Magnetic Field at BES Energies

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Chirality Retreat Meeting

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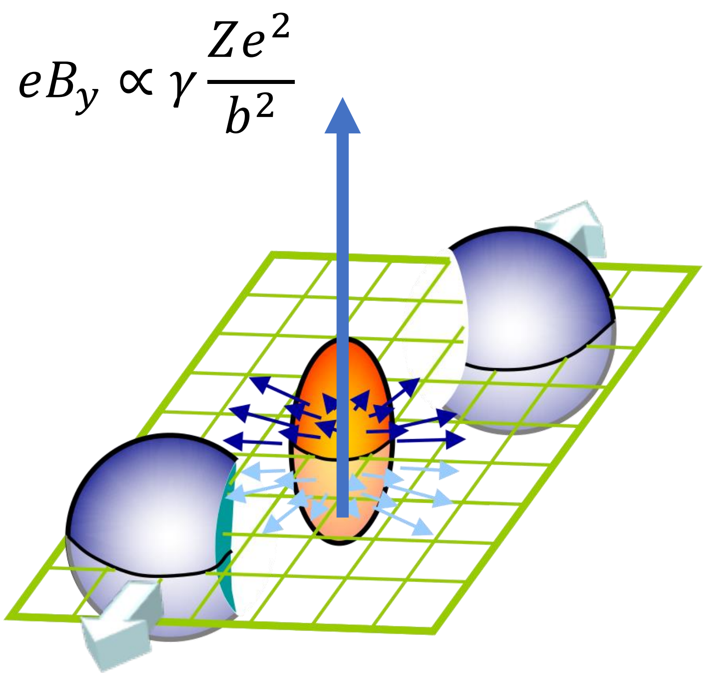
Contents

Introduction

- Strong Magnetic field



- **Strongest magnetic field**

  RHIC: $eB_y \sim 10^{18}$ Gauss @ 200 GeV
  LHC: $eB_y \sim 10^{19}$ Gauss @ 2760 GeV

- **Physics:**
  - Hyperon spin polarization
  - Vector meson spin alignment
  - Anomalous transport, e.g. CME, CMW…
  - Breit-Wheeler process $\gamma\gamma \to l^+ l^-$ and photon polarization
  - Direct flow of D^0 meson
  - …

Question:

Can we give some theoretical constrains on the magnetic field at the QGP evolution stage?

Review on magnetic field

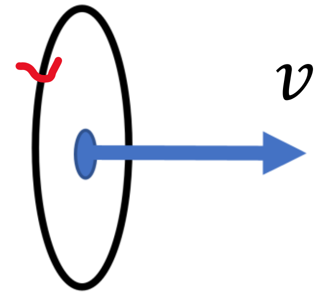
For a moving point charge

Lienard-Wiechert formula:

$$\mathbf{E}(t,\mathbf{x}) = \frac{q}{4\pi}\frac{\gamma_q \mathbf{R}}{[R^2 + (\gamma_q \mathbf{v}_q \cdot \mathbf{R})^2]^{3/2}},$$

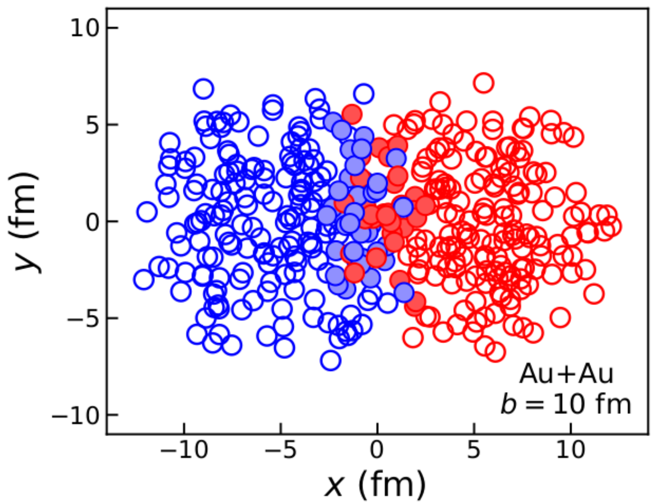
$$\mathbf{B}(t,\mathbf{x}) = \frac{q}{4\pi}\frac{\gamma_q \mathbf{v}_q \times \mathbf{R}}{[R^2 + (\gamma_q \mathbf{v}_q \cdot \mathbf{R})^2]^{3/2}},$$

$$\mathbf{R} = \mathbf{x} - \mathbf{x}_q$$



For heavy-ion collisions:

- Charge distribution:
  - Wood-Saxon distribution
  - Glauber sampling
- After collision:
  - Unwounded nucleons keep moving
  - Wounded nucleons slow down (stopping effect)
  - Full transport model or empirical formula
- Magnetic field evolves in:
  - Vacuum
  - Conductive medium

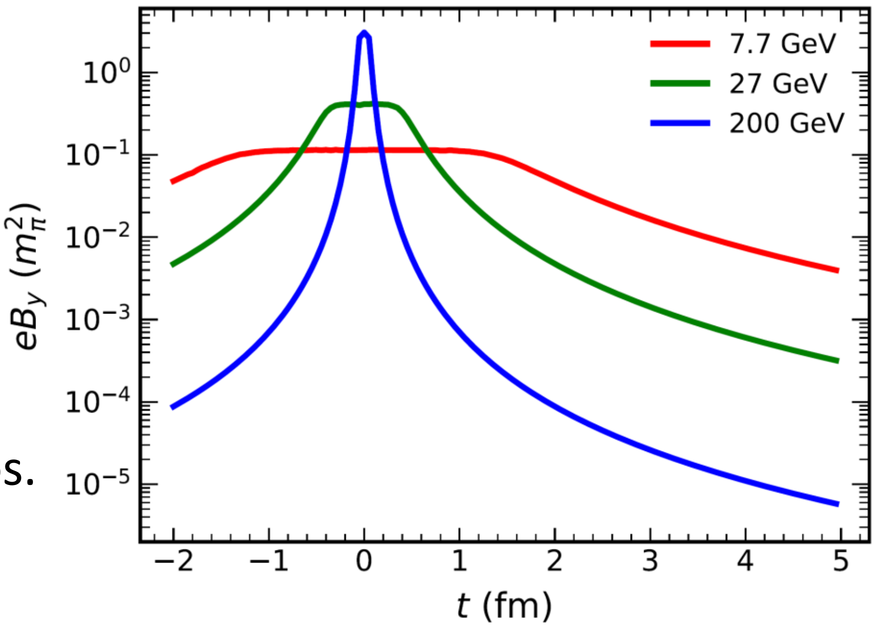


Skokov et al. 2009
Deng, Huang 2012
Tuchin 2013
McLerran, Skokov 2014
Gursoy, Kharzeev, Rajapopal 2014
Li, Sheng, Wang 2016
…

Magnetic field in vacuum

- Magnetic field drops rapidly in vacuum

$$B_y(t,\mathbf{x}) = \frac{q}{4\pi}\frac{\gamma v_z(x-x_0)}{[(x-x_0)^2+(y-y_0)^2+\gamma^2(z-z_0-v_z t)^2]^{3/2}},$$



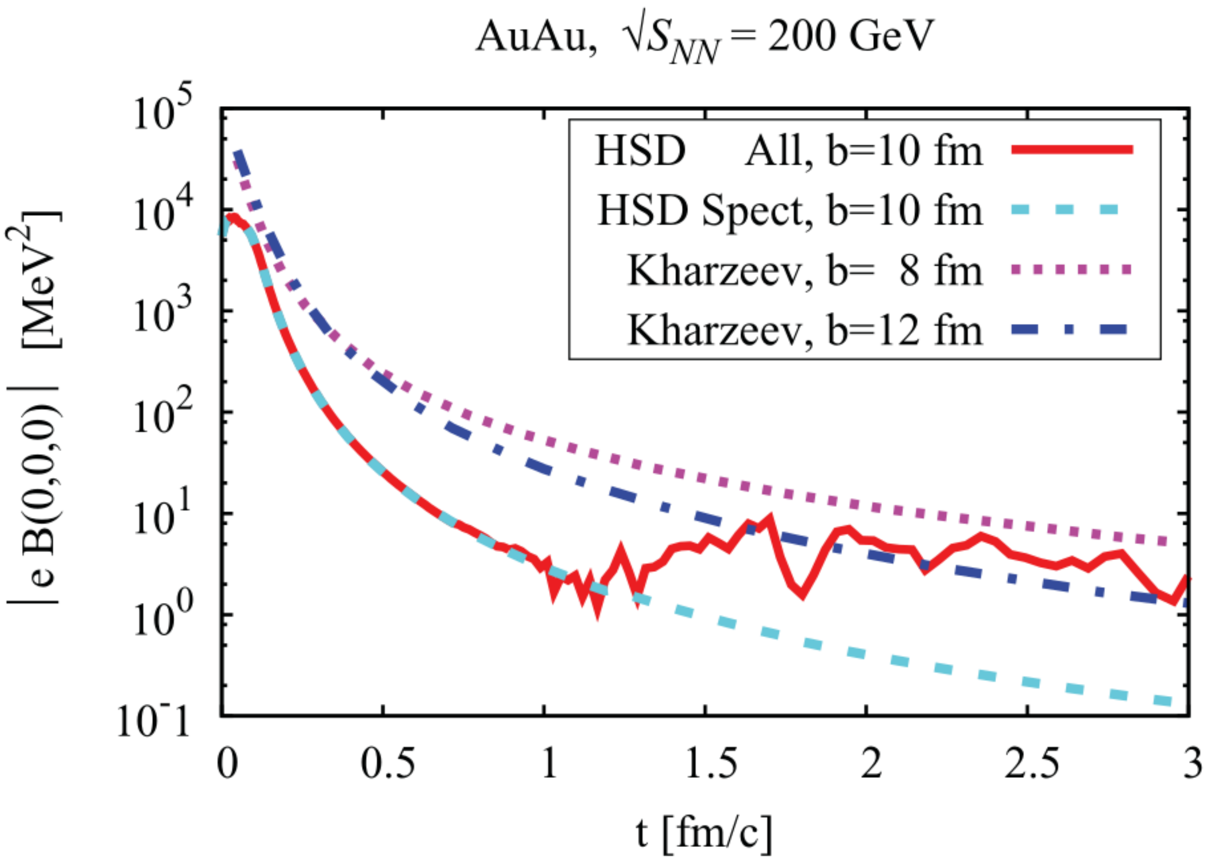
- Magnitude $B \propto \gamma \propto \sqrt{s}$
- Lifetime $t_B \propto 1/\gamma \propto 1/\sqrt{s}$

- The larger the field is, the faster it drops.

- In vacuum, and suppose nucleons are unwounded
  → symmetric evolution w.r.t. $t = 0$

Stopping effect

Simulate the EM field in vacuum by transport models
HSD, UrQMD, …



- It needs time to propagate the signal of nucleon's stopping (retarded potential)
- The stopping effect shows up only after $t > 1$ fm/c

V. Voronyuk, et.al, Phys. Rev. C 83, 054911

Electric conductivity of QGP medium

- Lattice calculations (T_c is critical temperature)

$$\sigma = (5.8 \pm 2.9)\ \frac{T}{T_c}\ \mathrm{MeV},$$

- Transport simulation by BAMPS

$$\sigma = \frac{1}{3T}\tau \sum_i {q_i}^2 n_i \qquad \tau = \frac{3}{2}\frac{1}{\sum_i n_i \sigma_{22}} \qquad \sigma \propto {}^{1}/_{\sigma_{22}}$$

where τ is the relaxation time for deviation from equilibrium

In the case of T = 255 MeV,

$\sigma_{22} = 1$ mb	$\sigma = 11.6$ MeV
$\sigma_{22} = 2$ mb	$\sigma = 5.8$ MeV

It is small compared to typical QCD scale of ~200 MeV

But it has obvious effect on the lifetime of EM field.

H. T. Ding, et.al, Phys. Rev. D 83, 034504
Z. Y. Wang, et.al, Phys. Rev. C 105, L041901

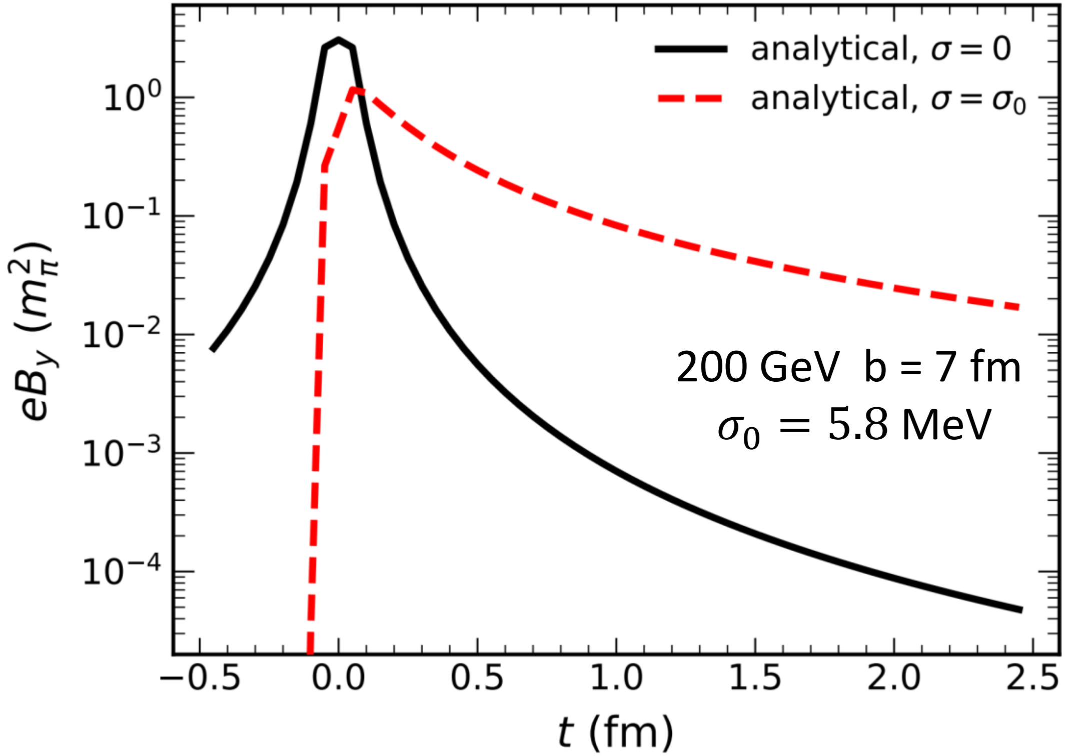
Magnetic field in conductive medium

Analytical formula:

$$\boxed{\mathbf{B}(t,\mathbf{x}) = \frac{q}{4\pi}\frac{\gamma\mathbf{v}\times\mathbf{R}}{\Delta^{3/2}}\left(1+\frac{\gamma\sigma}{2}|\mathbf{v}|\sqrt{\Delta}\right)e^{A}}$$

$$\Delta = R^2 + (\gamma\mathbf{v}\cdot\mathbf{R})^2$$

$$A = -\tfrac{\gamma\sigma}{2}(\gamma\mathbf{v}\cdot\mathbf{R} + |\mathbf{v}|\sqrt{\Delta})$$

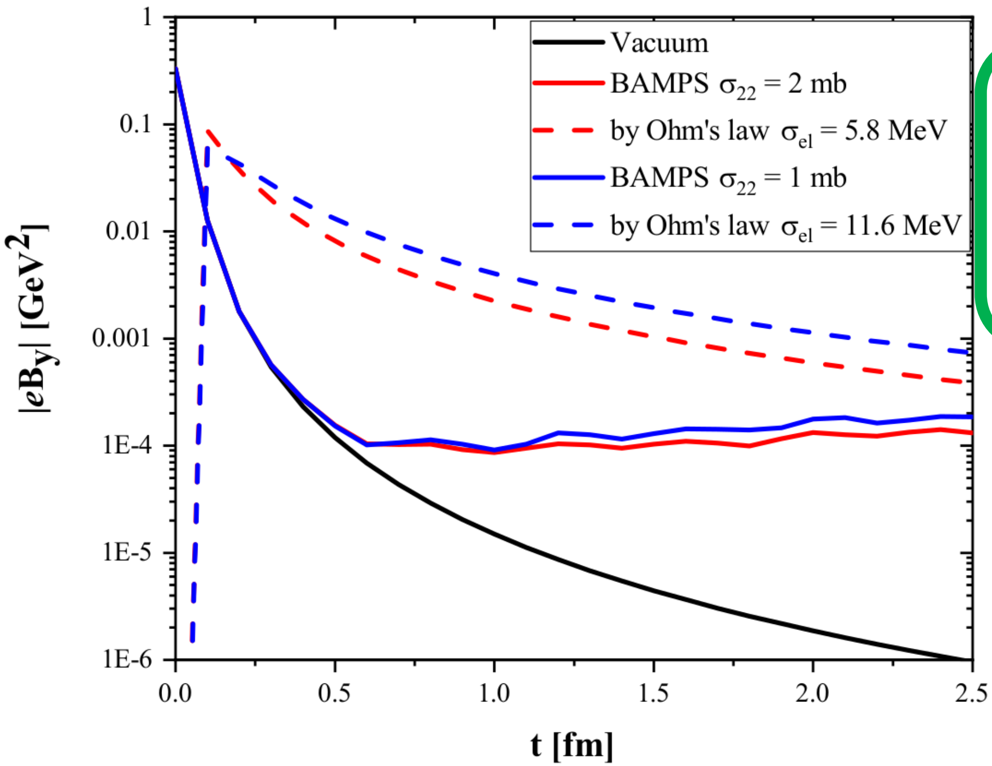


K. Tuchin, 2013
U. Gursoy, et.al, 2014
H. Li, et.al, 2016
…

- σ =const at both $t > 0$ and $t < 0$ (not realistic)
- Delay the field increasing at $t < 0$ (not realistic)
- Delay the field decreasing at $t > 0$ (expected, but over-estimated)

BAMPS simulation

- BAMPS simulates the response of quark matter to EB field, and solves Maxwell's equations.
- Significant suppression of the magnetic filed compared to the analytical formula.



They conclude:

incomplete electromagnetic response of hot QCD matter to external electromagnetic fields has been demonstrated by using parton cascade model BAMPS combined with the solution of Maxwell's equations. The numerical results showed the significant suppression of the induced magnetic field. This makes the search for any magnetic effects in heavy-ion experiments even more challenging.

Not apple-to-apple comparation

- BAMPS:

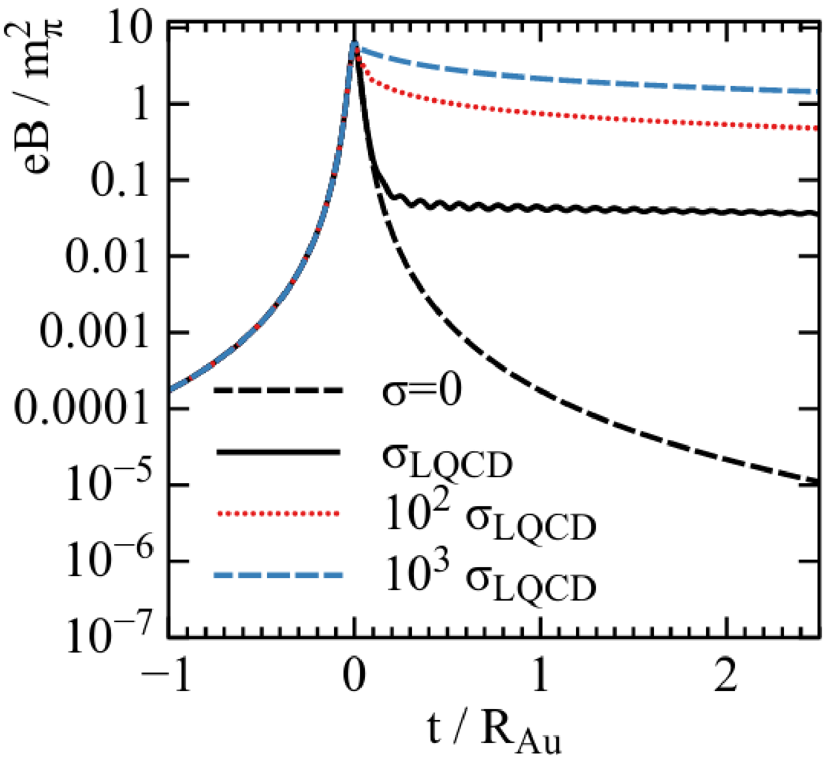
σ is switched on at $t = 0$

- Analytical Formula:

σ =const at both $t > 0$ and $t < 0$

Z. Y. Wang, J. X. Zhao, C. Greiner, Z. Xu, P. F. Zhuang, Phys. Rev. C 105, L041901

Solve Maxwell equation numerically



- Numerical solution of Maxwell's equations
- σ is switched on at $t = 0$
- Effect of σ is not obvious at early time (t < 1 fm/c) for $\sigma = 5.8$ MeV

L. McLerran, V. Skokov, Nucl. Phys. A 929 184-190

B. G. Zakharov, Phys. Lett. B, 737 262-266

Our model setup

Before collision (t<0):
Calculate the EM field in vacuum by L-W formula

After collision (t>0):
Numerically solve Maxwell's equations with non-zero σ

$$\begin{aligned}\nabla\cdot\mathbf{E} &= \rho,\\ \nabla\cdot\mathbf{B} &= 0,\\ \nabla\times\mathbf{E} &= -\partial_t\mathbf{B},\\ \nabla\times\mathbf{B} &= \mathbf{j}+\sigma\mathbf{E}+\partial_t\mathbf{E},\end{aligned}$$

We consider two cases

$$\sigma = \sigma_0\theta(t)$$

$$\sigma = \sigma_0\theta(t)/(1+t/t_0)^{1/3}$$

In both, σ is non-zero only after collision

Woods-Saxon distribution (rest frame)

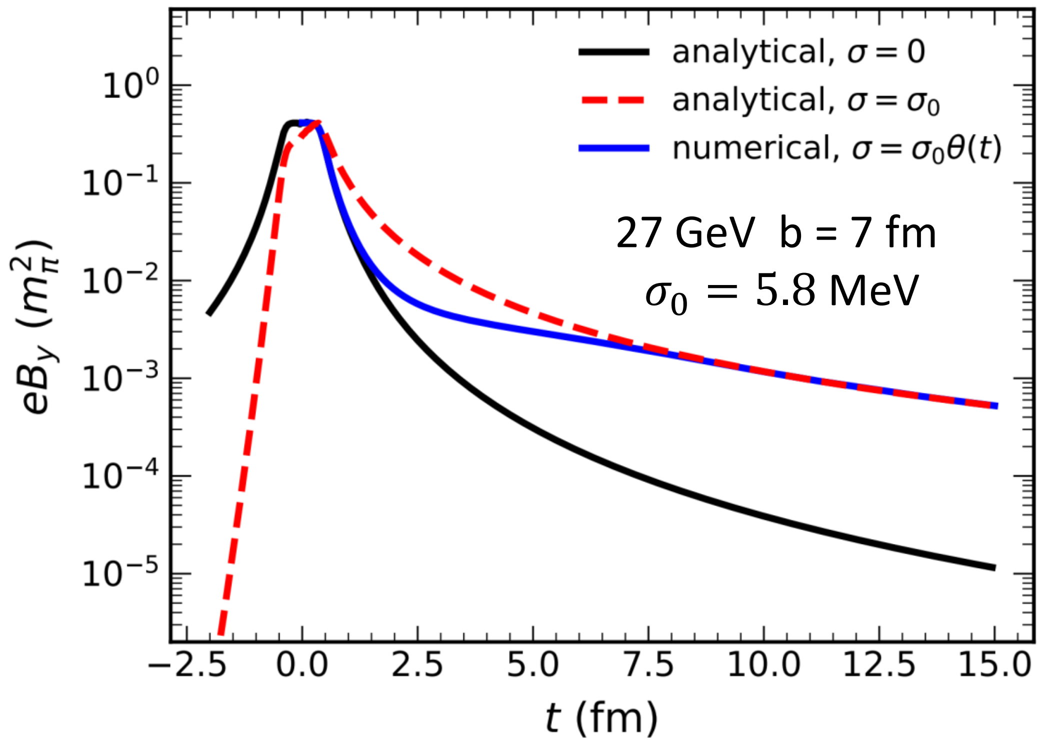
$$f(r) = \frac{\rho_0}{1+e^{[(r-R)/a]}}$$

lab frame

- $\rho^{\pm} = \gamma f\left(\sqrt{(x\mp b/2)^2 + y^2 + \gamma(z\mp v_z t)^2}\right)$
- $j_x^{\pm} = j_y^{\pm} = 0$
- $j_z^{\pm} = \pm\gamma v_z f\left(\sqrt{(x\mp b/2)^2 + y^2 + \gamma(z\mp v_z t)^2}\right)$

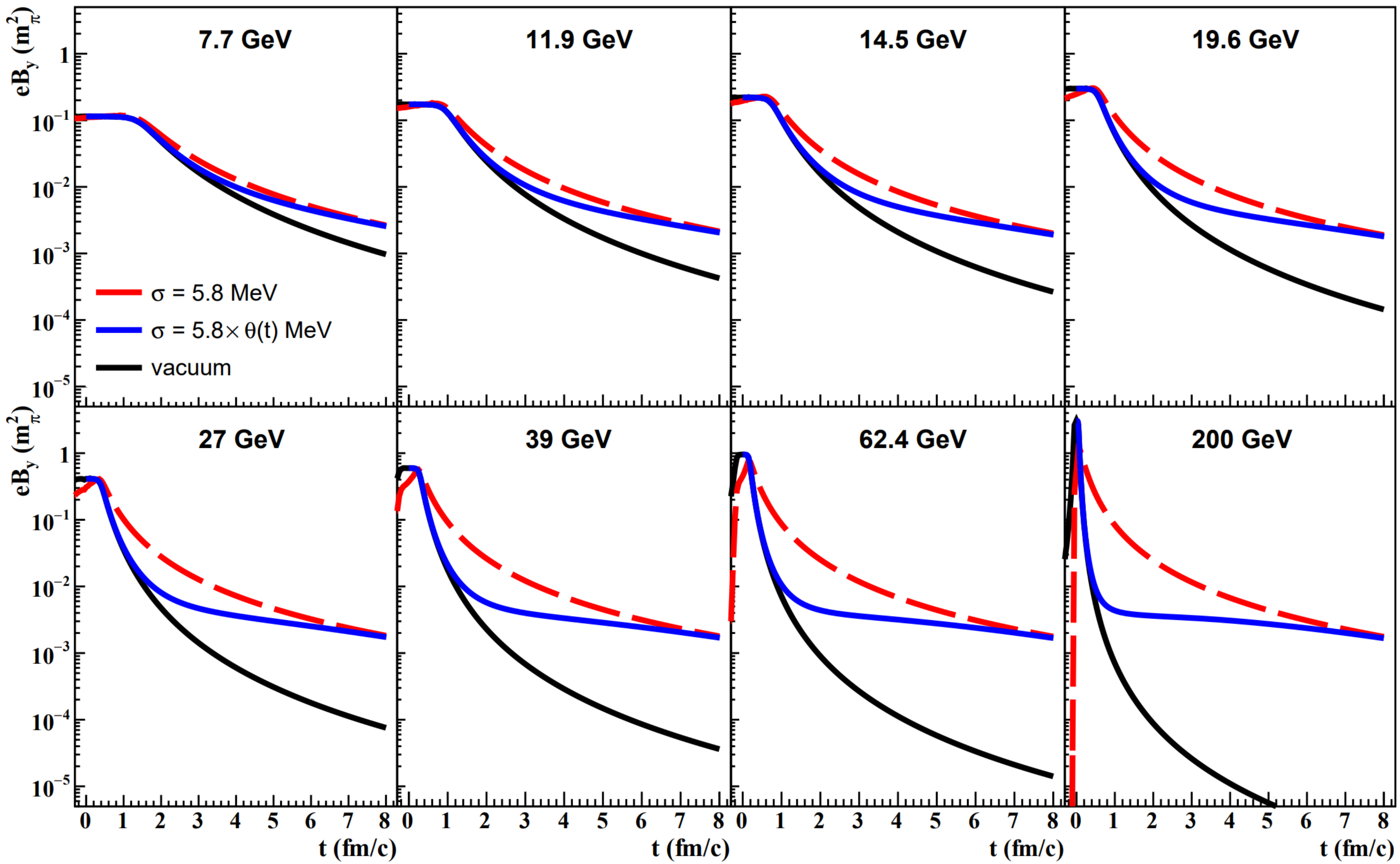
Simplification:
Nucleons fly freely without stopping

Analytical vs. Numerical



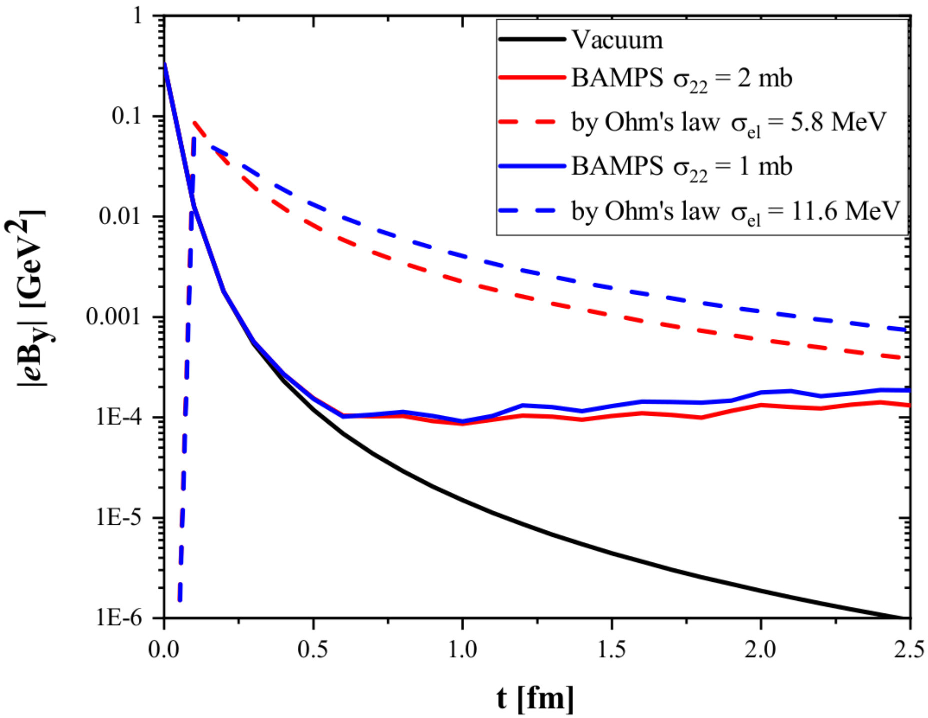
- Effect of σ is not obvious at early time (t < 1.5 fm/c)
- At early time, the analytical formula over estimates the field
- At very late time (t > 10 fm/c), it agrees with the numerical solution,
- but the time is too late (the field has became very small)

Analytical vs. Numerical

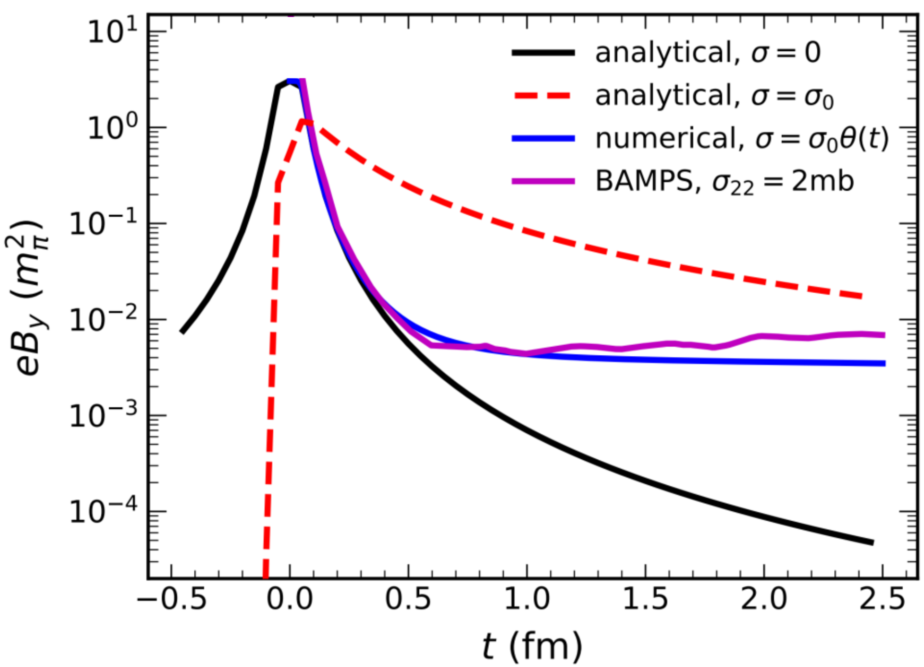


- ➢ The analytical formula over estimates the field at all energies

Compare with BAMPS calculation



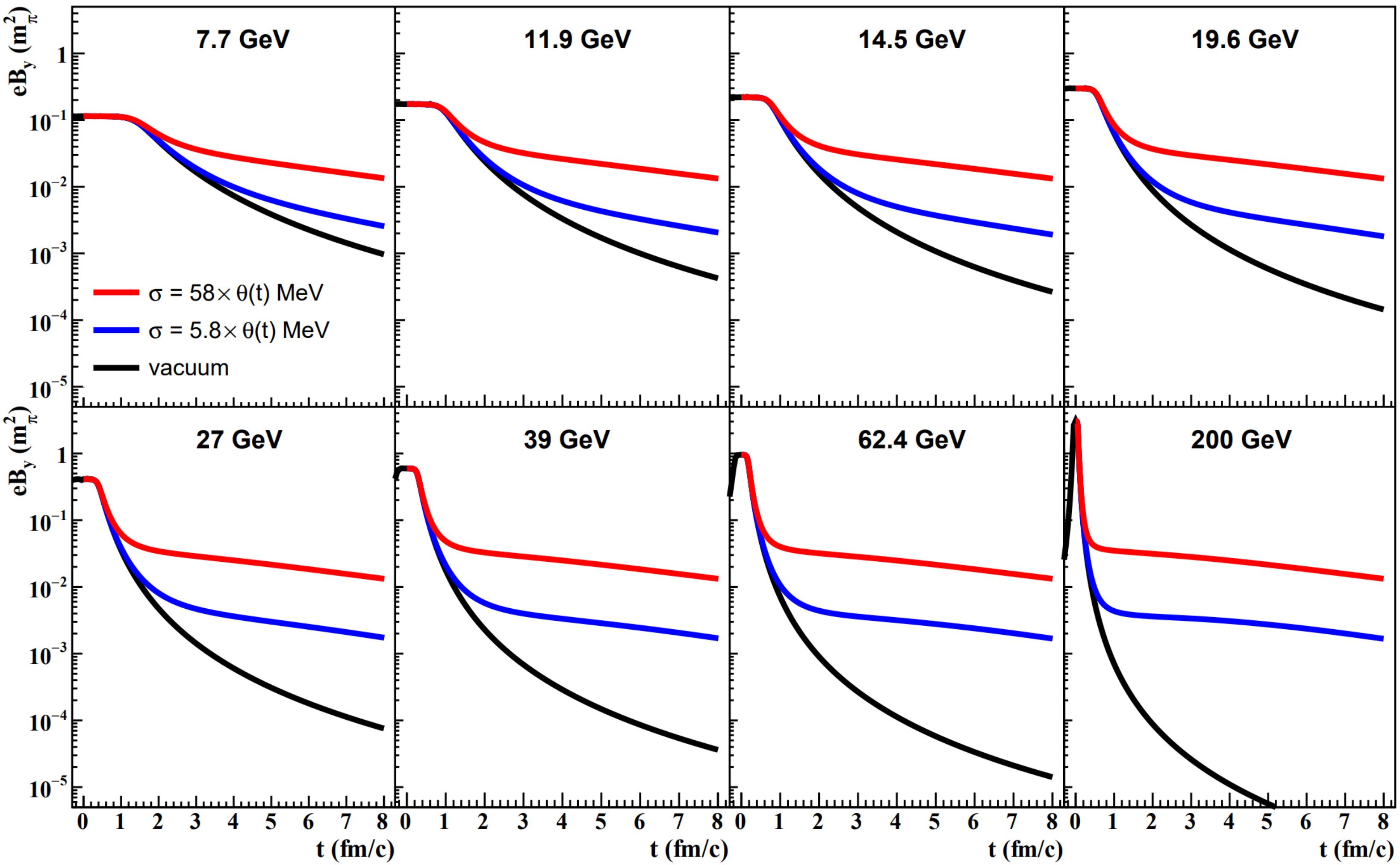
200 GeV, $\sigma_0 = 5.8$ MeV



- Numerical calculation agrees with BAMPS results (apple-to-apple comparation)
- Ohm’s law is valid in heavy ion collisions

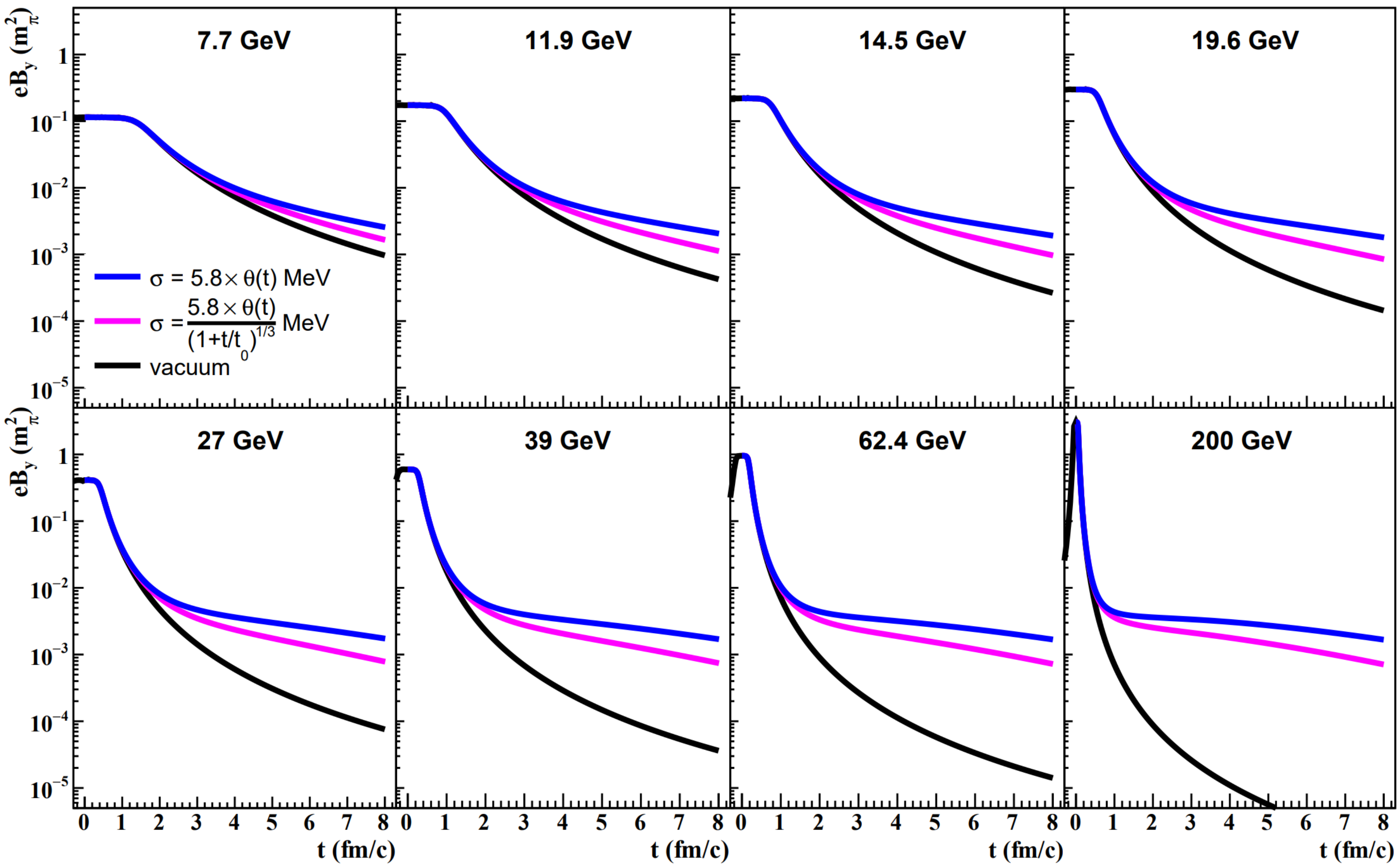
Z. Y. Wang, J. X. Zhao, C. Greiner, Z. Xu, P. F. Zhuang, Phys. Rev. C 105, L041901

Magnetic field with $\sigma = \sigma_0 \theta(t)$



- At late time, the field is sensitive to the σ value, and less sensitive to v_z
- We have used the same σ value at different energies
- For t > 4 fm/c, the field is at the order of $10^{-3} \sim 10^{-2}$ ($\sigma = 5.8$ MeV)

Magnetic field with $\sigma = \sigma_0\theta(t)/(1 + t/t_0)^{1/3}$



- The field also depends on σ's time behavior.
- When necessary, the numerical method can be applied to more accurate model.

Impact of the magnetic field

Effects at different time stage:

Dilepton production (initial time)

$t = 0$

Maximum magnetic field **observed!**

CME (early time)

t_{QGP}

Field drops fast at early time. So, how early QGP is formed?

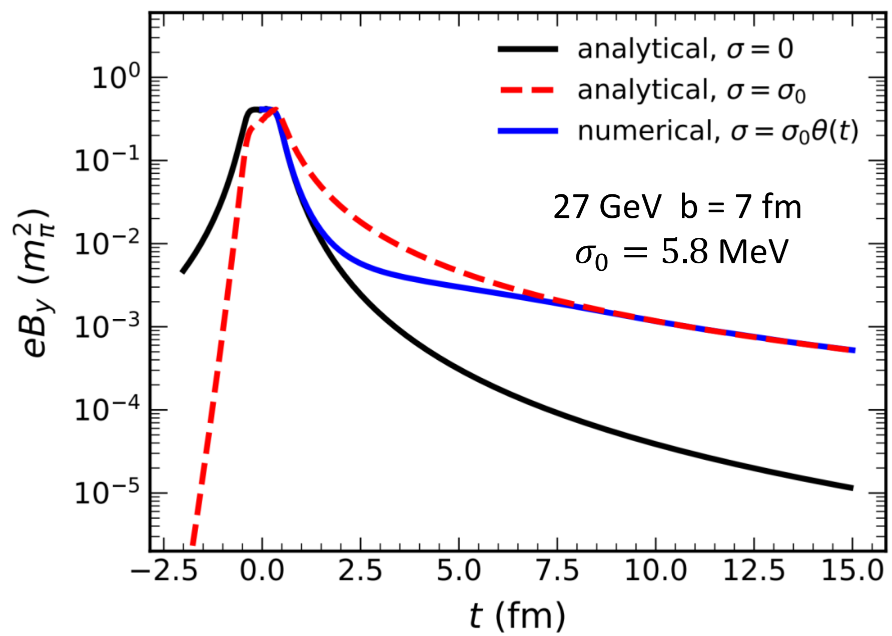
Direct flow (middle time)

More accurate model is needed to simulate it.

Spin polarization (late time)

t_{hadron}

The field is too small.

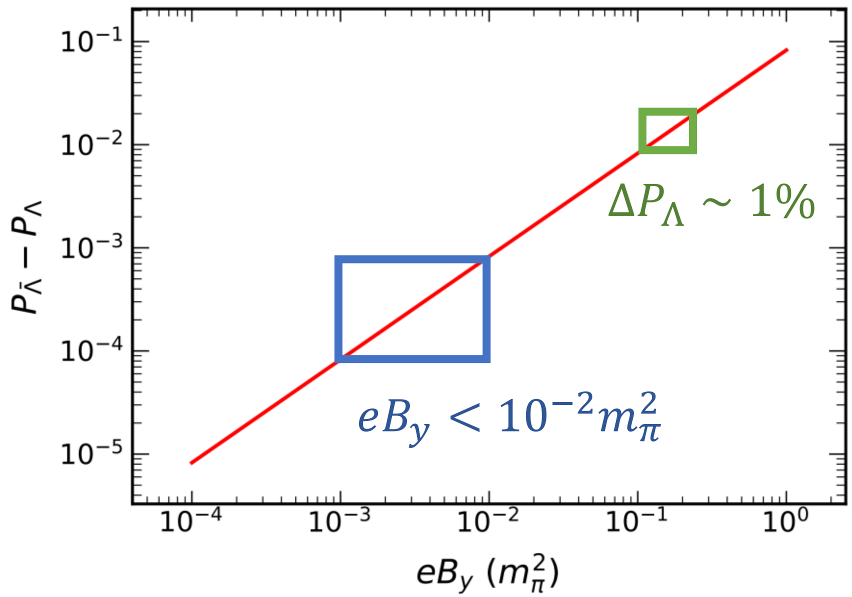


Global Λ polarization

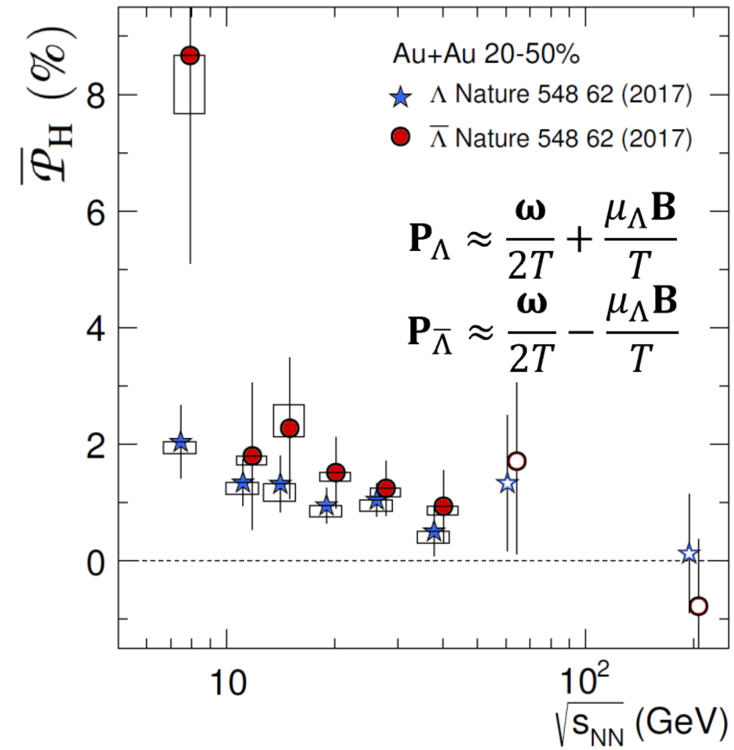
- Magnetic-field-induced global polarization:

$$P_\Lambda = -0.613\mu_N \frac{B}{T}$$

$$P_{\bar{\Lambda}} - P_\Lambda \approx 0.0826\left(\frac{eB}{m_\pi^2}\right)$$



STAR, Nature 548 62 (2017)



Other mechanism:
different space-time distributions of Λ and $\bar{\Lambda}$

A. Ayala, et al, Phys. Lett. B 810, 135818
O. Vitiuk, et al, Phys. Lett. B 803, 135298

Note: Λ and $\bar{\Lambda}$ flow is different at low energy

Spin anti-alignment

Quark and anti-quark are anti-polarized by magnetic field

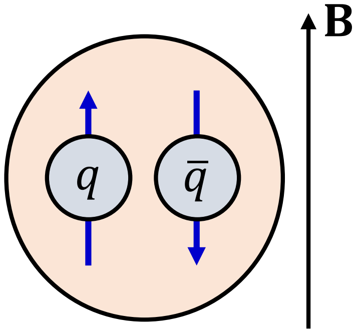
- ρ_{00} with respect to different directions:

$$\rho_{00}^{(x)} = \frac{1 - P_x^q P_x^{\bar{q}} + P_y^q P_y^{\bar{q}} + P_z^q P_z^{\bar{q}}}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}}$$

$$\rho_{00}^{(y)} = \frac{1 - P_y^q P_y^{\bar{q}} + P_x^q P_x^{\bar{q}} + P_z^q P_z^{\bar{q}}}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}}$$

$$\rho_{00}^{(z)} = \frac{1 - P_z^q P_z^{\bar{q}} + P_x^q P_x^{\bar{q}} + P_y^q P_y^{\bar{q}}}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}}$$

$$\mathbf{P}^q \parallel -\mathbf{P}^{\bar{q}} \propto \mathbf{B}$$



Y. Liu, C. Greiner, C. M. Ko, arxiv: 1403.4317
Y. G. Yang, et al, Phys. Rev. C 97, 034917

X. L. Xia, et al, Phys. Lett. B 817, 136325

ρ_{00} is effect of $\langle P_q P_{\bar{q}} \rangle$, instead of $\langle P_q \rangle \langle P_{\bar{q}} \rangle$

- $\langle P_q \rangle$ and $\langle P_{\bar{q}} \rangle$ are small (can be constrained by Λ polarization)
- $\langle P_q P_{\bar{q}} \rangle$, arising from fluctuation of magnetic/vorticity fields, may be not small

Our results do not rule out the magnetic-induced spin anti-alignment

$\langle B_y \rangle = 0$ does not put any limit on $\langle B_x^2 \rangle$, $\langle B_y^2 \rangle$, and $\langle B_z^2 \rangle$

Summary

We have numerically solved Maxwell's equations in conductive medium,

- The previous analytical formula over-estimated the magnetic field.

From the numerical solution,

- At early time: the field is similar with that in vacuum;
- At late time: the field decreases slower, but it is already very small.

It is important to distinguish effects at different time-stage.

- Can not lead to observable global Λ polarization
- Does not rule out spin (anti-)alignment

- For CME, depends on early time dynamics
- For direct flow of charged particle, more accurate model is needed

Thank you for attention!