Scattering amplitudes for Spinning Binaries

Andres Luna.

Brookhaven Lab (via Zoom), 23/03/23

UNIVERSITY OF COPENHAGEN

The Niels Bohr International Academy

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About

- Amplitudes only. PM only. Mostly up to G^2

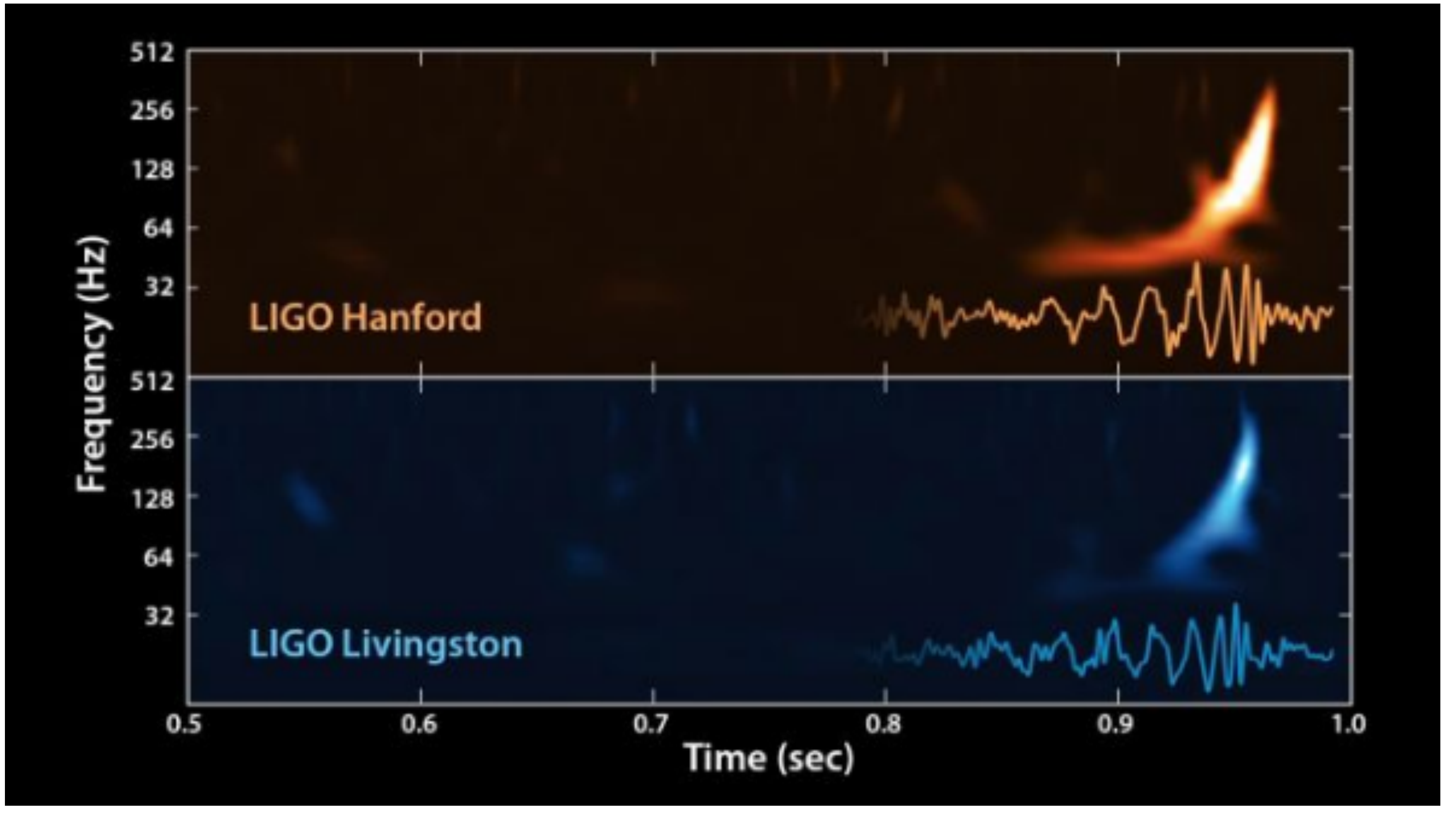
- For up to S^2, overlapping (and beyond) results from PM worldline theories (Headlined by Liu, Porto, Yang; Jakobsen, Mogull, Plefka, Steinhoff).

- Many contributors: Adamo, Aoude, Arkani-Hamed, Bautista, Bern, Bjerrum-Bohr, Buonanno, Cangemi, Chen, Chiodaroli, Chung, Cristofoli, Damgaard, Febres, Gonzo, Guevara, Haddad, Helset, Huang, Huang, Johansson, Kavanagh, Kim, KosmopoulosKraus, Lee, Lin, Maybee, Moynihan, Ochirov, O'Connell, Pichini, Roiban, Ross, Ruf, Sergola, Skowronek, Skvortsov, Shen, Steinhoff, Tourkine, Vines, White, Zeng, +

Outline

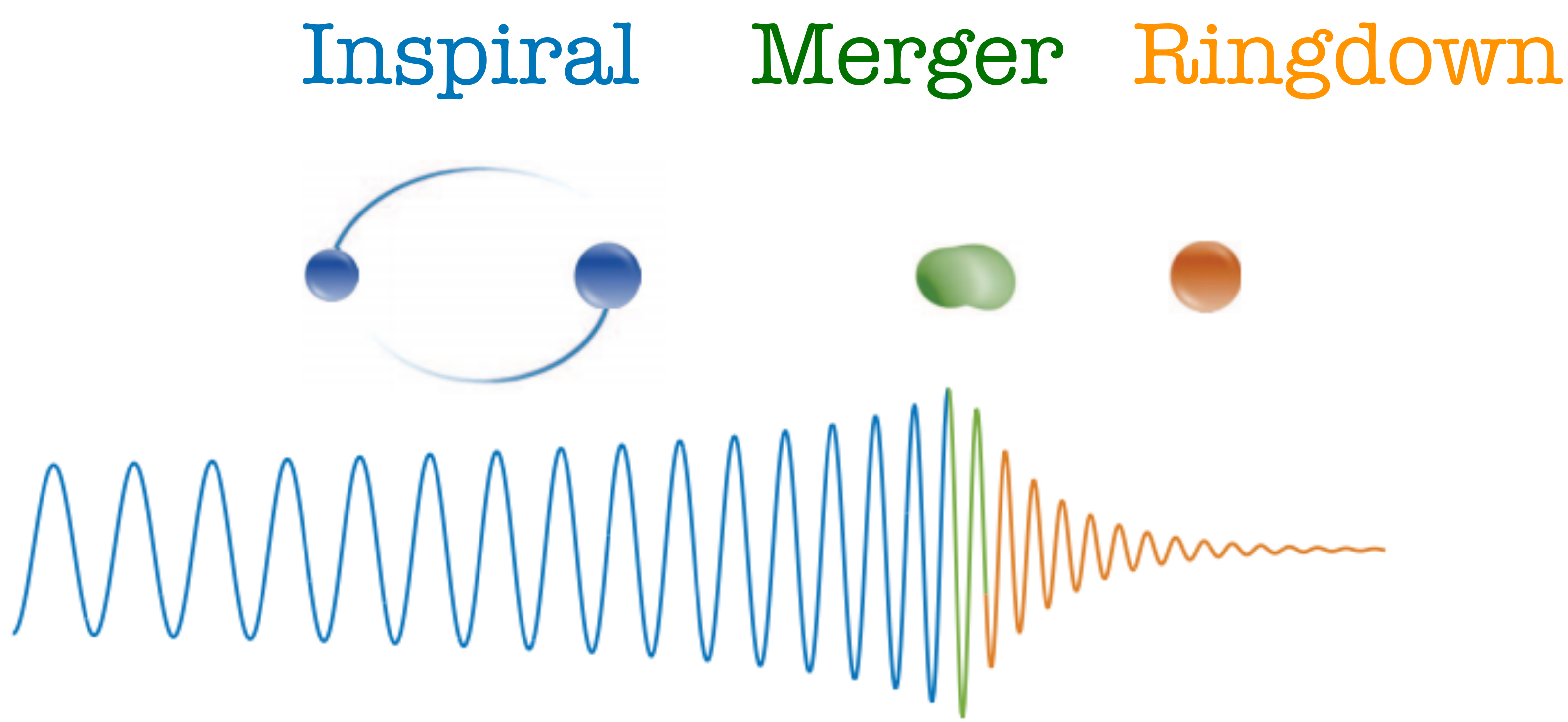
- Briefest intro
- Black holes from Amplitudes. Conjectures.
- Why do we trust these results? (Checks)
- Summary / Outlook

Gravitational Waves

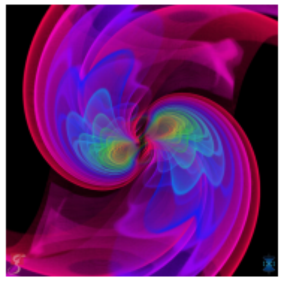


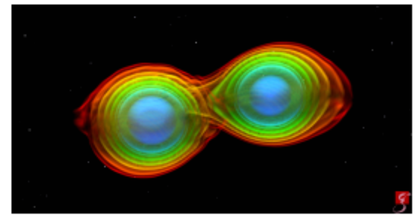
Recent detection of gravitational waves drives a demand for analytical methods for waveform templates.

How to apply Amplitudes methods to the field of gravitational waves?



The natural candidate is the inspiral phase, where perturbation theory is applied



The Need for High-Precision Gravitational Waveforms

Alessandra Buonanno

Max Planck Institute for Gravitational Physics

(Albert Einstein Institute)

Department of Physics, University of Maryland

"QCD Meets Gravity IV", NORDITA, Stockholm

Dec 10-14, 2018

Need more efficient ways to solve two-body problem, analytically

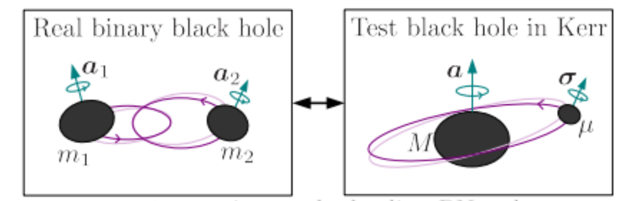
- In test-body limit, spinning EOB Hamiltonian includes **linear terms in spin of test body at all PN orders**.

(Barausse et al. 10, Barausse & AB 11, 12; Vines et al. 15)

$$(\sigma + \sigma^2)\left(1 + \frac{v^2}{c^2} + \cdots\right)$$

- Is EOB **mapping unique** at all orders?

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}{}^{\nu}}{\mu} - 1\right)}$$



exact mapping at the leading PN orders

Using **unbound orbits,** using **scattering angle** as adiabatic invariant, **at 1PM: mapping unique** & 2-body relativistic motion equivalent to 1-body motion in Kerr. *(Damour 16, Bini et al. 17-18, Vines 17)*

- Results at **leading PN order** but **all orders in spin.**

(Vines & Steinhoff 16; Vines & Harte 16, Siemonsen, Steinhoff & Vines 17)

see Steinhoff's & Damour's talks

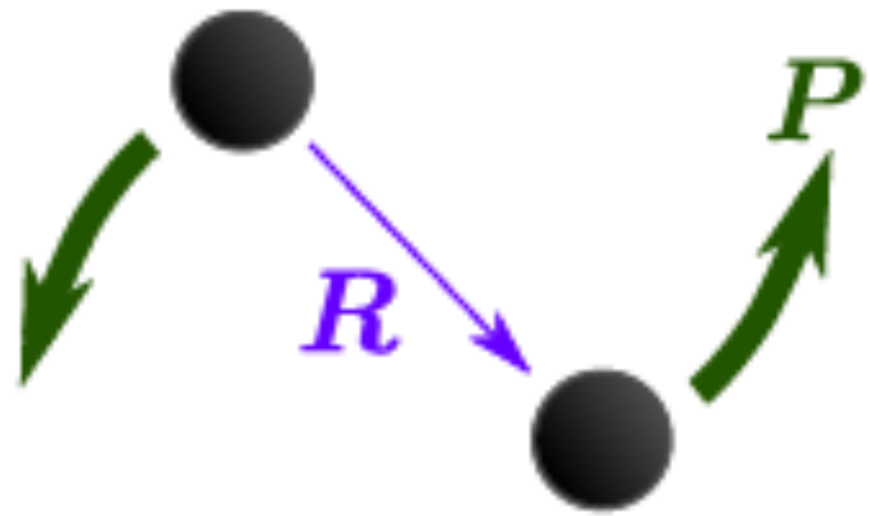
$$\frac{Gm}{rc^2}\left(1 + \frac{v^2}{c^2} + \cdots\right)$$

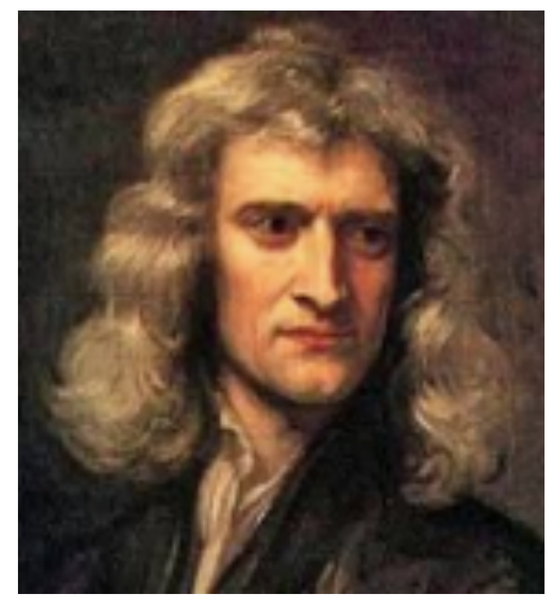
$GM/rc^2 \ll v^2/c^2 \sim 1$

$$\frac{v^2}{c^2}\left(S_i + S_i^2 + \cdots\right)$$

Need more efficient ways to solve two-body problem, analytically

The two-body problem





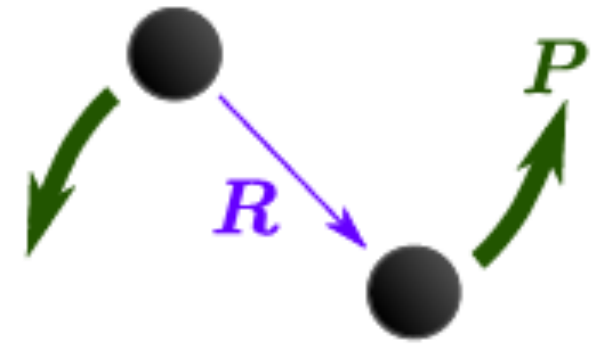
$$\frac{H}{\mu} = \boxed{\frac{P^2}{2} - \frac{Gm}{R}}$$

Newton $\sim \mathcal{O}(G)$

$m = m_A + m_B, \quad \nu = \mu/M \quad \mu = m_A m_B/m$

$$\frac{H}{\mu} = \frac{P^2}{2} - \frac{Gm}{R}$$

Newton $\sim \mathcal{O}(G)$



THE GRAVITATIONAL EQUATIONS AND THE PROBLEM OF MOTION

By A. Einstein, L. Infeld, and B. Hoffmann

(Received June 16, 1937)

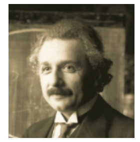
Introduction. In this paper we investigate the fundamentally simple question of the extent to which the relativistic equations of gravitation determine the motion of ponderable bodies.

degree of accuracy. In the present part we deal with the actual application of this method, carrying the calculation to such a stage that the main deviation from the Newtonian laws of motion is determined.

$$+\frac{1}{c^2}\left\{-\frac{P^4}{8}+\frac{3\nu P^4}{8}+\frac{Gm}{R}\left(-\frac{P_R^2\nu}{2}-\frac{3P^2}{2}-\frac{\nu P^2}{2}\right)+\frac{G^2m^2}{2R^2}\right\}$$

1PN: Einstein, Infeld, Hoffmann '38

$$m = m_A + m_B, \quad \nu = \mu/M \quad \mu = m_A m_B/m$$

virial theorem

$$v^2 \sim \frac{GM}{r} \ll 1$$

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN		
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)$	G
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)$	G^2
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)$	G^3
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)$	G^4
5PM					$(1$	$+v^2$	$+v^4$	$+v^6$	$+\cdots)$	G^5

$\vdots$

Post-Newtonian approximation

Using ADM Hamiltonian, EFT (NRGR) and Self-force

0PN: Newton 1666

1PN: Einstein, Infeld, Hoffmann '38

2PN: Ohta et. al. '73

3PN: Damour, Jaranowski, Schaefer, Blanchet, Faye ca. '97

4PN: Bini, Damour, Jaranowski, Schaefer, Blanchet, Faye, Marchand, Foffa, Sturani, Mastrolia, Sturm, Porto, Rothstein… ca. '13

5PN*: Bini, Damour, Geralico, Foffa, Mastrolia, Sturani, Torres-Bobadilla ca. '19

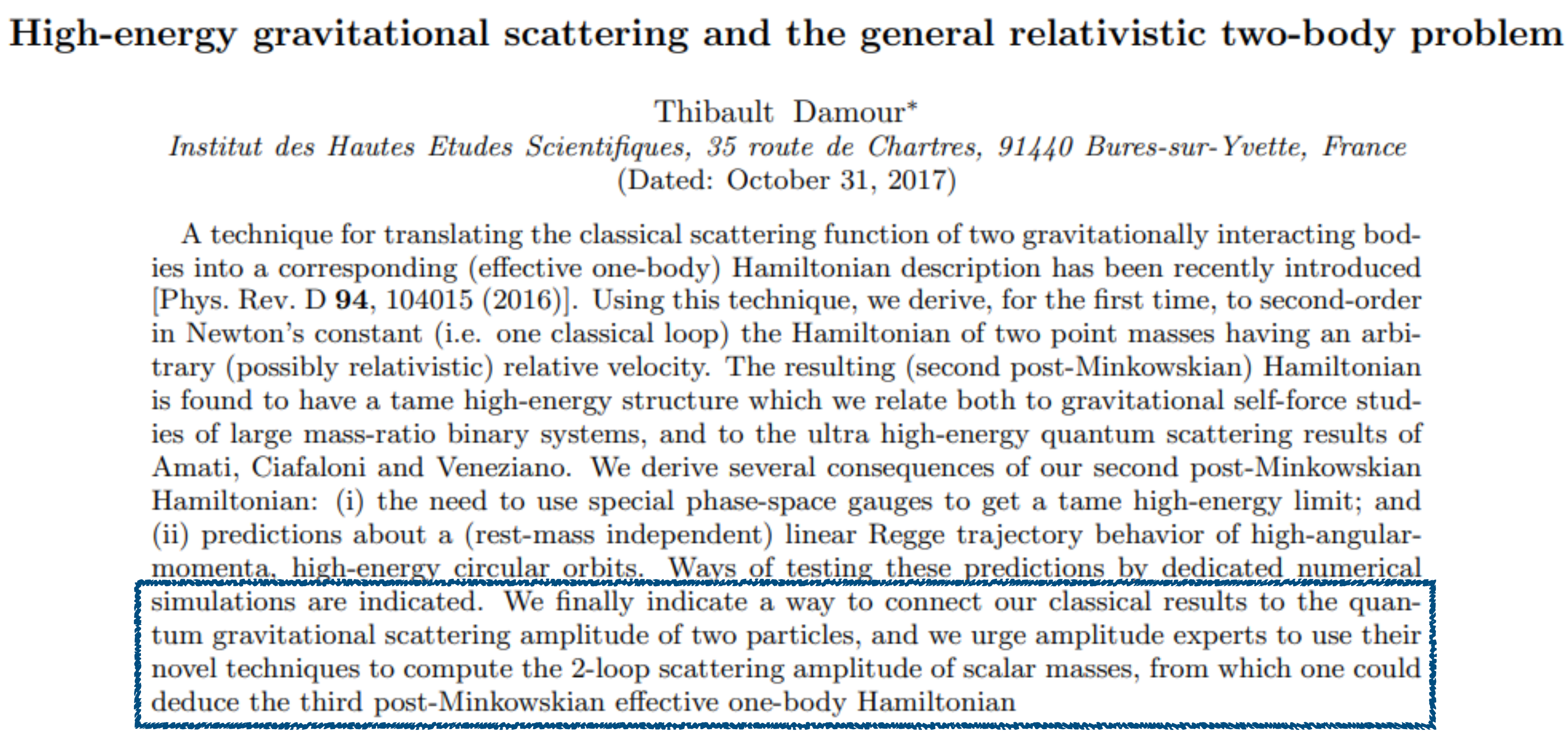
	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)\,G$
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)\,G^2$
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)\,G^3$
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)\,G^4$
5PM					$(1$	$+v^2$	$+v^4$	$+v^6$	$+\cdots)\,G^5$

$$\vdots$$

Post-Minkowskian approximation

Bertotti, Kerr, Plebanski, Portilla, Westpfahl, Goller, Bel, Damour, Deruelle, Ibanez, Martin, Ledvinka, Schaefer, Bicak…

An invitation


High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour*

Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France

(Dated: October 31, 2017)

A technique for translating the classical scattering function of two gravitationally interacting bodies into a corresponding (effective one-body) Hamiltonian description has been recently introduced [Phys. Rev. D **94**, 104015 (2016)]. Using this technique, we derive, for the first time, to second-order in Newton's constant (i.e. one classical loop) the Hamiltonian of two point masses having an arbitrary (possibly relativistic) relative velocity. The resulting (second post-Minkowskian) Hamiltonian is found to have a tame high-energy structure which we relate both to gravitational self-force studies of large mass-ratio binary systems, and to the ultra high-energy quantum scattering results of Amati, Ciafaloni and Veneziano. We derive several consequences of our second post-Minkowskian Hamiltonian: (i) the need to use special phase-space gauges to get a tame high-energy limit; and (ii) predictions about a (rest-mass independent) linear Regge trajectory behavior of high-angular-momenta, high-energy circular orbits. Ways of testing these predictions by dedicated numerical simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

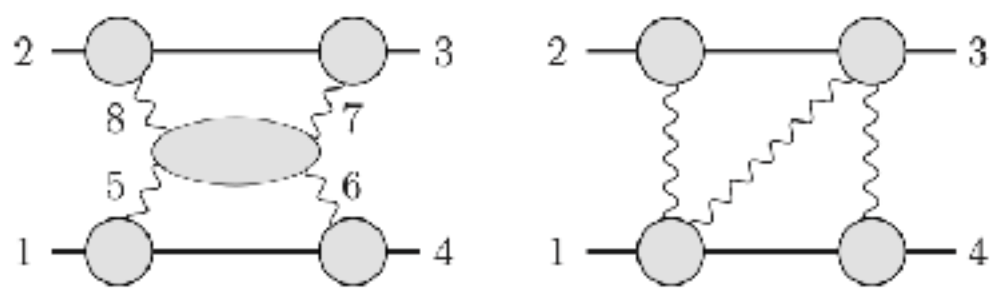
We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

Hamiltonian from Amplitudes

[Submitted on 14 Jan 2019]

Scattering Amplitudes and the Conservative Hamiltonian for Binary Systems at Third Post-Minkowskian Order

Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mikhail P. Solon, Mao Zeng



Unitarity: Loops from trees

$$A_4(1^s, 2^+, 3^-, 4^s) = i\,\frac{\langle 3|\,1\,|2]^2}{t_{23}\,t_{12}}\,,$$

$$A_4(1^-, 2^-, 3^+, 4^+) = i\,\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle}\,,$$

Spinor helicity

$$M_4(1,2,3,4) = -is_{12}A_4(1,2,3,4)\,A_4(1,2,4,3),$$

Double copy: Gravity = Gauge^2

$$\mathcal{M}_3 = \frac{\pi G^3 \nu^2 m^4 \log \boldsymbol{q}^2}{6\gamma^2\xi}\Bigg[3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3 - \frac{48\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} - \frac{18\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{(1+\gamma)(1+\sigma)}\Bigg] + \frac{8\pi^3 G^3 \nu^4 m^6}{\gamma^4\xi}\Big[3\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)F_1 - 32m^2\nu^2\left(1-2\sigma^2\right)^3 F_2\Big],$$

The requested 2-loop amplitude

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p},\boldsymbol{r}),$$

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i,$$

These Hamiltonian coefficients depend on the scattering Amplitudes

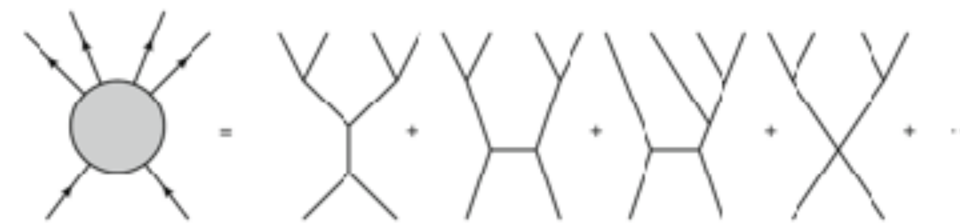
Amplitudes: Six gluons

THE CROSS SECTION FOR FOUR-GLUON PRODUCTION BY GLUON–GLUON FUSION

Stephen J. PARKE and T.R. TAYLOR

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510 USA

Received 13 September 1985



$$
\begin{aligned}
&|M(g^1, g^2, g^3, g^4, g^5, g^6)|^2 \\
&\quad = 2^{1-l}(N^2-1)^{-l}(A_0(123456) + A_0(135426) + A_0(126435) + A_0(156423) \\
&\qquad + A_0(125436) + A_0(136425) + A_0(134256) + A_0(124356) + A_0(145236) \\
&\qquad + A_0(146235) + A_2(123456) + A_2(135426) + A_2(123546) + A_2(136425) \\
&\qquad + A_2(156423) + A_2(125436) + A_2(126435) + A_2(563412) + A_2(526413) \\
&\qquad + A_2(123645) + A_2(135624) + A_2(623415) + A_2(523416) + A_2(125634) \\
&\qquad + A_2(523614)),
\end{aligned}
$$

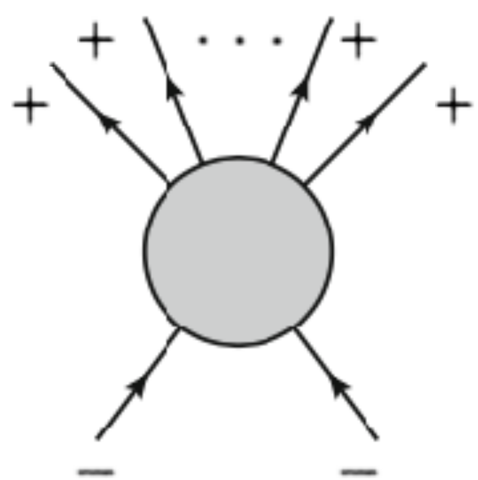
As the result of the computation of two hundred and forty Feynman diagrams, we obtain

$$
A_{\binom{0}{2}}(p_1, p_2, p_3, p_4, p_5, p_6)
= (\mathcal{D}^\dagger, \mathcal{D}^\dagger_\rho, \mathcal{D}^\dagger_\sigma, \mathcal{D}^\dagger_\tau)_{\binom{0}{2}} \cdot
\begin{pmatrix} K & K_\rho & K_\sigma & K_\tau \\ K_\rho & K & K_\tau & K_\sigma \\ K_\sigma & K_\tau & K & K_\rho \\ K_\tau & K_\sigma & K_\rho & K \end{pmatrix}
\cdot
\begin{pmatrix} \mathcal{D} \\ \mathcal{D}_\rho \\ \mathcal{D}_\sigma \\ \mathcal{D}_\tau \end{pmatrix}_{\binom{0}{2}}, \qquad (6)
$$

Simplicity: n-Gluons

Amplitude for *n*-Gluon Scattering

Stephen J. Parke and T. R. Taylor
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 17 March 1986)



Specifying helicities can lead to considerable simplifications

$$|\mathcal{M}_n(+++++\cdots)|^2 = c_n(g,N)[0+O(g^4)],$$
$$|\mathcal{M}_n(-++++\cdots)|^2 = c_n(g,N)[0+O(g^4)],$$
$$|\mathcal{M}_n(--+++\cdots)|^2 = c_n(q,N)[(p_1\cdot p_2)^4 \times \textstyle\sum_P [(p_1\cdot p_2)(p_2\cdot p_3)(p_3\cdot p_4)\cdots(p_n\cdot p_1)]^{-1} + O(N^{-2}) + O(g^2)],$$

In modern approaches, helicity Amplitudes are written in terms of spinors

$$A_n[1^+ \dots i^- \dots j^- \dots n^+] = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

Spinor helicity

Our vector expressions can be written in terms of spinors

$$k_{\alpha\dot{\beta}} = k_\mu \sigma^\mu_{\alpha\dot{\beta}} = |k\rangle_\alpha [k|_{\dot{\beta}} \qquad \varepsilon^\mu_+ = \frac{\langle r|\sigma^\mu|k]}{\sqrt{2}\langle r\,k\rangle}, \qquad \varepsilon^\mu_- = -\frac{[r|\bar{\sigma}^\mu|k\rangle}{\sqrt{2}[r\,k]}$$

They satisfy a bunch of relations

$$\langle p\,q\rangle = -\langle q\,p\rangle\,, \qquad [p\,q] = -[q\,p]$$

$$[p\,q]^* = \langle q\,p\rangle \qquad \langle p\,q\rangle\,[p\,q] \;=\; 2\,p\cdot q$$

$$\langle ri\rangle\langle jk\rangle + \langle rj\rangle\langle ki\rangle + \langle rk\rangle\langle ij\rangle \;=\; 0\,.$$

The approach can be generalized when massive vectors are involved

$$p_{\alpha\dot{\beta}} = p_\mu \sigma^\mu_{\alpha\dot{\beta}} = \epsilon_{ab}|p^a\rangle_\alpha [p^b|_{\dot{\beta}} \qquad \varepsilon^{ab}_{p\mu} = \frac{\langle p^{(a}|\sigma_\mu|p^{b)}]}{\sqrt{2}m}$$

Amplitudes for spinning binaries

A simplest Amplitude...

[Submitted on 14 Sep 2017 (v1), last revised 31 Oct 2021 (this version, v2)]

Scattering Amplitudes For All Masses and Spins

Nima Arkani-Hamed, Tzu-Chen Huang, Yu-tin Huang

$$\mathcal{M}_3^{(s)}(p_1, p_2, k^+) = \frac{\langle 12\rangle^{2s} x^2}{m^{2s-2}}$$

$$x = \sqrt{2}\,\frac{p \cdot \varepsilon}{m}$$

...matches a simplest object

The Kerr black hole!

Linearised Kerr black holes...

[Submitted on 18 Sep 2017 (v1), last revised 29 Sep 2017 (this version, v2)]

Scattering of two spinning black holes in post-Minkowskian gravity, to all orders in spin, and effective-one-body mappings

Justin Vines

$$\mathcal{S}_{\rm tot}[\Psi, h] = \mathcal{S}_{\rm grav}[h] + \mathcal{S}_{\rm int}[\Psi, h] + \mathcal{S}_{\rm kin}[\Psi]$$

$$\mathcal{S}_{\rm int}[\Psi, h] = \frac{1}{2}\int d^4x\, h_{\mu\nu}\, T^{\mu\nu}[\Psi]$$

For a linearized Kerr black hole

$$T^{\mu\nu}(x) = \int d\tau\, \hat{\mathcal{T}}^{\mu\nu}(p, a, \partial)\, \delta^4(x - z)$$

$$\hat{\mathcal{T}}^{\mu\nu}(p, a, \partial) = m\, \exp(a * \partial)^{(\mu}{}_{\rho}\, u^{\nu)} u^{\rho} \qquad (a * \partial)^{\mu}{}_{\nu} = \epsilon^{\mu}{}_{\nu\alpha\beta} a^{\alpha} \partial^{\beta}.$$

The stress energy tensor enters the Lagrangian

$$\mathcal{L}_{\rm int} = \frac{1}{2}\hat{\mathcal{T}}^{\mu\nu}(p_1, a_1, -\partial)\, h_{2\mu\nu}(x)\Big|_{x=z_1}$$

$$= \langle u_1 | \exp(a_1 * \partial)\ \mathcal{Q}\ \exp(a_2 * \partial) | u_2 \rangle \frac{G m_1 m_2}{r_2}\Big|_{x=z_1}$$

The interaction Lagrangian is translated into observables

$$\Delta p_{1\mu} = \int d\tau_1 \frac{\partial \mathcal{L}_{\rm int}}{\partial z_1^{\mu}} \qquad \Delta p_1^{\mu} = G m_1 m_2 \Re Z^{\mu}, \qquad Z_{\mu} = \frac{-2\Big[(2\gamma^2 - 1)\eta_{\mu\nu} + 2i\gamma\epsilon_{\mu\nu\alpha\beta} u_1^{\alpha} u_2^{\beta}\Big](b + i\Pi a_0)^{\nu}}{\sqrt{\gamma^2 - 1}\,(b + i\Pi a_0)^2}$$

…from Scattering amplitudes

[Submitted on 17 Dec 2018 (v1), last revised 9 Sep 2019 (this version, v3)]

Scattering of Spinning Black Holes from Exponentiated Soft Factors

Alfredo Guevara, Alexander Ochirov, Justin Vines

The stress energy tensor in momentum space

$$T^{\mu\nu}(-k) = 2\pi\delta(p\cdot k)p^{(\mu}\exp(a*ik)^{\nu)}{}_{\rho}p^{\rho}$$

Is matched by the three-point amplitude

$$h_{\mu\nu}(k)T^{\mu\nu}(-k) = \frac{1}{2}(2\pi)^2\delta(k^2)\delta(p\cdot k)\lim_{s\to\infty}\langle\mathcal{M}_3^{(s)}\rangle$$

$$\mathcal{M}_3^{(s)}(p_1,p_2,k^+) = \frac{\langle 12\rangle^{2s}x^2}{m^{2s-2}}$$

$$h_{\mu\nu} = 2\pi\delta(k^2)\varepsilon_\mu\varepsilon_\nu;$$

The amplitude is an exponential of the Lorentz generator

$$\hat{\mathcal{M}}_3^{(s)}(p_1,p_2,k^+) = \mathcal{M}_3^{(0)}\exp\left(i\frac{k_\mu\varepsilon_\nu^+J^{\mu\nu}}{p\cdot\varepsilon^+}\right)$$

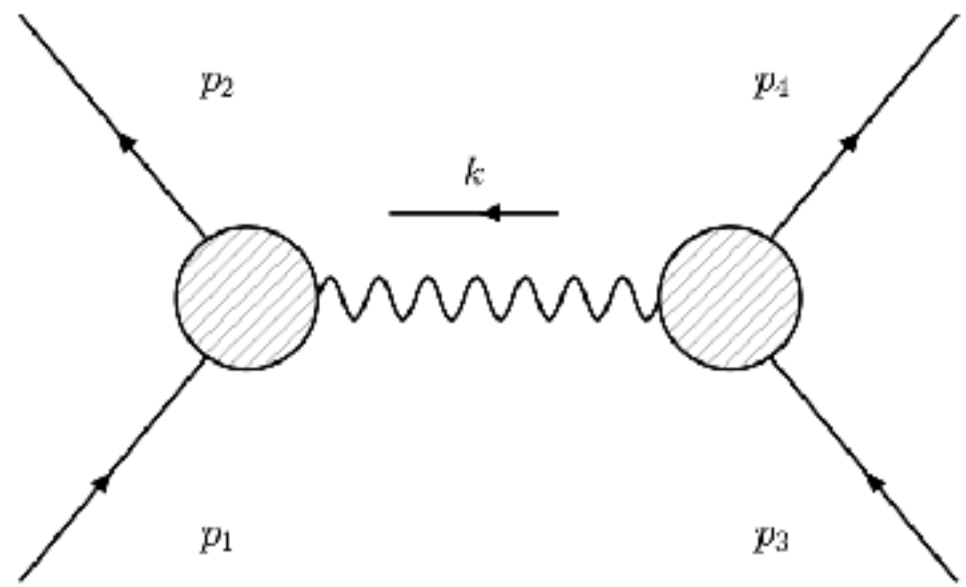
$$p_1 = p - k/2$$
$$p_2 = -p - k/2$$

Observables from Amplitudes

[Submitted on 17 Dec 2018 (v1), last revised 9 Sep 2019 (this version, v3)]

Scattering of Spinning Black Holes from Exponentiated Soft Factors

Alfredo Guevara, Alexander Ochirov, Justin Vines



From the 3-point amplitude, computed 1PM scattering angle

$$\theta = \frac{GE}{v^2}\left[\frac{(1+v)^2}{b+a_a+a_b} + \frac{(1-v)^2}{b-a_a-a_b}\right]$$

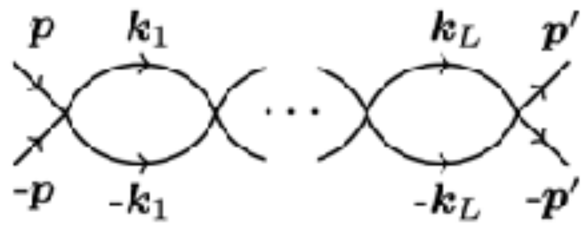
(Matches the one computed by Vines)

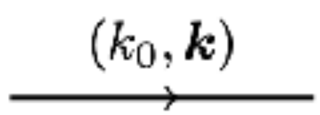
Observables from Amplitudes

Match an EFT to obtain a Hamiltonian

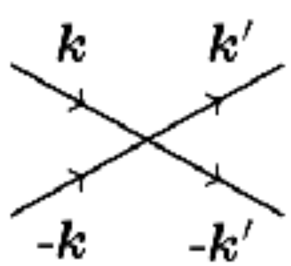
$$\mathcal{M}_n = \mathcal{M}_n^{\mathrm{EFT}}$$

$M_{\mathrm{EFT}}^{L\text{-loop}} =$





$$= \frac{i}{k_0 - \sqrt{\boldsymbol{k}^2 + m_{A,B}^2} + i0}$$

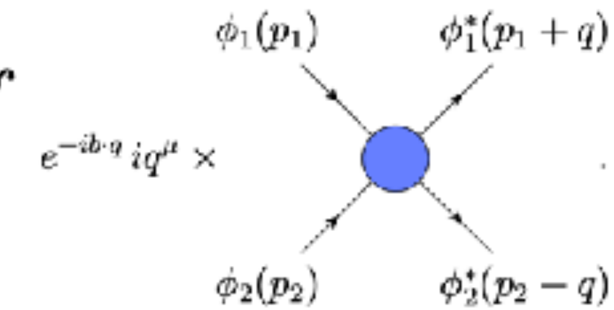


$$= -iV(\boldsymbol{k}, \boldsymbol{k}')\,,$$

KMOC formalism: observables directly from amplitudes

$$\langle \Delta p_1^\mu \rangle = \langle \psi | \, i[\mathbb{P}_1^\mu, T] \, | \psi \rangle + \langle \psi | \, T^\dagger [\mathbb{P}_1^\mu, T] \, | \psi \rangle$$

$$\langle \psi | \, i[\mathbb{P}_1^\mu, T] \, | \psi \rangle = \int e^{-ib\cdot q} \, i q^\mu \times$$



Eikonal-inspired methods

$$\theta_i = -\frac{E}{m_1 m_2 \sqrt{\sigma^2 - 1}} \partial_b \chi_i$$

$$\chi_1 = \frac{1}{4 m_1 m_2 \sqrt{\sigma^2 - 1}} \int \frac{d^{2-2\epsilon}\boldsymbol{q}}{(2\pi)^{2-2\epsilon}} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \mathcal{M}^{\mathrm{tree}}(\boldsymbol{q})$$

Breakthroughs of QFT methods in the PM

- Amplitudes

[Submitted on 20 Dec 2021]

Scattering Amplitudes, the Tail Effect, and Conservative Binary Dynamics at $O(G^4)$

Zvi Bern, Julio Parra-Martinez, Radu Roiban, Michael S. Ruf, Chia-Hsien Shen, Mikhail P. Solon, Mao Zeng

- PMEFT

[Submitted on 11 Oct 2022]

Radiation Reaction and Gravitational Waves at Fourth Post-Minkowskian Order

Christoph Dlapa, Gregor Kälin, Zhengwen Liu, Jakob Neef, Rafael A. Porto

- WQEFT

[Submitted on 19 Jan 2022 (v1), last revised 8 Apr 2022 (this version, v3)]

Conservative and radiative dynamics of spinning bodies at third post-Minkowskian order using worldline quantum field theory

Gustav Uhre Jakobsen, Gustav Mogull

- Eikonal

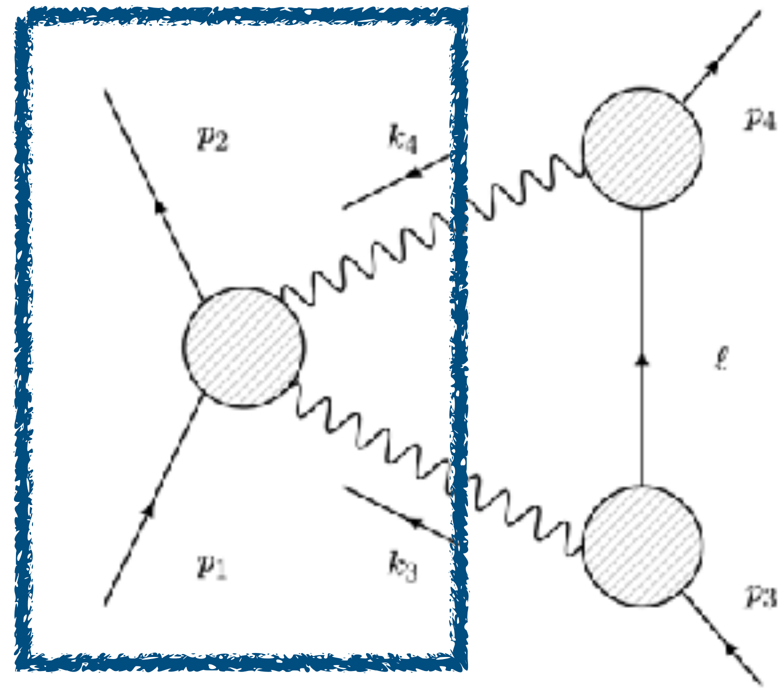
[Submitted on 21 Oct 2022]

Classical Gravitational Observables from the Eikonal Operator

Paolo Di Vecchia, Carlo Heissenberg, Rodolfo Russo, Gabriele Veneziano

2PM spinning observables

For 2PM, need the Compton Amplitude



What is Compton for Kerr?

The minimal coupling Compton is

$$\mathcal{M}_4^{(s)} = -\frac{\langle 4|1|3]^{4-2s}}{(2p_1 \cdot k_4)(2p_2 \cdot k_4)(2k_3 \cdot k_4)} \left([13]\langle 42\rangle + \langle 14\rangle[32]\right)^{2s}$$

Compton for Kerr

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GOV used it to produce an angle

$$\theta = \frac{GE}{v^2}\left[\frac{(1+v)^2}{b+a_a+a_b} + \frac{(1-v)^2}{b-a_a-a_b}\right] - \pi G^2 E \frac{\partial}{\partial b}\Big[m_b f(a_a, a_b) + m_a f(a_b, a_a)\Big] + \mathcal{O}(G^3)$$

$$f(\sigma, a) = \frac{1}{2a^2}\left(-b + \frac{(\jmath + \varkappa - 2a)^5}{4v\varkappa\left[(\jmath+\varkappa)^2 - (2va)^2\right]^{3/2}}\right) + \mathcal{O}(\sigma^5)$$

Stops at S^5

$$\jmath = vb + \sigma + a, \qquad \varkappa = \sqrt{\jmath^2 - 4va(b + v\sigma)}.$$

Compton for Kerr

The minimal coupling Compton is

$$\mathcal{M}_4^{(s)} = -\frac{\langle 4|1|3]^{4-2s}}{(2p_1\cdot k_4)(2p_2\cdot k_4)(2k_3\cdot k_4)}\Big([13]\langle 42\rangle + \langle 14\rangle[32]\Big)^{2s}$$

...sick!

How to produce amplitudes that reproduce the Kerr black hole beyond linear level, beyond S^4?

Curing the sick...

Problem already considered in...

[Submitted on 20 Dec 2018 (v1), last revised 22 Aug 2019 (this version, v3)]

The simplest massive S-matrix: from minimal coupling to Black Holes

Ming-Zhi Chung, Yu-tin Huang, Jung-Wook Kim, Sangmin Lee

but let us look at something more recent...

[Submitted on 30 Jul 2021 (v1), last revised 7 Mar 2022 (this version, v2)]

Compton Black-Hole Scattering for $s \leq 5/2$

Marco Chiodaroli, Henrik Johansson, Paolo Pichini

Using higher-spin theory ideas...

Lagrangians and Compton amplitudes from $P \cdot J = \mathcal{O}(m)$

$$e^{-1}\mathcal{L}_{\min} = \bar{\psi}_{\mu\nu}(i\nabla\!\!\!/ - m)\psi^{\mu\nu} + 2\bar{\psi}_{\mu\nu}\gamma^{\nu}(i\nabla\!\!\!/ + m)\gamma^{\rho}\psi^{\mu}_{\rho} - \frac{1}{2}\bar{\psi}^{\mu}_{\mu}(i\nabla\!\!\!/ - m)\psi^{\rho}_{\rho}$$
$$- (2\bar{\psi}^{\rho\mu}i\nabla_{\rho}\gamma^{\nu}\psi_{\mu\nu} + 2\bar{\psi}_{\mu\nu}\gamma^{\nu}i\nabla_{\rho}\psi^{\rho\mu}) + (\bar{\psi}^{\mu}_{\mu}i\nabla_{\rho}\gamma_{\sigma}\psi^{\rho\sigma} + \bar{\psi}^{\rho\sigma}\gamma_{\sigma}i\nabla_{\rho}\psi^{\mu}_{\mu})$$
$$+ m(\bar{\psi}^{\mu}_{\mu}\lambda + \bar{\lambda}\psi^{\mu}_{\mu}) - \frac{12}{5}\bar{\lambda}(i\nabla\!\!\!/ + 3m)\lambda\,,$$

...conjectured this Lagrangian is a black hole.

Curing the sick...

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Compton Black-Hole Scattering for $s \leq 5/2$

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Using higher-spin theory ideas...

Lagrangians and Compton amplitudes from $P \cdot J = \mathcal{O}(m)$

$$M(1\phi^{5/2}, 2\bar{\phi}^{5/2}, 3h^-, 4h^+) = iN_2 \frac{m^4 Q_4 + m^3 Q_3 + m^2 Q_2 + Q_0}{m^6 s_{12} t_{13} t_{14}}$$

$$Q_3 = 2N_2(\langle\mathbf{12}\rangle + [\mathbf{12}])\big([4|p_1|3\rangle^2(\langle\mathbf{12}\rangle - [\mathbf{12}])^2 + s_{12}N_4\big)\,,$$

$$N_2 = [4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle$$

$$N_4 = [\mathbf{1}4][\mathbf{2}4]\langle\mathbf{1}3\rangle\langle\mathbf{2}3\rangle$$

...conjectured this Amplitude is a black hole.

Is this right?

Mmm, we don’t think so.

S^4 Hamiltonian

[Submitted on 26 Nov 2021 (v1), last revised 18 Aug 2022 (this version, v3)]

The 2PM Hamiltonian for binary Kerr to quartic in spin

Wei-Ming Chen, Ming-Zhi Chung, Yu-tin Huang, Jung-Wook Kim

Look at their S^2 results...

$$M^{(2,0)} = A_{2,0}^{(1)}\mathcal{E}_a^2 + A_{2,0}^{(2)} q^2 (p_2 \cdot S_a)^2$$

$$A_{2,0}^{(1)} = \frac{\left(95\sigma^4 - 102\sigma^2 + 15\right) m_a + 4\left(15\sigma^4 - 15\sigma^2 + 2\right) m_b}{8\left(\sigma^2 - 1\right)^2 m_a^2}$$

$$A_{2,0}^{(2)} = \frac{\left(35\sigma^4 - 30\sigma^2 + 3\right) m_a + 4\left(3\sigma^2 - 1\right) m_b}{8\left(\sigma^2 - 1\right)^2}\,.$$

$$\mathcal{E}_a \equiv \epsilon^{\mu\nu\rho\sigma} p_{1\mu} p_{2\nu} q_\rho S_{a,\sigma}$$

...and ours

[Submitted on 19 Feb 2021]

Quadratic-in-Spin Hamiltonian at $\mathcal{O}(G^2)$ from Scattering Amplitudes

Dimitrios Kosmopoulos, Andres Luna

$$\mathcal{M}^{\triangle+\nabla} = \alpha_2^{(2,4)}(q\cdot S_1)^2 + \alpha_2^{(2,5)} q^2 S_1^2 + \alpha_2^{(2,6)} q^2 (p_2 \cdot S_1)^2$$

$$\alpha_2^{(2,4)} = -\frac{m_2^2}{16(\sigma^2-1)}\Big[-4m_2\big(-\sigma^2+1+C_{ES^2}(30\sigma^4-29\sigma^2+3)\big) - m_1\big(35\sigma^4-30\sigma^2-5+C_{ES^2}(155\sigma^4-174\sigma^2+35)\big)\Big],$$

$$\alpha_2^{(2,5)} = -\frac{m_2^2}{16(\sigma^2-1)}\Big[4m_2\big(15\sigma^4-17\sigma^2+2+C_{ES^2}(15\sigma^4-13\sigma^2+2)\big) + m_1\big(95\sigma^4-102\sigma^2+7+C_{ES^2}(95\sigma^4-102\sigma^2+23)\big)\Big],$$

$$\alpha_2^{(2,6)} = -\frac{1}{8(\sigma^2-1)^2}\Big[2m_2\big(15\sigma^4-14\sigma^2-1+C_{ES^2}(15\sigma^4-10\sigma^2+3)\big) + m_1\big(65\sigma^4-66\sigma^2+1+C_{ES^2}(65\sigma^4-66\sigma^2+17)\big)\Big].$$

Match for some value of Wilson coefficient (black hole).
But they have fewer structures!

Conjectures for black holes

[Submitted on 11 Mar 2022 (v1), last revised 29 Dec 2022 (this version, v3)]

Binary Dynamics Through the Fifth Power of Spin at $\mathcal{O}(G^2)$

Zvi Bern, Dimitrios Kosmopoulos, Andrés Luna, Radu Roiban, Fei Teng

We therefore conjecturally *define* the scattering amplitude of two Kerr black holes as the amplitude which realize the symmetry

$$a_i^\mu \to a_i^\mu + \xi_i q^\mu / q^2, \quad i = 1, 2$$

[Submitted on 11 Mar 2022 (v1), last revised 29 Mar 2022 (this version, v2)]

Searching for Kerr in the 2PM amplitude

Rafael Aoude, Kays Haddad, Andreas Helset

A final question remains, however: what contact terms in the Compton amplitude describe a Kerr black hole? While we cannot definitively answer this question within the bounds of this analysis, we have investigated the 2PM amplitude in the case where the correspondence in eq. (3.7) holds to higher orders in spin.

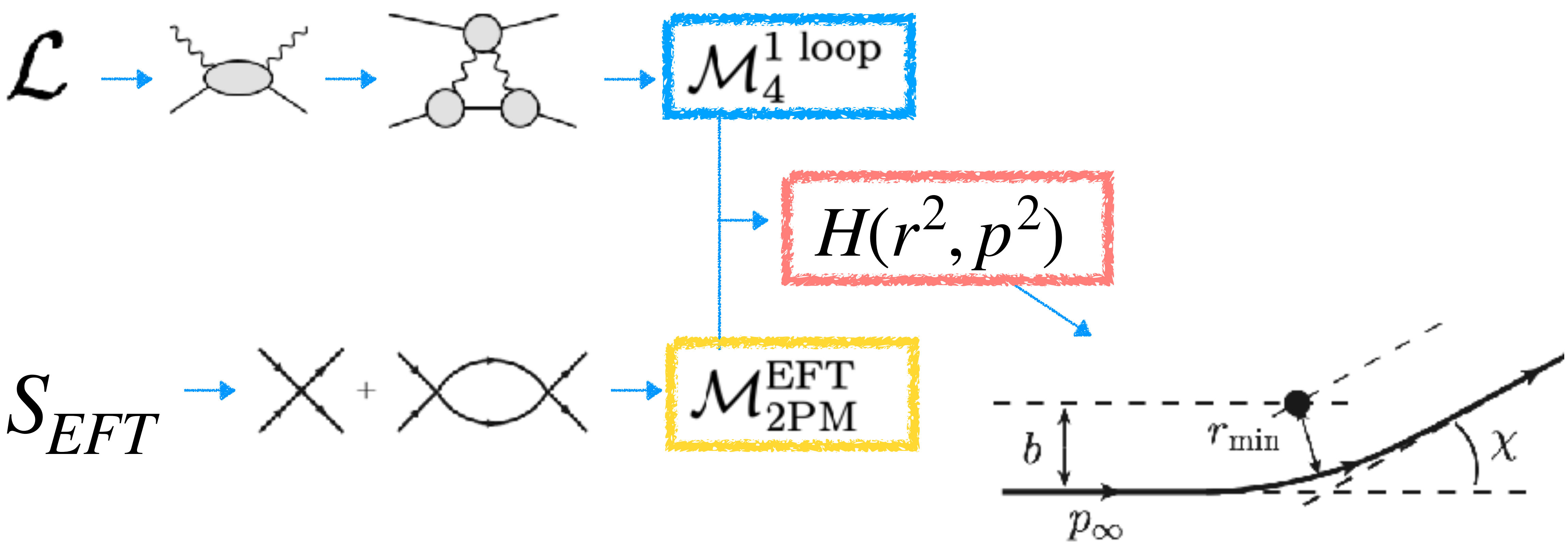
$$(q \cdot \mathfrak{a}_i)(q \cdot \mathfrak{a}_j) - q^2(\mathfrak{a}_i \cdot \mathfrak{a}_j), \qquad i, j = 1, 2,$$

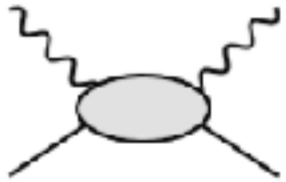
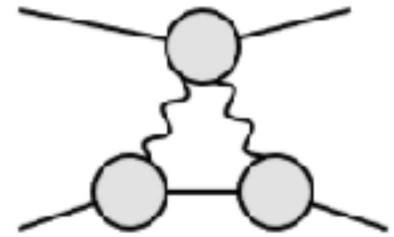
Spin Hamiltonians from Amplitudes

[Submitted on 6 May 2020]

Spinning Black Hole Binary Dynamics, Scattering Amplitudes and Effective Field Theory

Zvi Bern, Andres Luna, Radu Roiban, Chia-Hsien Shen, Mao Zeng



$\mathcal{L}$ →  →  → $\mathcal{M}_4^{1\text{ loop}}$

We consider a Lagrangian of rank-s tensor fields, minimally coupled to gravity

$$\mathcal{L}_{\min} = -R(e,\omega) + \frac{1}{2} g^{\mu\nu} \nabla(\omega)_\mu \phi_s \nabla(\omega)_\nu \phi_s$$

$${\phi_s}^{a_1 a_2 a_3 \ldots a_s}$$

Symmetric traceless tensor field

Covariant derivative

Lorentz Generator

$$\nabla(\omega)_\mu \phi_s = \partial_\mu \phi_s + \frac{i}{2} \omega_{\mu e f} M^{ef} \phi_s$$

Spin vector/tensor

$$\hat{S}^i = \frac{1}{2} \epsilon^{ijk} M_{jk}$$

$$S^i = \frac{1}{2} \epsilon^{ijk} S_{jk}$$

The spin tensor is obtained from the classical limit

$$\varepsilon(\boldsymbol{s}, p_1) M^{ab} \varepsilon(\boldsymbol{s}, p_2) = S(p_1, \boldsymbol{S})^{ab} \varepsilon(\boldsymbol{s}, p_1) \cdot \varepsilon(\boldsymbol{s}, p_2) + \mathcal{O}(q^0)$$

Effective Lagrangians

For non-minimal coupling, we look at the action by Levi-Steinhoff

$$L_{\rm SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

And consider its "covariantization". But EFT demands more operators

$$\mathcal{L}_C = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{\rm ES^{2n}}}{m^{2n}} \nabla_{f_{2n}} \ldots \nabla_{f_3} R_{af_1bf_2} \times \nabla^a \phi_s \, \mathbb{S}^{(f_1} \mathbb{S}^{f_2} \ldots \mathbb{S}^{f_{2n})} \nabla^b \phi_s$$
$$- \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{\rm BS^{2n+1}}}{m^{2n+1}} \nabla_{f_{2n+1}} \ldots \nabla_{f_3} \widetilde{R}_{(a|f_1|b)f_2} \times \nabla^a \phi_s \mathbb{S}^{(f_1} \mathbb{S}^{f_2} \ldots \mathbb{S}^{f_{2n+1})} \nabla^b \phi_s \,,$$

$$\mathcal{L}_H = -\sum_{n=1}^{\infty} \frac{(-1)^n (2n-1)}{(2n)!(2n+1)} \frac{H_{2n}}{m^{2n-2}} \nabla_{f_{2n}} \ldots \nabla_{f_3} R^{(a}{}_{f_1}{}^{b)}{}_{f_2} \times \phi_s M_a{}^{(f_1} M_b{}^{f_2} \mathbb{S}^{f_3} \ldots \mathbb{S}^{f_{2n})} \phi_s$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n n}{(2n+1)!(n+1)} \frac{H_{2n+1}}{m^{2n-1}} \nabla_{f_{2n+1}} \ldots \nabla_{f_3} \widetilde{R}^{(a}{}_{f_1}{}^{b)}{}_{f_2} \times \phi_s M_a{}^{(f_1} M_b{}^{f_2} \mathbb{S}^{f_3} \ldots \mathbb{S}^{f_{2n+1})} \phi_s \,.$$

Effective Amplitudes

$$\mathcal{M}^{\triangle+\nabla} = \frac{2\pi^2 G^2 \varepsilon_1 \cdot \varepsilon_4 \varepsilon_2 \cdot \varepsilon_3}{\sqrt{-q^2}} \sum_n \sum_i \alpha^{(n,i)} \mathcal{O}^{(n,i)} .$$

$$\alpha^{(2,i)} = \frac{m_1^2 m_2^2}{16(-1+\sigma^2)^2}(\gamma^{(2,i)} m_1 + \delta^{(2,i)} m_2) ,$$

$$\alpha^{(3,i)} = \frac{m_1^2 m_2^2 \sigma}{8(-1+\sigma^2)^2}(\gamma^{(3,i)} m_1 + 2\delta^{(3,i)} m_2) ,$$

	i		i		i	
$\mathcal{O}^{(2,i)}$	1	$\mathcal{E}_1^2$	2	$q^2(u_2\cdot a_1)^2$	3	$(q\cdot a_1)^2$
$\mathcal{O}^{(4,i)}$	1	$\mathcal{E}_1^4$	2	$q^2(u_2\cdot a_1)^2\mathcal{E}_1^2$	3	$q^4(u_2\cdot a_1)^4$
	4	$(q\cdot a_1)^2\mathcal{E}_1^2$	5	$q^2(q\cdot a_1)^2(u_2\cdot a_1)^2$	6	$(q\cdot a_1)^4$

$Z_{2,1} = C_2 + 1$	$Z_{2,2} = C_2 - 1$
$Z_{3,1} = 3C_2 + C_3$	$Z_{3,2} = C_2 - C_3$
$Z_{4,1} = 3C_2^2 + 4C_3 + C_4$ $Z_{4,2} = 3C_2^2 + C_4$	$Z_{4,3} = C_2^2 - C_4$ $Z_{4,4} = 3C_2^2 - 4C_3 + C_4$
$Z_{5,1} = 10C_2C_3 + 5C_4 + C_5$	$Z_{5,2} = 2C_2C_3 - C_4 - C_5$ $Z_{5,3} = 2C_2C_3 - 3C_4 + C_5$

i	$\gamma^{(2,i)}$	i	$\gamma^{(2,i)}$
1	$7 + 23C_2 - Z_{2,1}\sigma^2(102 - 95\sigma^2)$	3	$12Z_{2,2}(\sigma^2-1)^2(5\sigma^2-1)$
2	$5 - 11C_2 + 5Z_{2,1}\sigma^2(6 - 7\sigma^2)$		
i	$\gamma^{(3,i)}$	i	$\gamma^{(3,i)}$
1	$Z_{3,1}(5 - 9\sigma^2)$	3	$4Z_{3,2}(\sigma^2-1)(5\sigma^2-3)$
2	$Z_{3,1}(7\sigma^2 - 3)$		
i	$\gamma^{(4,i)}$	i	$\gamma^{(4,i)}$
1	$44C_3 + 59Z_{4,2} - Z_{4,1}\sigma^2(250-239\sigma^2)$	4	$12Z_{4,3}(1-\sigma^2)(23-102\sigma^2+95\sigma^4)$
2	$72C_3 - 78Z_{4,2} + Z_{4,1}\sigma^2(276-294\sigma^2)$	5	$12Z_{4,3}(\sigma^2-1)(11-30\sigma^2+35\sigma^4)$
3	$28C_3 - 9Z_{4,2} + 7Z_{4,1}\sigma^2(2-3\sigma^2)$	6	$24Z_{4,4}(\sigma^2-1)^3(5\sigma^2-1)$
i	$\gamma^{(5,i)}$	i	$\gamma^{(5,i)}$
1	$Z_{5,1}(7 - 13\sigma^2)$	4	$12Z_{5,2}(\sigma^2-1)(9\sigma^2-5)$
2	$2Z_{5,1}(11\sigma^2 - 5)$	5	$12Z_{5,2}(\sigma^2-1)(3-7\sigma^2)$
3	$Z_{5,1}(3\sigma^2 - 1)$	6	$8Z_{5,3}(\sigma^2-1)^2(3-5\sigma^2)$

$\delta_0^{(5,i)}$	1	[illegible]
	2	[illegible]
	3	[illegible]
	4	[illegible]
	5	[illegible]
	6	[illegible]
$\delta_1^{(5,i)}$	1	[illegible]
	2	[illegible]
	3	[illegible]
	4	[illegible]
	5	[illegible]
	6	[illegible]
$\delta_2^{(5,i)}$	1	[illegible]
	2	[illegible]
	3	[illegible]
	4	[illegible]
	5	[illegible]
	6	[illegible]
$\delta_3^{(5,i)}$	4	[illegible]
	5	[illegible]
	6	[illegible]
$\delta_4^{(5,i)}$	6	[illegible]

$$\delta^{(k,i)} = \sum_{l=0}^{5} \delta_l^{(k,i)} \sigma^{2l}$$

Using spin-shift symmetry

$$a_i^\mu \to a_i^\mu + \xi_i q^\mu / q^2, \quad i = 1, 2$$

$$\frac{1}{75}\delta_{\text{Kerr}}^{(5,1)} = 24(1 - 4\sigma^2) ,$$

$$\frac{1}{75}\delta_{\text{Kerr}}^{(5,2)} = 48(2 + \sigma^2) ,$$

$$\frac{1}{75}\delta_{\text{Kerr}}^{(5,3)} = 8(12 - 16\sigma^2 + 7\sigma^4) .$$

We conjectured this is a black hole.

On-shell approach

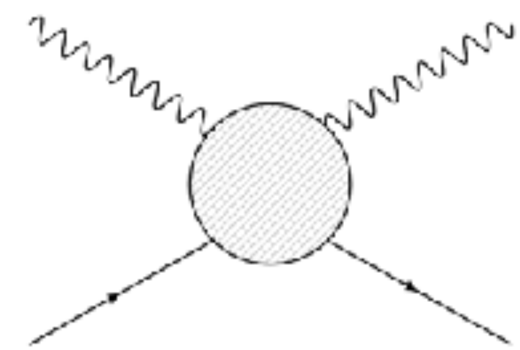
[Submitted on 11 Mar 2022 (v1), last revised 29 Mar 2022 (this version, v2)]

Searching for Kerr in the 2PM amplitude

Rafael Aoude, Kays Haddad, Andreas Helset

The (classical part of the) Compton amplitude

$$\mathcal{M}_4^{(s)} = -\frac{\langle 4|1|3]^{4-2s}}{(2p_1\cdot k_4)(2p_2\cdot k_4)(2k_3\cdot k_4)}\left([13]\langle 42\rangle + \langle 14\rangle[32]\right)^{2s}$$



...can be written in terms of the classical spin vector

$$\mathcal{A}^s = \mathcal{A}^0 \exp\left\{\left[q_4 - q_3 + \frac{(t_{14}-t_{13})}{y}w\right]\cdot \mathfrak{a}\right\}$$

$$w^\mu \equiv [4|\bar{\sigma}^\mu|3\rangle/2.$$

$$t_{1i} = (p_1 - q_i)^2 - m^2,\; s_{34} = (q_3+q_4)^2 \qquad y \equiv [4|p_1|3\rangle$$

$$\mathcal{M}^s_{\rm cl} = e^{(q_4-q_3)\cdot\mathfrak{a}}\sum_{n=0}^{2s}\frac{1}{n!}K_n, \qquad K_n \equiv \frac{y^4}{s_{34}t_{13}t_{14}}\left(\frac{t_{14}-t_{13}}{y}w\cdot\mathfrak{a}\right)^n$$

On-shell approach

Get the most general, spurious-pole-free Compton amplitude...

$$\mathcal{M}_{\rm cl}^{s} = e^{(q_4-q_3)\cdot\mathfrak{a}} \sum_{n=0}^{2s} \frac{1}{n!} \bar{K}_n,$$

$$\bar{K}_n \equiv \begin{cases} K_n, & n<4, \\ K_4 + m^2 (w\cdot\mathfrak{a})^4 d_0^{(4)}, & n=4, \\ K_3 L_{n-3} - K_2 \mathfrak{s}_2 L_{n-4} + m^2 (w\cdot\mathfrak{a})^4 \sum_{j=0}^{\lfloor (n-4)/2 \rfloor} d_j^{(n)} \mathfrak{s}_1^{n-4-2j} \mathfrak{s}_2^j, & n>4. \end{cases}$$

$$K_n \equiv \frac{y^4}{s_{34} t_{13} t_{14}} \left(\frac{t_{14}-t_{13}}{y} w\cdot\mathfrak{a} \right)^n \qquad L_m \equiv \sum_{j=0}^{\lfloor m/2 \rfloor} \binom{m+1}{2j+1} \mathfrak{s}_1^{m-2j} (\mathfrak{s}_1^2 - \mathfrak{s}_2)^j \qquad \begin{aligned} \mathfrak{s}_1 &\equiv (q_3 - q_4)\cdot\mathfrak{a}, \\ \mathfrak{s}_2 &\equiv -4(q_3\cdot\mathfrak{a})(q_4\cdot\mathfrak{a}) + s_{34}\mathfrak{a}^2 \end{aligned}$$

...such that the 2PM result satisfies the “black hole spin structure”?

$$(q\cdot\mathfrak{a}_i)(q\cdot\mathfrak{a}_j) - q^2(\mathfrak{a}_i\cdot\mathfrak{a}_j), \qquad i,j=1,2,$$

“minimal coupling”	from S=5/2 Lagrangian
$d_0^{(4)} = 0$	$d_0^{(4)} = -16/5$

Conjectures for black holes

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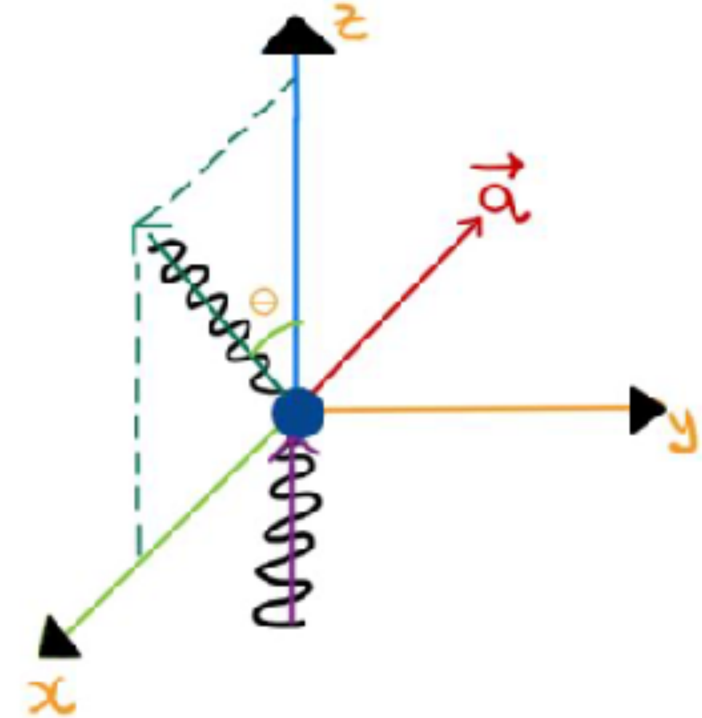
$$(q \cdot \mathfrak{a}_i)(q \cdot \mathfrak{a}_j) - q^2(\mathfrak{a}_i \cdot \mathfrak{a}_j), \qquad i, j = 1, 2,$$

Were these right?

Apparently not, either…

Compton from BHPT



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Scattering in Black Hole Backgrounds and Higher-Spin Amplitudes: Part II

Yilber Fabian Bautista, Alfredo Guevara, Chris Kavanagh, Justin Vines

A most general form of the solution
(compatible with crossing symmetry, locality, unitarity)

$$\langle A_4^S \rangle = \langle A_4^0 \rangle \times \left(e^{(2w+k_3-k_2)\cdot a} + P_\xi(k_2\cdot a, -k_3\cdot a, w\cdot a) \right)_{2S}$$

(This polynomial is up to S^5)

$$P_\xi = \sum_{m=0}^{2} \xi^{m-1} (w\cdot a)^{4-2m} (w\cdot a - k_2\cdot a)^m (w\cdot a + k_3\cdot a)^m r_{|a|}^{(m)} (k_2\cdot a, -k_3\cdot a, w\cdot a)$$

$$r_{|a|}^{(m)} = c_1^{(m)} + c_2^{(m)} (k_2{\cdot}a - k_3{\cdot}a) + c_3^{(m)} w{\cdot}a + c_4^{(m)} |a|\omega$$

Compton from BHPT

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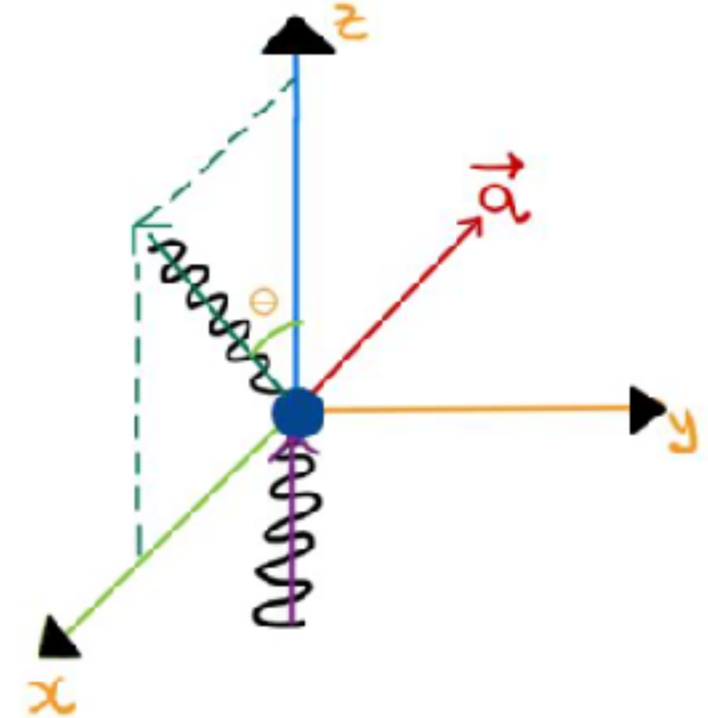


Matching solutions of the Teukolsky equation

$$f'^{\text{QFT}}_{lm}(\gamma) = f'^{\text{BHPT}}_{lm}(\gamma)\,,$$

$$f'^{\text{QFT}}_{lm}(\gamma) = \int d\Omega' \, {}_{-2}Y^*_{lm}(\theta,\phi')\langle A_4(\gamma,\theta,\phi')\rangle$$

$$\langle A_4^S\rangle = \langle A_4^0\rangle \times \left(e^{(2w+k_3-k_2)\cdot a} + P_\xi(k_2\cdot a, -k_3\cdot a, w\cdot a)\right)_{2S}$$

$$f'^{(\text{BHPT})}_{lm}(\gamma) = \sum_{m'} D^{l*}_{m'm}(\gamma)\, f_{lm'}$$

$$f_{lm} = \frac{2\pi}{i\omega}\sum_{P=\pm1}\left(e^{2i\delta^P_{lm}} - 1\right)$$

$$e^{2i\delta^P_{lm}} = (-1)^{l+1}\frac{\mathcal{C}_{lm} + 12iM\omega P}{16\omega^4}\frac{B^{\text{ref}}_{lm\omega}}{B^{\text{inc}}_{lm\omega}}$$

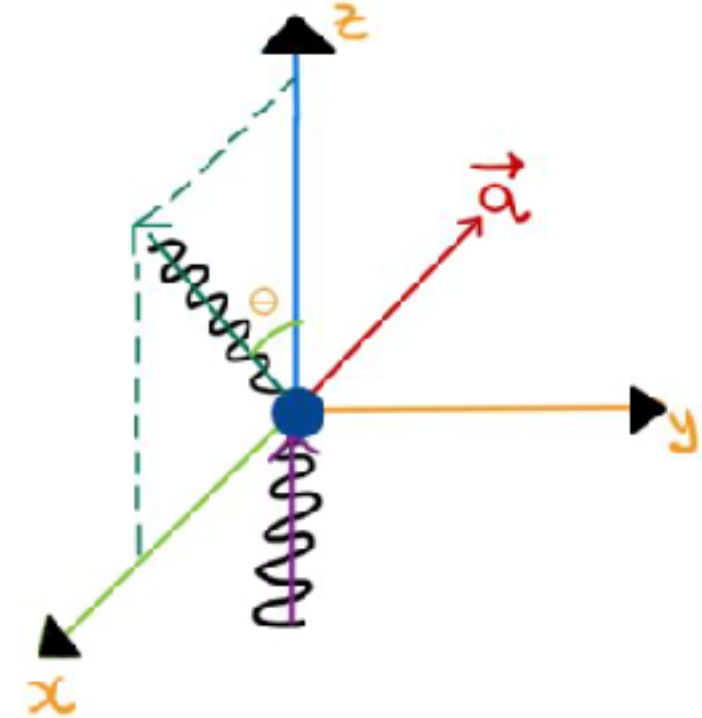
Spin	Spurious-pole	Free Coeffs.	Teukolsky Solutions
a^4		$c_1^{(i)},\ i=0,1,2$	$c_1^{(i)}=0,\ i=0,1,2$
a^5	$c_3^{(2)} = 4/15 - c_3^{(0)} + c_3^{(1)}$	$c_2^{(i)},\ i=0,1,2$ $c_3^{(i)},\ i=0,1$ $c_4^{(i)},\ i=0,1,2$	$c_2^{(i)}=0,\ i=0,1,2$ $c_3^{(0)}=\alpha\frac{64}{15},\ c_3^{(1)}=\alpha\frac{16}{3},$ $c_3^{(2)}=\frac{4}{15}(1+4\alpha),$ $c_4^{(0)}=\eta\alpha\frac{64}{15},$ $c_4^{(1)}=\eta\alpha\frac{16}{5},\ c_4^{(2)}=\eta\frac{4}{15}$

Compton from BHPT

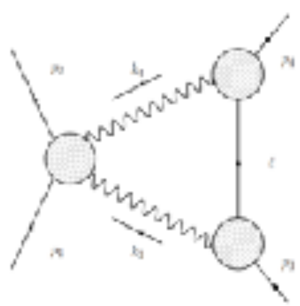
[Submitted on 15 Dec 2022 (v1), last revised 21 Feb 2023 (this version, v2)]

Scattering in Black Hole Backgrounds and Higher-Spin Amplitudes: Part II

Yilber Fabian Bautista, Alfredo Guevara, Chris Kavanagh, Justin Vines



Are these solutions compatible with the shift symmetry?



$$a_i^\mu \to a_i^\mu + \xi_i q^\mu / q^2, \quad i = 1, 2$$

Spin	Shift-Sym.	Free Coeffs.	Relation to [20]
a^4	$c_1^{(i)} = 0,\, i = 1,2$	$c_1^{(0)}$	$c_1^{(0)} = -\frac{d_0^{(4)}}{4!}$
a^5	$c_j^{(i)} = 0,\; i = 1,2,\; j = 2,3$ $c_3^{(0)} = \frac{4}{15},\; c_4^{(i)} = 0\,,\, i = 0,1,2$	$c_2^{(0)}$	$c_2^{(0)} = \frac{32+5d_0^{(4)} - d_0^{(5)}}{5!}$

it turns out, solutions of the Teukolsky equation, at the given order in spin a^n, do not preserve such symmetry for $n > 4$.

Why should we trust these results?

They have been checked against GR in several regimes

Checks: Post-Newtonian

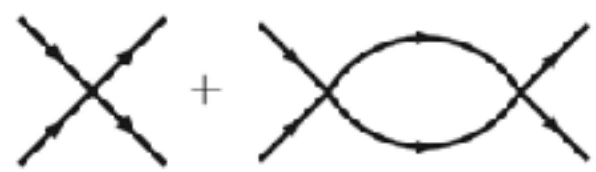
[Submitted on 14 Jul 2016 (v1), last revised 22 Sep 2021 (this version, v2)]

Complete conservative dynamics for inspiralling compact binaries with spins at the fourth post-Newtonian order

Michèle Levi, Jan Steinhoff

$$
\begin{aligned}
H^{S^4}_{\rm NNLO} = \nu^2 \Bigg\{ \Bigg[& \frac{\tilde L^4}{16\tilde r^7}(11-25\nu) + \frac{\tilde L^2}{16\tilde r^6}(159+33\nu) - \frac{24}{7\tilde r^5} + \frac{\tilde L^2 \tilde p_r^2}{16\tilde r^5}(7-74\nu) - \frac{9\tilde p_r^2}{16\tilde r^4}(3-5\nu) \\
& - \frac{\tilde p_r^4}{4\tilde r^3}(1+\nu)\Bigg]\vec S_1^2 - \Bigg[\frac{3\tilde L^4}{16\tilde r^7}(6-17\nu) + \frac{\tilde L^2}{16\tilde r^6}(329+61\nu) - \frac{37}{7\tilde r^5} + \frac{\tilde L^2\tilde p_r^2}{16\tilde r^5}(26-145\nu) \\
& - \frac{\tilde p_r^2}{16\tilde r^4}(3-85\nu) - \frac{\tilde p_r^4}{4\tilde r^3}(1+\nu)\Bigg](\vec n\cdot\vec S_1)^2 - \Bigg[\frac{\tilde L^4}{16\tilde r^7}(11-25\nu) + \frac{\tilde L^2}{8\tilde r^6}(91+11\nu) \\
& - \frac{\tilde L^2\tilde p_r^2}{16\tilde r^5}(1+85\nu)\Bigg](\vec S_1\cdot\vec\lambda)^2 - \Bigg[\frac{\tilde L^3\tilde p_r}{16\tilde r^6}(1+37\nu) - \frac{\tilde L\tilde p_r}{8\tilde r^5}(93+25\nu) \\
& - \frac{\tilde L\tilde p_r^3}{16\tilde r^4}(23-67\nu)\Bigg]\vec n\cdot\vec S_1\vec S_1\cdot\vec\lambda\Bigg\} + \nu^2 C_{1(\rm ES^2)}\Bigg\{\Bigg[\frac{\tilde L^4}{4\tilde r^7}(1+3\nu) - \frac{\tilde L^2}{4\tilde r^6}(22+5\nu) + \frac{55}{14\tilde r^5} \\
& - \frac{\tilde L^2\tilde p_r^2}{4\tilde r^5}(1-5\nu) + \frac{\tilde p_r^2}{4\tilde r^4}(9-13\nu) - \frac{\tilde p_r^4}{2\tilde r^3}(1+6\nu)\Bigg]\vec S_1^2 + \Bigg[\frac{7\tilde L^2}{4\tilde r^6}(4+\nu) - \frac{95}{14\tilde r^5} + \frac{\tilde L^2\tilde p_r^2}{8\tilde r^5}(7+6\nu) \\
& + \frac{3\tilde p_r^2}{4\tilde r^4}(9+11\nu) + \frac{\tilde p_r^4}{2\tilde r^3}(1+6\nu)\Bigg](\vec n\cdot\vec S_1)^2 - \Bigg[\frac{\tilde L^4}{4\tilde r^7}(1+2\nu) - \frac{\nu\tilde L^2}{2\tilde r^6} + \frac{\tilde L^2\tilde p_r^2}{8\tilde r^5}(5+18\nu)\Bigg](\vec S_1\cdot\vec\lambda)^2 \\
& + \Bigg[\frac{5\tilde L^3\tilde p_r}{8\tilde r^6} + \frac{\tilde L\tilde p_r}{4\tilde r^5}(8-11\nu) - \frac{\tilde L\tilde p_r^3}{8\tilde r^4}(1-6\nu)\Bigg]\vec n\cdot\vec S_1\vec S_1\cdot\vec\lambda\Bigg\} + \frac{\nu}{q}\Bigg\{\Bigg[-\frac{\tilde L^4}{16\tilde r^7}(11-59\nu+21\nu^2) \\
& - \frac{\tilde L^2}{16\tilde r^6}(139-93\nu-33\nu^2) - \frac{1}{2\tilde r^5}(7+2\nu) - \frac{\tilde L^2\tilde p_r^2}{16\tilde r^5}(7-40\nu+60\nu^2) - \frac{3\tilde p_r^2}{16\tilde r^4}(1+63\nu+15\nu^2) \\
& + \frac{\tilde p_r^4}{16\tilde r^3}(4-4\nu-5\nu^2)\Bigg]\vec S_1^2 + \Bigg[\frac{3\tilde L^4}{16\tilde r^7}(6-37\nu+15\nu^2) + \frac{\tilde L^2}{16\tilde r^6}(269-257\nu-61\nu^2) \\
& + \frac{\tilde L^2\tilde p_r^2}{16\tilde r^5}(26-101\nu+126\nu^2) + \frac{1}{2\tilde r^5}(7+15\nu) - \frac{\tilde p_r^2}{16\tilde r^4}(3+93\nu+85\nu^2) \\
& - \frac{\tilde p_r^4}{16\tilde r^3}(4-4\nu-5\nu^2)\Bigg](\vec n\cdot\vec S_1)^2 + \Bigg[\frac{\tilde L^4}{16\tilde r^7}(11-59\nu+21\nu^2) + \frac{\tilde L^2}{8\tilde r^6}(97-37\nu-11\nu^2) \\
& - \frac{\tilde L^2\tilde p_r^2}{16\tilde r^5}(1+35\nu-69\nu^2)\Bigg](\vec S_1\cdot\vec\lambda)^2 + \Bigg[\frac{\tilde L^3\tilde p_r}{16\tilde r^6}(1+47\nu-33\nu^2) - \frac{\tilde L\tilde p_r}{8\tilde r^5}(69-9\nu-25\nu^2) \\
& - \frac{\tilde L\tilde p_r^3}{16\tilde r^4}(23-65\nu+57\nu^2)\Bigg]\vec n\cdot\vec S_1\vec S_1\cdot\vec\lambda\Bigg\} + \frac{\nu}{q}C_{1(\rm ES^2)}\Bigg\{\Bigg[\frac{\tilde L^4}{16\tilde r^7}(1-23\nu+9\nu^2) \\
& + \frac{\tilde L^2}{8\tilde r^6}(64+73\nu-10\nu^2) - \frac{1}{4\tilde r^5}(13+27\nu) + \frac{\tilde L^2\tilde p_r^2}{8\tilde r^5}(7-34\nu+6\nu^2) - \frac{\nu\tilde p_r^2}{8\tilde r^4}(141+26\nu) \\
& + \frac{\tilde p_r^4}{16\tilde r^3}(13+20\nu-72\nu^2)\Bigg]\vec S_1^2 - \Bigg[\frac{\tilde L^3\tilde p_r}{8\tilde r^6}(5+6\nu-9\nu^2) + \frac{\tilde L\tilde p_r}{4\tilde r^5}(16-156\nu+11\nu^2) \\
& + \frac{\tilde L\tilde p_r^3}{8\tilde r^4}(-1-24\nu+27\nu^2)\Bigg]\vec n\cdot\vec S_1\vec S_1\cdot\vec\lambda - \Bigg[\frac{3\tilde L^4}{16\tilde r^7}(5-16\nu-3\nu^2) - \frac{\tilde L^2}{4\tilde r^6}(60+11\nu-7\nu^2) \\
& - \frac{1}{4\tilde r^5}(37+21\nu) + \frac{\tilde L^2\tilde p_r^2}{8\tilde r^5}(22-66\nu-21\nu^2) + \frac{3\tilde p_r^2}{8\tilde r^4}(35-26\nu-22\nu^2) \\
& + \frac{\tilde p_r^4}{16\tilde r^3}(23-36\nu-120\nu^2)\Bigg](\vec n\cdot\vec S_1)^2 + \Bigg[\frac{\tilde L^4}{4\tilde r^7}(1+3\nu-2\nu^2) + \frac{\tilde L^2}{8\tilde r^6}(4-57\nu+4\nu^2) \\
& + \frac{\tilde L^2\tilde p_r^2}{8\tilde r^5}(5+12\nu-15\nu^2)\Bigg](\vec S_1\cdot\vec\lambda)^2\Bigg\} + [1\leftrightarrow 2], \qquad (3.6)
\end{aligned}
$$

To compare with (overlapping parts of) them, we may compute Amplitudes from the Hamiltonian using EFT.



$$
\begin{aligned}
a_2^{(2,4)} = \frac{m_1 m_2^3 C_{ES^2}}{4(m_1+m_2)p^2} &+ \frac{m_2\big(10m_1^2 - 7m_1m_2 - 13m_2^2 + C_{ES^2}(32m_1^2 + 61m_1m_2 + 29m_2^2)\big)}{16m_1(m_1+m_2)} \\
&+ \frac{15m_1^4 - 73m_1^3m_2 - 361m_1^2m_2^2 - 343m_1m_2^3 - 82m_2^4}{64m_1^3m_2(m_1+m_2)}p^2 \\
&+ \frac{C_{ES^2}(93m_1^4 + 467m_1^3m_2 + 707m_1^2m_2^2 + 397m_1m_2^3 - 64m_2^4)}{64m_1^3m_2(m_1+m_2)}p^2 + \ldots,
\end{aligned}
$$

[Submitted on 25 Nov 2022]

Completing the Fifth PN Precision Frontier via the EFT of Spinning Gravitating Objects

Michèle Levi, Zhewei Yin

Our scattering angles agree with those for BHs in [41], that is when all spin-induced multipolar Wilson coefficients are set to unity, as stipulated in [24] for $C_{\rm ES^2} = C_{\rm BS^3} = C_{\rm ES^4} = 1$, and more generally postulated in our conjecture 2 in section 2 above, which also fixes $C_{\rm E^2S^4} = 1$.[3]

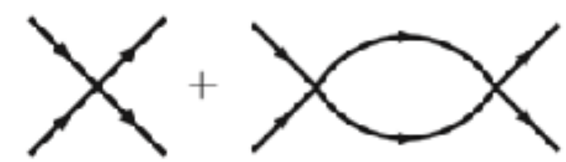
Checks: Test body Hamiltonian

[Submitted on 27 Jan 2016 (v1), last revised 28 Jun 2021 (this version, v3)]

Canonical Hamiltonian for an extended test body in curved spacetime: To quadratic order in spin

Justin Vines, Daniela Kunst, Jan Steinhoff, Tanja Hinderer

To compare with (overlapping parts of) them, we may compute Amplitudes from the Hamiltonian using EFT.



$$H_{S^2} = \frac{\sqrt{w}M}{2\hat{Q}r^3}\left[\vec{S}^2 k_1(\vec{x},\vec{P}) + (\vec{n}\cdot\vec{S})^2 k_2(\vec{x},\vec{P}) + \frac{(\vec{P}\cdot\vec{S})^2}{m^2}k_3(\vec{x},\vec{P}) + \frac{\vec{n}\cdot\vec{P}\,\vec{n}\cdot\vec{S}\,\vec{P}\cdot\vec{S}}{m^2}k_4(\vec{x},\vec{P})\right]$$

$$k_1 = -C + \frac{\vec{P}^2}{m^2}\left(3(1-C) - \frac{2m^2}{(\hat{Q}+m)^2}\right) + \frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2}\left(3C - 1 + w + 3(C-1)\frac{\hat{Q}(\hat{Q}+2m)}{m^2}\right),$$

$$k_2 = \frac{3C\hat{Q}^2}{m^2} + \frac{3\vec{P}^2}{m^2}\left(C + \frac{\vec{P}^2 - \hat{Q}(3\hat{Q}+4m)}{(\hat{Q}+m)^2}\right) + 3(1-\sqrt{w})^2(1+2\sqrt{w}+Cw)\frac{(\vec{n}\cdot\vec{P})^4}{m^2(\hat{Q}+m)^2}$$
$$+ \frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2}\left(-(1-\sqrt{w})^2 + 5(w-1)\frac{\vec{P}^2}{m^2} + 6(1-\sqrt{w})(1+C\sqrt{w})\frac{\hat{Q}(\hat{Q}+m)}{m^2}\right),$$

$$k_3 = 3(C-1)\left(1 + w\frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2}\right) + \frac{2m^2}{(\hat{Q}+m)^2},$$

$$k_4 = 6(1-C)\frac{(1+\sqrt{w})\hat{Q}^2 + (2+\sqrt{w})m\hat{Q}}{(\hat{Q}+m)^2} - 2(3C-1+\sqrt{w})\frac{m^2}{(\hat{Q}+m)^2} + 6(1-C)(1-\sqrt{w})w\frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2}$$

$$a_2^{(2,4)} = \frac{m_2{}^2\left(C_{ES^2}\left(30\gamma_1{}^4 - 29\gamma_1{}^2 + 3\right) - 30\gamma_1{}^4 + 59\gamma_1{}^2 - 24\gamma_1 - 5\right)}{16\gamma_1\left(\gamma_1{}^2 - 1\right)m_1},$$

$$a_2^{(2,5)} = -\frac{m_2{}^2\left(C_{ES^2}\left(15\gamma_1{}^4 - 13\gamma_1{}^2 + 2\right) - 15\gamma_1{}^4 + 43\gamma_1{}^2 - 24\gamma_1 - 4\right)}{16\gamma_1\left(\gamma_1{}^2 - 1\right)m_1},$$

$$a_2^{(2,6)} = \frac{m_2{}^2\left(C_{ES^2}\left(15\gamma_1{}^4 - 10\gamma_1{}^2 + 3\right) - 15\gamma_1{}^4 + 46\gamma_1{}^2 - 24\gamma_1 - 7\right)}{16\gamma_1\left(\gamma_1{}^2 - 1\right)^2 m_1{}^3},$$

$$a_2^{(2,\tilde{4})} = \frac{\left(95\gamma_1{}^4 - 102\gamma_1{}^2 + 15\right)m_1}{32\gamma_1\left(\gamma_1{}^2 - 1\right)}, \qquad a_2^{(2,\tilde{5})} = \frac{95\gamma_1{}^4 m_1 - 102\gamma_1{}^2 m_1 + 15m_1}{32\gamma_1 - 32\gamma_1{}^3},$$

$$a_2^{(2,\tilde{6})} = \frac{65\gamma_1{}^4 - 66\gamma_1{}^2 + 9}{16\gamma_1\left(\gamma_1{}^2 - 1\right)^2 m_1},$$

Checks: Test body / Self Force

[Submitted on 16 Sep 2019]

Test black holes, scattering amplitudes and perturbations of Kerr spacetime

Nils Siemonsen, Justin Vines

Consider a generalization of MPD equations

$$\frac{\mathrm{D}}{\mathrm{d}\tau}p_\mu + \frac{1}{2}R_{\mu\nu\kappa\lambda}\dot{z}^\nu S^{\kappa\lambda} = \frac{p\cdot\dot{z}}{2}\frac{\mathrm{D}}{\mathrm{D}z^\mu}\log\mathcal{M}^2,$$

$$\frac{\mathrm{D}}{\mathrm{d}\tau}S^{\mu\nu} - 2p^{[\mu}\dot{z}^{\nu]} = p\cdot\dot{z}\left(p^{[\mu}\frac{\partial}{\partial p_{\nu]}} + 2S^{[\mu}{}_\rho\frac{\partial}{\partial S_{\nu]\rho}}\right)\log\mathcal{M}^2,$$

With a dynamical mass function

$$\mathcal{M}^2_{\mathrm{GOV}} = m^2 + 2m^2u^\mu u^\nu\sigma^{\rho_1}\sigma^{\rho_2}\left(-\frac{1}{2!}R_{\mu\rho_1\nu\rho_2} + \frac{1}{3!}{}^*R_{\mu\rho_1\nu\rho_2;\rho_3}\sigma^{\rho_3} + \frac{1}{4!}R_{\mu\rho_1\nu\rho_2;\rho_3\rho_4}\sigma^{\rho_3}\sigma^{\rho_4}\right) + \mathcal{O}(\sigma^5)$$

Quadratic in curvature terms are considered

$$\begin{aligned}\frac{1}{m^2}\delta(\mathcal{M}^2)_4 = \;& C_{4\mathrm{A}}\,(\mathcal{E}_{\mu\nu}\sigma^\mu\sigma^\nu)^2 \\ & + C_{4\mathrm{B}}\,\mathcal{E}_{\mu\lambda}\mathcal{E}_\nu{}^\lambda\sigma^\mu\sigma^\nu(-\sigma^2) \\ & + C_{4\mathrm{C}}\,\mathcal{E}_{\kappa\lambda}\mathcal{E}^{\kappa\lambda}(-\sigma^2)^2 \\ & + C_{4\mathrm{D}}\,(\mathcal{B}_{\mu\nu}\sigma^\mu\sigma^\nu)^2 \\ & + C_{4\mathrm{E}}\,\mathcal{B}_{\mu\lambda}\mathcal{B}_\nu{}^\lambda\sigma^\mu\sigma^\nu(-\sigma^2) \\ & + C_{4\mathrm{F}}\,\mathcal{B}_{\kappa\lambda}\mathcal{B}^{\kappa\lambda}(-\sigma^2)^2, \\ & + C_{4\mathrm{G}}\,\ddot{\mathcal{E}}_{\mu\nu}\sigma^\mu\sigma^\nu(-\sigma^2),\end{aligned}$$

They enter in the combinations

$$C_{4\mathrm{a}} = C_{4\mathrm{A}} + C_{4\mathrm{B}}, \qquad C_{4\mathrm{c}} = C_{4\mathrm{C}}, \qquad C_{4\mathrm{e}} = C_{4\mathrm{E}} + \frac{C_{4\mathrm{F}}}{2},$$

Demanding good high energy behavior

$$C_{4\mathrm{a}} + 2C_{4\mathrm{c}} + C_{4\mathrm{e}} \doteq 0$$

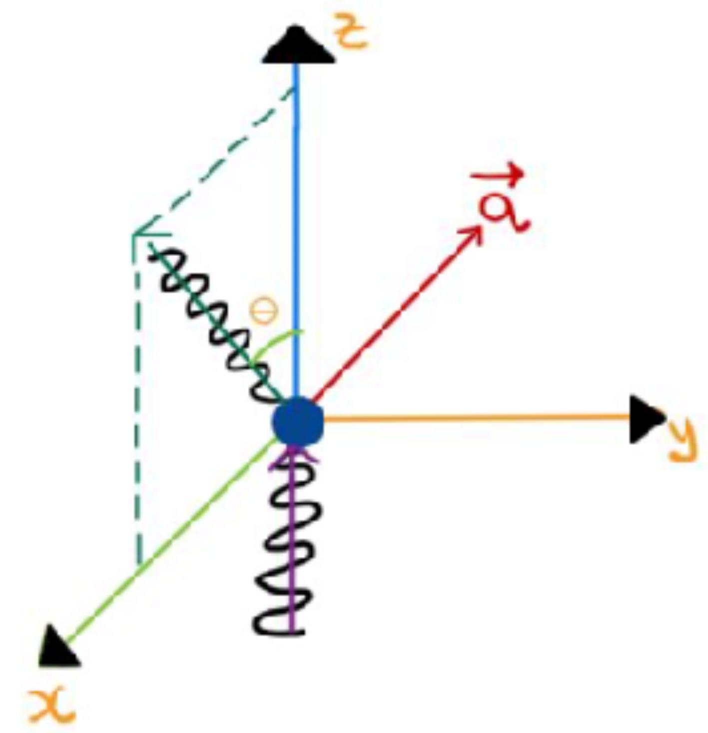
From "self-force" considerations.
(Detweiler redshift, circular orbit precession frequency)

$$C_{4a} + 6C_{4c} \doteq 0, \qquad C_{4a} + 3C_{4e} \doteq 0.$$

They're found to vanish

$$C_{4a} \doteq C_{4c} \doteq C_{4e} \doteq 0,$$

Checks: BHPT



[Submitted on 15 Dec 2022 (v1), last revised 21 Feb 2023 (this version, v2)]

Scattering in Black Hole Backgrounds and Higher-Spin Amplitudes: Part II

Yilber Fabian Bautista, Alfredo Guevara, Chris Kavanagh, Justin Vines

The Teukolsky solution...

Spin	Spurious-pole	Free Coeffs.	Teukolsky Solutions
a^4		$c_1^{(i)},\ i=0,1,2$	$c_1^{(i)}=0,\ i=0,1,2$

...results in shift symmetry up to S^4

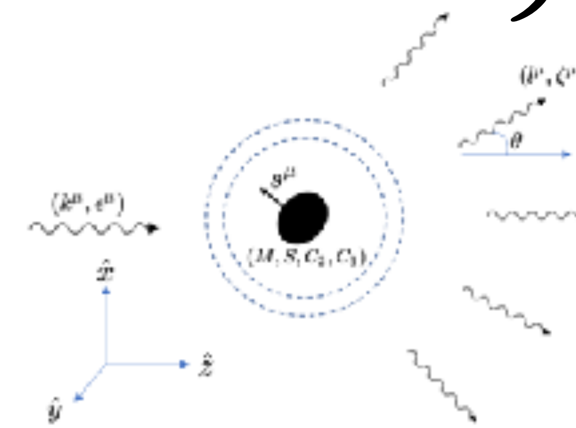
Spin	Shift-Sym.	Free Coeffs.	Relation to [20]
a^4	$c_1^{(i)}=0,\ i=1,2$	$c_1^{(0)}$	$c_1^{(0)}=-\frac{d_0^{(4)}}{4!}$

Checks: Other objects (Neutron Stars)

[Submitted on 5 Aug 2022 (v1), last revised 30 Aug 2022 (this version, v2)]

Scattering of gravitational waves off spinning compact objects with an effective worldline theory

M. V. S. Saketh, Justin Vines



$$\mathcal{M}_{++} = GM\omega\frac{\cos^4(\theta/2)}{\sin^2(\theta/2)}\Bigg(\exp[a\cdot(k+l-2w_{\rm S})] + \frac{C_2-1}{2}[(k-w_{\rm S})\cdot a]^2 + [(l-w_{\rm S})\cdot a]^2$$
$$+\frac{C_2-1}{2}[(k-w_{\rm S})\cdot a][(l-w_{\rm S})\cdot a][(k+l-2w_{\rm S})\cdot a] - (C_2-1)^2[(k-w_{\rm S})\cdot a][(l-w_{\rm S})\cdot a](w_{\rm S}\cdot a)$$
$$+\frac{C_3-1}{6}\{[(k-w_{\rm S})\cdot a]^3 + [(l-w_{\rm S})\cdot a]^3\}\Bigg).$$

We have verified that our Compton amplitudes for generic C_2 and C_3 (for both helicity configurations) are in agreement with the classical limits of the Compton amplitudes derived by the authors of Ref. [57] for use in their computation of the 2-to-2 scattering amplitudes, for a certain choice of their extra free parameter (specifically when their parameter H_2 equals 1).

$$\frac{\theta^{\rm NLO}_{S_1^3}}{\Gamma} = \tilde{v}\tilde{a}_1^3\Bigg[-\frac{4}{\tilde{b}}C_{1BS^3} + \frac{\pi}{\tilde{b}^2}\Bigg(\frac{15\nu}{4}\tilde{v}^2 + \Bigg(3\nu+6+\Bigg(-\frac{3\nu}{2}-\frac{27}{4}\Bigg)\tilde{v}^2\Bigg)C_{1ES^2}$$
$$+\Bigg(-6+\Bigg(\frac{27\nu}{4}-\frac{33}{4}\Bigg)\tilde{v}^2\Bigg)C_{1BS^3}$$
$$+\frac{\nu}{q}\Bigg(\frac{15}{4}\tilde{v}^2 + \Bigg(3-\frac{3}{2}\tilde{v}^2\Bigg)C_{1ES^2} + \frac{27}{4}\tilde{v}^2C_{1BS^3}\Bigg)\Bigg)\Bigg],$$

$$\frac{\theta^{\rm NLO}_{S_1^2S_2}}{\Gamma} = \tilde{v}\tilde{a}_1^2\tilde{a}_2\Bigg[-\frac{12}{\tilde{b}}C_{1ES^2} + \frac{\pi}{\tilde{b}^2}\Bigg(6\nu-12+\tilde{v}^2\Bigg(\frac{27\nu}{8}-\frac{99}{8}\Bigg)$$
$$+\Bigg(-3\nu+\Bigg(\frac{39\nu}{8}-\frac{207}{8}\Bigg)\tilde{v}^2-21\Bigg)C_{1ES^2}$$
$$+\frac{\nu}{q}\Bigg(6+\frac{27}{8}\tilde{v}^2+\Bigg(-3+\frac{39}{8}\tilde{v}^2\Bigg)C_{1ES^2}\Bigg)\Bigg)\Bigg].$$

[Submitted on 31 Oct 2022]

From the EFT of Spinning Gravitating Objects to Poincaré and Gauge Invariance

Michèle Levi, Roger Morales, Zhewei Yin

Summary

- Amplitudes in QFT can be used to get observables, for black holes and more general bodies.
- Several checks in different regimes (PN, test-body scattering, self-force, black hole perturbation theory).
- General agreement up to S^4 black hole, and S^3 generic bodies. But subtle beyond.

Outlook

- Can black hole S^5 be obtained from amplitudes? Effectively? Fundamentally?

- Double Copy? Neutron stars? Strong regimes?

- Phenomenological: More loops! There is some catch up to do.