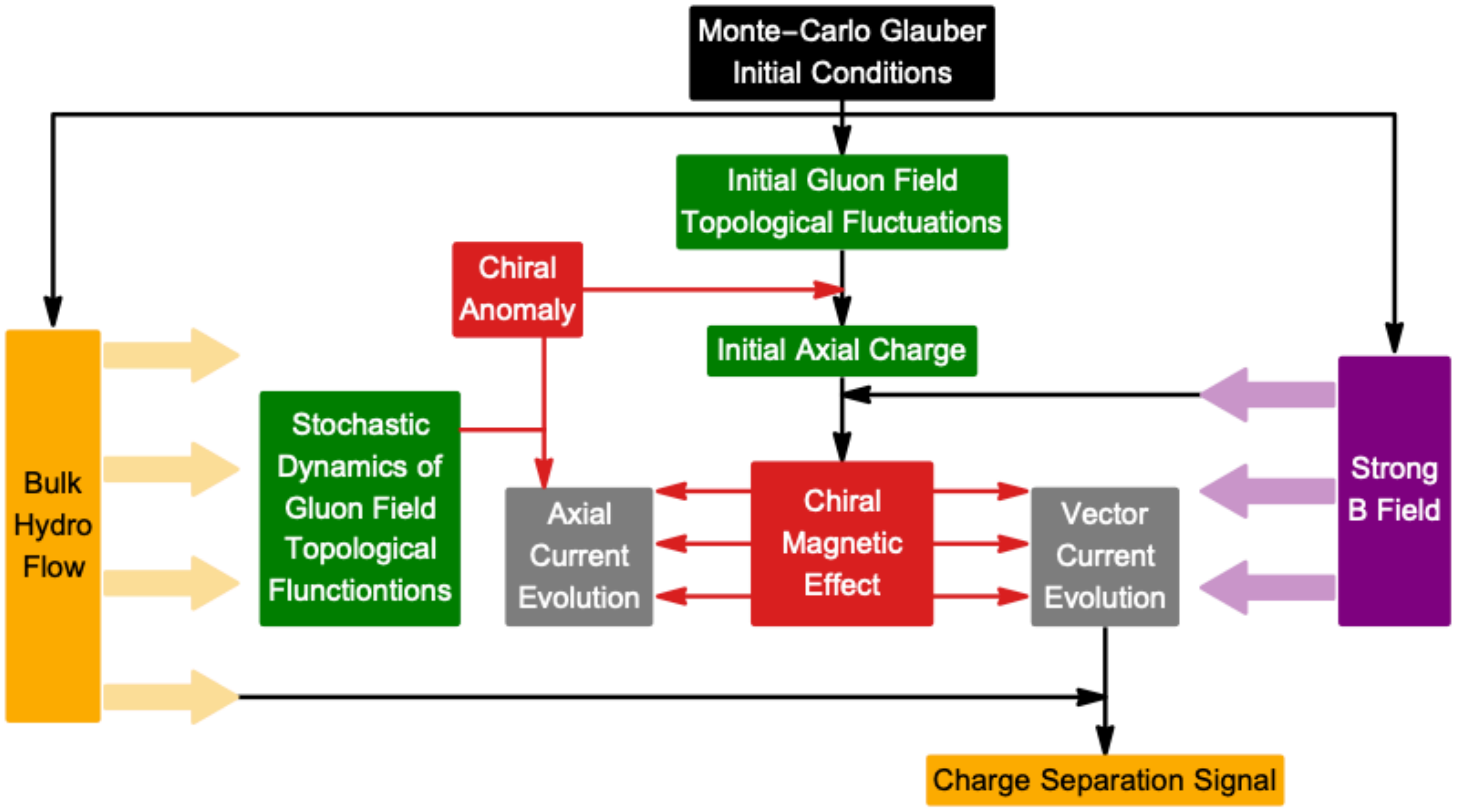
**Conclusion:** A realistic calculation of *B*(*x,t*) should be possible with current techniques.

A missing piece is a perturbative calculation of the electromagnetic response of the glasma during its equilibration toward a QGP. Since this occurs mainly before $\tau = 0.5$ fm/*c*, its accuracy is probably not a critical limitation for the overall uncertainty of the calculation.

A full (3+1)-dimensional simulation at the $\pm 50\,\%$ level should be possible, building on code infrastructure created by the BEST Collaboration.

It is clear that this would require a major effort.

This infrastructure has already been used to calculate the transport of the axial charge density in a heavy ion collision [Shi, Jiang, Lilleskov, Liao, Ann. Phys. 394 (2018) 50].



# Analysis of CME Limits - Axial number density 1

There are multiple sources to the Chern-Simons number density $\mathbf{E}^a \cdot \mathbf{B}^a$

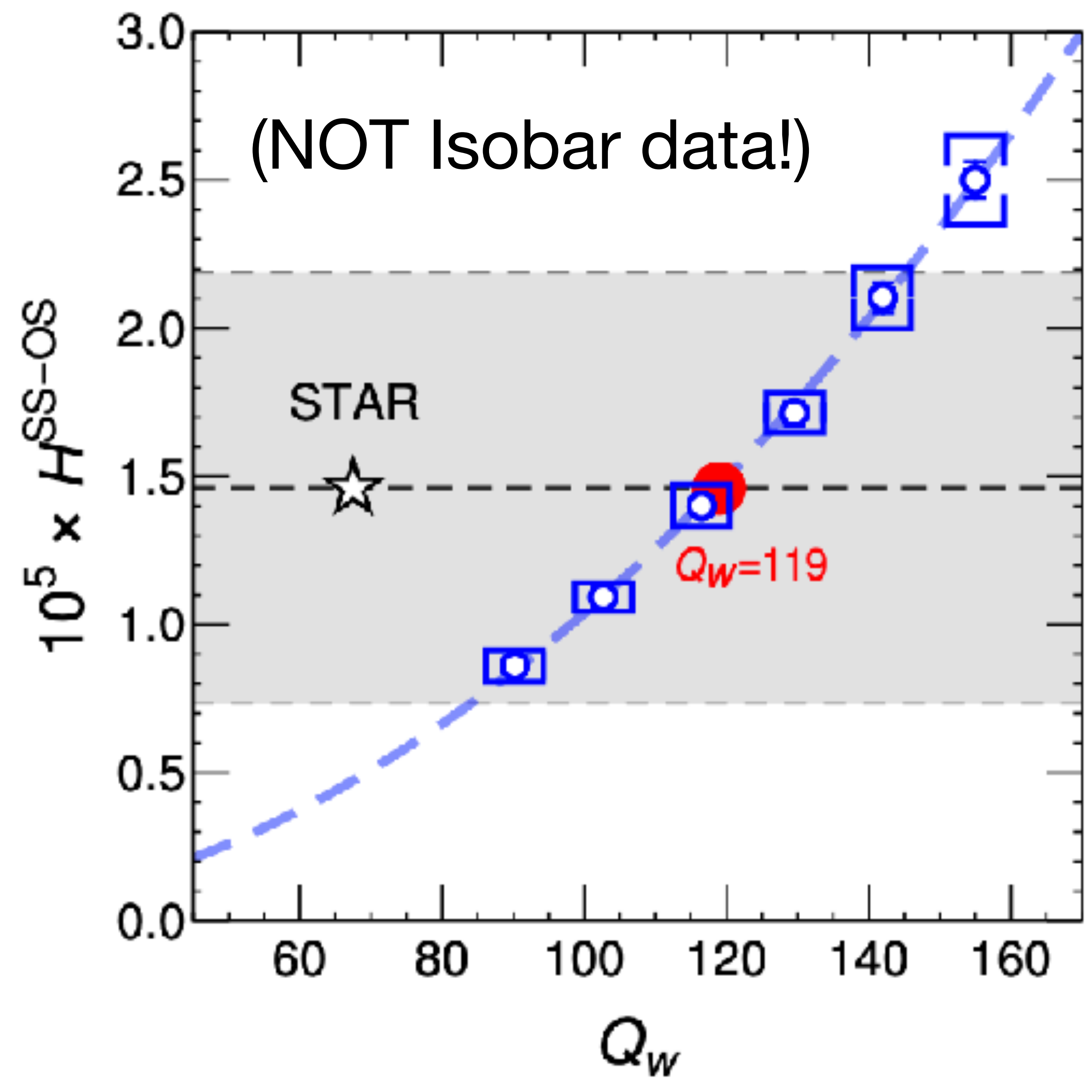
- Initial gluon configurations in the colliding nuclei (CGC) [Lappi and Schlichting, PRD 97 (2018) 034034]
- Fluctuations during the glasma phase [e.g. Kharzeev, Krasnitz & Venugopalan, PLB 545 (2002) 298; Guerrero-Rodriguez & Lappi, PRD 104 (2021) 014011]
- Chern-Simons number diffusion in the thermal QGP phase [Moore & Tassler, JHEP 02 (2011) 105]

Anomalous hydrodynamics with anomaly and relaxation term: [Shi, Jiang, Lilleskov, Liao, Ann. Phys. 394 (2018) 50]

$$\partial_\mu j_5^\mu = -\frac{N_c}{2\pi^2}\sum_f e_f^2 \vec{E}\cdot\vec{B} - \frac{n_5}{\tau_{\rm CS}}$$

Figure shows $H_{\text{SS-OS}}$ observable versus initial axial charge $Q_w$.

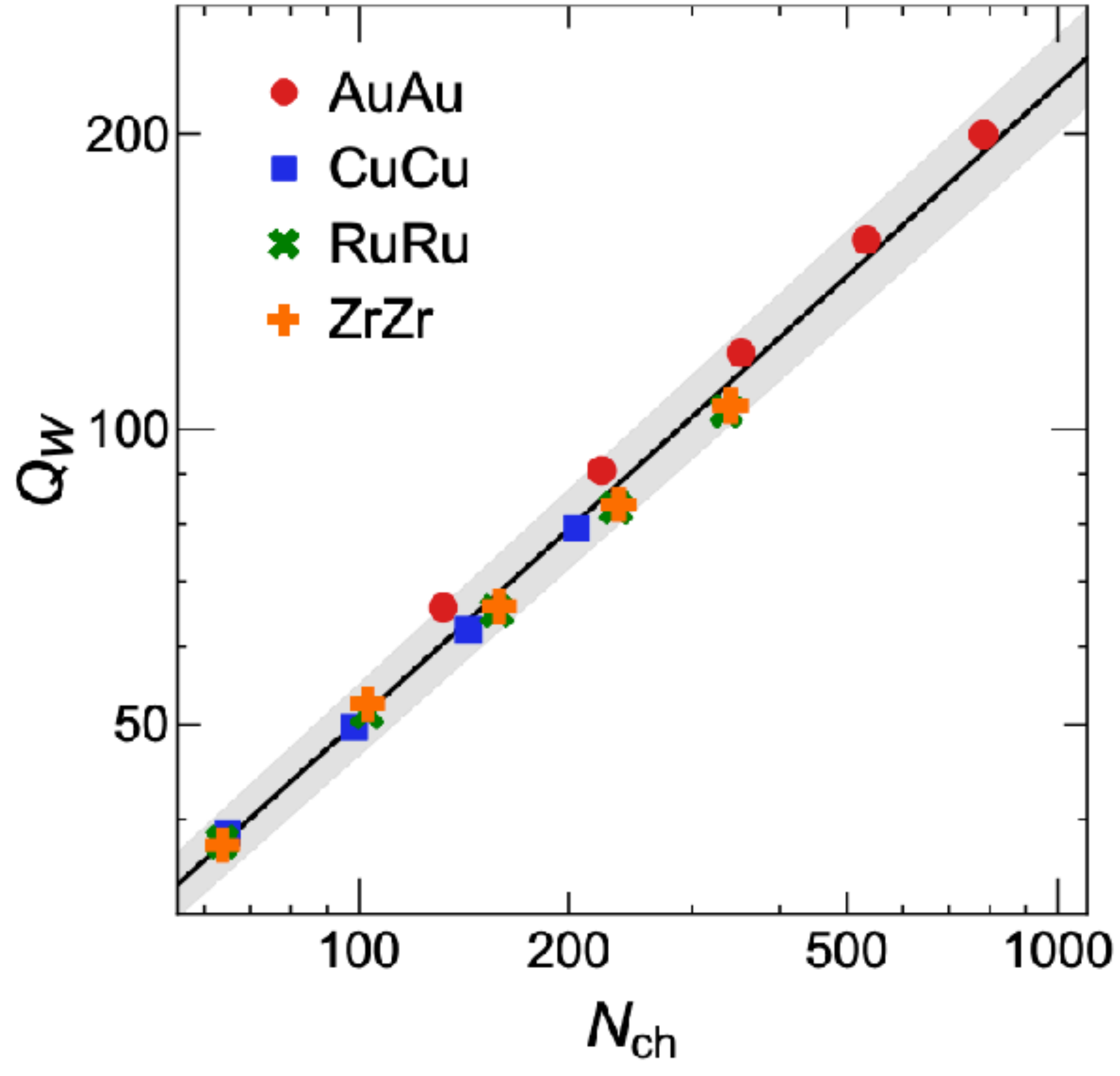
What is missing in this figure is the theoretical uncertainty from the uncertainties of magnetic field and axial density transport.



# Analysis of CME Limits - Axial number density 2

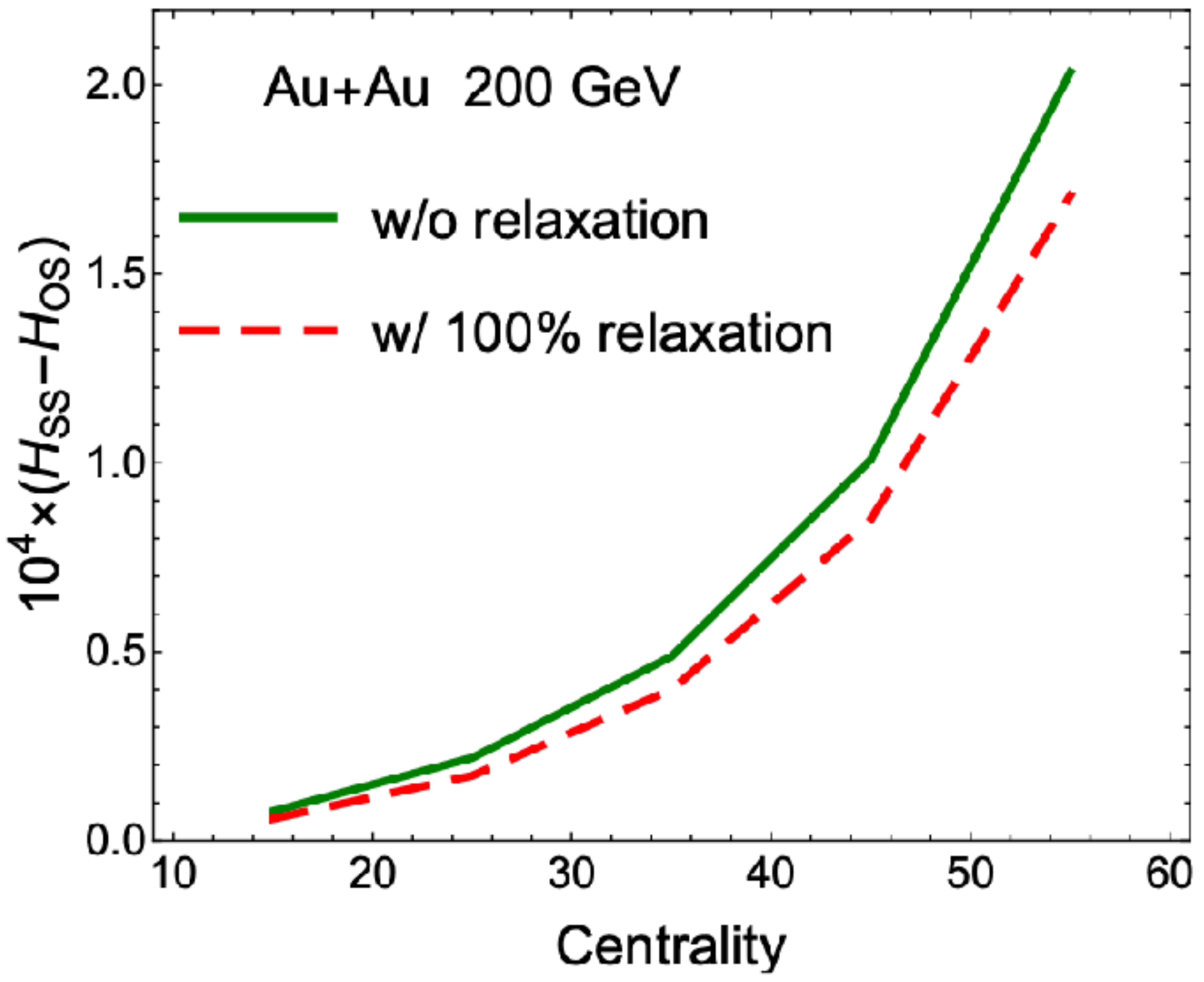
Initial winding number is determined by the number of glasma flux tubes and scaled with $N_{ch}$:

$$Q_w = \alpha N_{\rm ch}^{\beta} \text{ with } \alpha \approx 2.5, \beta \approx 2/3$$




Relaxation effect is found to be small

Shi, Jiang, Lilleskov, Liao, Ann. Phys. 394 (2018) 50



# Summary

- With reasonable effort, reliable estimates of the space- and time-dependent magnetic field including fully dynamical response of the QGP, should be possible to obtain.
- Experimental limits on CME can then set an upper bound on winding number fluctuations over the course of the heavy ion collision.
- Parameters: $Q_w^{\rm ini}$ (or $n_5/s$), diffusion constant and relaxation time for $n_5$. Others?
- How valuable would this information be?
- Are there other ways to determine the initial winding number?
- Certainly the tremendous effort invested in the experimental measurement justifies a commensurate theoretical effort.

- In the meantime, the experimental search will for nonzero anomalous chiral phenomena continue. A detected signal would be better than an upper limit.
- The latest predictions of B(t) also suggest that a detection of $B$ at the moment of hadronization via $\Lambda, \overline{\Lambda}$ polarization may be within reach.