LEX-EFT
Light Exotics Effective Field Theory

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General Methods for Model Agnostic Collider Phenomenology

Some Examples

- Machine Learning-Anomaly Detection
- SMEFT- EFT for SM operators, new physics offshell

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On Shell General EFT for Light Exotics LEX-EFT

Given a set of new states indexed by quantum numbers corresponding to symmetries, write all interactions with SM up to given dimension

- LEX-EFT offers a complete list of all possible interactions between light exotics and the Standard Model up to the desired order in effective cut-off (mass dimension). It is thus a guide for bSM precision and collider searches, it allows for the analysis of new event topologies, and it offers a comprehensive map of event kinematics without the burden of specifying UV-complete models.

- A complete LEX-EFT catalog would subsume other classes of exotic bSM models including supersymmetry, exotic Higgs models, and dark matter EFTs. Such a complete catalog may illuminate new interactions in these theories and thus new phenomenological channels for study.

- The LEX-EFT catalog would also bring to theoretical consideration bSM states that have not received model-building attention. It would thus cast a wider net over all of theory space. As we imagine the LEX-EFT approach would be closely followed up by a simplified model building approach, this would spark new theoretical innovation.

LEX Operators

$$\frac{1}{\Lambda^d}\,\lambda\,(\Phi_1\Phi_2\ldots)(\psi_1\psi_2\ldots)$$

SM fields

BSM fields

Effective cut off

operator dimension = d+4

Advantages of On-shell EFT

- *Kinematics and collider cross sections*: Using unique LEX operators allows one to keep track of process kinematics, which are vital in constructing collider searches for new physics. It also allows for the accurate computation of collider production cross sections, scaled by the relevant effective operator coefficients, up to the validity limits of the EFT. This allows full consideration of all processes involving production and decay of exotic states in collider searches.
- *Charge flow and validity of parameter space*: Constructing effective operators that are singlets under all gauge groups requires specification of the Clebsch-Gordan coefficients in operators linking light exotic fields to the SM. For any given set of fields, there may be multiple ways to perform charge contractions. Each of these contractions then corresponds to a unique operator, which gives a picture of the charge flow of the process involved. There may be naturally large coefficients associated with some operators, which drastically affects predictions of production cross sections in the theory. Moreover, we find that the range of validity of an effective operator may vary widely based on choice of charge contraction, even if the fields involved in the operators are the same.

Complementarity to Off-Shell EFT

- *Operator correlations*: A theory containing a specific LEX state has operators that may have correlations based on gauge invariance or other theoretical considerations. This approach works even with LEX states that are totally off-shell. The operator catalog for off-shell states leads to a specific list of correlated SMEFT operators that could be measured once the bSM states are integrated out.

- *Precision measurements at loop level*: Specifying the light exotic state appearing in a theory facilitates the computation of precision quantities such electroweak oblique parameters, lepton anomalous magnetic dipole moments, $b \to s\gamma$, etc., which may not be obvious from other operator catalogs.

Charge Flow: Constructing Singlets

Construct Charge Singlet Operators Under All gauge and global symmetry groups

SM :SU(3), SU(2), U(1) , BSM: U(1)′, SU(2)$_{R}$, etc.
Global:SU(N) flavor etc.

Fields are in representations $\mathbf{r}_i$ of a group

$$\frac{1}{\Lambda^d}\lambda\,(\Phi_1\Phi_2\ldots)(\psi_1\psi_2\ldots)$$

$\mathbf{r}_1$ $\mathbf{r}_2$

$$\mathbf{r}_1 \otimes \mathbf{r}_2 \otimes \cdots\cdot = 1$$

singlet

Iterative Construction of Singlets

Method for constructing group theory invariants from basic 2 field tensor product relations

$$\mathbf{r}_1 \otimes \mathbf{r}_2 = \mathbf{q}_1 \oplus \mathbf{q}_2 \oplus \ldots .$$

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example from SU(3)

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

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Method for constructing group theory invariants from basic 2 field tensor product relations

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example from SU(3)

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

quark

anti-quark

Higgs, photon, etc

gluon

Observation. If there exist invariant combinations of $n+1$ and $m+1$ fields transforming in the direct product representations $\mathbf{r}_1 \otimes \cdots \otimes \mathbf{r}_n \otimes \mathbf{p}$ and $\mathbf{q}_1 \otimes \cdots \otimes \mathbf{q}_m \otimes \mathbf{p}$ of a group, then there exists an invariant combination of $n+m$ fields in the reducible representation $\mathbf{r}_1 \otimes \cdots \otimes \mathbf{r}_n \otimes \bar{\mathbf{q}}_1 \otimes \cdots \otimes \bar{\mathbf{q}}_m$

If we consider the tensor product identities

$$\mathbf{a} \otimes \mathbf{b} \supset \mathbf{e} \quad \text{and} \quad \mathbf{c} \otimes \mathbf{d} \supset \mathbf{e}$$

We can construct the invariants

$$\mathbf{a} \otimes \mathbf{b} \otimes \bar{\mathbf{e}} \quad \text{and} \quad \mathbf{c} \otimes \mathbf{d} \otimes \bar{\mathbf{e}}$$

Taking the example example from SU(3)

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If we consider the two field tensor product identities

$$\mathbf{a} \otimes \mathbf{b} \supset \mathbf{e} \quad \text{and} \quad \mathbf{c} \otimes \mathbf{d} \supset \mathbf{e}$$

We can construct the three field invariants

$$\mathbf{a} \otimes \mathbf{b} \otimes \bar{\mathbf{e}} \quad \text{and} \quad \mathbf{c} \otimes \mathbf{d} \otimes \bar{\mathbf{e}}$$

Taking the example example from SU(3)

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{8}$$

Thus using the 2 field tensor products

$$\mathbf{a} \otimes \mathbf{b} \supset \mathbf{e} \text{ and } \mathbf{c} \otimes \mathbf{d} \supset \mathbf{e}$$

We can construct the four field invariant

$$\mathbf{a} \otimes \mathbf{b} \otimes \bar{\mathbf{c}} \otimes \bar{\mathbf{d}} \supset \mathbf{1}$$

Iterating this process we can construct invariants with any number of fields and thus build the operator catalog to the desired dimension in the effective cut off

Constructing 4 field invariants

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}}_{\rm a} \oplus \mathbf{6}_{\rm s},$$
$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8},$$
$$\mathbf{6} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{10},$$
$$\mathbf{6} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \mathbf{15},$$
$$\mathbf{6} \otimes \mathbf{6} = \bar{\mathbf{6}}_{\rm s} \oplus \mathbf{15}_{\rm a} \oplus \mathbf{15}'_{\rm s},$$
$$\mathbf{6} \otimes \bar{\mathbf{6}} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27},$$
$$\mathbf{8} \otimes \mathbf{3} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15},$$
$$\mathbf{8} \otimes \bar{\mathbf{6}} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15} \oplus \mathbf{24},$$
$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1}_{\rm s} \oplus \mathbf{8}_{\rm s} \oplus \mathbf{8}_{\rm a} \oplus \mathbf{10}_{\rm a} \oplus \overline{\mathbf{10}}_{\rm a} \oplus \mathbf{27}_{\rm s}$$

By iterating tensor products
We construct new invariant

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} = \mathbf{6} \otimes \mathbf{3}$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{6}}$$

With coefficient

$$[t^a_3]_j^{\ i}\, \bar{J}_{s\,ak}$$

Fundamentals contacted into 8

6-3-8 contraction

Within an N field invariant, sub-products of fields are in an intermediate representation

$$\underbrace{\left[\mathbf{r}_{\text{LEX}} \otimes \mathbf{r}_i \otimes \mathbf{r}_j \otimes \cdots \otimes \mathbf{r}_k\right]_{\mathbf{r}'}}_{N-n} \otimes \underbrace{\left[\mathbf{r}_l \otimes \cdots \otimes \mathbf{r}_m\right]_{\bar{\mathbf{r}}'}}_{n}$$

$$\text{with} \quad 1 \leq i, j, \ldots, m \leq M-1$$

With the remaining sub-product in the complementary representation to produce a singlet

There may be more than one way to construct the singlet out of the specified fields, there may be multiple ways to construct the intermediate representations

We now argue from induction. To build three-field invariants involving a LEX field, we need only consider the m possible bilinear tensor products of the LEX state with other representations allowed in the theory, $[\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i]_{\mathbf{r}'_j}$, to obtain the finite list of irreducible representations $\mathbf{r}'$ in the direct product. If any single field in the theory is in the conjugate representation $\bar{\mathbf{r}}'_j$, then we can directly contract indices to form an invariant:

$$[\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i]_{\mathbf{r}'_j} \otimes \bar{\mathbf{r}}'_j$$

With a list in hand of all m possible bilinear products $\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i$ in representations $\mathbf{r}'_j$, we can proceed to construct the four-field invariants. We find the direct products of the allowed representations $\mathbf{r}_k \otimes \mathbf{r}_l$ that are in a given conjugate representation $\bar{\mathbf{r}}'_j$ and contract these fields according to

$$[\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i]_{\mathbf{r}'_j} \otimes [\mathbf{r}_k \otimes \mathbf{r}_l]_{\bar{\mathbf{r}}'_j}$$

to obtain singlets. To proceed to five fields, we now consider all possible trilinear products of the form $\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i \otimes \mathbf{r}_j$. We note we have already found by exhaustion the representations of bilinear products of the first two fields in the previous step. In that step, the bilinears were in representations $\mathbf{r}'_j$ such that $\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i \supset \mathbf{r}'_j$. We can thus iterate the bilinear tensor products $\mathbf{r}'_j \otimes \mathbf{r}_j \supset \mathbf{r}'_k$ to find the representations $\mathbf{r}'_k$ of all trilinear products. We then find the remaining bilinear representations $\mathbf{r}_k \otimes \mathbf{r}_l$ that are in the conjugate representation $\bar{\mathbf{r}}'_k$ and contract *these* fields to form the five-field invariant. This process can be repeated indefinitely and will ultimately produce all possible terms — we only need to know the list of bilinear tensor products that involve relevant SM/LEX fields and the intermediate representations $\mathbf{r}'_j$, $\mathbf{r}'_k$, and so on.

Example SU(3) invariant

$$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$$

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quark

quark

LEX sextet

gluon

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$$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$$

quark

quark

LEX
sextet

gluon

$$\mathcal{L}_{\Phi qqg} \supset \frac{1}{\Lambda^2}\,\lambda^{IJ}_{qq}\,\Pi_s^{\;aij}\,\varphi^{\dagger s}\,(\overline{q^{c}_{\mathrm{R}}}_{Ii}\,\sigma^{\mu\nu}q_{\mathrm{R}Jj})\,G_{\mu\nu\, a} + \mathrm{H.c}$$

LEX
sextet

quark

gluon

Example SU(3) invariant

$$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$$

quark — quark — LEX sextet — gluon

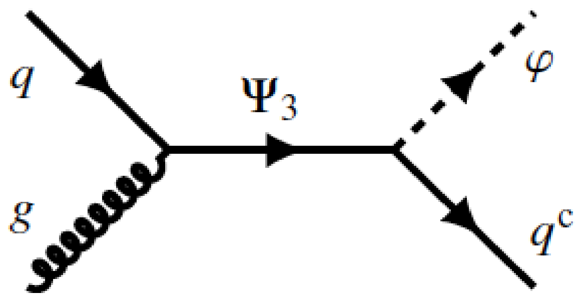
Clebsch-Gordoncoeff.

Lorentz

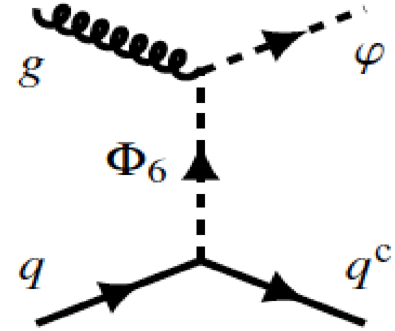
$$\mathcal{L}_{\Phi qqg} \supset \frac{1}{\Lambda^2} \lambda^{IJ}_{qq} \Pi_s^{a\,ij} \varphi^{\dagger s} (\overline{q^{c}_{\mathrm{R}}}_{Ii}\, \sigma^{\mu\nu} q_{\mathrm{R}Jj}) G_{\mu\nu\, a} + \mathrm{H.c}$$

LEX sextet — quark — gluon

$$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$$

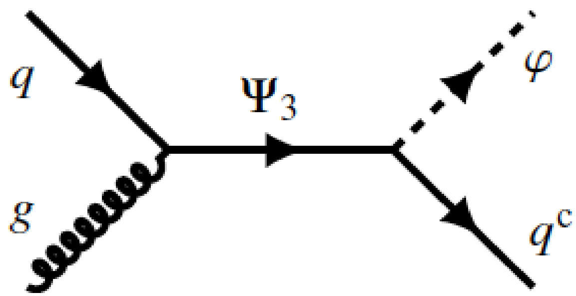


$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{8}$ and $\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}}$



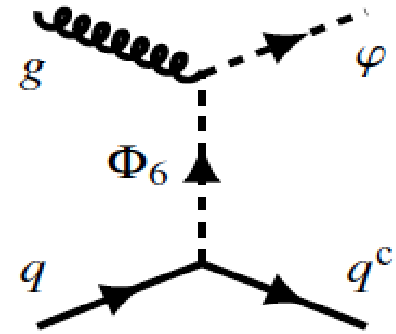
$\mathbf{6} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$ and $\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}}$.

$$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$$



$$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{8} \quad \text{and} \quad \mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}}$$

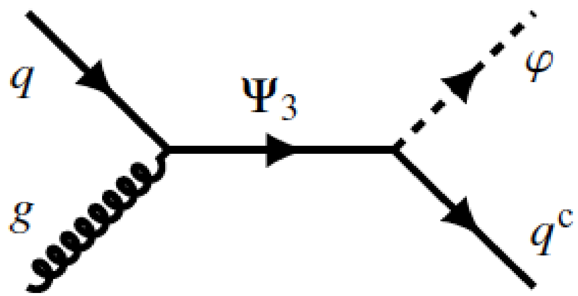
$$[\Pi_{\mathbf{3}}]_s{}^{a\,ij} = K_s{}^{ik}[t^a_{\mathbf{3}}]_k{}^j$$

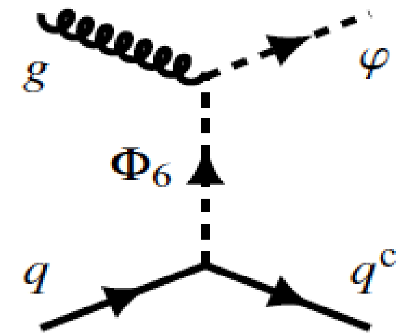


$$\mathbf{6} \otimes \bar{\mathbf{6}} \otimes \mathbf{8} \quad \text{and} \quad \mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}}.$$

$$[\Pi_{\mathbf{6}}]_s{}^{a\,ij} = K_r{}^{ij}[t^a_{\mathbf{6}}]_s{}^r$$

$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$





$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{8}$ and $\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}}$

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$$[\Pi_{\mathbf{3}}]_s^{\,a\,ij} = K_s^{\;ik}[t^a_{\mathbf{3}}]_k^{\;j}$$

$$[\Pi_{\mathbf{6}}]_s^{\,a\,ij} = K_r^{\;ij}[t^a_{\mathbf{6}}]_s^{\;r}$$

$$\sigma(qg \to \varphi q^c)$$

$$K_s^{\;ik}[t^a_{\mathbf{3}}]_k^{\;j}[t^a_{\mathbf{3}}]_j^{\;k'}\bar{K}^s_{\;ik'} = 8$$

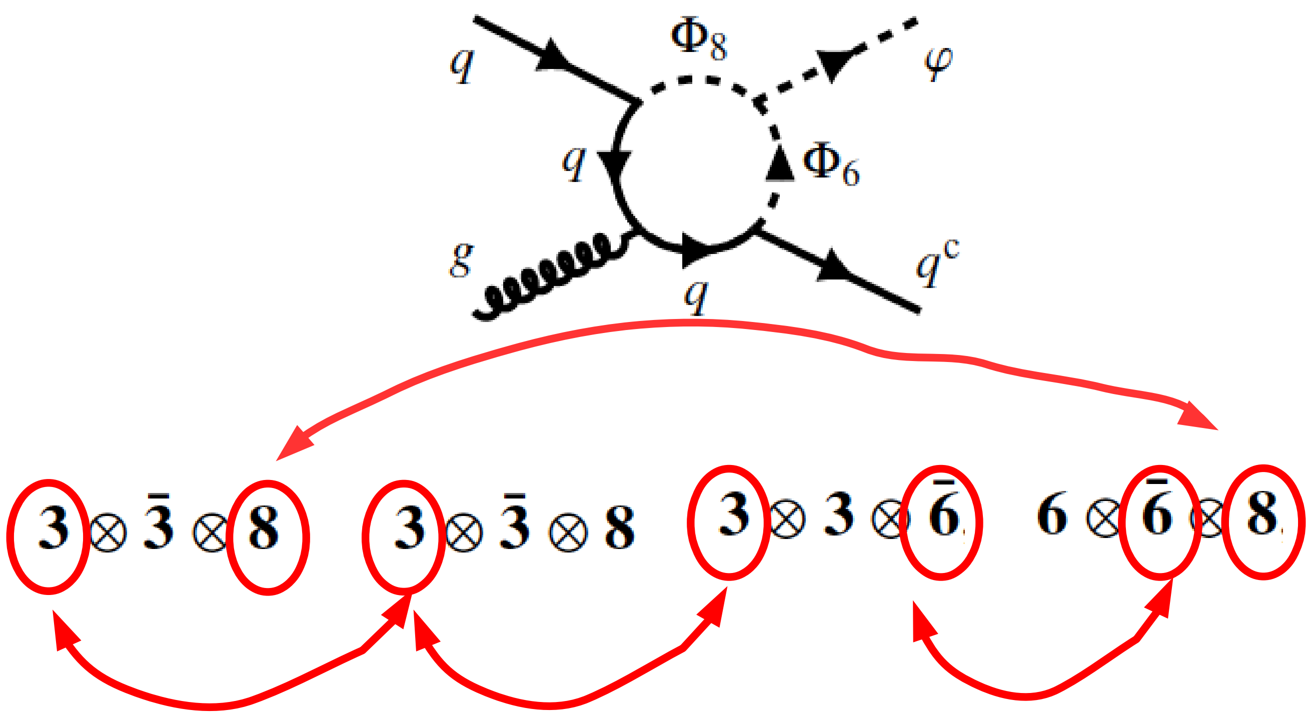
$$K_r^{\;ij}[t^a_{\mathbf{6}}]_s^{\;r}[t^a_{\mathbf{6}}]_{r'}^{\;s}\bar{K}^{r'}_{\;ij} = 20$$

$$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$$

$$[\Pi_{\text{loop}}]_s{}^{aij} = K_r{}^{jl}[t^a_{\mathbf{3}}]_l{}^k[t^b_{\mathbf{3}}]_k{}^i[t^b_{\mathbf{6}}]_s{}^r$$

$$\sigma(qg \to \varphi q^c)$$

$$[\Pi_{\text{loop}}]_s{}^{aij}[\bar{\Pi}_{\text{loop}}]^s{}_{aij} = \frac{20}{9}$$



Effect on EFT Validity from Unitarity

Consider the 2 to 2 process

$$qg \to \varphi q^{c}$$

With the perturbative unitarity bound

$$\Lambda \geq (\Pi_s{}^{a\,ij}\bar{\Pi}^s{}_{a\,ij})^{1/4}\left(\frac{\lambda_{qq}^{IJ}}{2\pi}\right)^{1/2}(\hat{s}-m_\varphi^2)^{1/2}$$

The validity limit on cut-offs between ssextet and loop

completions varies by a factor of

$$\left(\frac{[\Pi_{\mathbf{6}}]_s{}^{a\,ij}\,[\bar{\Pi}_{\mathbf{6}}]^s{}_{a\,ij}}{[\Pi_{\mathrm{loop}}]_s{}^{a\,ij}\,[\bar{\Pi}_{\mathrm{loop}}]^s{}_{a\,ij}}\right)^{1/4} = 9^{1/4} \approx 1.73$$

Production cross sections differ by a factor of 9

Kinematics

Consider the LEX spin 0 CP-even color sextet

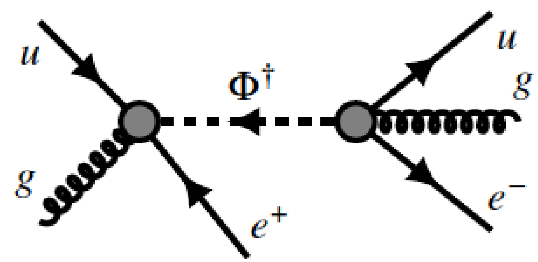
Field	$SU(3)_c \times SU(2)_L \times U(1)_Y$	B	L
Φ	$(\mathbf{6}, \mathbf{1}, \frac{1}{3})$	0	-1

Consider the LEX spin 0 CP-even color sextet

with interaction term

$$\mathcal{L}_{\Phi\ell^-} \supset \frac{1}{\Lambda^2} \lambda_{u\ell}^{IX} J^{s\,ia}\, \Phi_s \,(\overline{u^c_{\mathrm{R}Ii}}\, \sigma^{\mu\nu} \ell_{\mathrm{R}X})\, G_{\mu\nu\, a}$$

The collider production process is

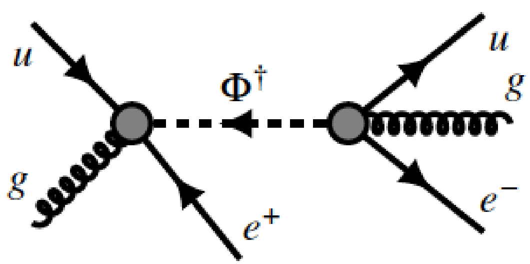


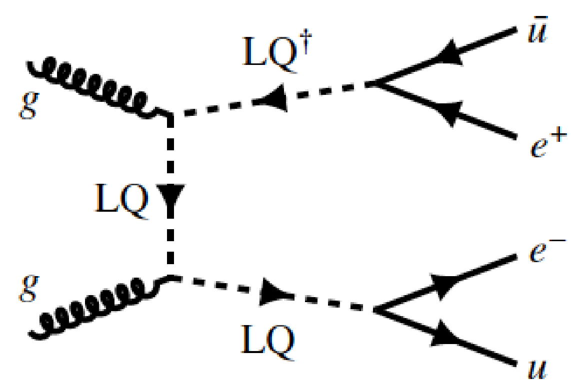
With final state $j\,j\,\ell^+\ell^-$

How is this covered by other exotics searches?

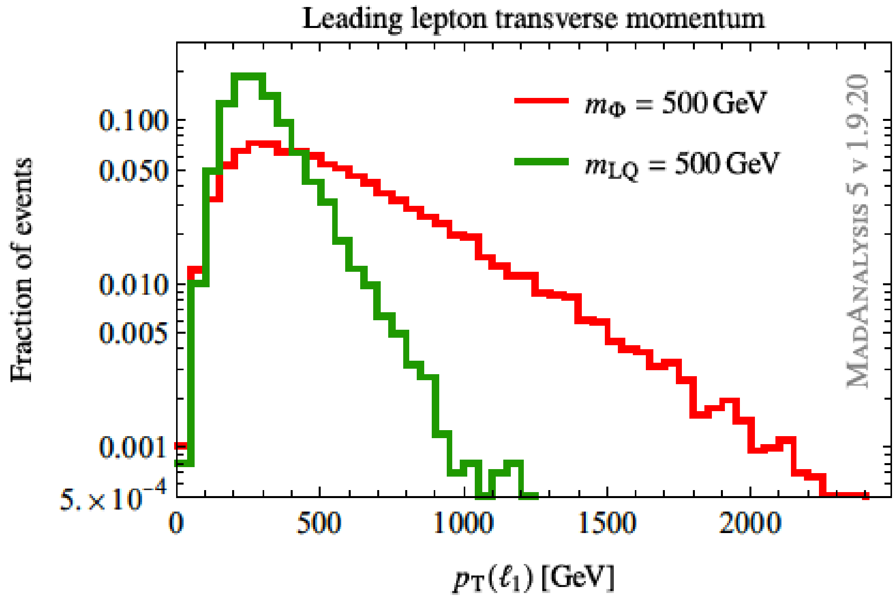
We can compare this to a similar BSM inclusive search for state for lepto-quarks

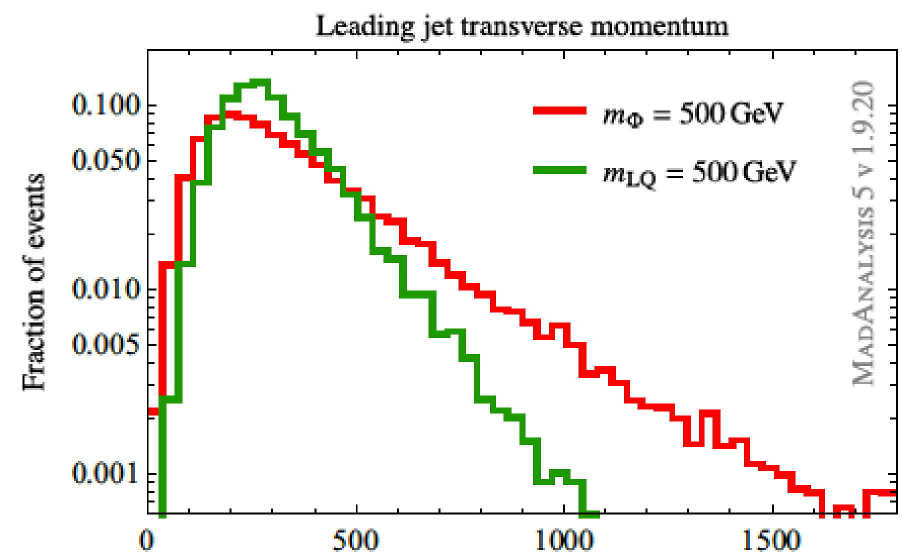
Field	$SU(3)_c \times SU(2)_L \times U(1)_Y$	B	L
Φ	$(\mathbf{6}, \mathbf{1}, \frac{1}{3})$	0	-1
Φ_{LQ}	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	$\frac{1}{3}$	1

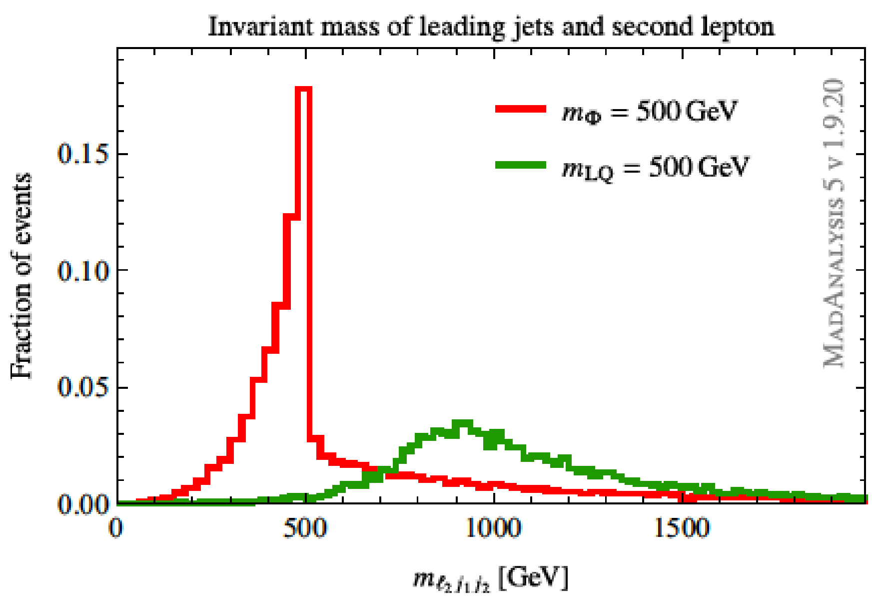


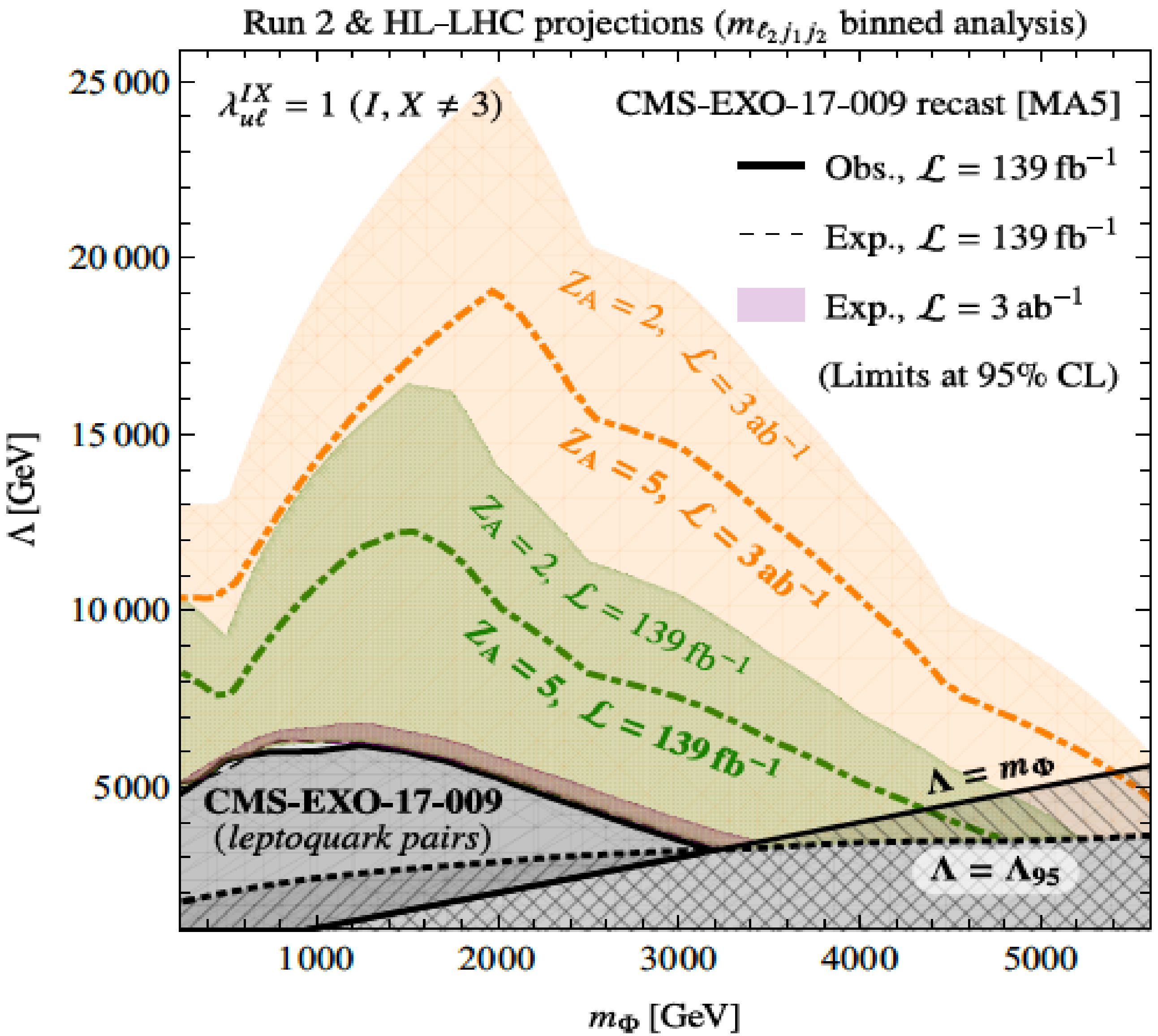


How do kinematics differ?


Leading lepton transverse momentum
Fraction of events
$p_T(\ell_1)$ [GeV]
m_Φ = 500 GeV
m_{LQ} = 500 GeV
MadAnalysis 5 v 1.9.20


Leading jet transverse momentum
Fraction of events
m_Φ = 500 GeV
m_{LQ} = 500 GeV
MadAnalysis 5 v 1.9.20


Invariant mass of leading jets and second lepton
Fraction of events
$m_{\ell_2 j_1 j_2}$ [GeV]
m_Φ = 500 GeV
m_{LQ} = 500 GeV
MadAnalysis 5 v 1.9.20



Run 2 & HL-LHC projections ($m_{\ell_2 j_1 j_2}$ binned analysis)
$\lambda_{u\ell}^{IX} = 1$ $(I, X \neq 3)$
CMS-EXO-17-009 recast [MA5]
Obs., $\mathcal{L} = 139$ fb^{-1}
Exp., $\mathcal{L} = 139$ fb^{-1}
Exp., $\mathcal{L} = 3$ ab^{-1}
(Limits at 95% CL)
$Z_A = 2$, $\mathcal{L} = 3$ ab^{-1}
$Z_A = 5$, $\mathcal{L} = 3$ ab^{-1}
$Z_A = 2$, $\mathcal{L} = 139$ fb^{-1}
$Z_A = 5$, $\mathcal{L} = 139$ fb^{-1}
CMS-EXO-17-009
(leptoquark pairs)
$\Lambda = m_\Phi$
$\Lambda = \Lambda_{95}$
Λ [GeV]
m_Φ [GeV]
25 000
20 000
15 000
10 000
5000
1000
2000
3000
4000
5000

Two approaches to Operator Catalogs

- **Field based**, pick an example LEX state with specific quantum numbers and write all possible operators up to desired mas dimension e.g. dim 6

- **Portal based**, pick a SM portal and write all possible LEX states that can couple through that portal (eg Higgs portal, lepton portal)

Example Catalog: Di-Boson Portal

Catalog all CP even spin zero scalars that couple to pairs of SM vector bosons

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SU(3) structure

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SU(3) structure

Operators with one gluon, LEX state must be octet of SU(3) LEX states may be SU(2) singlets or have SU(2) structure

Octets coupling to SU(2) tensor and Gluon

$(\mathbf{8}, \mathbf{2}, \frac{1}{2})$ $$\mathcal{L} \supset \frac{1}{\Lambda^2} H^{\dagger i} (\sigma^a)_i{}^j \phi_j^A W^{a\mu\nu} G_{\mu\nu}^A$$

$(\mathbf{8}, \mathbf{3}, 0)$ $$\mathcal{L} \supset \frac{1}{\Lambda} \phi^{Aa} W^{a\mu\nu} G_{\mu\nu}^A.$$

$(\mathbf{8}, \mathbf{1}, 0)$ $$\mathcal{L} \supset \frac{1}{\Lambda^3} (H^\dagger \sigma^a H) \phi^A W^{a\mu\nu} G_{\mu\nu}^A.$$

$(\mathbf{8}, \mathbf{4}, -\frac{1}{2})$ $$\phi_{ijk} H^k W^{ij\mu\nu} G_{\mu\nu}$$

$(\mathbf{8}, \mathbf{5}, 0)$ $$\phi_{ijkl} H^{\dagger i} H^j W^{kl\mu\nu} G_{\mu\nu}$$

$G_{\mu\nu}G^{\mu\nu}$ couplings

We use the tensor product relation

$$\mathbf{8}\otimes\mathbf{8}=\mathbf{1}_{\mathrm{s}}\oplus\mathbf{8}_{\mathrm{s}}\oplus\mathbf{8}_{\mathrm{a}}\oplus\mathbf{10}_{\mathrm{a}}\oplus\overline{\mathbf{10}}_{\mathrm{a}}\oplus\mathbf{27}_{\mathrm{s}}$$

$(\mathbf{27},\mathbf{3},0)$ $\quad [H^{\dagger i}\phi^{KL}_{IJij}H^{j}]\,G^{I\mu\nu}_{K}G^{J}_{L\mu\nu}$

$(\mathbf{10},\mathbf{3},0)$ $\quad \varepsilon^{KLM}[H^{\dagger i}\phi_{IJKij}H^{j}]\,G^{I\mu\nu}_{L}G^{J}_{M\mu\nu}$

LEX in Higher Dimensional Reps of SU(2)

In SU(2) n dimensional representation maps to algebra of spin J with

$$n = 2J + 1$$

For n=5, J is spin 2 object $J \in \{-2, -1, 0, 1, 2\}$

With tensor products

$$J \otimes L = J + L \oplus J + L - 1 \oplus \cdots \oplus J - L$$

An example product in SU(2)

$\mathbf{3} \otimes \mathbf{3}$ $\quad J = L = 1$ so $J \in \{0, 1, 2\}$

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{5}$$

All possible representations can be constructed by taking successive products of the fundamental These higher-dimensional representations may be denoted as symmetric tensors. Totally symmetric tensors of dimension d and rank r have

$$n = \frac{(d + r - 1)!}{r!\,(d - 1)!}$$

For SU(2), $d = 2$, so $n = r + 1$

5 of $SU(2)_L$ ⟶ ϕ_{ijkl}

$$\Phi = (\Phi^{++}, \Phi^{+}, \Phi^{0}, \Phi^{-}, \Phi^{--})$$

$$\frac{1}{\Lambda}\phi_{ijkl} W^{ij\mu\nu} W^{kl}_{\mu\nu} = \frac{1}{\Lambda}\Big(\Phi^{++} W^{-\mu\nu} W^{-}_{\mu\nu} + \Phi^{--} W^{+\mu\nu} W^{+}_{\mu\nu} - \sqrt{2}\Phi^{+} W^{-\mu\nu} W^{3}_{\mu\nu}$$

$$-\sqrt{2}\Phi^{-} W^{+\mu\nu} W^{3}_{\mu\nu} - \sqrt{\frac{2}{3}}\,\Phi^{0} W^{-\mu\nu} W^{+}_{\mu\nu} + \sqrt{\frac{2}{3}}\,\Phi^{0} W^{3\mu\nu} W^{3}_{\mu\nu}\Big) \quad ($$

Adding Higgs insertions

$\mathbf{2} \otimes \mathbf{6} \supset \mathbf{5}$ as with spins $\frac{1}{2} \otimes \frac{5}{2} = 1 \oplus \underline{2} \oplus 3$

With the product $\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{5}$

We can make the invariant Lagrangian term

$$\mathbf{2} \otimes \mathbf{6} \otimes \mathbf{3} \otimes \mathbf{3}$$

$$\phi_{ijklm} \, H^i W^{jk}_{\mu\nu} W^{lm\mu\nu}$$

Collider Processes

$$\mathcal{L} \supset \frac{1}{\Lambda} \phi_{ijkl} W^{\mu\nu ij} W^{kl}_{\mu\nu}$$

Yields associated production process

$$pp \to \Phi^{++} W^{-}$$

Through the same operator

$$\Phi^{++} \to W^{+} W^{+}$$

Tri-boson process

$$pp \to \phi^{++} W^{-} \to (W^{+} W^{+}) W^{-}$$

Consider the sextet

$$\Phi = (\Phi^{+++}, \Phi^{++}, \Phi^{+}, \Phi^{0}, \Phi^{-}, \Phi^{--})$$

Yields quark fusion process through W in the s channel

$$qq \rightarrow \Phi^{+++}\Phi^{--}$$

With mass splitting there may be a cascade decayl

$$\Phi^{+++} \rightarrow \Phi^{++}W^{+} \rightarrow \Phi^{+}W^{+}W^{+}$$

With operator

$$\mathcal{L} \supset \frac{1}{\Lambda^2}\, \phi_{ijklm} H^{m}\, W^{\mu\nu\, ij} W^{kl}_{\mu\nu}.$$

Allowing the charge 1 state to decay to WZ

Dimension	$SU(3)_c \times SU(2)_L \times U(1)_Y$	Operators
$5 \left[\times \frac{1}{\Lambda}\right]$	$(\mathbf{1}, \mathbf{5}, 0)$	$\phi_{ijkl}\, W^{ij\mu\nu} W^{kl}_{\mu\nu}$
	$(\mathbf{10}, \mathbf{1}, 0)$	$\varepsilon^{KLM} \phi_{IJK}\, G^{I\mu\nu}_{L} G^{J}_{M\mu\nu}$
	$(\mathbf{27}, \mathbf{1}, 0)$	$\phi^{KL}_{IJ}\, G^{I\mu\nu}_{K} G^{J}_{L\mu\nu}$
$6 \left[\times \frac{1}{\Lambda^2}\right]$	$(\mathbf{1}, \mathbf{4}, -\frac{1}{2})$	$\phi_{ijk}\, H^k\, W^{ij\mu\nu} B_{\mu\nu}$
	$(\mathbf{8}, \mathbf{4}, -\frac{1}{2})$	$\phi_{ijk}\, H^k\, W^{ij\mu\nu} G_{\mu\nu}$
	$(\mathbf{1}, \mathbf{4}, -\frac{1}{2})$	$\phi_{ijk}\, H_l\, W^{ij\mu\nu} W^{kl}_{\mu\nu}$
	$(\mathbf{1}, \mathbf{6}, -\frac{1}{2})$	$\phi_{ijklm}\, H^m\, W^{ij\mu\nu} W^{kl}_{\mu\nu}$
$7 \left[\times \frac{1}{\Lambda^3}\right]$	$(\mathbf{1}, \mathbf{5}, 0)$	$\phi_{ijkl}\, H^{\dagger i} H^j\, W^{kl\mu\nu} B_{\mu\nu}$
	$(\mathbf{8}, \mathbf{5}, 0)$	$\phi_{ijkl}\, H^{\dagger i} H^j\, W^{kl\mu\nu} G_{\mu\nu}$
	$(\mathbf{1}, \mathbf{7}, 0)$	$\phi_{ijklmn}\, H^{\dagger m} H^n\, W^{ij\mu\nu} W^{kl}_{\mu\nu}$
	$(\mathbf{10}, \mathbf{3}, 0)$	$\varepsilon^{KLM} [H^{\dagger i} \phi_{IJKij} H^j]\, G^{I\mu\nu}_{L} G^{J}_{M\mu\nu}$
	$(\mathbf{27}, \mathbf{3}, 0)$	$[H^{\dagger i} \phi^{KL}_{IJij} H^j]\, G^{I\mu\nu}_{K} G^{J}_{L\mu\nu}$

Future Directions

- Many catalogs to build *e.g. high reps of SU(2)*
- Explore existing catalogs, *e.g. di-boson portal, color sextet scalars*
- Search for classes of outstanding collider signatures
- Build UV completions

We now argue from induction. To build three-field invariants involving a LEX field, we need only consider the m possible bilinear tensor products of the LEX state with other representations allowed in the theory, $[\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i]_{\mathbf{r}'_j}$, to obtain the finite list of irreducible representations $\mathbf{r}'$ in the direct product. If any single field in the theory is in the conjugate representation $\bar{\mathbf{r}}'_j$, then we can directly contract indices to form an invariant:

$$[\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i]_{\mathbf{r}'_j} \otimes \bar{\mathbf{r}}'_j$$

With a list in hand of all m possible bilinear products $\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i$ in representations $\mathbf{r}'_j$, we can proceed to construct the four-field invariants. We find the direct products of the allowed representations $\mathbf{r}_k \otimes \mathbf{r}_l$ that are in a given conjugate representation $\bar{\mathbf{r}}'_j$ and contract these fields according to

$$[\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i]_{\mathbf{r}'_j} \otimes [\mathbf{r}_k \otimes \mathbf{r}_l]_{\bar{\mathbf{r}}'_j}$$

to obtain singlets. To proceed to five fields, we now consider all possible trilinear products of the form $\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i \otimes \mathbf{r}_j$. We note we have already found by exhaustion the representations of bilinear products of the first two fields in the previous step. In that step, the bilinears were in representations $\mathbf{r}'_j$ such that $\mathbf{r}_{\rm LEX} \otimes \mathbf{r}_i \supset \mathbf{r}'_j$. We can thus iterate the bilinear tensor products $\mathbf{r}'_j \otimes \mathbf{r}_j \supset \mathbf{r}'_k$ to find the representations $\mathbf{r}'_k$ of all trilinear products. We then find the remaining bilinear representations $\mathbf{r}_k \otimes \mathbf{r}_l$ that are in the conjugate representation $\bar{\mathbf{r}}'_k$ and contract *these* fields to form the five-field invariant. This process can be repeated indefinitely and will ultimately produce all possible terms — we only need to know the list of bilinear tensor products that involve relevant SM/LEX fields and the intermediate representations $\mathbf{r}'_j$, $\mathbf{r}'_k$, and so on.