Parity Solution to the Strong CP Problem and its Experimental Tests

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Outline

- Brief review of the strong CP problem
- Popular solutions to the problem
- Left-right symmetry and the Parity solution
- Model building and phenomenology
- Experimental tests
  - Vector-like quarks and leptons
  - Neutrino oscillations
  - W boson mass shift
  - Unitarity of CKM matrix and the "Cabibbo anomaly"
- Conclusions

The Strong CP Problem

- Strong interactions appear to conserve Parity (P) and Time Reversal (T) symmetries, and therefore also CP symmetry. However, QCD Lagrangian admits a source of P and T violation:

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \theta_{QCD} \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu} + \overline{q} \left(i\gamma^\mu D_\mu - m_q e^{i\theta_q \gamma_5} \right) q$$

- A chiral rotation on the quark field, $q \to e^{i\alpha\gamma_5/2} q$, can remove the phase of the quark mass as $\theta_q \to \theta_q - \alpha$. Due to the anomalous nature of this rotation, θ_{QCD} also changes to $\theta_{QCD} \to \theta_{QCD} + \alpha$

- The parameter

$$\overline{\theta} = \theta_{QCD} + \theta_q$$

  is invariant, and is physical

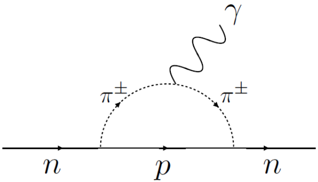
- With multiple flavors of quarks, the invariant physical parameter is

$$\overline{\theta} = \theta_{QCD} + \mathrm{ArgDet}(M_Q)$$

- $\overline{\theta}$ contributes to neutron Electric dipole moment (EDM)

Neutron EDM from $\overline{\theta}$

- In presence of $\overline{\theta}$ neutron will develop and EDM:



$$d_n \simeq \frac{e\,\overline{\theta}\,g_A\,c_+\,\mu}{8\pi^2 f_\pi^2}\mathrm{Log}\left(\frac{\Lambda^2}{m_\pi^2}\right) \simeq 3\times 10^{-16}\;\overline{\theta}\,\mathrm{e\ cm}$$

Here $\mu = \frac{m_u m_d}{m_u+m_d}$, $g_A \simeq 1.27$, $c_+ \simeq 1.6$, $\Lambda = 4\pi f_\pi$

- From $d_n < 1.8\times 10^{-26}$ e cm, one obtains $\Rightarrow \overline{\theta} < 10^{-10}$
- The extreme smallness of $\overline{\theta}$, a dimensionless parameter, is the strong CP problem
- Setting $\overline{\theta}$ to zero is unnatural, since weak interactions require $\mathcal{O}(1)$ CP violation in that sector

Popular Solutions to the Strong CP Problem

- Massless up quark: Since

$$\overline{\theta} = \theta_{QCD} + \mathrm{ArgDet}(M_Q),$$

  chiral rotations on any massless quark can remove it

- $m_u = 0$ is inconsistent with experimental data as well as lattice calculations

- Peccei-Quinn symmetry and the axion: Here $\overline{\theta}$ is promoted to a dynamical field. The potential for this field relaxes $\overline{\theta}$ to zero.

- An anomalous $U(1)_{PQ}$ symmetry is imposed, which is spontaneously broken as well explicitly broken by the QCD anomaly

- The effecitve interaction of the axion is given by

$$\mathcal{L} \supset \left(\frac{a}{f_a} + \theta\right) \frac{1}{32\pi^2} G\tilde{G}$$

- Parity solution: Since θ_{QCD} is odd under P, the strong P problem can be solved in P-symmetric theories without needing the axion

Parity Solution to the Strong P Problem

- Imagine Parity is spontaneously broken. $\Rightarrow$

$$\theta_{QCD} = 0 \text{ by Parity.}$$

- If the quark mass matrix is hermitian, also by Parity, then $\overline{\theta} = 0$ at tree-level.
- Quantum corrections could induce small nonzero $\overline{\theta}$.
- In left-right symmetric models, Parity symmetry is exact, with

$$q_L \leftrightarrow q_R, \qquad \Phi \leftrightarrow \Phi^\dagger$$

- Consequently, the Yukawa coupling $(Y_q\, \overline{q}_L\, \Phi\, q_R)$ is hermitian:

$$Y_q = Y_q^\dagger$$

- However, the quark mass matrix is

$$M_q = Y_q \langle \Phi \rangle$$

- It is a challenge to make the VEVs of Φ real.
- Initial attempts used discrete symmetries to achieve this goal.
  Mohapatra, Senjanovic (1978), Beg, Tsao (1978)

Left-Right Symmetric Models

- Gauge symmetry is extended to:

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

Pati, Salam (1974); Mohapatra, Pati (1975); Mohapatra, Senjanovic (1979)

- Fermions transform in a left-right symmetric manner:

$$Q_L\ (3,2,1,1/3) = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad Q_R\ (3,1,2,1/3) = \begin{pmatrix} u_R \\ d_R \end{pmatrix},$$

$$\Psi_L\ (1,2,1,-1) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \Psi_R\ (1,1,2,-1) = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}.$$

- Note the natural appearance of the right-handed neutrino, leading to small neutrino masses
- In standard LR theories, 3 types of Higgs fields are employed:

$$\Phi(1,2,2,0) = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix},\ \Delta_{L,R}(1,3(1),1(3),2) = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}_{L,R}$$

- Φ generates quark and lepton masses, $\Delta_{L,R}$ generate Majorana neutrino masses. Δ_R also breaks $SU(2)_R$ symmetry

Parity Solution to the Strong P Problem

- Parity symmetry can now be defined, under which

$$Q_L \leftrightarrow Q_R, \quad \Psi_L \leftrightarrow \Psi_R, \quad \Phi \to \Phi^\dagger, \quad \Delta_L \leftrightarrow \Delta_R$$

- Gauge fields transform under P as:

$$G^a_\mu(t,x) \to G^a_\mu(t,-x) \times s_\mu, \quad B_\mu(t,x) \to B_\mu(t,-x) \times s_\mu$$

$$W^a_{L,\mu}(t,x) \to W^a_{R,\mu}(t,-x) \times s_\mu, \quad W^a_{R,\mu}(t,x) \to W^a_{L,\mu}(t,-x) \times s_\mu$$

where $s_\mu = 1$ $(\mu = 0)$, $= -1$ $(\mu = 1,2,3)$

- Owing to this symmetry, $\theta_{QCD} = 0$

- Yukawa coupling matrices of quarks are hermitian also by P. Quark mass matrix is however not hermitian, since the $\langle \Phi \rangle$ is complex

- The Higgs potential of the standard left-right symmetric model has a single complex coupling ($\tilde{\Phi} = \tau_2 \Phi^* \tau_2$):

$$V \supset \left\{ \alpha_2 e^{i\delta_2} \left[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\mathrm{Tr}(\Delta_L \Delta_L^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger \Phi)\mathrm{Tr}(\Delta_R \Delta_R^\dagger) \right] + h.c. \right\}$$

- For nonzero phase δ_2, the VEVs of Φ would develop a relative phase of order one, spoiling the Parity solution to strong CP problem.

SUSY-Assistance to the Strong P Problem

- Supersymmetric Higgs sector would not admit such cross couplings in the potential, and could lead to real VEVs of Φ

- Several SUSY models have been constructed within left-right symmetry that solves the strong P problem

  Kuchimanchi (1996); Mohapatra, Rasin (1996); Mohapatra, Rasin, Senjanovic (1997); Babu, Dutta, Mohapatra (2002)

- Explicit SUSY LR models assume two copies of $\Phi(1,2,2,0)$ fields to generate CKM mixing angles

- If the theory has two hermitian flavor matrices Y_u and Y_d, and if all flavor singlets are real, the lowest order contribution to $\overline{\theta}$ would arise from:

$$c_1 \mathrm{Im}\mathrm{Tr}(Y_u^2 Y_d^4 Y_u^4 Y_d^2) + c_2 \mathrm{Im}\mathrm{Tr}(Y_d^2 Y_u^4 Y_d^4 Y_u^2)$$

SUSY and the Strong P Problem

- In SUSY LR models with two copies of $\Phi(1,2,2,0)$, all superpotential parameters are real due to P.
- In these models the coefficients $c_{1,2}$ are of order

$$c_{1,2} \sim \left(\frac{\ln(M_{W_R}/M_{W_L})}{16\pi^2} \right)^4$$

- They lead to and induced $\overline{\theta}$ of order

$$\overline{\theta} \sim 3 \times 10^{-27} (\tan\beta)^6 (c_1 - c_2)$$

Babu, Dutta, Mohapatra (2002)

- Argument similar to Eliis, Gaillard (1979) for SM contribution to $\overline{\theta}$
- If for some reason the phase of the quark mass matrix is zero in the Standard Model, it would arise via 7-loop diagrams, and would remain extremely small.

Solution with P Symmetry Alone

- Parity alone can solve the strong CP problem
- Key point is to go easy with the Higgs sector
- If only an $SU(2)_L$ doublet Higgs χ_L and an $SU(2)_R$ doublet Higgs χ_R are used for symmetry breaking, gauge rotations would guarantee that their VEVs are real
- Fermion mass generation is achieved via mixing of the usual fermions with vector-like fermions via χ_L and χ_R
- This class of left-right symmetric models belong to "universal seesaw" class Davidson, Wali (1987)
- Parity is softly broken by the mass terms of χ_L and χ_R, which leads to consistent phenomenology
- This setup can solve the strong P problem via parity symmetry alone. Babu, Mohapatra (1990)

Left-Right Symmetry with Universal Seesaw

- Gauge symmetry is extended to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$
- These models are motivated on several grounds:
  - Provide understanding of Parity violation
  - Better understanding of smallness of Yukawa couplings
  - Requires right-handed neutrinos to exist
  - Provide a solution to the strong CP problem via Parity
  - Naturally light *Dirac neutrinos* may be realized
  - Possible relevance to experimental anomalies

Davidson, Wali (1987) – universal seesaw
Babu, He (1989) – Dirac neutrino
Babu, Mohapatra (1990) – solution to strong CP problem via parity
Babu, Dutta, Mohapatra (2018) – R_{D^*} solution
Dunsky, Hall, Harigaya (2019) – spontaneous P breaking
Craig, Garcia Garcia, Koszegi, McCune (2020) – flavor constraints
Babu, He, Su, Thapa (2022) – neutrino oscillations with Dirac neutrinos
Harigaya, Wang (2022) – Baryogenesis
Babu, Dcruz (2022) – Cabibbo anomaly, W mass anomaly

Dcruz (2022) – Flavor constraints

Left-Right Symmetry with Small $\overline{\theta}$

- Fermion transformation: $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$:

$$Q_L\ (3,2,1,1/3) = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad Q_R\ (3,1,2,1/3) = \begin{pmatrix} u_R \\ d_R \end{pmatrix},$$

$$\Psi_L\ (1,2,1,-1) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \Psi_R\ (1,1,2,-1) = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}.$$

- Vector-like fermions are introduced to realize seesaw for charged fermion masses:

$$P(3,1,1,4/3), \quad N(3,1,1,-2/3), \quad E(1,1,1,-2)\ .$$

- Higgs sector is very simple:

$$\chi_L\ (1,2,1,1) = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}, \quad \chi_R\ (1,1,2,1) = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}$$

- $\langle \chi_R^0 \rangle = \kappa_R$ breaks $SU(2)_R \times U(1)_X$ down to $U(1)_Y$, and $\langle \chi_L^0 \rangle = \kappa_L$ breaks the electroweak symmetry with $\kappa_R \gg \kappa_L$

Seesaw for Charged Fermion Masses

- Yukaw interactions:

$$\mathcal{L} = y_u\ (\bar{Q}_L\tilde{\chi}_L + \bar{Q}_R\tilde{\chi}_R)P + y_d\ (\bar{Q}_L\chi_L + \bar{Q}_R\chi_R)N$$
$$+ y_\ell\ (\bar{\Psi}_L\chi_L + \bar{\Psi}_R\chi_R)E + h.c.$$

- Vector-like fermion masses:

$$\mathcal{L}_{\rm mass} = M_{P^0}\ \bar{P}P + M_{N^0}\ \bar{N}N + M_{E^0}\ \bar{E}E$$

- Seesaw for charged fermion masses:

$$M_F = \begin{pmatrix} 0 & y\kappa_L \\ y^\dagger\kappa_R & M \end{pmatrix} \Rightarrow m_f = \frac{y^2\kappa_L\kappa_R}{M}$$

- Under Parity, fields transform as:

$$Q_L \leftrightarrow Q_R, \quad \Psi_L \leftrightarrow \Psi_R, \quad F_L \leftrightarrow F_R, \quad \chi_L \leftrightarrow \chi_R$$

  Consquently $M_{F^0} = M_{F^0}^\dagger$

- $\theta_{QCD} = 0$ due to Parity; ArgDet$(M_U M_D) = 0$; induced $\overline{\theta} = 0$ at one-loop; small and finite $\overline{\theta}$ arises at two-loop

Vanishing $\overline{\theta}$ at one-loop

- Correction to the quark mass matrix:

$$\mathcal{M}_U = \mathcal{M}_U^0(1 + C)$$

- $\overline{\theta}$ is given by

$$\overline{\theta} = \mathrm{ArgDet}(1 + C) = \mathrm{ImTr}(1 + C) = \mathrm{ImTr}\, C_1$$

where a loop-expansion is used:

$$C = C_1 + C_2 + \ldots$$

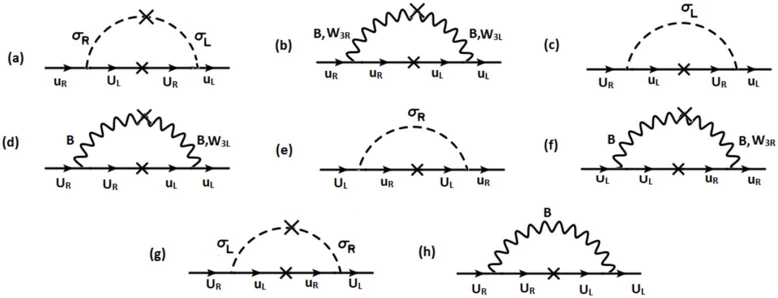
- The corrected mass matrix has a form:

$$\delta\mathcal{M}_U = \begin{bmatrix} \delta M_{LL}^U & \delta M_{LH}^U \\ \delta M_{HL}^U & \delta M_{HH}^U \end{bmatrix}$$

- From here $\overline{\theta}$ can be computed to be:

$$\overline{\theta} = \mathrm{ImTr}\left[-\frac{1}{\kappa_L \kappa_R} \delta M_{LL}^U (Y_U^\dagger)^{-1} M_U Y_U^{-1} + \frac{1}{\kappa_L} \delta M_{LH}^U Y_U^{-1} + \frac{1}{\kappa_R} \delta M_{HL}^U (Y_U^\dagger)^{-1} \right] .$$

Feynman Diagrams for induced $\overline{\theta}$



- Each diagram separately gives zero contribution to $\overline{\theta}$
- Induced value of $\overline{\theta}$ at two-loop is of order 10^{-11}
- Such a cancellation is not easy to achieve. For e.g., this typically does not occur in Nelson-Barr type models which utilize *CP* symmetry

Quality of the P Solution

- Quantum gravity is expected to violate all global symmetries, including Parity
- Leading Planck-scale induced correction to $\overline{\theta}$ arises from

$$\mathcal{L}^{d=5} = \frac{1}{M_{\rm Pl}} (\overline{Q}_L Q_R) \chi_R^\dagger \chi_L \ .$$

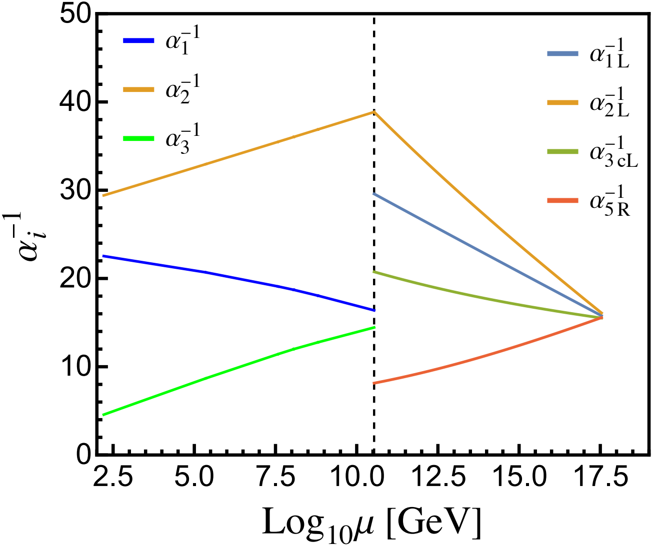
  Since this term is not expected to be Parity-symmetric, the resulting quark mass matrix is non-hermitian. If $M_{W_R} \leq 10^5$ GeV, however, the induced $\overline{\theta}$ from here is $< 10^{-10}$
- Contrast with the quality of axion, where a Planck induced operator $c\, S^5/M_{\rm Pl}$ should have the coefficient $c \leq 10^{-34}$ (or else $\overline{\theta}$ will shift away from zero by more than 10^{-10})
- P solution prefers low mass W_R, which may be experimentally probed

Matter Content from $SU(5)_L \times SU(5)_R$

$$\psi_{L,R} = \begin{bmatrix} D_1^c \\ D_2^c \\ D_3^c \\ e \\ -\nu \end{bmatrix}_{L,R} \qquad \chi_{L,R} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & U_3^c & -U_2^c & -u_1 & -d_1 \\ -U_3^c & 0 & U_1^c & -u_2 & -d_2 \\ U_2^c & -U_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -E^c \\ d_1 & d_2 & d_3 & E^c & 0 \end{bmatrix}_{L,R} ,$$

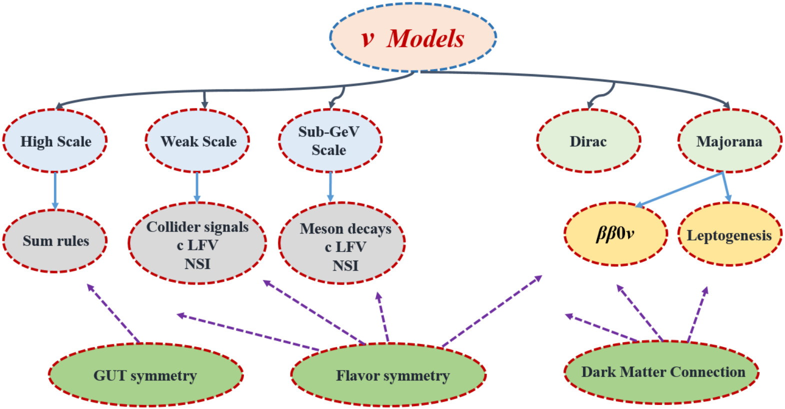
- All left-handed SM fermions are in $\{(10,1)+(\overline{5},1)\}$, while all right-handed SM fermions are in $\{(1,10)+(1,\overline{5})\}$
- There is ν_R in the theory, but no seesaw for neutrino sector
- Small *Dirac neutrino masses* arise as two-loop radiative corrections
- We have evaluated the flavor structure of the two-loop diagrams and shown consistency with neutrino data

Unification of Gauge Couplings



Babu, Mohapatra, Thapa (ongoing)

Roadmap for Neutrino Models



Dirac Neutrino Models

- Neutrinos may be Dirac particles without lepton number violation
- Oscillation experiments cannot distinguish Dirac neutrinos from Majorana neutrinos
- Spin-flip transition rates (early universe, stars) are suppressed by small neutrino mass:

$$\Gamma_{\mathrm{spin-flip}} \approx \left(\frac{m_\nu}{E}\right)^2 \Gamma_{\mathrm{weak}}$$

- Neutrinoless double beta decay discovery would establish neutrinos to be Majorana particles
- If neutrinos are Dirac, it would be nice to understand the smallness of their mass
- Models exist which explain the smallness of Dirac m_ν
- "Dirac leptogenesis" can explain baryon asymmetry

  Dick, Lindner, Ratz, Wright (2000)

Dirac Seesaw Models

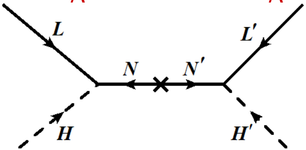
- Dirac seesaw can be achieved in Mirror Models
  Lee, Yang (1956); Foot, Volkas (1995); Berezhiani, Mohapatra (1995), Silagadze(1997)

- Mirror sector is a replica of Standard Model, with new particles transforming under mirror gauge symmetry:

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L ; \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} ; \quad L' = \begin{pmatrix} \nu' \\ e' \end{pmatrix}_L ; \quad H' = \begin{pmatrix} H'^+ \\ H'^0 \end{pmatrix}$$

- Effective dimension-5 operator induces small Dirac mass:

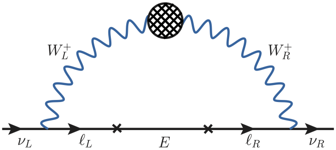
$$\frac{(LH)(L'H')}{\Lambda} \Rightarrow m_\nu = \frac{vv'}{\Lambda}$$

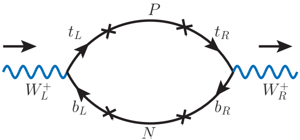


- $B - L$ may be gauged to suppress Planck-induced Weinberg operator $(LLHH)/M_{\rm Pl}$ that would make neutrino pseudo-Dirac particle

Naturally Light Dirac Neutrinos

- Higgs sector is very simple: $\chi_L(1,2,1,1/2) + \chi_R(1,1,2,1/2)$
- $W_L^+ - W_R^+$ mixing is absent at tree-level in the model
- $W_L^+ - W_R^+$ mixing induced at loop level, which in turn generates Dirac neutrino mass at two loop Babu, He (1989)





- Flavor structure of two loop diagram needs to be studied to check consistency
- Oscillation date fits well within the model regardless of Parity breaking scale Babu, He, Su, Thapa (2022)

Loop Integrals

$$M_{\nu^D} = \frac{-g^4}{2} y_t^2 y_b^2 y_\ell^2 \kappa_L^3 \kappa_R^3 \frac{r\ M_P M_N M_{E_\ell}}{M_{W_L}^2 M_{W_R}^2}\ I_{E_\ell}$$

$$I_{E_\ell} = \int\int \frac{d^4k d^4p}{(2\pi)^8} \frac{3M_{W_L}^2 M_{W_R}^2 + (p^2 - M_{W_L}^2)(p^2 - M_{W_R}^2)}{k^2(p+k)^2(k^2 - M_N^2)((p+k)^2 - M_P^2)p^2(p^2 - M_{E_\ell}^2)(p^2 - M_{W_L}^2)(p^2 - M_{W_R}^2)}$$

$$\begin{aligned}
G_1 = \frac{3}{(r_3-1)(r_4-1)(r_4-r_3)} \Bigg[& -\frac{\pi^2}{6}(r_1+r_2)(r_3-1)(r_3-r_4)(r_4-1) \\
& + r_3 r_4 (r_4 - r_3)\left(r_1 F\left[\frac{1}{r_1}, \frac{r_2}{r_1}\right] + r_2 F\left[\frac{1}{r_2}, \frac{r_1}{r_2}\right] + F\left[r_1, r_2\right]\right) \\
& - (r_4 - 1) r_4 \left(r_1 F\left[\frac{r_3}{r_1}, \frac{r_2}{r_1}\right] + r_2 F\left[\frac{r_3}{r_2}, \frac{r_1}{r_2}\right] + r_3 F\left[\frac{r_1}{r_3}, \frac{r_2}{r_3}\right]\right) \\
& + (r_3 - 1) r_3 \left(r_1 F\left[\frac{r_4}{r_1}, \frac{r_2}{r_1}\right] + r_2 F\left[\frac{r_4}{r_2}, \frac{r_1}{r_2}\right] + r_4 F\left[\frac{r_1}{r_4}, \frac{r_2}{r_4}\right]\right) \\
& + (r_3 - r_4)(r_3 - 1)(r_4 - 1)\left(r_2 Li_2\left[1 - \frac{r_1}{r_2}\right] + r_1 Li_2\left[1 - \frac{r_2}{r_1}\right]\right) \\
& + r_3 r_4 (r_3 - r_4)\left(Li_2[1 - r_1] + Li_2[1 - r_2] + r_1 Li_2\left[\frac{r_1 - 1}{r_1}\right] + r_2 Li_2\left[\frac{r_2 - 1}{r_2}\right]\right) \\
& + r_4 (r_4 - 1)\left(r_3 Li_2\left[1 - \frac{r_1}{r_3}\right] + r_3 Li_2\left[1 - \frac{r_2}{r_3}\right] + r_1 Li_2[1 - \frac{r_3}{r_1}] + r_2 Li_2[1 - \frac{r_3}{r_2}]\right) \\
& - r_3 (r_3 - 1)\left(r_4 Li_2\left[1 - \frac{r_1}{r_4}\right] + r_4 Li_2\left[1 - \frac{r_2}{r_4}\right] + r_1 Li_2[1 - \frac{r_4}{r_1}] + r_2 Li_2[1 - \frac{r_4}{r_2}]\right) \Bigg].
\end{aligned} \tag{-1}$$

Neutrino Fit in Two-loop Dirac Mass Model

Oscillation parameters	3σ **range** NuFit5.1	**Model prediction**			
		BP I (NH)	BP II (NH)	BP III (IH)	BP IV (IH)
$\Delta m_{21}^2 (10^{-5}\ \mathrm{eV}^2)$	6.82 - 8.04	7.42	7.32	7.35	7.30
$\Delta m_{23}^2 (10^{-3}\ \mathrm{eV}^2)$(IH)	2.410 - 2.574	-	-	2.48	2.52
$\Delta m_{31}^2 (10^{-3}\ \mathrm{eV}^2)$(NH)	2.43 - 2.593	2.49	2.46	-	-
$\sin^2\theta_{12}$	0.269 - 0.343	0.324	0.315	0.303	0.321
$\sin^2\theta_{23}$ (IH)	0.410 - 0.613	-	-	0.542	0.475
$\sin^2\theta_{23}$ (NH)	0.408 - 0.603	0.491	0.452	-	-
$\sin^2\theta_{13}$ (IH)	0.02055 - 0.02457	-	-	0.0230	0.0234
$\sin^2\theta_{13}$(NH)	0.02060 - 0.02435	0.0234	0.0223	-	-
δ_{CP} (IH)	192 - 361	-	-	271°	296°
δ_{CP} (NH)	105 - 405	199°	200°	-	-
m_{light} (10^{-3}) eV		0.66	0.17	0.078	4.95
M_{E_1}/M_{W_R}		917	321.3	639	3595
M_{E_2}/M_{W_R}		0.650	19.3	1.54	5.03
M_{E_3}/M_{W_R}		0.019	1.26	0.054	2.94

- Ten parameters to fit oscillation data
- Both normal ordering and inverted ordering allowed
- Dirac CP phase is unconstrained
- Left-right symmetry breaking scale is not constrained

Tests with N_{eff} in Cosmology

- Dirac neutrino models of this type will modify N_{eff} by about 0.14

$$\Delta N_{\text{eff}} \simeq 0.027 \left(\frac{106.75}{g_\star \left(T_{\text{dec}} \right)} \right)^{4/3} g_{\text{eff}}$$

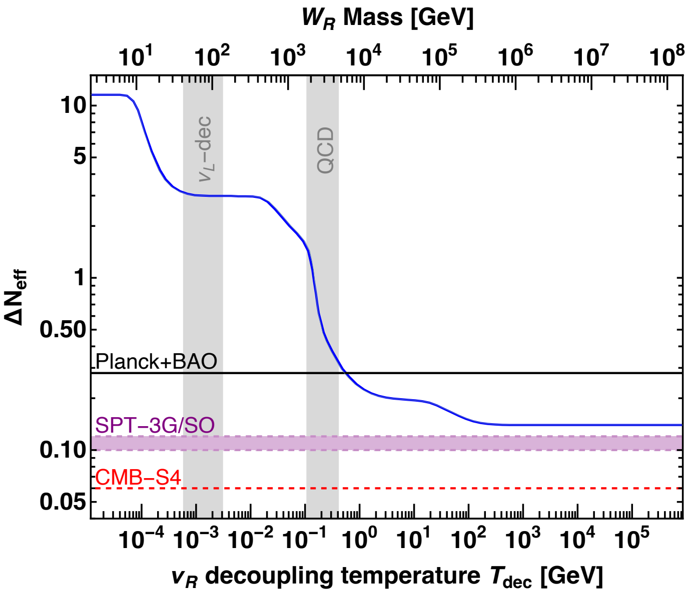
$$g_{\text{eff}} = (7/8) \times (2) \times (3) = 21/4$$

- Can be tested in CMB measurements: $N_{\text{eff}} = 2.99 \pm 0.17$ (Planck+BAO)

$$G_F^2 \left(\frac{M_{W_L}}{M_{W_R}} \right)^4 T_{\text{dec}}^5 \approx \sqrt{g^*(T_{\text{dec}})} \, \frac{T_{\text{dec}}^2}{M_{\text{Pl}}}$$

$$T_{\text{dec}} \simeq 400 \text{ MeV} \left(\frac{g_* \left(T_{\text{dec}} \right)}{70} \right)^{1/6} \left(\frac{M_{W_R}}{5 \text{ TeV}} \right)^{4/3}$$

- Present data sets a lower limit of 7 TeV on W_R mass



W_R Mass [GeV]
10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8
ΔN_{eff}
10
5
1
0.50
0.10
0.05
ν_L–dec
QCD
Planck+BAO
SPT–3G/SO
CMB–S4
10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5
ν_R decoupling temperature T_{dec} [GeV]

Pseudo-Dirac Neutrinos

- In any model with Dirac neutrinos, quantum gravity corrections could induce tiny Majorana masses via Weinberg operator
- The active-sterile neutrino mass splitting should obey $|\delta m^2| < 10^{-12}$ eV^2 from solar neutrino data – de Gouvea, Huang, Jenkins (2009)
- $B - L$ may be gauged in rder to control the small amount of Majorana mass. $(LLHH/M_{\rm Pl})$ won't be allowed due to $B - L$, but $(LLHH\varphi)/M_{\rm Pl}^2$ may be allowed – if φ has $B - L$ of $+2$
- In the current model $(\psi_R\psi_R\chi_R\chi_R)/M_{\rm Pl}$ is more important (if allowed), but $B - L$ gauging could forbid this operator, but may permit $(\psi_R\psi_R\chi_R\chi_R\varphi)/M_{\rm Pl}^2$
- Pseud-Dirac nature of neutrinos may be tested with high energy astrophysical neutrinos via (L/E)-dependent flavor ratios – Beacom, Bell, Hooper, Learned, Pakvasa, Weiler (2003)
- For $\langle\chi_R\rangle \sim \langle\varphi\rangle \sim 10^5$ GeV, $\Delta m^2 \approx 10^{-16}$ eV^2

IceCube Flavor Ratios for Pseudo-Dirac Neutrinos

- Flavor ratio at source from pion decay: $(\frac{1}{3},\ \frac{2}{3},\ 0)$
- For Dirac neutrinos these ratios become at detector $(\frac{1}{3},\ \frac{1}{3},\ \frac{1}{3})$
- For pseudo-Dirac neutrinos at the detector we have:

$$P_\beta = \frac{1}{3} + \delta P_\beta$$

$$\delta P_\beta = -\frac{1}{3}\left[|U_{\beta 1}|^2 \chi_1 + |U_{\beta 2}|^2 \chi_2 + |U_{\beta 3}|^2 \chi_3\right]$$

$$\chi_j = \sin^2\left(\frac{\Delta m_j^2 L}{4E}\right)$$

- NGC 1068 observation at IceCube probes $\delta m^2 \sim 10^{-21}$ eV^2

Carloni, Martínez-Soler, Argüelles, Babu, Dev (2022)

Anomalies and the P Symmetric Model

- Currently there are several experimental anomalies. The P symmetric model may be relevant to some of these
- Anomalies include:
  - Muon $g - 2$
  - R_D, R_{D^*} in B decays
  - W-boson mass shift
  - Cabibbo anomaly
- Not all anomalies find resolution here
- Notably, muon $g - 2$ is hard to explain, without further ingredients
- Cabibbo anomaly and W mass shift fit in nicely with testable predictions

Babu, Dcruz (2022)

Explaining the Cabibbo Anomaly

- The first row of the CKM matrix appears to show a 3 sigma deviation from unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5)$$

- The sum of the first column also deviates slightly from unity:

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.9970(18)$$

  Suggestive of mixing of up or down-quark with a vector-like quark

- Occurs naturally in the quark seesaw model. However, if the up-quark mixes with a heavy U-quark via

$$M_{\mathrm{up}} = \begin{bmatrix} 0 & y_u \kappa_L \\ y_u^* \kappa_R & M_U \end{bmatrix},$$

  $u_L - U_L$ mixing is too small, suppressed by u-quark mass.

- This is a consequence of Parity symmetry

Explaining the Cabibbo Anomaly (cont.)

- A way out: Mix down-quark with two of the D-quarks:

$$M_{\rm do} = \begin{bmatrix} 0 & y_d \kappa_L & 0 \\ y_d^* \kappa_L & M_1 & M_2 \\ 0 & M_2 & 0 \end{bmatrix}$$

- In this case large value of $y_d \kappa_L \sim 200$ GeV is allowed, without generating large u-quark mass. Note: $\mathrm{Det}(M_{\rm do}) = 0$
- Assume CKM angles arise primarily from down sector. Then the full 5×3 CKM matrix spanning (u, c, t) and (d, s, b, D_1, D_2) is:

$$V_{CKM}^T = \begin{bmatrix} c_L V_{ud} & c_L V_{us} & c_L V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ -s_L s_L' V_{ud} & -s_L s_L' V_{us} & -s_L s_L' V_{ub} \\ -s_L c_L' V_{ud} & -s_L c_L' V_{us} & -s_L c_L' V_{ub} \end{bmatrix}$$

- $s_L = 0.0387$ explains the apparent unitarity violation

Consistency with other constraints

- In order to get $s_L = 0.0387$, one of the U-quark mass should be below 5 TeV.
- Owing to the $d_L - D_L$ mixing, Z coupling to u_L is modified to

$$\left(\frac{g}{c_W}\right)\left(-\frac{1}{2}+\frac{1}{3}s_W^2-\frac{s_L^2}{2}\right)$$

- This shifts the Z hadronic width by about 1 MeV, which is consistent. The total Z width has an uncertainty of 2.3 MeV.
- There are no FCNC induced by Z boson at tree-level. The box diagram contribution to $K - \bar{K}$ mixing gets new contributions from VLQ, which is a factor of few below experimental value.
- Di-Higgs production via t-channel exchange of U quark is a possible way to test this model at LHC.

Explaining the W boson mass shift

- CDF collaboration recently reported a new measurement of W boson mass that is about 7 sigma away from SM prediction:

$$M_W^{\mathrm{CDF}} = (80,433.5 \pm 9.4)\ \mathrm{MeV}, \quad M_W^{\mathrm{SM}} = (80,357 \pm 6)\ \mathrm{MeV}$$

- Vector-like quark that mixes with SM quark can modify T, S, U parameters. This occurs in the quark seesaw model
- Needed mixing between SM quark and VLQ is or order 0.15. $t - T$ mixing alone won't suffice, as it is constrained by top mass.
- t-quark mixing with two VLQs with the mixing angle of order 0.15 can consistently explain the W mass anomaly
- Source of custodial $SU(2)$ violation is the $t_L - U_L$ mixing
- Mixing of light quarks with VLQs cannot explain the anomaly, since these mixings are constrained by Z hadronic width

W boson mass shift

- $(t,\ U_2,\ U_3)$ mass matrix:

$$M_{up} = \begin{pmatrix} 0 & 0 & y_t\kappa_L \\ 0 & m & M_1 \\ y_t\kappa_R & M_1 & M_2 \end{pmatrix}$$

- $m_t \to 0$ approximation is realized with $m \to 0$
- In the simplified verions with $M_2 = 0$, the oblique T-parameter is:

$$T = \frac{N_c M_T^2 s_L^4}{16\pi s_W^2 m_W^2}$$

Lavoura, Silva (1993); Dawson, Furlan (2012); Chen, Dawson, Furlan (2017)

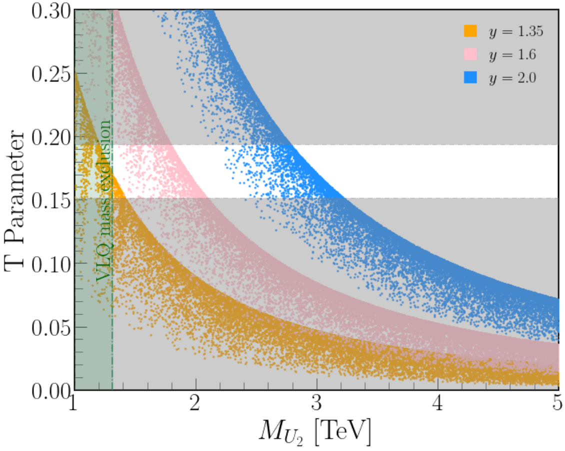
- $t_L - U_L$ mixing angle s_L is contrained from $|V_{td}|$ measurement to be $|s_L| < 0.17$
- $T = 0.16$ is obtained for $M_T = 2.1$ TeV. $T = \{0.15, 0.26\}$ needed to explain W mass shift implies $M_T = \{2.1, 2.6\}$ TeV

Generalized Expressions for T

$$
\begin{aligned}
T = \frac{N_c}{16\pi s^2 M_W^2} \Bigg[& \left(u_{11}^4 - 1\right) m_t^2 + \left(u_{21}^4 M_{U_2}^2 + u_{31}^4 M_{U_3}^2\right) \\
& + 2\left\{ u_{11}^4 - 1 + m_t^2 \left(\frac{1 - u_{11}^2}{m_t^2 - m_b^2} - \frac{u_{11}^2 u_{21}^2}{M_{U_2}^2 - m_t^2} - \frac{u_{11}^2 u_{31}^2}{M_{U_3}^2 - m_t^2} \right) \right\} m_t^2 \ln\left[\frac{m_t^2}{M_{U_3}^2}\right] \\
& + 2u_{21}^2 \left\{ u_{21}^2 + M_{U_2}^2 \left(-\frac{1}{M_{U_2}^2 - m_b^2} + \frac{u_{11}^2}{M_{U_2}^2 - m_t^2} - \frac{u_{31}^2}{M_{U_3}^2 - M_{U_2}^2} \right) \right\} M_{U_2}^2 \ln\left[\frac{M_{U_2}^2}{M_{U_3}^2}\right] \\
& + 2\left\{ \frac{u_{21}^2 + u_{31}^2}{m_t^2 - m_b^2} - \frac{u_{21}^2}{M_{U_2}^2 - m_b^2} - \frac{u_{31}^2}{M_{U_3}^2 - m_b^2} \right\} m_b^4 \ln\left[\frac{m_b^2}{M_{U_3}^2}\right] \Bigg],
\end{aligned}
$$

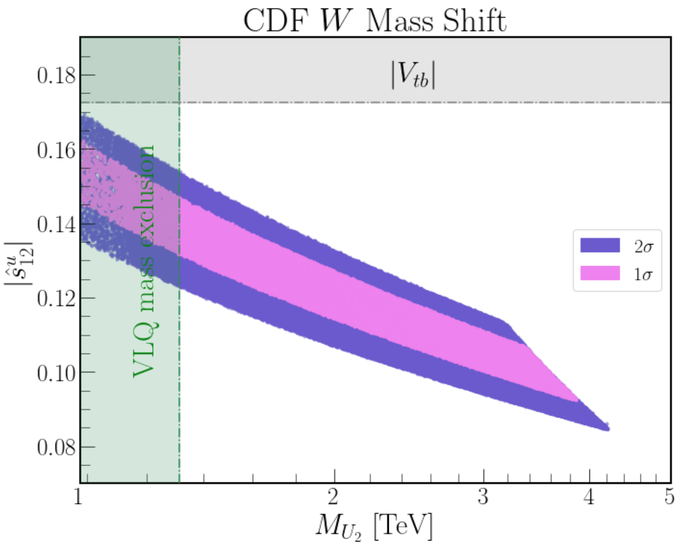
u_{ij} refer to the mixing matrix entries of U that diagoanlizes the 3x3 up-quark mass matrix

T Parameter vs VLQ Mass



Babu, Dcruz (2022)

M_W and VLQ Mass



Conclusions

- Strong CP problem is a strong indication for physics beyond the Standard Model
- Parity Symmetry alone can solve the problem. This is an alternative to the axion solution
- BSM theory should be left-right symmetric, so that P can be defined
- Models where P alone can solve the strong CP problem have a variety of testable consequences
- A second Higgs field and vector-like fermions are characteristics of these theories
- Dirac neutrinos, possibility of non-unitary CKM matrix, and a modified W boson mass can arise in these models
- Neutron EDM cannot be too small compared to experimental limits