Dissecting hadronisation corrections for collider physics

Silvia Ferrario Ravasio

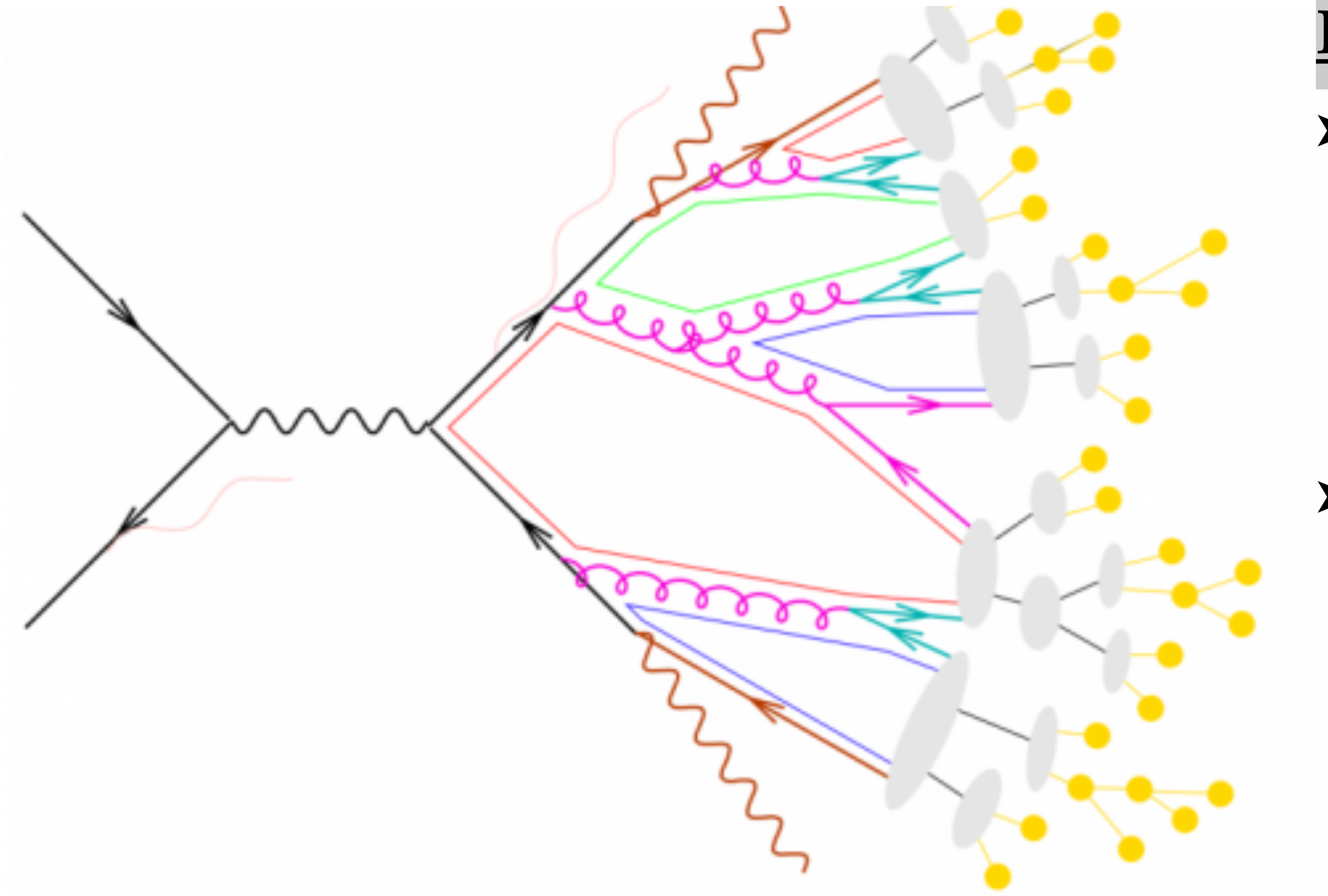
Based on

- *All-orders behaviour and renormalons in top-mass observables,*
  SFR, Nason, Oleari, *1810.10931*
- *Infrared renormalons in kinematic distributions for hadron collider processes,*
  SFR, Limatola, Nason, *2011.14114*
- *On linear power corrections in certain collider observables,*
  Caola, SFR, Limatola, Melnikov, Nason, *2108.08897*
- *Linear power corrections to e+e- shape variables in the three jet region,*
  Caola, SFR, Limatola, Melnikov, Nason, Ozcelik, *2108.08897*

Virtual HET seminar - BNL

26 January 2022

Collider events

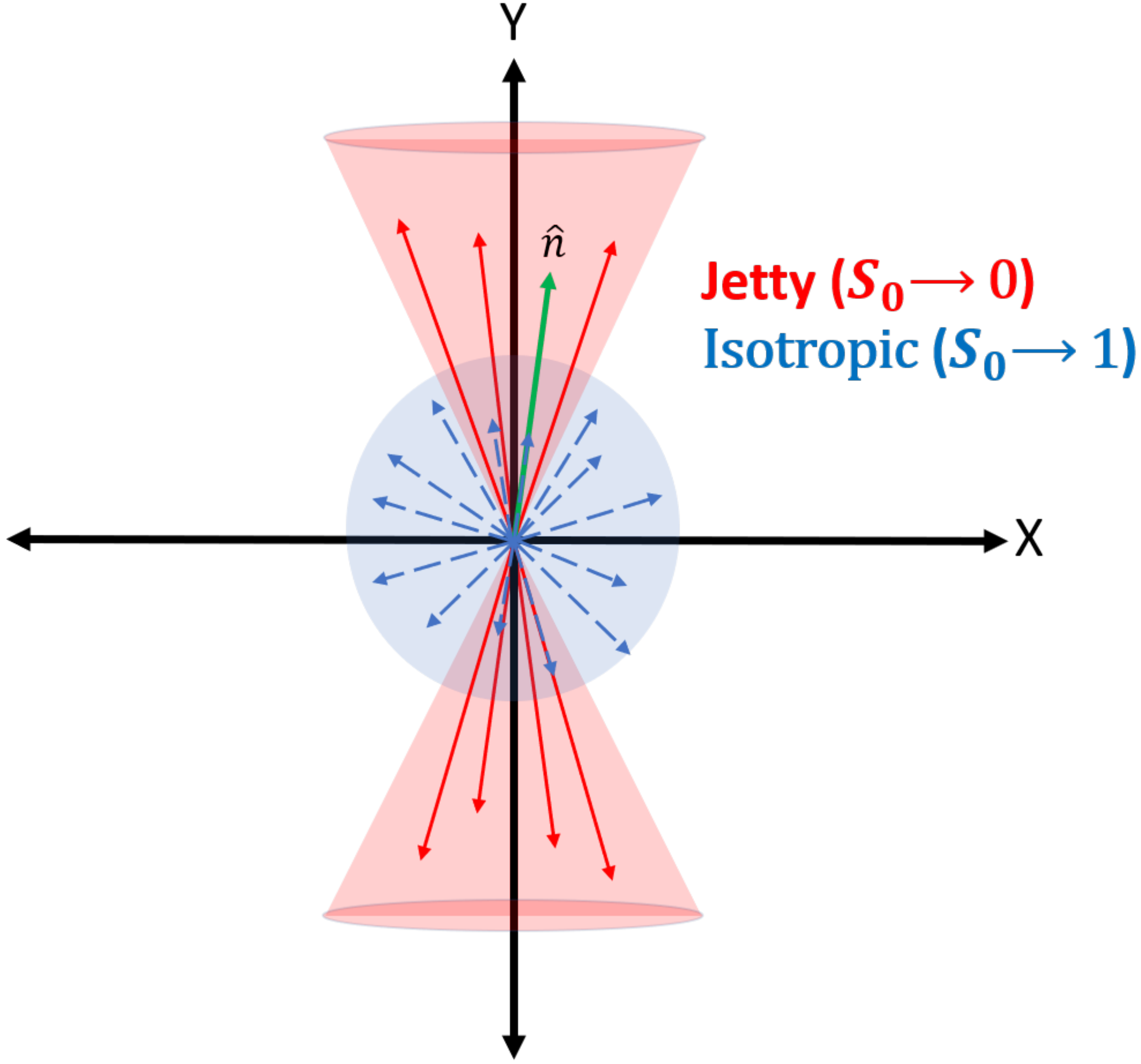


Ingredients to describe a collision

- Hard process (Q ~ 100 GeV): fixed order expansion in the strong coupling $\alpha_s(Q)$. First fully differential N3LO calulations last year.
- Multiple soft and/or collinear emissions, with $Q > k_\perp > \Lambda$, with Λ ~1 GeV. Tools: analytic resummation (more accurate, NNLL or N3LL) or parton shower algorithms (more flexible, but only LL)

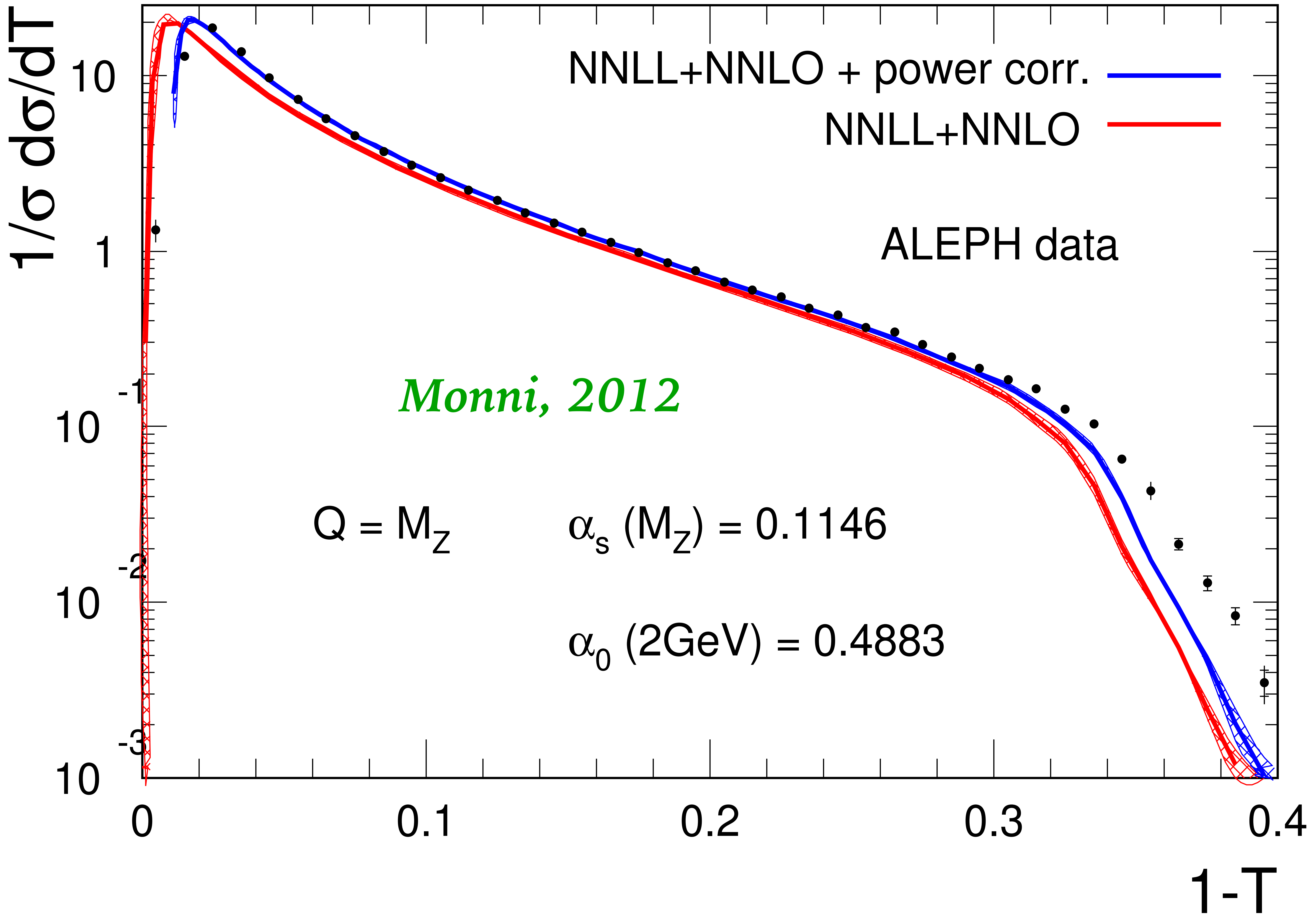
- **Hadronisation corrections:** phenomenological models (Lund or cluster) from Monte Carlo event generators, or analytic models

Event shapes



- State-of the art most precise calculations (NNLO, NNLL, N^3LL, N^3LO, . . .) are not interfaced to parton showers: e.g. Event shapes!
- The use of analytic hadronisation models is then recommended (estimating hadronisation from MC can lead inconsistencies)
- Event shapes measure the geometry of a collision: the more symmetric, the more radiation → very sensitive to the value of the strong coupling constant α_s
- Event shapes to perform **precise measurements of** α_s

Hadronisation models for event shapes

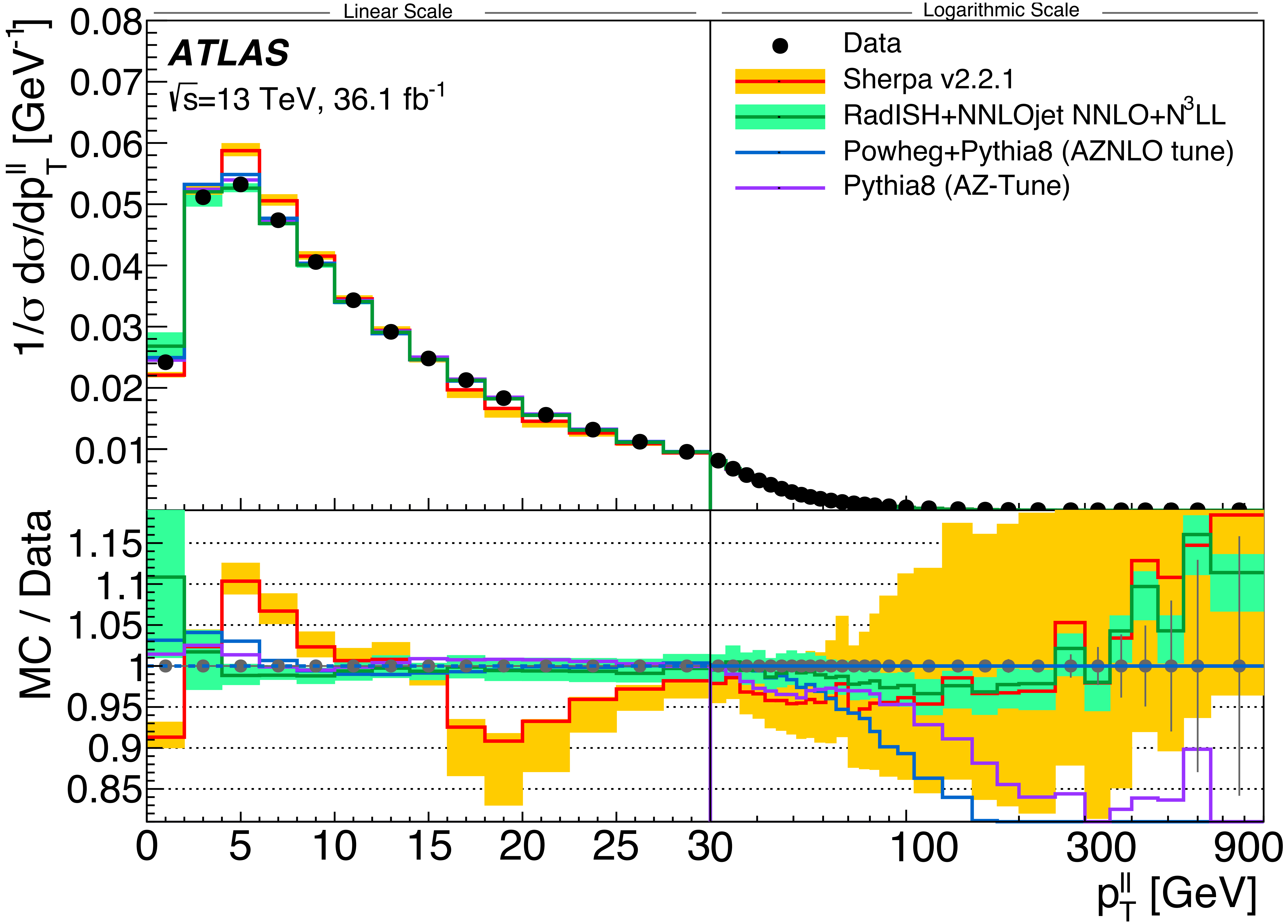


- **Non-perturbative linear-power corrections $\propto 1/Q$** required to fit the data!
- Analytic models: constant shift in the perturbative prediction

$$\Sigma(v) \to \Sigma\left(v - \underbrace{\mathcal{N}}_{\text{Universal}}\ \underbrace{\Delta V}_{\text{Obs dependent}}\right)$$

Is it really constant? We need to control linear NP corrections if we want percent or permille precision at $Q \approx 100$ GeV!

Transverse momentum of the Z boson



The transverse momentum of the Z boson is measured with **permille precision**.

But a **linear power corrections** can bring non-perturbative corrections of the order

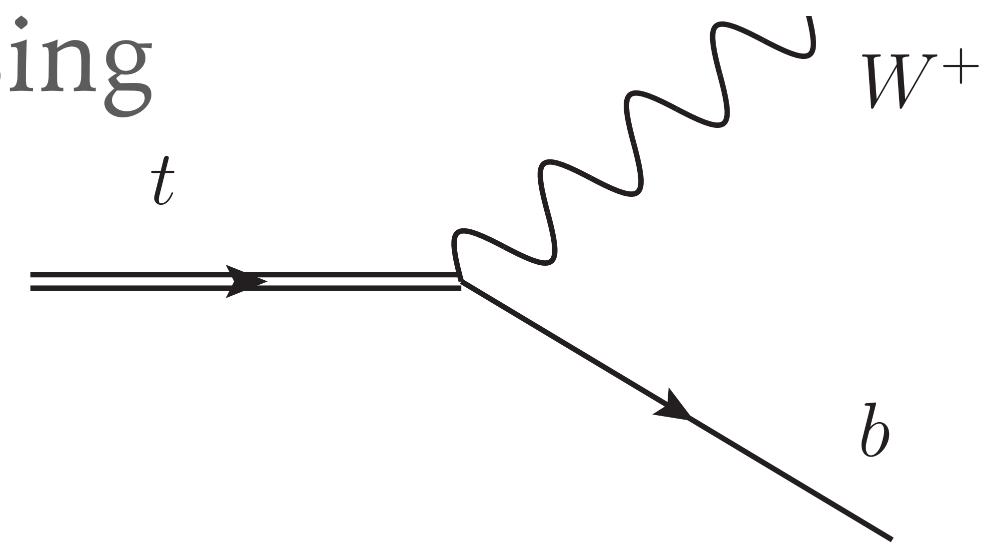
$$\frac{\Lambda}{p_{TZ}} = \frac{1\,\text{GeV}}{30\,\text{GeV}} \approx 3\,\%$$

This term can limit the theoretical precision of a perturbative calculation!

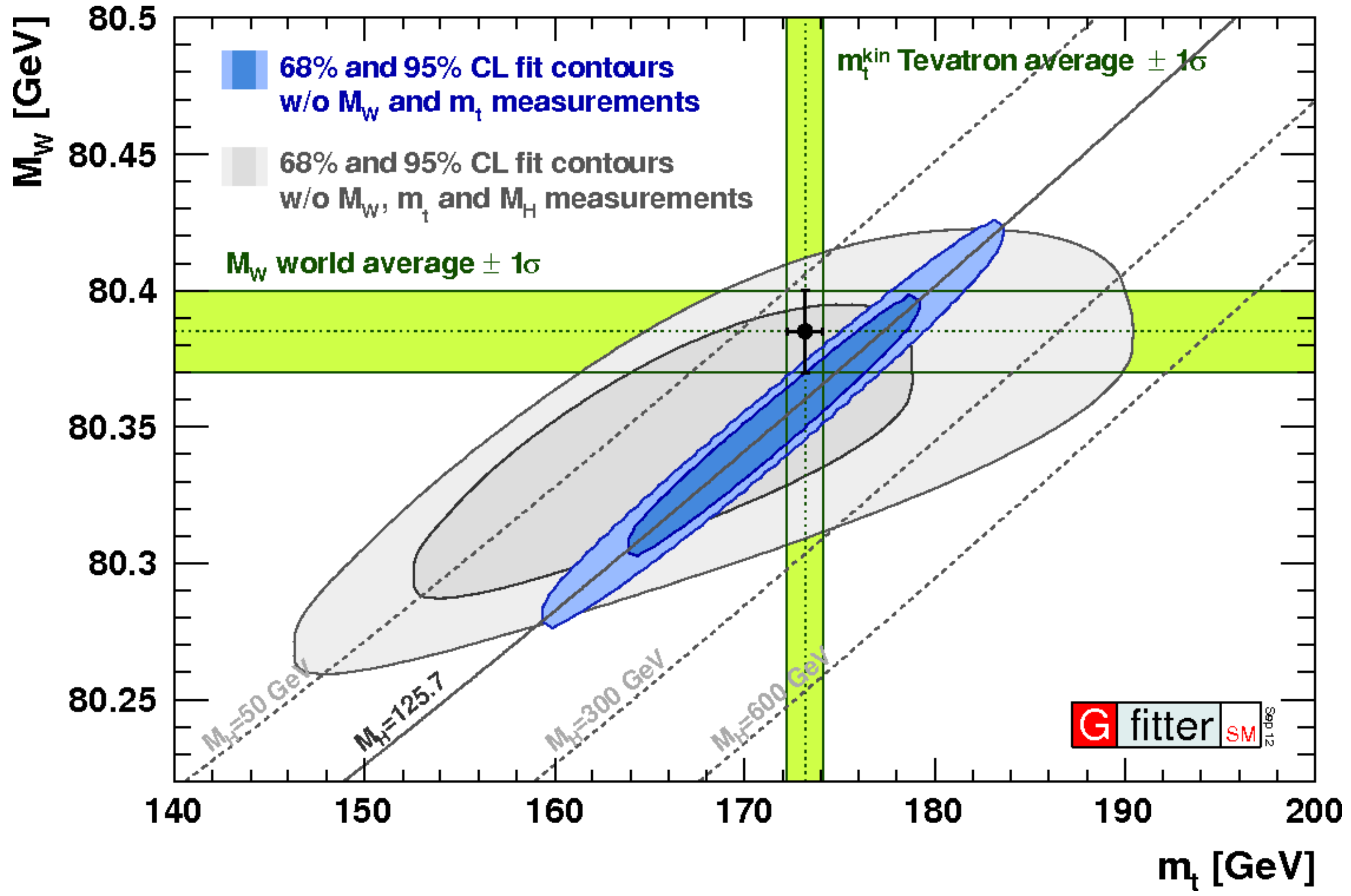
Top-quark mass and SM phenomenology

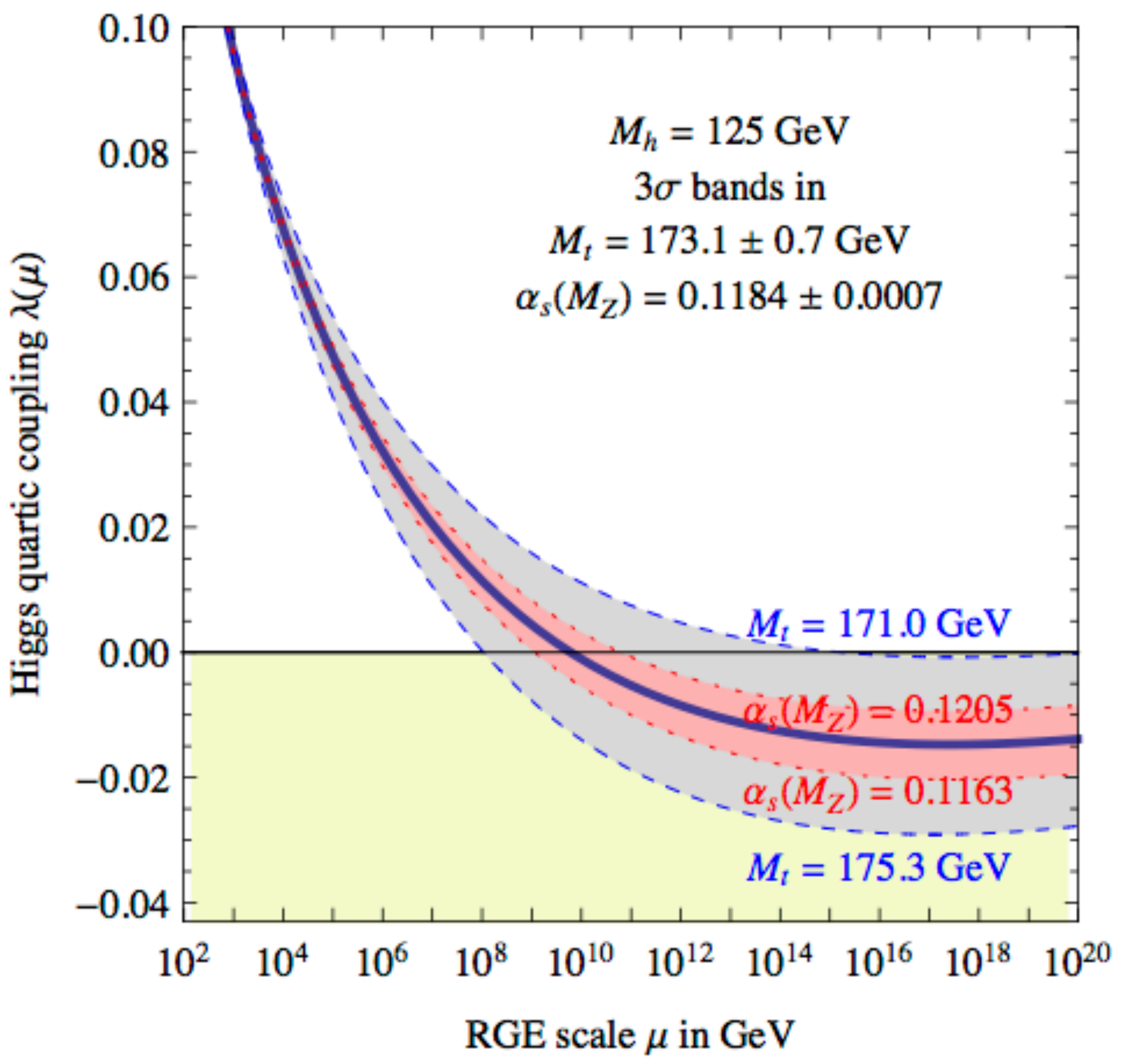
The top quark is the last quark observed so far, and its phenomenology is driven by its mass

- Only quark that **decays** instead of hadronising



- Its mass impacts many other **SM parameters** via loop corrections $(m_W, \lambda_{\text{Higgs}}, \ldots)$

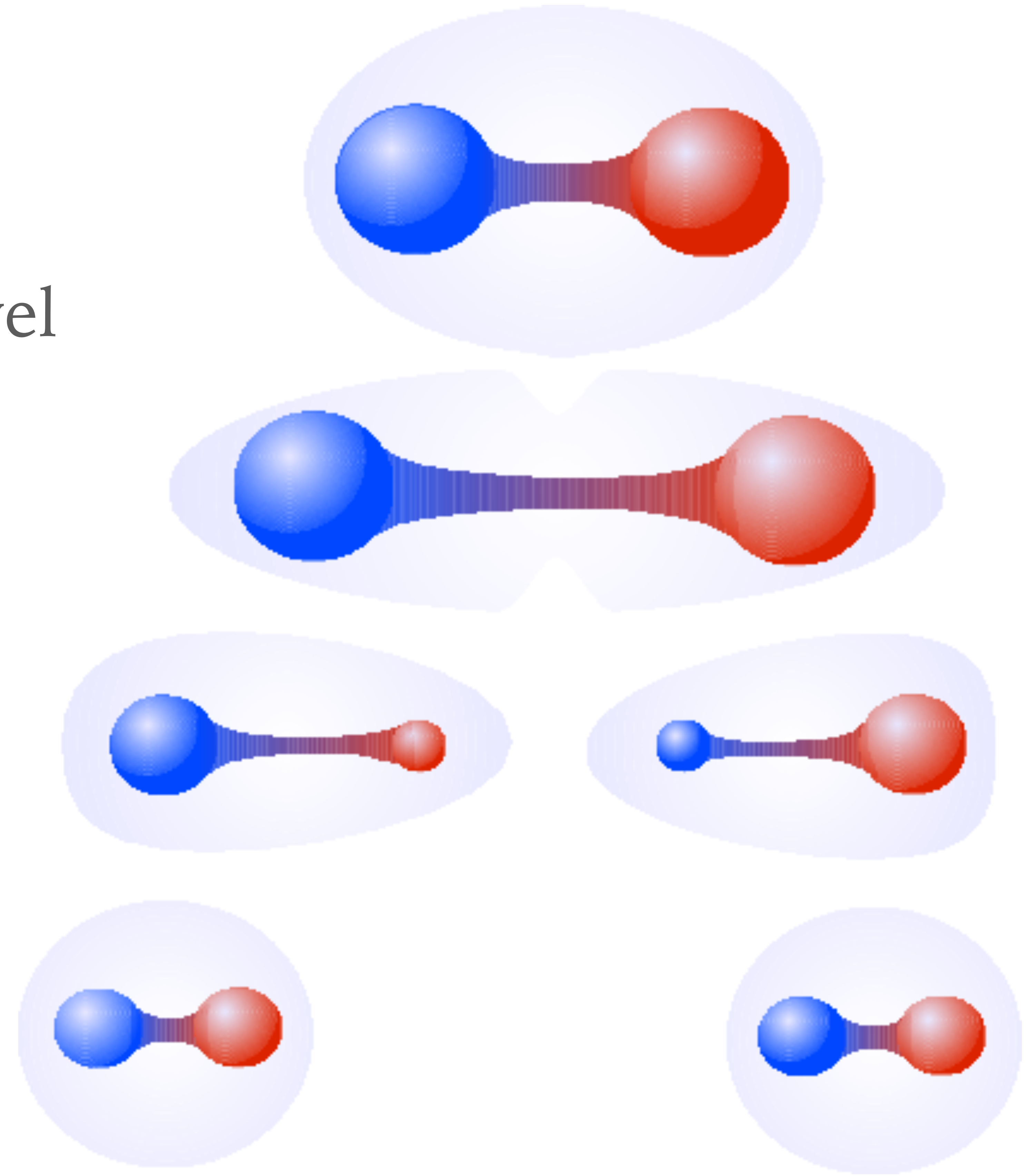




- It enters many **BSM scenarios**

Top pole mass

- Direct measurements most precise determination,
  CMS: $m_t = 172.44 \pm 0.13\ (\mathrm{stat}) \pm 0.47\ (\mathrm{syst})$ GeV
  ATLAS: $m_t = 172.61 \pm 0.25\ (\mathrm{stat}) \pm 0.41\ (\mathrm{syst})$ GeV
  projected future exp uncertainty 200 MeV: high precisions demands high level scrutiny of extracted m_t

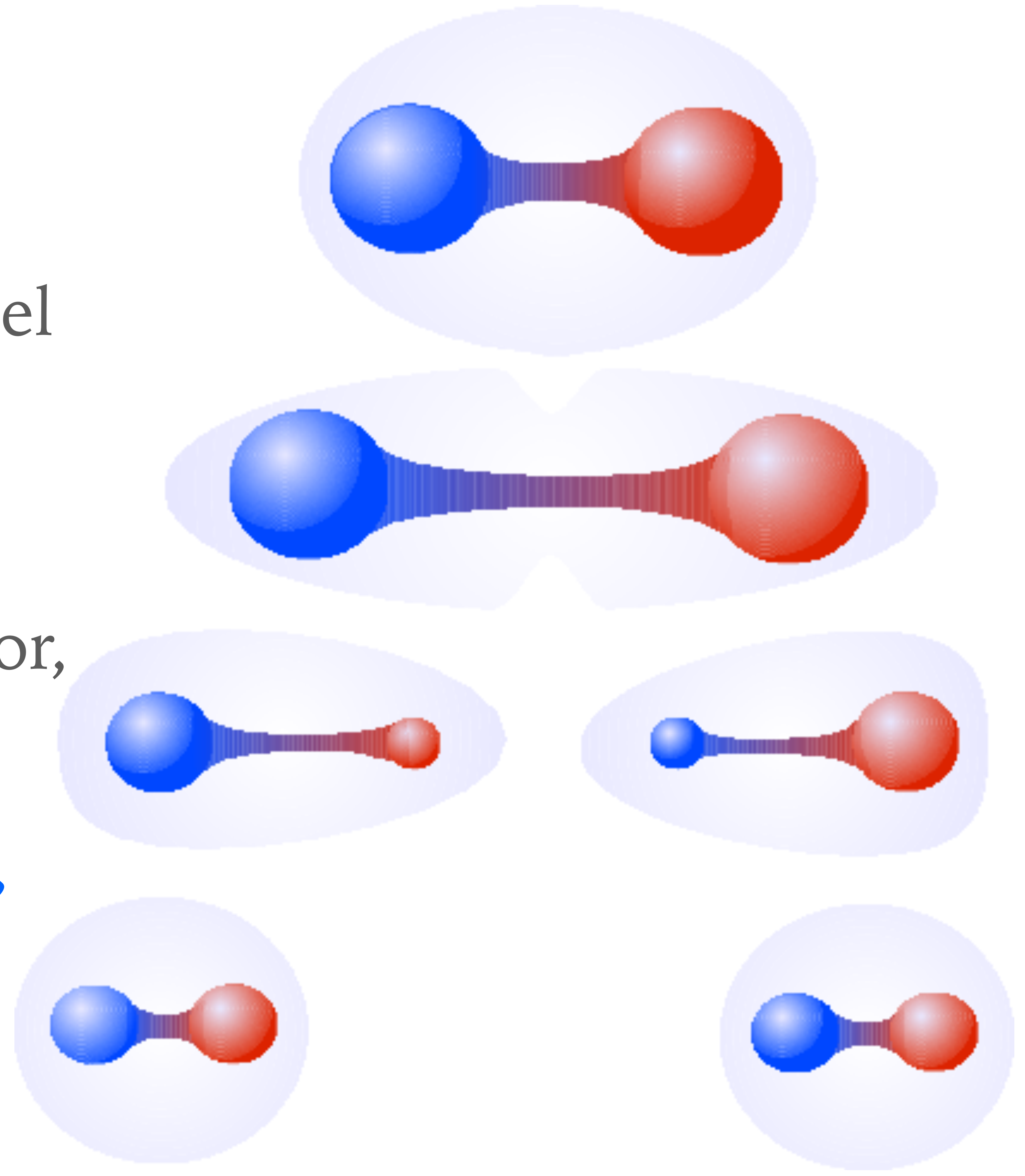


Top pole mass

- Direct measurements most precise determination,
  CMS: $m_t = 172.44 \pm 0.13\ (\text{stat}) \pm 0.47\ (\text{syst})$ GeV
  ATLAS: $m_t = 172.61 \pm 0.25\ (\text{stat}) \pm 0.41\ (\text{syst})$ GeV
  projected future exp uncertainty 200 MeV: high precisions demands high level scrutiny of extracted m_t

- m_t measurements are (related to the) pole mass, which is not very well-defined for a coloured object, as it is the location of the pole in the propagator, that corresponds to an asymptotic state. But there is confinement!

- For bottom and charm the divergent behaviour is already visible [Marquard, Smirnov, Smirnov, Steinhauser, 1502.01030]
  $m_c = 1.270 + 0.212 + 0.205 + 0.289 + 0.529 + \ldots$ GeV
  $m_b = 4.180 + 0.398 + 0.198 + 0.144 + 0.135 + \ldots$ GeV
  $m_t = 163.643 + 7.557 + 1.617 + 0.501 + 0.197 + \ldots$ GeV

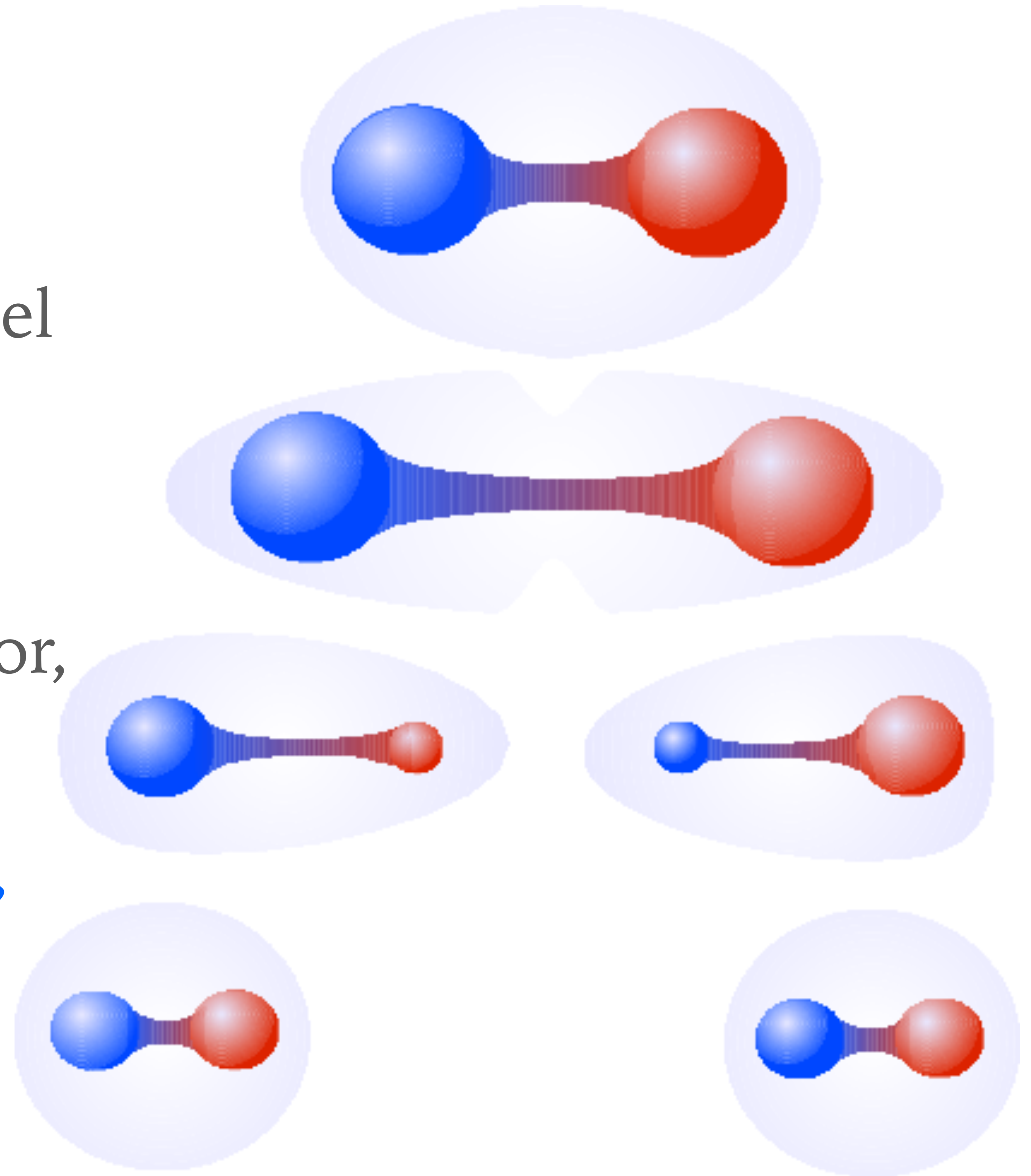


Top pole mass

- Direct measurements most precise determination,
  CMS: $m_t = 172.44 \pm 0.13$ (stat) $\pm$ 0.47 (syst) GeV
  ATLAS: $m_t = 172.61 \pm 0.25$ (stat) $\pm$ 0.41 (syst) GeV
  projected future exp uncertainty 200 MeV: high precisions demands high level scrutiny of extracted m_t

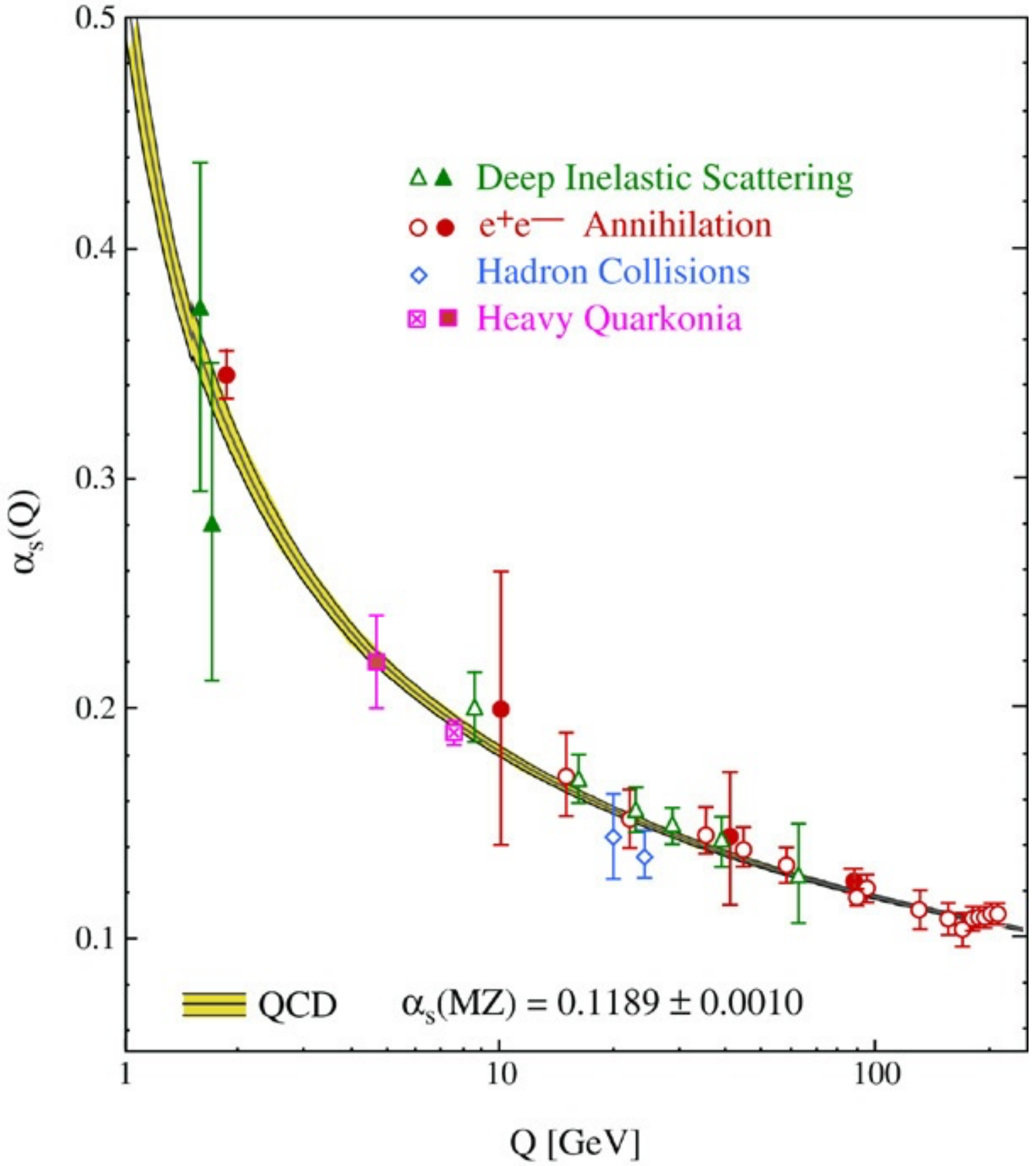
- m_t measurements are (related to the) pole mass, which is not very well-defined for a coloured object, as it is the location of the pole in the propagator, that corresponds to an asymptotic state. But there is confinement!

- For bottom and charm the divergent behaviour is already visible [Marquard, Smirnov, Smirnov, Steinhauser, 1502.01030]
  $m_c = 1.270 + 0.212 + 0.205 + 0.289 + 0.529 + \ldots$ GeV
  $m_b = 4.180 + 0.398 + 0.198 + 0.144 + 0.135 + \ldots$ GeV
  $m_t = 163.643 + 7.557 + 1.617 + 0.501 + 0.197 + \ldots$ GeV



- Top pole-mass ambiguity estimated to be between 100 and 250 MeV [Beneke, Marquard, Nason, Steinhauser, 1605.03609] [Hoang, Lepenik, Preisser, 1706.08526]. How does it impact top-related observables? Which renormalisation scheme yields the best large-orders behaviour?

Estimating non-perturbative power corrections



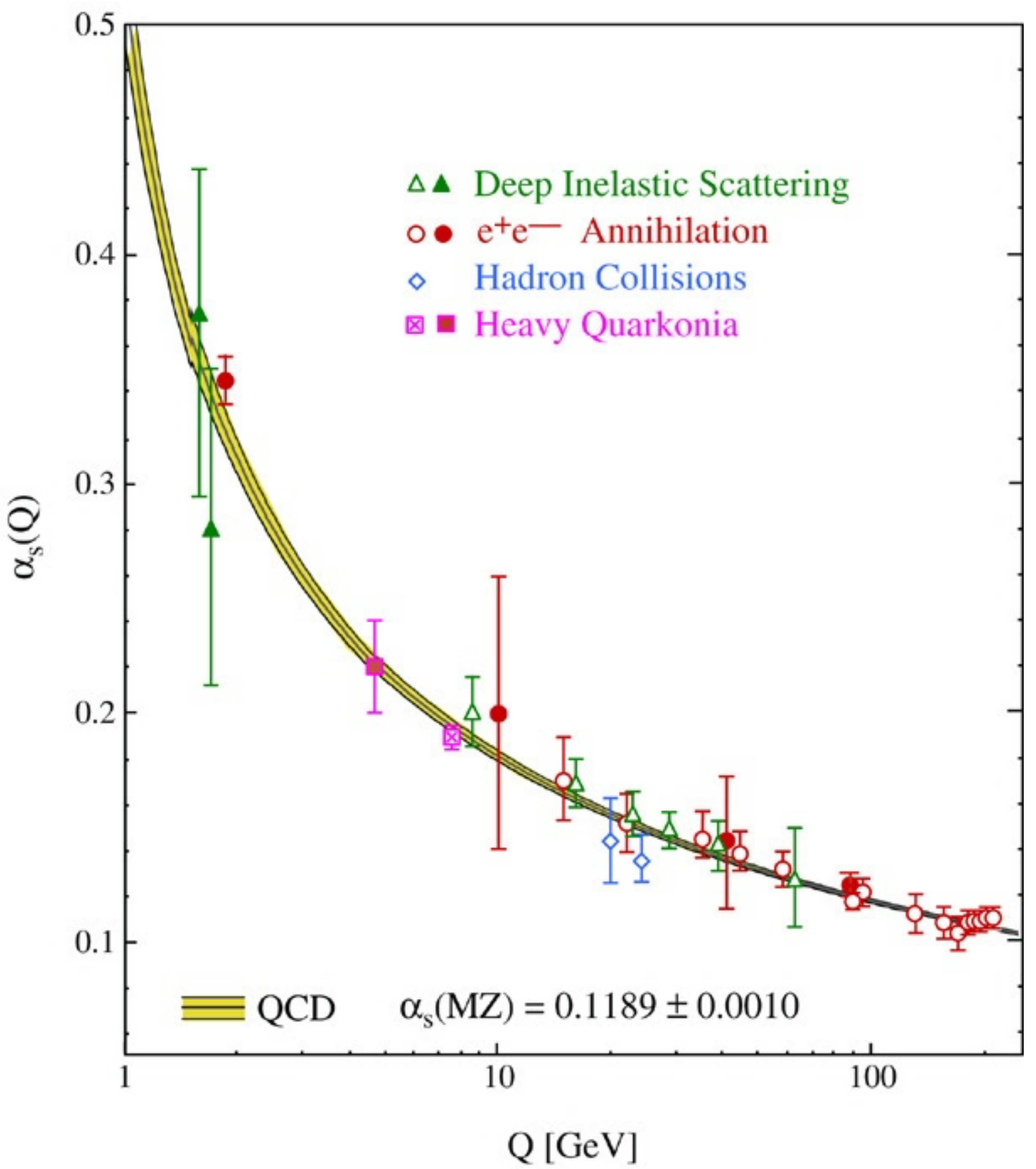
- Several sources of non-perturbative corrections, e.g. the **Landau pole** Λ in the QCD coupling constant

$$\alpha_s(Q) = \frac{1}{2b_0 \log \frac{Q}{\Lambda}}, \qquad b_0 = \frac{11 C_A}{12\pi} - \frac{n_l T_R}{3\pi} > 0$$

which leads to an **intrinsic ambiguity** when integrating over soft momenta

$$\int_0^Q dk\, k^{p-1} \alpha_s(k) = Q^p \times \frac{p}{2\, b_0} \sum_{n=0}^{\infty} \left(\frac{2\, b_0}{p} \alpha_s(Q) \right)^{n+1} n!$$

Estimating non-perturbative power corrections



- Several sources of non-perturbative corrections, e.g. the **Landau pole Λ** in the QCD coupling constant

$$\alpha_s(Q) = \frac{1}{2b_0 \log \frac{Q}{\Lambda}}, \qquad b_0 = \frac{11C_A}{12\pi} - \frac{n_l T_R}{3\pi} > 0$$

which leads to an **intrinsic ambiguity** when integrating over soft momenta

$$\int_0^Q dk\, k^{p-1} \alpha_s(k) = Q^p \times \frac{p}{2\,b_0} \sum_{n=0}^{\infty} \left(\frac{2\,b_0}{p} \alpha_s(Q) \right)^{n+1} n!$$

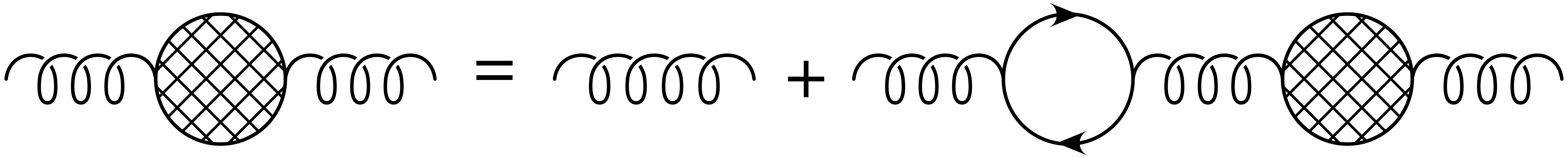
- The ambiguity has to cancel with contributions arising from physics beyond perturbation theory: estimate of **non-perturbative effects**. The smallest term in the series is

$$Q^p \sqrt{\frac{\alpha_s(Q) p\pi}{b_0}} e^{-\frac{p}{2b_0\alpha_s}} \approx \boxed{\sqrt{\frac{\alpha_s(Q) p\pi}{b_0}} \Lambda^p}$$

Non-perturbative power correction. Often dubbed RENORMALONS

The large number-of-flavours limit

- Ambiguity related to the appearance of the Landau pole can be studied in the large number of flavour n_f limit, which allows to perform all-orders computations exactly

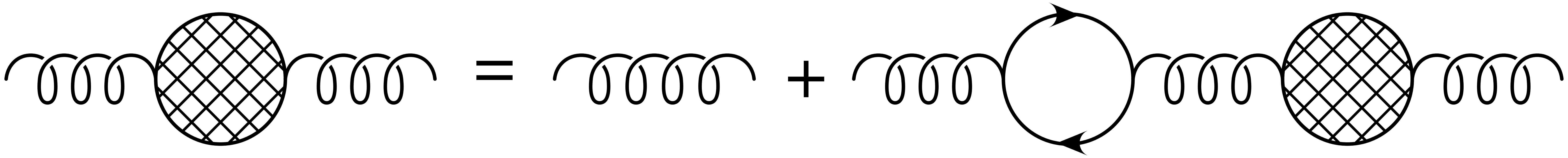


$$\frac{-ig^{\mu\nu}}{k^2+i\eta} \to \frac{-ig^{\mu\nu}}{k^2+i\eta} \times \frac{1}{1+\Pi(k^2+i\eta,\mu^2,\epsilon)-\Pi_{\rm ct}}$$

$$\Pi(k^2+i\eta,\mu^2)-\Pi_{\rm ct} = \alpha_s(\mu)\left(-\frac{n_f T_R}{3\pi}\right)\left[\log\left(\frac{|k^2|}{\mu^2}\right) - i\pi\theta(k^2) - \frac{5}{3}\right] + \mathcal{O}(\epsilon)$$

The large number-of-flavours limit

- Ambiguity related to the appearance of the Landau pole can be studied in the large number of flavour n_f limit, which allows to perform all-orders computations exactly



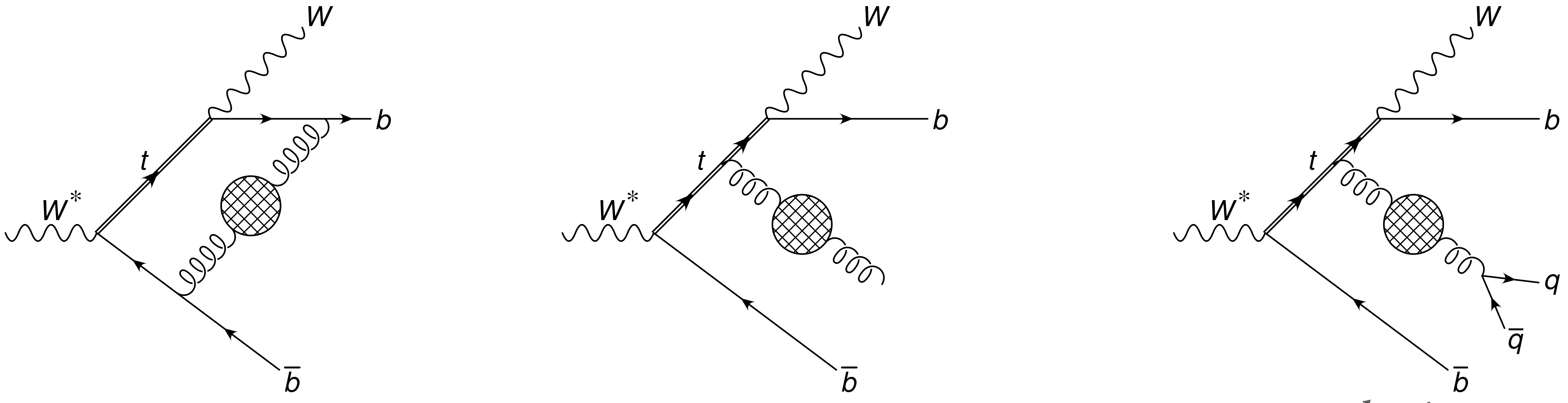
$$\frac{-ig^{\mu\nu}}{k^2+i\eta} \to \frac{-ig^{\mu\nu}}{k^2+i\eta} \times \frac{1}{1+\Pi(k^2+i\eta,\mu^2,\epsilon)-\Pi_{ct}}$$

$$\Pi(k^2+i\eta,\mu^2)-\Pi_{ct} = \alpha_s(\mu)\left(-\frac{n_f T_R}{3\pi}\right)\left[\log\left(\frac{|k^2|}{\mu^2}\right)-i\pi\theta(k^2)-\frac{5}{3}\right]+\mathcal{O}(\epsilon)$$

- Naive non-abelianisation at the end of the calculation (large b_0)

$$\Pi(k^2+i\eta,\mu^2)-\Pi_{ct} \to \alpha_s(\mu)\underbrace{\left(\frac{11C_A}{12\pi}-\frac{n_l T_R}{3\pi}\right)}_{b_0}\left[\log\left(\frac{|k^2|}{\mu^2}\right)-i\pi\theta(k^2)-C\right]$$

The large number-of-flavours limit for realistic collider processes

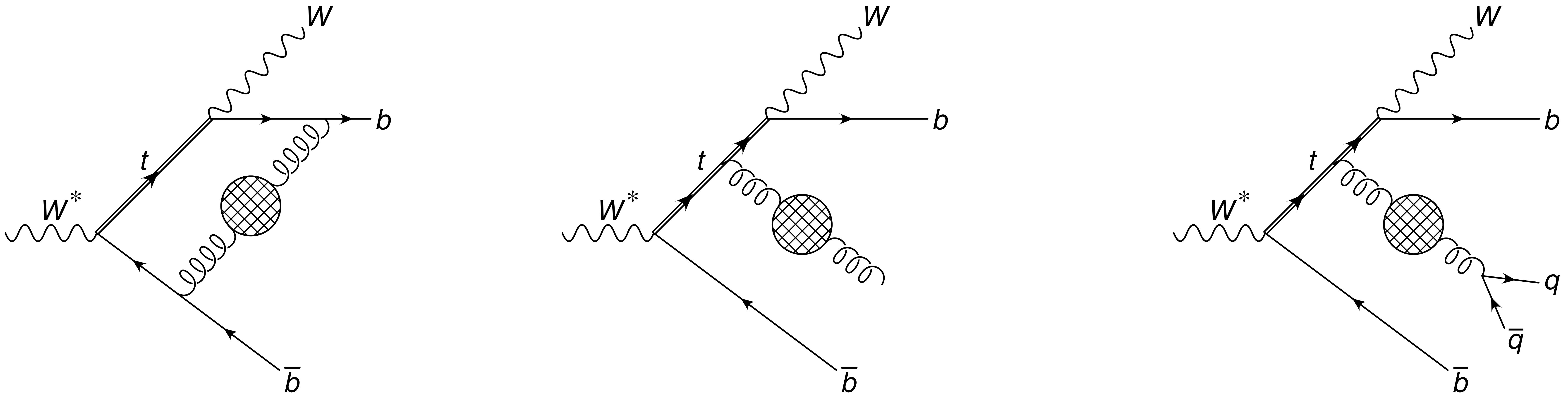


$$O = \int d\Phi \frac{d\sigma(\Phi)}{d\Phi} O(\Phi) = O_{\rm LO} - \frac{1}{\pi b_0} \int_0^\infty d\lambda \frac{d}{d\lambda}\left[\frac{T(\lambda)}{\alpha_s(\mu)}\right] \overbrace{\arctan\left[\pi b_0 \alpha_s\left(\lambda e^{-C/2}\right)\right]}^{\alpha_{\rm eff}(\lambda),\quad \textit{Beneke, '98}}$$

- λ can be thought as gluon **mass / virtuality**
- $T(\lambda) = \int d\Phi_b V_\lambda(\Phi_b) O(\Phi_b) + \frac{\lambda^2}{\pi b_0} \int d\Phi_{q\bar{q}} R_{q\bar{q}}(\Phi_{q\bar{q}}) O(\Phi_{q\bar{q}}) \delta(m^2_{q\bar{q}} - \lambda^2)$
- $T(\lambda) \xrightarrow{\lambda \to 0} O_{\rm NLO}$

SFR, Nason, Oleari, *1810.10931*

The large number-of-flavours limit for realistic collider processes



$$O = \int d\Phi \frac{d\sigma(\Phi)}{d\Phi} O(\Phi) = O_{\rm LO} - \frac{1}{\pi b_0}\int_0^\infty d\lambda \frac{d}{d\lambda}\left[\frac{T(\lambda)}{\alpha_s(\mu)}\right] \overbrace{\arctan\left[\pi\, b_0\, \alpha_s\left(\lambda e^{-C/2}\right)\right]}^{\alpha_{\rm eff}(\lambda),\quad \textit{Beneke, '98}}$$

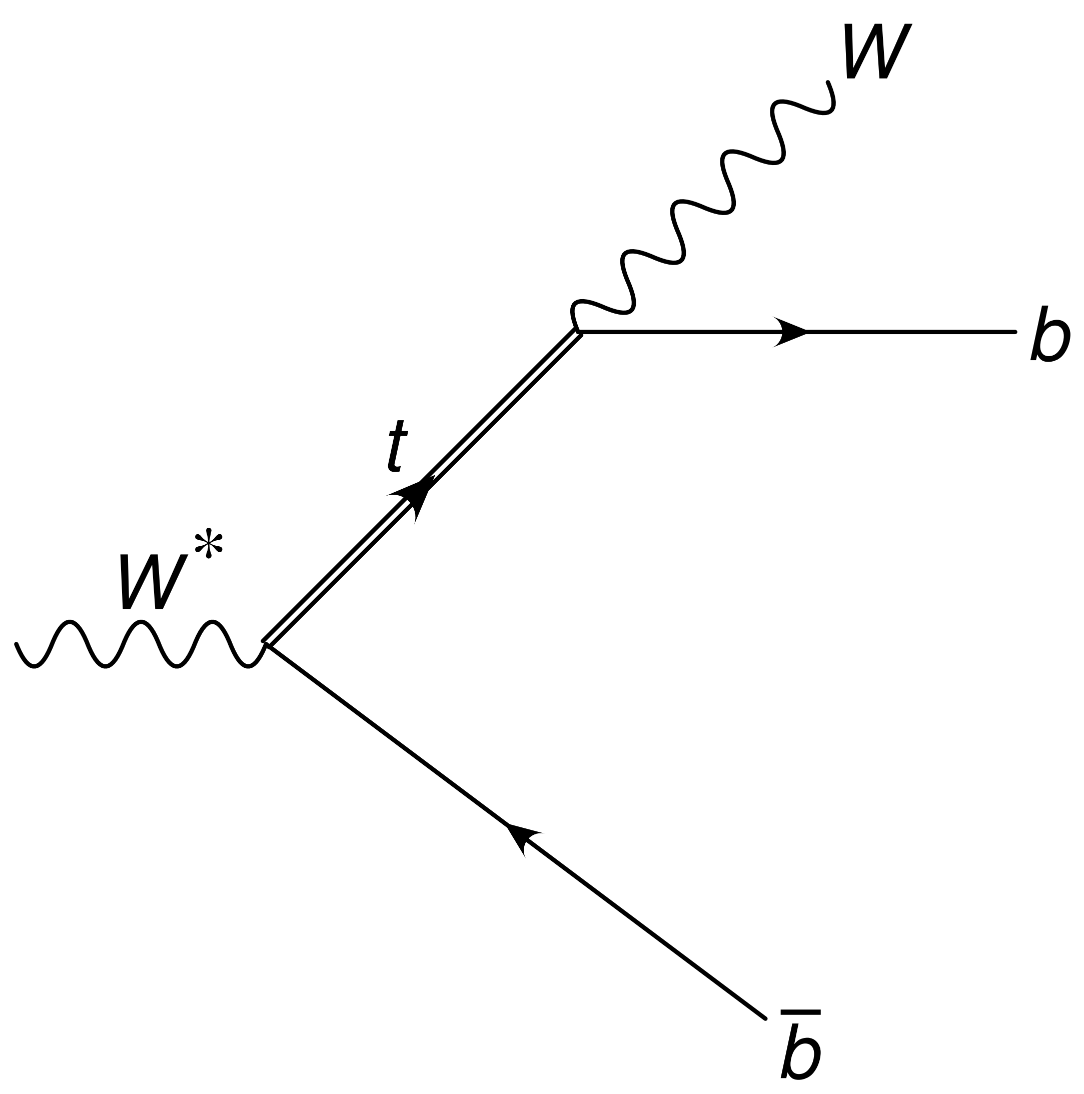
If $\frac{1}{\alpha_s(\mu)}\frac{dT(\lambda)}{d\lambda}\Big|_{\lambda=0} = -A \neq 0$, the low-$\lambda$ contribution leads to $(a = b_0\alpha_s)$

Linear power correction/ renormalon

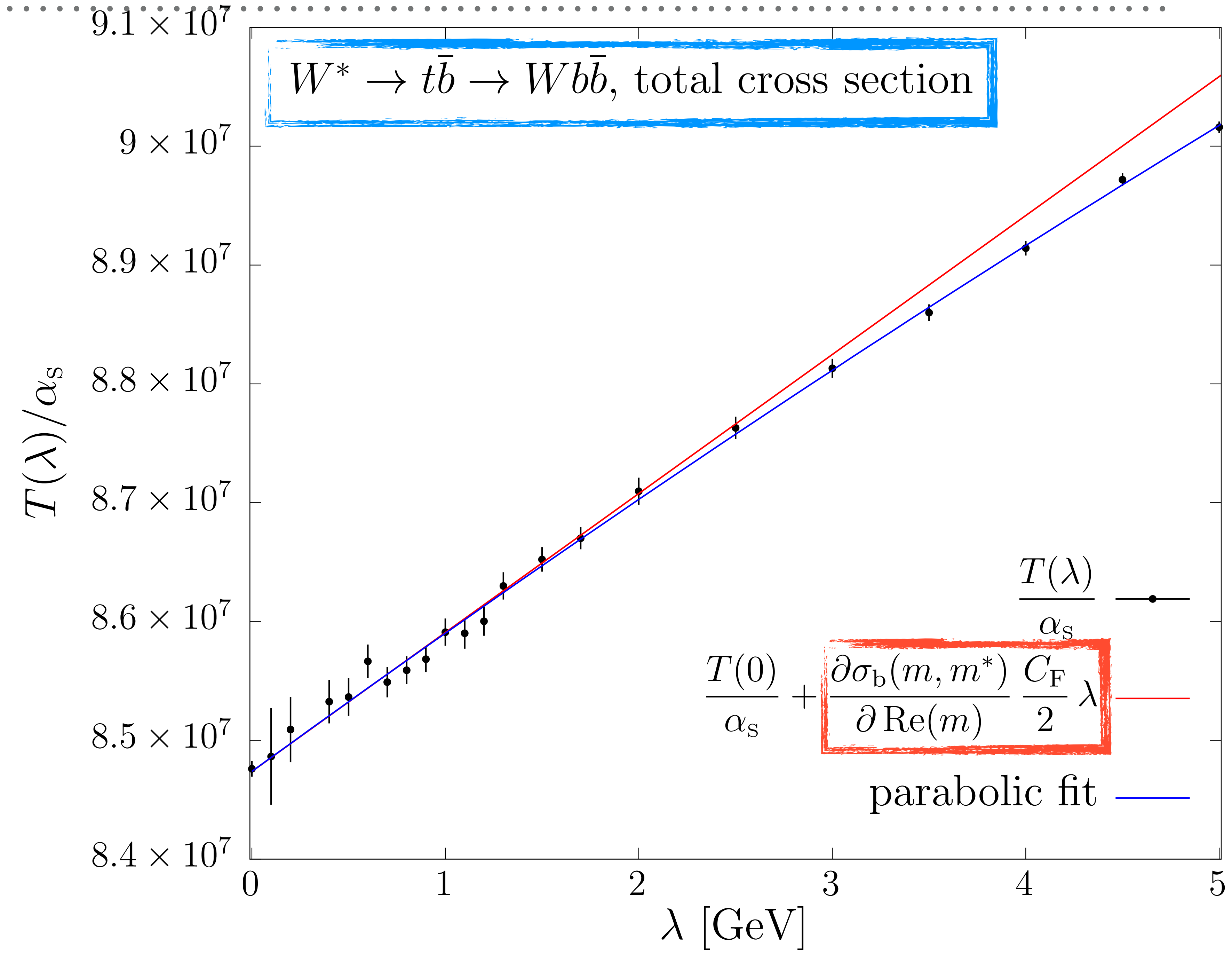
$$\underbrace{\frac{1}{\pi b_0}\arctan(\pi a) + \alpha_s \int_0^1 dz \frac{\pi a z \cos(\pi z/2) - \sin(\pi z/2)}{1+(z\pi a)^2}}_{\text{analytic}} + \underbrace{\frac{1}{\pi b_0} PV \int_0^\infty dt \frac{\exp\left(-\frac{t}{2a}\right)}{1-t}}_{\text{Borel sum + PV for pole}} - 2\underbrace{\frac{\mathbf{1}}{\mathbf{2b_0}}\exp\left(-\frac{\mathbf{1}}{\mathbf{2a}}\right)}_{\text{ambiguity} = \frac{\Lambda}{2b_0\mu}}$$

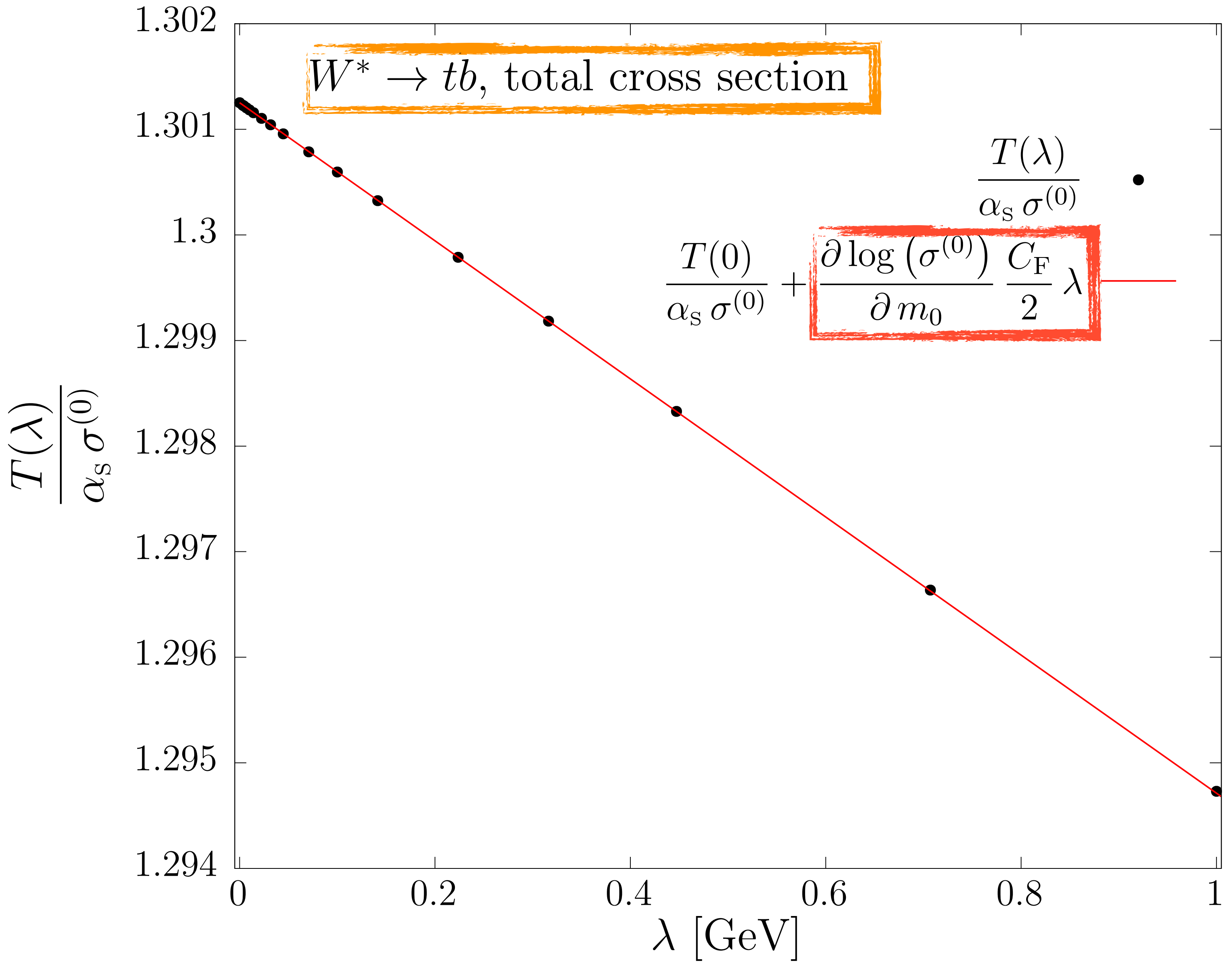
SFR, Nason, Oleari, *1810.10931*

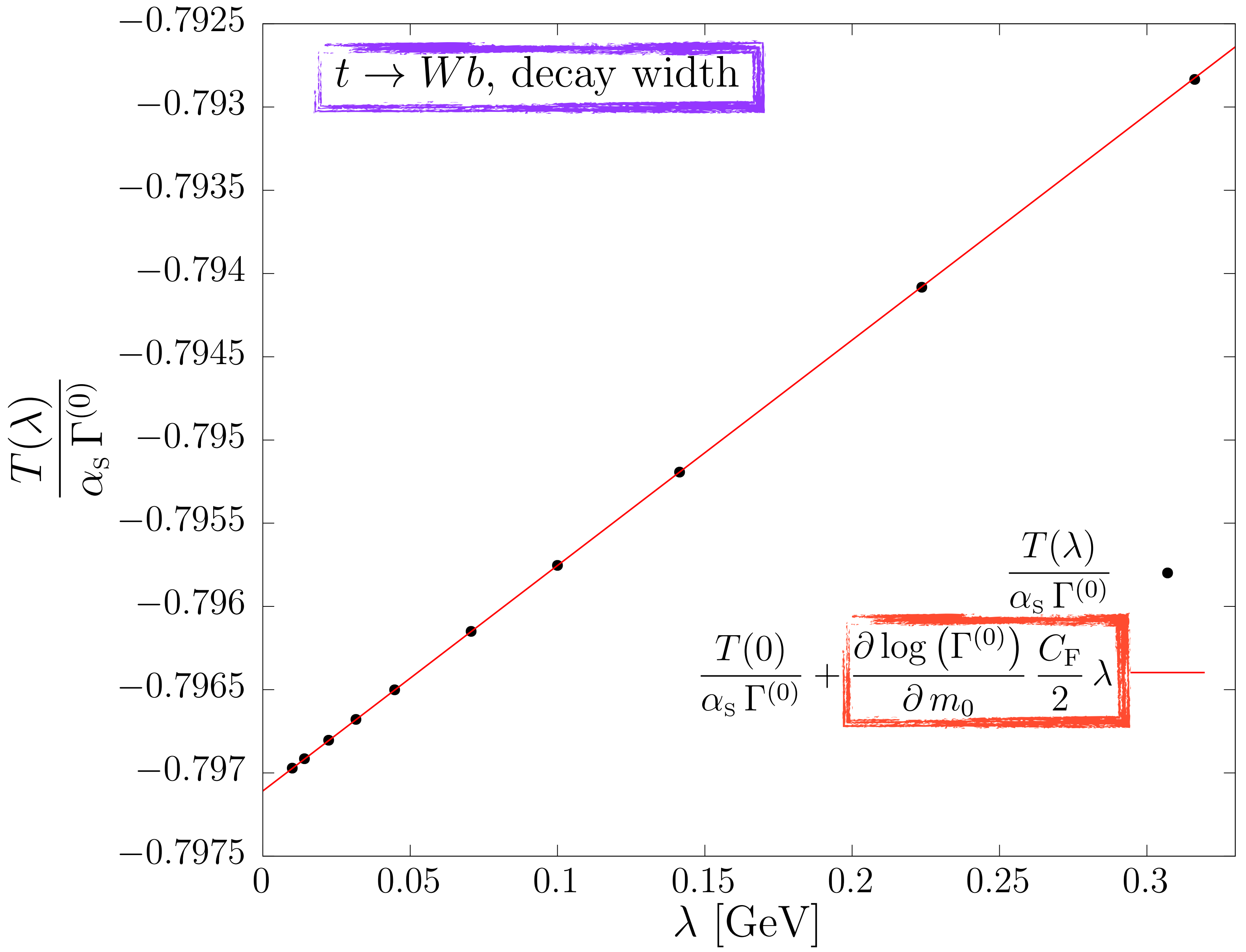
Single-top production and decay: total cross-section [SFR, Nason, Oleari, 1810.10931]



If we use the **complex pole scheme** to compute the total cross section, $T(\lambda)$ has a **linear slope**. The linear slope is caused by the pole mass counterterm, and disappears if using the $\overline{\textbf{MS}}$ scheme







Same holds in the **narrow width approximation,** where the cross section factorises between top production and decay

Single-top production and decay: leptonic observables [SFR, Nason, Oleari, 1810.10931]

Energy of the W boson (in the lab frame)

The **top width Γ_t** drastically changes the small-λ behaviour of $T(\lambda)$. A finite-width removes the linear renormalon in the $\overline{\mathrm{MS}}$ scheme, and reduces it in the pole scheme.

$$A = \frac{1}{\alpha_s}\frac{dT(\lambda)}{d\lambda}\bigg|_{\lambda=0}$$

Γ_t	slope (pole)	slope ($\overline{\mathrm{MS}}$)
NWA	0.53 (2)	0.46 (2)
10 GeV	0.058 (8)	0.004 (8)
20 GeV	0.061 (2)	0.001 (2)

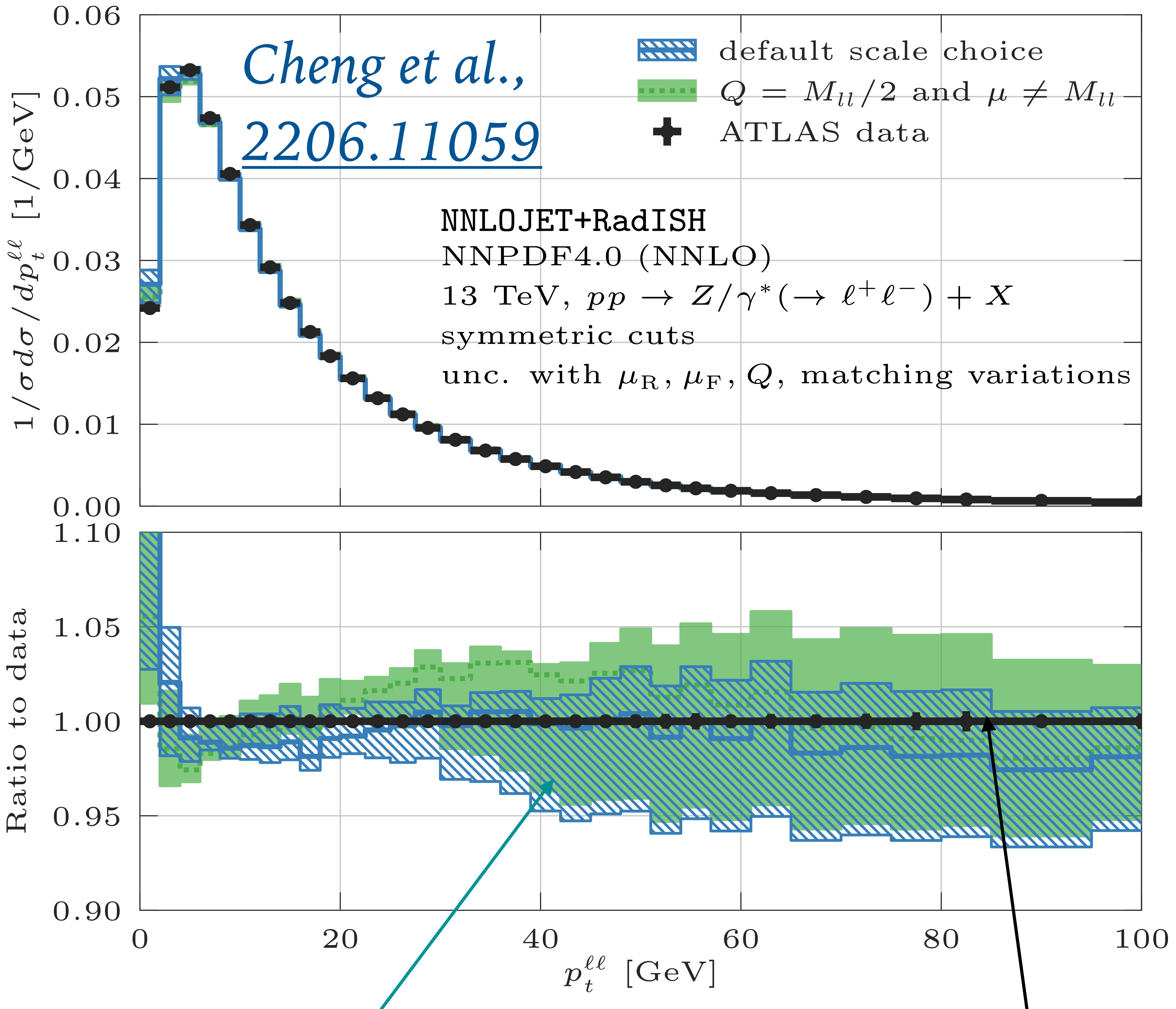
$c_i\alpha_S^i$ [MeV]	pole	$\overline{\mathrm{MS}}$
$i=4$	−94 (6)	−78 (6)
$i=5$	−44 (5)	−35 (5)
$i=6$	−22 (4)	−17 (4)
$i=7$	−13 (4)	−8(4)
$i=8$	−9 (4)	−4 (4)
$i=9$	−7 (4)	−2 (4)
$i=10$	−6 (5)	−1 (5)
$i=11$	−7 (6)	0 (6)
$i=12$	−9 (9)	1 (9)

$\Gamma_t = 1.33\ GeV$

$$O - O_{\mathrm{LO}} \approx A \underbrace{\int_0^\infty d\lambda\, \alpha_s(\lambda)}$$

to be sensitive to scales of order Γ_t, we need to go till order $i = 1 + \log(m_t/\Gamma_t) \approx 6$. For lower orders, the pole scheme is not appreciably worse than the $\overline{\mathrm{MS}}$!

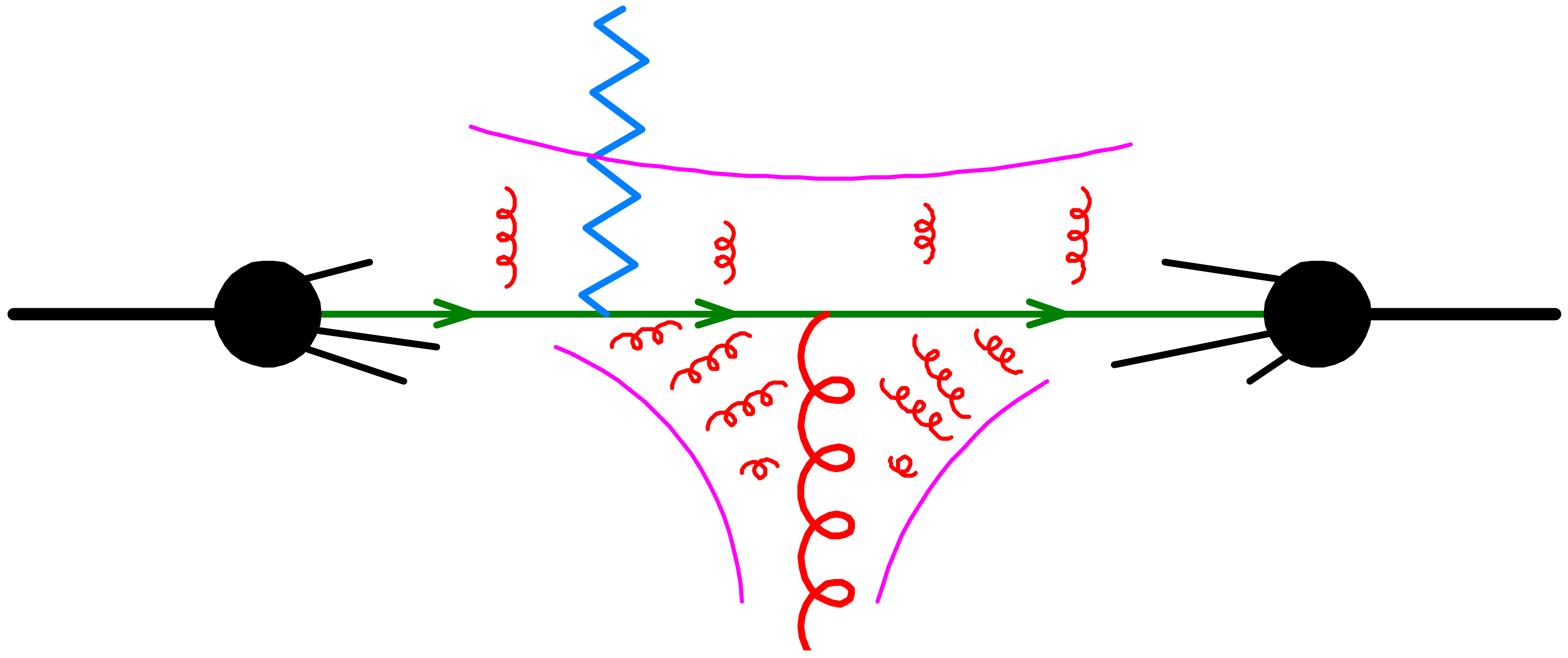
Transverse momentum of the Z boson



Theory uncertainty of the N3LO(+N3LL) calculation $\sim 5\,\%$

Experimental uncertainty at the permille level

Focus on the moderate-large value of $p_{T,Z}$: here the Z is recoiling against a hard jet

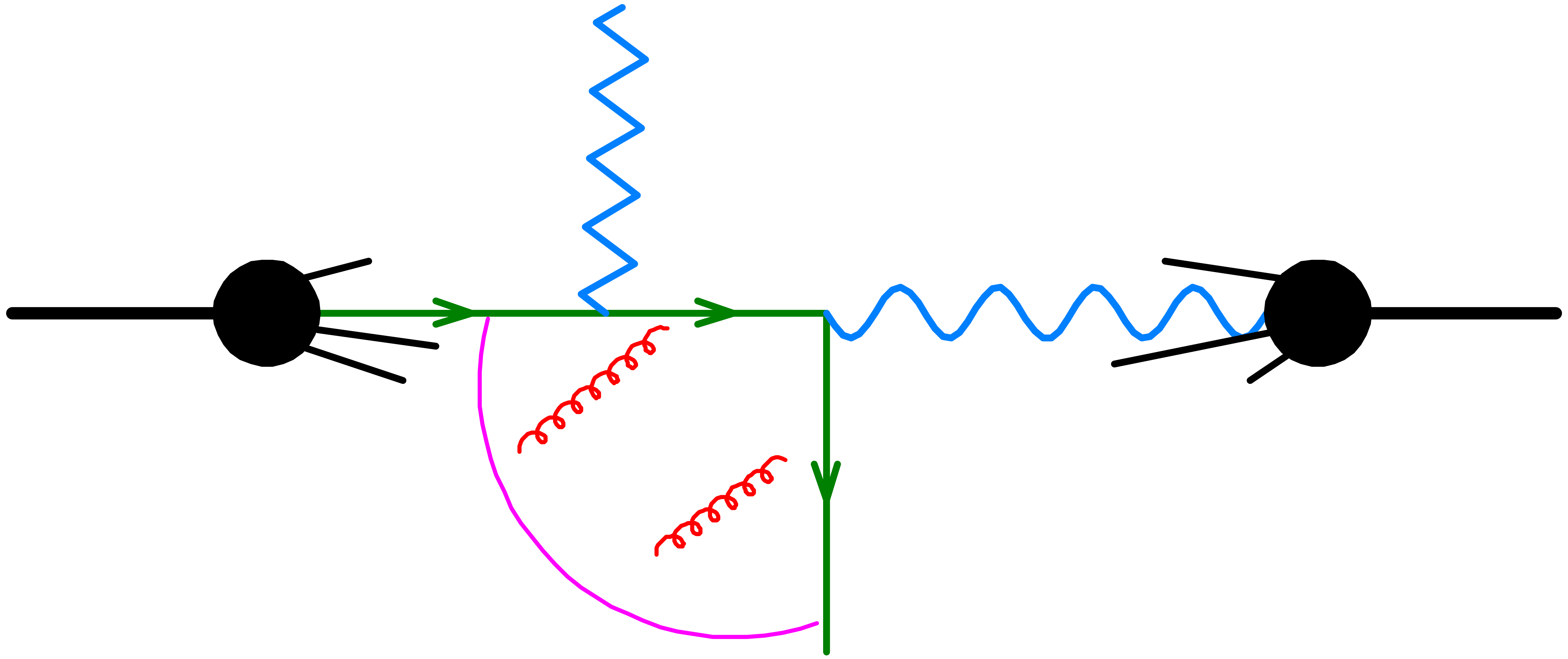


The soft radiation pattern is not azimuthally symmetric. If renormalons are related to soft emission, they may affect the $p_{T,Z}$ linearly by recoil: $\dfrac{\Lambda}{p_{TZ}} = \dfrac{1\,\text{GeV}}{30\,\text{GeV}} \approx 3\,\%$

Zpt in the large number of flavours

[SFR, Limatola, Nason, 2011.14114]

To address the problem in the large-n_f approach:
Consider a simplified process with the same features (i.e. asymmetric azimuthal soft radiation) that does not involve gluons at LO

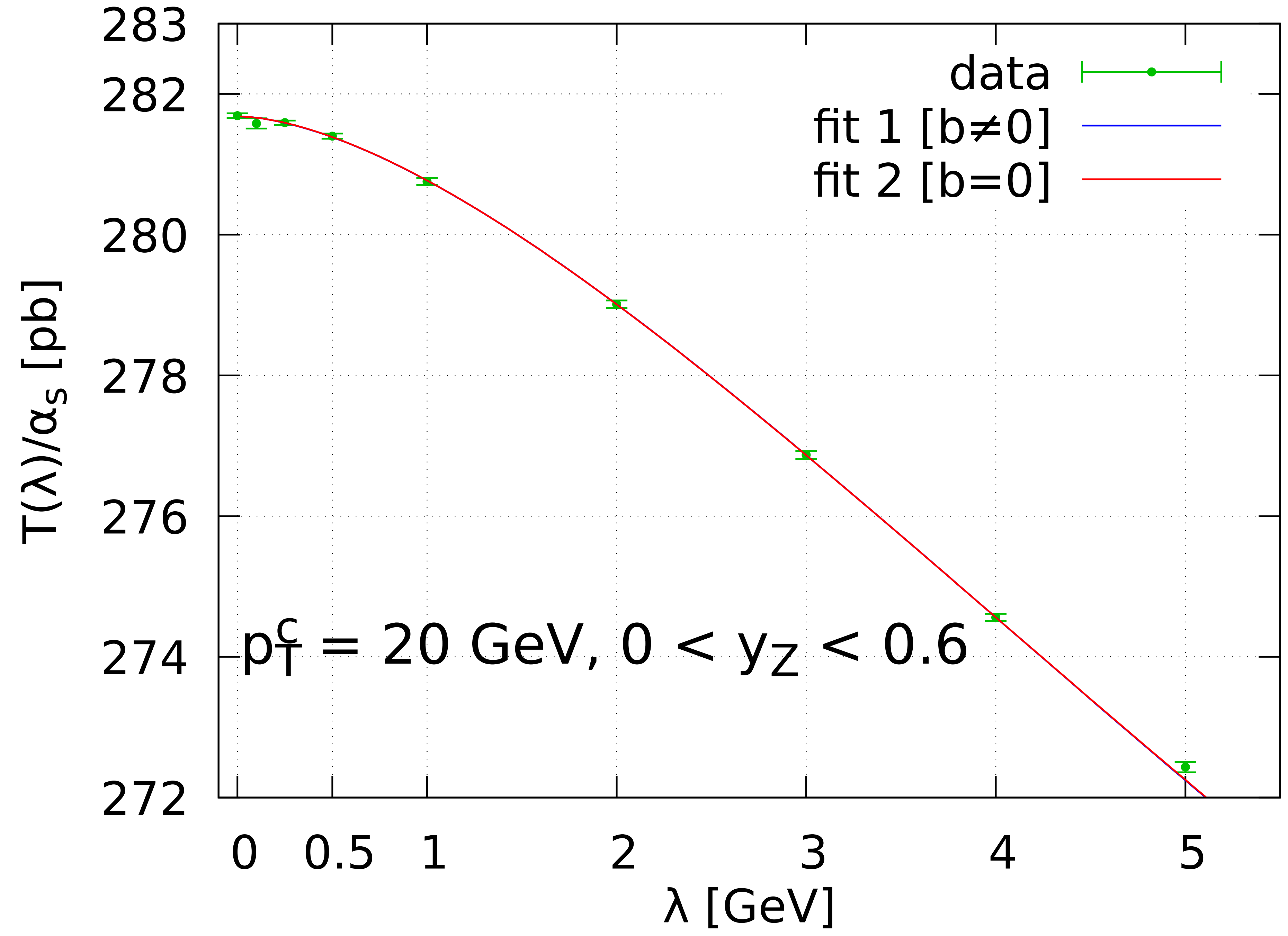


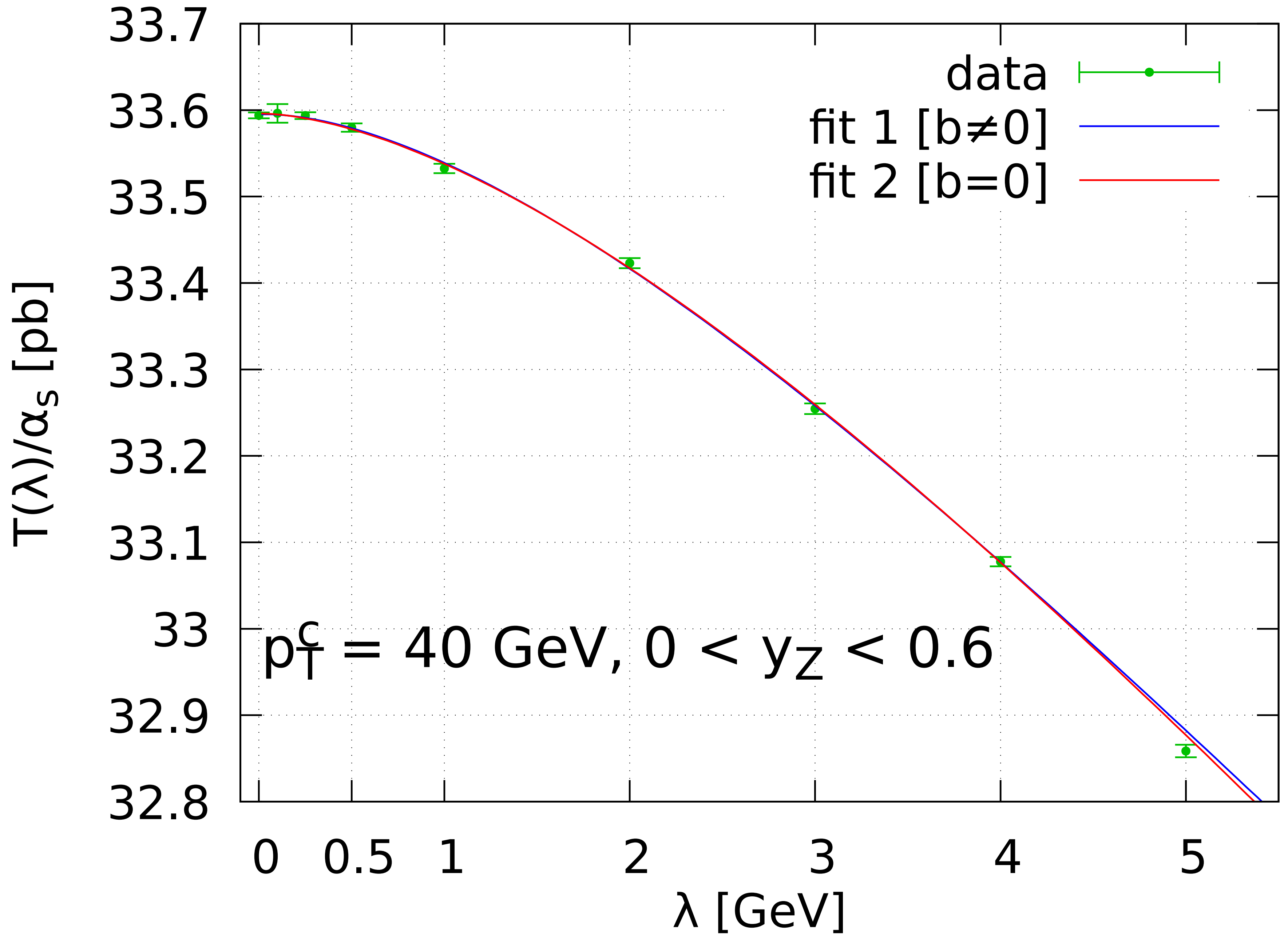
Also for $\gamma q \to Zq$ the radiation pattern is not azimutally symmetric. If we find here linear corrections in the $p_{T,Z}$ spectrum, it is likely to be there also in $q\bar{q} \to Zg$

Zpt in the large number of flavours

[SFR, Limatola, Nason, 2011.14114]

$$\Sigma\,(p_{T,Z} > p_T^c, 0 < y_Z < 0.6)$$

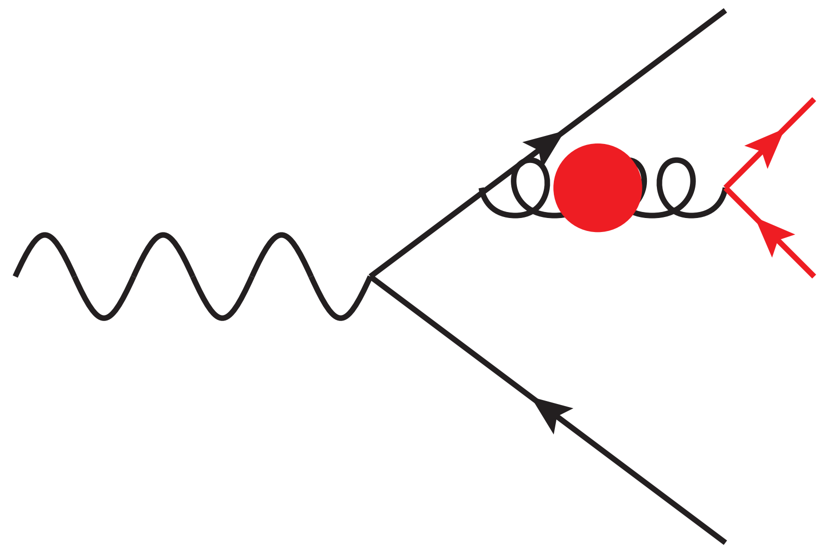




- As for the **total cross section** [Beneke, Braun, hep-ph/9506452] and the **rapidity distribution** [Dasgupta, hep-ph/9911391] there is no sign of a linear renormalon
- In 2011.14114 we only produced a numerical evidence: can we find an analytic argument, to understand **under what conditions the linear mass dependence cancel in an (abelian) theory with massive gluons, in the context of a single gluon emission or exchange?**

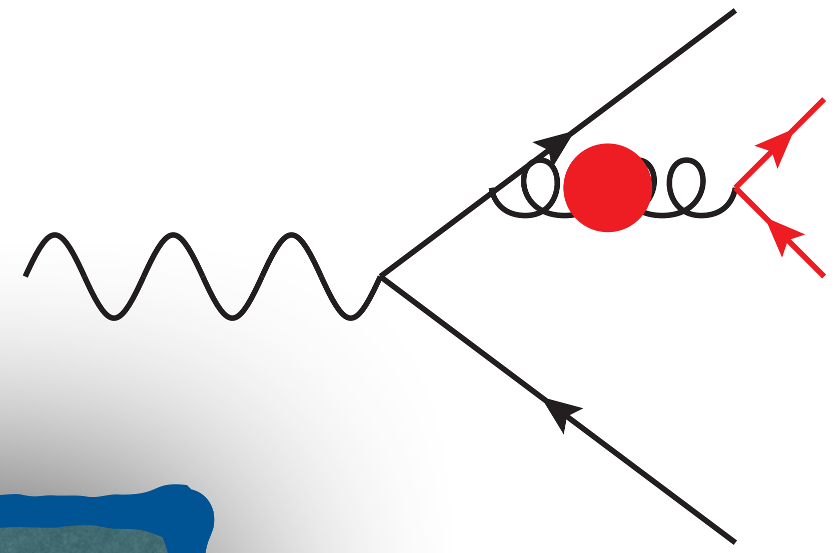
Linear renormalons for collider observables [Caola, SFR, Limatola, Melnikov, Nason, 2011.14114]

- In 2011.14114 we find that the only term that can lead to a linear mass dependence, is the one arising from the emission of a **soft gluon of fixed offshellness λ** that decays into a pair of soft quarks



- If we can **integrate inclusively** over the **radiation phase space**, no linear λ dependence arise!
- **Now the absence of linear renormalon can be inferred for all distributions that can be integrated in radiation at fixed underlying Born**
  - Total $e^+e^- \to$ hadrons (well known)
  - DIS structure functions (well known)
  - Drell-Yan inclusive and rapidity distributions
  - The Z transverse momentum distribution, for moderate or large p_{TZ}

Linear renormalons for collider observables [Caola, SFR, Limatola, Melnikov, Nason, 2011.14114]

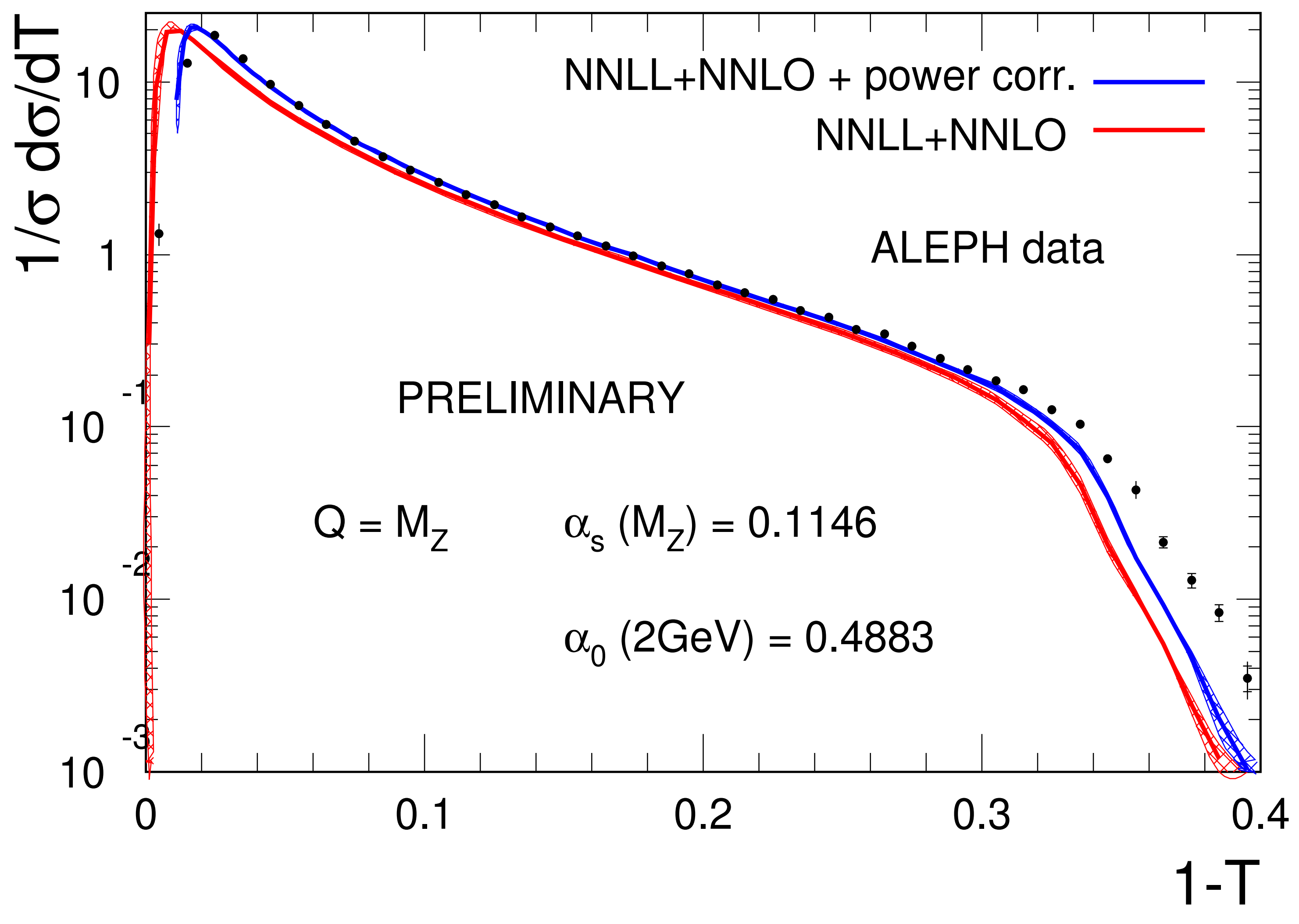


- In 2011.14114 we find that the only term that can lead to a linear mass dependence, is the one arising from the emission of a **soft gluon of fixed offshellness λ** that decays into a pair of soft quarks

Can we use our findings to get a better estimate of linear power corrections for cases where we know they do exists (e.g. event shapes)?

- If we can **integra** ... endence arise!
- **Now the absenc** ... **that can be integrated in rac**
  - Total $e^+e^- \to$ h
  - DIS structure functions (well known)
  - Drell-Yan inclusive and rapidity distributions
  - The Z transverse momentum distribution, for moderate or large p_{TZ}

Linear power corrections in event shapes



- Event shapes (**thrust, C-parameter. . .**) have linear power corrections

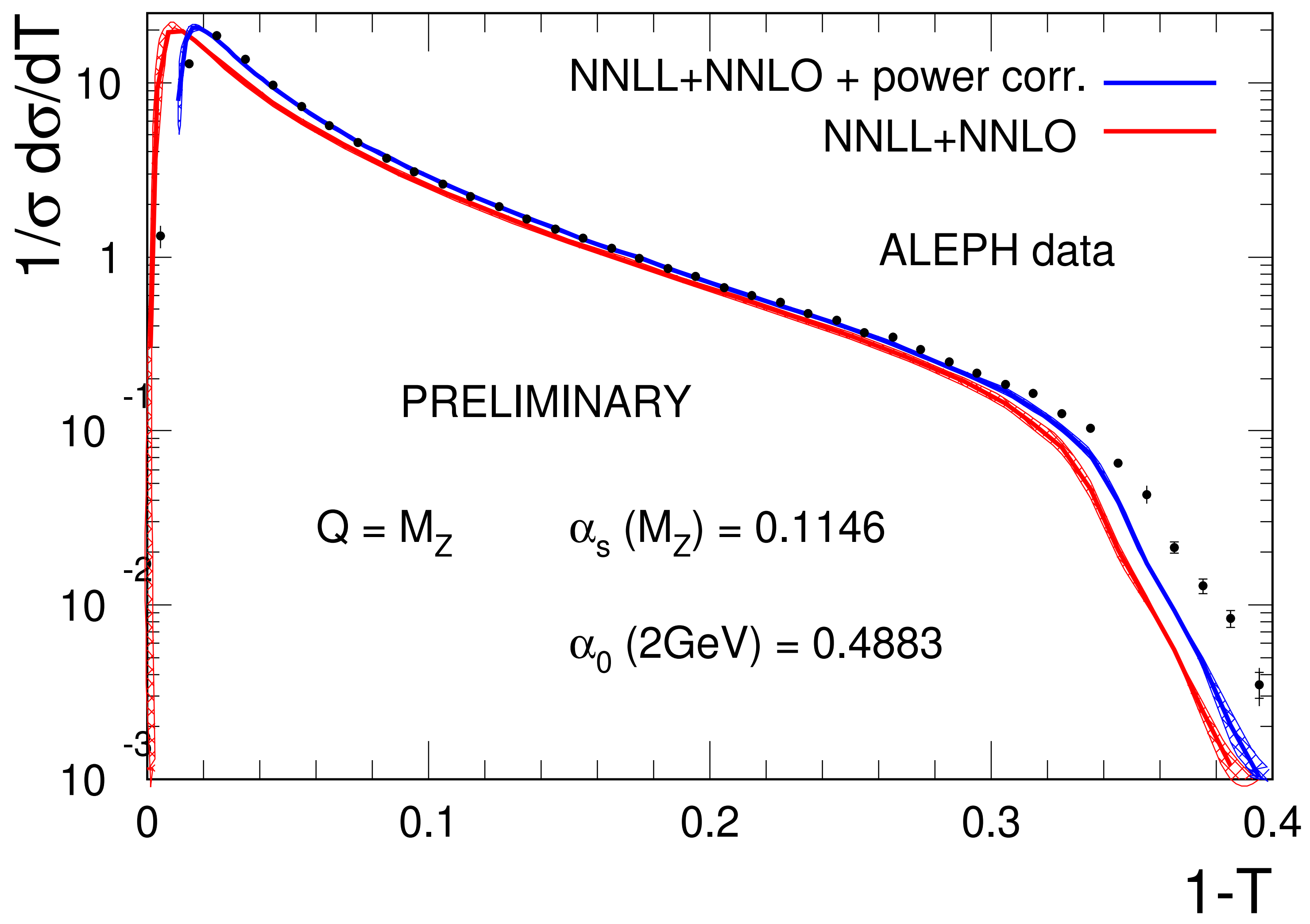
$$T = \max_{\vec{n}} \sum_i \frac{|\vec{p}_i \cdot \vec{n}|}{\sqrt{s}}, \quad C = 3 - \frac{3}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot Q)(p_j \cdot Q)}$$

- **Strong coupling constant** determinations lead

α_s =0.1179(10) world average

α_s =0.1135(10) from **Thrust** [Abbate et al., Phys. Rev. D **86** (2012), 094002]

Linear power corrections in event shapes



- Event shapes (**thrust, C-parameter. . .**) have linear power corrections

$$T = \max_{\vec{n}} \sum_i \frac{|\vec{p}_i \cdot \vec{n}|}{\sqrt{s}}, \quad C = 3 - \frac{3}{2}\sum_{i,j}\frac{(p_i \cdot p_j)^2}{(p_i \cdot Q)(p_j \cdot Q)}$$

- **Strong coupling constant** determinations lead

  α_s =0.1179(10) world average

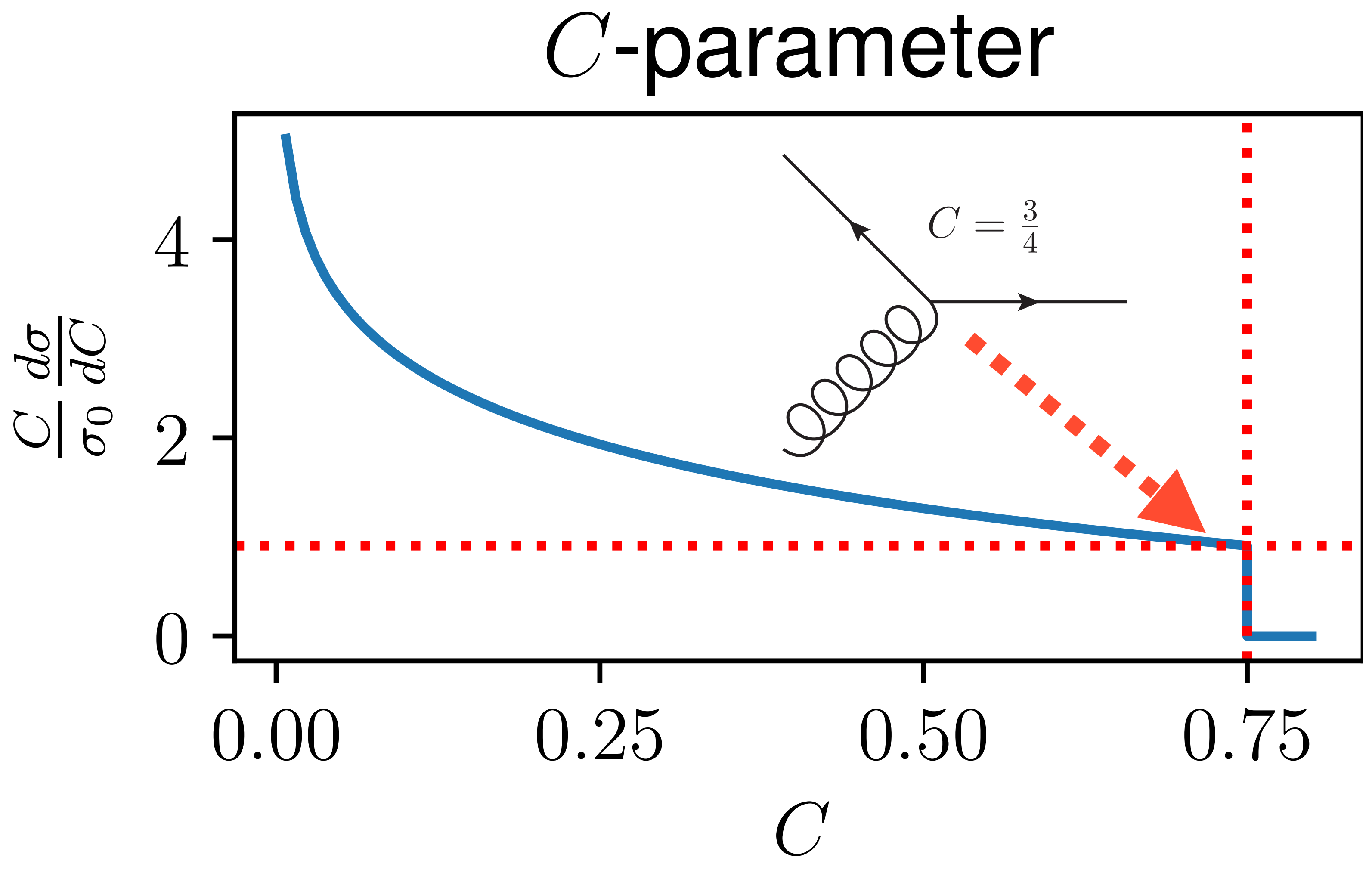
  α_s =0.1135(10) from **Thrust** [Abbate et al., Phys. Rev. D **86** (2012), 094002]

- Linear power corrections for $V = 0$ (i.e. in the two jet limit) known for a long time [Nason, Seymour hep-ph/9506317, Dokshitzer, Webber hep-ph/9704298, Dokshitzer et al. hep-ph/9802381] and assumed to be valid also for $V \gg 0$

- But for the **C-parameter** it was recently showed

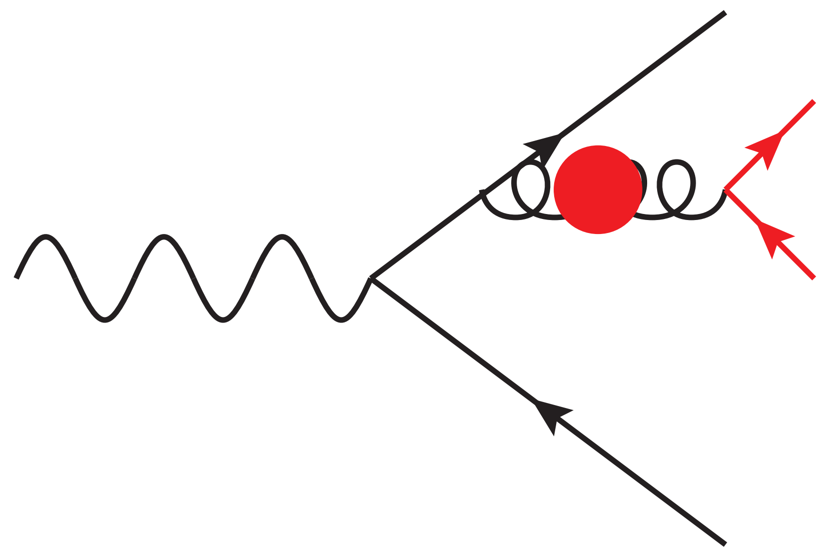
$$\frac{\text{Linear power correction at C=0.75}}{\text{Linear power correction at C=0}} = 0.48$$

[Luisoni, Monni, Salam 2012.00622]



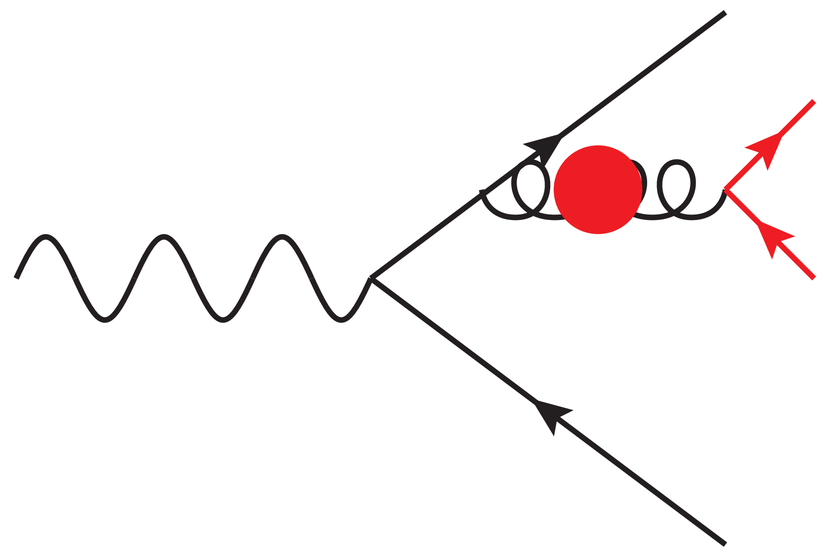
Linear power corrections in event shapes in the two jet limit

- Linear power corrections can only arise from diagrams containing a soft gluer that splits into a $q\bar{q}$ pair



$$T(\lambda) \approx \frac{\lambda^2}{\pi b_0} \int d\Phi_{q\bar{q}} \delta(m_{q\bar{q}}^2 - \lambda^2) R_{q\bar{q}} \left[V(\Phi_{q\bar{q}}) - \underbrace{V(\Phi_{\rm b})}_{0} \right]$$

Linear power corrections in event shapes in the two jet limit



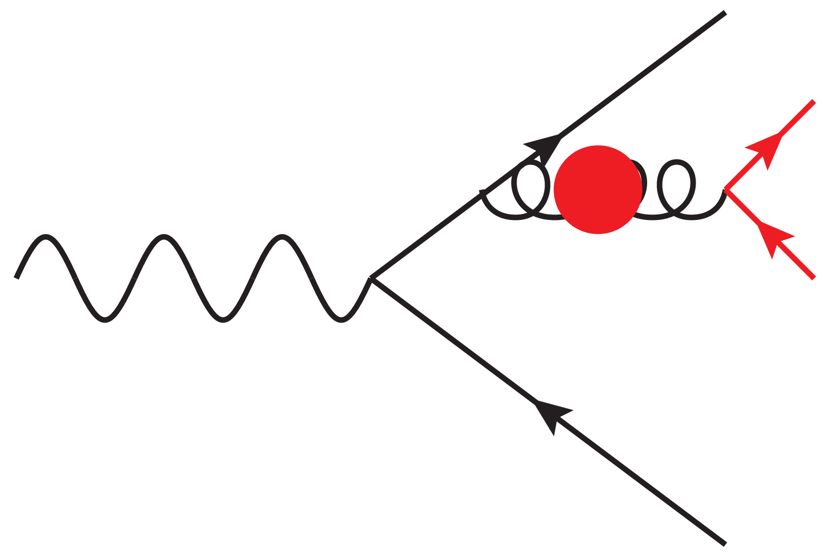
- Linear power corrections can only arise from diagrams containing a soft gluer that splits into a $q\bar{q}$ pair

$$T(\lambda) \approx \frac{\lambda^2}{\pi b_0} \int d\Phi_{q\bar{q}} \delta(m_{q\bar{q}}^2 - \lambda^2) R_{q\bar{q}} \left[V(\Phi_{q\bar{q}}) - \underbrace{V(\Phi_{\rm b})}_{0} \right]$$

- For many observables, such as thrust and C-parameter, in the two-jet limit $V \propto k_t e^{-|\eta|}$: the collinear limit is exponentially suppressed, we can approximate $R_{q\bar{q}}$ with the **leading soft approximation**

Linear power corrections in event shapes in the two jet limit

- Linear power corrections can only arise from diagrams containing a soft gluer that splits into a $q\bar{q}$ pair



$$T(\lambda) \approx \frac{\lambda^2}{\pi b_0} \int d\Phi_{q\bar{q}} \delta(m_{q\bar{q}}^2 - \lambda^2) R_{q\bar{q}} \left[V(\Phi_{q\bar{q}}) - \underbrace{V(\Phi_{\rm b})}_{0} \right]$$

- For many observables, such as thrust and C-parameter, in the two-jet limit $V \propto k_t e^{-|\eta|}$: the collinear limit is exponentially suppressed, we can approximate $R_{q\bar{q}}$ with the **leading soft approximation**

- Event shapes are **additive** observables: in the soft limit $V(1,2) \approx V(1) + V(2)$, so we have

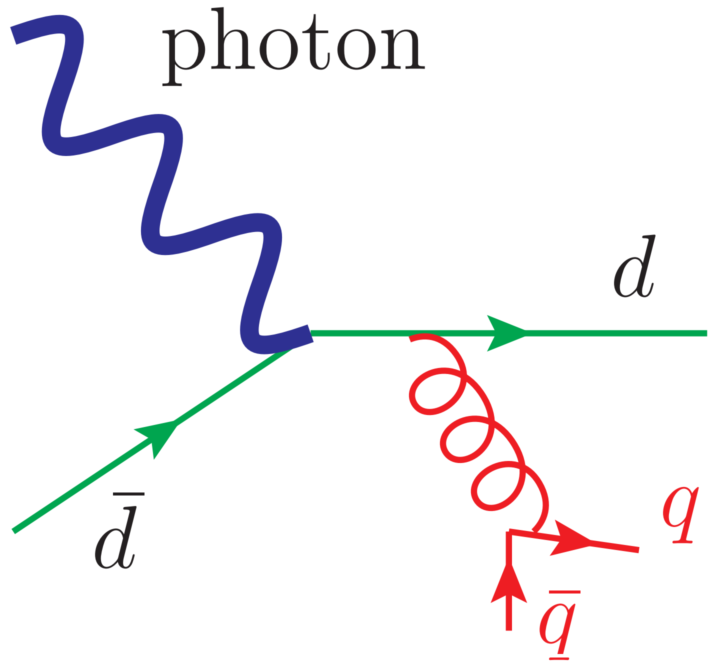
$$T(\lambda) \approx \frac{\lambda^2}{\pi b_0} \int d\Phi_{q\bar{q}} \delta(m_{q\bar{q}}^2 - \lambda^2) R_{q\bar{q}}^{\rm soft} \, \Delta V(q,\bar{q}) = \mathcal{M} \lambda \underbrace{\frac{2 C_F \alpha_s}{\pi} \int_0^Q \frac{dp_T}{p_T} \int_{\log(p_T/Q)}^{-\log(p_T/Q)} d\eta \, \delta(p_T - \lambda)}_{\text{massless soft gluon emission probability}} \, \Delta V(\{p_T, \eta\})$$

where $\mathcal{M}$ is a universal factor, dubbed **Milan factor** [Dokshitzer et al. hep-ph/9802381], $\Delta V(\{p_T, \eta\})$ is the shift in the event shape due to the emission of a massless gluon of given transverse momentum p_T and rapidity η

Large nf approximation for event shapes in the three-jet limit

[Caola, SFR, Limatola, Melnikov, Nason, 2011.14114, + Ozcelik 2108.08897]

- To be able to use our simple abelian model away from the two jet limit, we consider the toy process $\gamma^* \to d\bar{d}\gamma$, and the emission of a $q\bar{q}$ pair from the $d\bar{d}$ dipole



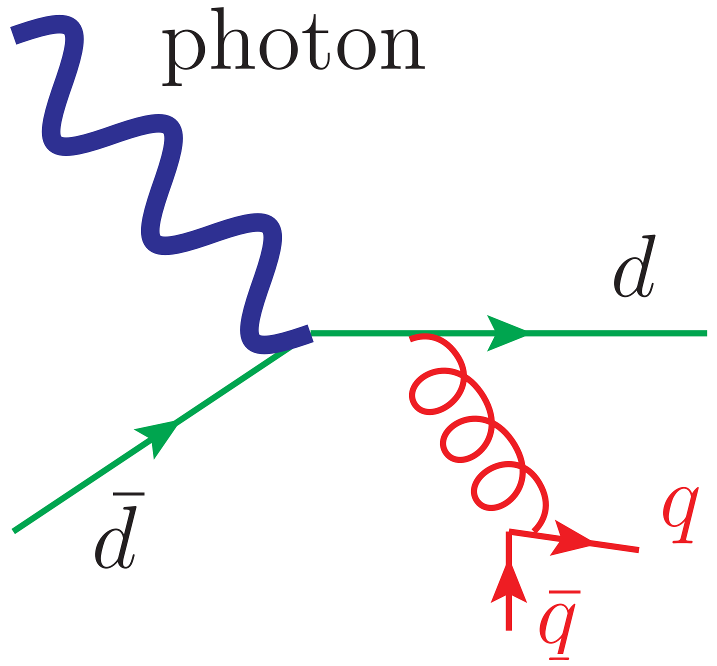
$$T(\lambda; \Phi_0) \approx \frac{\lambda^2}{\pi b_0} \int d\Phi_{q\bar{q}} \delta(m_{q\bar{q}}^2 - \lambda^2) R_{q\bar{q}} \left[V(\Phi_{q\bar{q}}) - V(\Phi_{\rm b}) \right] \delta(\Phi_{\rm b} - \Phi_0)$$

  - Conversely to the two jet case, here there is a non-trivial **underlying Born phase** space! i.e. there are multiple ways of reshuffling the momenta of the photon and of the $d, \bar{d}$ to ensure momentum conservation when removing the $q\bar{q}$ pair, each of them leading to a different value for $V(\Phi_0)$!

Large nf approximation for event shapes in the three-jet limit

[Caola, SFR, Limatola, Melnikov, Nason, 2011.14114, + Ozcelik 2108.08897]

- To be able to use our simple abelian model away from the two jet limit, we consider the toy process $\gamma^* \to d\bar{d}\gamma$, and the emission of a $q\bar{q}$ pair from the $d\bar{d}$ dipole



$$T(\lambda; \Phi_0) \approx \frac{\lambda^2}{\pi b_0} \int d\Phi_{q\bar{q}} \delta(m_{q\bar{q}}^2 - \lambda^2) R_{q\bar{q}} \left[V(\Phi_{q\bar{q}}) - V(\Phi_{\rm b}) \right] \delta(\Phi_{\rm b} - \Phi_0)$$

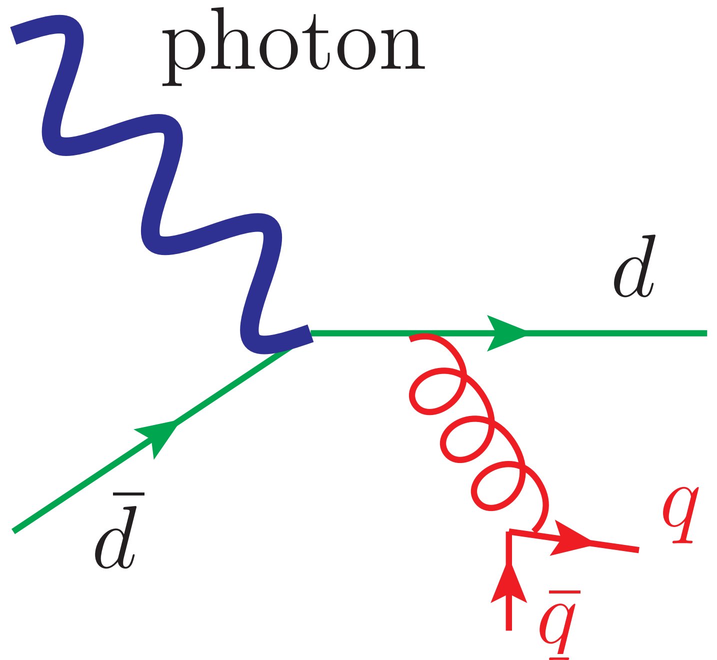
  - Conversely to the two jet case, here there is a non-trivial **underlying Born phase** space!
    i.e. there are multiple ways of reshuffling the momenta of the photon and of the $d, \bar{d}$ to ensure momentum conservation when removing the $q\bar{q}$ pair, each of them leading to a different value for $V(\Phi_0)$!

  - **In 2011.14114 we learnt that we can choose any mapping that is smooth and analytic in the soft limit (i.e. it depends only linearly on the gluon momentum, at least for the longitudinal components)**

Large nf approximation for event shapes in the three-jet limit

[Caola, SFR, Limatola, Melnikov, Nason, 2011.14114, + Ozcelik 2108.08897]

- To be able to use our simple abelian model away from the two jet limit, we consider the toy process $\gamma^* \to d\bar{d}\gamma$, and the emission of a $q\bar{q}$ pair from the $d\bar{d}$ dipole



$$T(\lambda;\Phi_0) \approx \frac{\lambda^2}{\pi b_0}\int d\Phi_{q\bar{q}}\delta(m_{q\bar{q}}^2-\lambda^2)R_{q\bar{q}}\left[V(\Phi_{q\bar{q}})-V(\Phi_b)\right]\delta(\Phi_b-\Phi_0)$$

  - Conversely to the two jet case, here there is a non-trivial **underlying Born phase** space! i.e. there are multiple ways of reshuffling the momenta of the photon and of the $d, \bar{d}$ to ensure momentum conservation when removing the $q\bar{q}$ pair, each of them leading to a different value for $V(\Phi_0)$!
  - **In 2011.14114 we learnt that we can choose any mapping that is smooth and analytic in the soft limit (i.e. it depends only linearly on the gluon momentum, at least for the longitudinal components)**

- Solved the recoil issue, everything proceeds as in the two jet limit, since

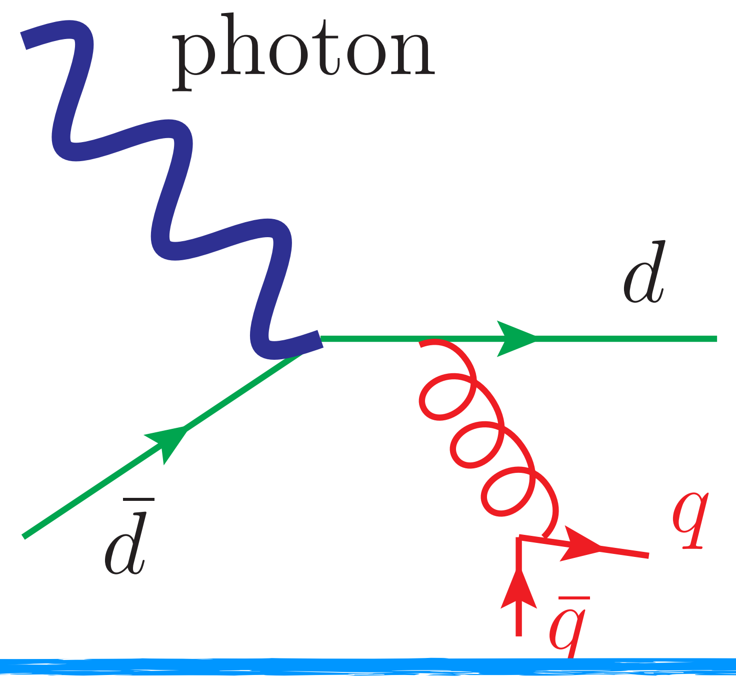
$$\Delta V(\{p_T,\eta,\phi\};\Phi_0) = \frac{p_T}{Q}f(\eta,\phi;\Phi_0), \qquad \text{with } \lim_{\eta\pm\infty}\int\frac{d\phi}{2\pi}f(\eta,\phi;\Phi_0) \propto e^{-|\eta|}$$

And we get

$$T(\lambda;\Phi_0) = \mathscr{M}\lambda\frac{2C_F\alpha_s}{\pi}\int_0^{m_{d\bar{d}}}\frac{dp_T}{p_T}\int_{\log(p_T/m_{d\bar{d}})}^{-\log(p_T/m_{d\bar{d}})}d\eta\int_0^{2\pi}\frac{d\phi}{2\pi}\delta(p_T-\lambda)\ \Delta V(\{p_T,\eta,\phi;\},\Phi_0)$$

Linear power corrections in event shapes in the three-jet limit [Caola, SFR, Limatola, Melnikov, Nason, 2011.14114, + Ozcelik 2108.08897]

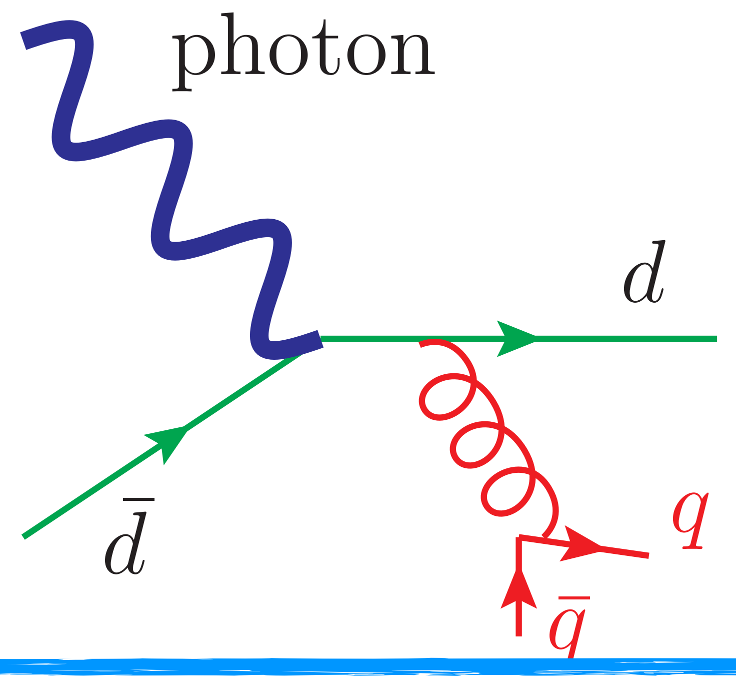
- We have our recipe to calculate the non-perutbative corrections in our simplified abelian model for the process $d\bar{d}\gamma$, we want to convert it to the real QCD word to handle $d\bar{d}\mathbf{g}$. How?



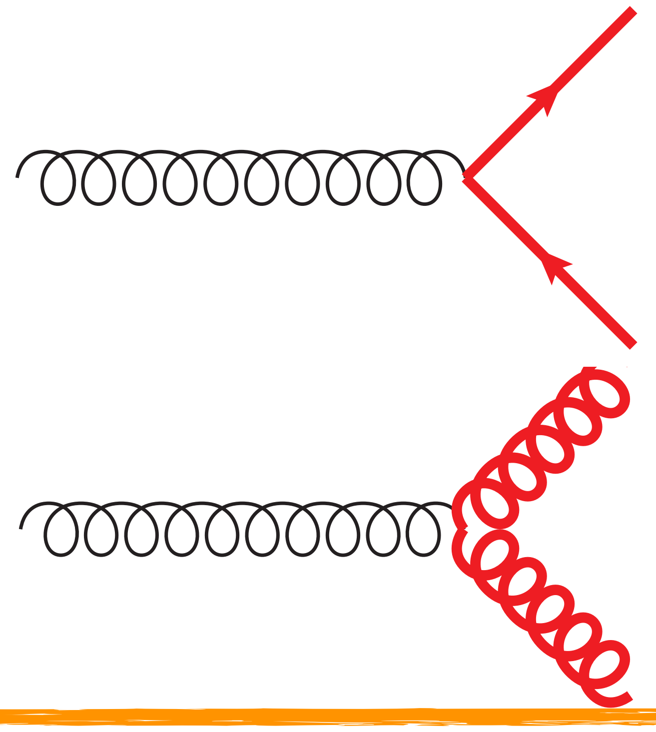
Linear power corrections in event shapes in the three-jet limit

[Caola, SFR, Limatola, Melnikov, Nason, 2011.14114, + Ozcelik 2108.08897]

- We have our recipe to calculate the non-perutbative corrections in our simplified abelian model for the process $d\bar{d}\gamma$, we want to convert it to the real QCD word to handle $d\bar{d}\mathbf{g}$. How?



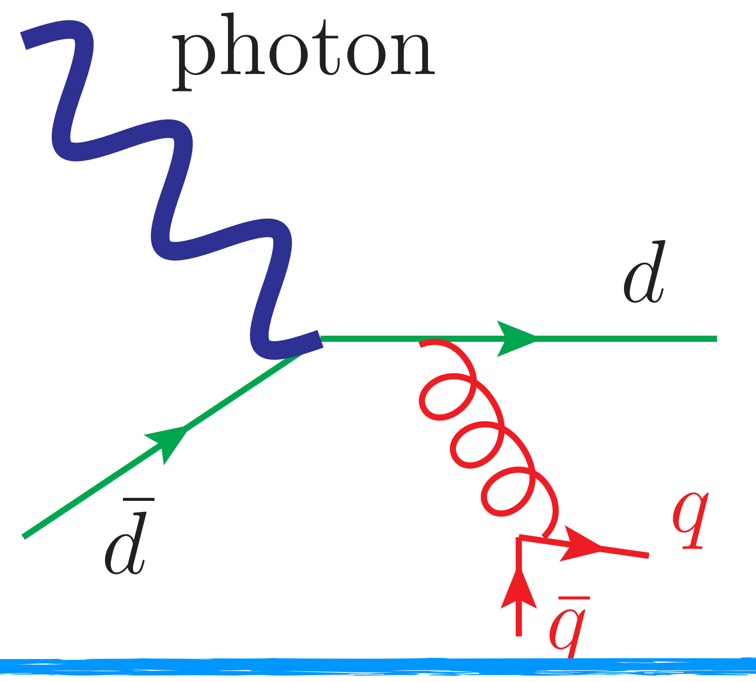
A. The Milan factor $\mathcal{M}$ relates the calculation with a **massless gluon of $p_T = \lambda$** to the one containing an off shell gluon of mass λ that splits into a $q\bar{q}$ pair, but it is customary in the literature to include also the effect of a $g^* \to gg$ splitting



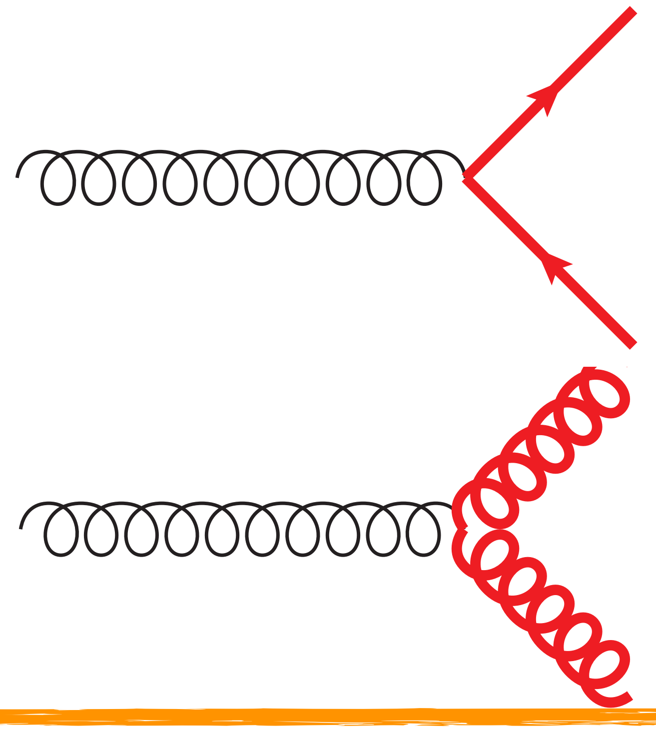
Linear power corrections in event shapes in the three-jet limit

[Caola, SFR, Limatola, Melnikov, Nason, 2011.14114, + Ozcelik 2108.08897]

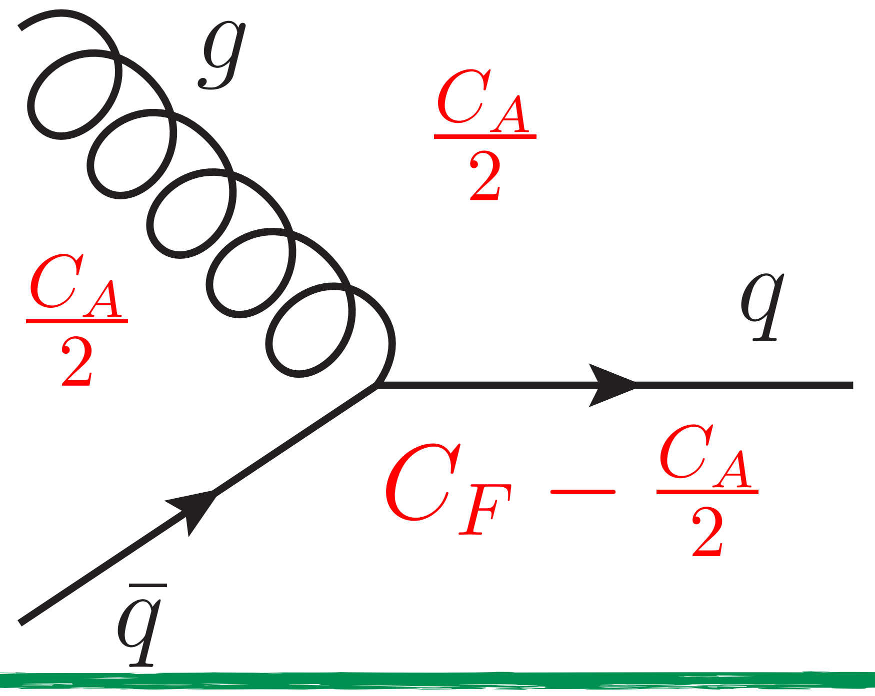
- We have our recipe to calculate the non-perutbative corrections in our simplified abelian model for the process $d\bar{d}\gamma$, we want to convert it to the real QCD word to handle $d\bar{d}\mathbf{g}$. How?



A. The Milan factor $\mathcal{M}$ relates the calculation with a **massless gluon of $p_T = \lambda$** to the one containing an off shell gluon of mass λ that splits into a $q\bar{q}$ pair, but it is customary in the literature to include also the effect of a $g^* \to gg$ splitting



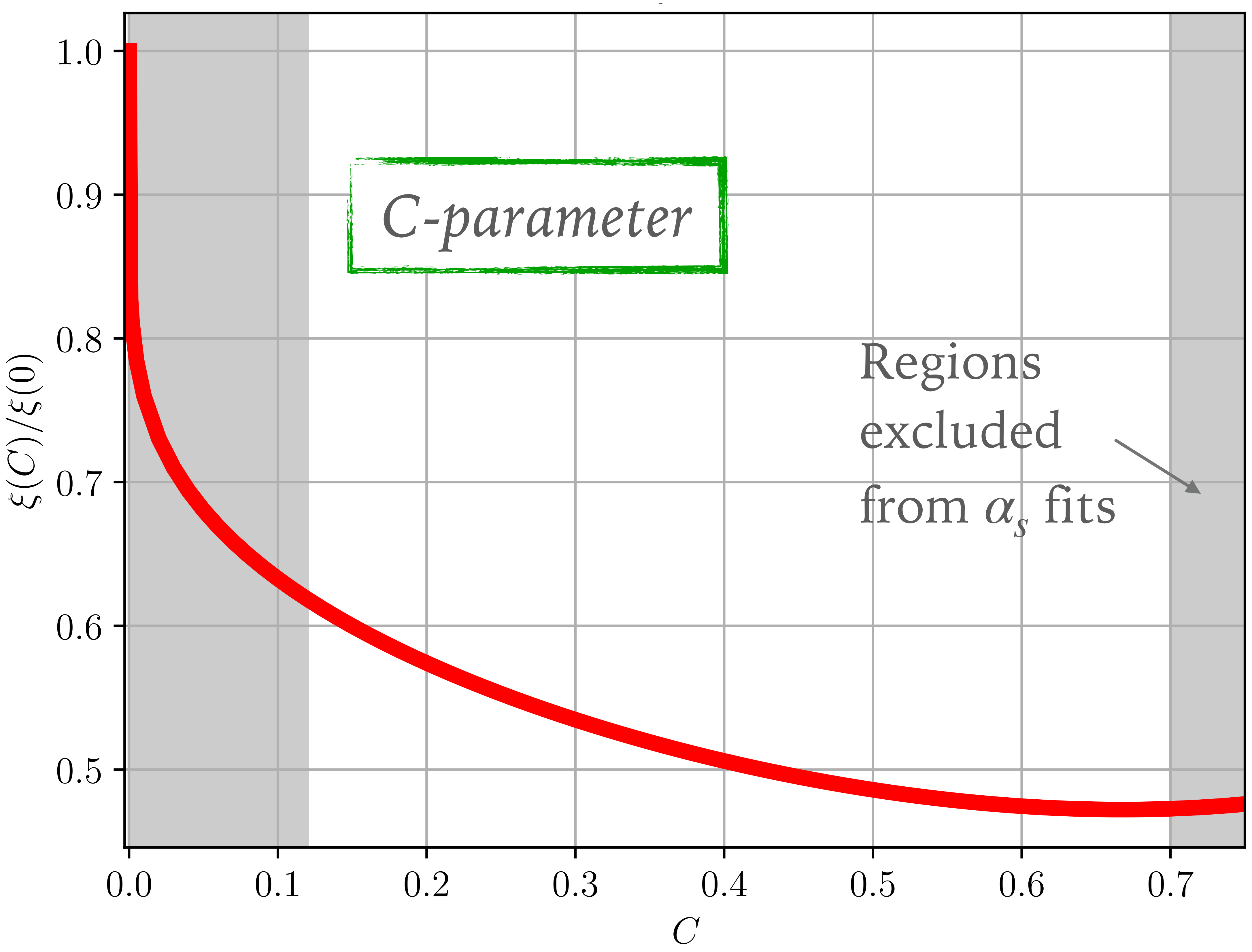
B. We assume the **massless gluon of $p_T = \lambda$** can be emitted from all the colour-dipoles in the process, and we replace the colour factor C_F with the one appropriate for each dipole

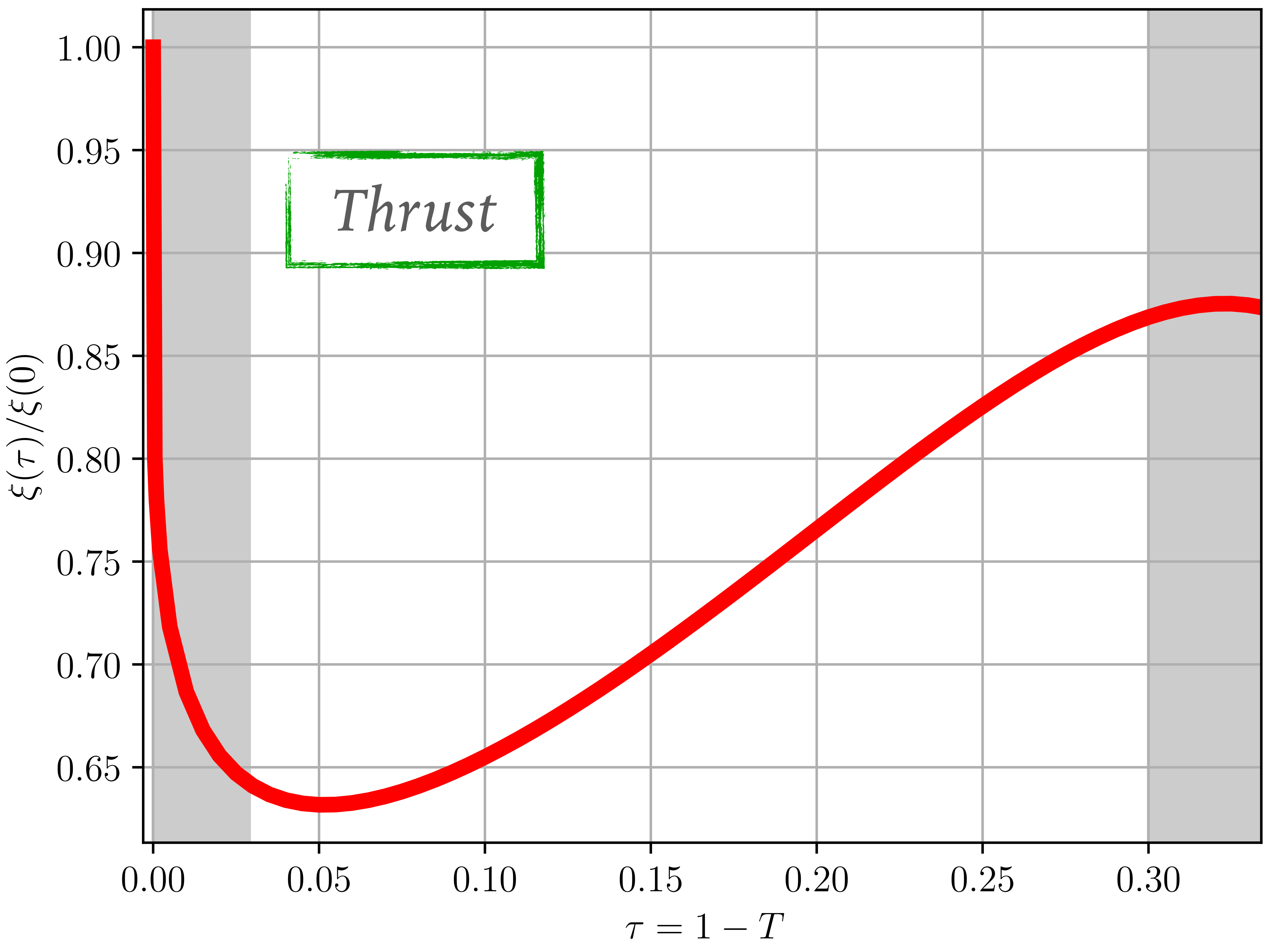


Linear power corrections in event shapes in the three-jet limit

[Caola, SFR, Limatola, Melnikov, Nason, Ozcelik 2108.08897]

Ratio between the linear power correction computed with our model $\xi(V)$ vs the one in the two jet limit $\xi(0)$ for the cumulative distribution



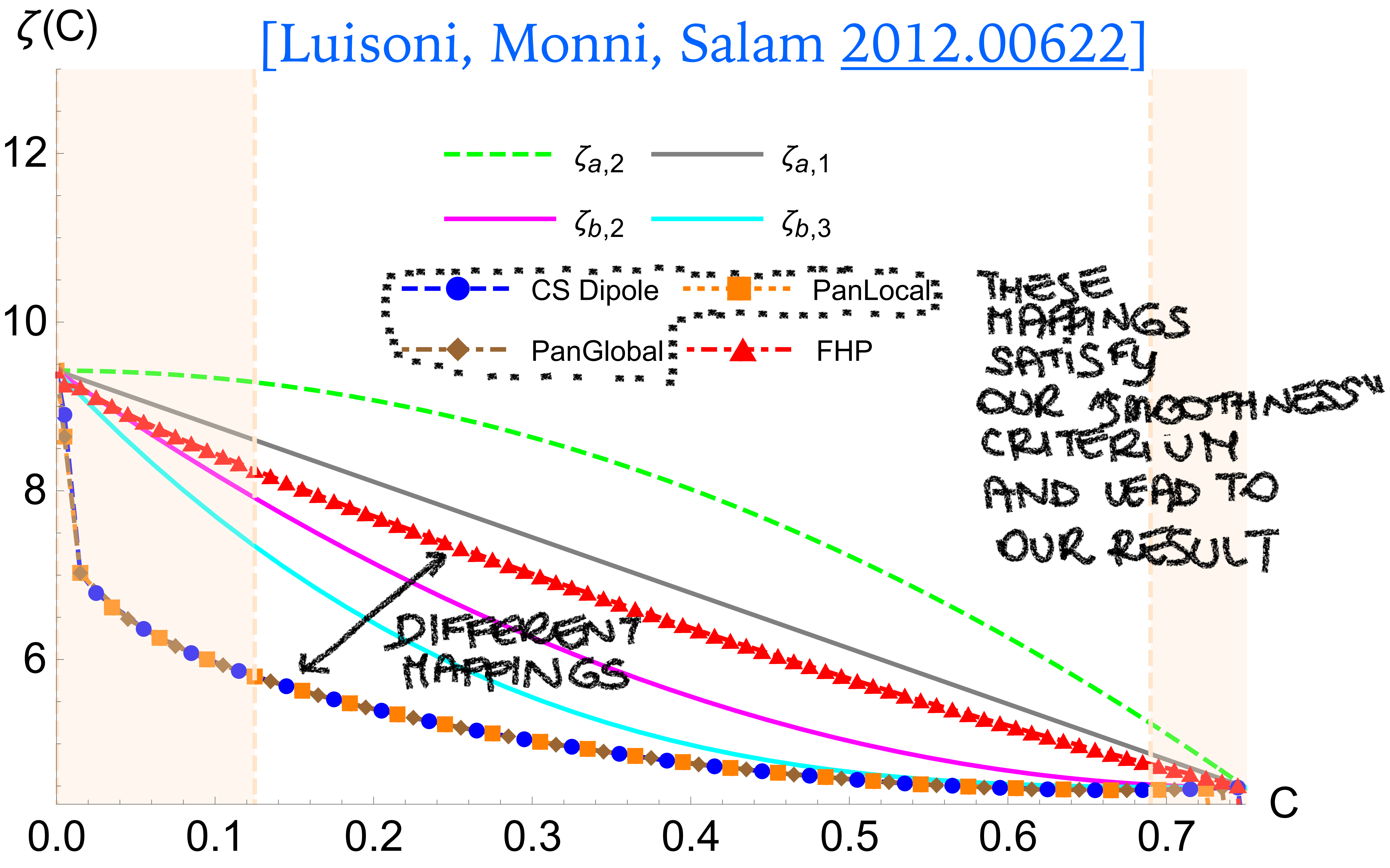


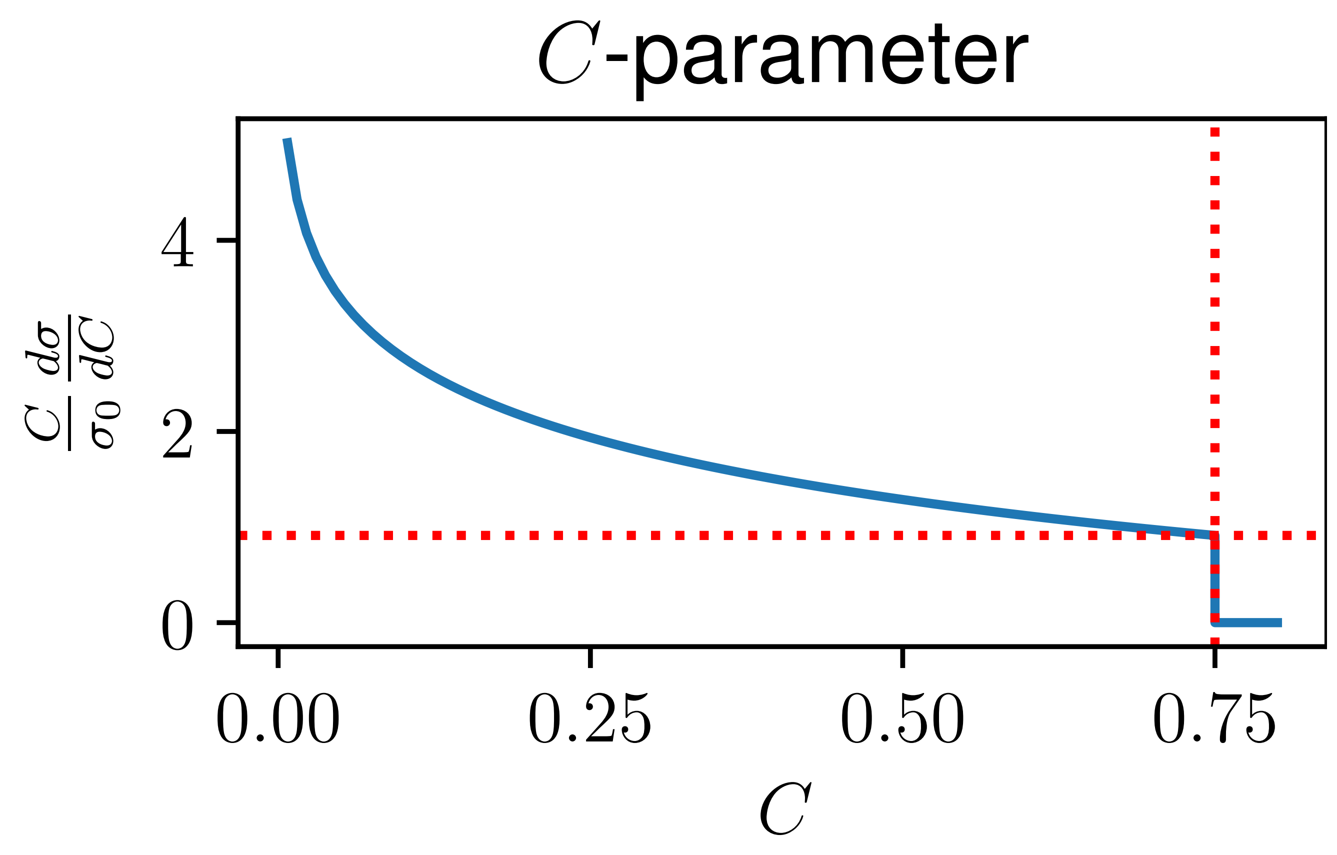
Non-negligible kinematic dependence of the power correction!

Linear power corrections in event shapes in the three-jet limit

[Caola, SFR, Limatola, Melnikov, Nason, 2011.14114, + Ozcelik 2108.08897]

Our result seems the 'trivial' extension of the two jet limit case. Why it was not obtained before?

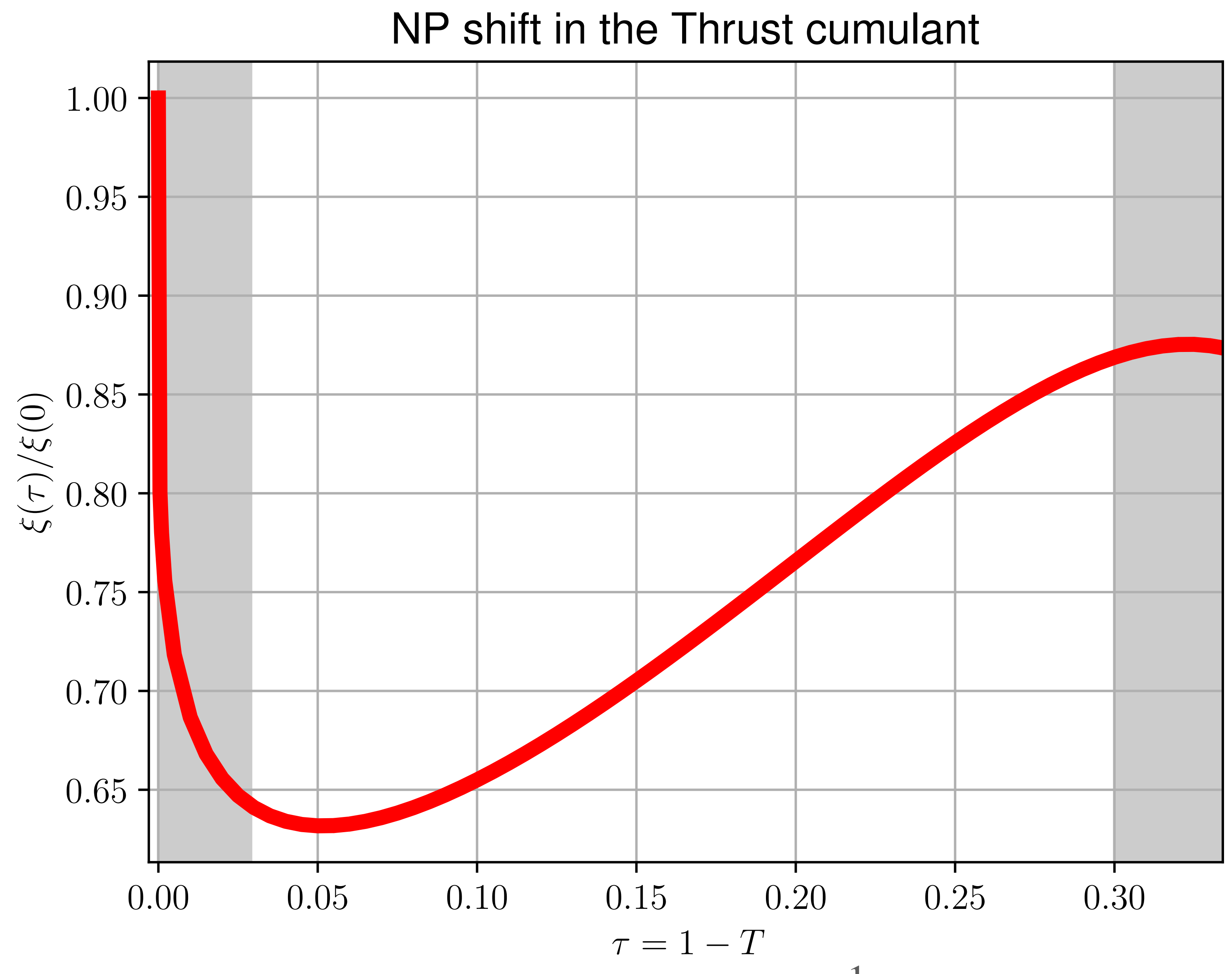


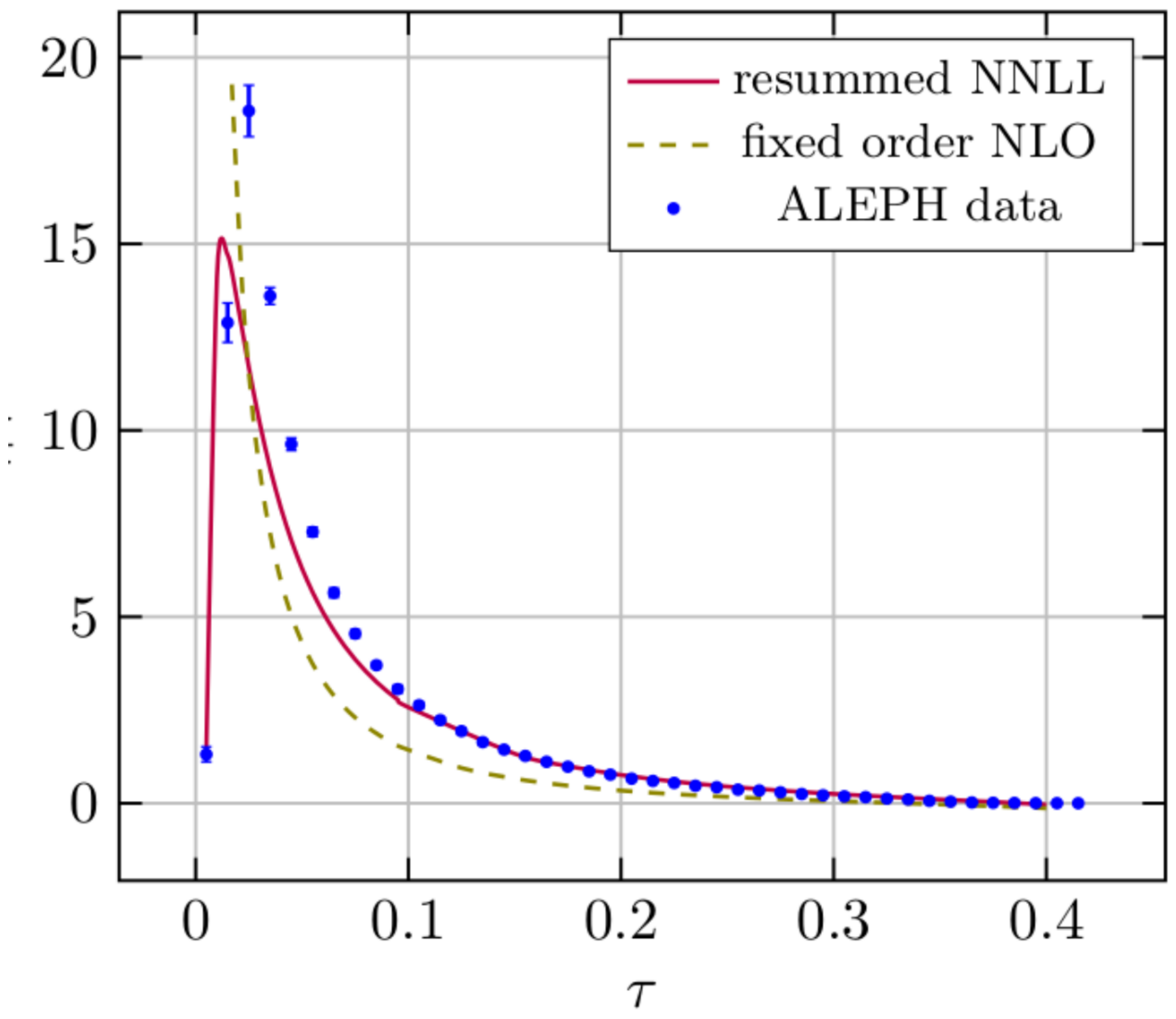


The two-jet limit and $C = 0.75$ are special point, where every mapping $\Phi_n \rightarrow \Phi_{n+1}$ leads to the same linear power correction. Not true elsewhere!

Limitations of our approach

[Caola, SFR, Limatola, Melnikov, Nason, 2011.14114, + Ozcelik 2108.08897]



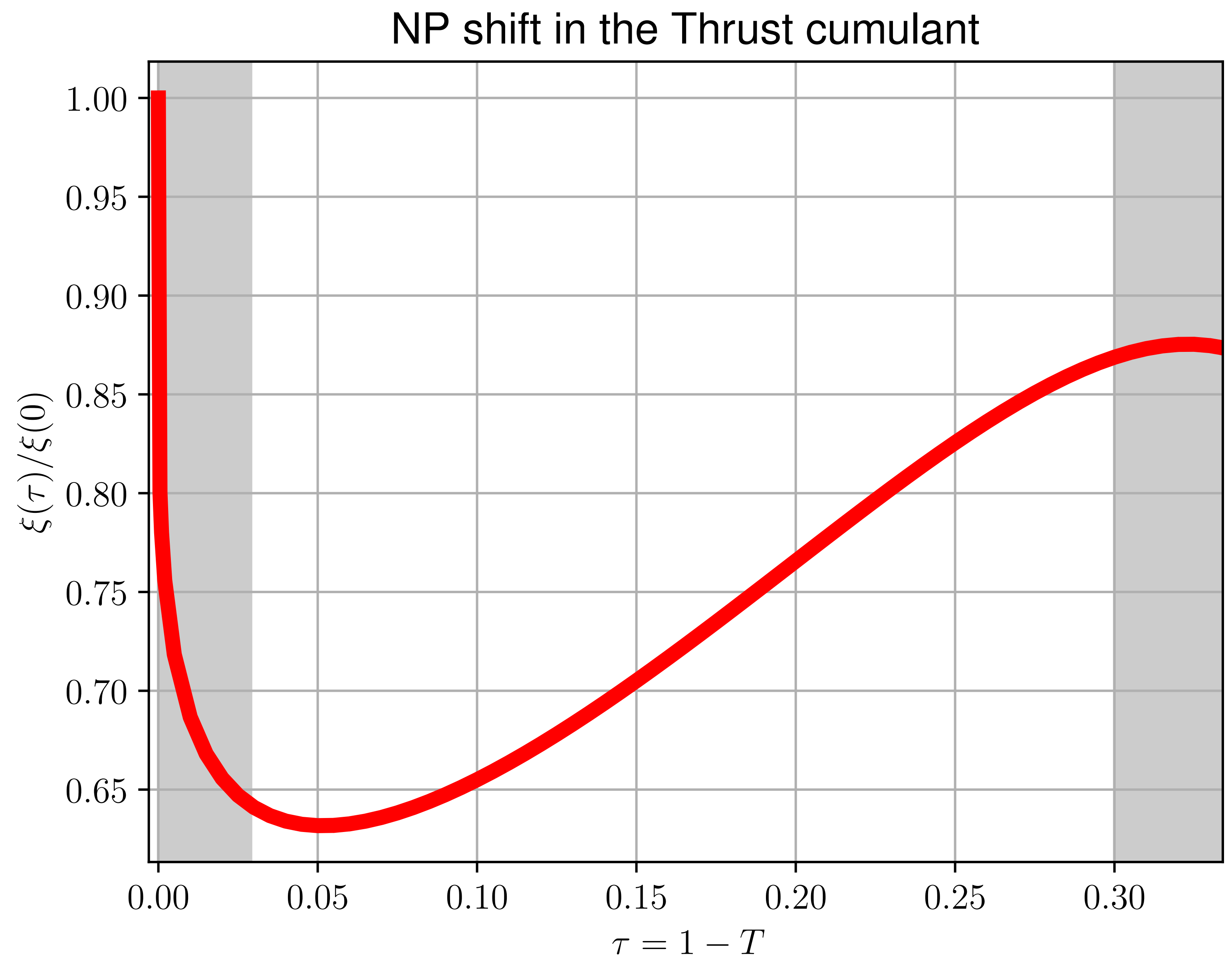


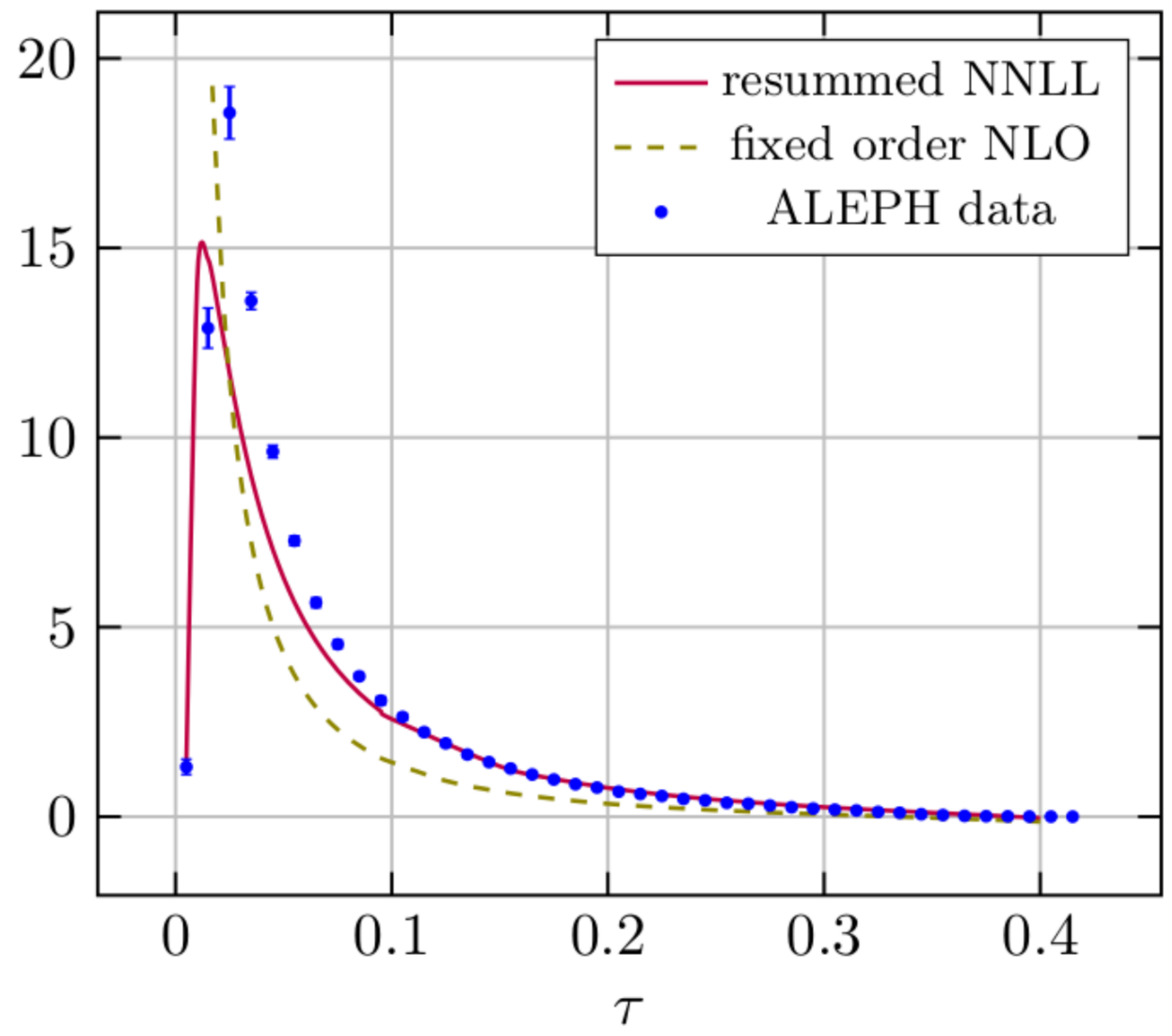
$$\Delta V(v) = \left[\int d\Phi \delta(V(\Phi) - v) \frac{d\sigma^{\rm pert}}{d\Phi}\right]^{-1} \int d\Phi \delta(V(\Phi) - v) \frac{d\sigma^{\rm pert}}{d\Phi} \frac{2\mathscr{M}C_i}{\pi} \int \frac{dp_T}{p_T} dy \frac{d\varphi}{2\pi} \Delta V(p_T, \varphi, \eta; \Phi)\, \delta(p_T - \lambda)$$

- In our model, there is a rapid and abrupt change of the power correction in the vicinity of the two-jet limit
- This is because within our simplified abelian model, we can only dress leading order calculations with non-perturbative gluers emissions, but a LO calculation for 3 jets is not reliable for $V \to 0$, as it misses important log enhanced contributions!

Limitations of our approach

[Caola, SFR, Limatola, Melnikov, Nason, 2011.14114, + Ozcelik 2108.08897]

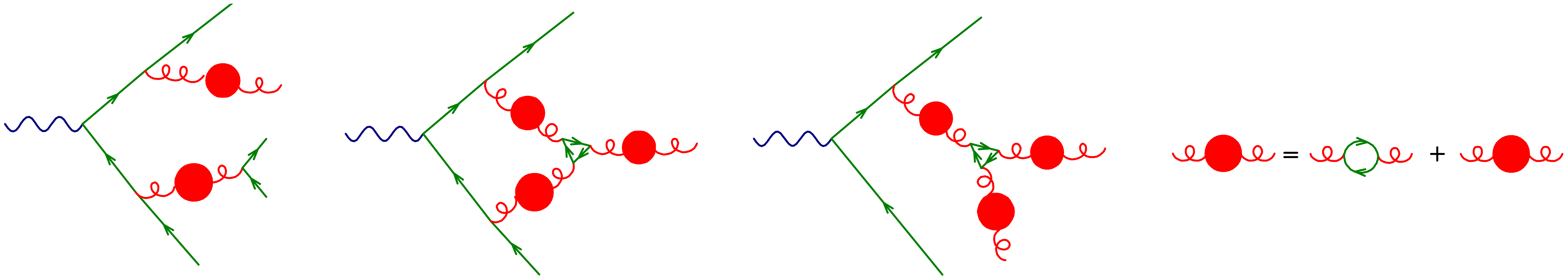




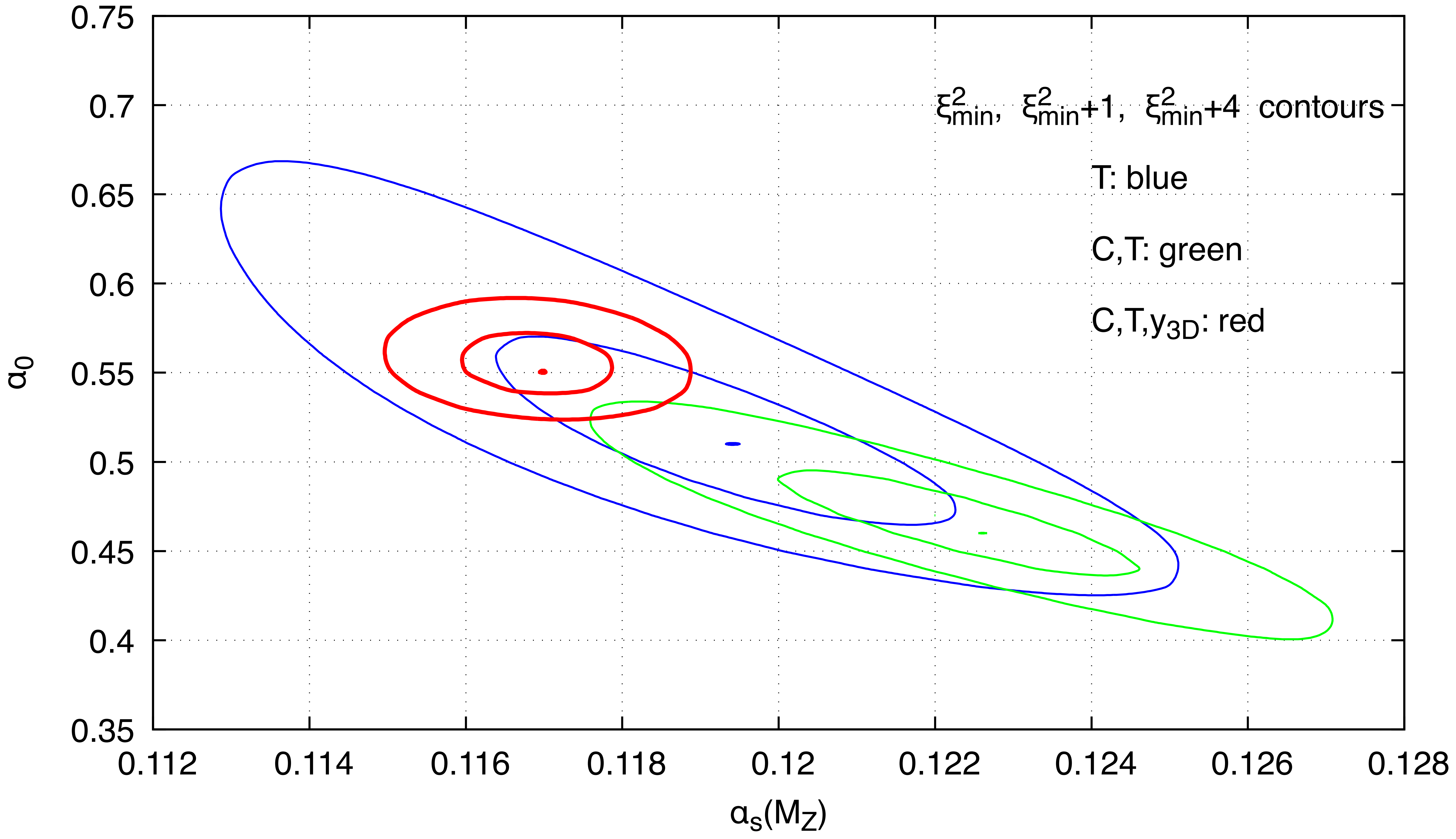
- In our model, there is a rapid and abrupt change of the power correction in the vicinity of the two-jet limit
- This is because within our simplified abelian model, we can only dress leading order calculations with non-perturbative gluers emissions, but a LO calculation for 3 jets is not reliable for $V \to 0$, as it misses important log enhanced contributions!

$$\Delta V(v) = \left[\left[\int d\Phi \delta(V(\Phi) - v) \frac{d\sigma^{\rm pert}}{d\Phi}\right]^{-1} \int d\Phi \delta(V(\Phi) - v) \frac{d\sigma^{\rm pert}}{d\Phi} \frac{2\mathscr{M} C_i}{\pi} \int \frac{dp_T}{p_T} dy \frac{d\varphi}{2\pi} \Delta V(p_T, \varphi, \eta; \Phi)\, \delta(p_T - \lambda)\right.$$

- But going beyond LO to calculate $d\sigma^{\rm pert}$ also demands the inclusion of other contributions we dunno how to handle!



Preliminary fit of the strong coupling [Nason, Zanderighi, 2301.03607]



Fit only in the 3-jet region, using NNLO calculation without resummation.

"number of variations of our procedure can lead easily to differences of the order of a percent"

→ despite event shapes will probably never lead to a competitive estimate of α_s, this is the simplest context where we can explore the interplay between perturbative and non-perturbative effects in jet-production processes.

Many thanks to G. Zanderighi for the plot!

Conclusions and outlook

➤ It is of outmost importance to **tame hadronisation** corrections if we aim at 1% accuracy

A. **When** do we expect linear power corrections?

B. **How do we estimate** linear power corrections?

Conclusions and outlook

- It is of outmost importance to **tame hadronisation** corrections if we aim at 1% accuracy
  A. **When** do we expect linear power corrections?
  B. **How do we estimate** linear power corrections?
- The **large-n_f limit** provides a simplified framework where we can get insights from QCD first principles
- Within this framework we can investigate the all-orders behaviour of processes that do not involve gluons at the lowest perturbative order

Conclusions and outlook

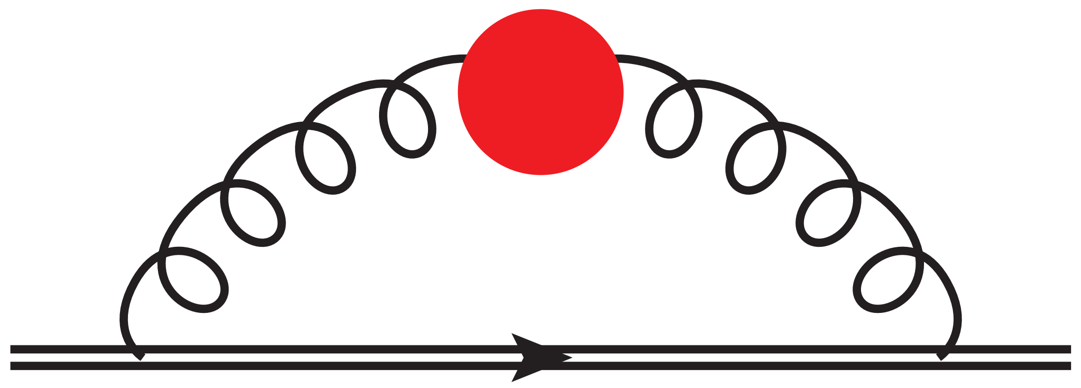
- It is of outmost importance to **tame hadronisation** corrections if we aim at 1% accuracy
  A. **When** do we expect linear power corrections?
  B. **How do we estimate** linear power corrections?
- The **large-n_f limit** provides a simplified framework where we can get insights from QCD first principles
- Within this framework we can investigate the all-orders behaviour of processes that do not involve gluons at the lowest perturbative order
  A. We investigate the perturbative convergence of observables used to infer the **top mass**, renormalising the top mass in the pole and in the $\overline{\mathrm{MS}}$ scheme
  B. We showed **inclusive observables** do not have linear power corrections
  C. We gained more insights on the calculation of non-perturbative corrections for **event shapes in the three-jet region** … although some arbitrariness is token to “non-abelianise” our result, and we do not have yet a final recipe!

Backup

Top pole-mass ambiguity in the large number-of-flavours limit

- The relation between the pole and th $\overline{\text{MS}}$ mass can be computed using the large-b_0 approximation

$$m_p - \overline{m}(\mu_m) = \text{Fin}\Big[$$



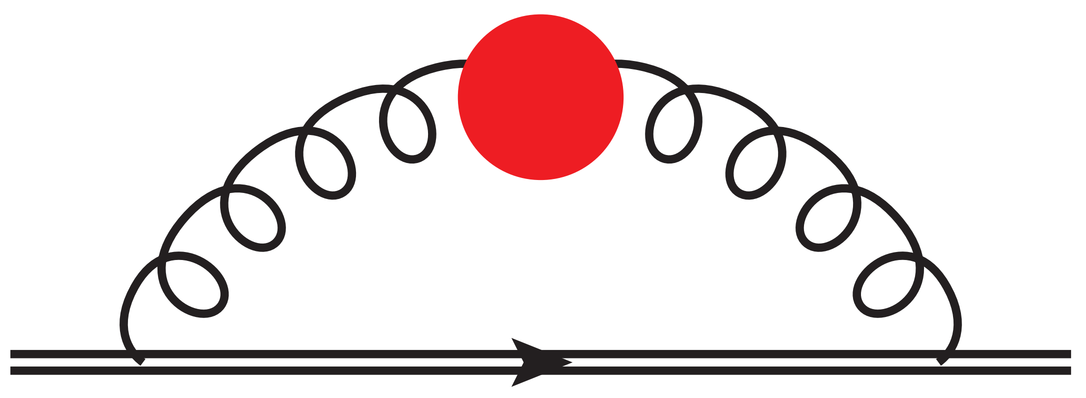
$$\Big]$$

[Ball, Beneke, Braun, hep-ph/9502300]

$$m_p - \bar{m}(\mu_m) = 7.557 + 2.345 + 0.584 + 0.241 + 0.127 + 0.085 + 0.067 + \mathbf{0.063} + 0.067 + \ldots \text{GeV}$$

Top pole-mass ambiguity in the large number-of-flavours limit

- The relation between the pole and th $\overline{\text{MS}}$ mass can be computed using the large-b_0 approximation

$$m_p - \overline{m}(\mu_m) = \text{Fin}\left[\right.$$  $$\left.\right]$$

[Ball, Beneke, Braun, hep-ph/9502300]

$$m_p - \bar{m}(\mu_m) = 7.557 + 2.345 + 0.584 + 0.241 + 0.127 + 0.085 + 0.067 + \mathbf{0.063} + 0.067 + \ldots \text{GeV}$$

- Asymptotic behaviour is known

[Beneke, Braun, hep-ph/9402364]

$$c_{n+1} \to N\,\bar{m}(m)\,(2b_0)^n \frac{\Gamma\left(1+n+b\right)}{\Gamma(1+b)}\left(1+\sum_{k=1}^{\infty}\frac{s_k}{n}\right) \quad \text{with } b = \frac{b_1}{2b_0^2},\, s_i = s_i(b_0, b_1, \ldots)$$

- We can fit N from the already known coefficients, getting

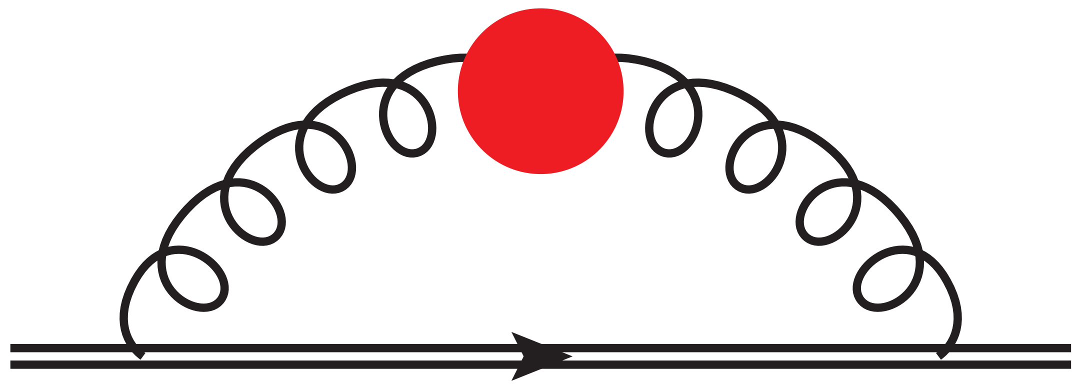
$$m_p - \bar{m}(\mu_m) = \underbrace{7.577 + 1.617 + 0.501 + 0.197}_{\text{exact}} + 0.112 + 0.079 + 0.066 + \boxed{\mathbf{0.064}} + 0.071 + \ldots \text{ GeV}$$

[Beneke, Marquard, Nason, Steinhauser, 1605.03609]

Light quark mass effects not included, they increase by roughly a factor of 2 this number.

Top pole-mass ambiguity in the large number-of-flavours limit

- The relation between the pole and th $\overline{\text{MS}}$ mass can be computed using the large-b_0 approximation

$$m_p - \overline{m}(\mu_m) = \text{Fin}\Bigg[$$  $$\Bigg]$$

[Ball, Beneke, Braun, hep-ph/9502300]

$$m_p - \bar{m}(\mu_m) = 7.557 + 2.345 + 0 \quad \ldots \quad 0.067 + \mathbf{0.063} + 0.067 + \ldots \text{GeV}$$

How do we propagate this uncertainty to observables used to infer the top mass?

- Asymptotic behaviour is known … Beneke, Braun, hep-ph/9402364]

$$c_{n+1} \to N\,\bar{m}(m)\,(2b_0)^n \frac{\Gamma\left(1+n+\ldots\right.}{\Gamma(1+b\ldots} \quad \ldots \frac{b_1}{2b_0^2},\ s_i = s_i(b_0, b_1, \ldots)$$

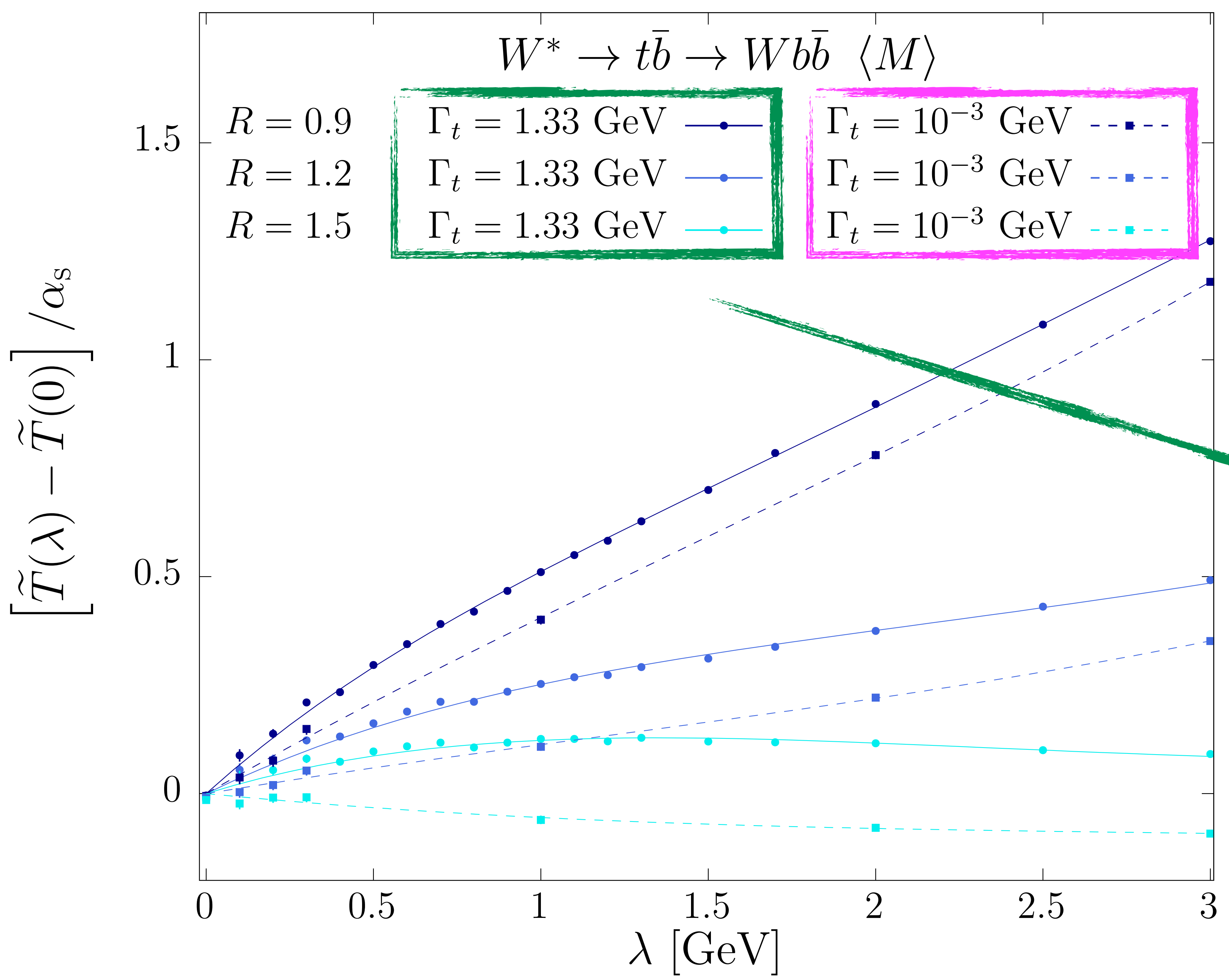
- We can fit N from the already known coefficients, getting

$$m_p - \bar{m}(\mu_m) = \underbrace{7.577 + 1.617 + 0.501 + 0.197}_{\text{exact}} + 0.112 + 0.079 + 0.066 + \boxed{\mathbf{0.064}} + 0.071 + \ldots \text{ GeV}$$

[Beneke, Marquard, Nason, Steinhauser, 1605.03609]

Light quark mass effects not included, they increase by roughly a factor of 2 this number.

Single-top production and decay: reconstructed-top mass [SFR, Nason, Oleari, 1810.10931]

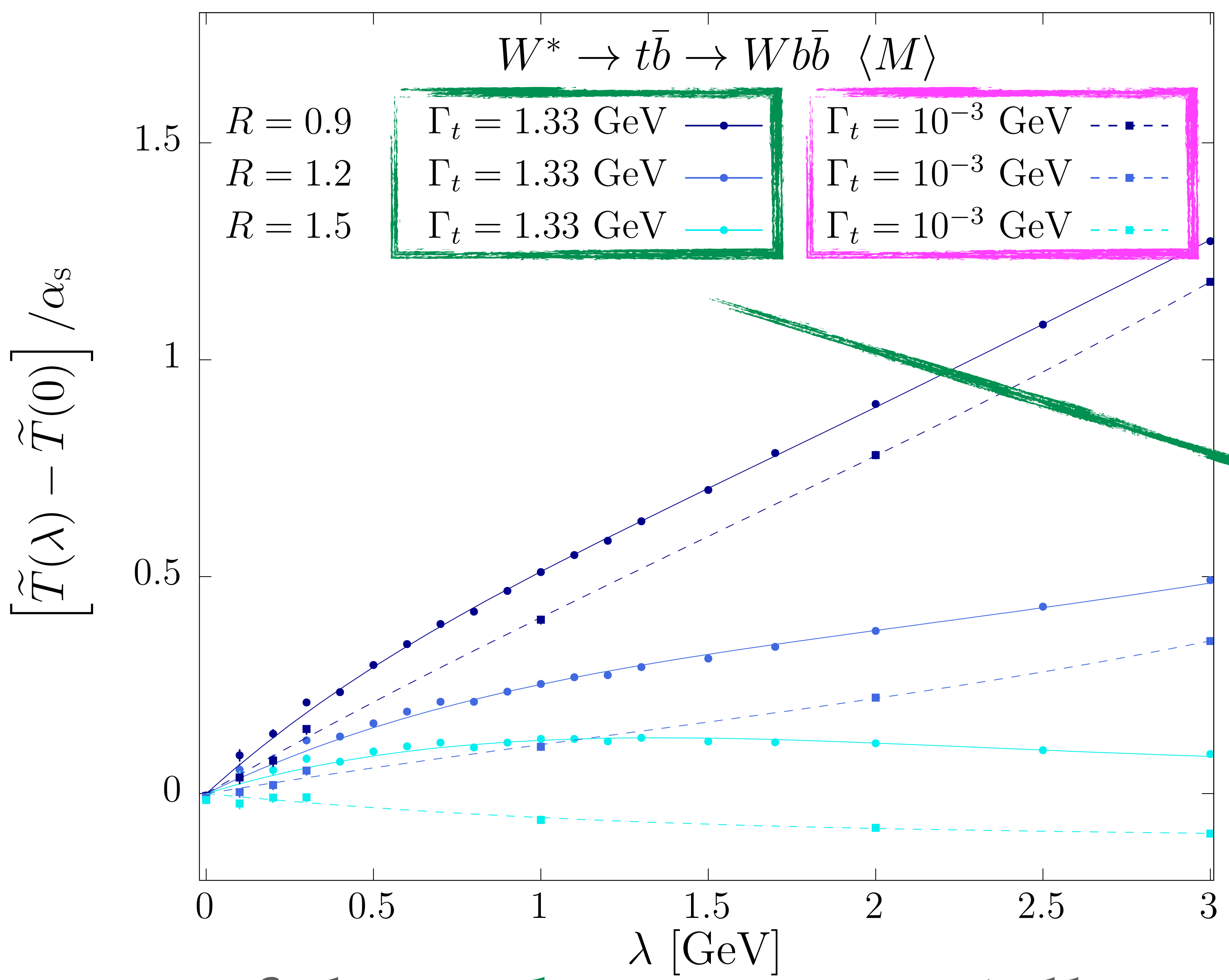


In NWA, pole mass = mass of the top decay products.

Linear slope due to finite size of the b-jet cone radius. For $R \to \pi/2$, the slope is 0 when using the pole mass.

Finite width effects induce a small slope.

Single-top production and decay: reconstructed-top mass [SFR, Nason, Oleari, 1810.10931]



In NWA, pole mass = mass of the top decay products.

Linear slope due to finite size of the b-jet cone radius. For $R \to \pi/2$, the slope is 0 when using the pole mass.

Finite width effects induce a small slope.

The scheme change induces a larger slope

Perturbative order	*Pole mass as a function of the $\overline{\text{MS}}$ mass*	*Mass of the top decay products, R=1.5, pole mass*	*Mass of the top decay products, R=1.5, $\overline{\text{MS}}$ mass*
4	+171	−6(1)	+163(1)
5	+89	−10(1)	+79(1)
6	+60	−11(1)	+49(1)
7	+47	−11(1)	+35(1)
8	**+44**	−12(1)	+31(1)
9	+46	−15(1)	+31(1)
10	+55	−19(1)	+36(1)

The use of the **pole mass** partially cancels the linear renormalon present in M_{Wb_j}, leading to a better perturbative series than the one in the $\overline{\text{MS}}$ scheme. This is why $m_{\text{pole}}(\bar{m}(m))$ and $M_{W,b_j}(\bar{m}(m))$ have similar perturbative expansion