Entanglement Entropy in (1+1)-d with Defects

Fei Yan

Rutgers University

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Quantum Entanglement

- Quantum information and quantum computing: quantum entanglement as a resource for manipulating computational tasks.
- Condensed matter physics:
  - ⋆ Diagnose quantum critical phenomena
  - ⋆ Dynamics of strongly correlated quantum systems
  - ⋆ Characterize quantum phases of matter
- High energy theory:
  - ⋆ Measures of degrees of freedoms under RG flows in QFTs
  - ⋆ Holography

Quantum Defects

- Defects are important tools in describing nature:
  • quantum impurities • boundaries of finite-size systems
  • domain walls separating differently-ordered regions

- Essential for understanding QFTs:
  •line defects • surface defects • domain walls

Outlook

Entanglement entropy in (1+1)-d with defects:

- Entanglement entropy in 1d critical quantum spin chains with defects:
  - Transverse field Ising (TFI) model
  - Three-state Potts model

  [wip w/ L. Grans-Samuelsson, A. Roy, H. Saleur]

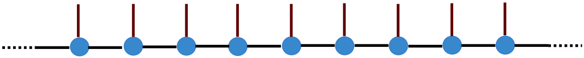
- Field theoretical replica trick method

Today: bipartite quantum systems, total system in its ground state

Spin chains and quantum simulation

- Digital quantum simulation: quench dynamics
- Analog quantum simulation: programmable Rydberg atom arrays

Transverse Field Ising model



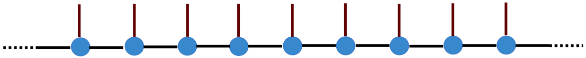
$$H = -\sum_{i=1}^{N-1} \sigma_i^z \sigma_{i+1}^z - h \sum_{i=1}^{N} \sigma_i^x \quad \text{open b.c.}$$

$\mathcal{H} = (\mathbb{C}^2)^{\otimes N}$

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- $\mathbb{Z}_2$ symmetry, $Q_{\mathbb{Z}_2} = \prod_i \sigma_i^x$, $[H, Q_{\mathbb{Z}_2}] = 0$.

Transverse Field Ising model



$$H = -\sum_{i=1}^{N-1} \sigma_i^z \sigma_{i+1}^z - h \sum_{i=1}^{N} \sigma_i^x$$ open b.c.

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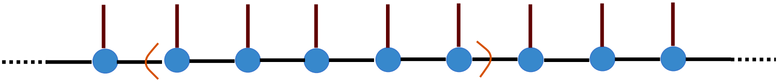
- $\mathbb{Z}_2$ symmetry, $Q_{\mathbb{Z}_2} = \prod_i \sigma_i^x$, $[H, Q_{\mathbb{Z}_2}] = 0$.
- $h > 1$: disordered phase (paramagnetic), gapped, symmetry preserving
  $h < 1$: ordered phase (ferromagnetic), gapped, symmetry breaking
- $h = 1$, 2nd-order quantum phase transition $\rightarrow$ Ising CFT

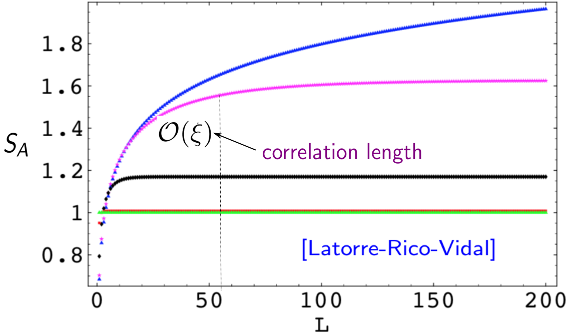
Ground State Entanglement Entropy in TFI



A: spin block of L sites

$\mathcal{H}_{\text{tot}} = \mathcal{H}_A \otimes \mathcal{H}_{A^c}$

$S_A := -\text{Tr}[\rho_A \log \rho_A]$ $\qquad \rho_A := \text{Tr}_{A^c} |\Psi_g\rangle\langle\Psi_g|$ ← ground state



$h = 1$

$$S_A = \frac{1}{6} \log L + S_0$$

$c/3$

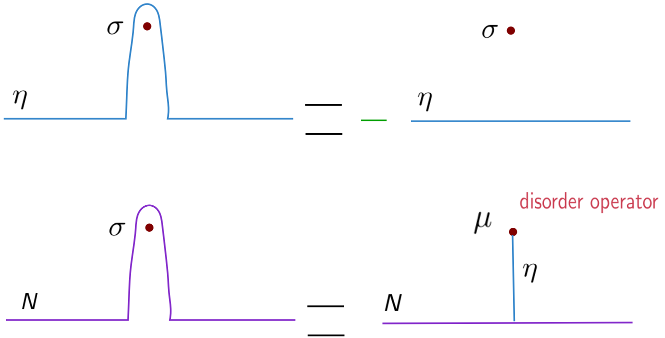
[Holzhey-Larsen-Wilczek],[Calabrese-Cardy]

Topological lines in the Ising CFT

- $c = 1/2$ diagonal RCFT, three Virasoro primaries: $1_{0,0}$, $\epsilon_{\frac{1}{2},\frac{1}{2}}$, $\sigma_{\frac{1}{16},\frac{1}{16}}$
- Three Verlinde lines: I, $\mathbb{Z}_2$ line η, Kramers-Wannier duality line N

$$\eta^2 = I, \;\; N\eta = \eta N = N, \;\; N^2 = 1 + \eta$$

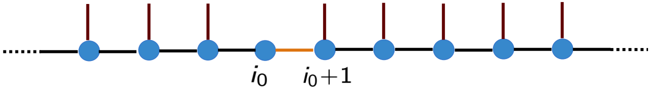
[Moore-Seiberg],[Tambara-Yamagami],[Frohlich-Fuchs-Runkel-Schweigert],[Chang-Lin-Shao-Wang-Yin]...



Topological defects in critical TFI

The $\mathbb{Z}_2$-defect η and the duality defect N in critical transverse field Ising chain:
[Grimm-Schütz],[Oshikawa-Affleck],[Aasen-Mong-Fendley],[Hauru-Evenbly-Ho-Gaiotto-Vidal],[Roy-Saleur]

- $$H_{\eta,i_0} = -\sum_{i=1, i\neq i_0}^{N-1} \sigma_i^z \sigma_{i+1}^z - \sum_{i=1}^{N} \sigma_i^x + \sigma_{i_0}^z \sigma_{i_0+1}^z$$
  top. $\leftrightarrow$ $\exists$ local unitary relating H_{η,i_0} to H_{η,i_0+k}. $\sigma_{i_0}^x H_{\eta,i_0} \sigma_{i_0}^x = H_{\eta,i_0-1}$
- $$H_{N,i_0} = -\sum_{i=1, i\neq i_0}^{N-1} \sigma_i^z \sigma_{i+1}^z - \sum_{i=1, i\neq i_0}^{N} \sigma_i^x - \sigma_{i_0}^y \sigma_{i_0+1}^z$$



Continuous families of defects in critical TFI

$\exists$ two continuous families of defects in crit. transverse field Ising:
[Henkel-Patkós-Schlottmann],[Abraham-Ko-Svrakic],[Grimm],[Affleck-Oshikawa],...

- energy-type: $H_{b,i_0} = -\sum_{i=1, i\neq i_0}^{N-1} \sigma_i^z \sigma_{i+1}^z - \sum_{i=1}^{N} \sigma_i^x - b\sigma_{i_0}^z \sigma_{i_0+1}^z$

  $b \in (-\infty, \infty)$: $b = 1 \leftrightarrow$ identity line, $b = -1 \leftrightarrow \mathbb{Z}_2$-line η

  $b = 0, \pm\infty \leftrightarrow$ totally reflective

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- duality-type: $H_{\tilde{b},i_0} = -\sum_{i=1,i\neq i_0}^{N-1} \sigma_i^z \sigma_{i+1}^z - \sum_{i=1,i\neq i_0}^{N} \sigma_i^x - \tilde{b}\sigma_{i_0}^y \sigma_{i_0+1}^z$

  $\tilde{b} \in [0, \infty)$: $\tilde{b} = 1 \leftrightarrow$ KW duality line N, $b = 0, \infty \leftrightarrow$ totally reflective

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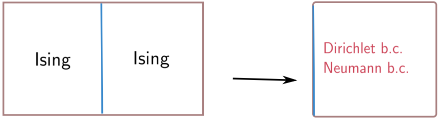
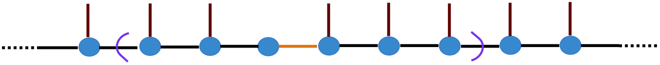


$\text{Ising}^2 \approx \mathbb{Z}_2$-orbifold of free cpt. boson

Ground state EE in critical TFI with defects

[Affleck-Oshikawa],[Saleur],[Sakai-Satoh],[Bachas-Brunner-Roggenkamp],[Herzog-Nishioka],[Brehm-Brunner],[Eisler-Peschel,] [Gutperle-Miller],[Calabrese-Mintchev-Vicari],[Klich-Vaman-Wong],[Roy-Saleur],[Rogerson-Pollmann-Roy],...



A: spin block of L sites

$$S_{A,D} = \frac{c}{3}\log L + S_D^0$$

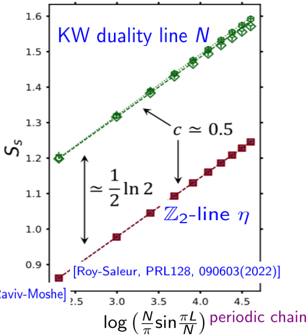
$$S_D^0 = \delta + \log(g_D)$$

g-function

[Cardy],[Ishibashi],[Affleck-Oshikawa],[Affleck-Ludwig],[Harvey-Kachru-Moore-Silverstein]

$$g_{UV} > g_{IR}$$

[Affleck-Ludwig],[Kutasov-Marino-Moore],[Friedan-Konechny],[Casini-Landea-Torroba],[Cuomo,Komargodski,Raviv-Moshe]



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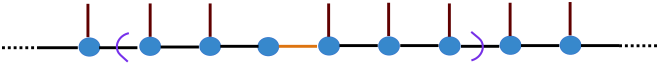

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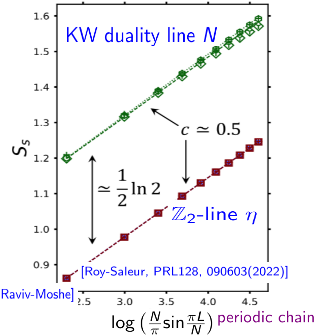
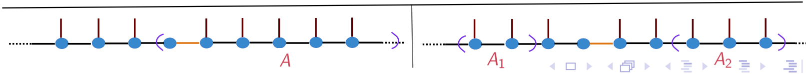
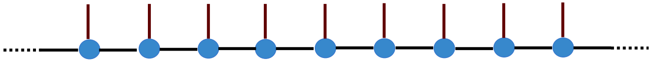




The three-state Potts model



$$H = -\sum_{i=1}^{N-1}\left(\sigma_i^\dagger\sigma_{i+1} + \sigma_{i+1}^\dagger\sigma_i\right) - h\sum_{i=1}^{N}\left(\tau_i + \tau_i^\dagger\right)$$ open b.c. $\mathcal{H} = (\mathbb{C}^3)^{\otimes N}$

measure spin

$$\sigma = \begin{pmatrix}1 & 0 & 0\\ 0 & w & 0\\ 0 & 0 & w^2\end{pmatrix}, \quad \tau = \begin{pmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}, \quad w = \mathrm{e}^{2\pi\mathrm{i}/3}$$

shift spin

$$\sigma_i^3 = \tau_i^3 = 1, \sigma_i\tau_i = w\tau_i\sigma_i$$

$S_3 = \mathbb{Z}_3 \rtimes \mathbb{Z}_2$

$\mathbb{Z}_3$-charge

$$Q = \prod_{i=1}^{N}\tau_i^\dagger$$

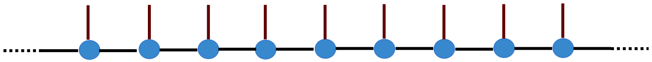
$$Q\sigma_i Q^\dagger = w\sigma_i, \quad Q\tau_i Q^\dagger = \tau_i$$

$\mathbb{Z}_2$ charge conjugation

$$C = \prod_{i=1}^{N} c_i, \quad c_i = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$$

$$C\sigma_i C = \sigma_i^\dagger, \quad C\tau_i C = \tau_i^\dagger$$

The three-state Potts model



$$H = -\sum_{i=1}^{N-1}\left(\sigma_i^\dagger \sigma_{i+1} + \sigma_{i+1}^\dagger \sigma_i\right) - h\sum_{i=1}^{N}\left(\tau_i + \tau_i^\dagger\right)$$ open b.c. $\mathcal{H} = (\mathbb{C}^3)^{\otimes N}$

$$\sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & w & 0 \\ 0 & 0 & w^2 \end{pmatrix}, \quad \tau = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad w = e^{2\pi i/3}$$

measure spin (σ), shift spin (τ)

$$\sigma_i^3 = \tau_i^3 = 1, \quad \sigma_i\tau_i = w\tau_i\sigma_i$$

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$\mathbb{Z}_3$-charge: $Q = \prod_{i=1}^{N} \tau_i^\dagger$

$$Q\sigma_i Q^\dagger = w\sigma_i, \quad Q\tau_i Q^\dagger = \tau_i$$

$\mathbb{Z}_2$ charge conjugation: $C = \prod_{i=1}^{N} c_i, \quad c_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$$C\sigma_i C = \sigma_i^\dagger, \quad C\tau_i C = \tau_i^\dagger$$

- $h > 1$: disordered phase, $h < 1$: ordered phase
  $h = 1$, 2nd order quantum phase transition $\rightarrow$ 3-state Potts CFT
- $c = 0.8$, non-diagonal Virasoro $M(6,5)$, 12 Virasoro primaries
  [Mong-Clarke-Alicea-Lindner-Fendley]

Ground state EE in critical Potts with topological defects

$\exists$ 17 simple topological defects in Potts CFT [Petkova-Zuber],[Chang-Lin-Shao-Wang-Yin],...
generated by 1, $\mathbb{Z}_3$-defect η, $\mathbb{Z}_2^C$-defect C, KW duality defect N, W defect

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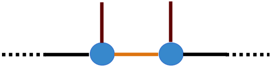
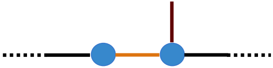


$\mathbb{Z}_3$-defect η $\quad e^{2\pi i/3}\sigma^\dagger_{i_0}\sigma_{i_0+1} + \text{h.c.}$

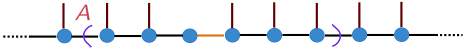
$\mathbb{Z}_2^C$-defect C $\quad \sigma^\dagger_{i_0}\sigma^\dagger_{i_0+1} + \text{h.c.}$



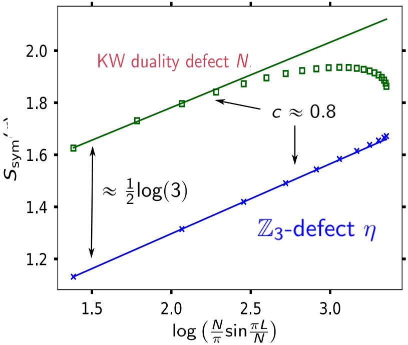
KW duality defect N

$$e^{-i\pi/3}\sigma_{i_0}\tau_{i_0}\sigma^\dagger_{i_0+1} + \text{h.c.}$$

$\mathbb{Z}_3$ Tambara-Yamagami
$N^2 = 1 + \eta + \eta^2,\ N\eta = \eta N = N$



$$S_{A,D} = \frac{c}{3}\log L + S_D^0 = \frac{c}{3}\log L + \log(g_D) + \delta$$



Intermediate Summary

Entanglement entropy in 1d critical quantum spin chains with defects:

- Transverse field Ising: energy defects, duality defects
- Three-state Potts: topological defects
- Extract physical data from the scaling behavior of EE:
  central charge, defect g-function, defect strength

The replica trick

Suppose $\mathcal{H}_{\text{tot}} = \mathcal{H}_A \otimes \mathcal{H}_{A^c}$ where A is the subsystem, $\rho_{\text{tot}} = |\Psi\rangle\langle\Psi|$

- Entanglement entropy $S_A = -\text{Tr}_A [\rho_A \log \rho_A]$, $\rho_A := \text{Tr}_{A^c} \rho_{\text{tot}}$

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- $S_A = -\lim_{n\to 1} \partial_n \log \text{Tr}_A \left(\rho_A^n\right) = \lim_{n\to 1} S_A^{(n)}$

  Rényi entropy $S_A^{(n)} = \frac{1}{1-n} \log \text{Tr}_A \left(\rho_A^n\right)$

  $S_A^{(n)}$ defined for $n \in \mathbb{Z}_+$, analytic continuation to $\mathbb{R}_+$ in taking $n \to 1$.

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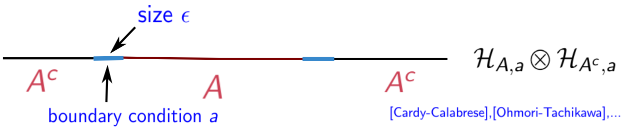


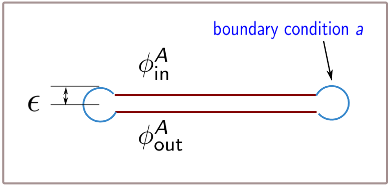
[Cardy-Calabrese],[Ohmori-Tachikawa],...

Path integral formulation of the replica trick

[Calabrese-Cardy],[Holzhey-Larsen-Wilczek],...,[Ohmori-Tachikawa],...

$$\rho_A := \mathrm{Tr}_{A^c} |\Psi_g\rangle\langle\Psi_g|$$

$$\langle \phi^A_{\text{out}} | \rho_A | \phi^A_{\text{in}} \rangle = \frac{1}{Z} \times$$



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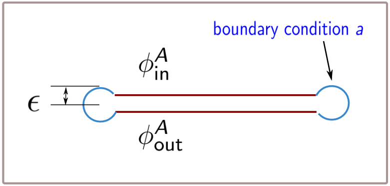
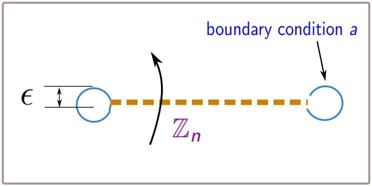


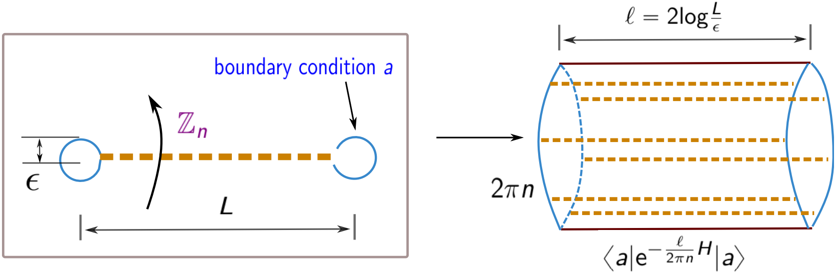
$$\mathrm{Tr}_A\left(\rho_A^n\right) = \frac{1}{(Z)^n}\times$$



$$= \frac{Z_n}{(Z)^n}$$

Z_n: p.f. on the n-fold cover

Example: a single interval in 2d CFT



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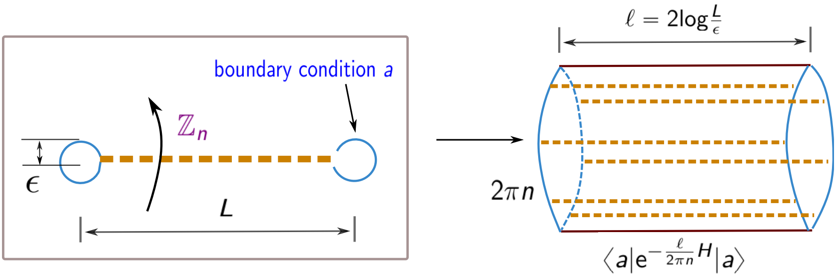


$$\mathrm{Tr}_A\left(\rho_A^n\right) = \left(\frac{L}{\epsilon}\right)^{-2\Delta_n} g_a^{2(1-n)}$$

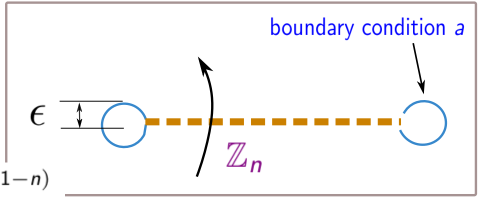
$$\Delta_n = \frac{c}{12}\left(n - \frac{1}{n}\right) \qquad g_a := \langle 0 | a \rangle$$

$$S^{(n)} = \frac{c}{6}\frac{n+1}{n}\log\frac{L}{\epsilon} + 2\log\left(g_a\right)$$

$$S = \frac{c}{3}\log\frac{L}{\epsilon} + 2\log\left(g_a\right) = \frac{c}{3}\log\frac{L}{\epsilon'}$$

The twist fields

$$\mathrm{Tr}_A\left(\rho_A^n\right) = \frac{1}{(Z)^n} \times$$



$$= \frac{\epsilon^{2\Delta_n} g_a^{2(1-n)}}{L^{2\Delta_n}}$$

$$= \langle \Phi_n(z_1 = 0) \overline{\Phi}_n(z_2 = L) \rangle$$

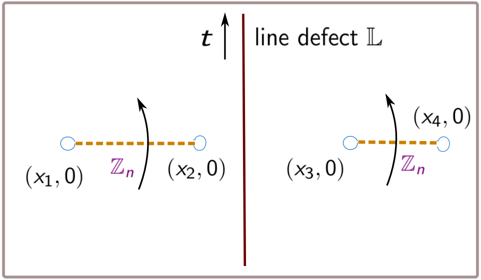
replica theory: $T^{\otimes n}/\mathbb{Z}_n$

twist field: primary scalars with $h = \bar{h} = \frac{1}{2}\Delta_n$

The replica trick with line defects in 2d CFT

- Example: line defect $\mathbb{L}$, subsystem $A = [x_1, x_2] \cup [x_3, x_4]$

$$\mathrm{Tr}_A\left(\rho_A^n\right) = \frac{1}{(Z)^n} \times$$

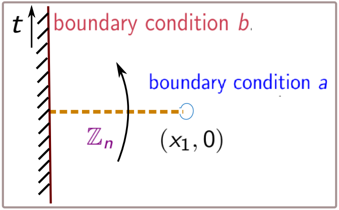


$$= \langle \Phi_n(x_1)\overline{\Phi}_n(x_2)\Phi_n(x_3)\overline{\Phi}_n(x_4)\rangle_{\mathbb{L}_n}$$

BCFT: an interval touching the boundary

- 2d CFT on $\{(x,t)|x \geqslant 0\}$, with boundary condition b. Ground state entropy for the subsystem $A = [0, x_1]$.

$$\mathrm{Tr}_A(\rho_A^n) = \frac{1}{(Z)^n} \times$$



$$= \frac{\epsilon^{\Delta_n} g_a^{1-n} g_b^{1-n}}{(2x_1)^{\Delta_n}} = \langle \overline{\Phi}_n(x_1) \rangle_b$$

$$S = \frac{c}{6} \log \frac{2x_1}{\epsilon} + \log(g_a) + \log(g_b) = \frac{c}{6} \log \frac{2x_1}{\epsilon'} + \log(g_b)$$

BCFT: an interval away from the boundary

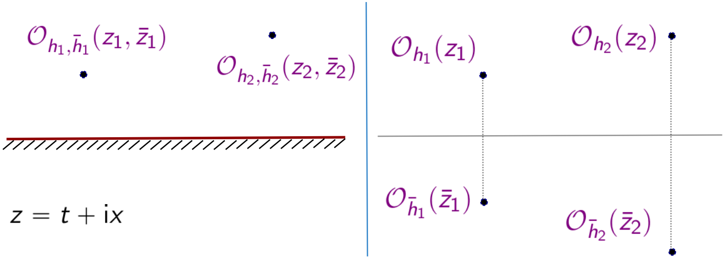
- 2d CFT on $\{(x,t)|x \geqslant 0\}$, with boundary condition b. Ground state entropy for the subsystem $A = [x_1, x_2]$.
  [Sully-Raamsdonk-Wakeham],...

$$\mathrm{Tr}_A(\rho_A^n) = \langle \Phi_n(x_1)\overline{\Phi}_n(x_2)\rangle_{b_n}$$

BCFT: an interval away from the boundary

- 2d CFT on $\{(x,t)|x \geqslant 0\}$, with boundary condition b. Ground state entropy for the subsystem $A = [x_1, x_2]$.
  [Sully-Raamsdonk-Wakeham],...

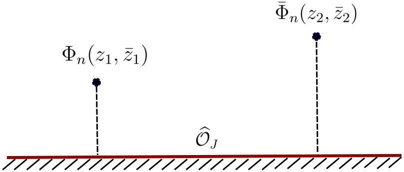
$$\mathrm{Tr}_A(\rho_A^n) = \langle \Phi_n(x_1)\overline{\Phi}_n(x_2)\rangle_{b_n}$$

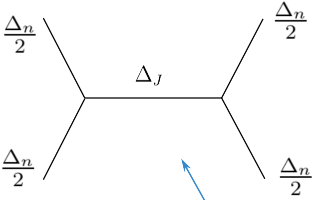


$$\langle \Phi_n(x_1)\overline{\Phi}_n(x_2)\rangle_{b_n} = \left(\frac{\eta}{4x_1x_2}\right)^{\Delta_n} F(\eta)$$

$$\eta = \frac{(z_1-\bar{z}_1)(z_2-\bar{z}_2)}{(z_1-\bar{z}_2)(z_2-\bar{z}_1)} \in [0,1]$$

The boundary channel





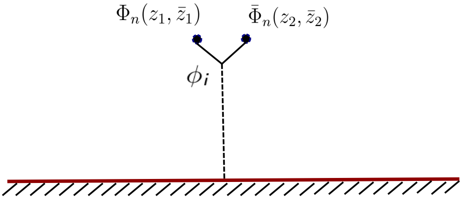
$$\langle \Phi_n(z_1, \bar{z}_1)\bar{\Phi}_n(z_2, \bar{z}_2)\rangle_{b_n} = \left(\frac{\eta}{4x_1x_2}\right)^{\Delta_n} \sum_J \mathcal{B}^{b_n}_{\Phi_n,\widehat{\mathcal{O}}_J}\mathcal{B}^{b_n}_{\bar{\Phi}_n,\widehat{\mathcal{O}}_J}\mathcal{F}(nc, \Delta_J; \frac{\Delta_n}{2}|\eta)$$

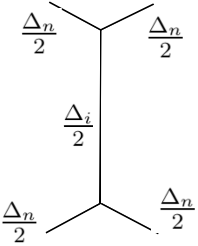
bdy primaries

$$\Phi_n(z, \bar{z}) = \sum_J \frac{\mathcal{B}^{b_n}_{\Phi_n,\widehat{\mathcal{O}}_J}}{(2x)^{\Delta_n - \Delta_J}}\hat{\mathcal{O}}_J(t) + \text{desc}$$

$\eta \to 0$: $$\frac{\mathcal{A}^{b_n}_{\Phi_n}\mathcal{A}^{b_n}_{\bar{\Phi}_n}}{(4x_1x_2)^{\Delta_n}} = \boxed{\frac{g_b^{2-2n}\epsilon'^{2\Delta_n}}{(4x_1x_2)^{\Delta_n}}}$$

The bulk channel





$$\langle \Phi_n(z_1, \bar{z}_1)\bar{\Phi}_n(z_2, \bar{z}_2)\rangle_{b_n} = \left(\frac{1-\eta}{|z_1 - z_2|^2}\right)^{\Delta_n} \sum_i \underline{C^{\phi_i}_{\Phi_n,\bar{\Phi}_n}} \mathcal{A}^{b_n}_{\phi_i} \underline{\mathcal{F}(nc, \frac{\Delta_i}{2}; \frac{\Delta_n}{2}|1-\eta)}$$

bulk primaries → i 3-pt coeff. → $C^{\phi_i}_{\Phi_n,\bar{\Phi}_n}$

$\eta \to 1$

$$\frac{C^{I}_{\Phi_n,\bar{\Phi}_n}\mathcal{A}^{b_n}_{I}}{|z_1 - z_2|^{2\Delta_n}} = \boxed{\frac{\epsilon'^{2\Delta_n}}{|z_1 - z_2|^{2\Delta_n}}}$$

BCFT entropy: special limits

[Sully-Raamsdonk-Wakeham]

Rényi entropy

$$\eta \to 0: \; S_A^{(n)} = \frac{c}{12}\frac{n+1}{n}\log\left(\frac{2x_1}{\epsilon'}\right) + \frac{c}{12}\frac{n+1}{n}\log\left(\frac{2x_2}{\epsilon'}\right) + 2\log g_b$$

$$\eta \to 1: \; S_A^{(n)} = \frac{c}{6}\frac{n+1}{n}\log\frac{|x_2 - x_1|}{\epsilon'}$$

Entanglement entropy

$$\eta \to 0: \; S_A = \frac{c}{6}\log\left(\frac{4x_1x_2}{\epsilon'^2}\right) + 2\log g_b$$

$$\eta \to 1: \; S_A = \frac{c}{3}\log\left(\frac{|x_1 - x_2|}{\epsilon'}\right)$$

BCFT entropy: special limits

[Sully-Raamsdonk-Wakeham]

Rényi entropy

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$$\eta \to 1: \; S_A^{(n)} = \frac{c}{6}\frac{n+1}{n}\log\frac{|x_2 - x_1|}{\epsilon'}$$

Entanglement entropy

$$\eta \to 0: \; S_A = \frac{c}{6}\log\left(\frac{4x_1x_2}{\epsilon'^2}\right) + 2\log g_b$$

$$\eta \to 1: \; S_A = \frac{c}{3}\log\left(\frac{|x_1 - x_2|}{\epsilon'}\right)$$

- Intermediate scaling behavior of $S_A^{(n)}$ and S_A for $0 < \eta < 1$?

Summary

Entanglement entropy in (1+1)-d with defects:

- Entanglement entropy in 1d critical quantum spin chains with defects:
  - Transverse field Ising (TFI) model
  - Three-state Potts model
- Field theoretical replica trick method

Thank You!