

# QED Radiative Corrections for the Bethe-Heitler Process and DVCS

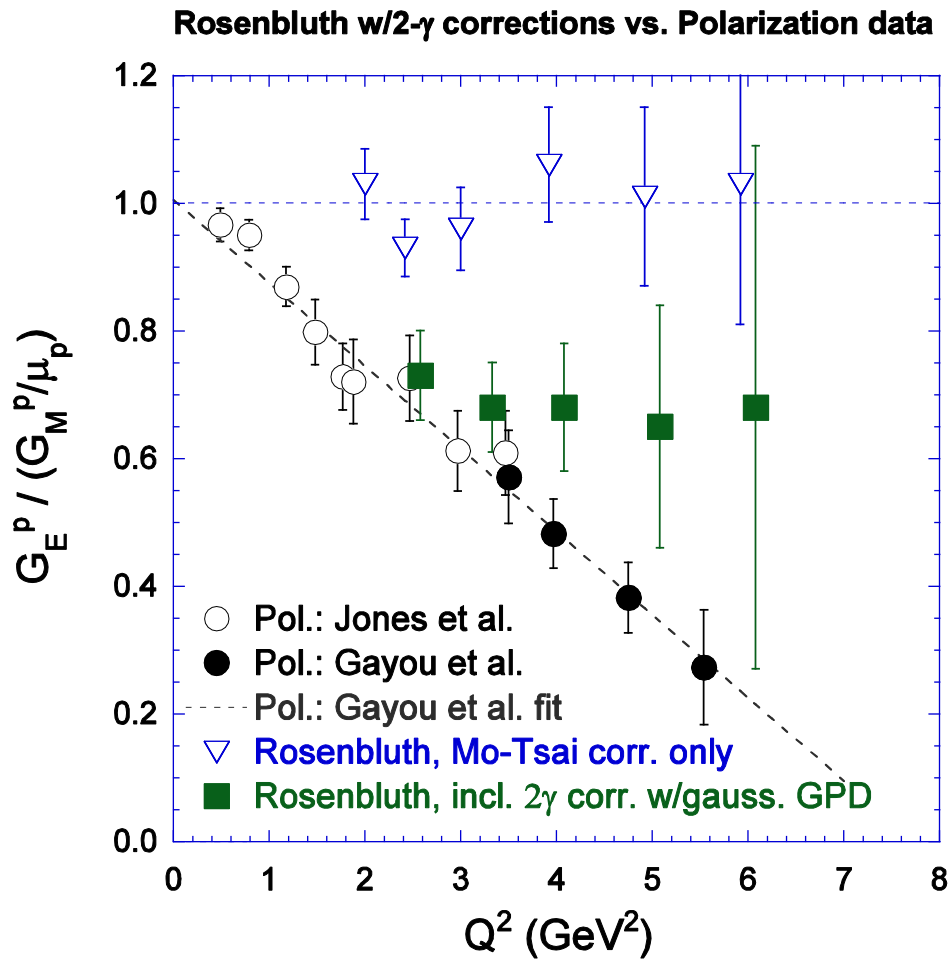
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# Updated Ge/Gm plot

AA, Brodsky, Carlson, Chen, Vanderhaeghen,

Phys.Rev.Lett.93:122301, 2004; Phys.Rev.D72:013008, 2005



# Full Calculation of Bethe-Heitler Contribution

*Additional work by AA et al., using MASCARAD (Phys.Rev.D64:113009,2001)  
Full calculation including soft and hard bremsstrahlung*

Radiative leptonic tensor in full form

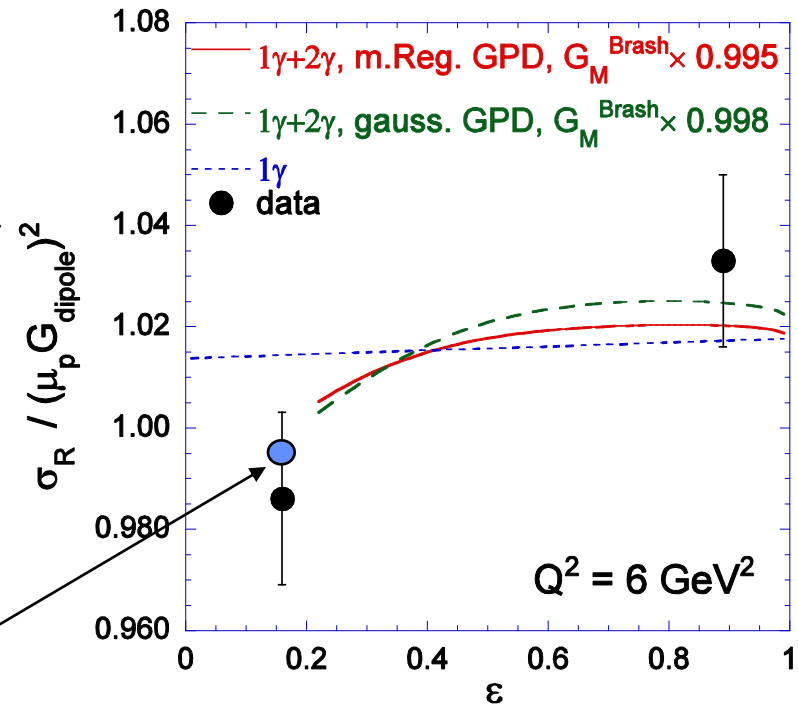
AA et al, *PLB 514, 269 (2001)*

$$L^r_{\mu\nu} = -\frac{1}{2} \text{Tr}(\hat{k}_2 + m)\Gamma_{\mu\alpha}(1 + \gamma_5 \hat{\xi}_e)(\hat{k}_1 + m)\bar{\Gamma}_{\alpha\nu}$$

$$\Gamma_{\mu\alpha} = \left( \frac{k_{1\alpha}}{k \cdot k_1} - \frac{k_{2\alpha}}{k \cdot k_2} \right) \gamma_\mu - \frac{\gamma_\mu \hat{k} \gamma_\alpha}{2k \cdot k_1} - \frac{\gamma_\alpha \hat{k} \gamma_\mu}{2k \cdot k_2}$$

$$\Gamma_{\alpha\nu} = \left( \frac{k_{1\alpha}}{k \cdot k_1} - \frac{k_{2\alpha}}{k \cdot k_2} \right) \gamma_\nu - \frac{\gamma_\alpha \hat{k} \gamma_\nu}{2k \cdot k_1} - \frac{\gamma_\nu \hat{k} \gamma_\alpha}{2k \cdot k_2}$$

Cross section for ep elastic scattering



Additional effect of full soft+hard brem  $\rightarrow$  +1.2% correction to  $\epsilon$ -slope

# Radiative Corrections for Exclusive Processes

- Photon emission is a part of any electron scattering process:  
accelerated charges radiate
- Exclusive electron scattering processes such as  $p(e, e' h_1)h_2$  are in fact inclusive  $p(e, e' h_1)h_2 n\gamma$ ,  
where we can produce an infinite number of low-energy photons
- But low-energy photons do not affect polarization observables, thanks to Low theorem

# Rad. Correction to Single-Spin Asymmetries of VCS

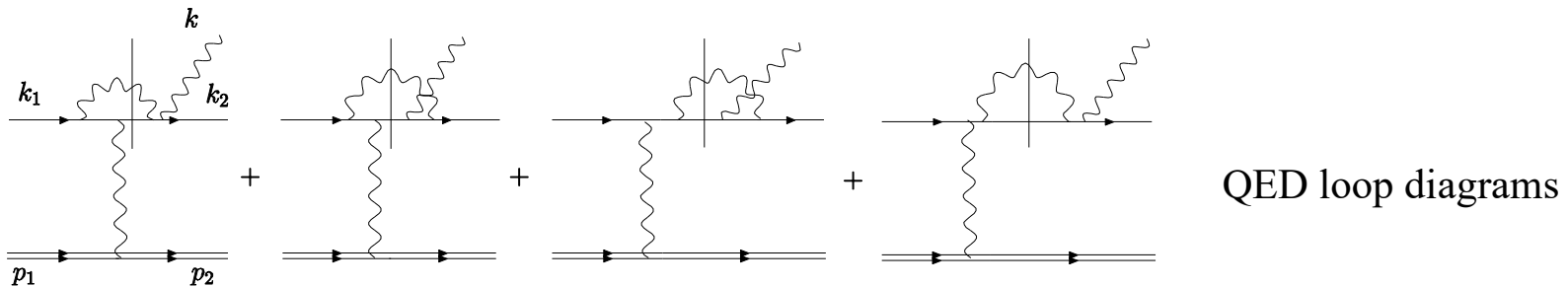
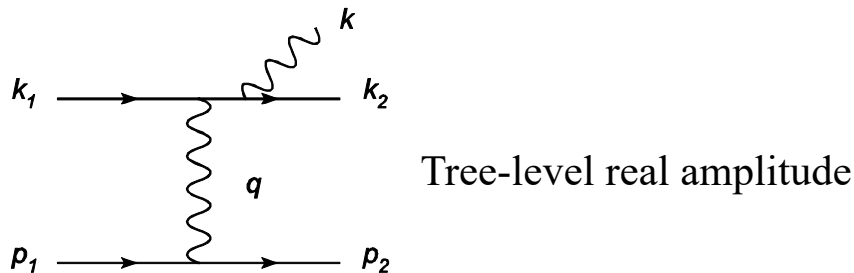
- Evaluation of QED radiative corrections for single-spin asymmetries in Virtual Compton Scattering experiments with CLAS (see also earlier calculations by Vanderhaeghen et al. (2000) for beam SSA in VCS)
  - Since VCS is studied through interference with Bethe-Heitler process, its properties need to be understood precisely
  - If the QED correction to the asymmetries is a few per cent, it alters interpretation of VCS measurements in terms of GPDs
  - Earlier calculations for a related process of radiative Moller scattering  $e^-+e^- \rightarrow e^-+e^-+\gamma$  show large SSA (up to 20%, see Arbuzov et al., Phys. Atom. Nuclei, **59**, 841 (1996))

# Single-Spin Asymmetries in Scattering Processes in QED (early history)

- N. F. Mott, *Proc. R. Soc. (London)*, **A124**, 425 (1929), noticed that polarization and/or asymmetry is due to spin-orbit coupling in the Coulomb scattering of electrons. (Extended to high energy ep-scattering by AA et al., 2002-2005).
- Asymmetries in photon radiation and pair production:
  - Olsen & Maximon (1959), Johnson & Rozics (1964), Motz, Olsen & Koch (1964), Kolbenstwedt & Olsen (1964)

# Feynman Diagrams

- SSA in Bethe-Heitler process is due to interference between (real) tree-level amplitude and QED loops =  $O(\alpha)$  correction that contain absorptive parts



# Formalism

AA, Konchatnij, Merenkov, *Single-spin asymmetries in the Bethe-Heitler process  $e^- + p \rightarrow e^- + \text{gamma} + p$  from QED radiative corrections*, J.Exp.Theor.Phys.102:220-233, 2006; hep-ph/0507059

- Beam SSA

$$A^e = \frac{\alpha}{4\pi} \frac{\text{Re}(P_{\mu\nu}^{(1)} H_{\mu\nu})}{B_{\mu\nu} H_{\mu\nu}}$$

- $H_{\mu\nu}$  and  $B_{\mu\nu}$  are standard hadronic and leptonic tensors in the leading order
- $P_{\mu\nu}$  is calculated from loop diagrams using Cutkosky cuts and doing analytic 2-dimensional integration

$$P_{\mu\nu}^{(1)} = i(k_1 k_2 q \nu) [B_1 \tilde{k}_{1\mu} + B_2 \tilde{k}_{2\mu}] - i(k_1 k_2 q \mu) [B_1^* \tilde{k}_{1\nu} + B_2^* \tilde{k}_{2\nu}]$$



# Formalism (cont.)

$$B_1 = \frac{4}{st} \left[ \frac{4u}{a} \left( 1 - \frac{q^2}{a} L_{qu} \right) + \frac{3t}{b} L_{qt} + \frac{(u^2 - 2s^2 - su)}{cu} L_{sq} + \right. \\ \left. \frac{b}{c} \left( 1 + \frac{s}{c} L_{sq} \right) + \frac{1}{s} (2c - s) L_{tu} - \left( 1 + \frac{2c^2}{st} + \frac{6b}{c} + \frac{2s^2}{ut} \right) L_{qu} + \right. \\ \left. \frac{2b^2}{ut} L_{su} + \left( -1 + \frac{uc}{s^2} - \frac{2t}{s} \right) G + \left( \frac{b}{t} + \frac{b^2}{t^2} \right) \tilde{G} + 3 \right]$$

$$B_2 = -B_1(s \leftrightarrow t), \tilde{G} = G(s \leftrightarrow t)$$

$$a = s + t, b = s + u, c = t + u$$

$$G = L_{qu} (L_q + L_u - 2L_t) - \frac{\pi^2}{3} - 2Li_2 \left( 1 - \frac{q^2}{u} \right) + 2Li_2 \left( 1 - \frac{t}{q^2} \right)$$

$$L_{xy} = L_x - L_y, L_q = \log \left( \frac{-q^2}{m^2} \right), L_u = \log \left( \frac{-u}{m^2} \right), L_t = \log \left( \frac{-u}{m^2} \right), L_s = \log \left( \frac{-s}{m^2} \right)$$

# Expression for beam SSA

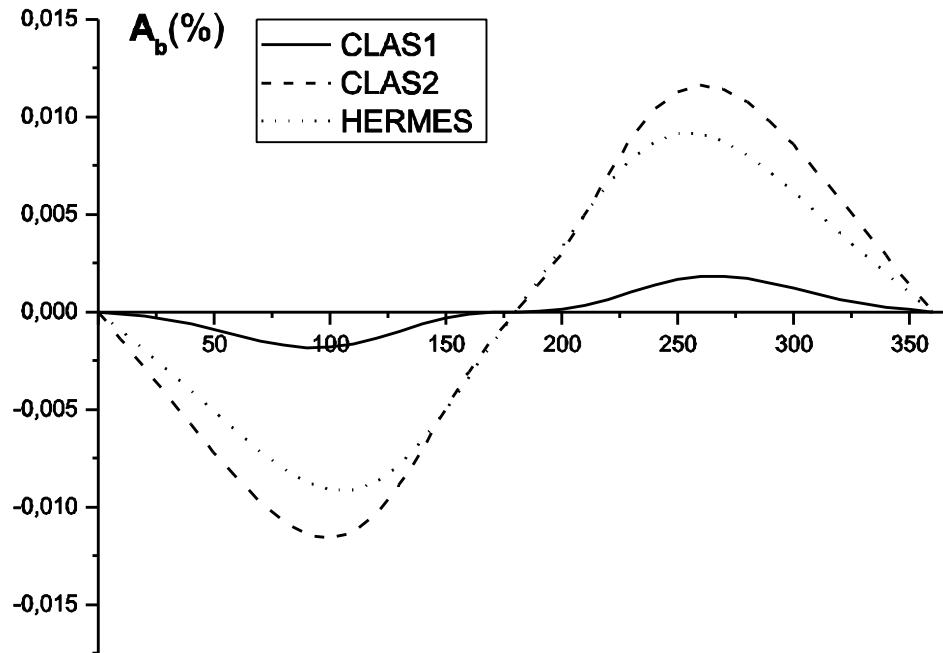
$$P_{\mu\nu}^{(1)} H_{\mu\nu} = \frac{2\pi(k_1 k_2 q p_1)}{st} (F_1^2 - \frac{q^2}{4M^2} F_2^2) [(2V - s + q^2) \bar{B}_1 + (2X - s - u) \bar{B}_2],$$

$$\bar{B}_1 = \frac{2(u^2 - 2s^2 - su)}{uc} + \frac{2bc}{c^2} + \frac{4b^2}{t^2} - \frac{4b}{t} \left(1 + \frac{b}{t}\right) \log\left(1 + \frac{t}{u}\right),$$

$$\bar{B}_2 = \frac{6s}{c} - \frac{2(2b - t)}{t} + 4\left(-1 + \frac{ub}{t^2} - \frac{s}{t}\right) \log\left(1 + \frac{t}{u}\right)$$

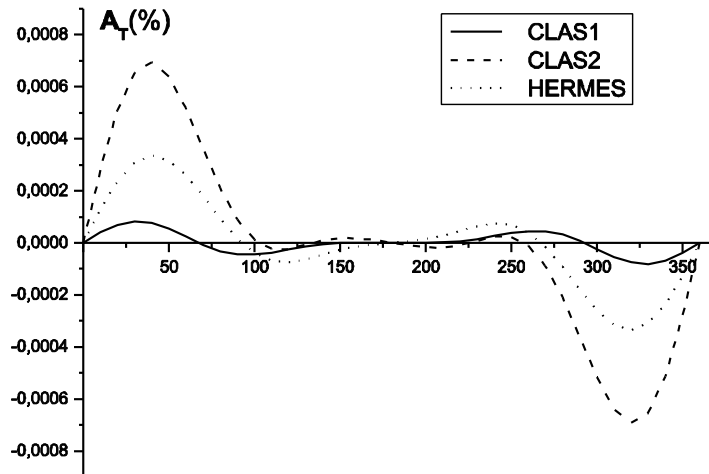
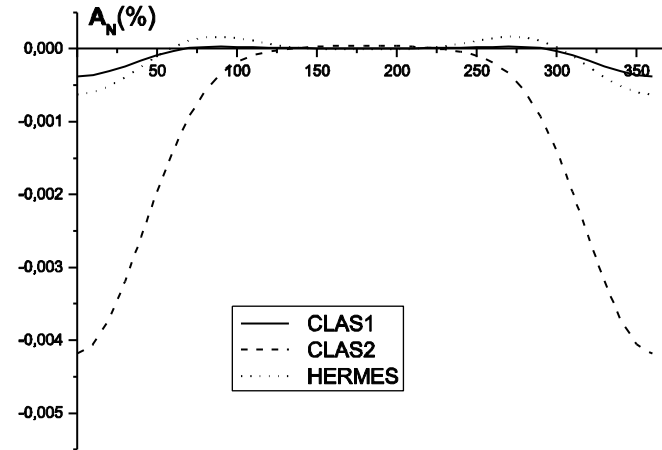
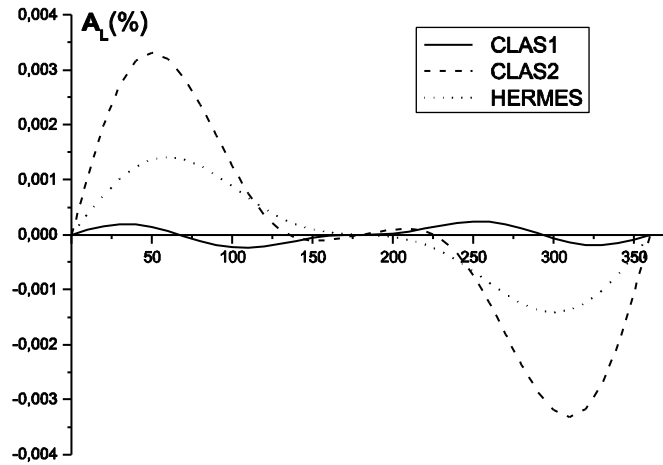
- Results are expressed in terms of analytic functions of Mandelstam invariants
- Free of infrared and mass singularities
- No large logarithms appear
- In addition to  $\alpha$ , proportional to  $q^2$  that is small in DVCS kinematics
- Similar formulas obtained for target SSA; similar suppression takes place

# Numerical results



Asymmetry less than 0.015% due to  $O(\alpha)$ +additional kinematic suppression

# Single-spin target asymmetries



Target asymmetries do not exceed  $10^{-4}$

# RC to Polarized DVCS

- Akushevich, Il'yichev PRD **98**, 013005 (2018) : brem corrections to DVCS cross section and asymmetry, (leading-log approx, both e and p detected)

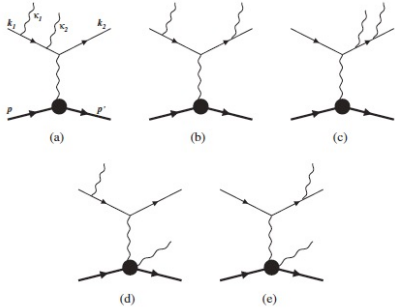


FIG. 2. Feynman graphs with two real photons in the final state: both photons produced by leptons (a), (b), and (c) and by leptons and hadrons (d) and (e).

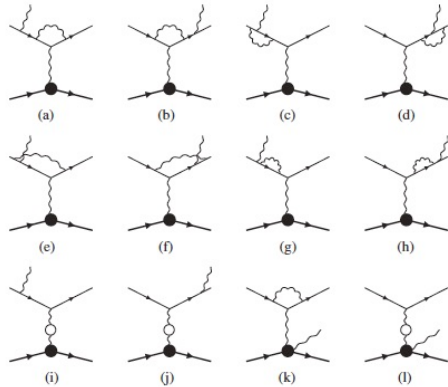


FIG. 4. The one-loop Feynman graphs for the BH (a-h) and DVCS (k) amplitudes containing the infrared divergence and the graphs for the vacuum polarization (i), (j), and (l).

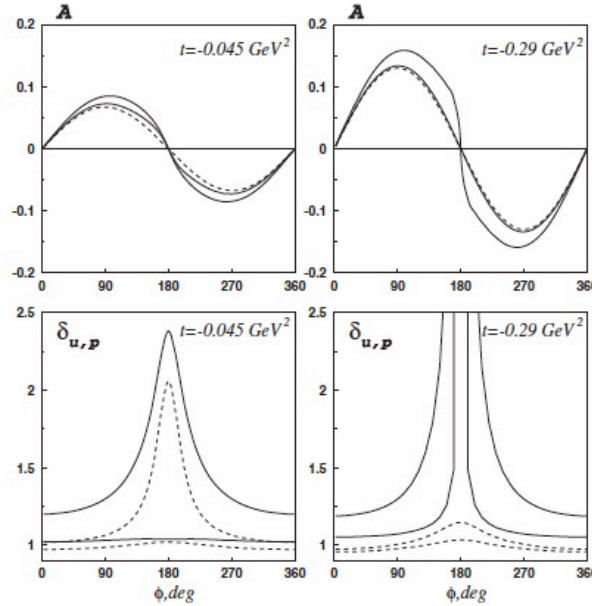


FIG. 5. The  $\phi$ -dependence of the asymmetry (upper) and RC factors (lower plots). The dashed curve at the upper plots gives the  $\sigma_{1Y}$  and the solid curve shows the observed cross sections with  $V_{\text{cut}}^2 = 0.3 \text{ GeV}^2$  (the curve closer to dashed curve) and without cuts. Dashed and solid curves at the bottom plots show  $\delta_{u,p}$  with and without the cut, respectively. The curves with higher values corresponds to  $\delta_p$ , i.e.,  $\delta_p > \delta_u$ . Kinematical variables used for this example were  $x = 0.1$ ,  $Q^2 = 2 \text{ GeV}^2$ , and  $E_{\text{beam}} = 11 \text{ GeV}$ .

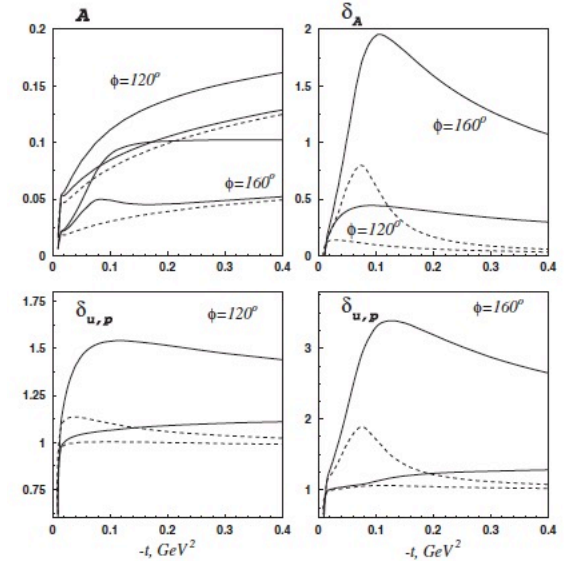
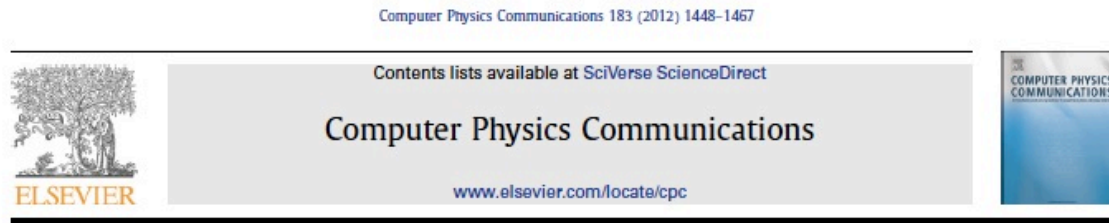


FIG. 6. The  $t$ -dependence of the asymmetry (upper left), RC to asymmetry (upper right), and RC factors (lower plots). Dashed curve for  $A$  gives the  $\sigma_{1Y}$  and solid curves show the observed cross sections with  $V_{\text{cut}}^2 = 0.3 \text{ GeV}^2$  (the curve closer to dashed curve) and without cuts. Dashed and solid curves at the other three plots show  $\delta_{A,u,p}$  with and without the cut, respectively. Kinematical variables used for this example were  $x = 0.1$ ,  $Q^2 = 2 \text{ GeV}^2$ , and  $E_{\text{beam}} = 11 \text{ GeV}$ .

# Leading-Order Bethe Heitler

- Developed for Rad Corrections in elastic  $ep \rightarrow ep$  process.



## Monte Carlo generator ELRADGEN 2.0 for simulation of radiative events in elastic $ep$ -scattering of polarized particles<sup>☆</sup>

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### ABSTRACT

The structure and algorithms of the Monte Carlo generator ELRADGEN 2.0 designed to simulate radiative events in polarized  $ep$ -scattering are presented. The full set of analytical expressions for the QED radiative corrections is presented and discussed in detail. Algorithmic improvements implemented to provide faster simulation of hard real photon events are described. Numerical tests show high quality of generation of photonic variables and radiatively corrected cross section. The comparison of the elastic radiative tail simulated within the kinematical conditions of the BLAST experiment at MIT BATES shows a good agreement with experimental data.

#### Program summary

Program title: ELRADGEN 2.0

Catalogue identifier: AELO\_v1\_0

Program summary URL: [http://cpc.cs.qub.ac.uk/summaries/AELO\\_v1\\_0.html](http://cpc.cs.qub.ac.uk/summaries/AELO_v1_0.html)

Program obtainable from: CPC Program Library, Queen's University, Belfast, N. Ireland

# VCS, DVCS

- Radiatively-corrected Bethe-Heitler process could be a potential source of systematics for measurements of SSA with JLAB, EIC and other facilities
- It is demonstrated that QED loop corrections have negligibly small effect ( $<10^{-3}$ - $10^{-4}$ ) on measured SSA in BH-VCS interference
  - Photon loops connecting electron and proton legs should be included, but they are model-dependent
- Spin asymmetries are much less sensitive to rad.corrections than unpolarized cross sections (the latter is  $\sim 20\%$  effect)
- Corrections due to (double)bremsstrahlung were calculated by Vanderhaeghen et al Phys.Rev.C62:025501,2000 in a soft-photon approximation.- estimated corrections to cross section  $\sim 25\%$ ; to single-spin asymmetry  $\sim 5\%$