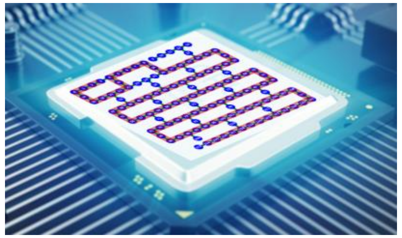
From Bell inequalities to quantum information science and technology: some perspectives on the 2022 Nobel Prize in Physics



Tzu-Chieh Wei

C. N. Yang Institute for Theoretical Physics

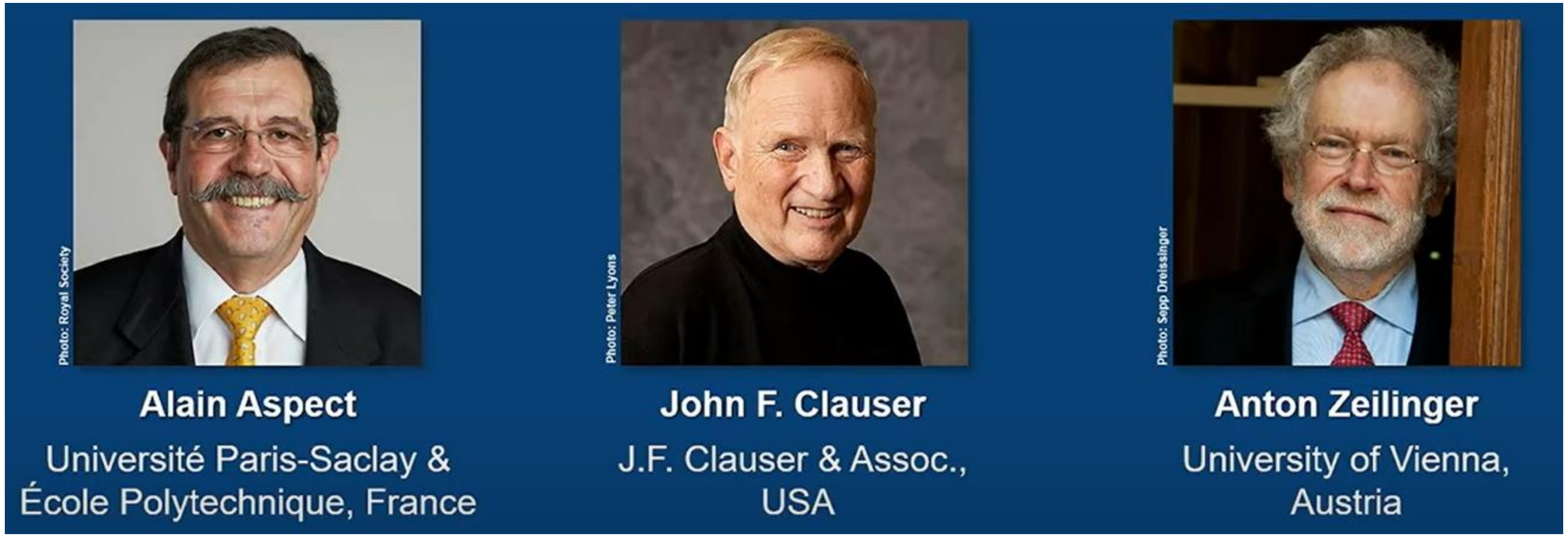
Stony Brook University

C²QA Co-design Center for Quantum Advantage

NSF

The Nobel Prize in Physics 2022

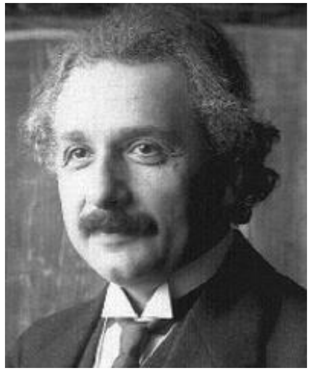
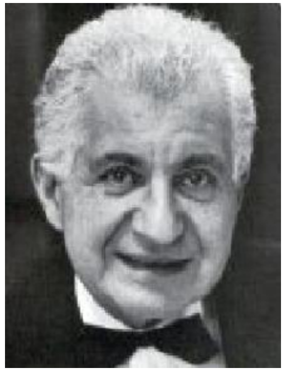
Awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"



credit: nobelprize.org

What’s it about?

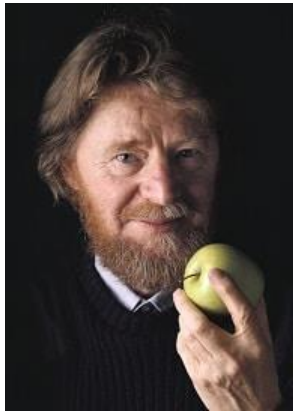
It is about whether quantum mechanics is a `complete’ theory and are there alternative ones that reproduce quantum mechanical prediction. A story begins with founders of QM and still continues …


Albert Einstein, Boris Podolsky, Nathan Rosen

Niels Bohr

Erwin Schrödinger



John Stewart Bell

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

....

Raju and Anton (et al.)

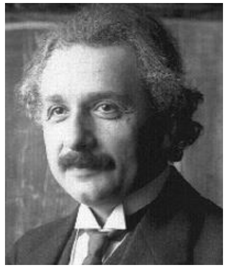
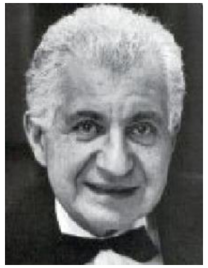
From Raju to me: “PS: I was fortunate to be hosted by Zeilinger on a visit to Vienna in 2018 as part of a committee he convened. I am attaching a picture - you can see GHZ on the board :-)”

Talk Outline

- Introduction
- EPR paradox and Bell's inequalities
- Entanglement as a resource
- Emergence of Quantum Information Science and Technology
- Selected own research
- Conclusion

EPR: Quantum mechanics is not complete

A complete theory: there is an element of reality—it is possible to predict a physical quantity without disturbing the system


- ❑ But in QM for two non-commuting operators $[A,B] \neq 0$, knowledge of A precludes knowledge of B

➔ EPR concluded: either (1) QM is not complete or
(2) these two quantities cannot have simultaneous reality

They used a specific example and argued (2) is wrong thus QM is not complete

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} e^{(2\pi i/h)(x_1 - x_2 + x_0)p} dp \sim \delta(x_1 - x_2 + x_0)$$

- Measure momentum of particle 1: e.g. p_1=p, then know p_2=-p (without measuring particle 2)
- Measure position of particle 1: e.g. x_1, then know x_2= x_1 +x_0 (without measuring particle 2)

Bohr’s response

EPR commented, “Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality only when they can be simultaneously measured or predicted.”

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. Bohr, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

Bohr: “In fact, it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, the coexistence of which might at first sight appear irreconcilable with the basic principles of science.”

➔ making the choice to measure one, e.g., position excludes the possibility of measuring the other, e.g., momentum ➔ the inference of predetermined position and momentum values for the second particle was not valid.

More of nonlocality, less of physical reality

❑ Nonlocality

Einstein wrote the Schroedinger, "it [the paper] did not come out as well as I had originally wanted; rather, the essential thing was, so to speak, smothered by the formalism."

What bothered Einstein more is "**non-locality**" or the "**spooky action at a distance**"

→ the real state of particle B could not depend on which kind of measurement was done in A

❑ Entanglement

Schroedinger then coined the word "entanglement" to describe the correlations between two particles that interact and then separate, as in the EPR experiment

Referred to it as "the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought."

Alternative formulation of EPR: a singlet state

PHYSICAL REVIEW VOLUME 108, NUMBER 4 NOVEMBER 15, 1957

Discussion of Experimental Proof for the Paradox of Einstein, Rosen, and Podolsky

D. Bohm and Y. Aharonov
Technion, Haifa, Israel
(Received May 10, 1957)


Bohm and Aharonov described EPR paradox using a singlet state

See also in Bohm, Quantum Theory (book, 1951)

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle_{AB} - |\downarrow\uparrow\rangle_{AB})$$

- Measure σ_z ($S_z = \frac{\hbar}{2}\sigma_z$) on A: e.g. +1 ➔ can infer B's σ_z is -1
- Measure σ_x ($S_x = \frac{\hbar}{2}\sigma_x$) on A: e.g. -1 ➔ can infer B's σ_x is +1

➔ This formulation helped Jonh Bell to construct an inequality to show inconsistence between "local realistic hidden-variable theories" and quantum mechanics

Bell’s inequality

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
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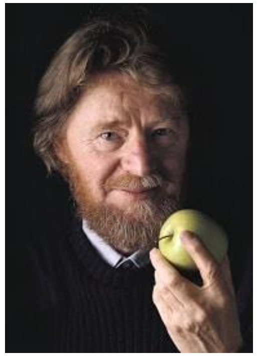
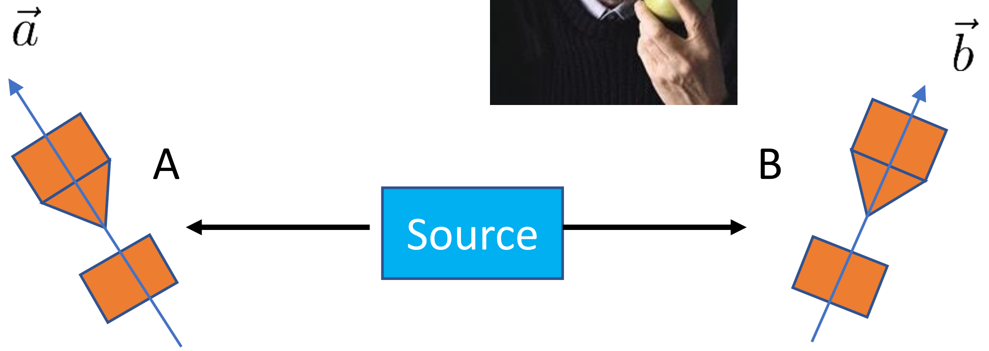




- For the singlet state, A measures along axis ***a*** and B measures along axis ***b***, the prediction from QM is

$$\langle\psi^-|\vec{\sigma}\cdot\vec{a}\otimes\vec{\sigma}\cdot\vec{b}|\psi^-\rangle = -\vec{a}\cdot\vec{b}$$

- For a local realistic theory, there is a local hidden variable λ with distribution ρ(λ) so that the `physical reality’ is

$$A(\vec{a},\lambda) = +1 \text{ or } -1, \quad B(\vec{b},\lambda) = +1 \text{ or } -1$$ So that $$P(\vec{a},\vec{b}) = \int d\lambda\, \rho(\lambda) A(\vec{a},\lambda) B(\vec{b},\lambda) = -\vec{a}\cdot\vec{b}$$

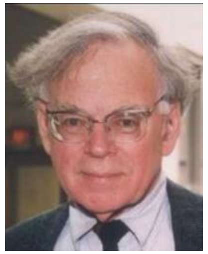
- Bell showed that for small numbers ϵ and δ

$$4(\epsilon + \delta) \geq \sqrt{2} - 1$$ where $$|\bar{P}(\vec{a},\vec{b}) + \vec{a}\cdot\vec{b}| \leq \epsilon \qquad |\overline{\vec{a}\cdot\vec{b}} - \vec{a}\cdot\vec{b}| \leq \delta$$

Clauser-Horne-Shimony-Holt (CHSH) inequality

- ❑ Bell's original inequality cannot be tested experimentally






VOLUME 23, NUMBER 15 PHYSICAL REVIEW LETTERS 13 OCTOBER 1969

PROPOSED EXPERIMENT TO TEST LOCAL HIDDEN-VARIABLE THEORIES*

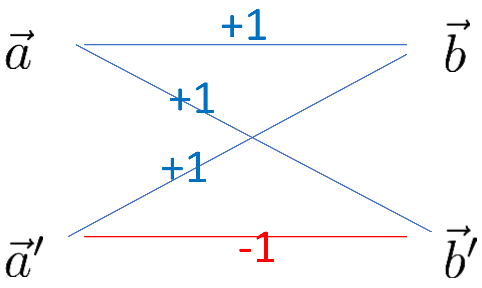
John F. Clauser† Michael A. Horne Abner Shimony Richard A. Holt

(Received 4 August 1969)

- ➢ CHSH proposed an inequality satisfied by LHV theories

$$|P(a,b)-P(a,c)| \leq 2-P(b',b)-P(b',c).$$

recall: $P(\vec{a},\vec{b}) = \int d\lambda\, \rho(\lambda) A(\vec{a},\lambda) B(\vec{b},\lambda)$



$$S \equiv |P(\vec{a},\vec{b}) + P(\vec{a},\vec{b}') + P(\vec{a}',\vec{b}) - P(\vec{a}',\vec{b}')|$$

$$\leq \int d\lambda\, \rho(\lambda) \Big| A(\vec{a},\lambda)B(\vec{b},\lambda) + A(\vec{a},\lambda)B(\vec{b}',\lambda) + A(\vec{a}',\lambda)B(\vec{b},\lambda) - A(\vec{a}',\lambda)B(\vec{b}',\lambda) \Big|$$

$$\leq 2$$

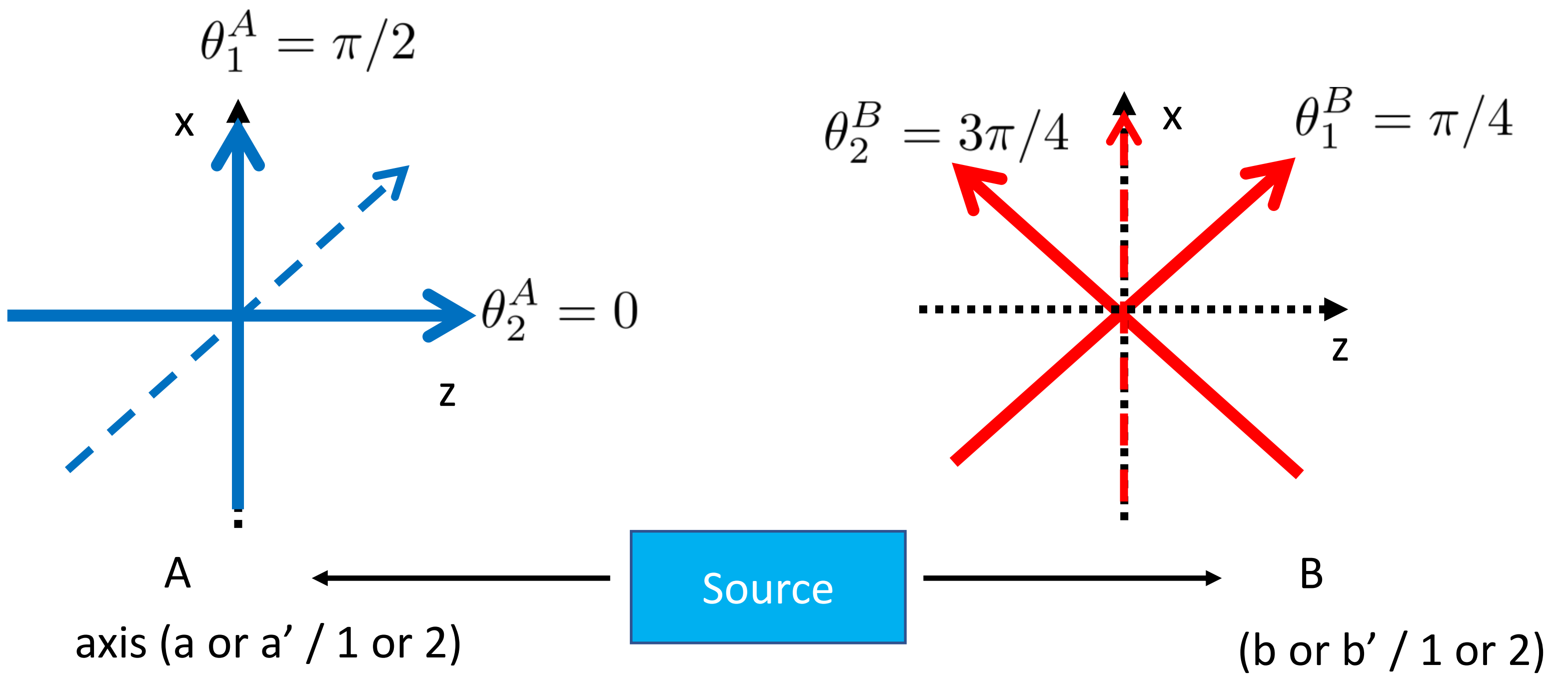
➔ Inequality can be tested experimentally

note: $A(B+B') + A'(B-B') = \pm 2$

Singlet state violates CHSH inequality

$$S \equiv |P(\vec{a},\vec{b}) + P(\vec{a},\vec{b}') + P(\vec{a}',\vec{b}) - P(\vec{a}',\vec{b}')| \leq 2 \text{ for LHV theories}$$

$$P_{\rm QM}(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b} \Longrightarrow \max_\psi B_{\rm QM}(\psi) = 2\sqrt{2} = \left| -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right|$$



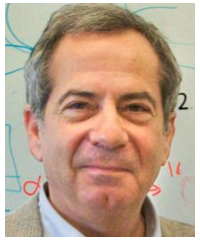
Measurement axes for spins (angles will be halved for photon polarization)

Experimental tests of CHSH-Bell inequality

VOLUME 28, NUMBER 14 PHYSICAL REVIEW LETTERS 3 APRIL 1972

Experimental Test of Local Hidden-Variable Theories*

Stuart J. Freedman and John F. Clauser
Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 4 February 1972)

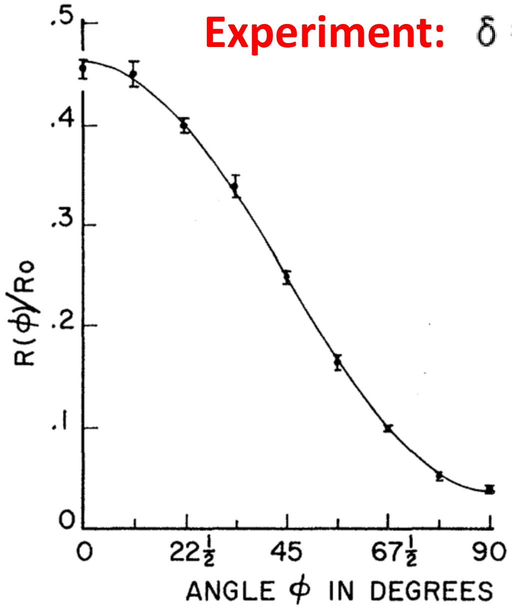


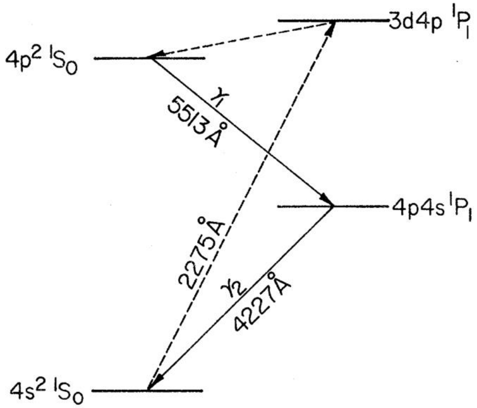
The results of the measurements of the correlation $R(\varphi)/R_0$, corresponding to a total integration time of ~200 h, are shown in Fig. 3. All error limits are conservative estimates of 1 standard deviation. Using the values at $22\frac{1}{2}°$ and $67\frac{1}{2}°$, we obtain $\delta = 0.050 \pm 0.008$ in clear violation of inequality (3).[12] Furthermore, we observe no evidence for a deviation from the predictions of quantum mechanics, calculated from the measured polarizer efficiencies and solid angles, and shown as the solid curve in Fig. 3. We consider these results to be strong evidence against local hidden-variable theories.

LHV theories:

$$\delta = |R(22\tfrac{1}{2}°)/R_0 - R(67\tfrac{1}{2}°)/R_0| - \tfrac{1}{4} \leq 0$$

Experiment: $\delta = 0.050 \pm 0.008$





Two photons in atomic cascade of calcium

Improved experiments on CHSH-Bell inequality

VOLUME 47, NUMBER 7 PHYSICAL REVIEW LETTERS 17 AUGUST 1981

Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France



$\delta_{exp} = 5.72\times 10^{-2} \pm 0.43\times 10^{-2}$

eived 30 March 1981)

VOLUME 49, NUMBER 2 PHYSICAL REVIEW LETTERS 12 JULY 1982

Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities

$S_{QM} = 2.70 \pm 0.05$

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Laboratoire associé au Centre National de la Recherche Scientifique, Université Paris-Sud, F-91406 Orsay, France

(Received 30 December 1981)

VOLUME 49, NUMBER 25 PHYSICAL REVIEW LETTERS 20 DECEMBER 1982

Experimental Test of Bell's Inequalities Using Time-Varying Analyzers

Alain Aspect, Jean Dalibard,[a] and Gérard Roger
Institut d'Optique Théorique et Appliquée, F-91406 Orsay Cédex, France

(Received 27 September 1982)



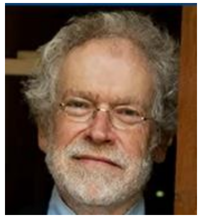
Violation CHSH-Bell inequality at space-like separation

PRL

VOLUME 81 7 DECEMBER 1998 NUMBER 23

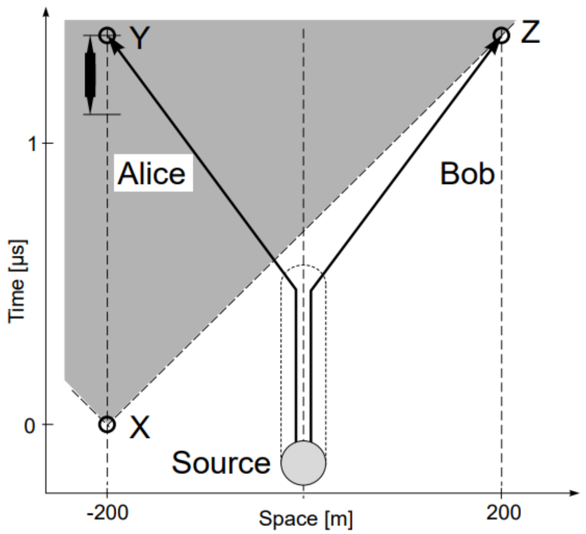
Violation of Bell's Inequality under Strict Einstein Locality Conditions

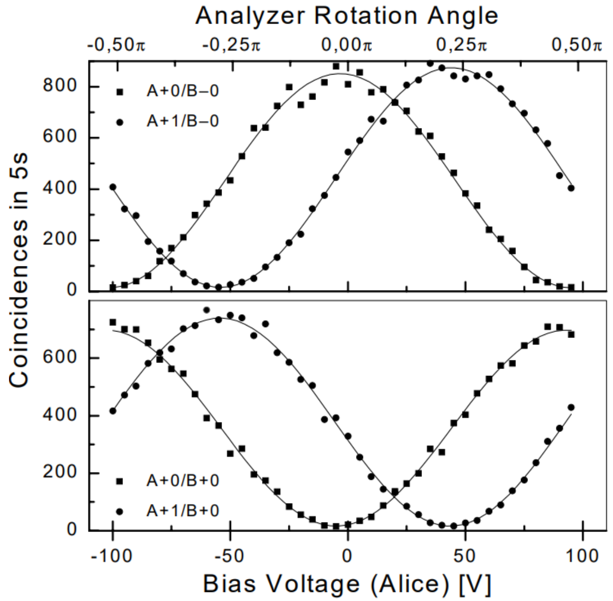
Gregor Weihs, Thomas Jennewein, Christoph Simon, Harald Weinfurter, and Anton Zeilinger
Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria
(Received 6 August 1998)



Anton Zeilinger

.... The necessary spacelike separation of the observations is achieved by sufficient physical distance between the measurement stations, by ultrafast and random setting of the analyzers, and by completely independent data registration. [S0031-9007(98)07901-0]





$$S = 2.73 \pm 0.02$$

Violation of local realistic theory at a single shot

- Greenberger, Horne and Zeilinger (GHZ) consider the four-particle entangled state and showed that no local hidden variable theory can reproduce the correlation

$$|1, 0\rangle = (|\uparrow\uparrow\downarrow\downarrow\rangle - |\downarrow\downarrow\uparrow\uparrow\rangle) / \sqrt{2}$$

- Later Greenberger, Horne, Shimony and Zeilinger (GHSZ) [1990] consider a 3-particle entangled state and showed violation without inequality
  - For classical local theory, one attributes this to local properties:
    $x_1x_2x_3=+1$, $y_1y_2x_3=-1$, $y_1x_2y_3=-1$, $x_1y_2y_3=-1$ (where $x,y= \pm1$)

[Mermin, 1990]

  - But contradiction arises when we multiply all four equalities:
    1= -1 ! (experiments show QM is correct)

[Pan et al. (Zeilinger's group), Nature 2000]

(book, 1989)

Bell's Theorem, Quantum Theory and Conceptions of the Universe

Going Beyond Bell's Theorem

Daniel M. Greenberger, Michael A. Horne, Anton Zeilinger

Pages 69-72

$$|\mathrm{GHSZ}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle)$$

$$X \otimes X \otimes X|\mathrm{GHSZ}\rangle = (+1)|\mathrm{GHSZ}\rangle$$

$$Y \otimes Y \otimes X|\mathrm{GHSZ}\rangle = (-1)|\mathrm{GHSZ}\rangle$$

$$Y \otimes X \otimes Y|\mathrm{GHSZ}\rangle = (-1)|\mathrm{GHSZ}\rangle$$

$$X \otimes Y \otimes Y|\mathrm{GHSZ}\rangle = (-1)|\mathrm{GHSZ}\rangle$$

Talk Outline

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- Entanglement as a resource
- Emergence of Quantum Information Science and Technology
- Selected own research
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Change of view: Entanglement as a resource!

❑ Ekert: "Quantum cryptography based on Bell's theorem" [PRL 1991]

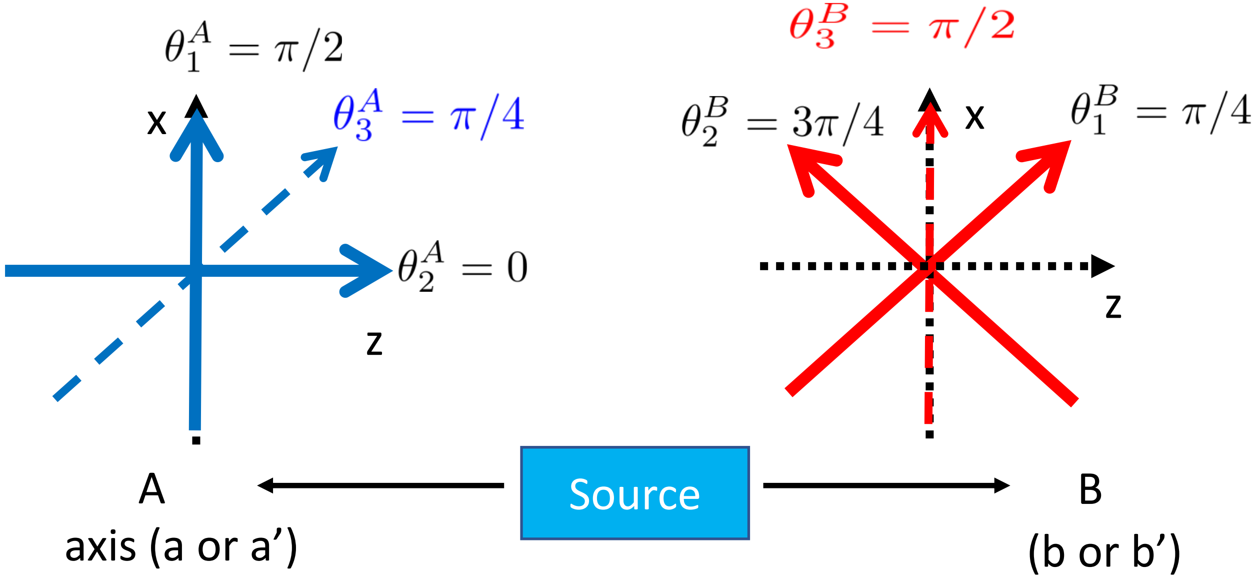
Artur Ekert

➔ Add a third axis of measurement on each side

➔ Alice & Bob randomly measure in three different directions

1. For the axes in the same direction, outcomes are anti-correlated → can establish a common secret binary string

2. The rest of data can be used to check violation of Bell inequality → degree of security

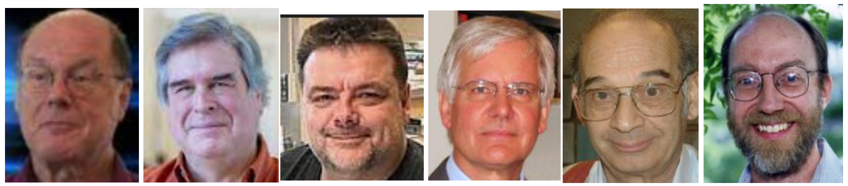


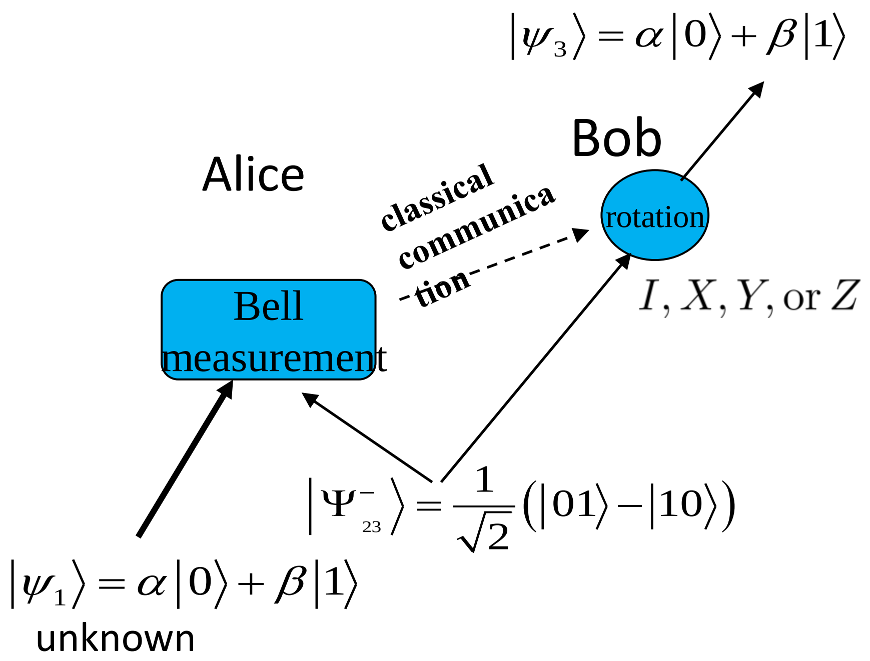
$$P_{\rm QM}(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b} \Longrightarrow \max_\psi B_{\rm QM}(\psi) = 2\sqrt{2}$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle_{AB} - |\downarrow\uparrow\rangle_{AB})$$

Singlet state enables "teleportation"

- ❑ Bennett, Brassard, Crepeau, Jozsa, Peres & Wootters: "Teleporting an unknown state via dual classical and EPR channels" [PRL 1992]



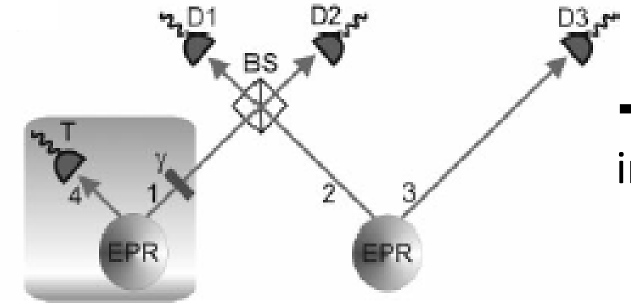


Experiments: Zeilinger's group, Nature 1997
& F. De Martini's group, PRL 1998

- ❑ This leads to entanglement swapping

Theory: Zukowski, Zeilinger, Horne & Ekert, PRL '93

Experiment: Pan, Bouwmeester, Weinfurter & Zeilinger, PRL ,98



➔ Basis for quantum repeater in a quantum internet

Quantum Teleportation (analysis)

$$|\psi\rangle_1 \otimes |\Phi^+\rangle_{23} = \frac{1}{\sqrt{2}}(a|0\rangle + b|1\rangle) \otimes (|00\rangle + |11\rangle)$$

$$= \frac{1}{\sqrt{2}}(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle)$$

$$|\Phi^\pm\rangle \equiv \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

$$|\Psi^\pm\rangle \equiv \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

$$= \frac{1}{2}\left\{a(|\Phi^+\rangle + |\Phi^-\rangle) \otimes |0\rangle + a(|\Psi^+\rangle + |\Psi^-\rangle) \otimes |1\rangle + b(|\Psi^+\rangle - |\Psi^-\rangle) \otimes |0\rangle + b(|\Phi^+\rangle - |\Phi^-\rangle) \otimes |1\rangle\right\}$$

$$= \frac{1}{2}\left\{|\Phi^+\rangle \otimes (a|0\rangle + b|1\rangle) + |\Phi^-\rangle \otimes (a|0\rangle - b|1\rangle) + |\Psi^+\rangle \otimes (a|1\rangle + b|0\rangle) + |\Psi^-\rangle \otimes (a|1\rangle - b|0\rangle)\right\}$$

$I|\psi\rangle$ $\quad Z|\psi\rangle$ $\quad X|\psi\rangle$ $\quad iY|\psi\rangle$

The unknown information a & b is preserved in the third particle, but depending on the outcome of the 'Bell-state' measurement in the basis of $\Phi^{\pm}$ & $\Psi^{\pm}$

Four possible outcomes, Alice informs Bob: (1) Φ^+ → apply identity (nothing); (2) Φ^- → apply Z to particle 3; (3) Ψ^- → apply X to particle 3; (4) Ψ^-→ apply –iY to particle 3

➔ Recover ψ at particle 3

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Quantum computation can be powerful

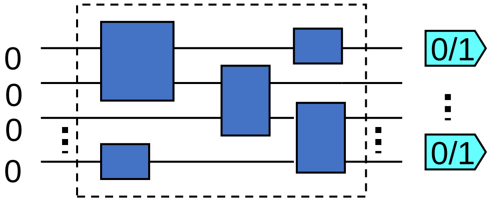
Feynman: they can simulate other quantum systems more efficiently than classical computers

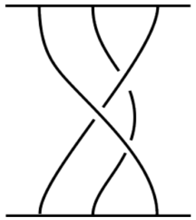
Shor: quantum algorithm for factoring (almost) exponentially faster than classical algorithms

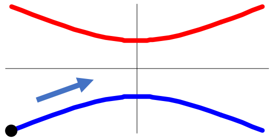
Grover: quantum search algorithm offers quadratic speedup for unstructured data search

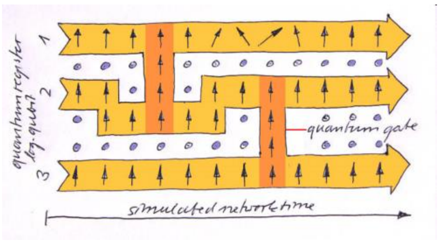
➔ There is a zoo of quantum algorithms, https://quantumalgorithmzoo.org/

Different quantum computing frameworks: circuit model, topological, adiabatic and measurement-based, etc.





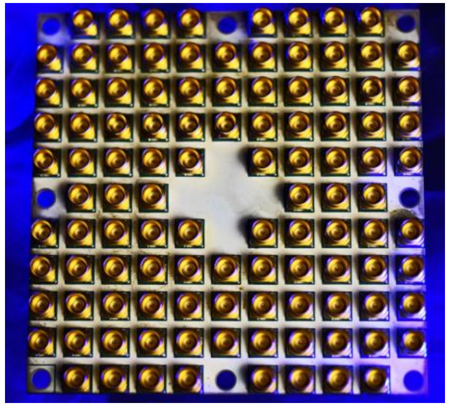




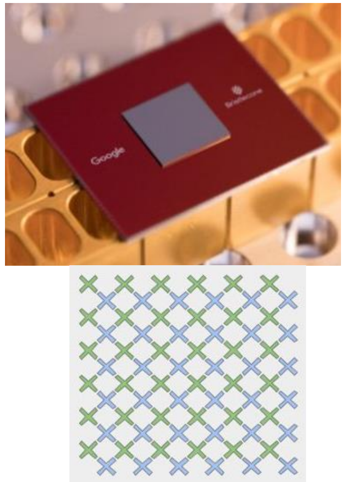
Zoo of quantum computers

IBM 127-Qubit Q Computer

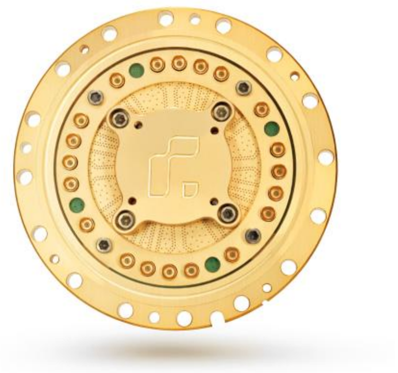
Intel 49-Qubit QC



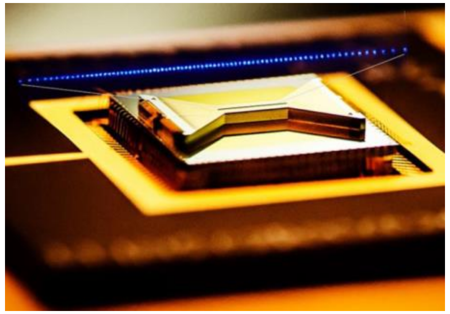
Google 72-Qubit QC



Rigetti 80-Qubit QC

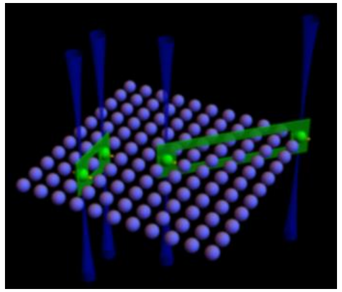


IonQ 160-Qubit QC
[trapped ions]

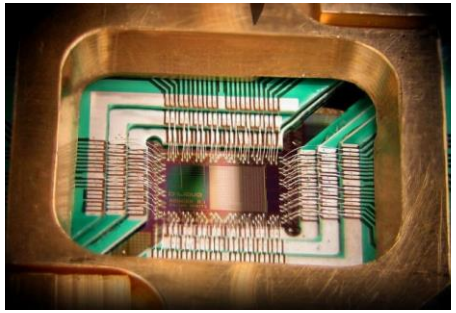


Quantinuum 20 qubits
[trapped ions]

QuEra 256-atom
quantum simulator

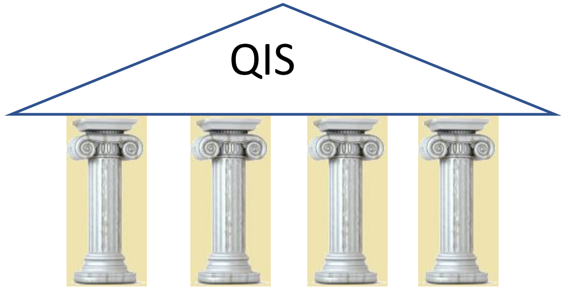


D-Wave 5000+
qubit annealer



Quantum information science and technology

Four pillars: Quantum communication, quantum computation, quantum simulations and quantum sensing & metrology (per European Flagship)



➔ Increased funding and investment

New directions may emerge: e.g. quantum machine learning

Applications are also important

Both for solving scientific problems or realistic applications in various areas

Quantum-ready workforce development is crucial for sustainment

Note: SBU is launching a QIST Master’s program in Fall 2023

NISQ devices and quantum advantage

Preskill: current machines are noisy intermediate-scale quantum (**NISQ**) devices

➔ A milestone is to demonstrate quantum supremacy/advantage

Notable experiments:

Google quantum supremacy with 53 qubits [Nature 2019]

Quantum walk with 62 qubits [Pan's group, Science 2021]

Creation of toric code state with 31 qubits [Google QAI, Science 2021]

Xanadu's boson sampling with 219 photons in 216 modes [Nature 2022]

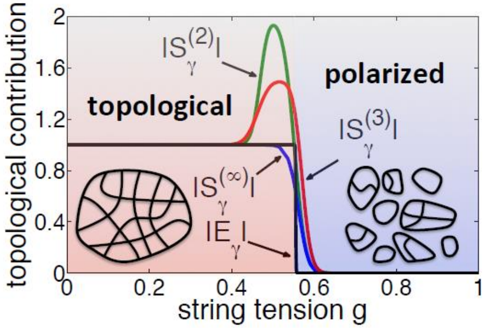
"Traversable Worm Hole" on Google's machine [Nature 2022]

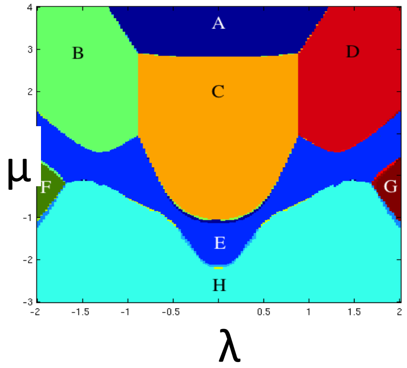
Talk Outline

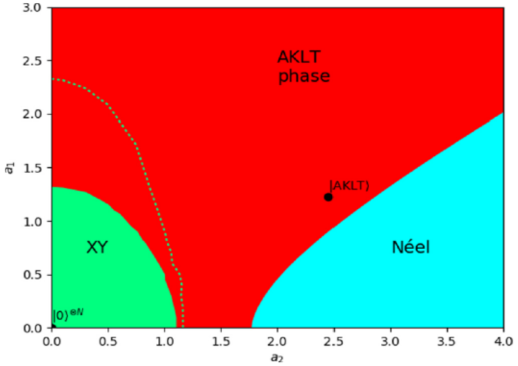
- Introduction
- EPR paradox and Bell's inequalities
- Entanglement as a resource
- Emergence of Quantum Information Science and Technology
- Selected own research
- Conclusion

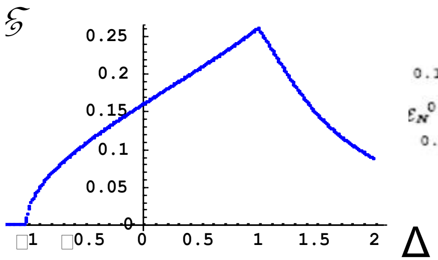
Some snapshots of my research interests

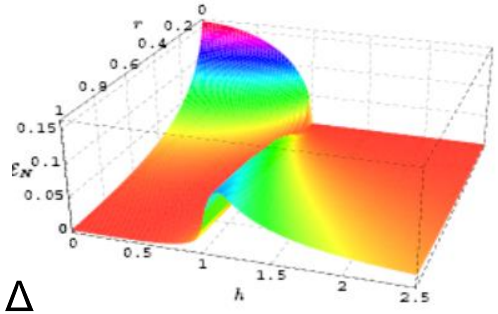
❑ Quantum entanglement in many-body systems and phase transitions



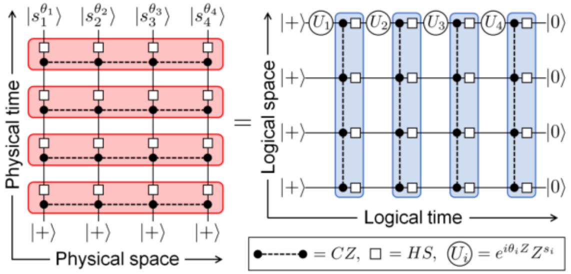


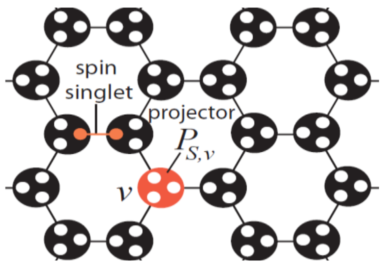


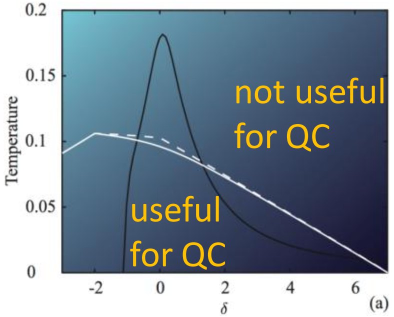


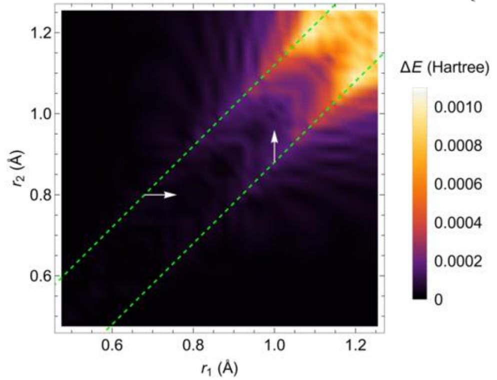


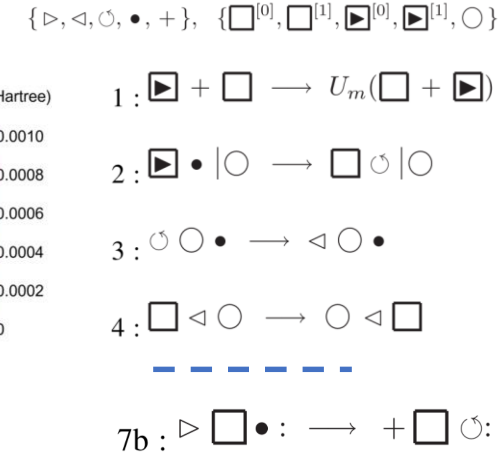
❑ Quantum computation (different frameworks) and simulations



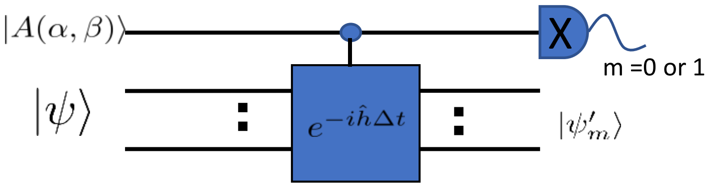


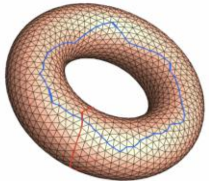


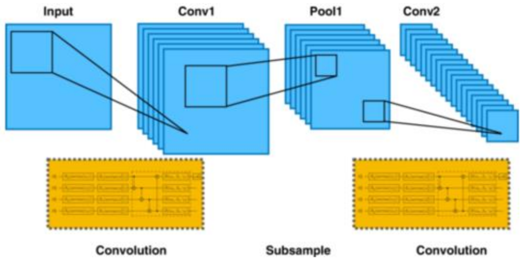




❑ Quantum algorithms & quantum machine learning

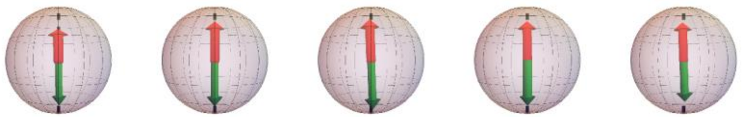




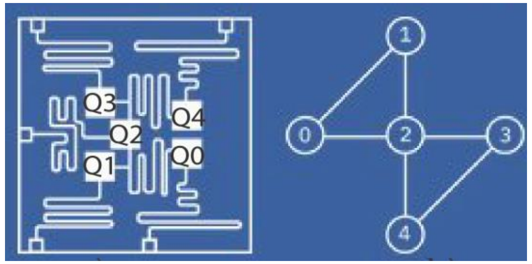


Several works on running quantum devices

- “Detector Tomography on IBM Quantum Computers and Mitigation of Imperfect Measurement”

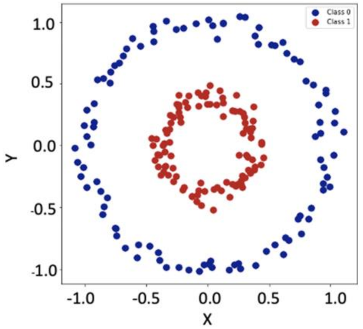


[Chen, Farahzad, Yoo & Wei, PRA **100**,052315 (2019)]



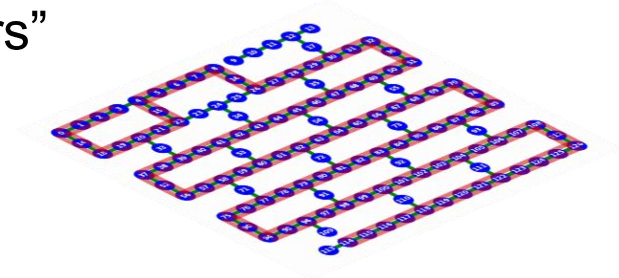
- “Unified framework for quantum classification”

[Nghiem, Chen & Wei,
PR Research **3**,033056 (2021)]



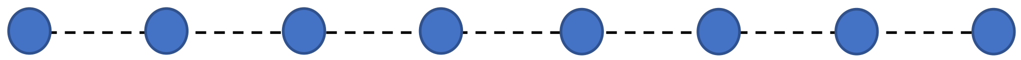
- “Simulating large-size quantum spin chains on cloud superconducting quantum computers”

[Yu, Zhao & Wei, arXiv:2207.09994]

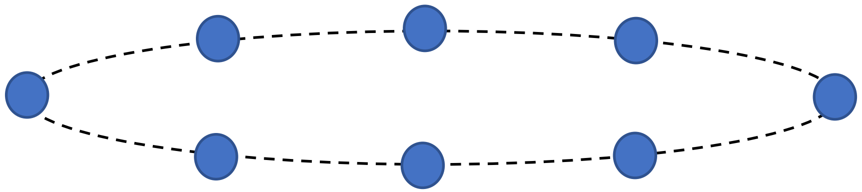


Our recent work: realizing large XXZ spin chains

(a)



(b)



$$\hat{H}_{\mathrm{XXZ}}(\Delta) = \sum_{j=1}^{N-1} \hat{h}_{XXZ}^{[j,j+1]}(\Delta) = \sum_{j=1}^{N-1} \left(\sigma_x^{[j]}\sigma_x^{[j+1]} + \sigma_y^{[j]}\sigma_y^{[j+1]} + \Delta\sigma_z^{[j]}\sigma_z^{[j+1]} \right)$$

Δ=1 is the antiferromagnetic Heisenberg chain

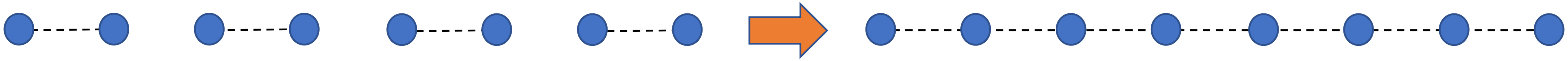
Δ<-1 is the ferromagnetic phase → GS: ↑↑ ⋯ ↑ or ↓↓ ⋯ ↓

We will focus on Δ>-1:

(1) -1< Δ < 1 is the gapless/critical phase

(2) Δ>1 is the antiferromagnetic phase

Connecting singlet pairs to XXZ ground states



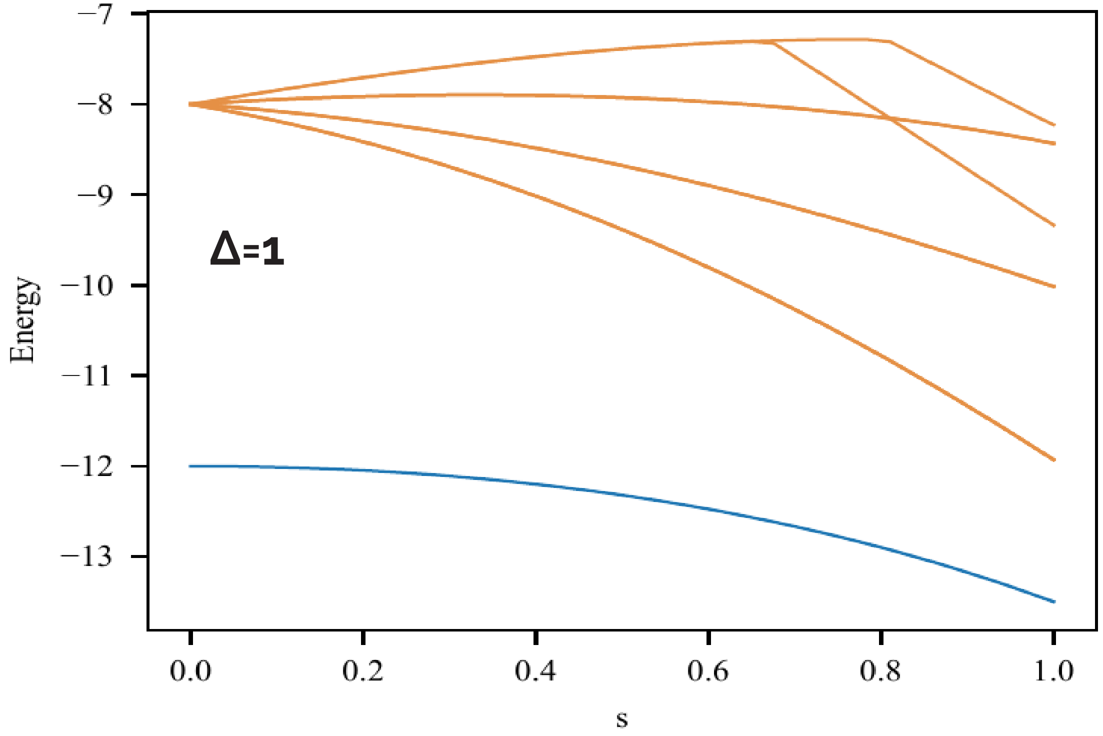
$$\hat{H}_{\text{odd}} = \sum_{j=1}^{N/2-1} \left(\sigma_x^{[2j-1]}\sigma_x^{[2j]} + \sigma_y^{[2j-1]}\sigma_y^{[2j]} + \Delta\sigma_z^{[2j-1]}\sigma_z^{[2j]} \right)$$

$$\hat{H}_{\text{XXZ}}(\Delta) = \sum_{j=1}^{N-1} \hat{h}_{XXZ}^{[j,j+1]}(\Delta)$$

➔ Product of singlet pairs is GS of H_{odd}

➔ Can be connected to GS of H_{XXZ} using adiabatic connection:

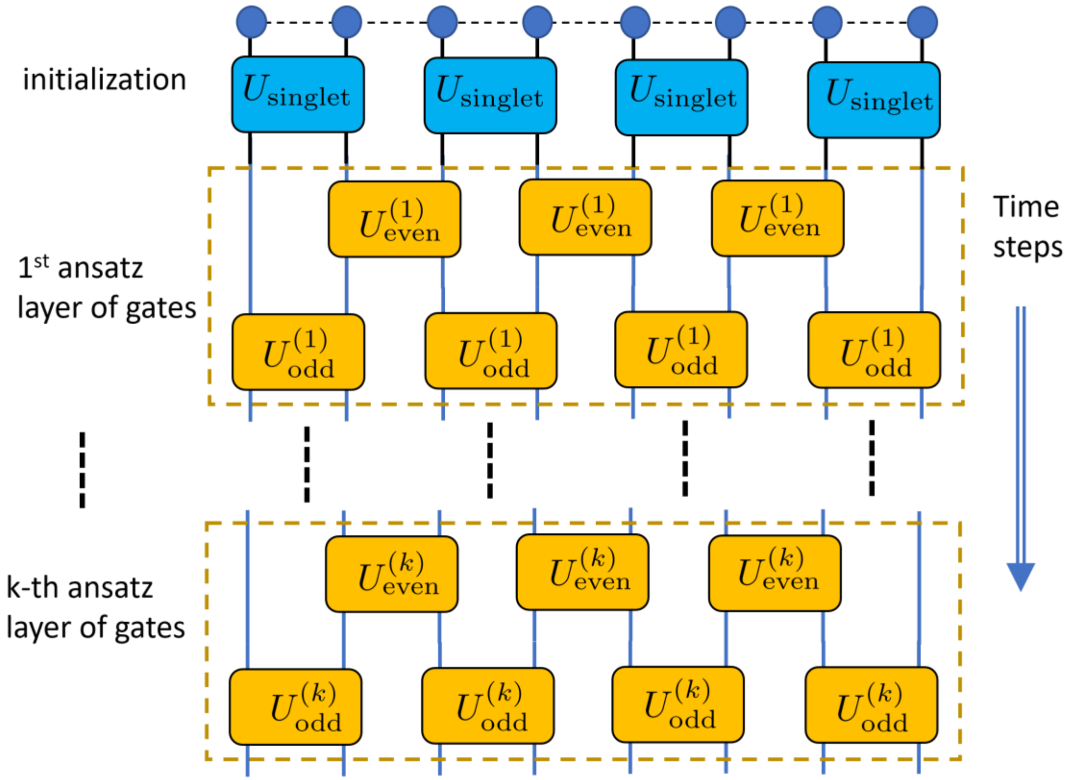
$$\hat{H}(s) \quad = \quad (1-s)\hat{H}_{\text{odd}} + s\,\hat{H}_{\text{XXZ}} = \hat{H}_o(s) + \hat{H}_e(s)$$



Turn Trotterized evolution into variational circuits

$$\hat{H}(s) = (1-s)\hat{H}_{\text{odd}} + s\,\hat{H}_{\text{XXZ}} = \hat{H}_o(s) + \hat{H}_e(s)$$

$$\mathbf{U}_{\text{evo}} \approx \prod_{l=1}^{N_L} e^{-i\hat{H}(s_l)\delta s} \approx \prod_{l=1}^{N_L} \left(e^{-i\hat{H}_{\text{e}}(s_l)\delta s} e^{-i\hat{H}_{\text{o}}(s_l)\delta s} \right)$$



➔ Make {*s*, *δs*} variational

$$|\psi_{\text{ansatz}}(\{\theta\})\rangle = \bigotimes_{l=1}^{N_L} \left[U^{(l)}_{\text{even}}(\{\theta_e\}) U^{(l)}_{\text{odd}}(\{\theta_o\}) \right] |\psi_{\text{singlets}}\rangle$$

$$U^{(l)}_{\text{even/odd}}(\{\theta\}) = \bigotimes_{j \in \text{even/odd}} \left[e^{-i\theta^{(l)}_{e/o,x}\, \sigma_x^{[j]}\sigma_x^{[j+1]} - i\theta^{(l)}_{e/o,y}\, \sigma_y^{[j]}\sigma_y^{[j+1]} - i\theta^{(l)}_{e/o,z}\, \sigma_z^{[j]}\sigma_z^{[j+1]}} \right]$$

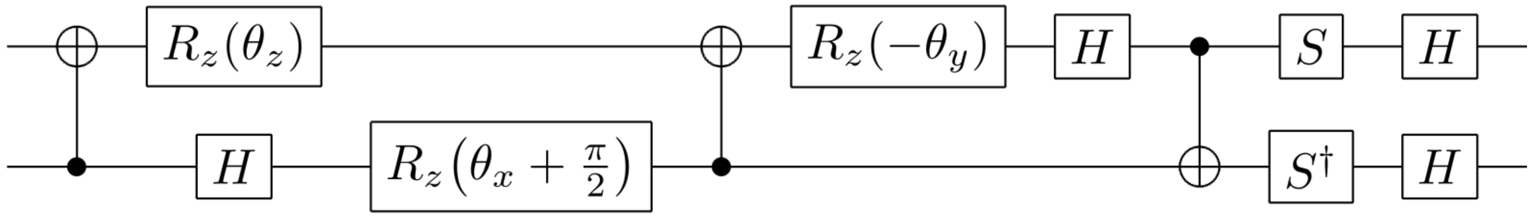
Rxyz Gate

$$|\psi_{\text{ansatz}}(\{\theta\})\rangle = \bigotimes_{l=1}^{N_L} \left[U_{\text{even}}^{(l)}(\{\theta_e\})U_{\text{odd}}^{(l)}(\{\theta_o\})\right]|\psi_{\text{singlets}}\rangle$$

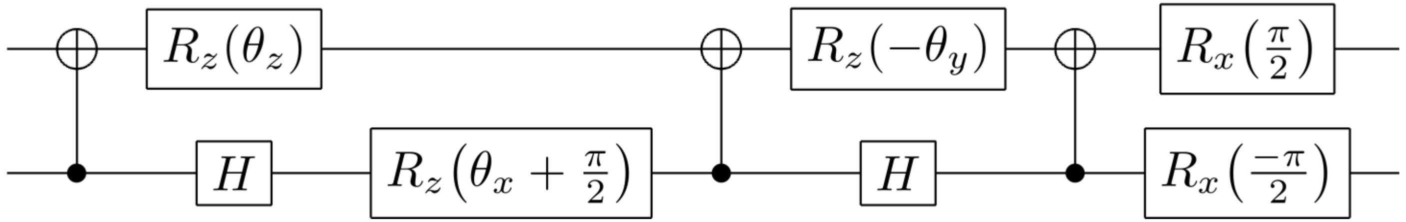
$$U_{\text{even/odd}}^{(l)}(\{\theta\}) = \bigotimes_{j\in\text{even/odd}} \left[e^{-i\theta_{e/o,x}^{(l)}\,\sigma_x^{[j]}\sigma_x^{[j+1]}-i\theta_{e/o,y}^{(l)}\,\sigma_y^{[j]}\sigma_y^{[j+1]}-i\theta_{e/o,z}^{(l)}\,\sigma_z^{[j]}\sigma_z^{[j+1]}}\right]$$

$$R_{xyz}(\theta_x,\theta_y,\theta_z) \equiv e^{-i(\theta_x/2)\sigma_x\otimes\sigma_x-i(\theta_y/2)\sigma_y\otimes\sigma_y-i(\theta_z/2)\sigma_z\otimes\sigma_z}$$

$=$



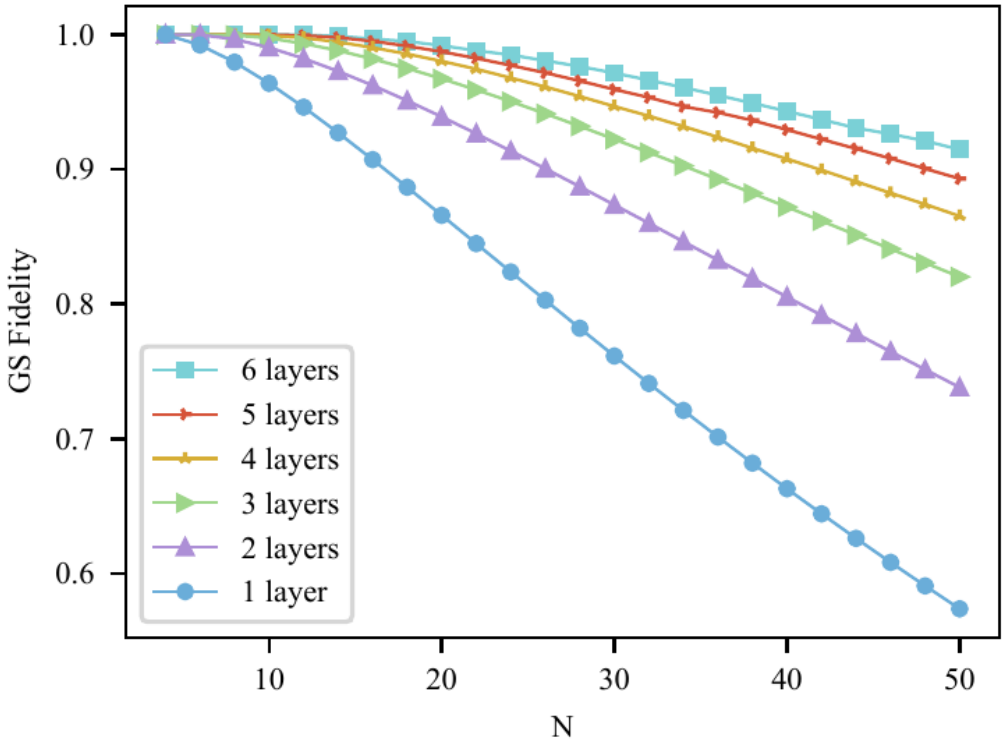
$=$



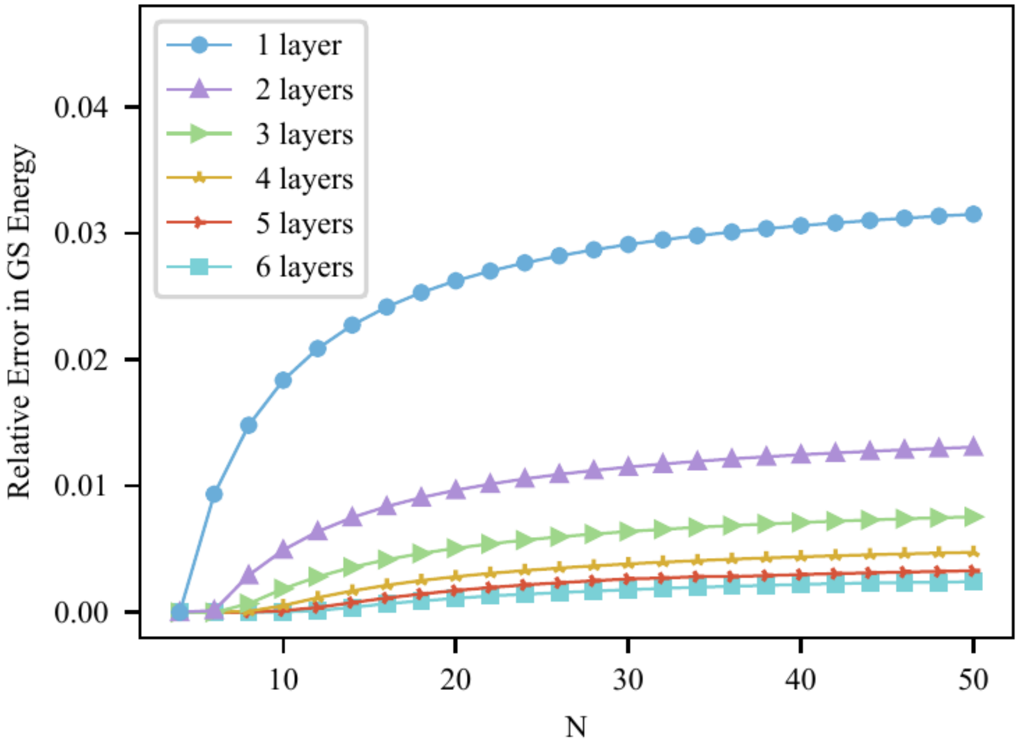
How good is the ansatz?

➔ Use Ground-state Fidelity and relative error in GS energy

$$f \equiv |\langle \psi_{\mathrm{gs}} | \psi_{\mathrm{ansatz}}(\{\theta^*\})\rangle|$$



$$\epsilon \equiv |E_{\mathrm{ansatz}}(\{\theta^*\}) - E_{\mathrm{gs}}|/|E_{\mathrm{gs}}|$$

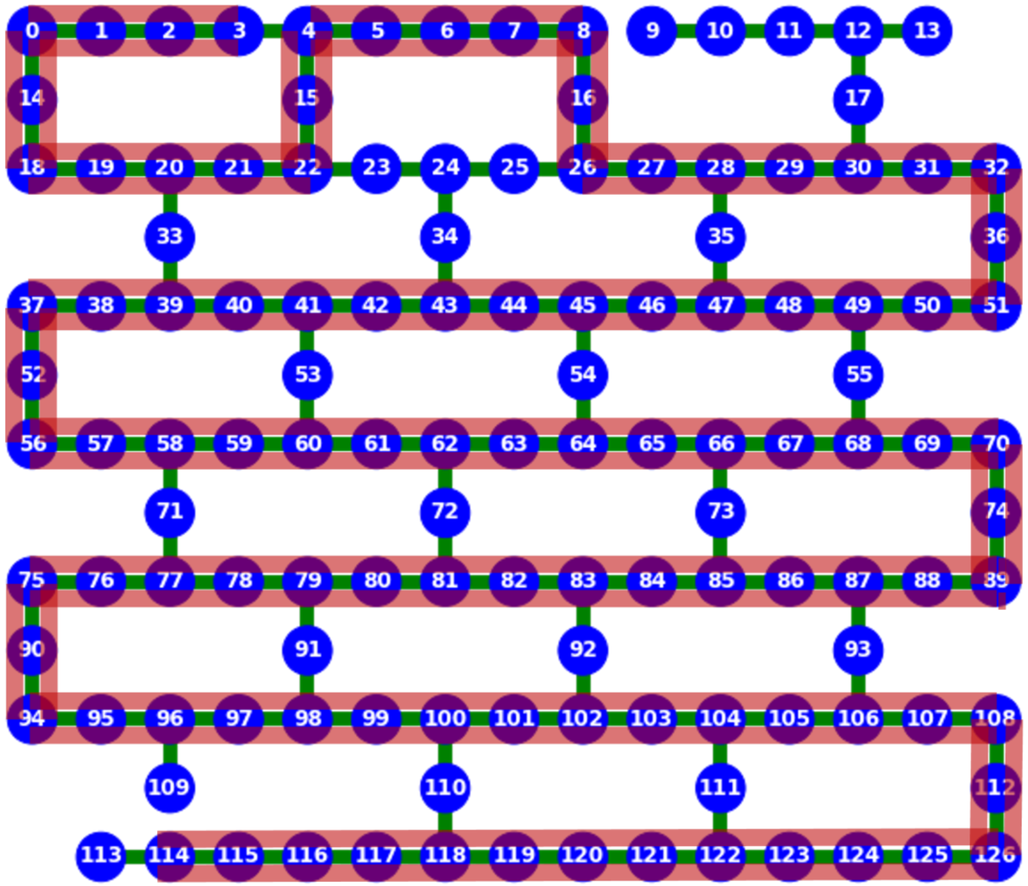


*Note: we use matrix-product state method to compute the GS and optimal variational parameters

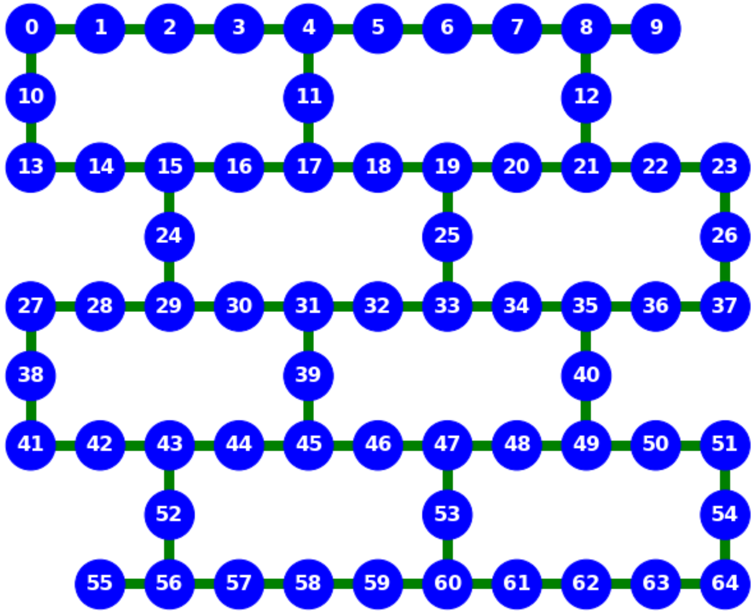
Experimental platform in the cloud

IBM cloud quantum computers: used 9 machines/backends with 3 different layouts

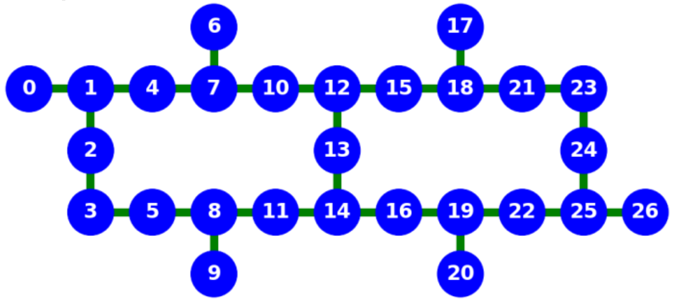
Ibm_washington: 127 qubits



Ibmq_brooklyn: 65 qubits

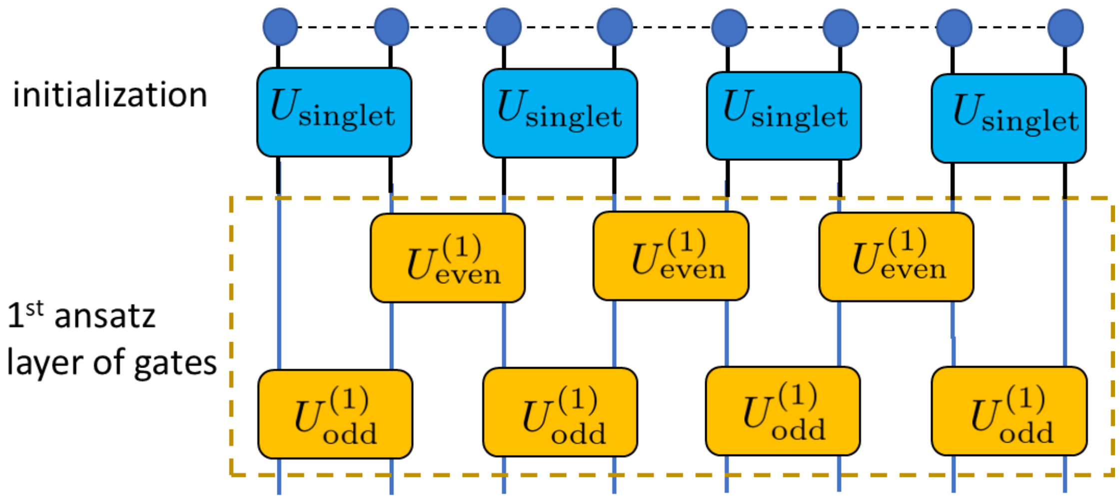


7 other 27 qubit machines:



Execute numerically optimized variational circuits

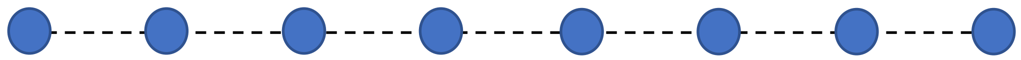
- Employ just one-layer ansatz in the experiment (N_L=1)



$$|\psi_{\text{ansatz}}(\{\theta\})\rangle = \bigotimes_{l=1}^{N_L} \left[U^{(l)}_{\text{even}}(\{\theta_e\})U^{(l)}_{\text{odd}}(\{\theta_o\})\right]|\psi_{\text{singlets}}\rangle$$

$$U^{(l)}_{\text{even/odd}}(\{\theta\}) = \bigotimes_{j\in\text{even/odd}} \left[e^{-i\theta^{(l)}_{e/o,x}\,\sigma^{[j]}_x\sigma^{[j+1]}_x - i\theta^{(l)}_{e/o,y}\,\sigma^{[j]}_y\sigma^{[j+1]}_y - i\theta^{(l)}_{e/o,z}\,\sigma^{[j]}_z\sigma^{[j+1]}_z}\right]$$

Measure energy?



$$\hat{H}_{\mathrm{XXZ}}(\Delta) = \sum_{j=1}^{N-1} \hat{h}_{XXZ}^{[j,j+1]}(\Delta)$$

$$= \sum_{j=1}^{N-1} \left(\sigma_x^{[j]}\sigma_x^{[j+1]} + \sigma_y^{[j]}\sigma_y^{[j+1]} + \Delta\sigma_z^{[j]}\sigma_z^{[j+1]} \right)$$

Three different approaches:

1. Measure Pauli X, Y and Z separately
2. Perform "quantum state tomography" on all nearest-neighbor qubit pairs ➔ $\rho_{i,i+1}$
3. Use "Bell-state" measurement (*most economic of all three*)

➔ For nearest-neighbor XXZ term

$|\Psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$ has energy $-2-\Delta$, the triplet $|\Psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$ has energy $2-\Delta$, and both $|00\rangle$ and $|11\rangle$ (or equivalently $|\Phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$) have energy Δ

Readout error mitigation

Prior characterization M of probability "measured outcome vs. input state"
➔ relates ideal distribution and measured distribution

$$\vec{P}_{\text{measured}} = \mathcal{M}\vec{P}_{\text{ideal}}$$

➔ Can invert to obtain supposedly ideal outcome distribution
➔ used for reading out measured & readout-error mitigated energy

Zero-noise extrapolation (ZNE)

Consider possibly stretching the depth of our ansatz U by inserting a few (UU^{-1})

$$|\psi_{m\equiv(2n+1)}\rangle = \mathbf{U}(\mathbf{U}^{-1}\mathbf{U})^n|0...0\rangle$$

Ideally, $|\psi_{m=1}\rangle$ and $|\psi_{m=2n+1}\rangle$ should have same energy E_m

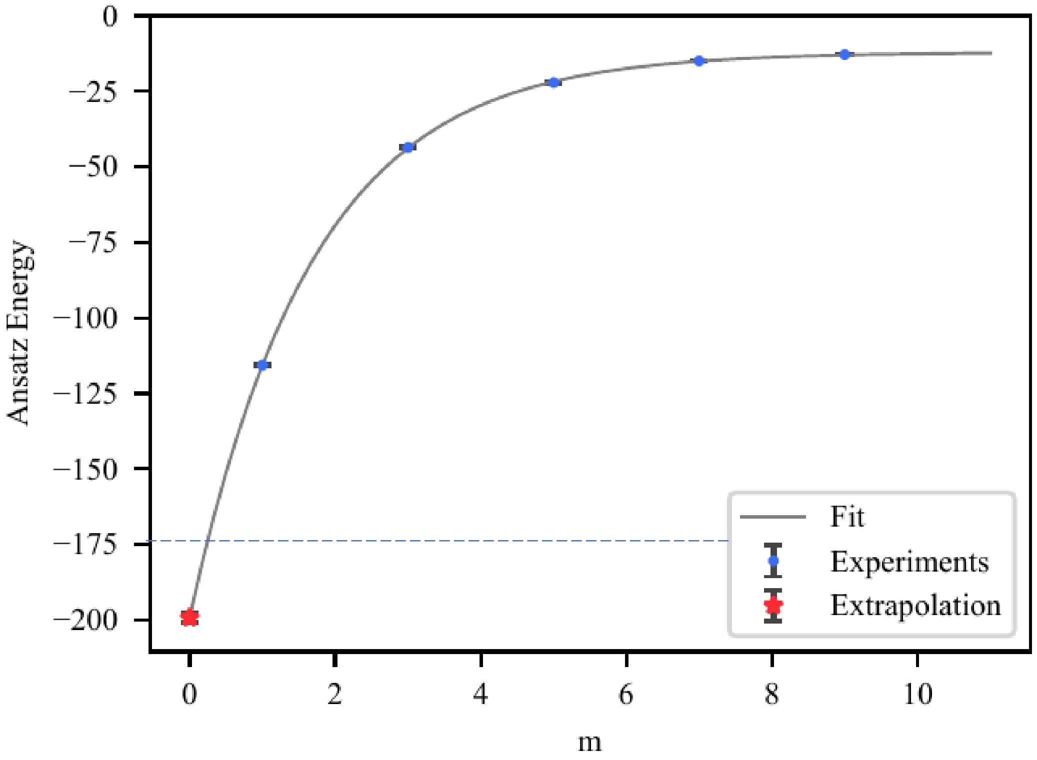
➔ Extrapolate to $E_{m=0}$ (supposedly noiseless limit)

Zero-noise extrapolation (ZNE) --- 102 qubits

$|\psi_{m\equiv(2n+1)}\rangle = \mathbf{U}(\mathbf{U}^{-1}\mathbf{U})^n|0...0\rangle$ energy E_m

- E.g. 102-qubit Heisenberg chain

Optimal ansatz energy with “redundant” repetition



- Expect: -174.41 (anatz) and -180.056 (MPS GS)
- m=1 → obtain ≈ -115.65 ☹
- ZNE → obtain ≈ -199.2 ☹

$$\text{with fitting: } f_E(m) = a\exp(-bm) + c$$

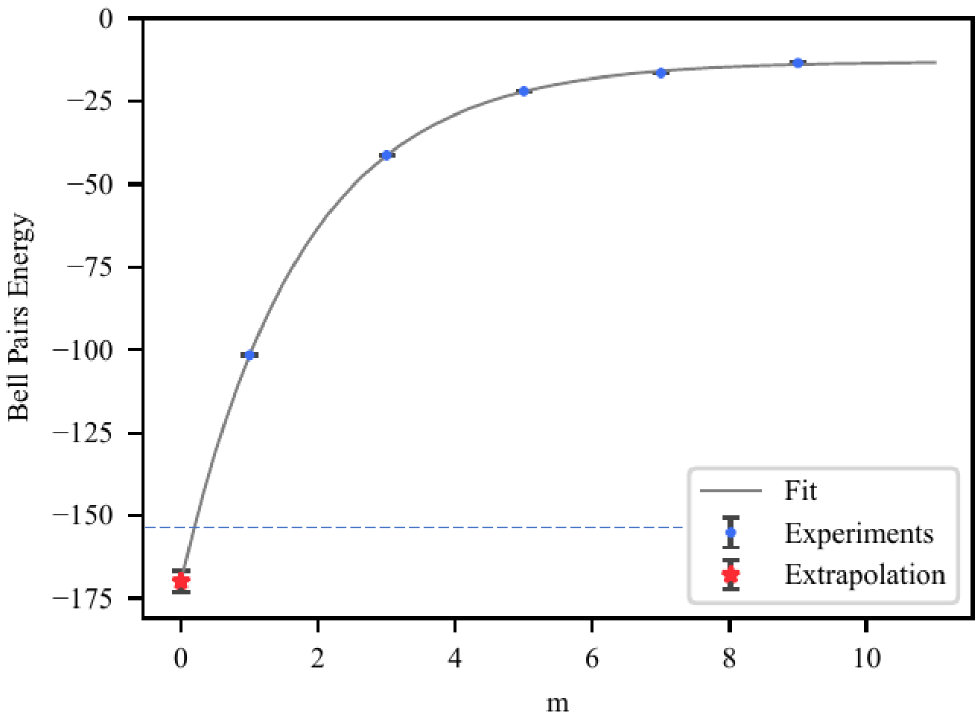
➔ Nowhere near expected values?!!

Reference-state ZNE --- 102 qubits

$$|\psi_{m\equiv(2n+1)}\rangle = \mathbf{U}(\mathbf{U}^{-1}\mathbf{U})^n|0...0\rangle \quad \text{energy } E_m$$

Bell pairs energy with "redundant" repetition

(by setting all variational parameters to zero)



- Expect: -153 (exact)
- m=1 → obtain ≈ -101.66 ☹
- ZNE → obtain ≈ -169.8 ☹

→ Nowhere near expected values!!

❖ Can infer the rescale factor (-169.8)/(-153)

→ use this to correct ansatz energy:

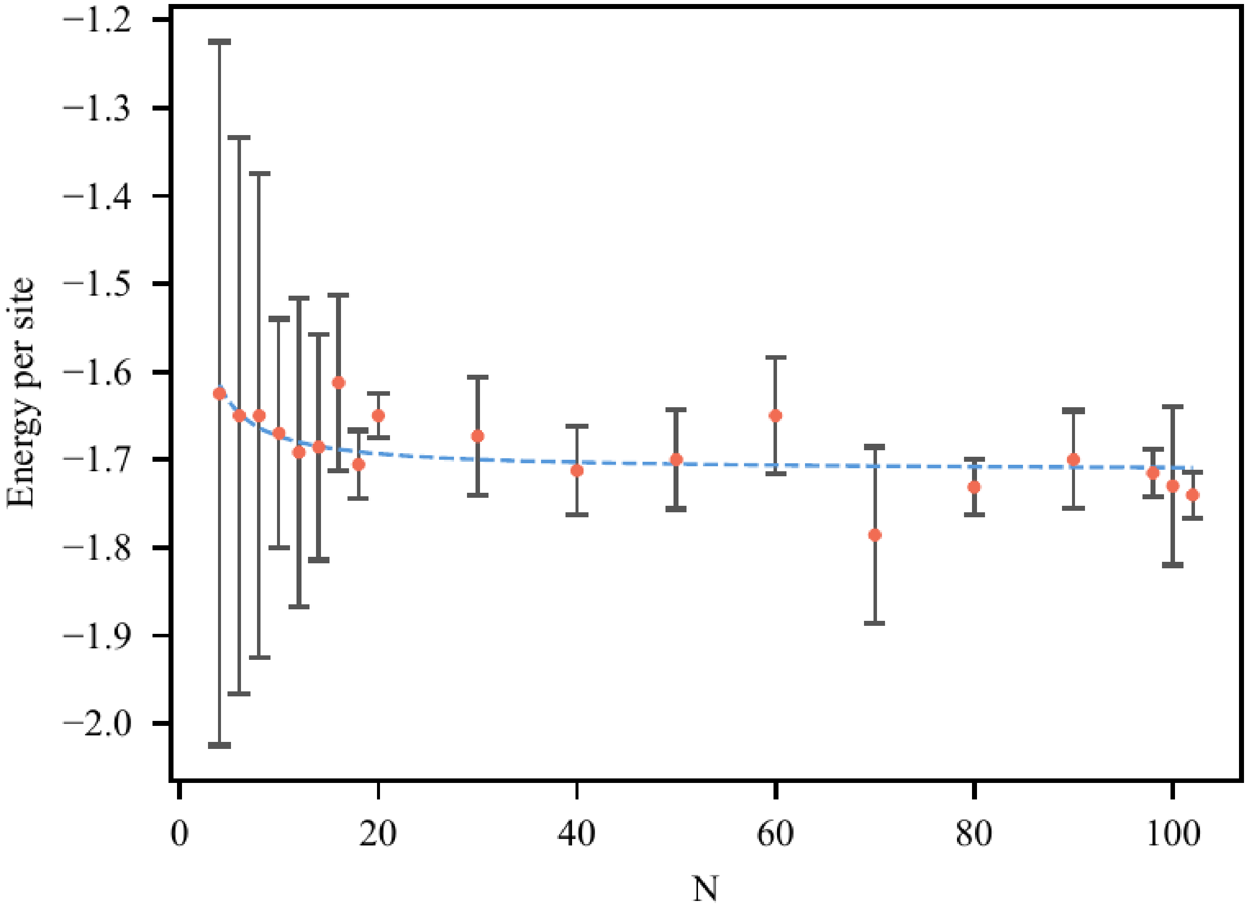
$$-199.2/(169.8/153) \approx -179$$

→ Agrees with expected -174.41

Reference-state ZNE --- 4 to 102 qubits (Heisenberg)

Experimental energy density extrapolated to thermodynamic limit $-1.713 + 0.393/N$

vs. Bethe ansatz solution $4(\ln 2 - 1) \approx -1.773$

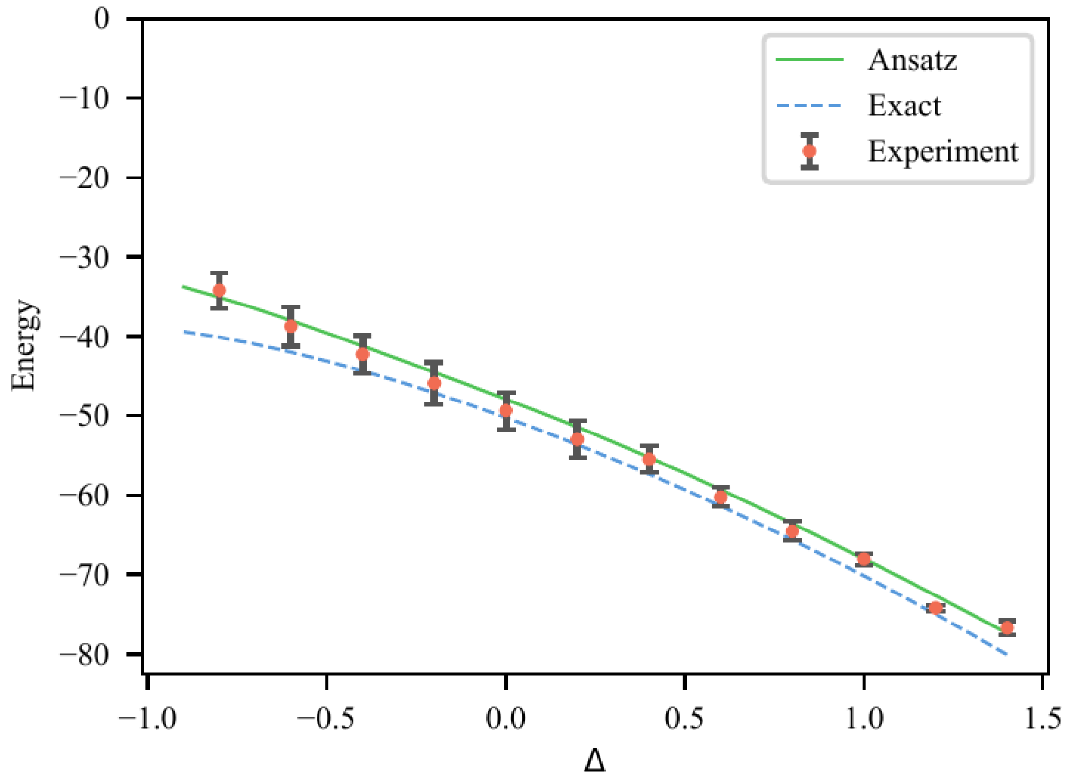


Cloud Experiments/Realizations: XXZ model

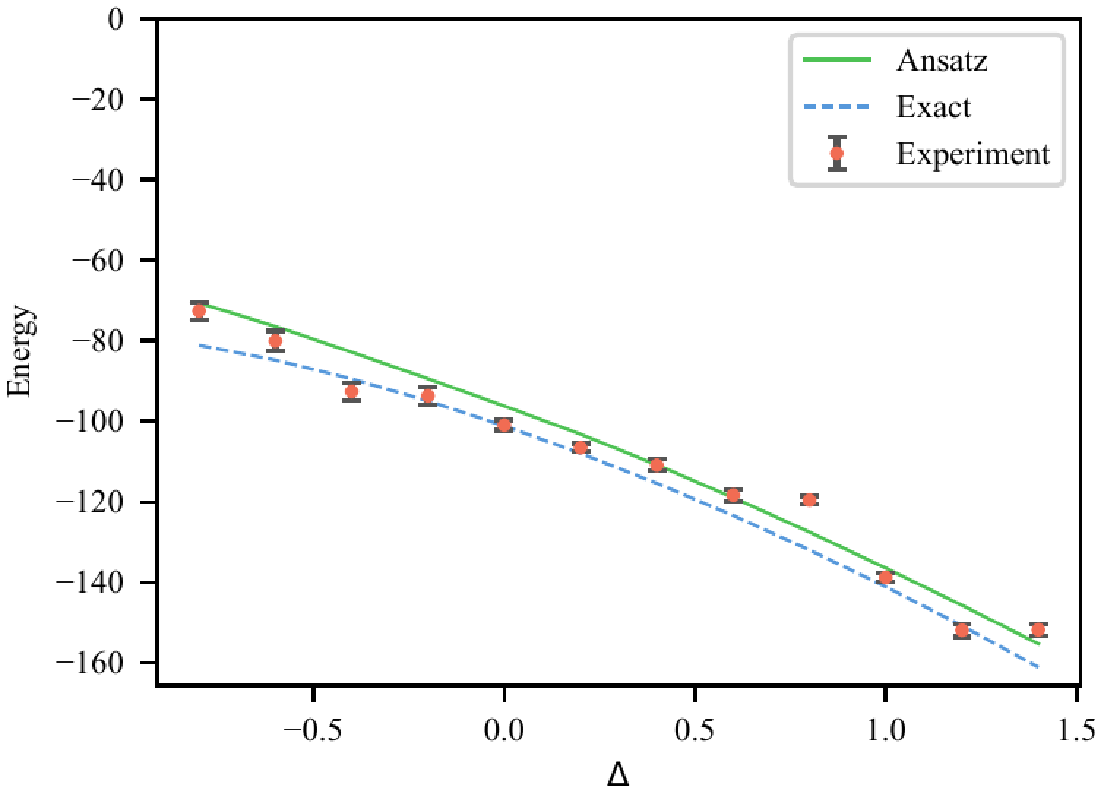
Hongye Yu

Yusheng Zhao

40 qubits (on ibmq_brooklyn)



80 qubits (on ibm_washington)



Conclusion

- Described some history of EPR paradox and Bell inequalities
- Some important developments in quantum information science
- Discussed selected own research
- An exciting era for quantum information science and technology