Dark matter direct detection from single phonons to nuclear recoils

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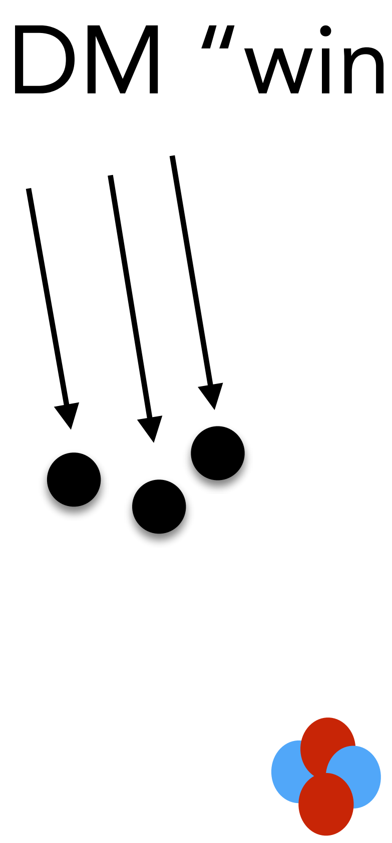
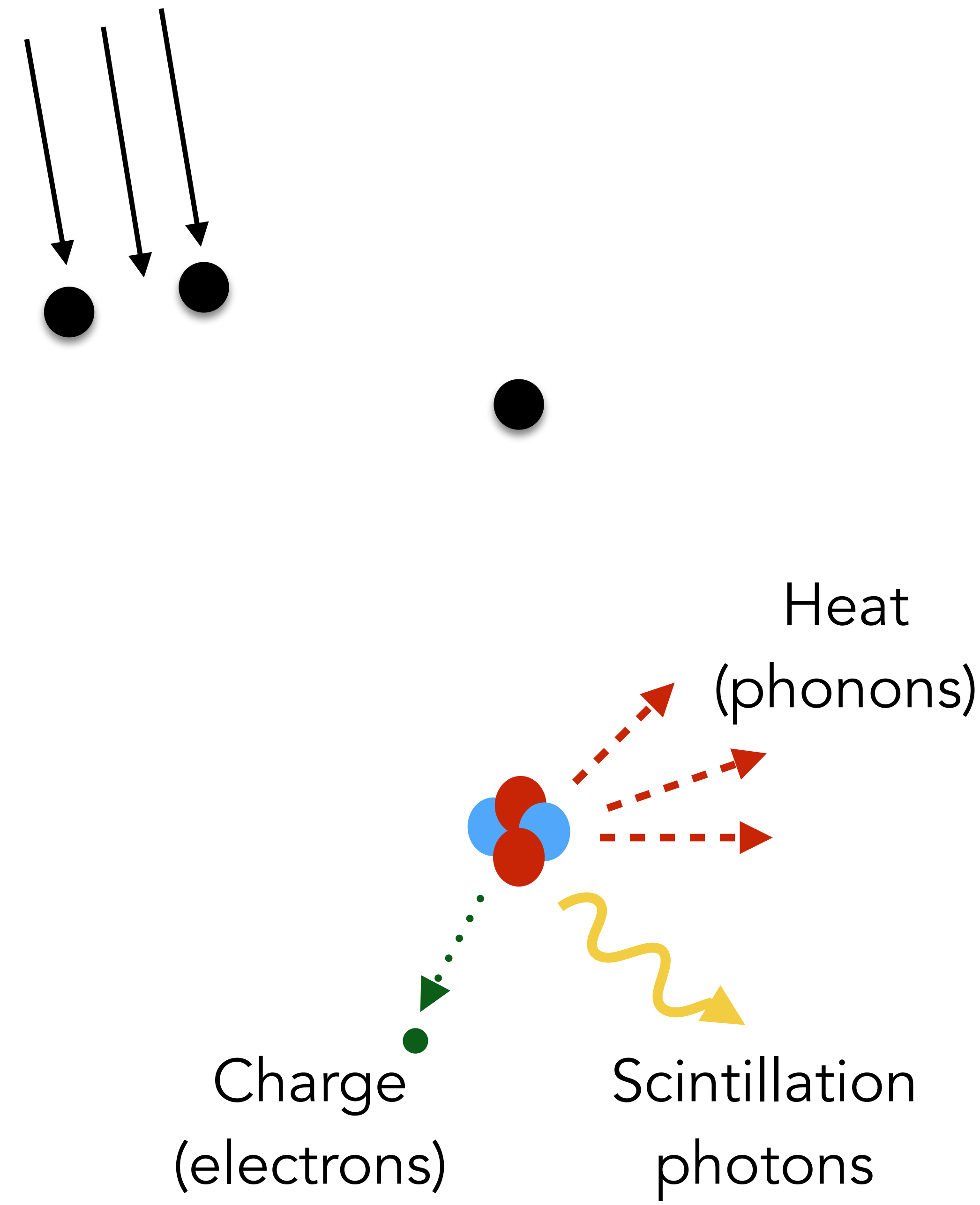
February 9, 2023
Brookhaven Seminar

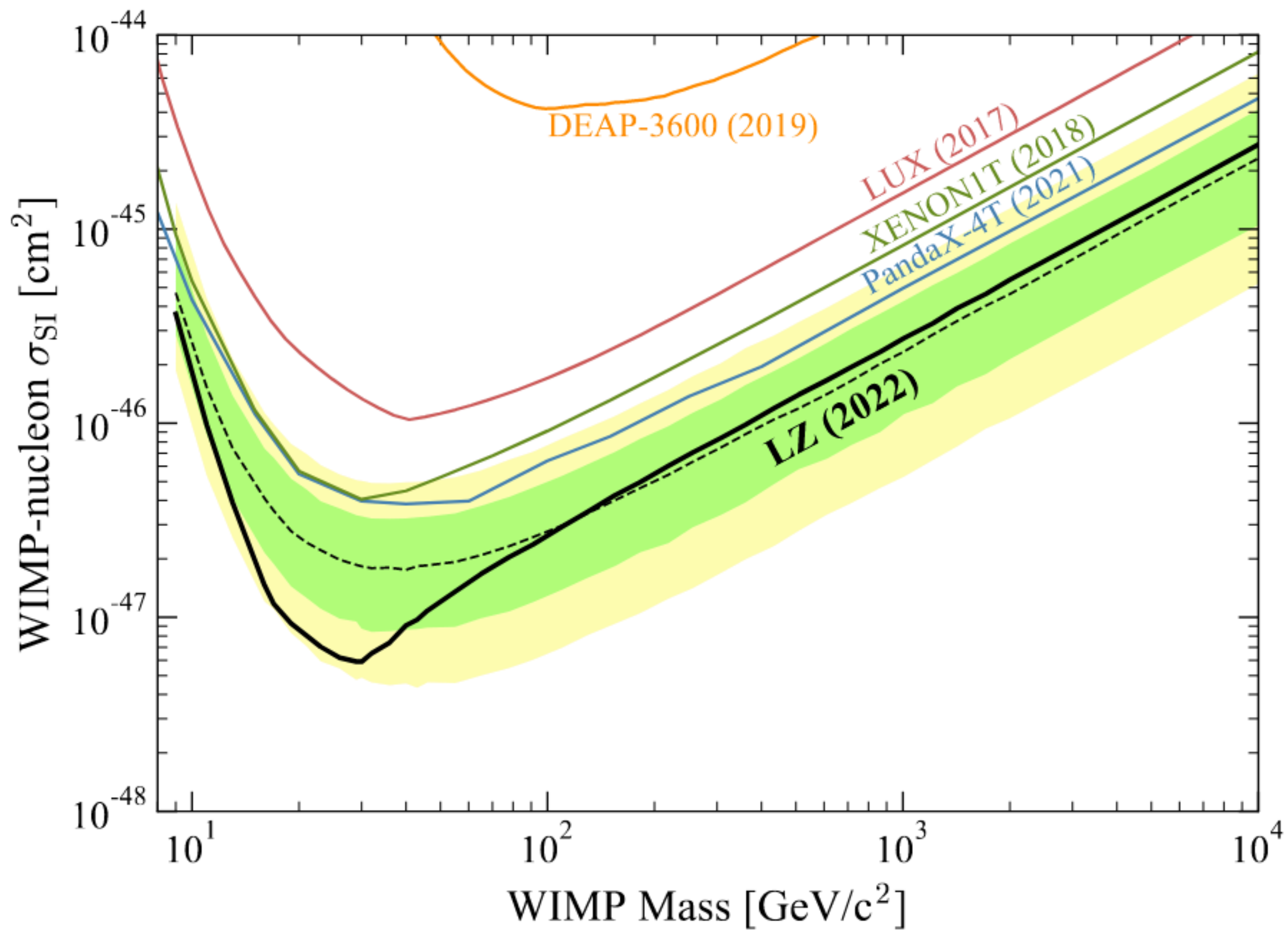


Direct detection of nuclear recoils

DM “wind”



Strong constraints on interaction cross section when DM mass comparable to nucleus mass



(Proton mass = 0.938 GeV)

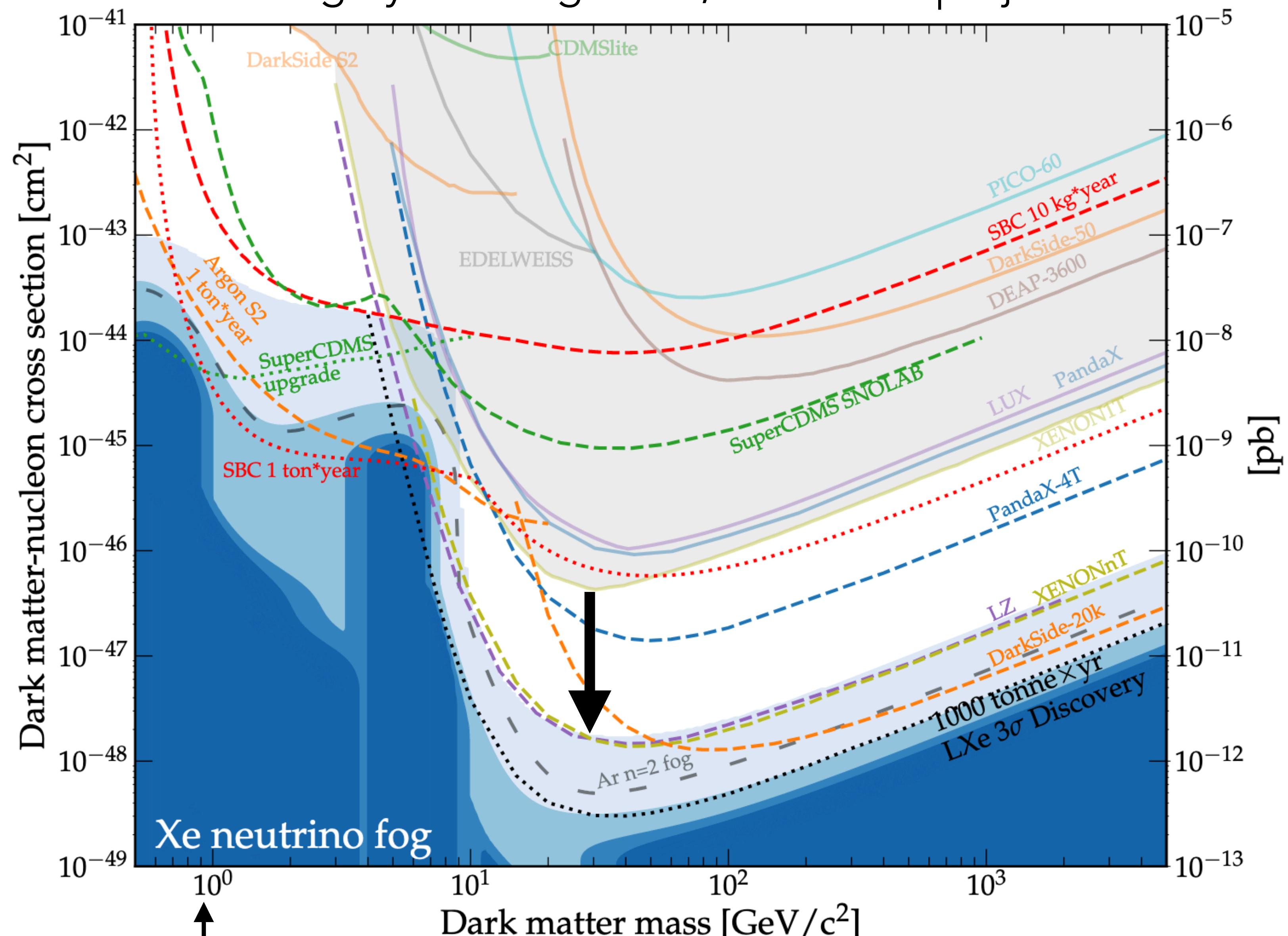
Direct detection of nuclear recoils

Landscape of direct detection above 1 GeV

XENON-nT

Many decades of progress searching for nuclear recoils

Shaded gray: existing limits; others are projections

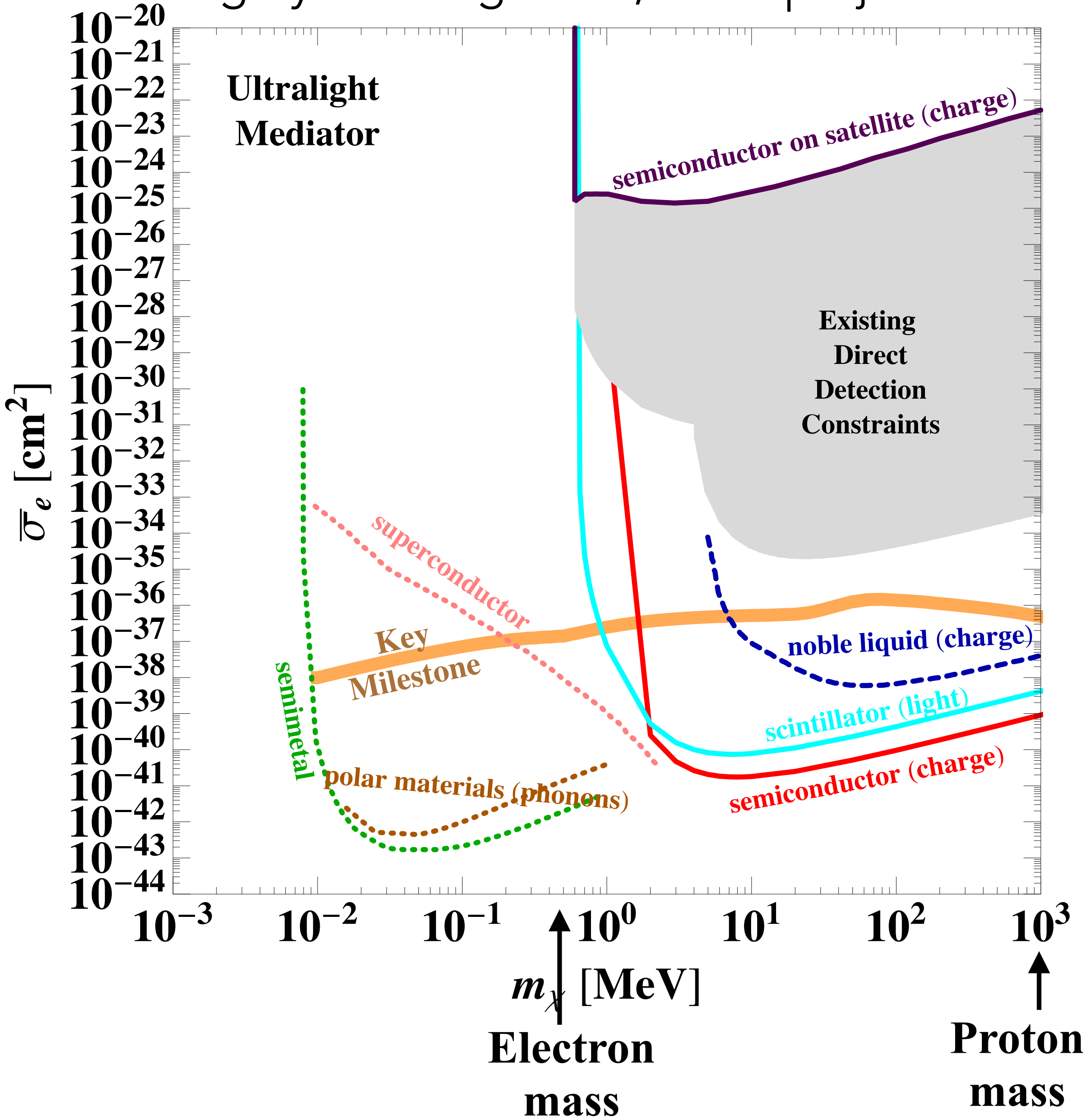


Proton mass

From 2203.08084

Landscape of direct detection below 1 GeV

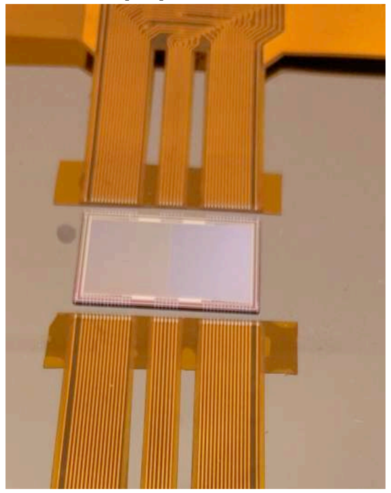
Shaded gray: existing limits; lines: projections

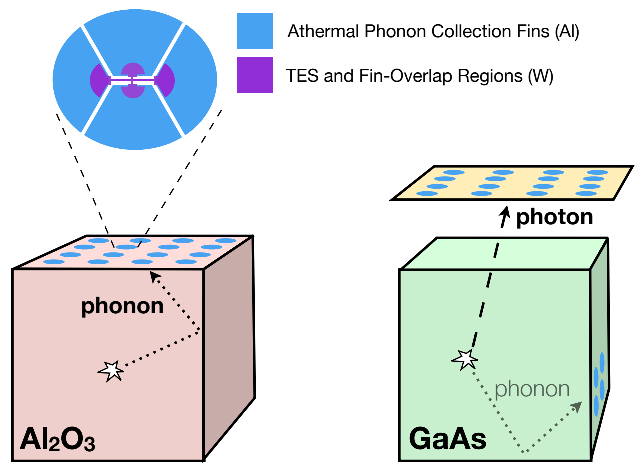


Rapid development in recent decade

- New particle candidates
- New experiments
- **New theory for direct detection**

Skipper CCD

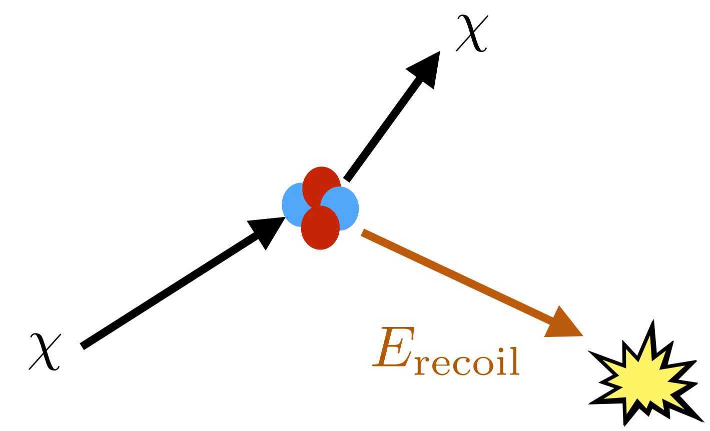




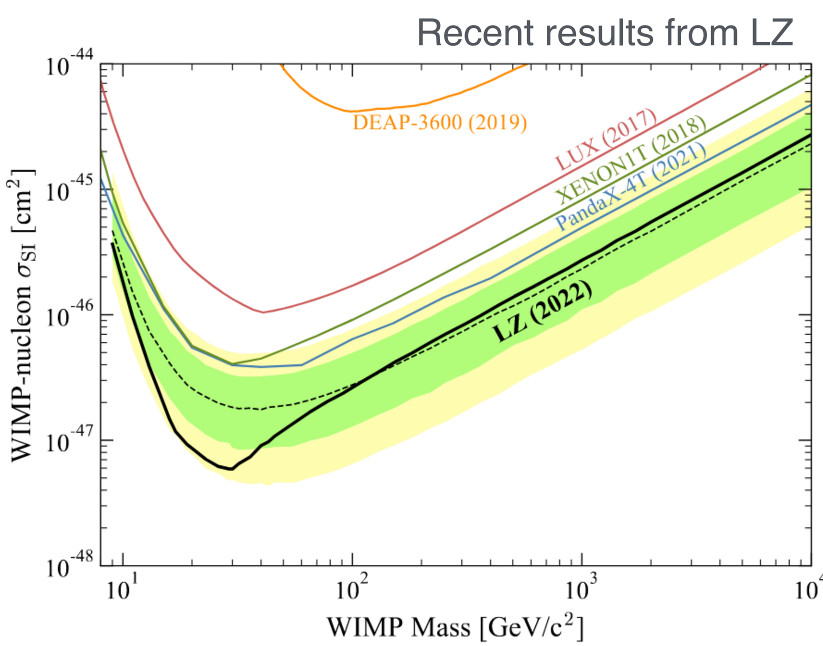
Basic Research Needs Report:
"Dark Matter Small Projects New Initiatives"

Dark matter mass

keV MeV GeV TeV



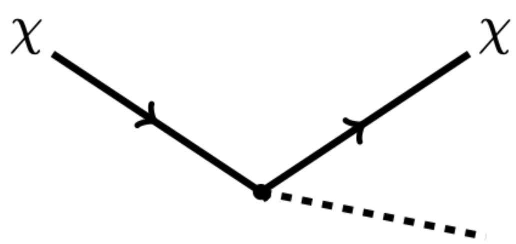
Nuclear recoils



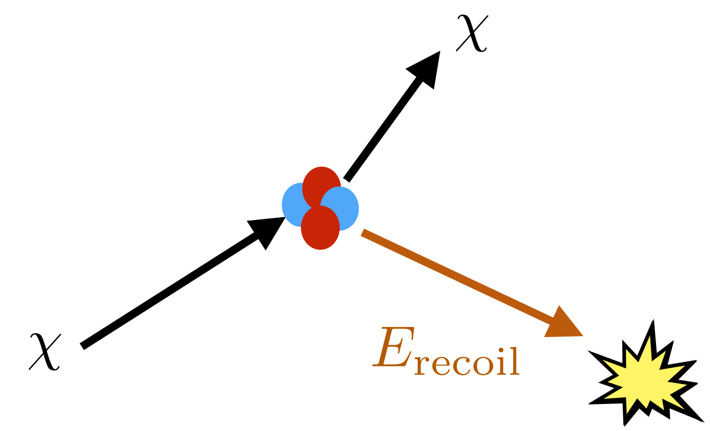
O(keV) thresholds

Dark matter mass

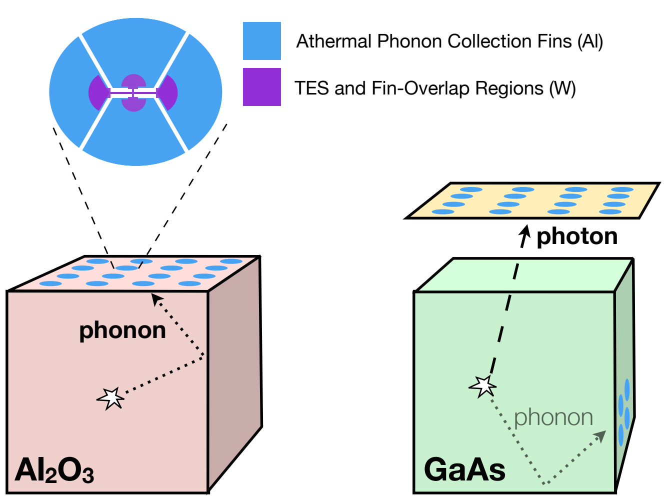
keV MeV GeV TeV



Single phonon excitation

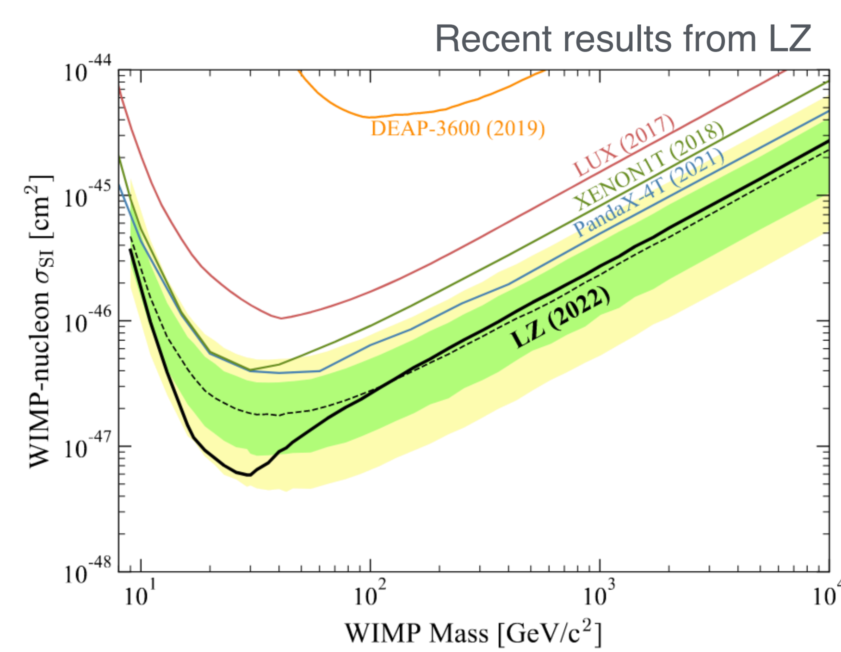


Nuclear recoils



From TESSERACT white paper

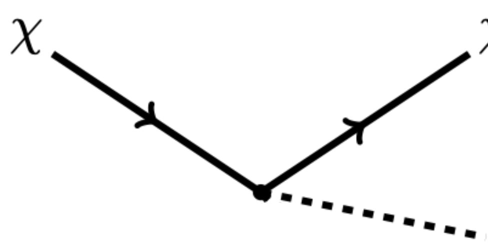
Single phonons excitations with energy 1-100 meV (SPICE)



O(keV) thresholds

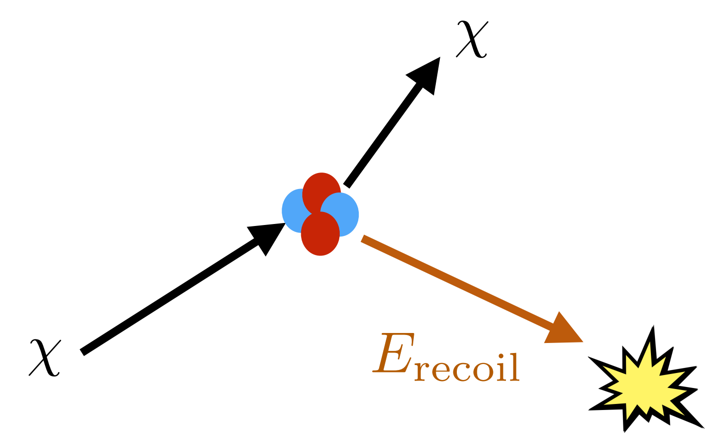
Dark matter mass

keV MeV GeV TeV

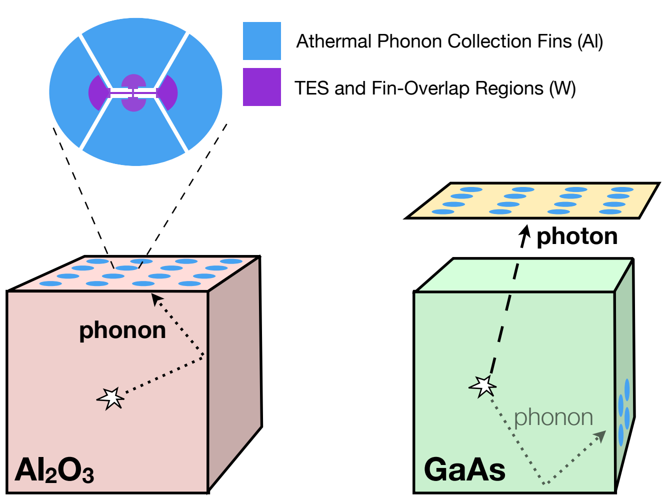


Single phonon excitation

?



Nuclear recoils



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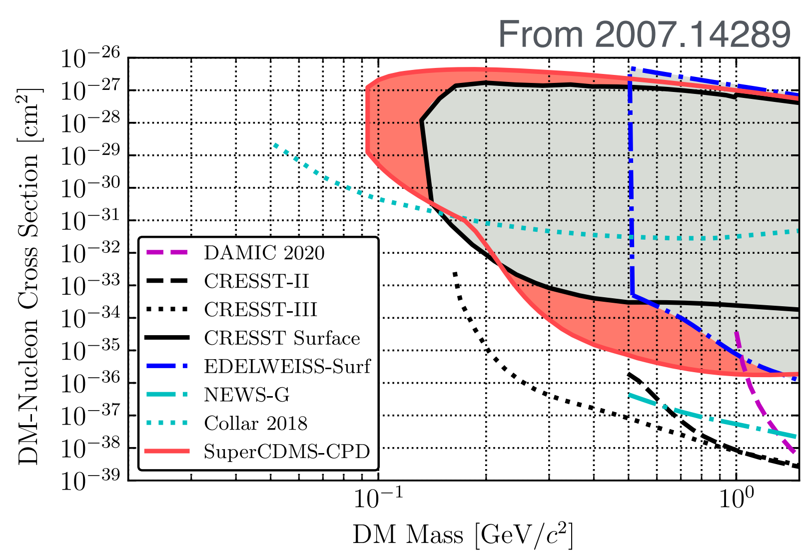
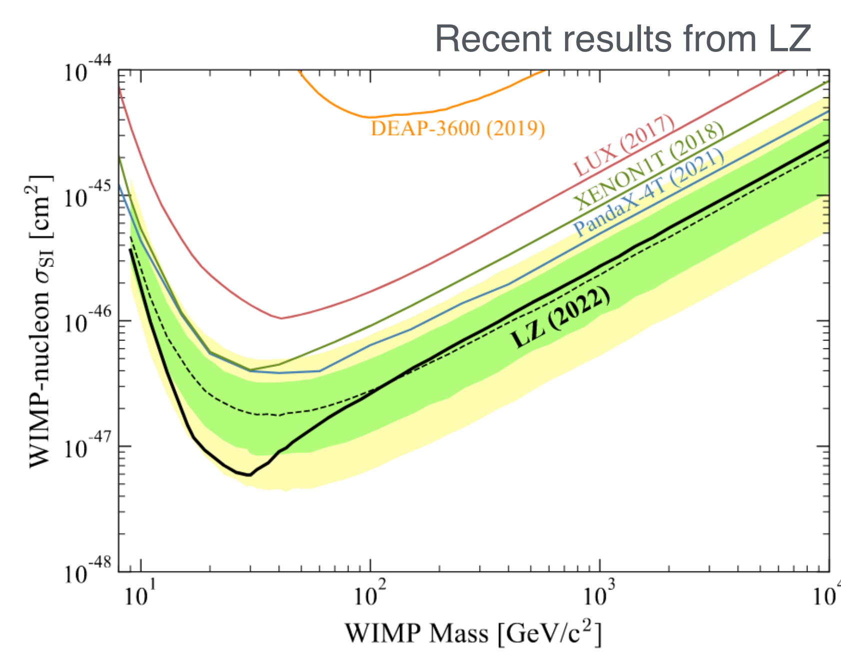
Single phonons excitations with energy 1-100 meV (SPICE)

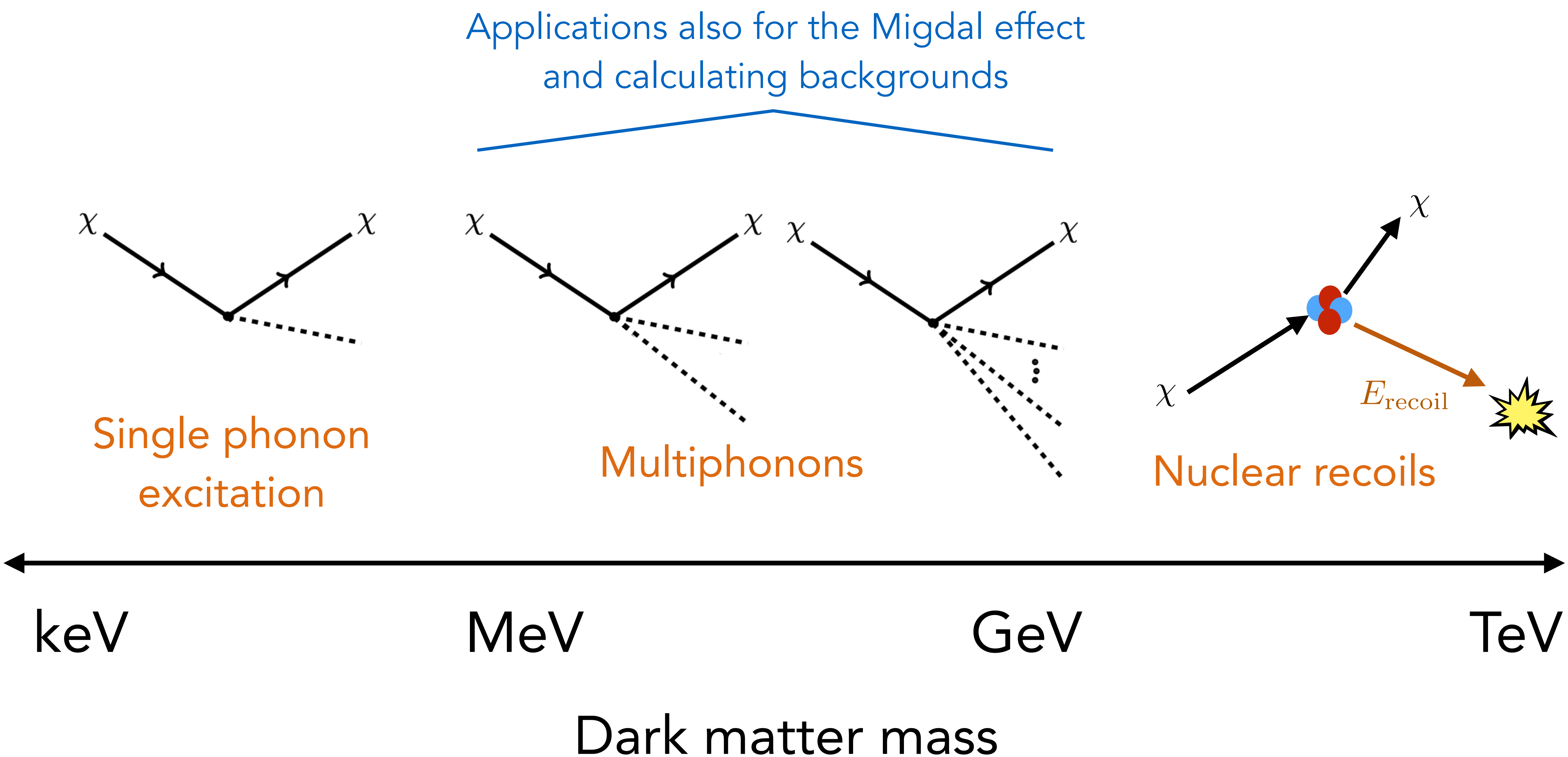


O(10) eV thresholds



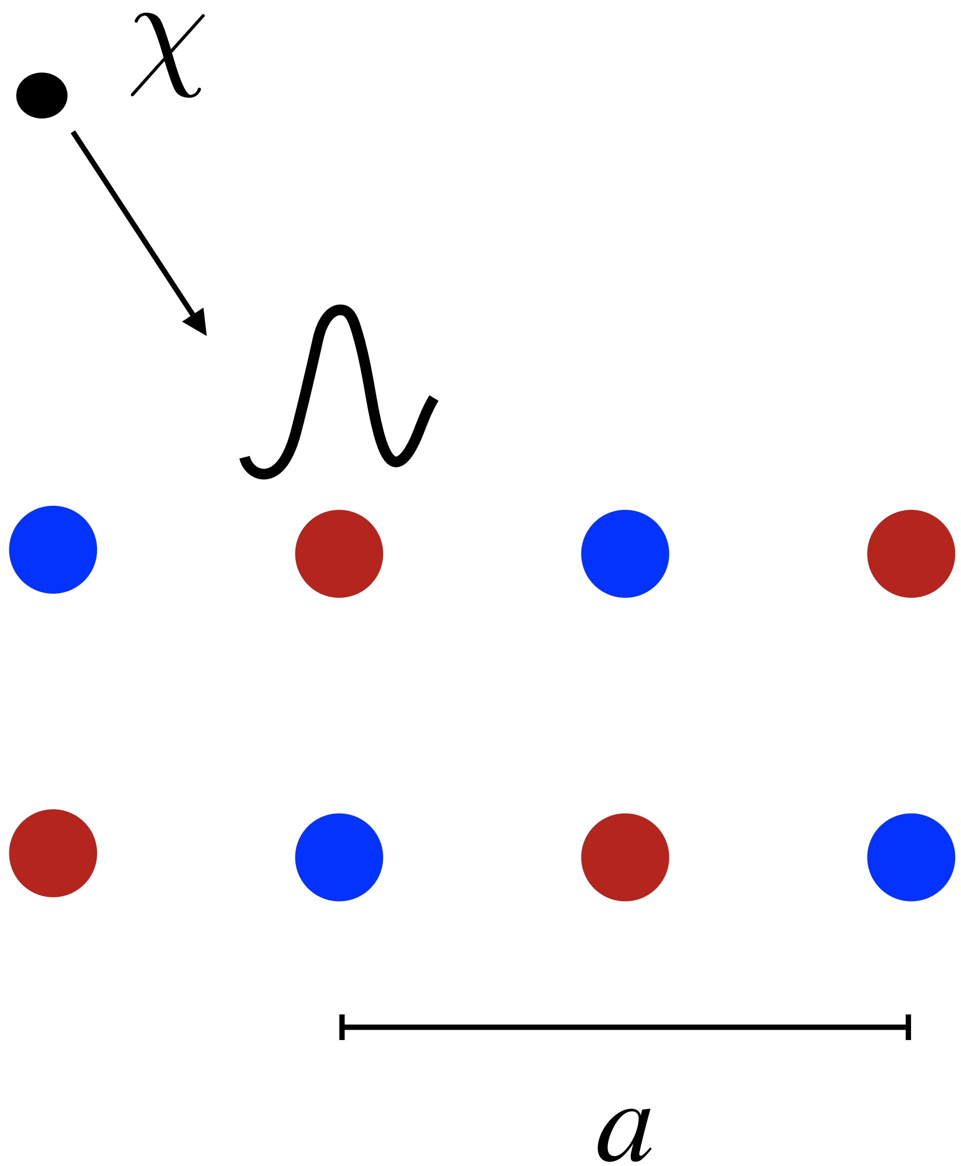
O(keV) thresholds

DM-nucleus scattering in crystals



Brian Campbell-Deem, Knapen, TL, Ethan Villarama 2205.02250
Campbell-Deem, Cox, Knapen, TL, Melia 1911.03482
Knapen, Kozaczuk, TL 2011.09496

What does DM-nucleus scattering look like in a crystal?



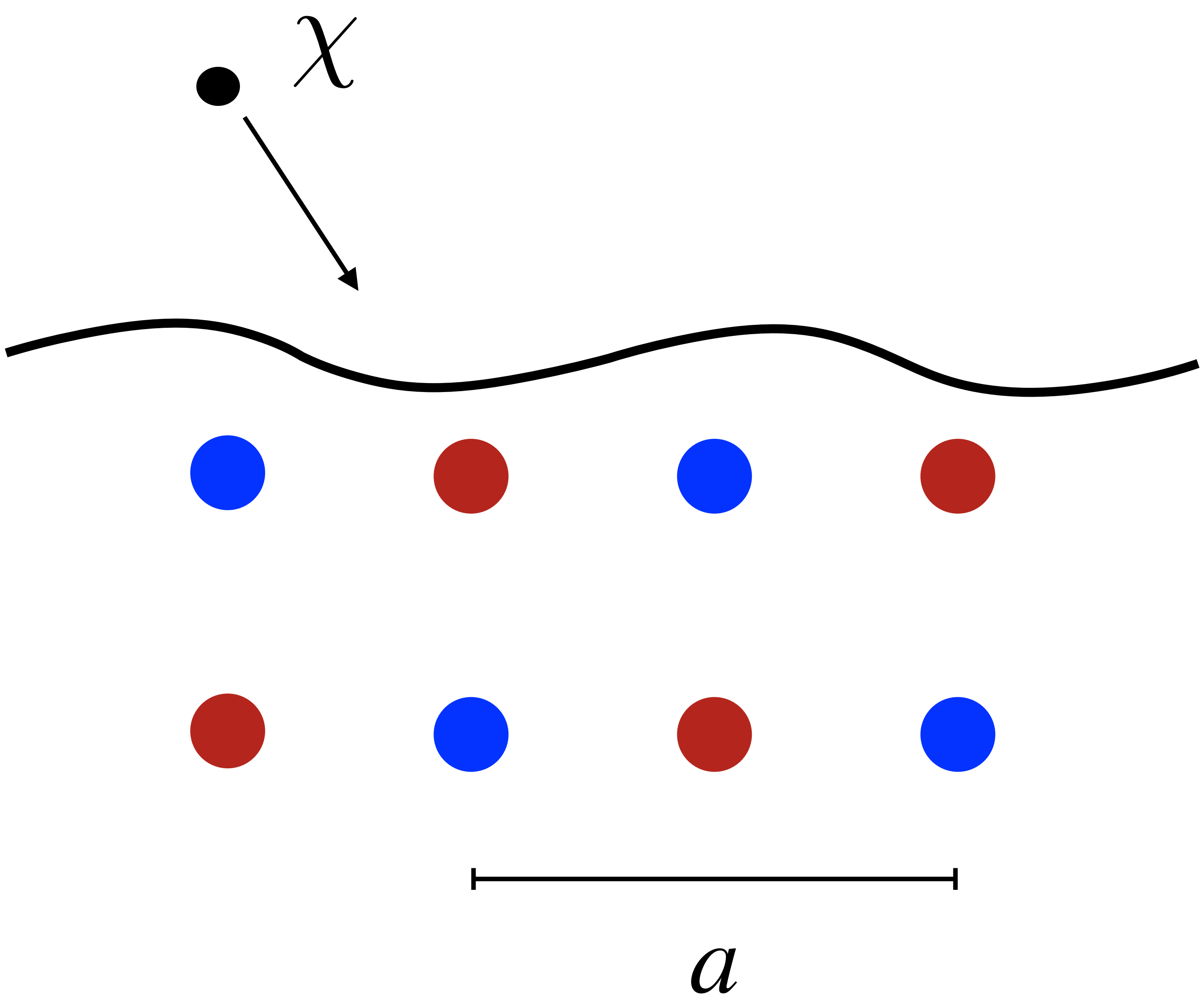
When momentum transfer

$$q \gg q_{\rm BZ} = \frac{2\pi}{a} \sim \text{few keV}$$

and $\omega \gg \bar{\omega}_{\rm phonon} \sim 10\text{-}100 \text{ meV}$

DM scatters off an individual nucleus

What does DM-nucleus scattering look like in a crystal?



When momentum transfer

$$q \ll q_{\rm BZ} = \frac{2\pi}{a}$$

and $\omega \sim \bar{\omega}_{\rm phonon}$

DM excites collective excitations = phonons

DM scattering rate

DM-mediator form factor

$$\frac{d\sigma}{d^3\mathbf{q}\,d\omega} \propto \sigma_{\chi p}\,|\tilde{F}_{\rm med}(q)|^2\, S(\mathbf{q},\omega)\,\delta(\omega - \mathbf{q}\cdot\mathbf{v} + \tfrac{q^2}{2m_\chi})$$

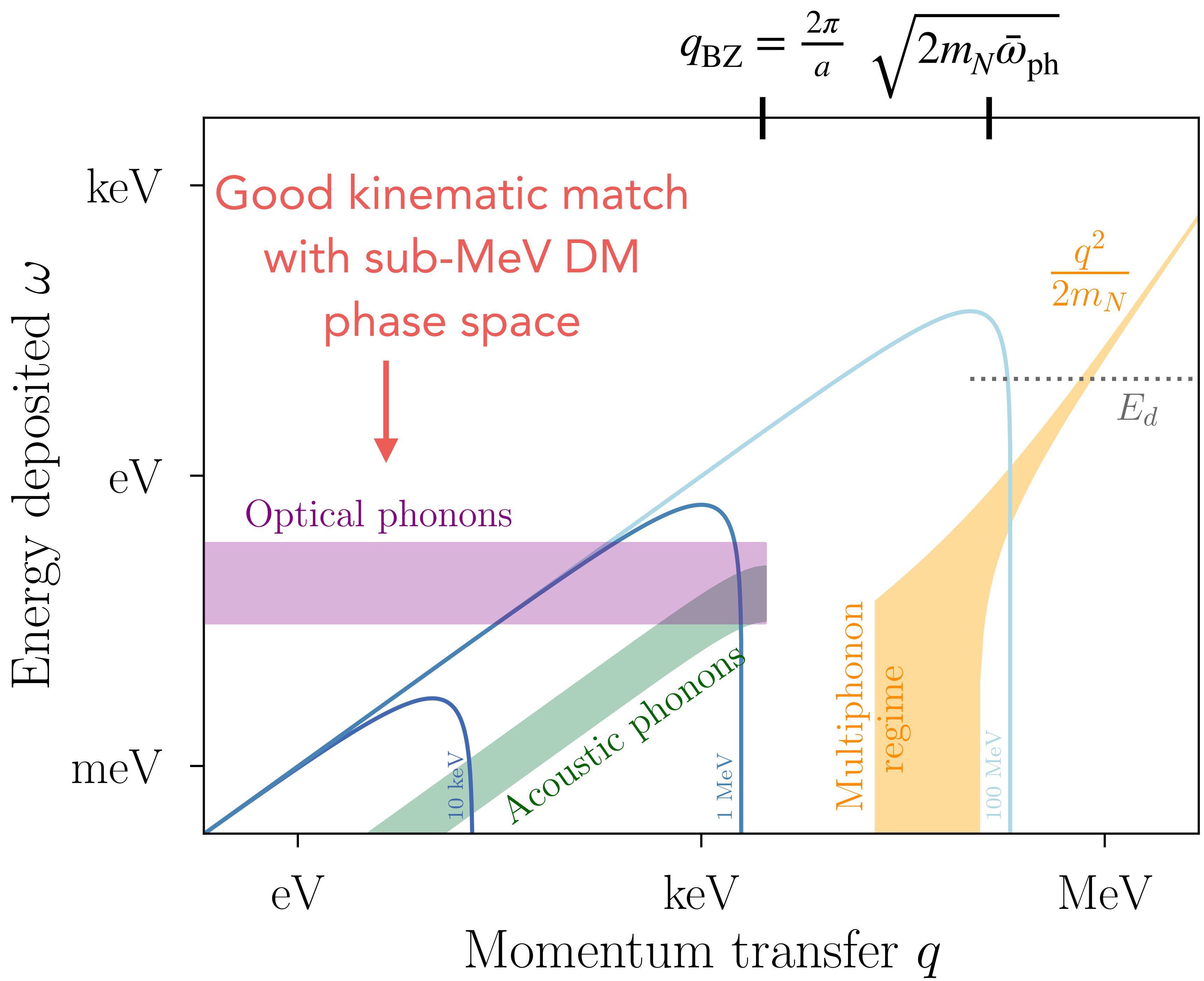
Dynamic structure factor captures response of target

For free nuclei and spin-independent interactions:

$$S(\mathbf{q},\omega) \propto A_N^2\,\delta\left(\omega - \tfrac{q^2}{2m_N}\right)$$

Goal: understand $S(\mathbf{q},\omega)$ from the single phonon to the nuclear recoil regime

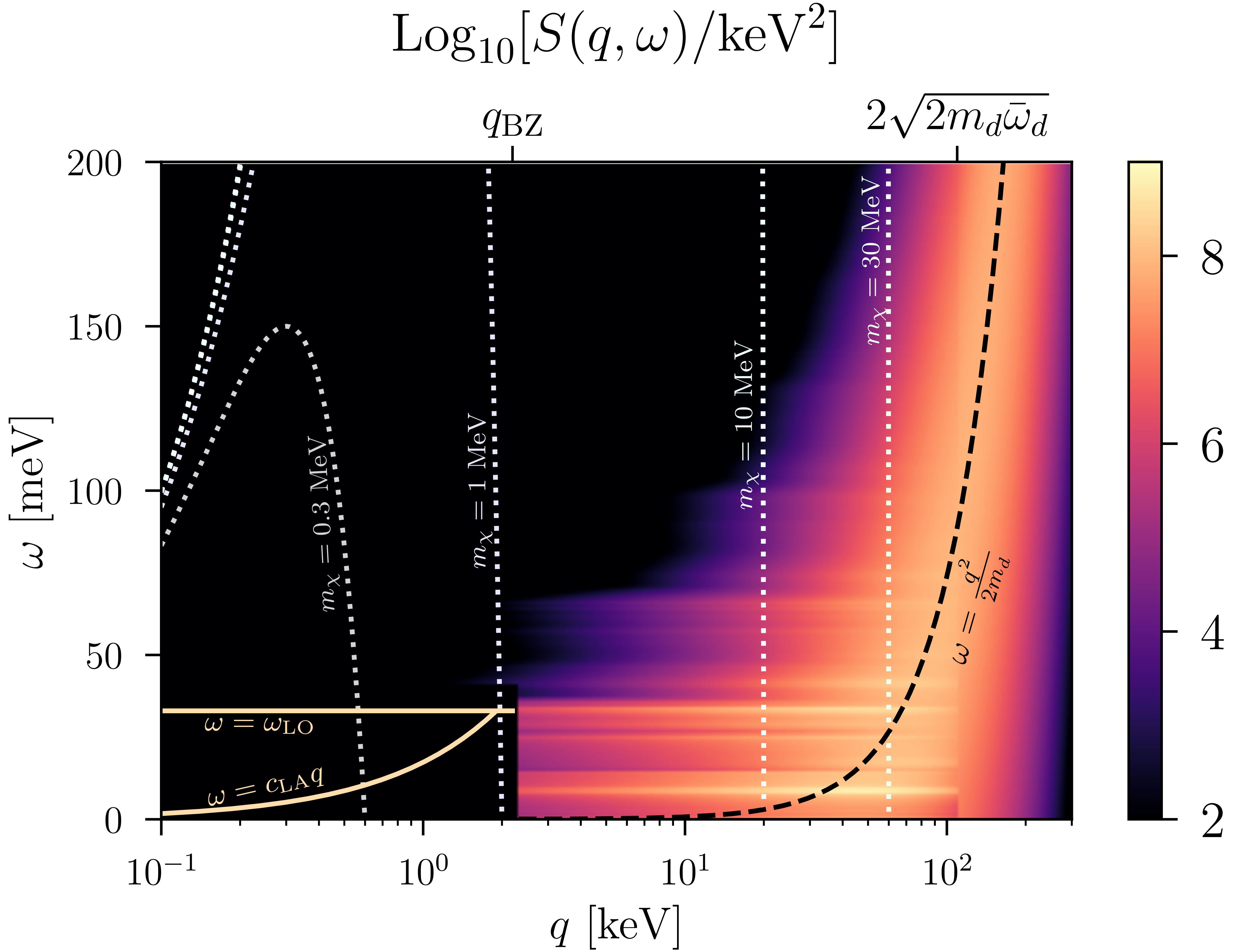
DM-nucleus scattering in a crystal



Kahn, TL 2108.03239

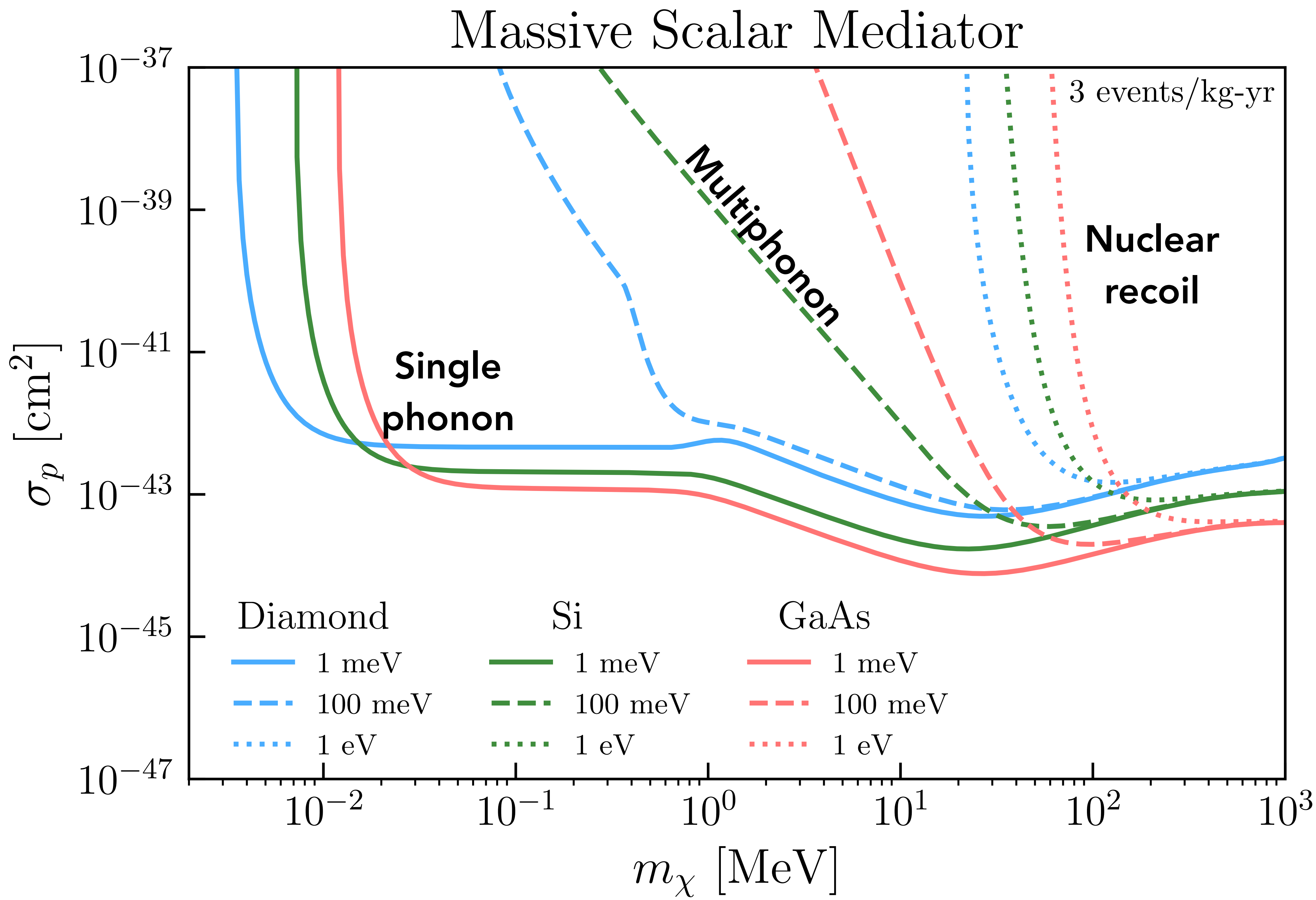
Trickle, Zhang, Zurek, Inzani, Griffin 1910.08092

Structure factor for GaAs



Campbell-Deem, Knapen, TL, Villarama 2205.02250

Reach for sub-GeV dark matter

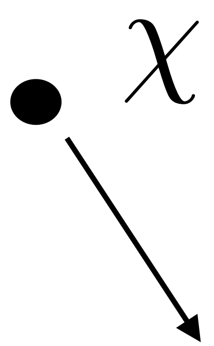


Campbell-Deem, Knapen, TL, Villarama 2205.02250

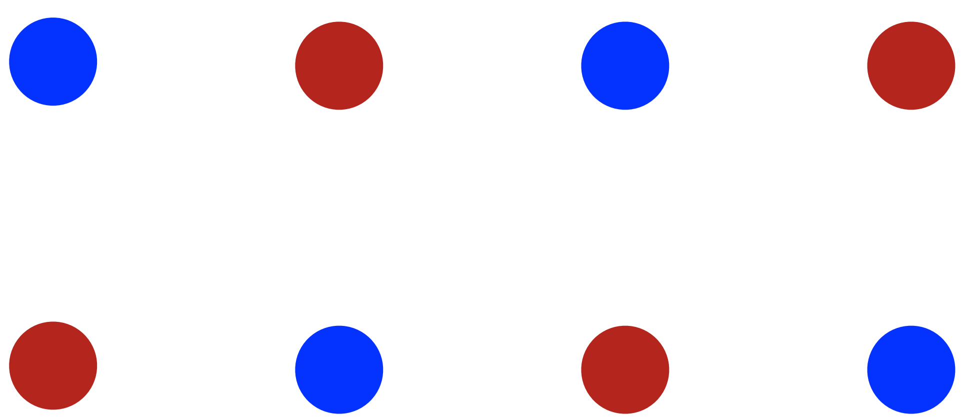
Now for the details

DM-nucleus interaction

Assuming spin-independent interactions



f_J - effective coupling strength between DM and ion J



Short range potential

$$\sigma_{\chi p} = 4\pi b_p^2$$

$$V(\mathbf{r}) \propto b_p \sum_J f_J \, \delta(\mathbf{r} - \mathbf{r}_J)$$

In Fourier space

$$V(\mathbf{q}) \propto b_p \sum_J f_J \, e^{i\mathbf{q}\cdot\mathbf{r}_J}$$

For spin-independent interactions: $f_J = A_J$

Dynamic structure factor

$$S(\mathbf{q},\omega) \equiv \frac{2\pi}{V}\sum_f \left|\sum_J \langle \Phi_f | f_J e^{i\mathbf{q}\cdot\mathbf{r}_J} |0\rangle\right|^2 \delta\left(E_f - \omega\right)$$

$$= \frac{1}{V}\sum_{J,\,J'}^{N} f_J f_{J'}^* \int_{-\infty}^{\infty} dt\, \langle e^{-i\mathbf{q}\cdot\mathbf{r}_{J'}(0)} e^{i\mathbf{q}\cdot\mathbf{r}_J(t)}\rangle e^{-i\omega t}$$

Contains interference terms between different atoms in lattice
— allows for collective excitations

Phonons appear through positions of ions:

$$\mathbf{r}_J(t) = \mathbf{r}_J^0 + \mathbf{u}_J(t)$$

Quantized phonon field given in terms of phonon dispersions $\omega_{\nu\mathbf{k}}$ and eigenvectors $\mathbf{e}_{\nu\mathbf{k}}$

$$\mathbf{u}_J(t) \sim \sum_\nu \sum_\mathbf{k} \frac{1}{\sqrt{2Nm_N\omega_{\nu,\mathbf{k}}}}\left(\mathbf{e}_{\nu\mathbf{k}}\hat{a}_{\nu,\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}_J^0 - i\omega_{\nu,\mathbf{k}}t} + \mathrm{c.c.}\right)$$

Single phonon excitation

Studied extensively in literature:

$$S^{n=1}(\mathbf{q},\omega) \sim \sum_{J,J'} f_J f_{J'} \int dt\, \langle \mathbf{q}\cdot\mathbf{u}_J(0)\ \mathbf{q}\cdot\mathbf{u}_{J'}(t)\rangle e^{-i\omega t}$$

Griffin, Knapen, TL, Zurek 1807.10291; Griffin, Inzani, Trickle, Zhang, Zurek 1910.10716
Griffin, Hochberg, Inzani, Kurinsky, TL, Yu 2020; Coskuner, Tickle, Zhang, Zurek 2102.09567

Momentum conservation up to crystal momentum G

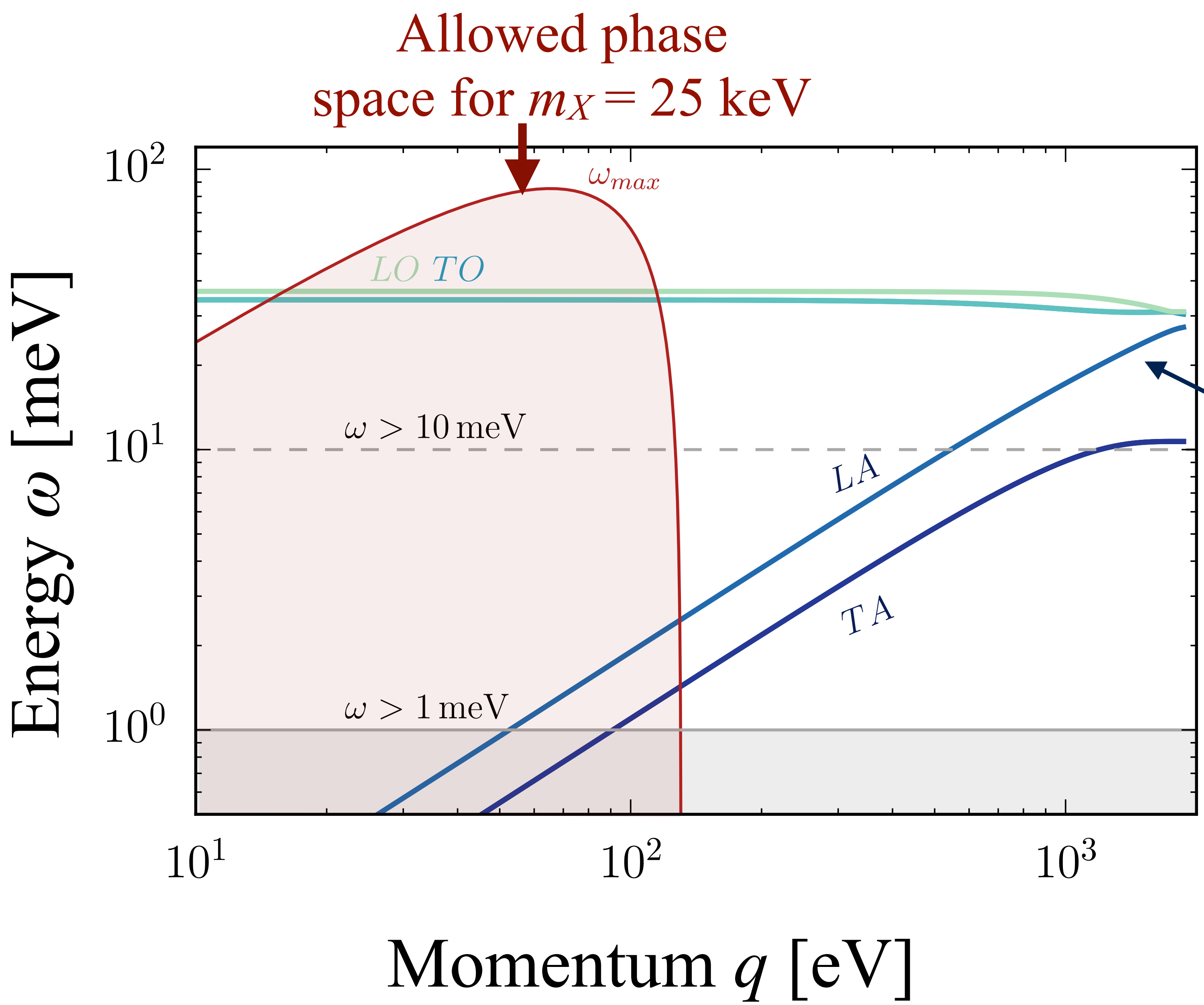
G = 0 for q < keV (first Brillouin Zone)

$$S(\mathbf{q},\omega) = \frac{2\pi}{\Omega}\sum_{\nu\mathbf{k}} \delta_{\mathbf{k},\mathbf{q}+\mathbf{G}}\,\delta(\omega-\omega_{\nu\mathbf{k}})\left|\sum_d f_d\,\frac{\mathbf{q}\cdot\mathbf{e}_{\nu,d,\mathbf{k}}}{\sqrt{2m_d\omega_{\nu\mathbf{k}}}}e^{-W_d(q)}\right|^2$$

[1-phonon]

Phonon form factor for spin-independent interaction to excite a phonon in branch ν with momentum k

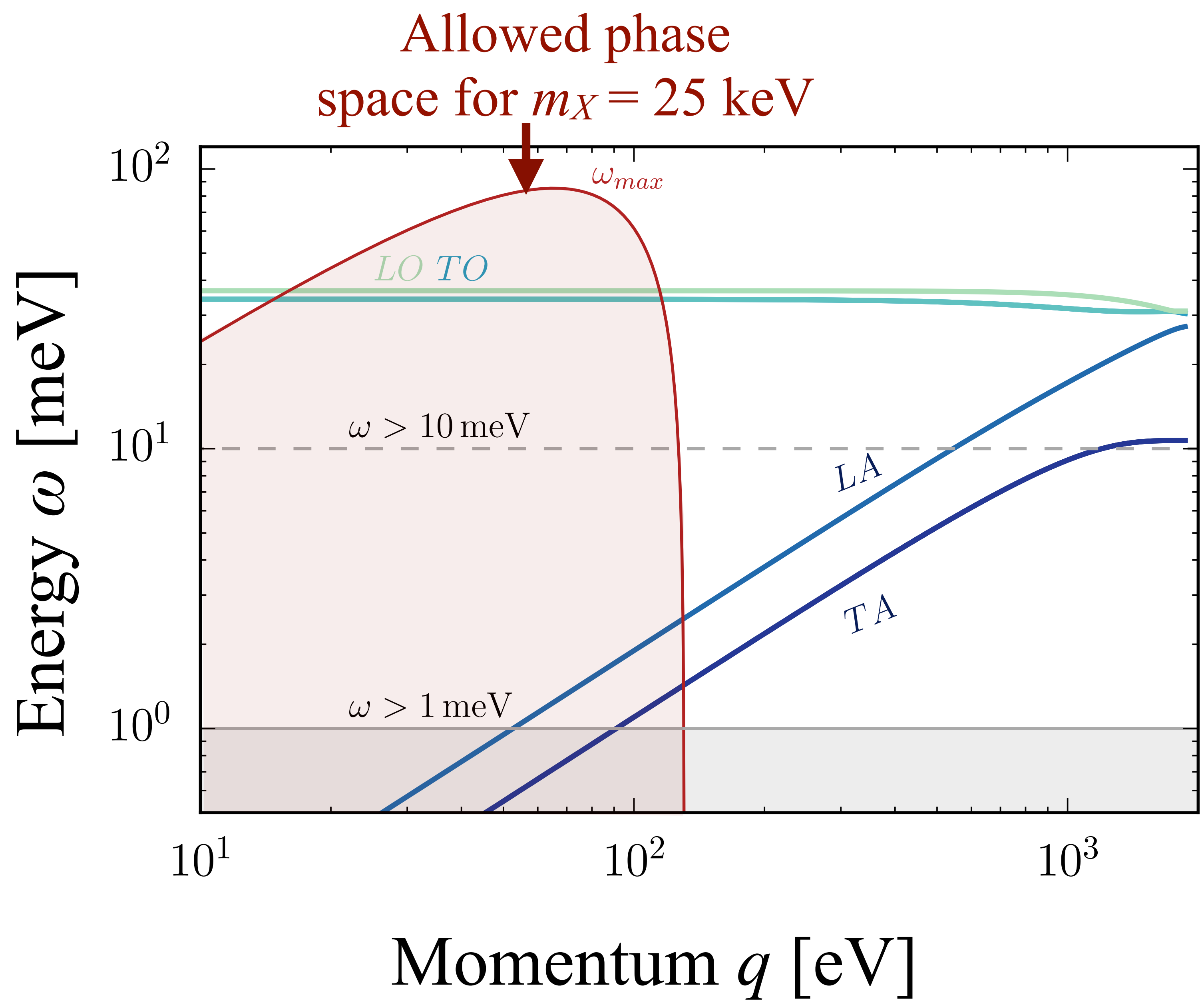
Single phonon excitations

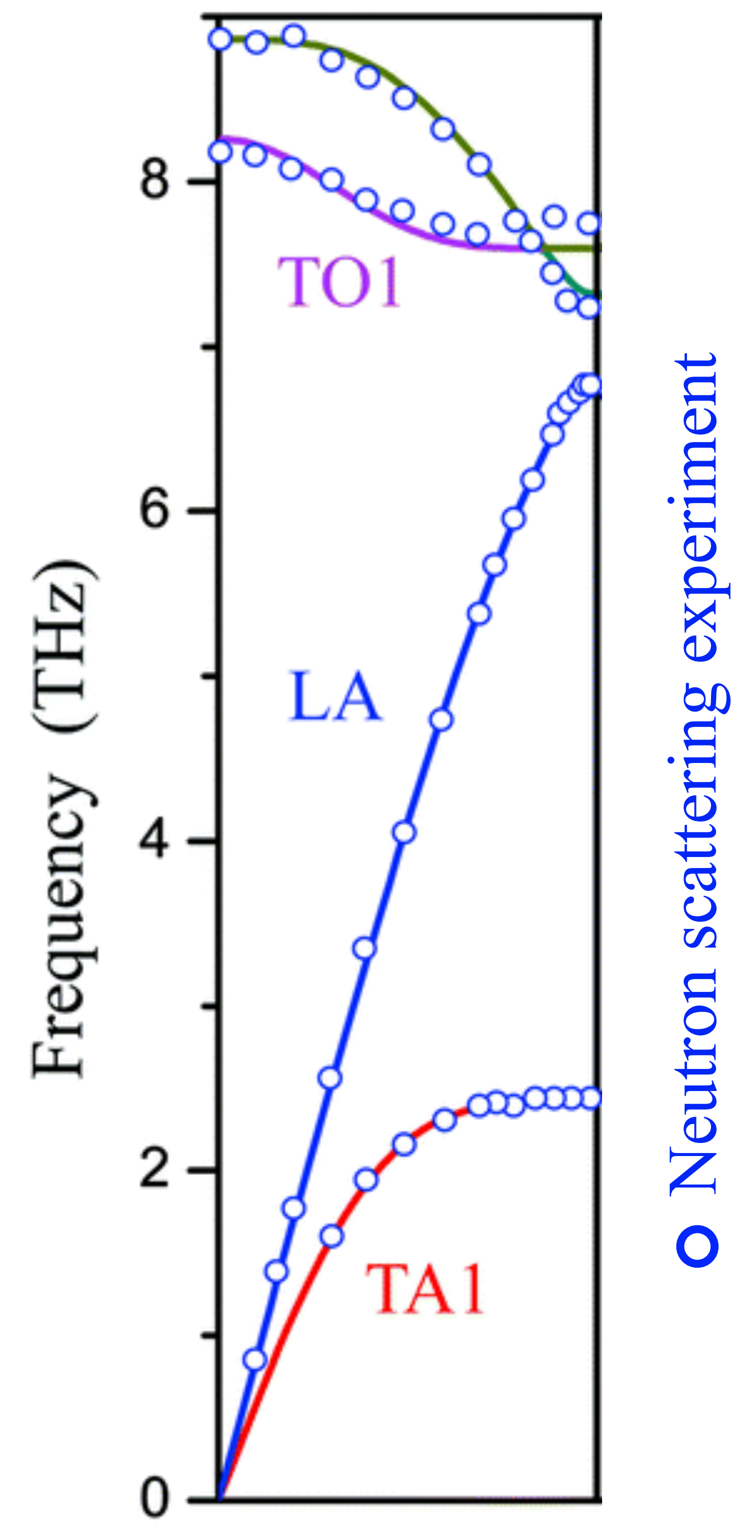


Resonant response of material at single-phonon dispersions (calculated with DFT)

Knapen, TL, Pyle, Zurek PRB 2018
Griffin, Knapen, TL, Zurek PRD 2020

Single phonon excitations



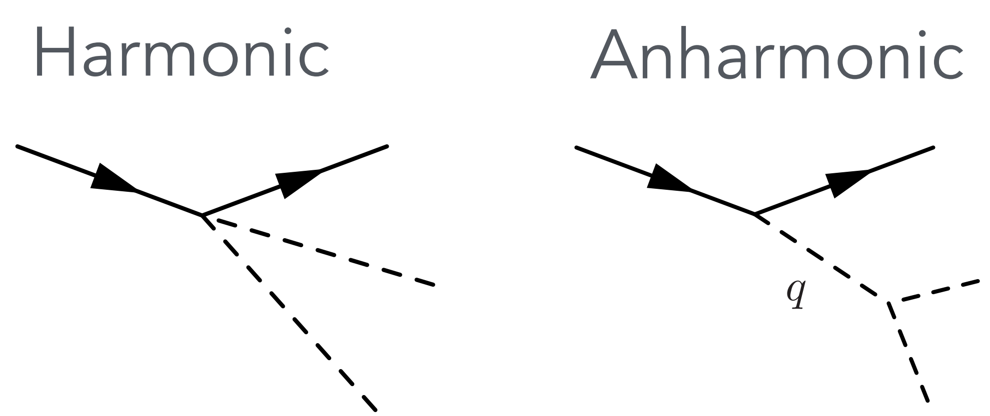


Knapen, TL, Pyle, Zurek PRB 2018
Griffin, Knapen, TL, Zurek PRD 2020

Going to larger momentum

Expansion in $q^2/(M_N\omega)$ (and anharmonic interactions):

$$S(\mathbf{q},\omega) = \text{(0-phonon)} + \text{(1-phonon)} + \text{(2-phonon)} + \cdots$$



Quickly becomes more complicated to evaluate for more than 1 phonon

Our approach: use harmonic & incoherent approximations

Incoherent approximation for $q > q_{\mathrm{BZ}}$ or $n > 1$ phonons

Neglect interference terms entirely:

$$S(\mathbf{q},\omega) = \frac{1}{V}\sum_{J,\,J'}^{N} f_J f_{J'}^{*} \int\limits_{-\infty}^{\infty} dt\, \langle e^{-i\mathbf{q}\cdot\mathbf{r}_{J'}(0)} e^{i\mathbf{q}\cdot\mathbf{r}_J(t)}\rangle e^{-i\omega t}$$
$$\approx \frac{1}{V}\sum_{J}^{N}(f_J)^2 \int\limits_{-\infty}^{\infty} dt\, \langle e^{-i\mathbf{q}\cdot\mathbf{u}_J(0)} e^{i\mathbf{q}\cdot\mathbf{u}_J(t)}\rangle e^{-i\omega t}$$

Given in terms of auto-correlation function

Motivation for $q > q_{\mathrm{BZ}}$: scatter off individual nuclei at large q

Motivation for $n > 1$: momentum gets distributed over multiple phonons, and the motions of individual atoms will be less correlated.

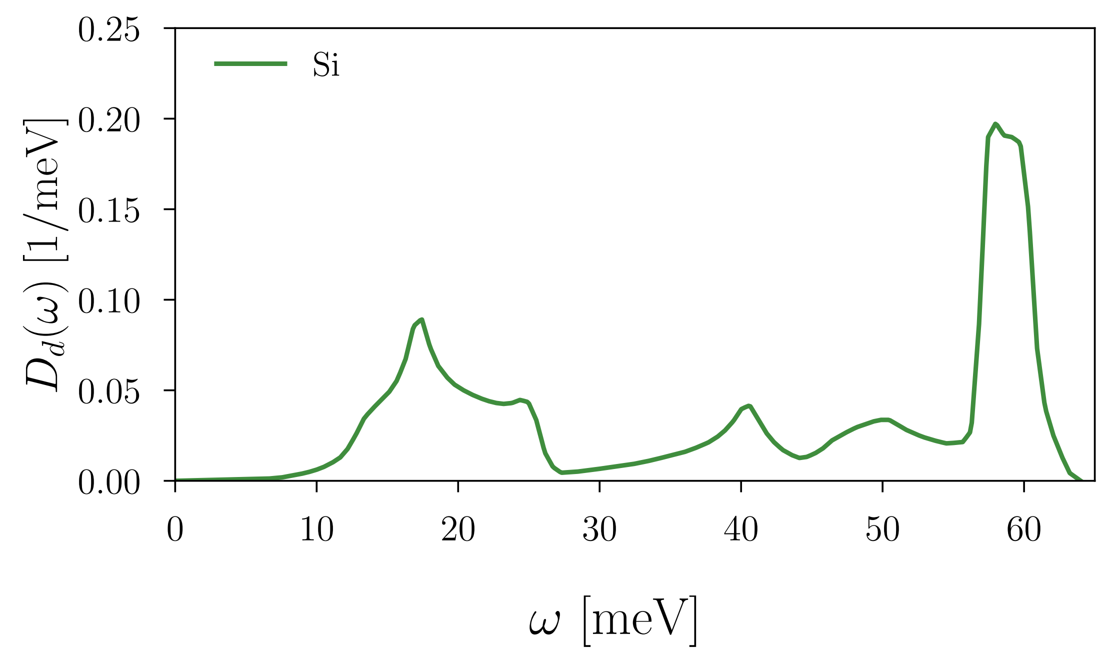
Harmonic approximation

Auto-correlation can be approximated using the phonon density of states:

$$\mathbf{u}_J(t) \sim \sum_{\mathbf{q}} \frac{1}{\sqrt{2m_N\omega_{\mathbf{q}}}} \left(\hat{a}^\dagger_{\mathbf{q}}\mathbf{e}_{\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{r}_J + i\omega_{\mathbf{q}}t} + \text{h.c.}\right)$$

$$\langle \mathbf{q}\cdot\mathbf{u}_J(0)\mathbf{q}\cdot\mathbf{u}_J(t)\rangle \approx \frac{q^2}{2m_N}\int d\omega' \frac{D(\omega')}{\omega'} e^{i\omega' t}$$

In the harmonic, isotropic limit



$$S(\mathbf{q},\omega) = \sum_d \frac{f_d^2}{\Omega} e^{-2W_d(\mathbf{q})} \int_{-\infty}^{\infty} dt\, e^{\frac{q^2}{2m_d}\int d\omega' \frac{D_d(\omega')}{\omega'} e^{i\omega' t}} e^{-i\omega t}$$

Debye-Waller factor $\quad W_d(\mathbf{q}) = \frac{q^2}{4m_d}\int d\omega' \frac{D_d(\omega')}{\omega'}$

Phonon expansion

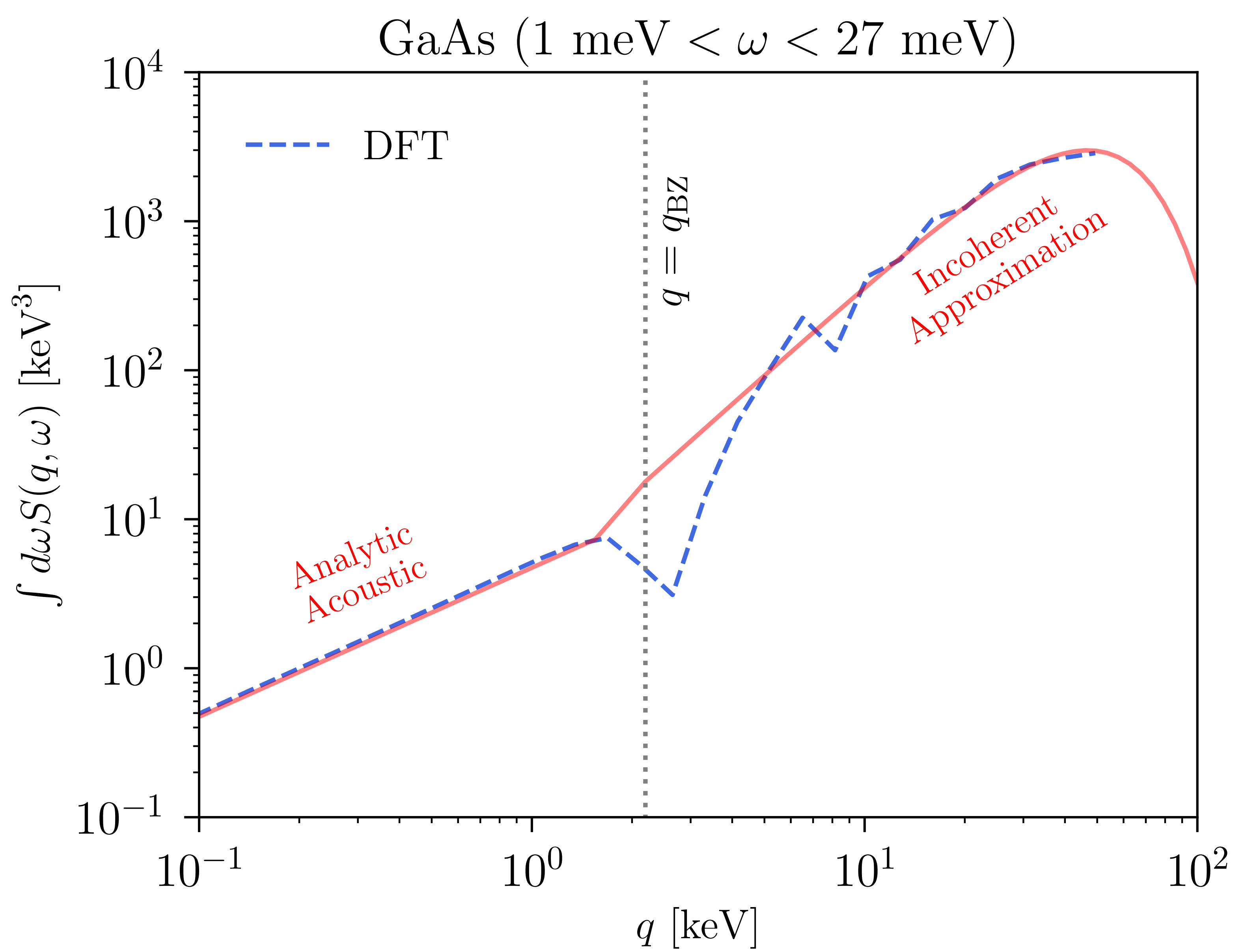
Expansion of structure factor in powers of q:

$$S(q,\omega) \propto \sum_J e^{-2W_J(q)} (f_J)^2 \sum_n \frac{1}{n!} \left(\frac{q^2}{2m_N}\right)^n \left(\prod_{i=1}^{n} \int d\omega_i \frac{D(\omega_i)}{\omega_i}\right) \delta\left(\sum_j \omega_j - \omega\right)$$

$$\sim \left(\frac{q^2}{2m_N \bar{\omega}_{\rm ph}}\right)^n$$

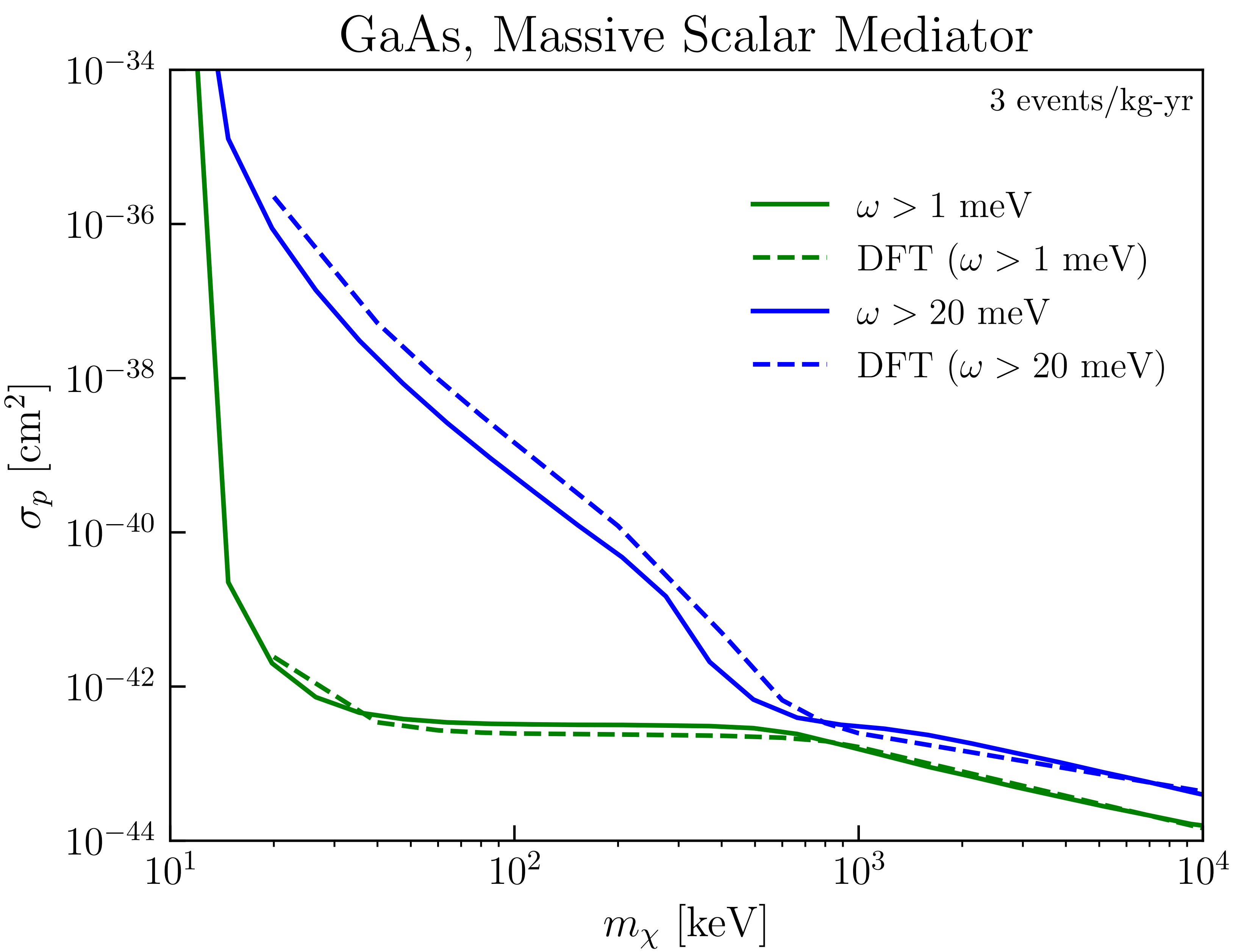
$q \approx \sqrt{2m_N \bar{\omega}_{\rm ph}}$ for many phonons to contribute

Comparison with full (DFT) calculation for n=1 phonon

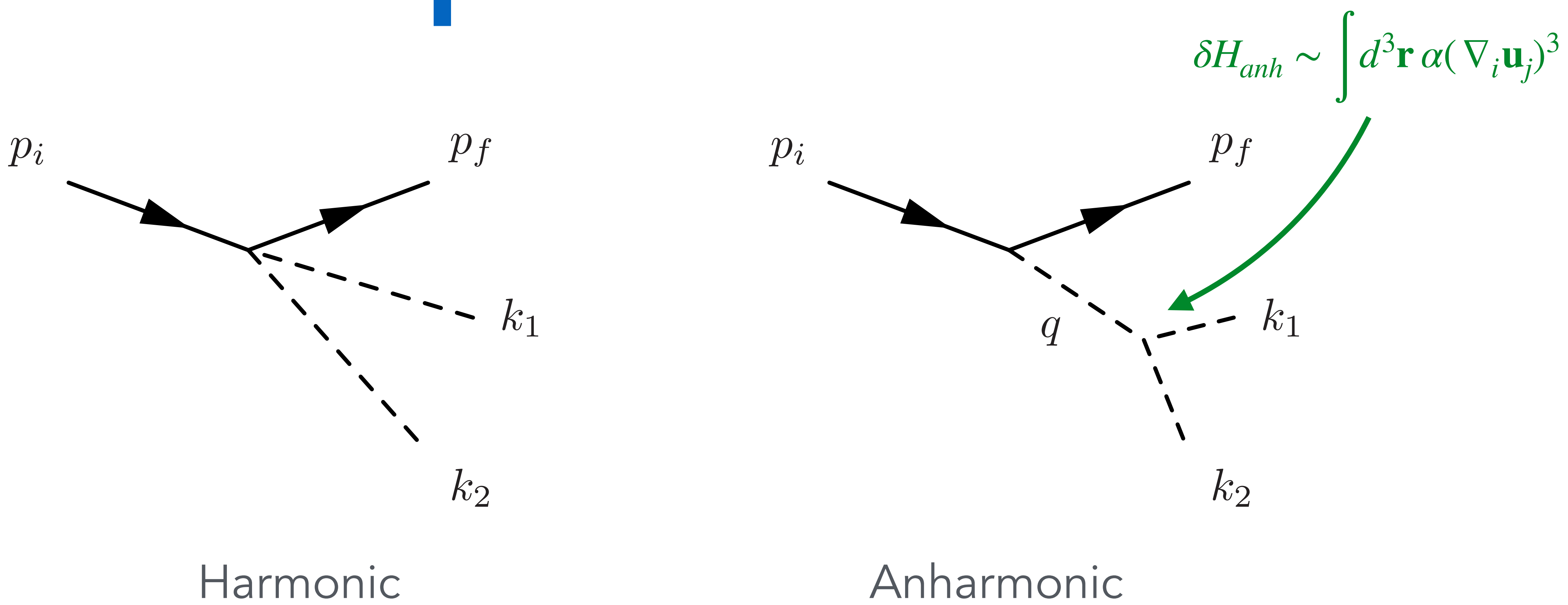


Incoherent approximation captures integrated structure factor

Comparison with full (DFT) calculation for n=1 phonon



2 phonons

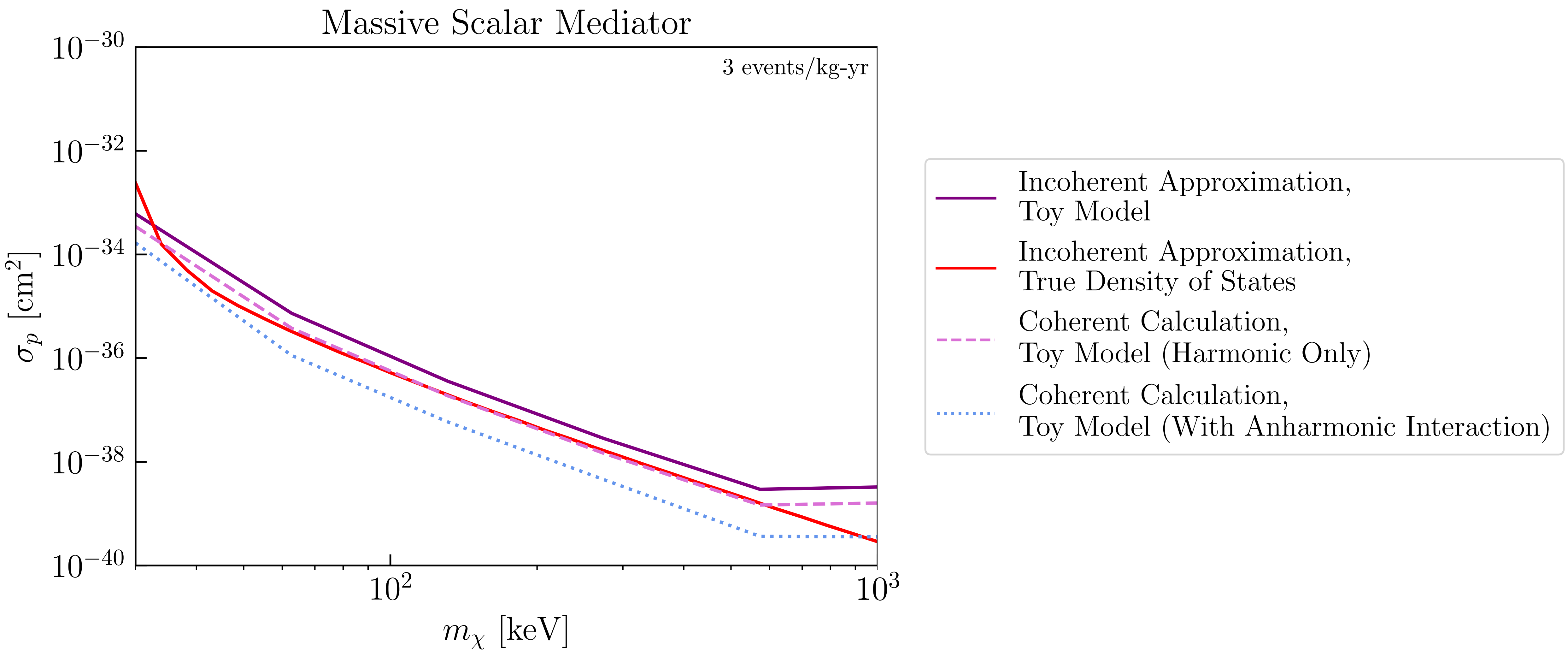


Calculated with long-wavelength ($q \ll q_{\rm BZ}$) approximation in crystals

Campbell-Deem, Cox, Knapen, TL, Melia 1911.03482

How does this EFT approach compare with the incoherent + harmonic approximation?

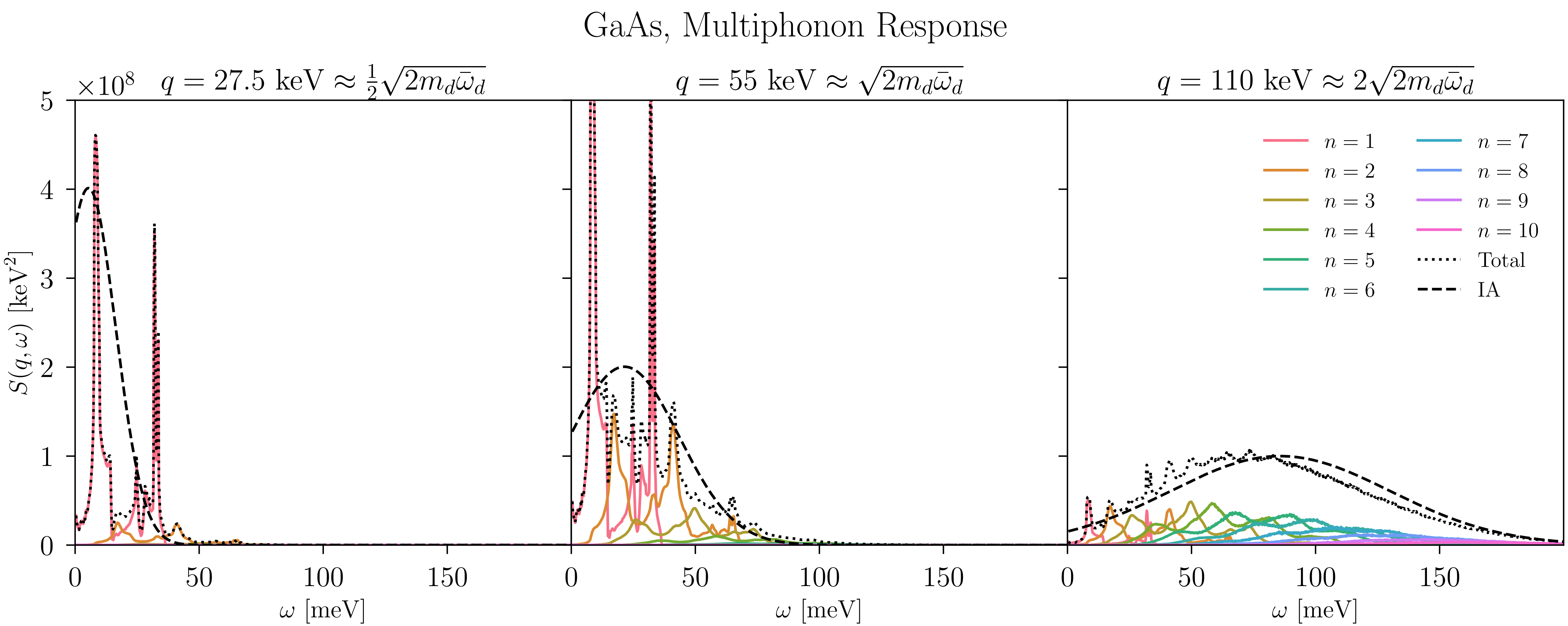
GaAs 2-phonon, ($\omega > 40$ meV)



Incoherent + harmonic approximation works to within a factor of few for $q < q_{\rm BZ}$, comparing to harmonic crystal result. The difference is larger if anharmonic interactions are included.

This should work better with higher q and n.

Multiphonons become important around $q = \sqrt{2m_N\bar{\omega}_{ph}}$



$q = \frac{1}{2}\sqrt{2m_N\bar{\omega}_{ph}}$: dominated by n=1 phonon	$q = \sqrt{2m_N\bar{\omega}_{ph}}$: contributions from n=1,2,3,4…	$q = 2\sqrt{2m_N\bar{\omega}_{ph}}$: can be approximated by Gaussian envelope (Impulse Approximation)

Impulse approximation

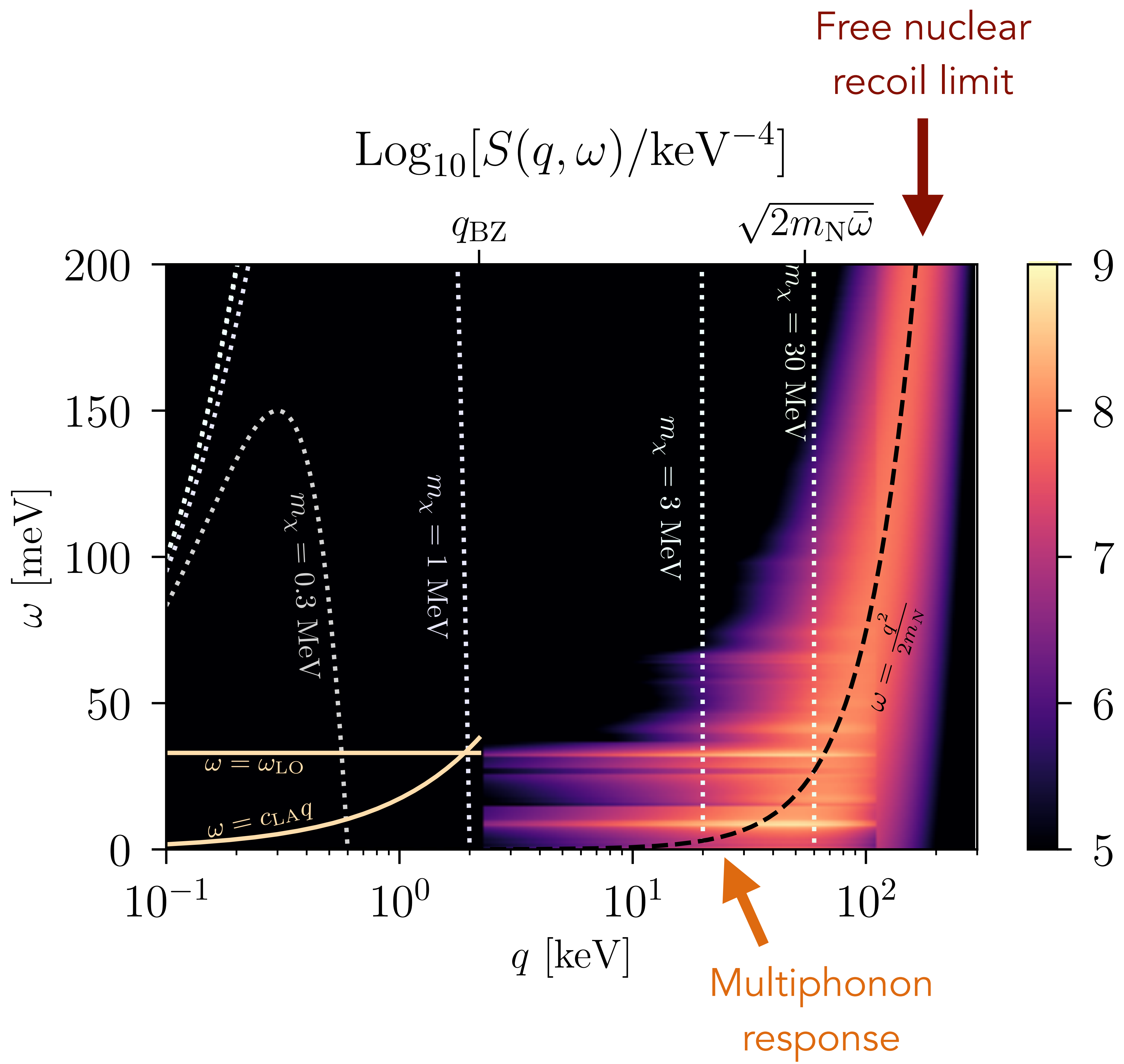
When $q \gg \sqrt{2m_N\bar{\omega}_{\text{ph}}}$, "re-sum" the n-phonon contributions and directly evaluate by stationary phase approximation:

$$S(\mathbf{q},\omega) = \sum_d \frac{f_d^2}{\Omega} e^{-2W_d(\mathbf{q})} \int_{-\infty}^{\infty} dt\, e^{\frac{q^2}{2m_d}\int d\omega' \frac{D_d(\omega')}{\omega'} e^{i\omega' t}} e^{-i\omega t}$$

Peaked about free nuclear recoil dispersion as $\omega \gg \bar{\omega}_{\text{ph}}$:

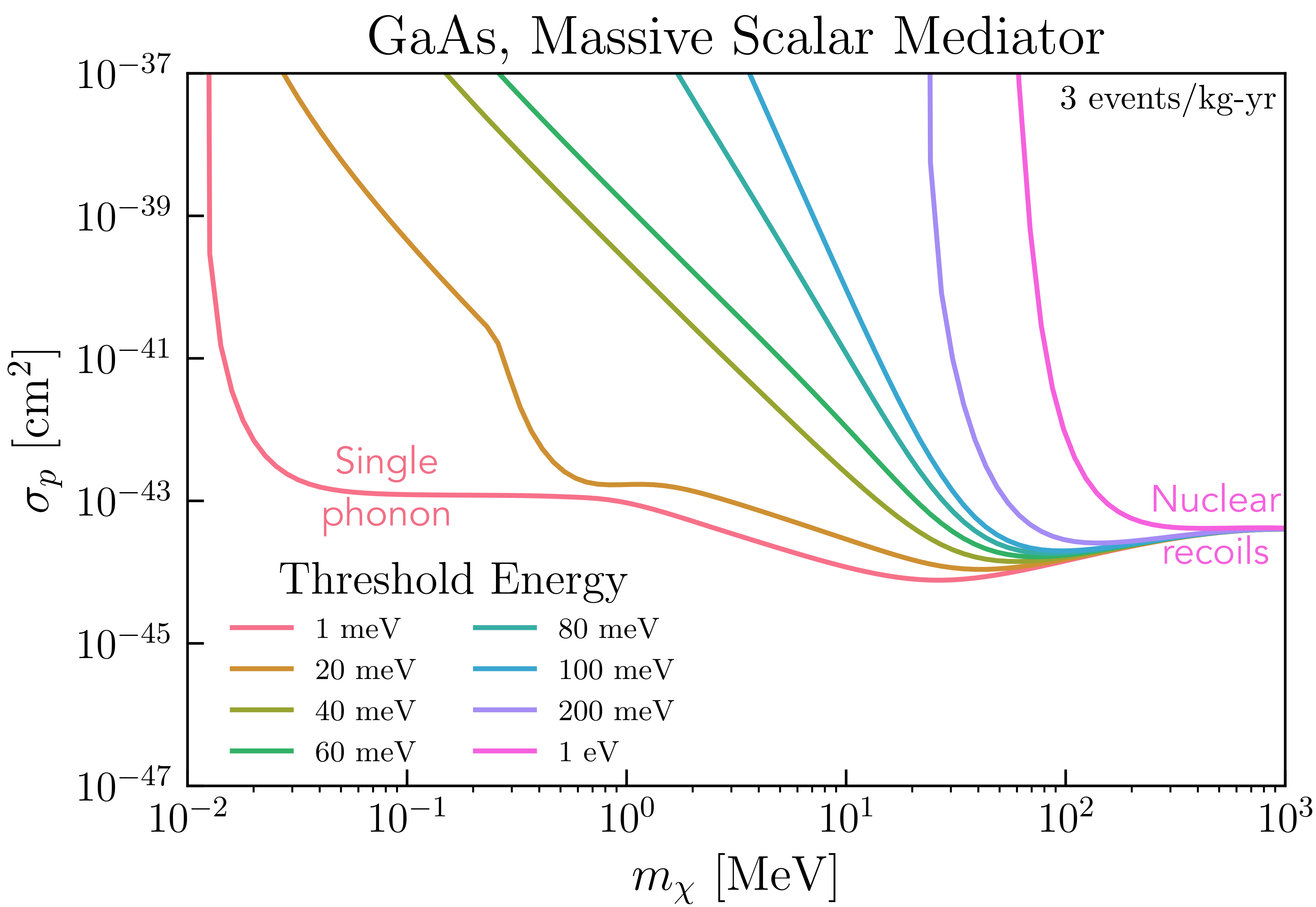
$$S^{\text{IA}}(q,\omega) \propto \sum_J f_J^2 \sqrt{\frac{2\pi}{\Delta^2}} \exp\left(-\frac{(\omega - \frac{q^2}{2m_N})^2}{2\Delta^2}\right), \quad \Delta^2 = \frac{q^2\bar{\omega}_{\text{ph}}}{2m_N}$$

With $\Delta/\omega \to 0$ in the high energy limit



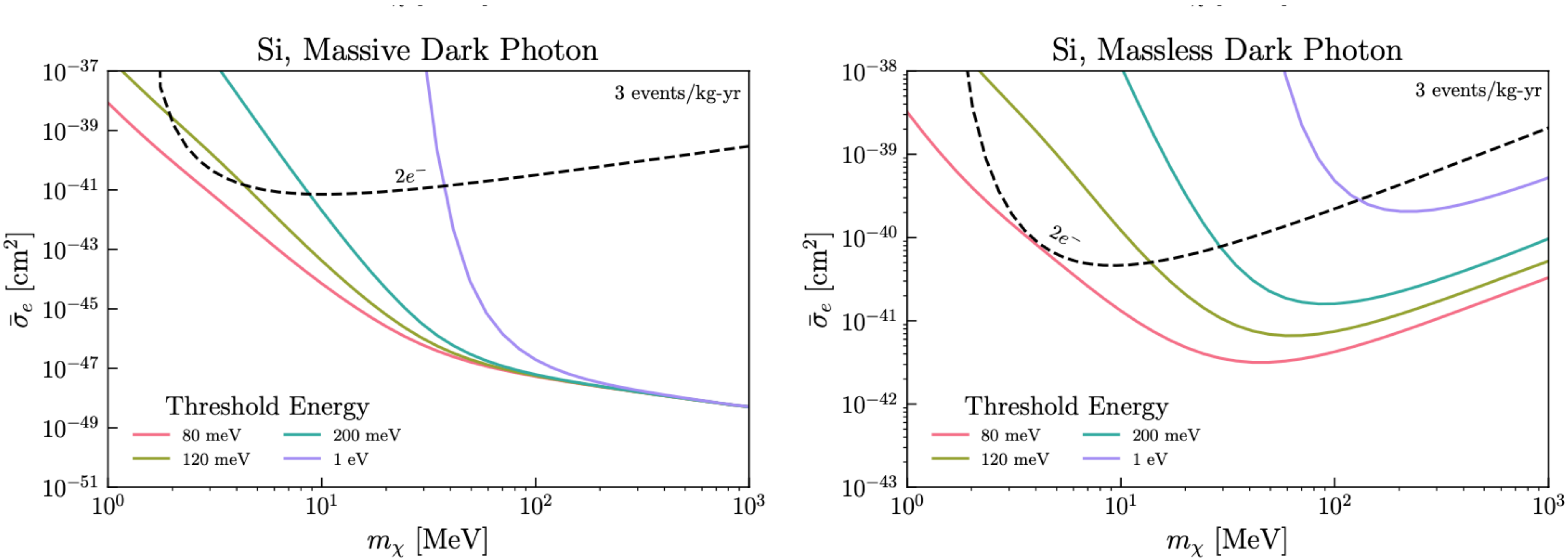
Campbell-Deem, Knapen, TL, Villarama 2205.02250

DM scattering rate



Campbell-Deem, Knapen, TL, Villarama 2205.02250
Harmonic oscillator model: Kahn, Krnjaic, Mandava 2011.09477

Dark photon mediator



Coupling given by *q*-dependent effective charge *Z(q)*

Single phonon reach estimated by dielectric response or directly computed in DFT

Future steps

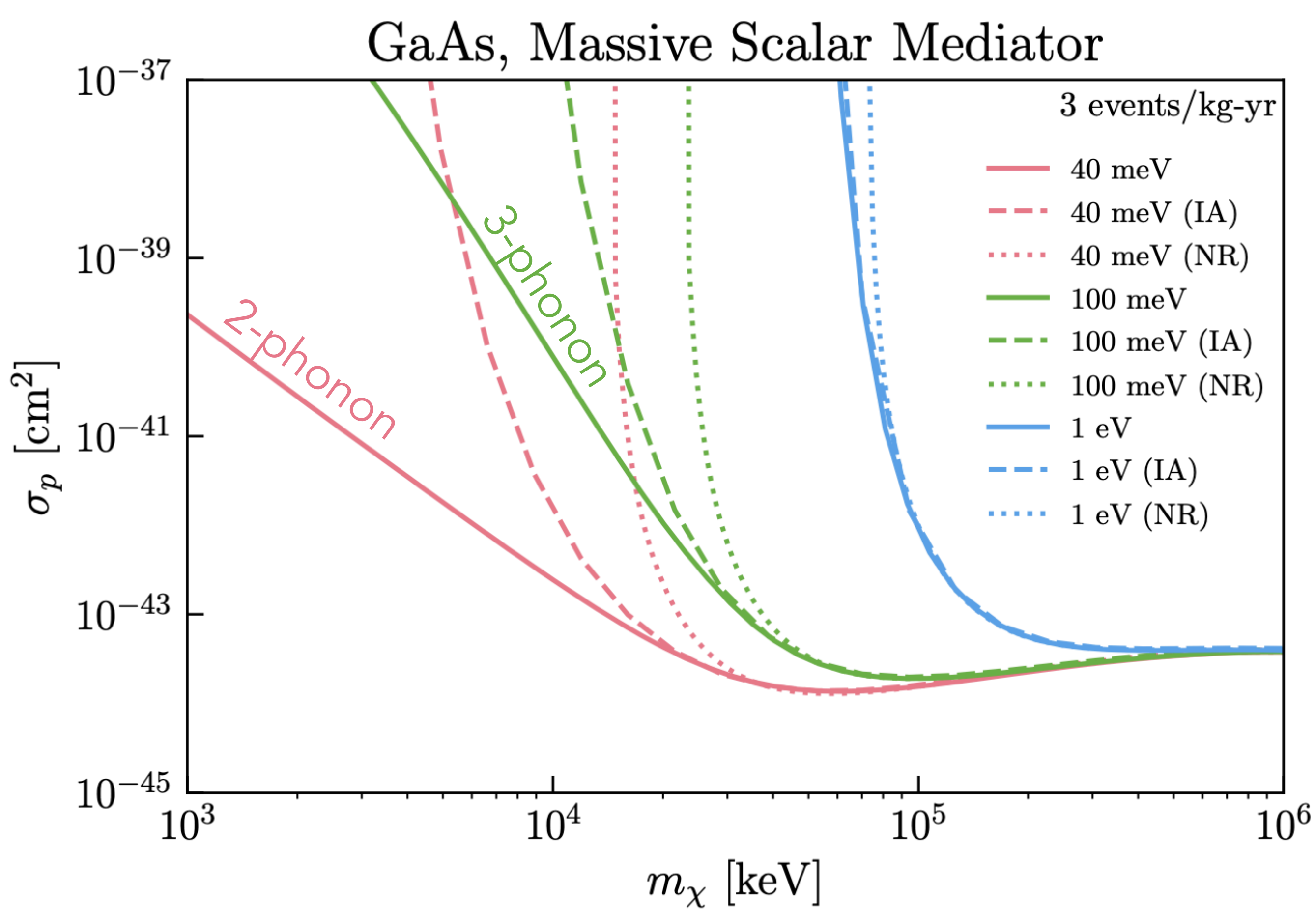
Pinning down $S(q, \omega)$:

Quantify theoretical uncertainties and validity of approximations (harmonic approximation)

Detailed look at two (or three) phonon rates

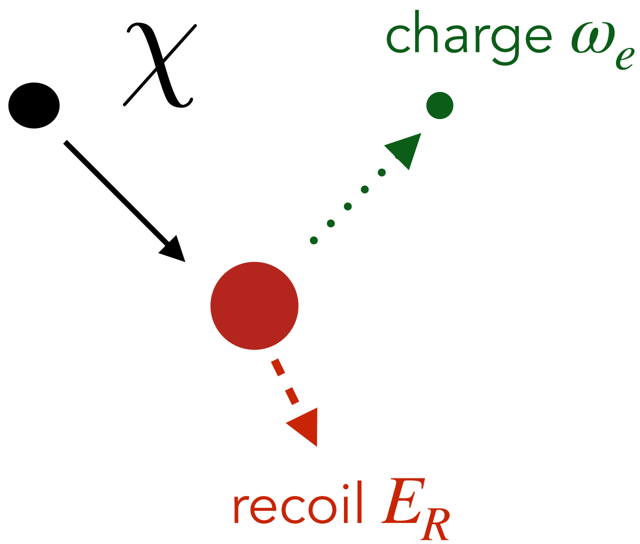
Experimental calibration?

Above eV scale, rates pretty quickly converge to the impulse approximation, nuclear recoils



Migdal effect

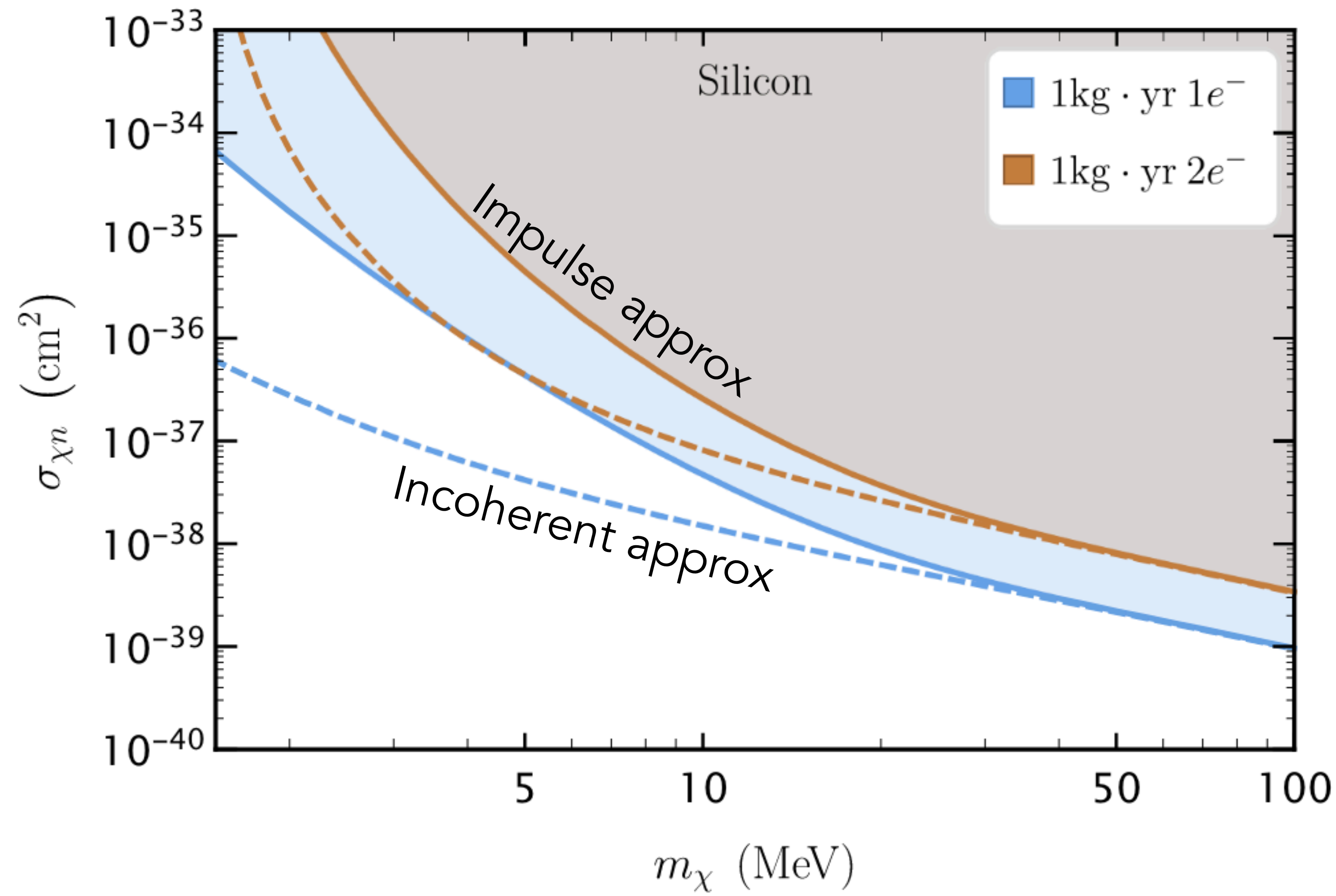
DM-nucleus scattering with charge emission



$$\frac{d\sigma}{dE_R d\omega_e} \approx \frac{d\sigma_N}{dE_R}\frac{dP}{d\omega_e}$$

↑ DM-nucleus scattering

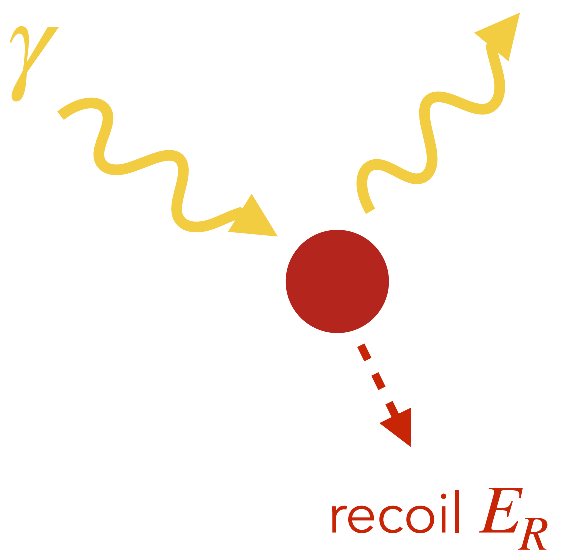
↑ Probability for charge excitation



From Liang, Mo, Zheng, Zhang 2205.03395
Knapen, Kozaczuk, Lin 2011.09496

Backgrounds

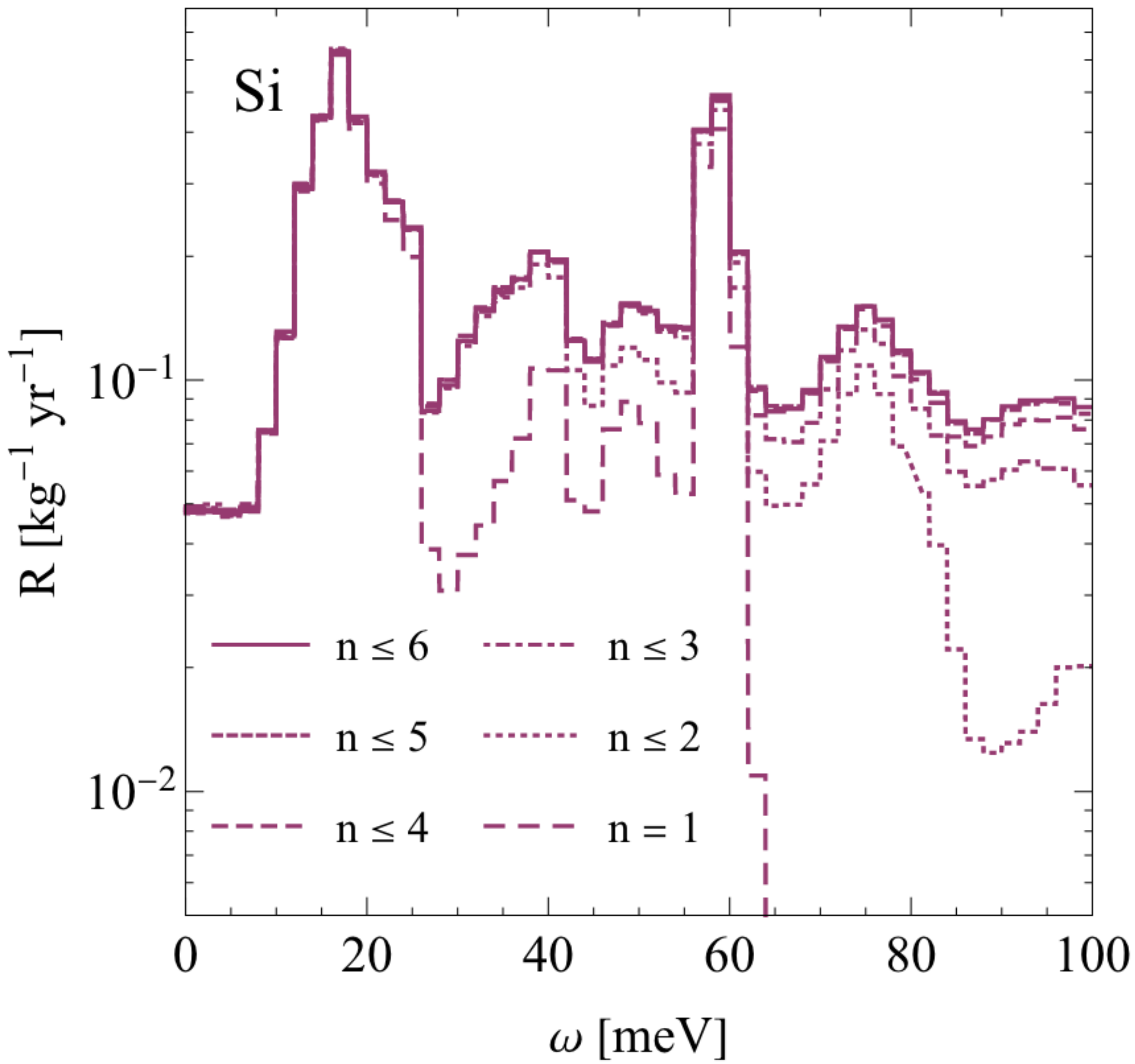
Coherent scattering of high energy (~MeV) photons off ions



$$\frac{d\sigma}{d\Omega d\omega} = \frac{d\sigma}{d\Omega}(\mathbf{q}, E_\gamma)\, S(\mathbf{q}, \omega)$$

Coherent ion scattering

Structure factor for nuclear recoils



A. Robinson 1610.07656

Figure from Berghaus, Essig, Hochberg, Shoji, Sholapurkar 2112.09702

DM scattering in crystals