Electroweak renormalization and the treatment of vacuum expectation values

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“Trail” map for the talk

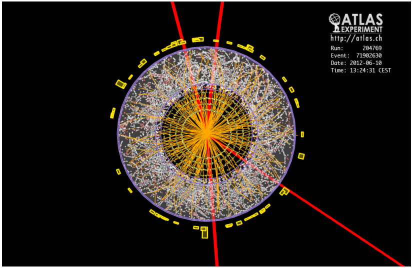
Important questions

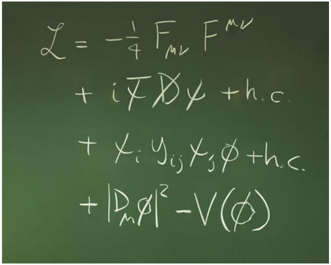
Important questions:

- What are the input parameters of my theory?
- What is their meaning?
- What is their relation to observables?

Renormalization scheme

Physical observables $\longleftrightarrow$ Original parameters





Renormalization scheme

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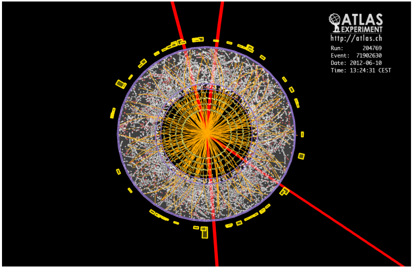

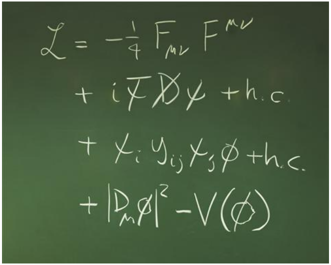

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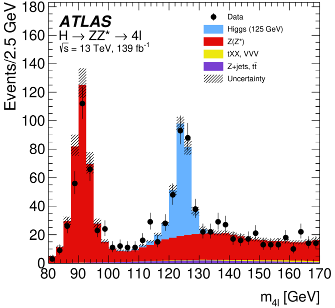
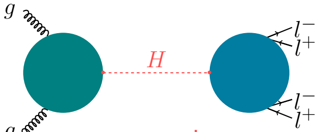


$$\mathcal{L} = (\partial_\mu H)(\partial^\mu H) - \frac{1}{2} m^2 H^2$$

$+$ interaction terms

$+$ other fields



$$\propto \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}0^+}$$

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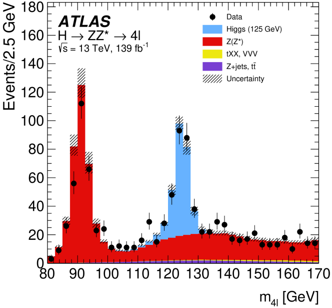

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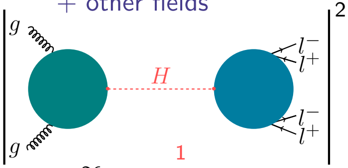

$$\propto \frac{1}{(p^2 - m^2)^2}$$

Renormalization scheme

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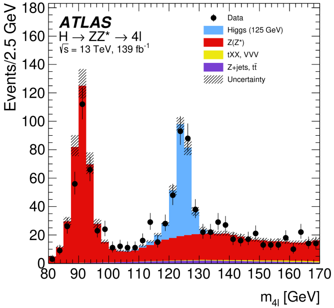

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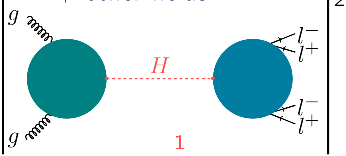

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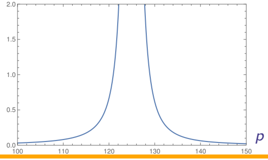
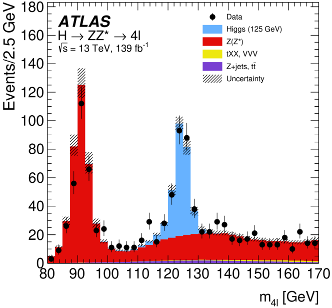


Renormalization scheme

Physical observables ⟷ Original parameters



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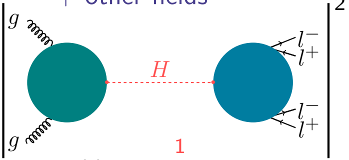

$$\propto \frac{1}{(p^2 - m^2)^2}$$

m_{4l}

$= \sqrt{\sum_\ell p_\ell^2}$

$=$ four-lepton mass

$p_\ell =$ 4-momentum of one lepton

Renormalization scheme

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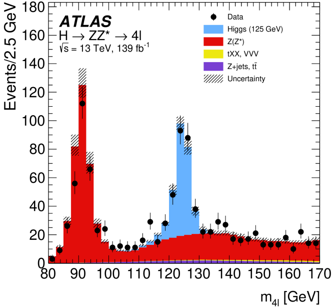

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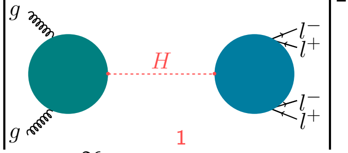


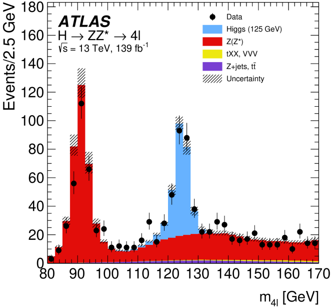
$$\propto \frac{1}{(p^2 - m^2)^2}$$

divergent at $p = m$

$\rightarrow$ physical?

Renormalization scheme

Physical observables $\overset{\text{quantum corrections}}{\longleftrightarrow}$ Original parameters



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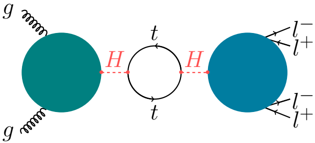
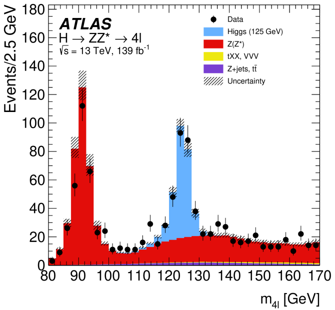


Renormalization scheme

quantum corrections

Physical observables ⟷ Original parameters



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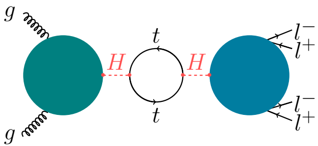
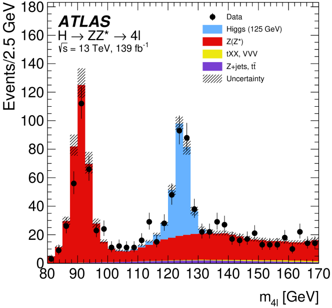


$$\propto \frac{\mathrm{i}}{p^2 - \left(m^2 + \Delta m^2 + \mathrm{i} m \Gamma\right) + \mathrm{i}0^+}$$

Renormalization scheme

quantum corrections

Physical observables $\longleftrightarrow$ Original parameters



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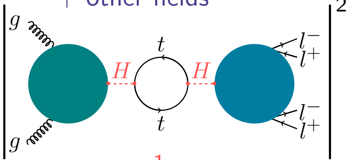
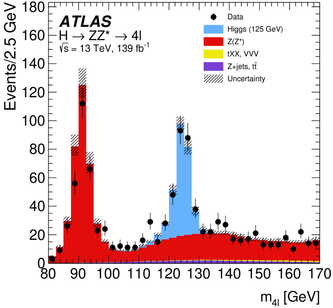


$$\propto \frac{1}{(p^2 - (m^2 + \Delta m^2))^2 + m^2 \Gamma^2}$$

Renormalization scheme

quantum corrections

Physical observables ⟷ Original parameters



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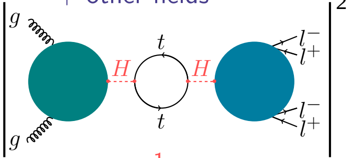

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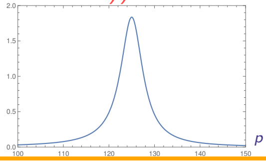

Renormalization scheme

quantum corrections

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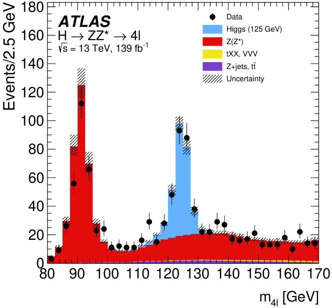

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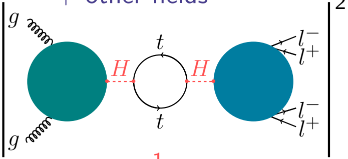

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Renormalization scheme

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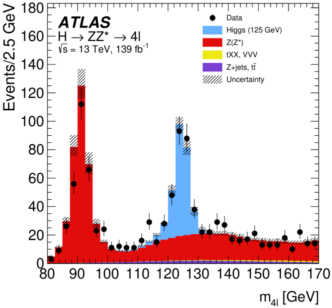

$$\mathcal{L} = (\partial_\mu H)(\partial^\mu H) - \frac{1}{2} m^2 H^2$$

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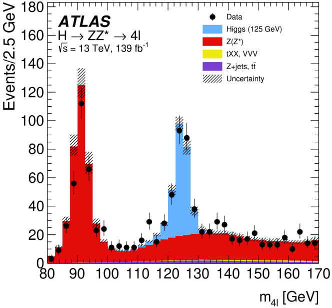
$\Rightarrow$ Establish meaning of parameters

Renormalization scheme

Physical observables ⟷ Original parameters
(quantum corrections)



$$\mathcal{L} = (\partial_\mu H)(\partial^\mu H) - \frac{1}{2} m^2 H^2$$

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$\Rightarrow$ Establish meaning of parameters

$\Rightarrow$ Choice of expansion point of the perturbation series

Assumptions for the following

- Chosen regularization scheme is good
  (see e.g. [Gnendinger et al, arXiv:1705.01827]),
  in particular:
  - ⋆ mathematically consistent
  - ⋆ compatible with QFT-properties of unitarity and causality
  - ⋆ symmetries are preserved
    otherwise: symmetry-restoring counterterms might be needed

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→ Multiplicative renormalization:

Parameters: $g_j^0 = \quad Z_{g_j} g_j \quad = \quad g_j + \delta g_j^{(1)} + \delta g_j^{(2)} + \ldots$

Fields: $f_j^0 = (\sqrt{Z_f})_{jk} f_k = (\delta_{jk} + \frac{1}{2}(\delta^{(1)} Z_f)_{jk} + \ldots) f_k$

Criteria for reasonable renormalization conditions

- Numerical stability of the perturbative expansion should be ensured.
- No "dead corners" in parameter space
  $\rightarrow$ avoid singularities in physical observables
- S-matrix elements
  $=$ gauge-independent functions of renormalized parameters
- Simple: Avoid dependence on a specific physical process
- Good decoupling properties in case of heavy particles

formulated for mixing angles in [Denner, Dittmaier, Lang 1808:03466; Freitas, Stöckinger hep-ph/0205281]

Some issues

- Bookkeeping and number of parameters
- Mixing angles
- **Treatment of tadpole contributions**

  (connected to the renormalization of vacuum expectation values)

Vacuum expectation values

Higgs potential in the SM:

$$V = -\frac{\mu_2^2}{2}\left(\Phi^\dagger\Phi\right) + \frac{\lambda_2}{4}\left(\Phi^\dagger\Phi\right)^2$$

At leading order $(\mu_2^2 > 0)$: Minimum at $\left(\Phi^\dagger\Phi\right) = \frac{2\mu_2^2}{\lambda_2}$

Common choice:

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \eta + \mathrm{i}\chi) \end{pmatrix}$$

v = vacuum expectation value of Higgs doublet

η = physical Higgs field

ϕ^+ = charged Goldstone-boson field

χ = neutral Goldstone-boson field

Vacuum expectation values

In matrix notation: (for later convenience):

Higgs potential in the SM:

$$V = -\frac{\mu_2^2}{2}\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right] + \frac{\lambda_2}{16}\left(\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right]\right)^2$$

At leading order ($\mu_2^2 > 0$): Minimum at $\frac{1}{2}\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right] = \frac{2\mu_2^2}{\lambda_2}$

Vacuum expectation values

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Higgs doublet in matrix notation

$$\mathbf{\Phi} = \frac{1}{\sqrt{2}}\left[(v+\eta)\mathbb{1} + 2\mathrm{i}\boldsymbol{\phi}\right], \qquad \boldsymbol{\phi} \equiv \frac{\phi_j\sigma_j}{2} = \frac{\vec{\phi}\cdot\vec{\sigma}}{2}$$

Relations between $\phi^\pm$, χ and ϕ_j, $j = 1, \ldots, 3$:

$$\phi^\pm = \frac{1}{\sqrt{2}}(\phi_2 \pm \mathrm{i}\phi_1)$$
$$\chi = -\phi_3$$

Vacuum expectation values

In matrix notation: (for later convenience):

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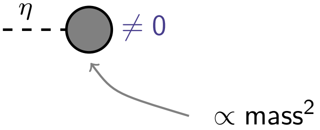
Inserting $\mathbf{\Phi}$ in V leads to term $\propto \eta$:

$$V = \cdots + \underbrace{\frac{1}{4}v(-4\mu_2^2 + \lambda_2 v^2)}_{=-t_\eta}\eta$$

Vacuum expectation values and tadpole contributions

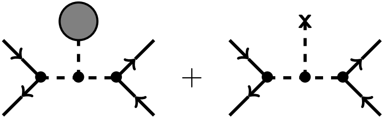
Beyond leading order:

Tadpole contributions in $V \propto \eta$ occur:

$\Gamma^\eta = T_\eta =$  $\neq 0$

Convenient choice:

$\Gamma^\eta_R = T_\eta + \delta t_\eta =$  $\stackrel{!}{=} 0$

$\Rightarrow$ e.g.  $= 0$

Issues with tadpole contributions

- On-shell conditions
  (= parameters defined via physical observables):

  Tadpole contributions cancel
  ⇒ independent on tadpole treatment

- $\overline{\text{MS}}/\overline{\text{DR}}$ conditions:

  ⋆ $\overline{\text{MS}}/\overline{\text{DR}}$ masses (also in SM!)

  ⋆ $\overline{\text{MS}}/\overline{\text{DR}}$ mixing angles

  depend on tadpole treatment.

Common treatments of tadpoles

Parameter Renormalized Tadpole Scheme (PRTS): [Böhm, Hollik, Spiesberger 86; Denner 0709.1075]

Fleischer-Jegerlehner Tadpole Scheme (FJTS): [Fleischer, Jegerlehner 80; Actis, Ferroglia, Passera, Passarino hep-ph/0612122]

Common treatments of tadpoles

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- Expansion about the loop-corrected minimum of the effective potential

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$$t_\eta + \delta t_\eta^{\mathrm{PRTS}} + T_\eta \underset{\substack{\uparrow \\ \text{minimum condition}}}{=} 0$$

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renormalization condition

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  Ensuring $\delta t_\eta^{\mathsf{FJTS}} + T_\eta = 0$ by shifting $\eta \to \eta + \Delta v$

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Fleischer-Jegerlehner Tadpole Scheme (FJTS): [Fleischer, Jegerlehner 80; Actis, Ferroglia, Passera, Passarino hep-ph/0612122]

- Expansion about the minimum of the potential at leading order

  $t_{\eta,0} \overset{!}{=} 0 \Rightarrow$ no gauge dependencies

  Ensuring $\delta t_\eta^{\mathsf{FJTS}} + T_\eta = 0$ by shifting $\eta \to \eta + \Delta v$

  $\Rightarrow$ Term $\propto \eta$ in V: $\frac{1}{2} M_h^2 \eta^2 \to M_h^2 \Delta v\, \eta + \ldots$

Common treatments of tadpoles

Parameter Renormalized Tadpole Scheme (PRTS): [Böhm, Hollik, Spiesberger 86; Denner 0709.1075]

- Expansion about the loop-corrected minimum of the effective potential:

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$\Rightarrow -M_h^2 \Delta v + T_\eta = 0$

relatively large corrections

see also [Jegerlehner et al 1212.4319; Kniehl et al 1503.02138; Kataev et al 2201.12073]

Combining good properties?

PRTS:

+ Expansion about loop-corrected minimum of the effective potential

  $\rightarrow$ relatively small corrections

− Gauge dependence

FJTS:

+ Gauge independence

− Expansion about minimum of the potential at leading order

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Combining good properties?

Idea: [Dittmaier, HR 2203.07236; 2206.01479]

- Expansion about the corrected minimum of the effective potential
- Use field basis in which $v + h(x)$ is gauge-invariant

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[Lee, Zinn-Justin '72; ...; Grosse-Knetter, Körgeler hep-ph/9212268; Dittmaier, Grosse-Knetter hep-ph/9501285]

$\rightarrow$ Switch to non-linear (nl) representation:

$$\mathbf{\Phi} = \frac{1}{\sqrt{2}}(v + h)U(\boldsymbol{\zeta}) \quad \text{with } U(\boldsymbol{\zeta}) \equiv \exp\left(\frac{2\mathrm{i}\boldsymbol{\zeta}}{v}\right)$$

h = physical Higgs field, $\boldsymbol{\zeta} \equiv \frac{\zeta_j \sigma_j}{2}$ = Goldstone matrix, $j = 1, 2, 3$

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Gauge transformation: $\boldsymbol{\Phi} = \underbrace{S(\boldsymbol{\theta})}_{\exp(\frac{\mathrm{i}}{2} g_2 \theta_j \sigma_j)} \boldsymbol{\Phi} \underbrace{S_Y(\theta_Y)}_{\exp(\frac{\mathrm{i}}{2} g_1 \theta_Y \sigma_3)}$

θ_a = gauge parameters
g_n = gauge couplings

$$\Rightarrow h \rightarrow h, \qquad U(\boldsymbol{\zeta}) \rightarrow S(\boldsymbol{\theta})U(\boldsymbol{\zeta})S_Y(\theta_Y)$$

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$\to$ Switch to non-linear (nl) representation:

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h = physical Higgs field, $\zeta \equiv \frac{\zeta_j \sigma_j}{2}$ = Goldstone matrix, $j = 1, 2, 3$

Then: $\mathbf{\Phi}^\dagger \mathbf{\Phi} = \frac{1}{2}(v+h)^2$

$\to$ no Goldstone-boson terms in the Higgs potential

Where do the ζ fields appear?

Kinetic term:

$$\mathcal{L}_{\mathrm{H,kin}} = \frac{1}{2}\mathrm{tr}\left[(D_\mu \mathbf{\Phi})^\dagger (D^\mu \mathbf{\Phi})\right]$$

with the covariant derivative

$$D^\mu \mathbf{\Phi} = \partial^\mu \mathbf{\Phi} - \mathrm{i} g_2 \mathbf{W}^\mu \mathbf{\Phi} - \mathrm{i} g_1 \mathbf{\Phi} B^\mu \frac{\sigma_3}{2}, \qquad \mathbf{W}^\mu \equiv \frac{W_j^\mu \sigma_j}{2}$$

then

$$\begin{aligned}\mathcal{L}_{\mathrm{H,kin}} &= \frac{1}{4}\mathrm{tr}\left[\left(\partial_\mu h + \mathrm{i} g_2 \mathbf{C}_\mu^{(\mathrm{u})}(v+h)\right)\left(\partial^\mu h - \mathrm{i} g_2 \mathbf{C}^{(\mathrm{u}),\mu}(v+h)\right)\right] \\ &= \frac{1}{2}(\partial h)^2 + \frac{g_2^2}{8}(v+h)^2\, \vec{C}_\mu^{(\mathrm{u})} \cdot \vec{C}^{(\mathrm{u}),\mu}\end{aligned}$$

with

$$\mathbf{C}_\mu^{(\mathrm{u})} \equiv U(\zeta)^\dagger\, \mathbf{W}_\mu\, U(\zeta) + \frac{\mathrm{i}}{g_2} U(\zeta)^\dagger\, \partial_\mu U(\zeta) + \frac{g_1}{g_2} B_\mu \frac{\sigma_3}{2}$$

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then

$$\mathcal{L}_{\mathrm{H,kin}} = \frac{1}{2}(\partial h)^2 + \frac{(v+h)^2}{2v^2}\Big\{(\partial_\mu \vec{\zeta}) \cdot (\partial^\mu \vec{\zeta}) + \frac{g_2^2 v^2}{4} \vec{C}_\mu \cdot \vec{C}^\mu$$

$$+ g_1 g_2 B_\mu \left[-W_3^\mu \zeta^2 + (\vec{W}^\mu \cdot \vec{\zeta})\zeta_3\right] - g_2^2 v \vec{C}_\mu \cdot (\vec{W}^\mu \times \vec{\zeta})$$

$$- g_2 v \vec{C}_\mu \cdot \partial^\mu \vec{\zeta} - g_2 (\vec{C}_\mu - 2\vec{W}_\mu) \cdot (\vec{\zeta} \times \partial^\mu \vec{\zeta})\Big\} + \mathcal{O}(\zeta^3)$$

with $\vec{C}^\mu = \left(W_1^\mu, W_2^\mu, \frac{Z^\mu}{\cos\theta_W}\right)^{\mathrm{T}}$

$Z^\mu \hat{=}$ Z boson
$\theta_W \hat{=}$ weak mixing angle

Where do the ζ fields appear?

Gauge-fixing Lagrangian:

$$\mathcal{L}_{\mathrm{gf}} = -\frac{1}{2\xi_A}\left(\partial_\mu A^\mu\right)^2 - \frac{1}{2\xi_A}\left(\partial_\mu Z^\mu + \xi_Z M_Z \zeta_3\right)^2 \\ -\frac{1}{\xi_W}\left(\partial_\mu W^{+\,\mu} - \mathrm{i}\xi_W M_W \zeta^+\right)\left(\partial_\nu W^{-\,\nu} + \mathrm{i}\xi_W M_W \zeta^-\right)$$

with

$\zeta^\pm = (\zeta_2 \pm \mathrm{i}\zeta_1)/\sqrt{2}$

$W^\pm_\mu = (W_{1,\mu} \mp \mathrm{i} W_{2,\mu})/\sqrt{2} \;\hat{=}\; \mathrm{W}^\pm$ boson

$A^\mu \;\hat{=}$ photon

$\xi_A,\ \xi_Z,\ \xi_W \;\hat{=}$ gauge parameters

$M_Z,\ M_W \;\hat{=}$ Z- and W-boson masses

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$$-\frac{1}{\xi_W}\left(\partial_\mu W^{+\,\mu} - \mathrm{i}\xi_W M_W \zeta^+\right)\left(\partial_\nu W^{-\,\nu} + \mathrm{i}\xi_W M_W \zeta^-\right)$$

$\Rightarrow$ ζ fields enter also ghost Lagrangian

Relations

between component fields of linear and non-linear representation:

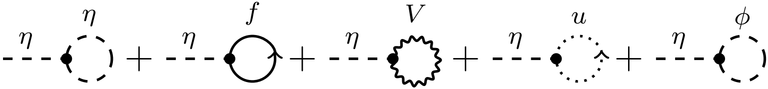
$$\eta = \cos\left(\frac{\zeta}{v}\right)(v+h) - v = h - \frac{\zeta^2}{2v}\left(1+\frac{h}{v}\right) + \mathcal{O}(\zeta^4)$$

$$\vec{\phi} = \frac{\sin\left(\frac{\zeta}{v}\right)}{\zeta}(v+h)\vec{\zeta} \qquad = \left(1+\frac{h}{v}\right)\vec{\zeta} + \mathcal{O}(\zeta^3)$$

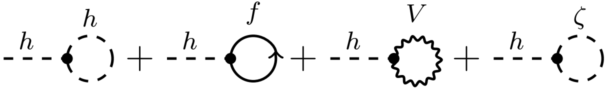
$\Rightarrow (\eta, \vec{\phi})$ and $(h, \vec{\zeta})$ agree up to higher powers in $\zeta \equiv |\vec{\zeta}|$.

Tadpole contributions

In the linear representation:

$\Gamma^{\eta} = T^{\eta} =$ 

In the non-linear representation:

$\Gamma^{h}_{\mathrm{nl}} = T^{h}_{\mathrm{nl}} =$ 

$\rightarrow$ gauge independent

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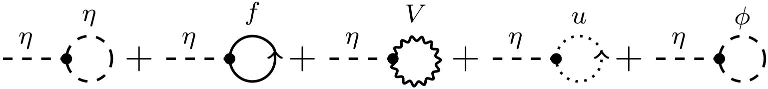
$\Gamma^\eta = T^\eta =$ 

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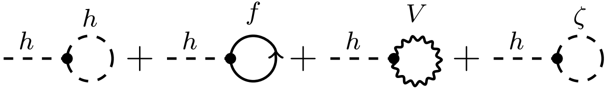
$\Gamma^h_{\mathrm{nl}} = T^h_{\mathrm{nl}} =$ 

$\rightarrow$ gauge independent

$$\Gamma^\eta = \Gamma^h_{\mathrm{nl}} + M_H^2 \Delta v_\xi, \qquad \Delta v_\xi = \frac{1}{16\pi^2 v}\left\{\frac{1}{2}A_0(\xi_Z M_Z^2) + A_0(\xi_W M_W^2)\right\}$$

ξ_a = gauge parameters, A_0 = 1-point scalar integral

Gauge-Invariant Vacuume expectation value Scheme (GIVS)

[Dittmaier, HR 2203.07236; 2206.01479]

Decomposition of tadpole contributions: see also [Dūdėnas, Löschner 2010.15076]

$$\delta t^{\mathrm{GIVS}} = \delta t_1^{\mathrm{GIVS}} + \delta t_2^{\mathrm{GIVS}}$$

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Fix parts via:

PRTS part: $\delta t_1^{\mathrm{GIVS}} = -T_h^{\mathrm{nl}}$ → leads to relatively small NLO contributions

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FJTS part: $\eta \to \eta + \Delta v_\xi$

$$\delta t_2^{\mathrm{GIVS}} = -M_h^2 \Delta v_\xi = T_h^{\mathrm{nl}} - T_\eta$$ → compensates gauge dependence

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Comparison of Schemes

Conversion from on-shell mass M^{OS} to $M^{\overline{\text{MS}}}$ via bare mass M_0:

$$M_0 = M^{\text{OS}} + \delta M^{\text{OS}} = M^{\overline{\text{MS}}} + \delta M^{\overline{\text{MS}}}$$

Difference between on-shell and $\overline{\text{MS}}$ mass:

$$\Delta M^{\overline{\text{MS}}-\text{OS}} = \delta M^{\text{OS}} - \delta M^{\overline{\text{MS}}} = \delta M^{\text{OS}}\big|_{\text{finite}}$$

	M^{OS}[GeV]	$\Delta M_{\text{EW}}^{\overline{\text{MS}}-\text{OS}}$[GeV]		
		FJTS	PRTS	GIVS
W boson	80.379	−2.22	0.82	0.74
Z boson	91.1876	−0.77	1.25	1.14
Higgs boson	125.1	6.34	3.16	2.80
top quark	172.4	10.75	0.99	0.54
bottom quark	4.93	−1.79	0.10	0.13
τ lepton	1.77686	−0.93	−0.028	−0.015

GIVS leads to much smaller corrections than FJTS!

GIVS for BSM

GIVS can be formulated for BSM:

Already available for: [Dittmaier, HR 2206.01479]

- Singlet extension of the SM: non-linear representation: straightforward
- Two-Higgs-Doublet Model (2HDM)

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- Two-Higgs-Doublet Model (2HDM)
  linear representation of Higgs doublets in matrix form ($n = 1, 2$):

$$\mathbf{\Phi}_n = \frac{1}{\sqrt{2}}\left[(v_n + \eta_n)\mathbb{1} + 2\mathrm{i}\phi_n\right], \qquad \phi_n \equiv \frac{\phi_{nj}\sigma_j}{2}$$

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  non-linear representation of Higgs doublets ($n = 1, 2$):

$$\mathbf{\Phi}_n = U(\zeta)\,\mathbf{\Phi}_n^{(\mathrm{u})} \quad \text{with} \quad \mathbf{\Phi}_n^{(\mathrm{u})} = \frac{1}{\sqrt{2}}\left[(v_n + h_n)\mathbb{1} + \mathrm{i} c_{nj}\sigma_j\rho_j\right]$$

j= 1, 2, 3

Requiring canonical field normalization:

$$c_{1j} = c_1 = -\frac{v_2}{v} \equiv -\sin\beta, \quad c_{2j} = c_2 = \frac{v_1}{v} \equiv \cos\beta, \quad v \equiv \sqrt{v_1^2 + v_2^2}$$

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  Relation between the fields of the two representations:

$$\eta_n = \cos\left(\frac{\zeta}{v}\right)(v_n + h_n) - v_n - \sin\left(\frac{\zeta}{v}\right) c_n \frac{\vec{\rho}\cdot\vec{\zeta}}{\zeta}$$

$$\vec{\phi}_n = c_n \cos\left(\frac{\zeta}{v}\right)\vec{\rho} + \sin\left(\frac{\zeta}{v}\right)\left[(v_n + h_n)\frac{\vec{\zeta}}{\zeta} + c_n \frac{\vec{\rho}\times\vec{\zeta}}{\zeta}\right]$$

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- Singlet extension of the SM:
- Two-Higgs-Doublet Model (2HDM)
  Higgs potential:

$$
\begin{aligned}
V = & \frac{m_{11}^2}{2} \mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1\right] + \frac{m_{22}^2}{2} \mathrm{tr}\left[\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2\right] - m_{12}^2 \mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2\right] \\
& + \frac{\lambda_1}{8}\left(\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1\right]\right)^2 + \frac{\lambda_2}{8}\left(\mathrm{tr}\left[\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2\right]\right)^2 + \frac{\lambda_3}{4} \mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1\right] \mathrm{tr}\left[\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2\right] \\
& + \lambda_4 \mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2 \Omega_+\right] \mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2 \Omega_-\right] + \frac{\lambda_5}{2}\left[\left(\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2 \Omega_+\right]\right)^2 + \left(\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2 \Omega_-\right]\right)^2\right]
\end{aligned}
$$

with $\Omega_\pm = \frac{1}{2}(1 \pm \sigma_3)$

GIVS for BSM

Numerical example: $h \to 4f$ in the 2HDM: [Dittmaier, HR 2206.01479]

Ren. scheme	tadpoles	A1 LO	A1 NLO
OS12 (α, β)		$0.89832(3)$	$0.96194(7)^{-0.1\%}_{+0.1\%}$
$\overline{\text{MS}}(\alpha, \beta)$	FJTS	$0.89996(3)^{+0.7\%}_{-7.4\%}$ $\Delta_{\text{OS12}} = +0.2\%$	$0.96283(7)^{+0.8\%}_{-0.2\%}$ $\Delta_{\text{OS12}} = +0.1\%$
$\overline{\text{MS}}(\alpha, \beta)$	PRTS	$0.89035(3)^{-2.8\%}_{+0.9\%}$ $\Delta_{\text{OS12}} = -0.9\%$	$0.96103(7)^{+1.2\%}_{+0.4\%}$ $\Delta_{\text{OS12}} = -0.1\%$
$\overline{\text{MS}}(\alpha, \beta)$	GIVS	$0.89082(3)^{-2.7\%}_{+0.9\%}$ $\Delta_{\text{OS12}} = -0.8\%$	$0.96106(7)^{+1.2\%}_{+0.5\%}$ $\Delta_{\text{OS12}} = -0.1\%$
$\overline{\text{MS}}(\lambda_3, \beta)$	FJTS	$0.89246(3)^{-15.1\%}_{+1.6\%}$ $\Delta_{\text{OS12}} = -0.7\%$	$0.96108(7)^{+17.3\%}_{+1.9\%}$ $\Delta_{\text{OS12}} = -0.1\%$
$\overline{\text{MS}}(\lambda_3, \beta)$	PRTS/GIVS	$0.89156(3)^{-8.4\%}_{+1.7\%}$ $\Delta_{\text{OS12}} = -0.8\%$	$0.96111(7)^{+3.8\%}_{+2.1\%}$ $\Delta_{\text{OS12}} = -0.1\%$

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$\Rightarrow$ Improvement for $\overline{\text{MS}}$ defined mixing angles

Depending on process and parameter region:

On-shell definitions via ratios of form factors still better

e.g. for $H \to 4\,f$ $\quad$ $H =$ heavy CP-even Higgs boson in 2HDM

[Denner, Dittmaier, Lang 1808:03466]

Summary

- Renormalization scheme:

  Fixes relations between physical observables and original parameters

- Challenge:

  Find simple gauge-independent conditions

  $\rightarrow$ numerically stable results in the whole parameter space

- Treatment of tadpoles:

  $\rightarrow$ in pure OS scheme not relevant

  $\rightarrow$ in $\overline{\text{MS}}/\overline{\text{DR}}$ schemes:

  Hybrid scheme GIVS combines advantages of PRTS and FJTS:

  Numerical stability and gauge independence

Scenario A1

$M_h = 125$ GeV, $M_H = 300$ GeV, $M_{A_0} = M_{H^+} = 460$ GeV,

$\lambda_5 = -1.9$, $\tan\beta = 2$, $\cos(\beta - \alpha) = 0.1$

$\mu_0 = \frac{1}{5}(M_h + M_H + M_A + 2M_{H^+})$