Fermilab

Alexander von Humboldt
Stiftung/Foundation

# A NLL accurate Parton Shower algorithm in Sherpa

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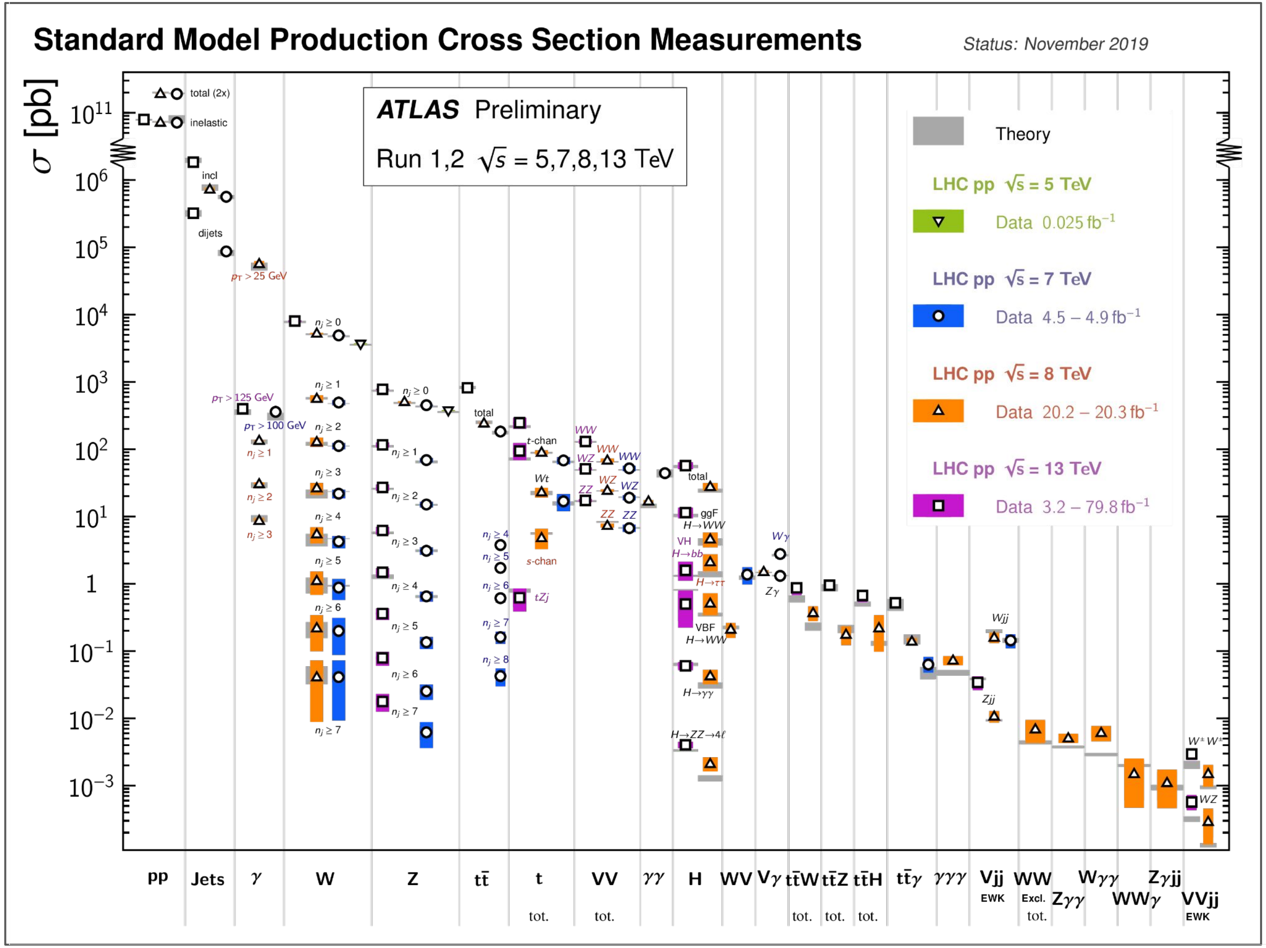
2208.06057, in collaboration with Stefan Höche, Frank Krauss, Daniel Reichelt & Marek Schönherr

# Quest for precision

Measurements and theory predictions reached incredible levels of precision

However, with increasing statistics theoretical uncertainties will become dominant for many processes




https://twiki.cern.ch/twiki/bin/view/AtlasPublic/StandardModelPublicResults

# Event Generators

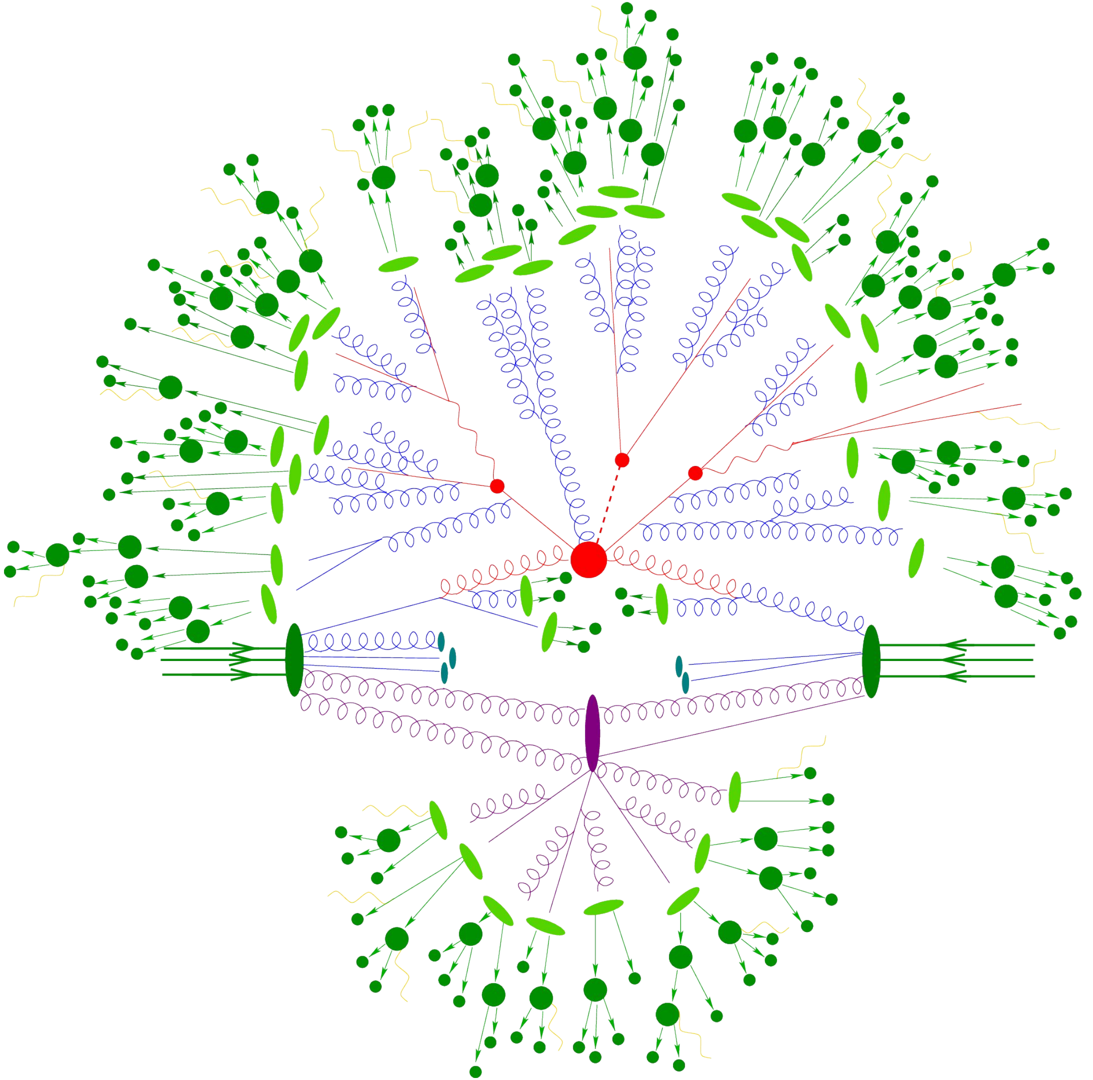
Crucial for precision Collider Physics

Short distance physics:

- Hard Process
- Parton Shower

Long distance physics:

- Underlying Interaction
- Hadronization
- QED FSR
- Hadron Decays



# Event Generators

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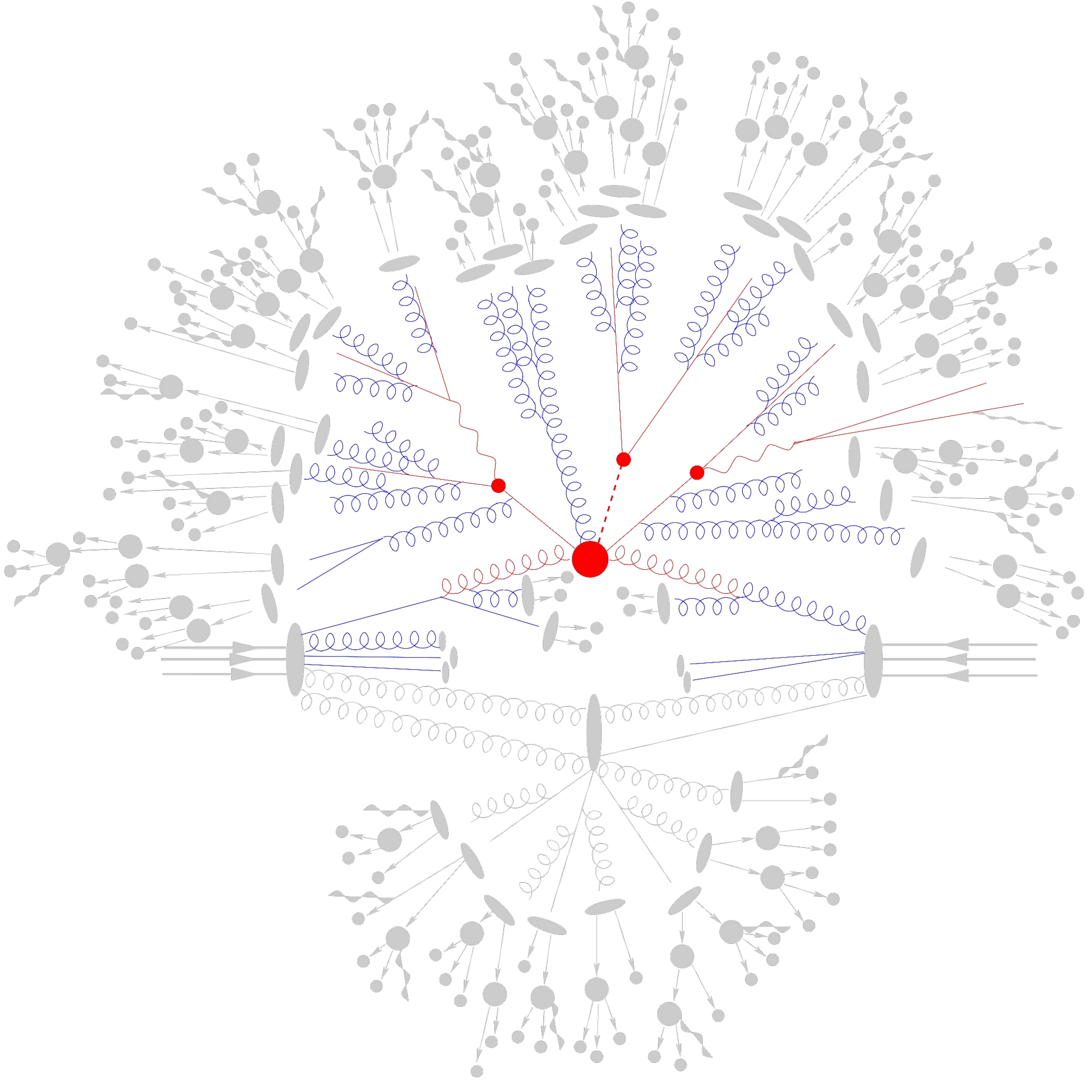
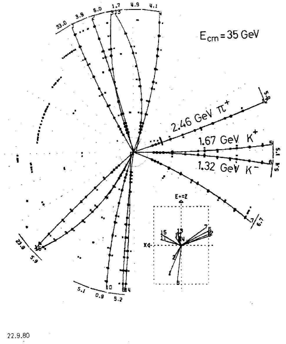
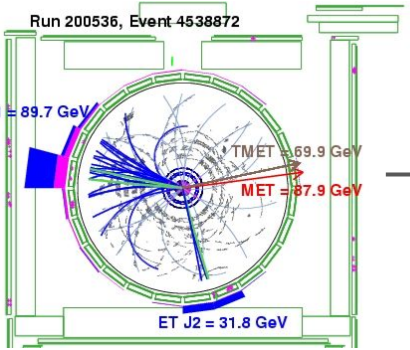
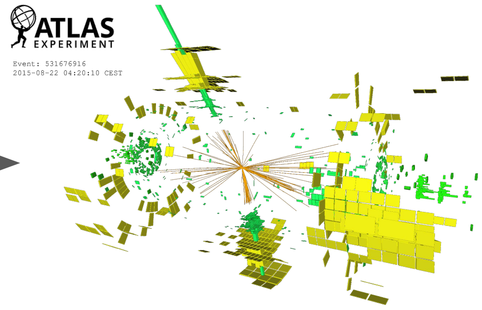


# Timeline

- 1985: Color coherent evolution; Backward evolution
- 1990: ME corrections; Color dipole formalism; Spin correlations
- 1995: Color dipole showers
- 2000
- 2005: LO merging; NLO matching
- 2010: Dipole-like showers; Antenna showers; NLO merging
- 2015: Color ME corrections; NNLO matching; Threshold effects; NLO splitting kernels; Higher-order PS framework
- 2018: Accuracy quantification; Amplitude evolution
- 2022: NLL & NGLs in one framework; Efficient 1/Nc corrections



# Timeline

Color coherent evolution
Backward evolution
ME corrections
Color dipole formalism
Spin correlations
Color dipole showers
LO merging
NLO matching
Dipole-like showers
Antenna showers
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Color ME corrections
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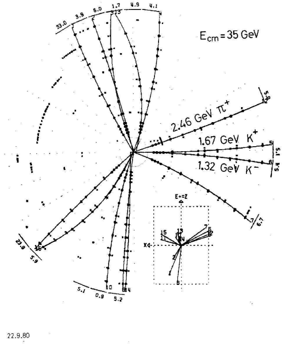
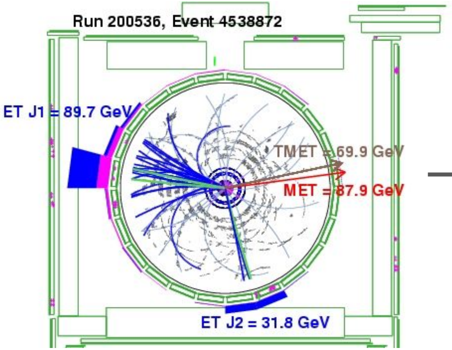
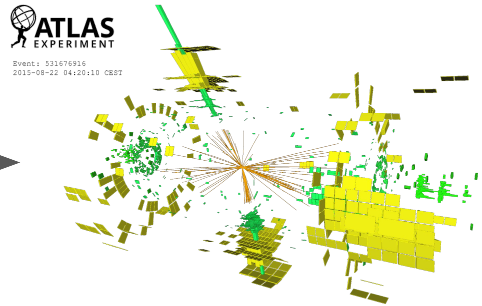
Accuracy quantification
Amplitude evolution
NLL & NGLs in one framework
Efficient 1/Nc corrections

1985 — 1990 — 1995 — 2000 — 2005 — 2010 — 2015 — 2018 — 2022

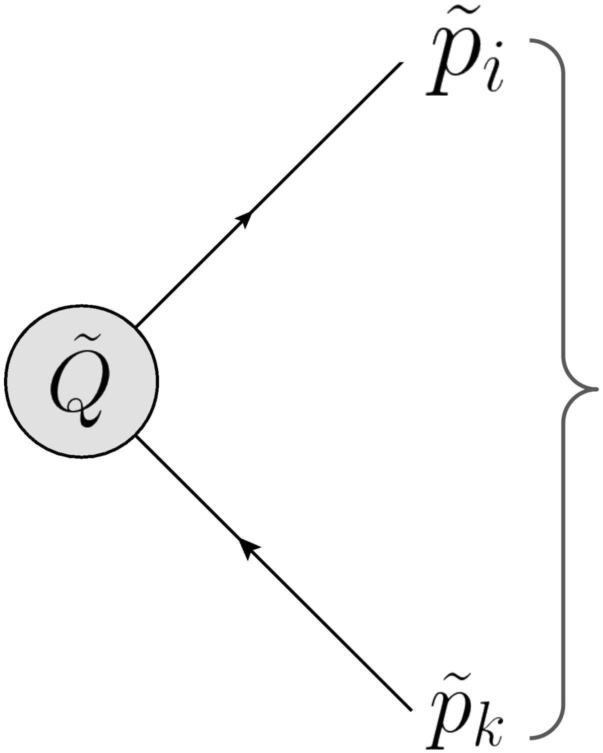


# Parton Showers

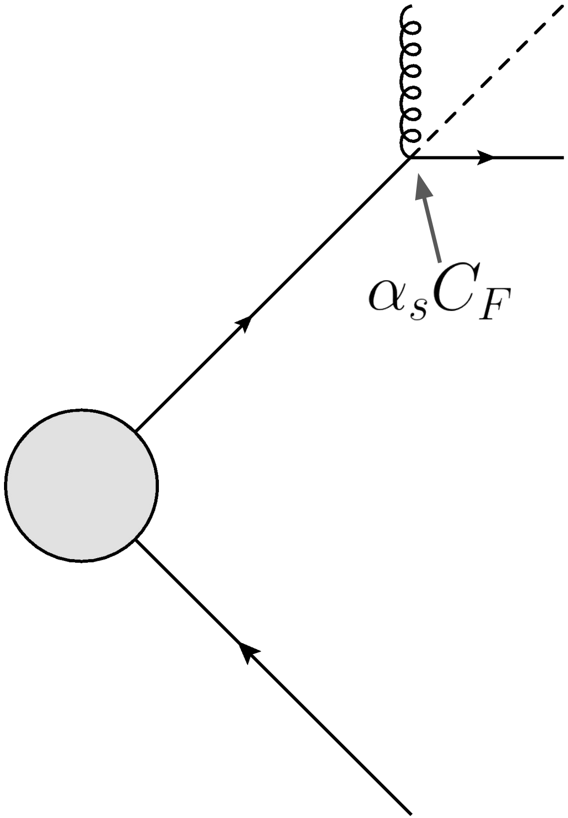
Start with fixed order configuration,
e.g. ee ⟶ qq, qq ⟶ ll, eq ⟶ eq




Color dipole

# Parton Showers

Add one gluon emission




Gluon emission most likely in singular limits!

# Parton Showers

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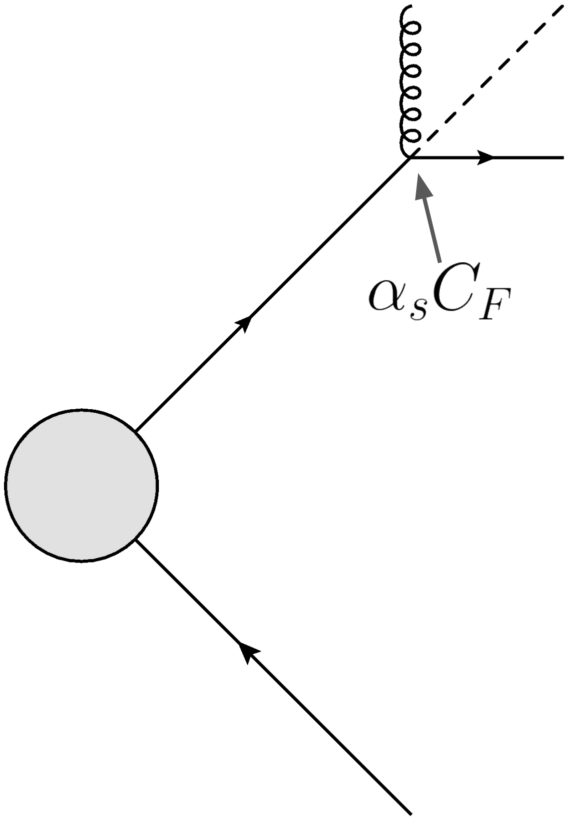


Gluon emission most likely in singular limits!

Gluon soft: $|\mathcal{M}_{qqg}|^2 \approx 2\frac{p_i p_k}{(p_i p_j)(p_j p_k)}|\mathcal{M}_{qq}|^2$

Gluon collinear: $|\mathcal{M}_{qqg}|^2 \approx \frac{P_{qq}(z)}{2(p_i p_j)}|\mathcal{M}_{qq}|^2$

# Parton Showers

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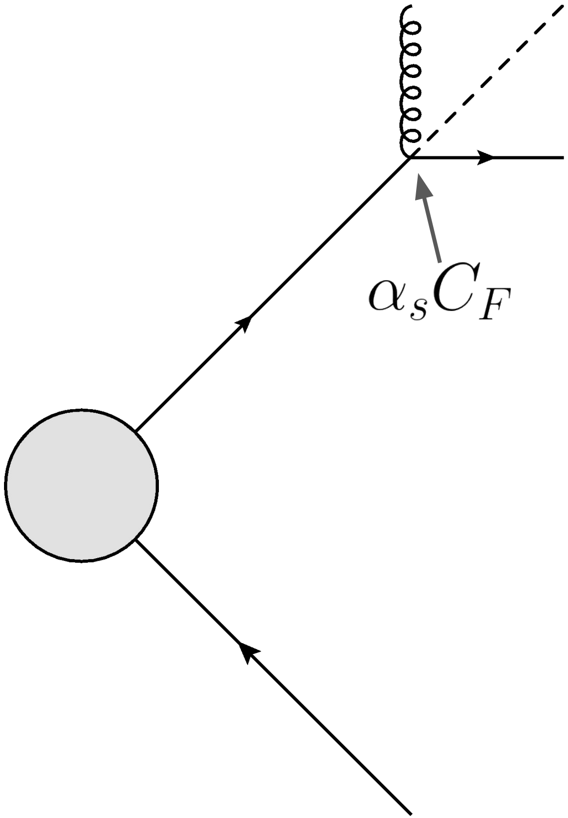


Gluon emission most likely in singular limits!

Depends on both dipole members

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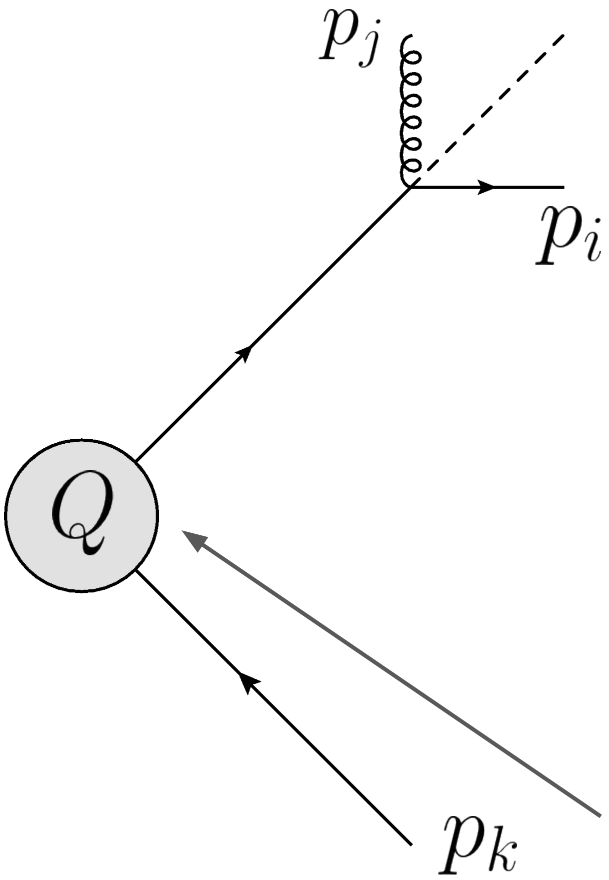
Gluon collinear: $|\mathcal{M}_{qqg}|^2 \approx \frac{P_{qq}(z)}{2(p_i p_j)}|\mathcal{M}_{qq}|^2$

Depends only on radiating parton

Problem #1:
Double counting in iterated limit!

# Parton Showers

Add one gluon emission



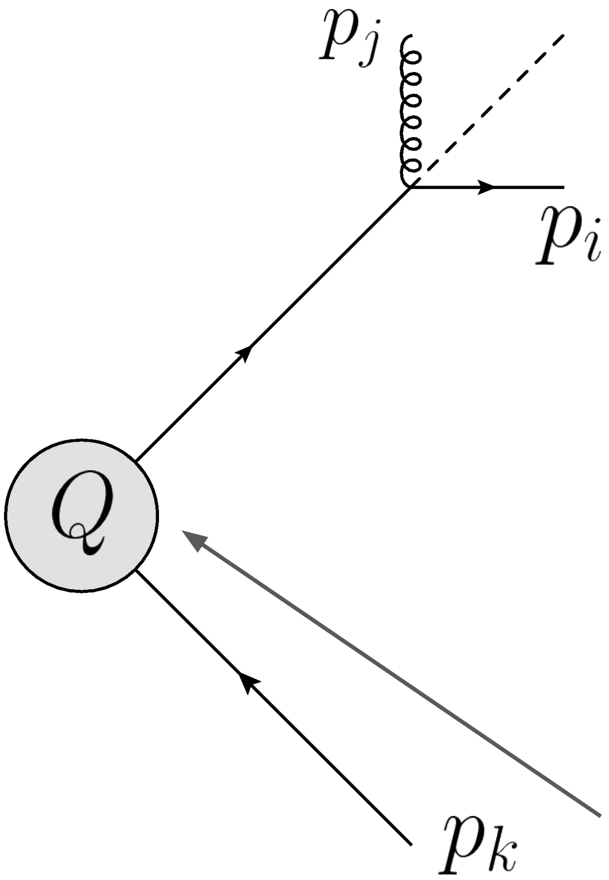

Need momentum mapping between on-shell momenta:

$\tilde{p}_i$, $\tilde{p}_k$ ⇒ $p_i$, $p_k$, $p_j$

Also $\tilde{Q}$ may change

# Parton Showers

Add one gluon emission




Problem #2:
Suitable momentum mapping

Need momentum mapping between on-shell momenta:

$$\tilde{p}_i, \tilde{p}_k \Rightarrow p_i, p_k, p_j$$

Conditions:

$$p_i \to z\tilde{p}_i$$

$$p_j \to (1-z)\tilde{p}_i$$

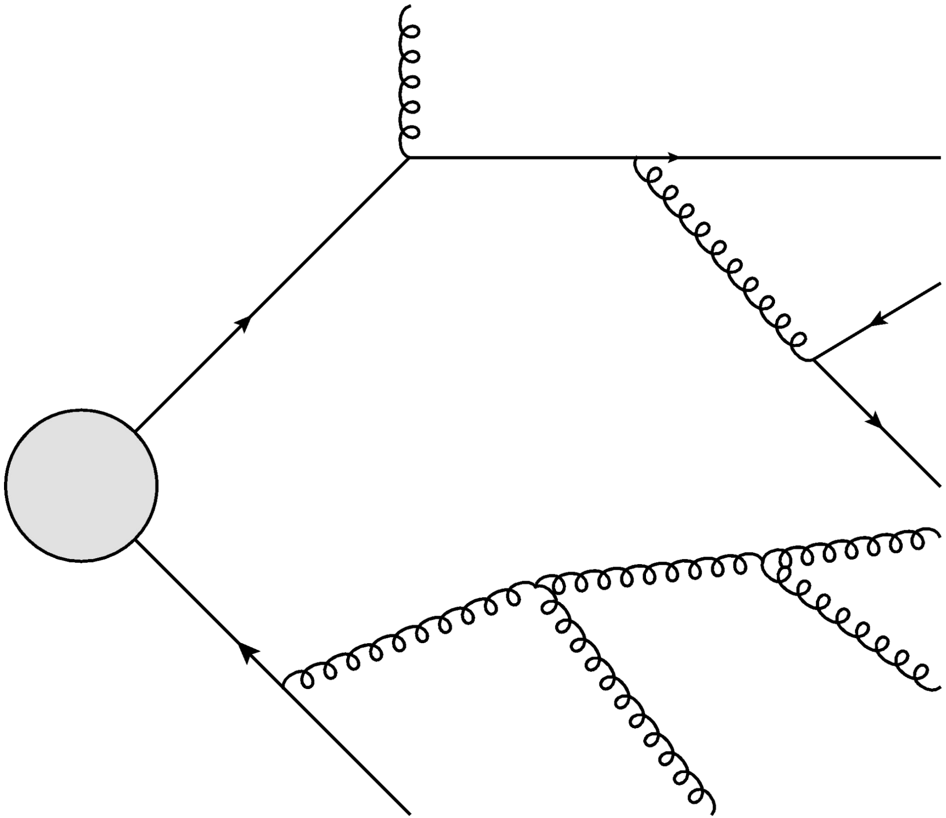
in collinear limit, and

$$Q^2 = \tilde{Q}^2$$

Also $\tilde{Q}$ may change

# Parton Showers

Repeatedly add emissions



Problem #3:
When do we stop?
→ Evolution variable

Problem #4:
Evolution resums large logarithms, but at which accuracy?

Problem #5:
How do we handle NLO calculations?

# NLL Showers

Criteria for NLL accuracy at leading color outlined in:
[Dasgupta,Dreyer,Hamilton,Monni,Salam,Soyez] 2002.11114

Where do the logarithms come from?
(see also [Banfi, Salam, Zanderighi] hep-ph/0407286)

Shower needs to reproduce results of analytic resummation of rIRC observables

Depends on logarithmic variables of emission pairs:

| Energies/Angles | Distinctly different | Comparable |
|---|---|---|
| Distinctly different | LL | NLL |
| Comparable | NLL | NNLL |

Shower needs to reproduce the correct tree-level ME squared in these regions

# Soft Radiation

Factorisation in the soft limit:

$$ {}_n\langle 1,\ldots,n|1,\ldots,n\rangle_n = -8\pi\alpha_s \sum_{i,k\neq j} {}_{n-1}\langle 1,\ldots,\not{j},\ldots,n\big|\mathbf{T}_i\mathbf{T}_k\, w_{ik,j}\big|1,\ldots,\not{j},\ldots,n\rangle_{n-1} $$

Eikonal factor:

$$ w_{ik,j} = \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{1}{E_j^2}\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} $$

Implementing the Eikonal in the collinear limit leads to double-counting of soft singularities

[Marchesini, Webber] *Nucl.Phys.B* 310 (1988) 461-526

# Soft Radiation

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**Additive matching of singularities:**

$$ W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j} $$

$$ \tilde{W}^i_{ik,j} = \frac{1}{2}\left(W_{ik,j} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{jk}}\right) $$

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$W_{ik,j}$

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$$ W_{ik,j} $$

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**With $p_i$ on z-axis:**

$$ \cos\theta_{jk} = \cos\theta^i_j \cos\theta^i_k + \sin\theta^i_j \sin\theta^i_k \cos\phi^i_{jk} $$

**Implementing the Eikonal in the collinear limit leads to double-counting of soft singularities**

[Marchesini, Webber] *Nucl.Phys.B* 310 (1988) 461-526

# Soft Radiation

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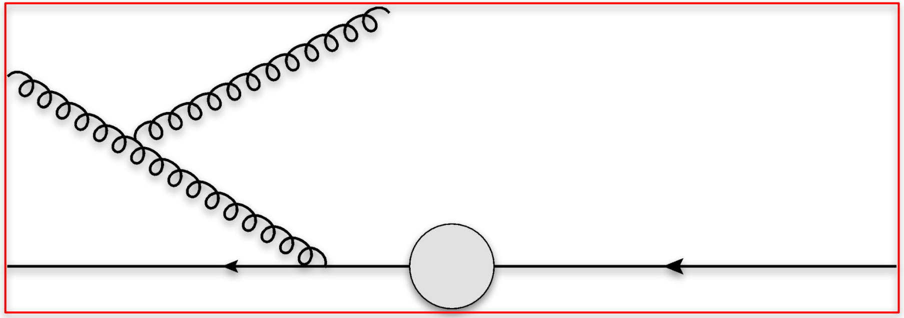
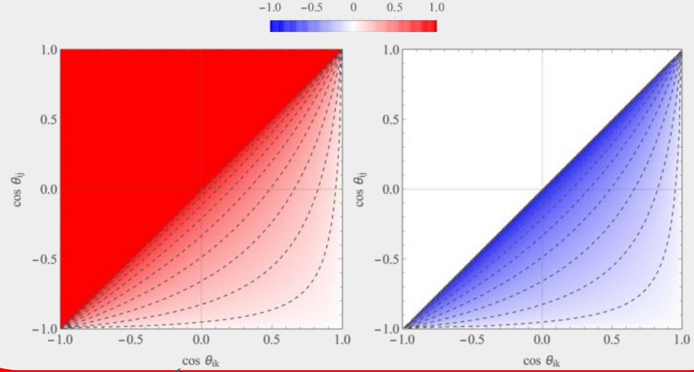


**Option 2: Implement radiator differentially**




**Option 1:**
Angular Ordering → Spoils NGLs

**Azimuthal average:**

$$ \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi^i_{jk}\tilde{W}^i_{ik,j} = \frac{\theta(\theta_{ik}-\theta_{ij})}{1-\cos\theta_{ij}} $$

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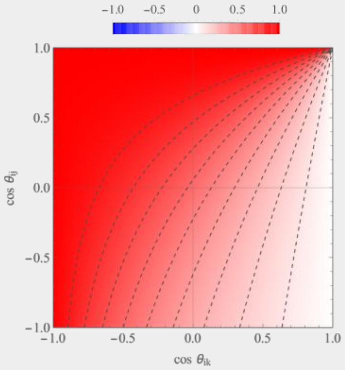
**Multiplicative matching of singularities:**

$$ W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j} $$

$$ \bar{W}^i_{ik,j} = W_{ik,j}\frac{1-\cos\theta_{jk}}{2-\cos\theta_{ij}-\cos\theta_{jk}} $$

[Catani, Seymour] hep-ph/9605323

**Implement radiator differentially**




**Azimuthal average:**

$$ \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi^i_{jk}\bar{W}^i_{ik,j} = \frac{1}{\sqrt{(A^i_{ik,j})^2-(B^i_{ik,j})^2}} $$

$$ A^i_{ij,k} = \frac{2-\cos\theta_{ij}(1+\cos\theta_{ik})}{1-\cos\theta_{ik}} $$

$$ B^i_{ij,k} = \frac{\sqrt{(1-\cos^2\theta_{ij})(1-\cos^2\theta_{ik})}}{1-\cos\theta_{ik}} $$

# Soft Radiation

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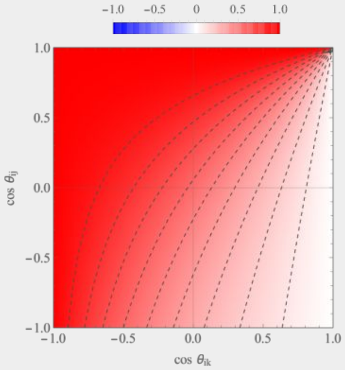



$$\frac{1}{2p_ip_j}P_{(ij)i}(z) \to \frac{1}{2p_ip_j}P_{(ij)i}(z)+\delta_{(ij)i}\left[\frac{\bar{W}^i_{ik,j}}{E_j^2} - w^{(\mathrm{coll})}_{ik,j}(z)\right]$$

Splitting functions depend on direction of color spectator! N.b.: only leading color

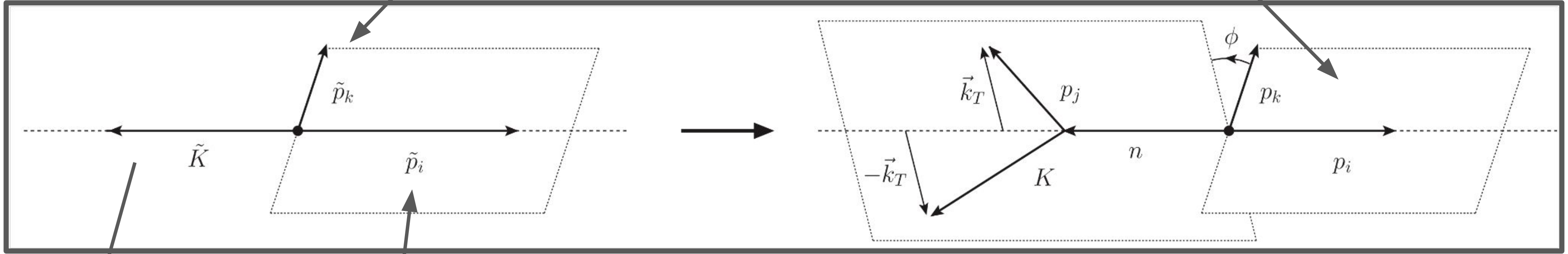
# Momentum Mapping

Main Idea:
maintain directions of hard particles exactly

$$p_i = z\tilde{p}_i \qquad z = \frac{p_i n}{(p_i + p_j)n}$$

$$p_k = \tilde{p}_k$$

Color Spectator




Hard system

Emitter

Need to find $K$ and $p_j$ such that:

$$K^2 = \tilde{K}^2 \quad p_j \to (1-z)\tilde{p}_i$$

Shift:
$n = \tilde{K} + (1-z)\tilde{p}_i$

# Momentum Mapping

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maintain directions of hard particles exactly

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$$p_k = \tilde{p}_k$$

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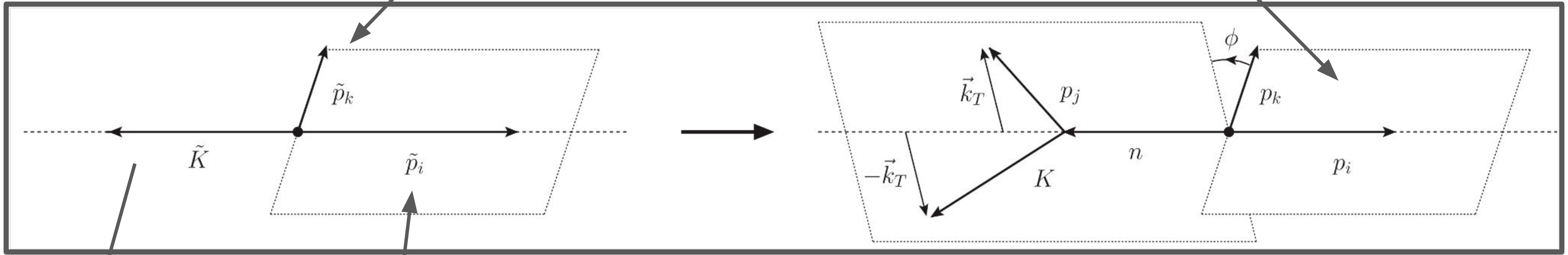



Hard system

Emitter

$$v = \frac{p_i p_j}{p_i \tilde{K}} \qquad \kappa = \frac{\tilde{K}^2}{2\tilde{p}_i \tilde{K}}$$

$$p_j = (1-z)\tilde{p}_i + v\left(\tilde{K} - (1-z+2\kappa)\tilde{p}_i\right) + k_\perp$$

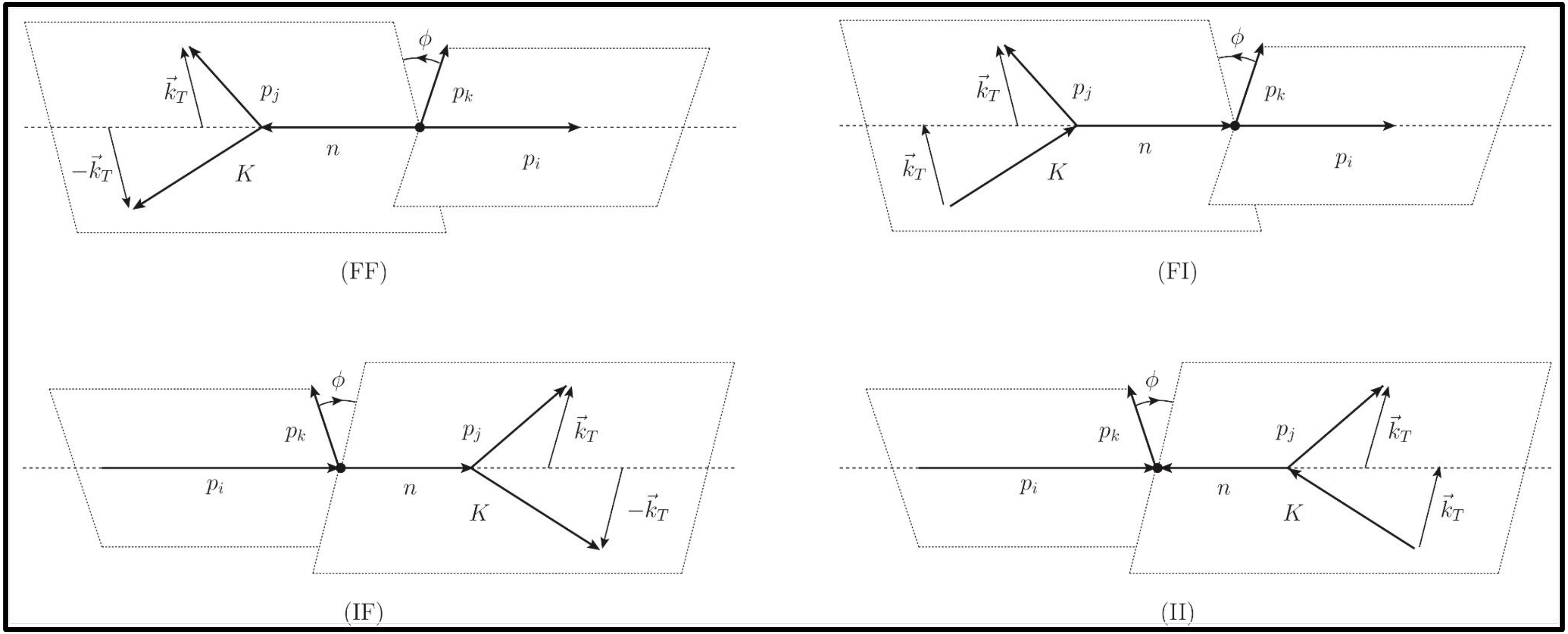
$$K = \tilde{K} - v\left(\tilde{K} - (1-z+2\kappa)\tilde{p}_i\right) - k_\perp$$

Recoil distributed to remaining momenta through Lorentz Transformation:

$$p_l^\mu \to \Lambda^\mu_\nu(K, \tilde{K}) p_l^\nu$$

# Recoil

Momentum mapping works for initial and final state emitters/spectator
→ e+ e-, pp, DIS, … all treated on same footing



# Recoil

Recoil distributed to remaining momenta through Lorentz Transformation:

$$p_l^\mu \to \Lambda^\mu_\nu(K, \tilde{K}) p_l^\nu$$

Define

$$X^\mu = p_j^\mu - (1-z)\,\tilde{p}_i^\mu$$
$$= v\big(\tilde{K}^\mu - (1 - z + 2\kappa)\,\tilde{p}_i^\mu\big) + k_\perp^\mu$$

At most $\mathcal{O}(k_\perp)$ in logarithmically enhanced region

# Recoil

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$$= v\big(\tilde{K}^\mu - (1-z+2\kappa)\,\tilde{p}_i^\mu\big) + k_\perp^\mu$$

$$\Lambda^\mu{}_\nu(K, \tilde{K}) = g^\mu_\nu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

$$A^\nu = 2\left[\frac{(\tilde{K}-X)^\nu}{(\tilde{K}-X)^2} - \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}\right] \qquad B^\nu = \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}$$

# Recoil

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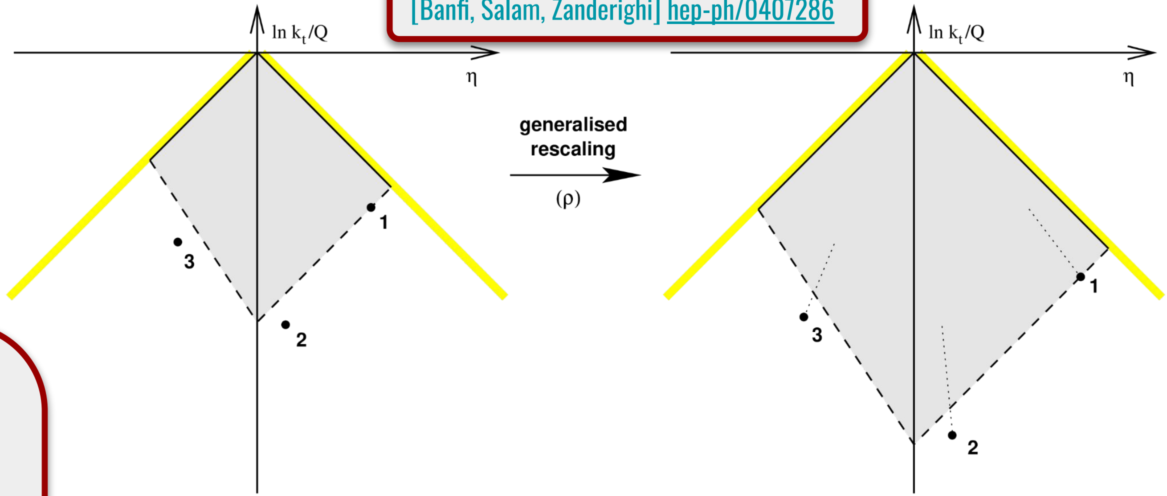
Suppressed by

$$\mathcal{O}(k_\perp/K)$$

$$\Lambda^\mu_\nu \approx g^\mu_\nu + \frac{K_\rho X_\sigma}{K^2}T^{\mu\rho\sigma}_\nu + \mathcal{O}(k_\perp^2)$$

# Recoil

[Banfi, Salam, Zanderighi] hep-ph/0407286




For one emission kinematic variables in the Lund plane scale like:

$$k_{t,l} \to k'_{t,l} = k_{t,l}\rho^{(1-\xi_l)/a+\xi_l/(a+b)}$$

$$\eta_l \to \eta'_l = \eta - \xi_l \frac{\ln \rho}{a+b}$$

$$\xi_l = \frac{\eta_l}{\eta_{l,\max}}$$

where a = 1 and b = 0 for Alaric

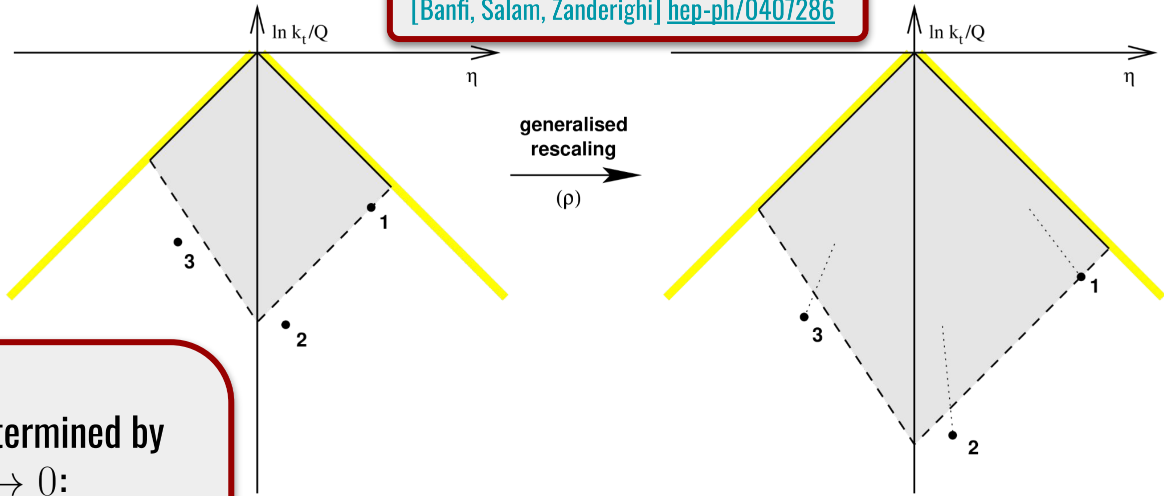
Working in the rest frame of the color dipole, the other momenta scale like:

$$\tilde{p}_l^0 \sim \rho^{1-\xi_l}$$
$$\tilde{p}_l^{1,2} \sim \rho$$
$$\tilde{p}_l^3 \sim \rho^{1-\xi_l}$$

for $\rho \to 0$

# Recoil

[Banfi, Salam, Zanderighi] hep-ph/0407286




Scaling under an additional emission is determined by the Lorentz transformation in the limit $\rho \to 0$:

$$\Delta p_l^\mu = 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}\,\tilde{K}^\mu - \frac{\tilde{p}_l X}{\tilde{K}^2}\,\tilde{K}^\mu + \frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}\,X^\mu$$

Scaling becomes:

$$\Delta p_l^0 \sim \rho^{1-\xi_l}X^0 + \rho^{2-\xi_l-\max(\xi_i,\xi_j)}\tilde{K}^0 + \rho^{1-\xi_l}X^0 \sim \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \qquad \tilde{p}_l^0 \sim \rho^{1-\xi_l}$$

$$\Delta p_l^{1,2} \sim \rho^{1-\xi_l}X^{1,2} \sim \rho^{2-\xi_l} \qquad \tilde{p}_l^{1,2} \sim \rho$$

$$\Delta p_l^3 \sim \rho^{1-\xi_l}X^3 \sim \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \qquad \tilde{p}_l^3 \sim \rho^{1-\xi_l}$$

# Evolution

Evolution variable:

$t = 2E_j^2(1-\cos\theta_j^i) = v(1-z)2\tilde{p}_i\tilde{K}$

Corresponds to Lund plane $k_t^2 \;\to\; \beta_{\mathrm{PS}} = 0$ in the generalized rescaling limit

Rewrite partial radiators:

$$\bar{w}_{ik,j}^i = \frac{\bar{W}_{ik,j}^i}{E_j^2} = \frac{\bar{W}_{ik,j}}{p_i p_j}$$

$$\bar{W}_{ik,j}^i = \frac{z}{1-z}(1-\cos\theta_j^i)\bar{W}_{ik,j}$$

Bounded from above by 2

$\bar{w}_{ik,j}^i \leq 2w_{ik,j}^{(\mathrm{coll})}$

Mild dependence on azimuthal angle

Differential splitting probabilities:

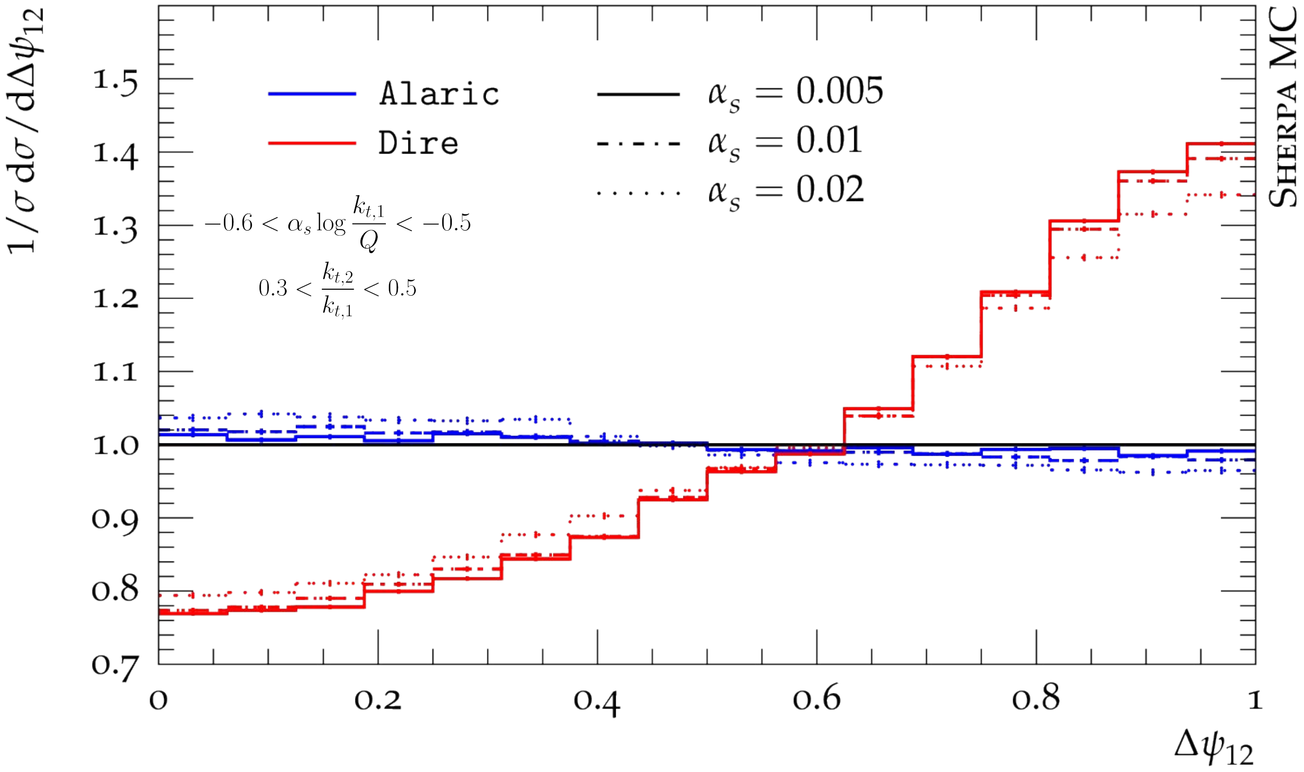
$$\mathrm{d}P_{ik,j}^{i\,(\mathrm{soft})}(t,z,\phi) = \mathrm{d}t\ \mathrm{d}z\ \frac{\mathrm{d}\phi}{2\pi}\frac{\alpha_s}{2\pi t}2C_i\bar{W}_{ik,j}$$

$$\mathrm{d}P_{ik,j}^{i\,(\mathrm{coll})}(t,z,\phi) = \mathrm{d}t\ \mathrm{d}z\ \frac{\mathrm{d}\phi}{2\pi}\frac{\alpha_s}{2\pi t}C_{\tilde{i}i}$$

Soft-subtracted splitting functions

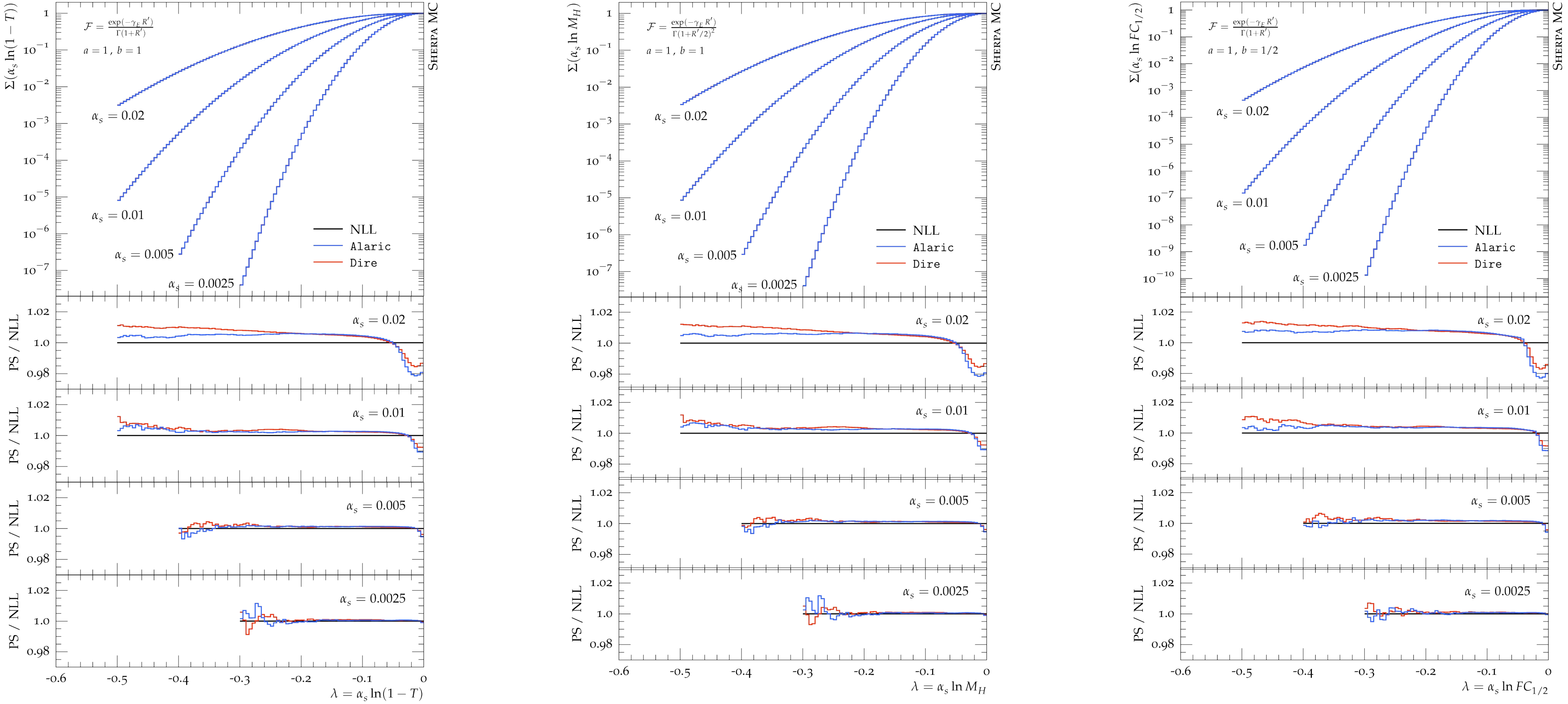
# Numerical Tests

Azimuthal angle between two Lund plane declusterings
Tests soft and rapidity separated emissions



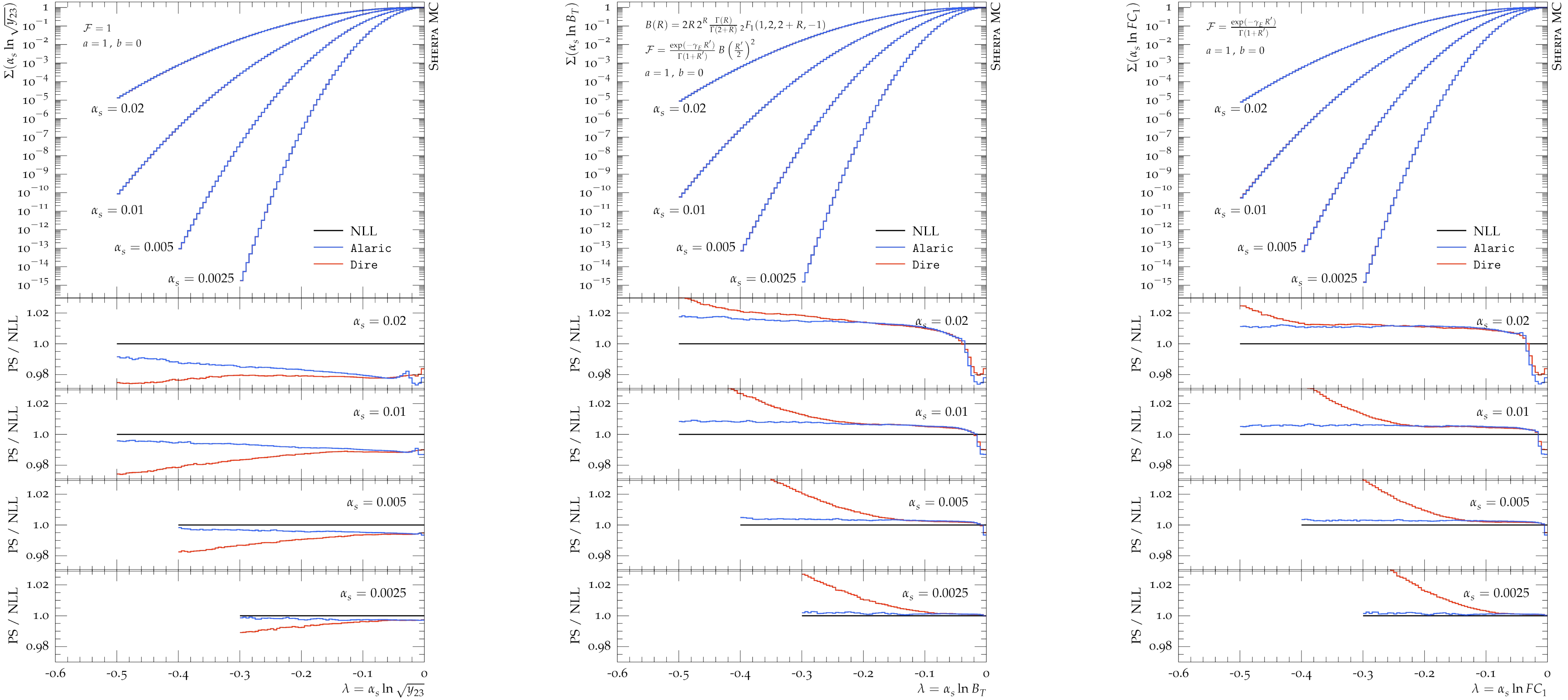
# Numerical Tests

For Thrust, Heavy Jet mass and Fractional Energy Correlators with b = 1, both Dire and Alaric are NLL



# Numerical Tests

For the Two-Jet rate, total Broadening and FC with b = 1 Alaric and Dire differ



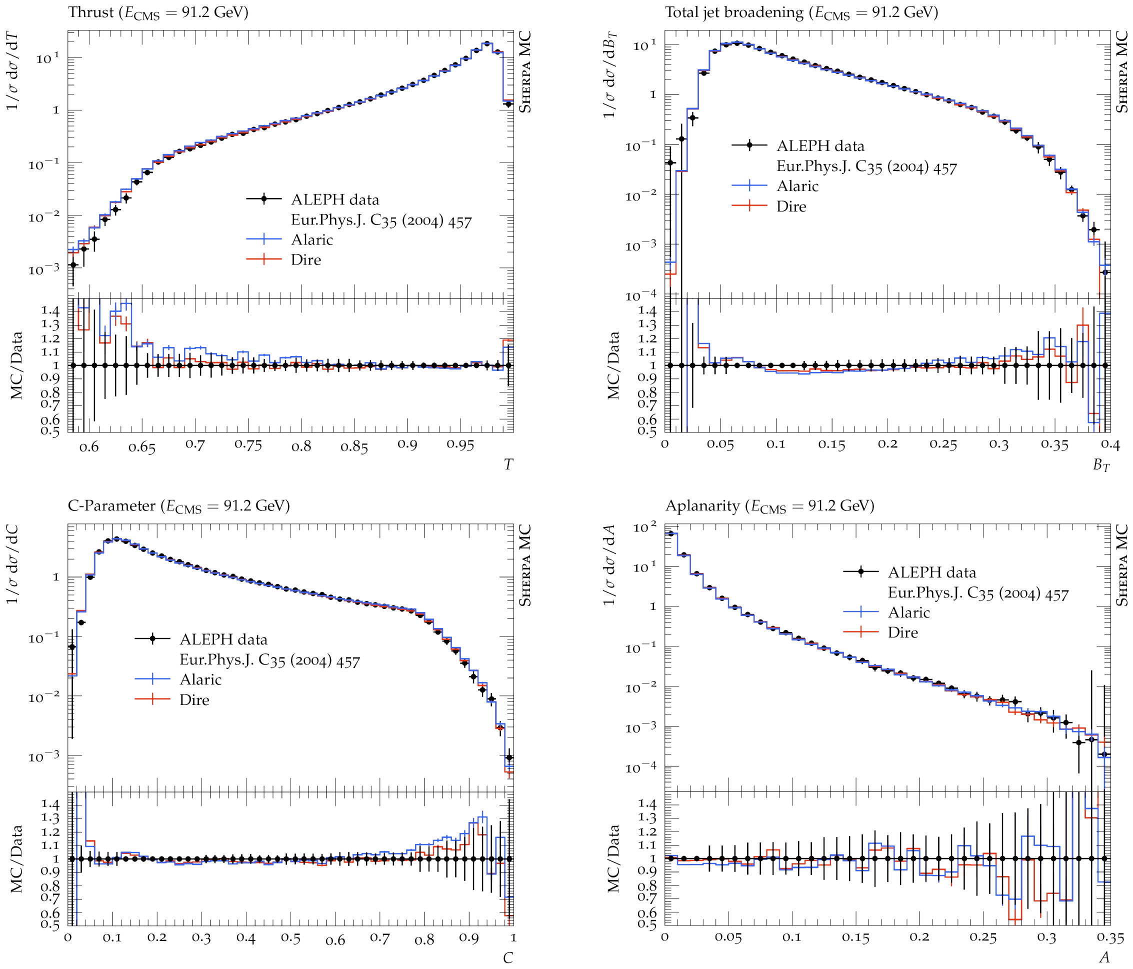
# Let’s look at Data

Details:

- CMW scheme
- Massless b- and c-quarks
- Flavour thresholds
- Hadronization through Lund string fragmentation

Comments:

- Perturbative region to the right, except for thrust
- Some deviations for Broadening and Aplanarity
- MECs and massive quarks will improve situation



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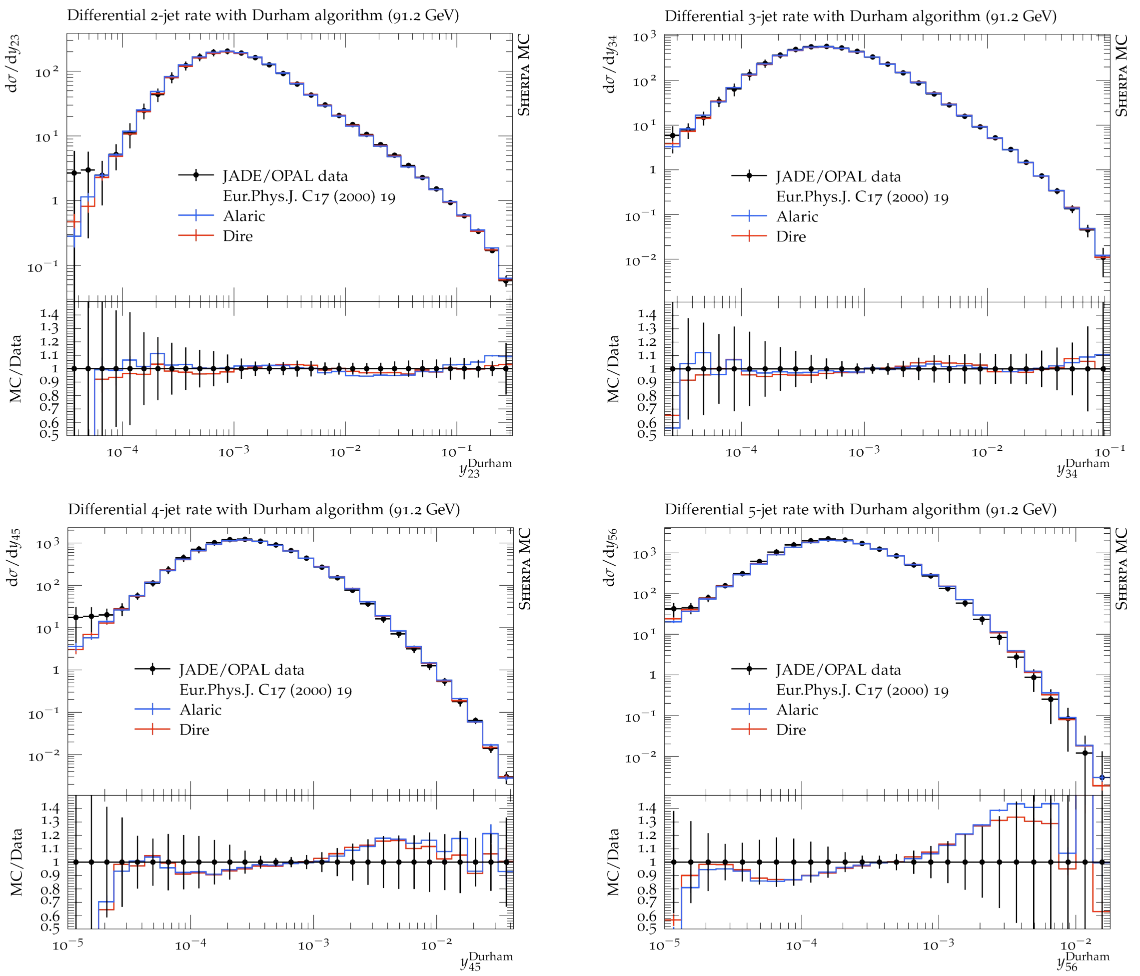
- CMW scheme
- Massless b- and c-quarks
- Flavour thresholds
- Hadronization through Lund string fragmentation

Comments:

- Perturbative region to the right
- b-quark mass corresponds to $y \approx 2.8 \times 10^{-3}$



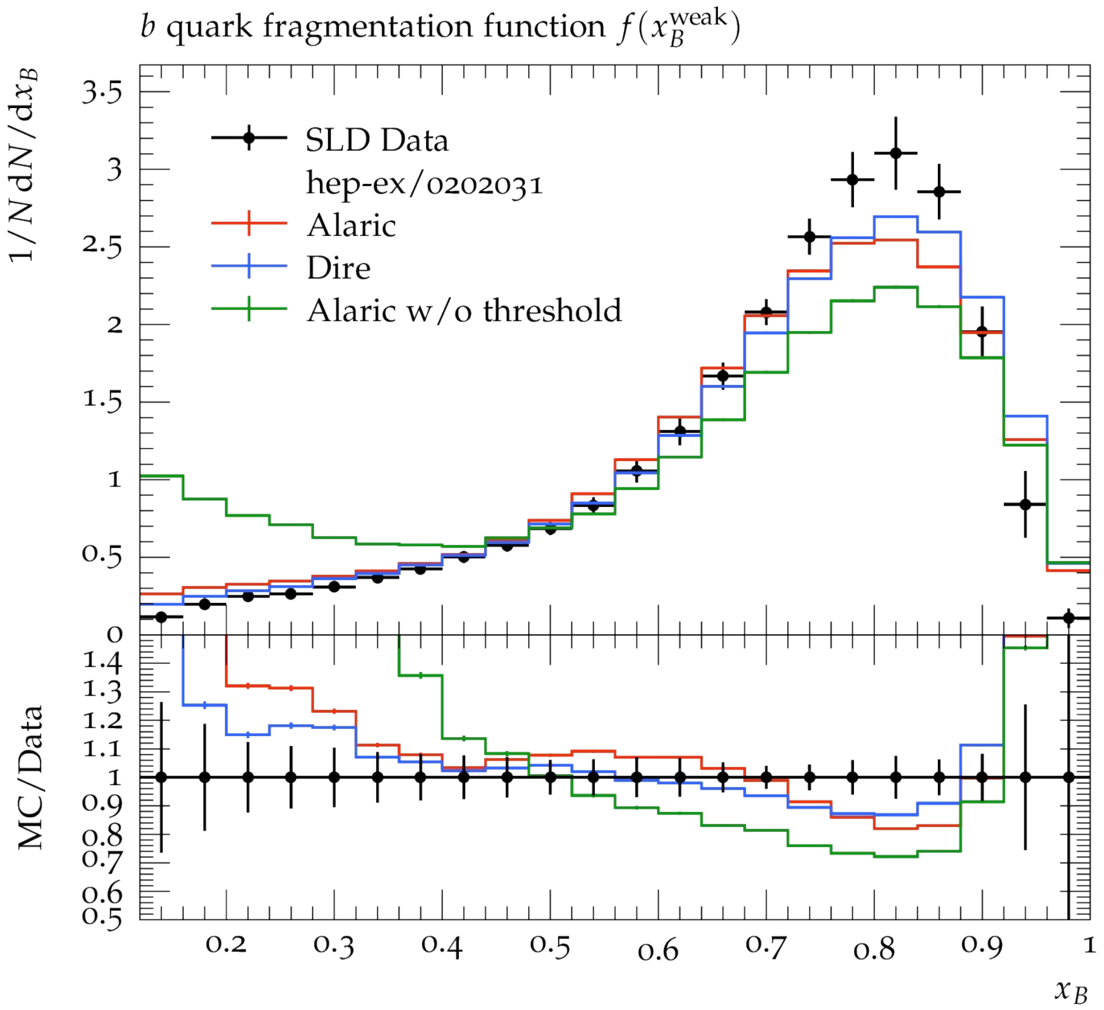
# Let's look at Data

Details:

- CMW scheme
- Massless b- and c-quarks
- Flavour thresholds
- Hadronization through Lund string fragmentation

Comments:

- Low values of x dominated by $g \to bb$
- Large values of x dominated by $b \to bg$ and hadronization



# NLO Matching

If we compare NLO calculation and PS expanded to first order in the strong coupling:

- NLO calculation: contains virtual corrections, one hard, soft or collinear emission
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→ Find procedure such that PS treats soft and collinear emissions, FO calculation treats hard emissions and there is a smooth crossover in between them

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$$\sigma^{(\mathrm{NLO})} = \int \mathrm{d}\Phi_n \, [B + V] + \int \mathrm{d}\Phi_{n+1} \, R$$

Explicit poles

Divergent in soft and collinear limits

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$$\sigma^{(\mathrm{NLO})} = \int \mathrm{d}\Phi_n \left[B + V + \int \mathrm{d}\Phi_{+1} S\right] + \int \mathrm{d}\Phi_{n+1} \left[R - S\right]$$

Depends on momentum mapping, poles need to be made explicit

Contains Eikonal and splitting functions

Two common schemes:

- [Catani, Seymour] hep-ph/9605323
- [Frixione, Kunszt, Signer] hep-ph/9512328

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In the MC@NLO scheme, the subtraction terms are chosen to be the PS evolution kernels → Need to compute integrated terms with our momentum mapping
[Frixione, Webber] hep-ph/0204244

# NLO Matching

Alaric shares many similarities with
Catani-Seymour identified particle subtraction
$\rightarrow$ MC@NLO matching straightforward
Follow [Höche, Liebschner, Siegert] 1807.04348

Combined integrated subtraction term for identified parton production with a partonic fragmentation function:

$$\int_{m+1} \mathrm{d}\sigma^S + \int_m \mathrm{d}\sigma^C = \frac{1}{2} \sum_{i=g,q,\bar{q}} \sum_{\tilde{\imath}=1}^{m} \int_0^1 \frac{\mathrm{d}z}{z^{2-2\epsilon}} \int_m \mathrm{d}\sigma^B\left(p_1, \ldots, \frac{p_i}{z}, \ldots, p_m\right) \otimes \hat{\mathbf{I}}^{(\mathrm{FS})}_{\tilde{\imath}i}$$

Insertion operator:

$$\hat{\mathbf{I}}^{(\mathrm{FS})}_{\tilde{\imath}i} = \delta(1-z)\mathbf{I}_{\tilde{\imath}i} + \mathbf{P}_{\tilde{\imath}i} + \mathbf{H}_{\tilde{\imath}i}$$

$$\mathbf{I}_{\tilde{\imath}i}(p_1, \ldots, p_i, \ldots, p_m; \epsilon) = -\frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}}\mathbf{T}_k}{\mathbf{T}^2_{\tilde{\imath}}} \left(\frac{4\pi\mu^2}{2p_ip_k}\right)^{\epsilon} \mathcal{V}_{\tilde{\imath}i}(\epsilon)$$

$$\mathbf{P}_{\tilde{\imath}i}\left(p_1, \ldots, \frac{p_i}{z}, \ldots, p_m; z; \mu_F\right) = \frac{\alpha_s}{2\pi} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}}\mathbf{T}_k}{\mathbf{T}^2_{\tilde{\imath}}} \ln \frac{z\mu_F^2}{2p_ip_k} \delta_{\tilde{\imath}i} P_{\tilde{\imath}i}(z)$$

$$\mathbf{H}_{\tilde{\imath}i}(p_1, \ldots, p_i, \ldots, p_m; n; z) = -\frac{\alpha_s}{2\pi} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}}\mathbf{T}_k}{\mathbf{T}^2_{\tilde{\imath}}} \left[\tilde{K}^{\tilde{\imath}i}(z) + \bar{K}^{\tilde{\imath}i}(z) + 2P_{\tilde{\imath}i}(z)\ln z + \mathcal{L}^{\tilde{\imath}i}(z; p_i, p_k, n)\right]$$

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Non-trivial integral:

$$\int_0^1 \mathrm{d}z\, \mathbf{H}_{\tilde{\imath}i}(p_1, \ldots, p_i, \ldots, p_m; n; z) = -\frac{\alpha_s}{2\pi} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}}\mathbf{T}_k}{\mathbf{T}_{\tilde{\imath}}^2} \left\{ \mathcal{K}^{\tilde{\imath}i} + \delta_{\tilde{\imath}i} \,\mathrm{Li}_2\left(1 - \frac{2\tilde{p}_i\tilde{p}_k\, \tilde{K}^2}{(\tilde{p}_i\tilde{K})(\tilde{p}_k\tilde{K})}\right) - \int_0^1 \mathrm{d}z\, P_{\mathrm{reg}}^{qq}(z) \ln \frac{n^2 \tilde{p}_i\tilde{p}_k}{2z(\tilde{p}_i n)^2} \right\}$$

# Towards higher orders

NNLO Subtraction:

- Iterated soft+collinear limits
- Genuine double soft+triple collinear limits
- Overlapping singularities
- Integrated counterterms complicated

By now, several schemes have been developed

Nested soft collinear subtraction scheme comes with analytical expressions for integrated CTs and subtraction terms solely based on Eikonal factors and splitting functions

[Caola, Melnikov, Röntsch] 1702.01352

NNLL in Parton Showers:

- Must handle emissions for which multiple logarithmic variables are of similar size
- Higher-order kernels [Höche, Prestel] 1705.00742
- Comparison to analytic resummation

Momentum mapping will be crucial to match to fixed order calculations. Ideally, repeat MC@NLO one order higher...

Can we modify an existing subtraction scheme for this purpose?

# Conclusion

2208.06057

- We presented a new NLL accurate Parton Shower Algorithm: Alaric
- First dipole-like algorithm to disentangle colour and kinematics
- Strict positivity of evolution kernels
- Momentum mapping preserves directions of hard partons

Additional developments:

- Initial state emitter and spectator ✅
- Initial state emitter and final state spectator ✅
- Implementation in Sherpa ✅

Add massive quarks, higher order corrections, spin correlations, subleading colour,...