Quantum simulation in time evolution and bound state problems

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Quantum Journal Club, BNL

Nov 21st, 2022

This talk is based on
1) Barata, Du, Li, WQ, Salgado, Phys.Rev.D106, 074013; arXiv:2208.06750 (2022)
2) WQ, Basili, Pal, Luecke, Vary; Phys.Rev.Research; arXiv:2112.01927 (2021)

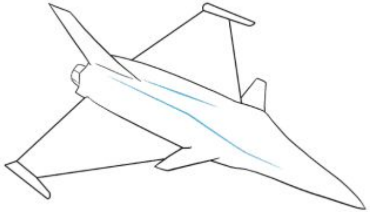
Outline

1. Quantum simulation of jet in a medium

2. Variational quantum eigensolvers to bound state structures

What is jet quenching?

Slide from Li's talk at Qiskit Fall Fest at USC (2022)



"Jet"
a rapid stream

"Quenching"
a rapid cooling process

What is jet quenching?

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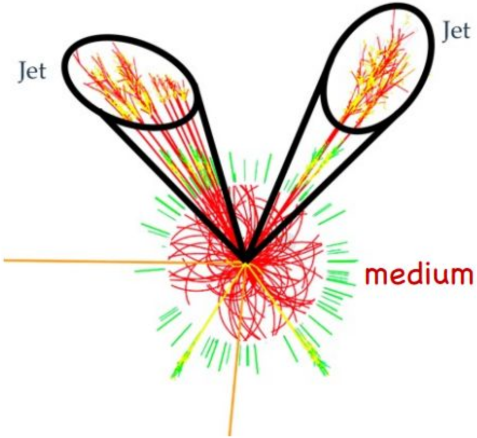
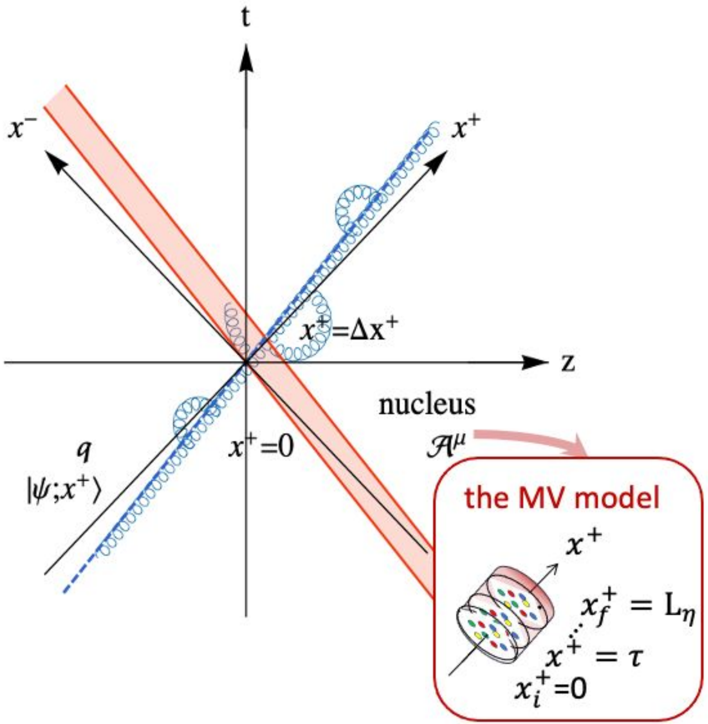


In heavy ion collisions, a jet is a cone-shaped beam of energetic particles. When propagating through the hot medium, it loses energy due to jet-medium interaction, a phenomenon known as “jet quenching”.

Jet evolution: Physical setup

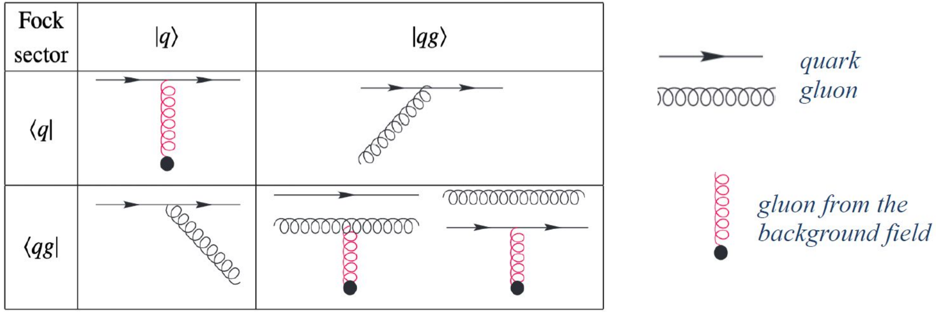
Li, Lappi, Zhao, PRD104,056014 (2021)
Li, Zhao, Maris, Chen, Li, Tuchin, Vary, PRD101,076016 (2020)



High-energy quark moving close to the light cone scattering on a dense nucleus medium

The light-front Hamiltonian in the $|q\rangle + |qg\rangle$ Fock sector:

$$P^-(x^+) = P^-_{\mathrm{KE}} + V(x^+) = P^-_{\mathrm{KE}} + \left\{V_{qg} + V_{\mathcal{A}}(x^+)\right\}$$



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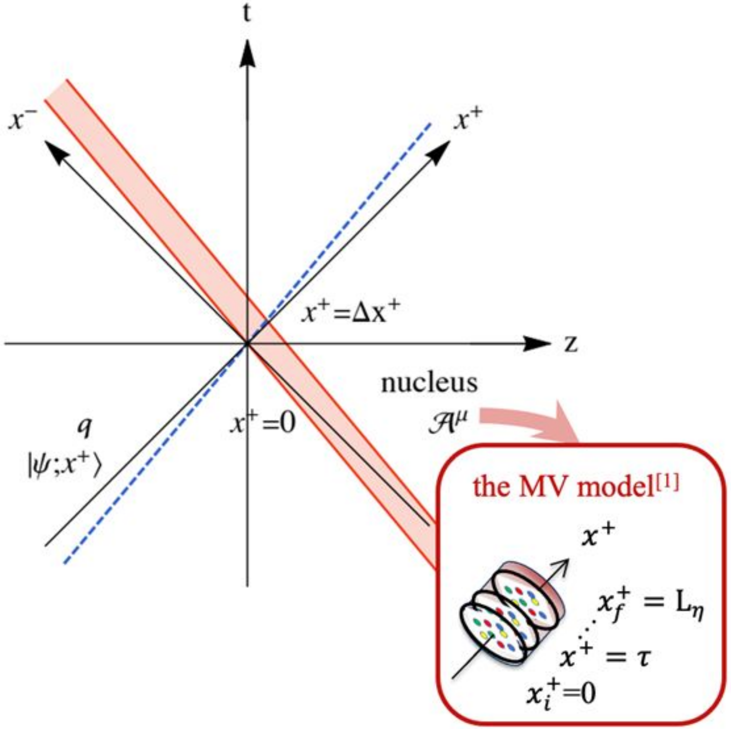

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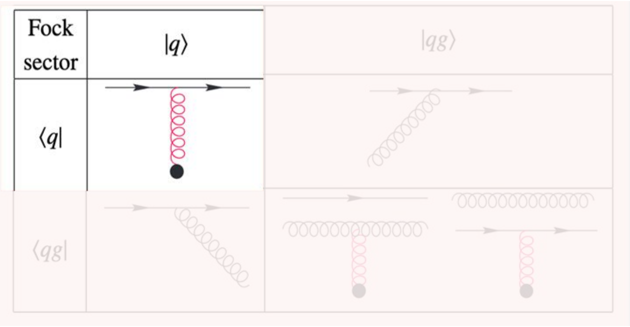
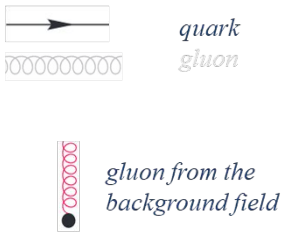




Background field and Evolution

The light-front Hamiltonian has a quark kinetic energy (KE) + background potential (A) terms:

$$P^-(x^+) = P^-_{\mathrm{KE}} + V_{\mathcal{A}}(x^+) = \frac{p_\perp^2}{p^+} + g\mathcal{A}(x^+)\cdot T$$

McLerran, Venugopalan, PRD49, 2233; PRD49, 3352; PRD50, 2225 (1994)

The stochastic background field uses the McLerran-Venugopalan (MV) model

$$\langle\!\langle \rho_a(\vec{x}_\perp, x^+)\rho_b(\vec{y}_\perp, y^+)\rangle\!\rangle = g^2\tilde{\mu}^2\delta_{ab}\delta^2(\vec{x}_\perp - \vec{y}_\perp)\delta(x^+ - y^+) \qquad (m_g^2 - \nabla_\perp^2)\mathcal{A}_a^-(\vec{x}_\perp, x^+) = \rho_a(\vec{x}_\perp, x^+)$$

Light-front time evolution of the probe, decomposed as sequence of unitary operators

$$\begin{aligned}|\psi_{L_\eta}\rangle =& U(L_\eta;0)\,|\psi_0\rangle \\ \equiv& \mathcal{T}_+ e^{-i\int_0^{L_\eta} \mathrm{d}x^+\, P^-(x^+)}\,|\psi_0\rangle\end{aligned} \qquad U(L_\eta;0) = \prod_{k=1}^{N_t} U(x_k^+; x_{k-1}^+)$$

Quantum Simulation Algorithm

Wiesner, 9603028 (1996); Zalka, 9603026 (1996)

1. **Define problem Hamiltonian**
2. **Encode Hamiltonian onto basis**
3. Prepare initial states
4. Evolution
5. Measurement protocol

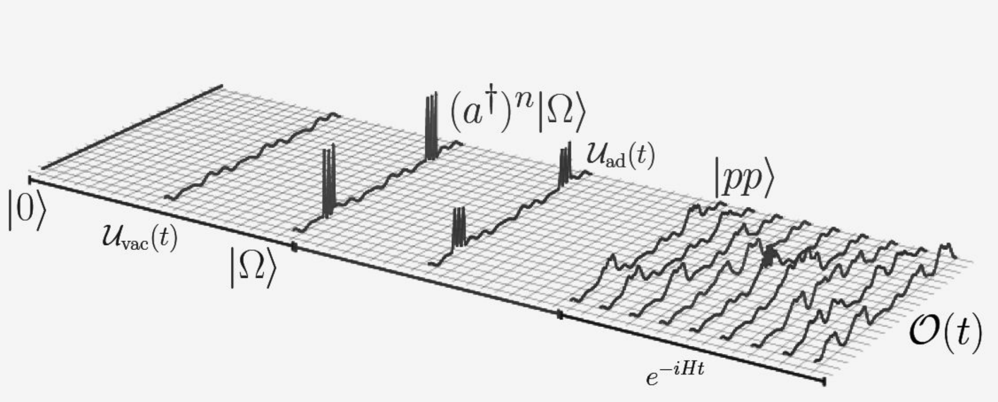


Image from Lamm's talk at Fermilab (2021)

Basis space encoding

Here, we have 2D momentum/position space + 1D color space, requiring a total number of states:

$$n_{\text{tot}} = (2N_\perp)^2 N_c$$

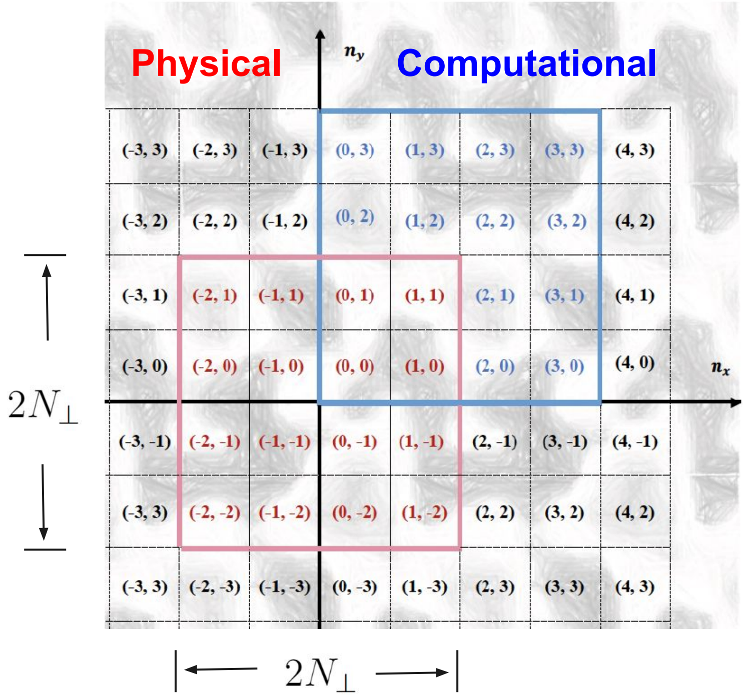
On the circuit, each spatial dimension requires $n_Q = \log_2 2N_\perp$ **Total qubits:** $2n_Q + \lceil \log N_c \rceil$

For example, in SU(2) theory or $N_c = 2$

$$\begin{aligned} |\psi\rangle &= |q_{2n_Q} \cdots q_{n_Q+1}\rangle \otimes |q_{n_Q+1} \cdots q_1\rangle \otimes |q_0\rangle \\ &= |n_x\rangle \otimes |n_y\rangle \otimes |c\rangle \\ &= |k_x\rangle \otimes |k_y\rangle \otimes |c\rangle \end{aligned}$$

$$\begin{aligned} &|q(k_x = 1, k_y = 2, c = 0)\rangle \\ &= |01, 10, 0\rangle = |01\rangle \otimes |10\rangle \otimes |0\rangle \end{aligned}$$

Periodical lattice



The lattice is periodical; used for both pos and mom space via quantum fourier transform (FT)

For convenience, we work with the **computational lattice**, and relate to the **physical lattice** by

$$(n_x, n_y) \Longleftrightarrow (n_x + i2N_\perp, n_y + j2N_\perp)$$

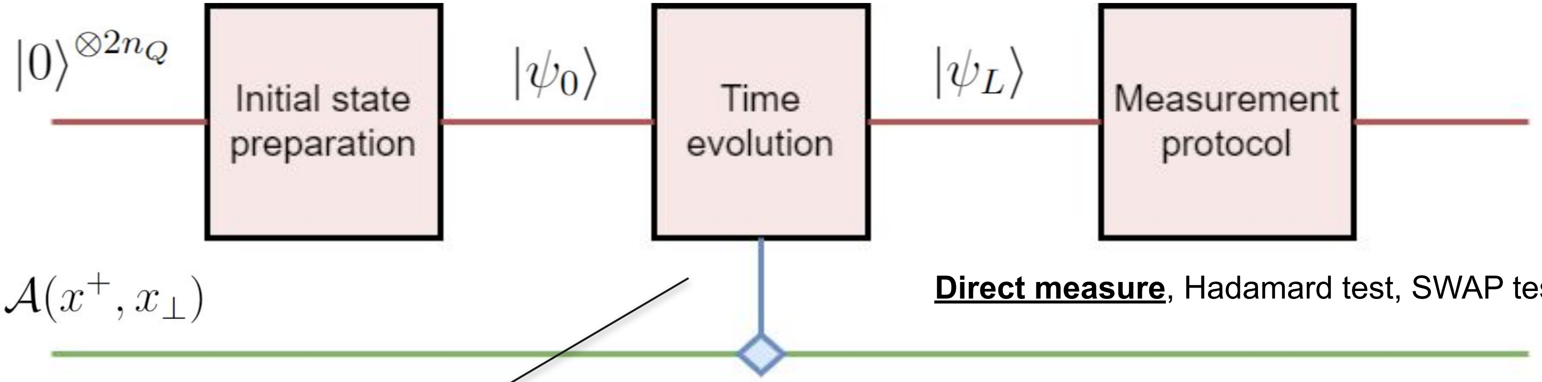
pos lattice spacing: $a_\perp = L_\perp / N_\perp$

mom lattice spacing: $d_p = \pi / L_\perp$

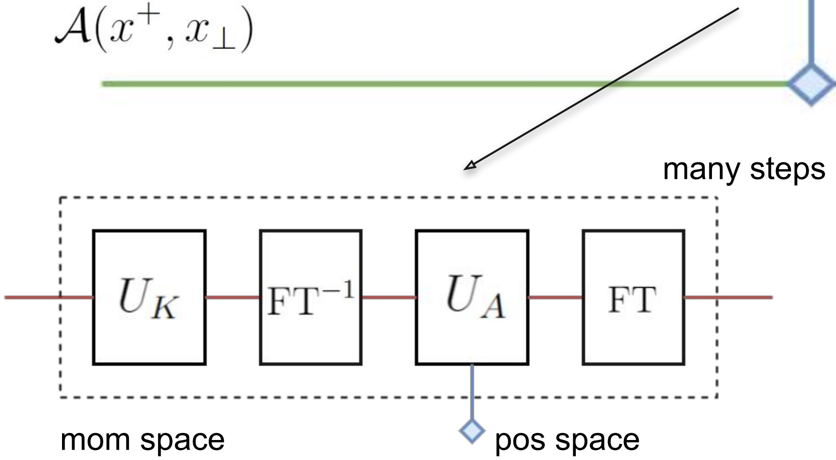
Quantum simulation of jet evolution workflow:

colorless case

$N_c = 1$



Direct measure, Hadamard test, SWAP test, etc



FT allows convenient simulation in the respective basis

$$U_K(\delta x^+)\,|\boldsymbol{p}\rangle = \exp(-i\delta x^+ p_\perp^2/p^+)\,|\boldsymbol{p}\rangle$$

$$U_A(x^+, \delta x^+)\,|\boldsymbol{x}\rangle = \exp(-ig\delta x^+ \mathcal{A}(x^+, \boldsymbol{x})\cdot T)\,|\boldsymbol{x}\rangle$$

Color implementation

In the simplest colorful case of two SU(2) colors

$$A(x) \cdot T = \sum_{a=X,Y,Z} A^a \sigma^a / 2$$

$$\exp(-ig\delta x^+ A(x) \cdot T) = \exp\{-ig\delta x^+ (A^X \otimes \sigma^X + A^Y \otimes \sigma^Y + A^Z \otimes \sigma^Z)\}$$

Block diagonal

Implementations:

- Exact method using property of the exponential of a Pauli vector $e^{ia(\hat{n}\cdot\vec{\sigma})} = I\cos a + i(\hat{n}\cdot\vec{\sigma})\sin a$
- Exponent matrix approximation (Pade approximation)
- Linear-order (and higher), allows for separation of color and pos space: circuit modularization

$$\exp\left(-iA(x)\sigma^Z\right)|\boldsymbol{x}\rangle|c\rangle = \begin{cases} e^{-iA(x)}|\boldsymbol{x}\rangle|c\rangle \ , \text{if } c=0 \\ e^{+iA(x)}|\boldsymbol{x}\rangle|c\rangle \ , \text{if } c=1 \end{cases}$$

Simulation parameters:

- We study **both evolutions with and without color**, initial state: $(p_x, p_y) = (0, 0)$
- Duration of static medium: $L_\eta = 50\,\mathrm{GeV}^{-1} \approx 10\,\mathrm{fm}$
- 5 stochastic fields are used for configuration average
- Two sets of lattice grids, 32 X 32 (10 qubits) and 64 X 64 (12 qubits)
- Fix g = 1, physical IR regulator $m_g = 0.8\,\mathrm{GeV}(\ll Q_s)$
- Selected values of saturation scales

$$Q_s^2 \equiv C_F \frac{g^4 \mu^2 L_\eta}{2\pi} \qquad C_F = (N_c^2 - 1)/2N_c$$

$$\lambda_{IR} = \frac{\pi}{N_\perp a_\perp} \ll m_g \ll Q_s \ll \lambda_{UV} = \frac{\pi}{a_\perp}$$

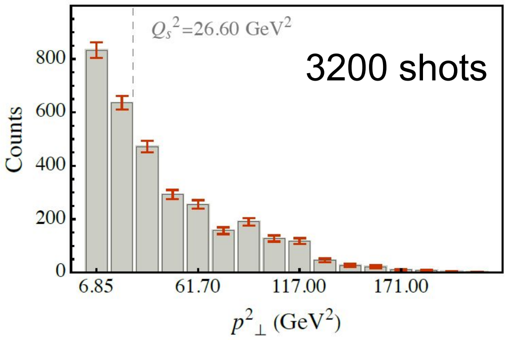
Range coverage condition

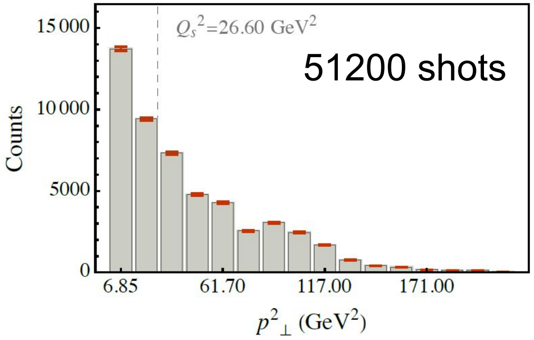
$$L_\eta < \tau_{\mathrm{sat}} \equiv \langle p^2 \rangle_{\mathrm{asy}} / \hat{q} \approx \frac{2\pi^2}{3a_\perp^2 \hat{q}}$$

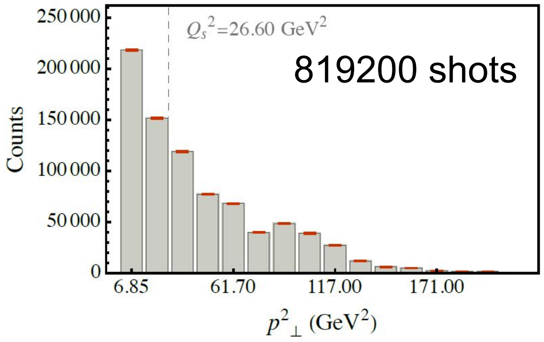
Broadening coverage condition

Event simulations (collapse of quantum states)

1. sampling noise, statistical uncertainty

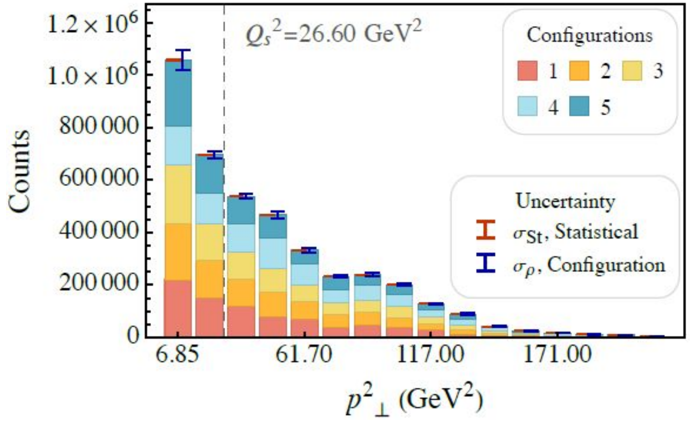


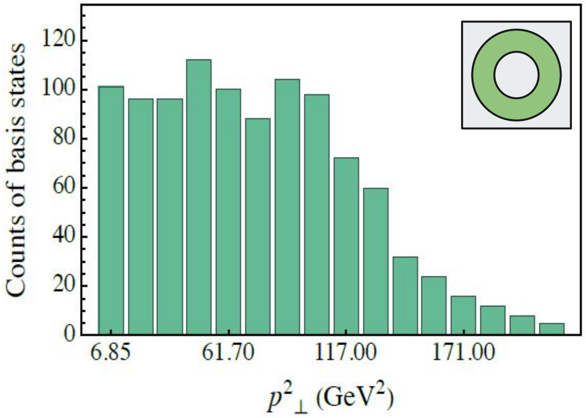




Event simulations (collapse of quantum states)

2. configuration uncertainty

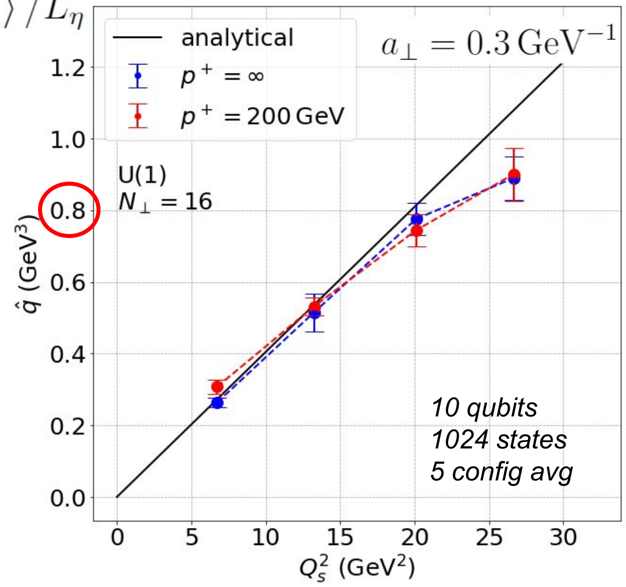


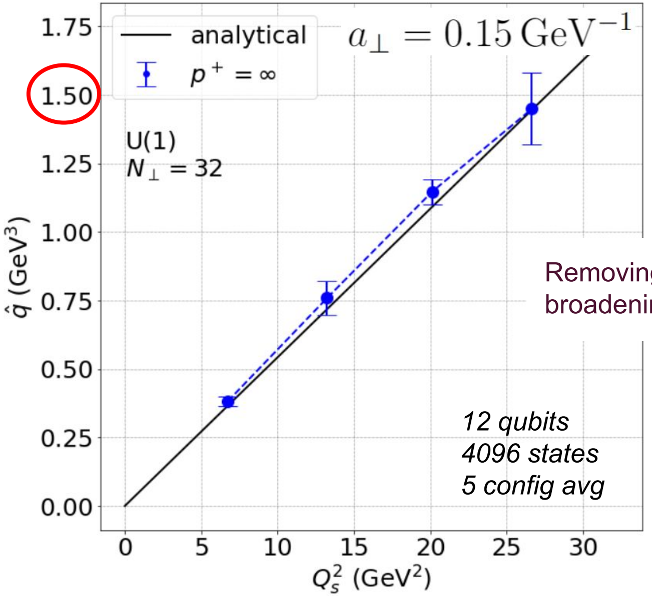


Results: colorless U(1) case

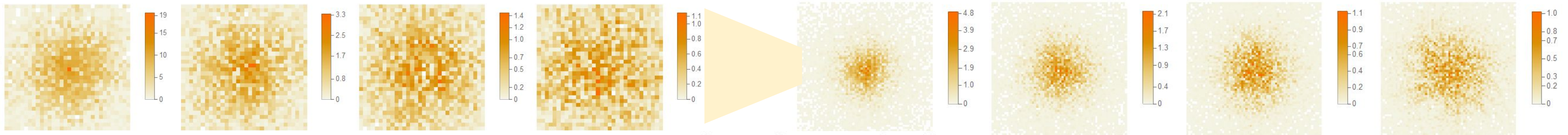
819200 shots
noiseless QASM simulation

$\hat{q} = \langle \boldsymbol{p}_\perp^2 \rangle / L_\eta$





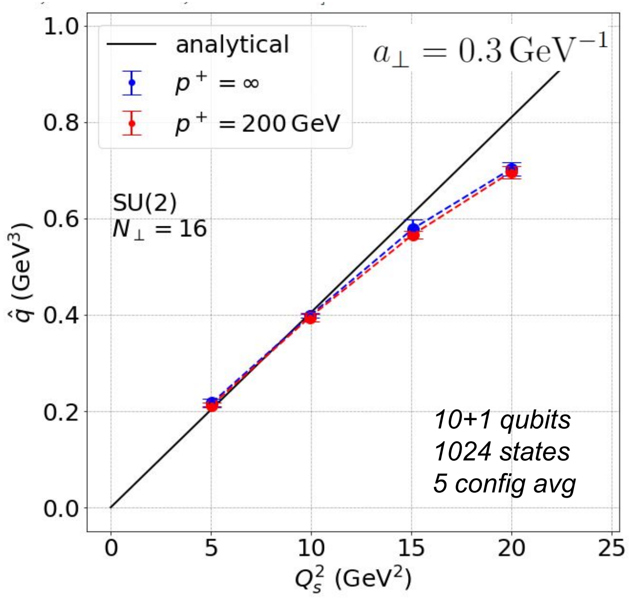
Removing cutoff, mom broadening increases

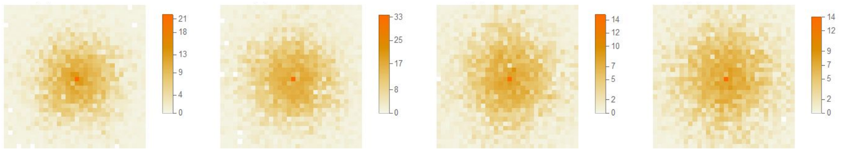


$p_\perp = [-\pi/a_\perp, \pi/a_\perp]$

Results: colorful SU(2) case

819200 shots
noiseless QASM simulation





Analytical

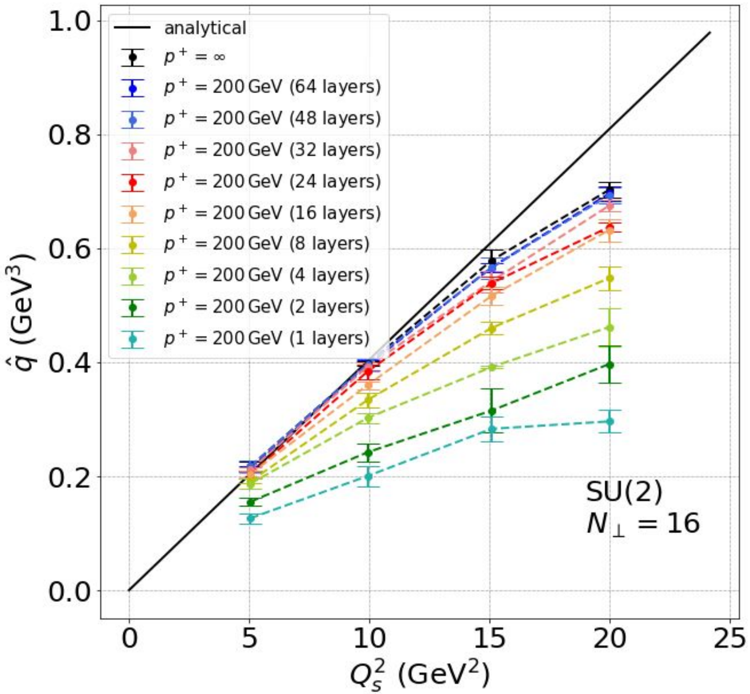
$$\hat{q} = \frac{g^4}{4\pi} C_F \tilde{\mu}^2 \left\{ \log\left(1 + \frac{\frac{\pi^2}{a_\perp^2}}{m_g^2}\right) - \frac{1}{1 + \frac{a_\perp^2 m_g^2}{\pi^2}} \right\}$$

identical curve in eikonal limit for U(1) and SU(2) as function of Q_s^2

Simulation

$$\hat{q} = \langle \boldsymbol{p}_\perp^2 \rangle / L_\eta$$

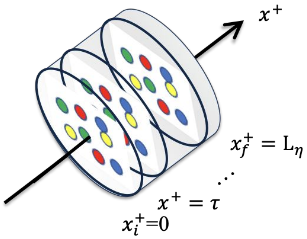
Medium is layer dependent



Correlation is approximately satisfied with increasing layers

Lappi, EPJ.C55:285-292; 0711.3039 (2007)

$$\langle\langle \rho_a(n^x, n^y, n_\tau)\rho_b(n'^x, n'^y, n'_\tau)\rangle\rangle = g^2\tilde{\mu}^2\delta_{ab}\frac{\delta_{n^x,n'^x}\delta_{n^y,n'^y}}{a_\perp^2}\frac{\delta_{n_\tau,n'_\tau}}{\tau}$$



Also preparing for evolving or responsive medium in future

Effects in anisotropic mediums

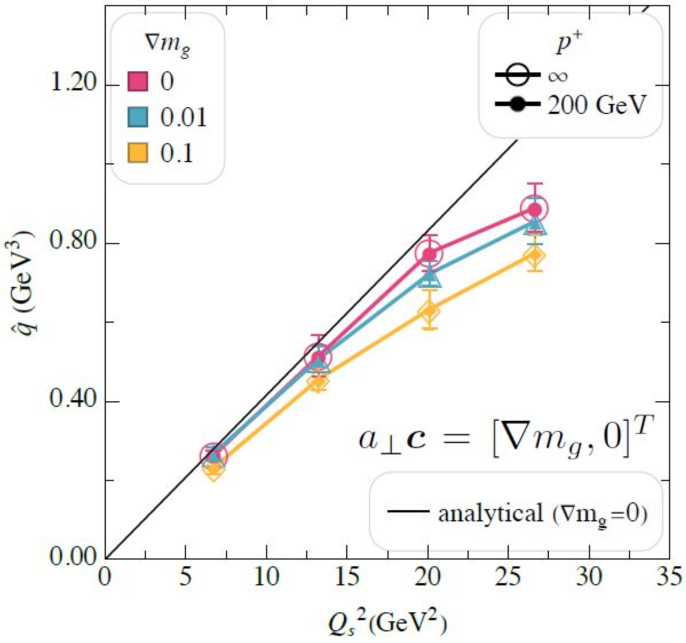
M. Li, Zhao, Maris, Chen, Li, Tuchin, Vary, PRD101,076016; 2002.09757 (2020)

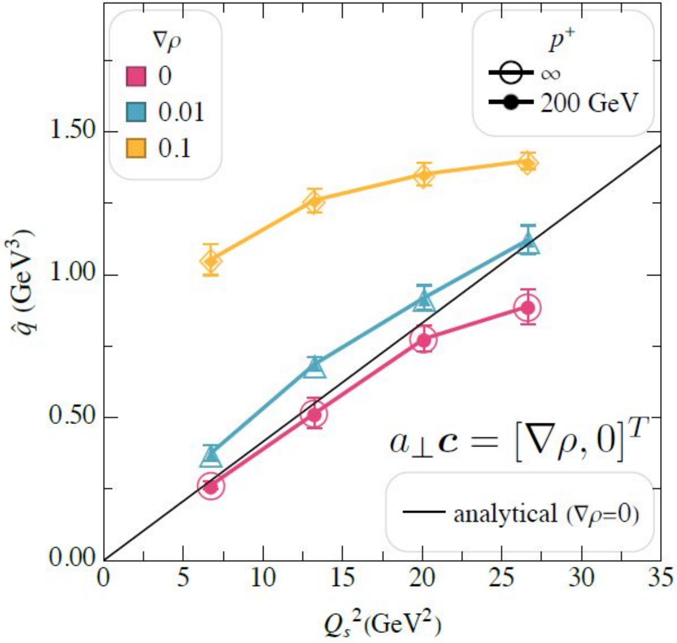
Transverse gradient $m_g^2(\boldsymbol{x}) = m_g^2(1 + \boldsymbol{c} \cdot \Delta \boldsymbol{x})$

Sadofyev, Sievert, Vitev, 2104.09513 (2022)

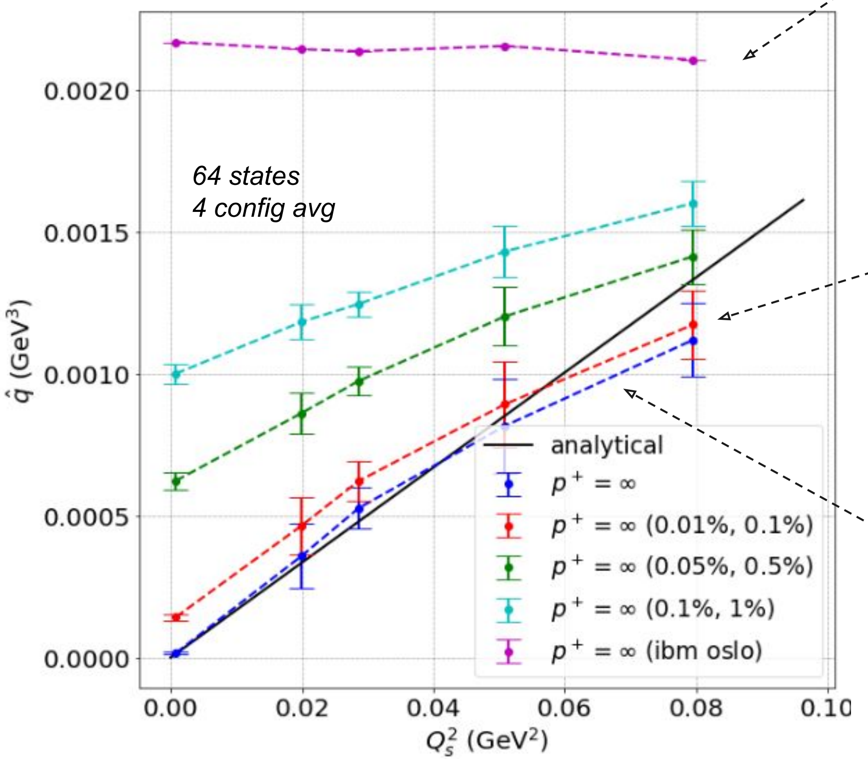
Profile function

$$A_a^-(x^+, \boldsymbol{x}) \to A_a^-(x^+, \boldsymbol{x})\rho(\boldsymbol{x}) \quad \rho(\boldsymbol{x}) = 1 + \boldsymbol{c} \cdot \Delta \boldsymbol{x}$$

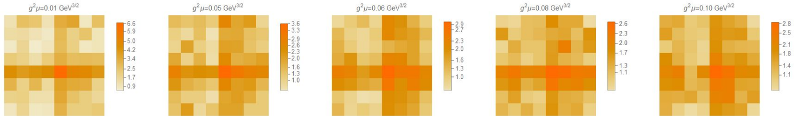




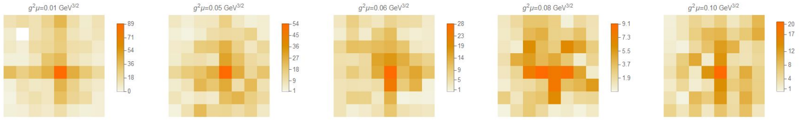
Towards practical simulations



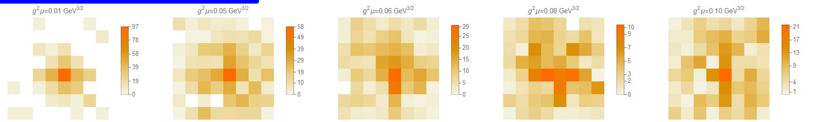
IBM oslo computer



customized noise model with errors (0.01% for 1-qubit gates)



exact simulation



Beyond this work: 1. Multi-particle jet

- Incorporate multiple Fock sectors is key next step for this project. It enables study of gluon absorption and emission

$$|\psi\rangle = c_1 |q\rangle + c_2 |qg\rangle + c_3 |qgg\rangle + \cdots$$

Li, Lappi, Zhao, PRD104, 056014; 2107.02225 (2021)

- Qubit encoding in single particle basis states

$$|\psi\rangle = \underbrace{|q\rangle\cdots|q\rangle}_{n} \otimes \underbrace{|g\rangle\cdots|g\rangle}_{m} \longrightarrow |\tilde{\psi}\rangle = \prod_{i=1}^{m}\Big(|e_{g_i}\rangle \otimes |g_i\rangle\Big) \otimes \prod_{i=1}^{n}\Big(|e_{q_i}\rangle \otimes |q_i\rangle\Big)$$

$$|q_{\text{dressed}}\rangle = |z_g\rangle \otimes |g\rangle \otimes |q\rangle$$

with longitudinal mom, SU(2) color & light-front helicity, we need

$$N_{tot} = \underbrace{(2N_\perp)^2 \times 2 \times 2}_{\text{quark degrees of freedom}} + \underbrace{N_z \times (2N_\perp)^4 \times (2\cdot 2) \times (2\cdot 3)}_{\text{quark-gluon degrees of freedom}} \longrightarrow (10 + 4\log N_\perp + \log N_z)\ \text{qubits}$$

2. Resource-efficient quantum simulation

- Current publicly available: Noisy intermediate-scale quantum computers (NISQ)

ibm_nairobi

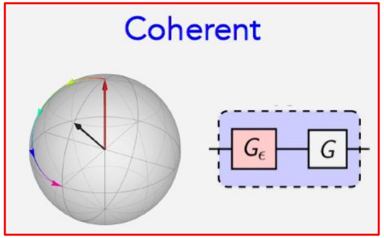
Details

7 Qubits	Status:	Online	Avg. CNOT Error:	1.024e-2
32 QV	Total pending jobs:	27 jobs	Avg. Readout Error:	2.923e-2
2.6K CLOPS	Processor type ⓘ:	Falcon r5.11H	Avg. T1:	104.68 us
	Version:	1.0.24	Avg. T2:	62.85 us
	Basis gates:	CX, ID, RZ, SX, X	Providers with access:	1 Providers ↓
	Your usage:	2952 jobs		

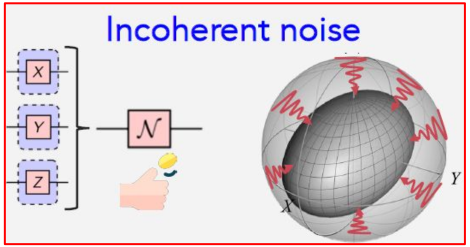
Overall performance, comparable to a circuit size of 5 by 5

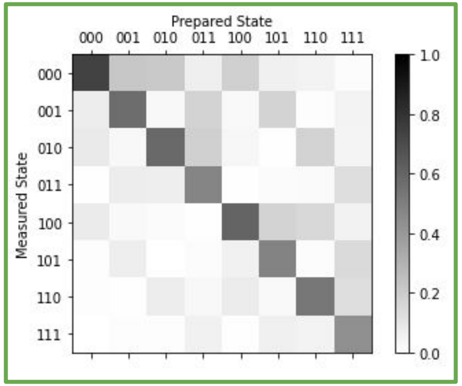
T1 relaxation time (|1> => |0>)

T2 dephasing time (|+> => |->)



images from Minev's talk at QGSS 2022





2. Resource-efficient QS (cont)

- Our current circuit at N = 4 alone has around ~1000 gates when transpiled in IBM computers

- Possible improvements:

  1. Approximate unitary gate implementation (abs. tol = 1e-6 % => 1 %)

  2. Implementation of the potential fields on the quantum circuit, i.e., polynomial-time discretization method, with well-controlled approx

     Kassal, Jordan, Love, Mohseni, Aspuru-Guzik, 0801.2986 (2008)

  3. New algorithms to exploit block-diagonal unitary matrix on circuit?

  4. New quantum gate that controls direction of evolution?

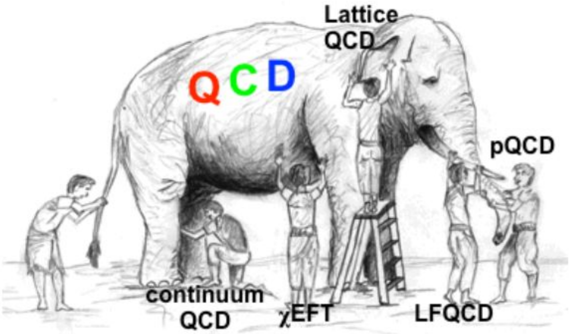
  5. Approximate QFT (limited improvement)

     Barencoa, Ekerta, Suominenb, Torma, 9601018 (1996)

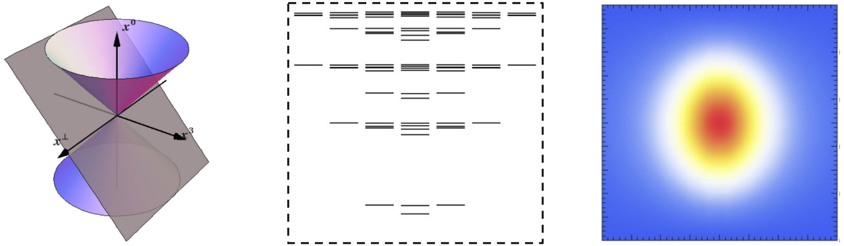
  6. Variational quantum simulation (VQS)

     Yuan, Endo, Zhao, Li, Benjamin, Quantum 3, 191; 1812.08767 (2019)
     Li, Benjamin, PRX 7, 021050; 1611.09301 (2016)

How to study hadron structures?



QCD is the underlying theory



Light-front Hamiltonian formalism

Light-front Hamiltonian formalism

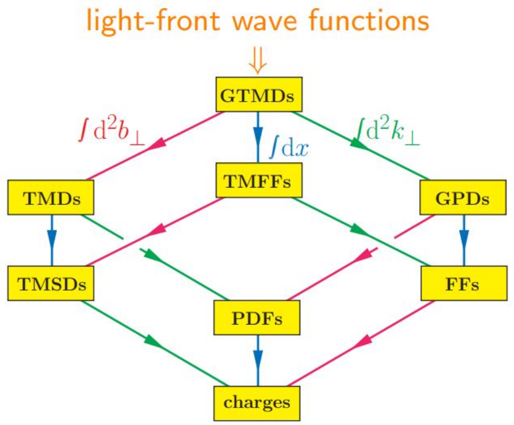
Basis Light-front Quantization (BLFQ)

Vary, Honkanen, Li, Maris, Brodsky, Harindranath, Teramond, Sternberg, Ng, Yang, PRC 81:035205; 0905.1411 (2010)

- Light-front dynamics
  simple dispersion relation

- Hamiltonian approach
  eigenvalues => mass spectrum
  eigenfunctions => observables

- Basis function approach
  exploit symmetry



Applications to various systems: cc, bb, bc, qq, baryon, etc

Access to different observables: form factors, (semi-)leptonic decay, PDFs, GPDs, diffraction production

Image from Li at DNP 2017

Light mesons

WQ, Jia, Li, Vary, PRC 102, 055207; 2005.13806 (2020)

In the application of the light mesons within the valence $|q\bar{q}\rangle$ Fock sector

$$H_{\mathrm{eff},\gamma_5} = \overbrace{\underbrace{\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\mathbf{r}_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}} + V_g}^{H_{\mathrm{eff}}} + H_{\gamma_5}$$

m_q ($m_{\bar{q}}$): quark (antiquark) mass
κ : confining strength
V_g : one-gluon exchange
H_{γ_5} : pseudoscalar contact interaction

Li, Maris, Vary, PRD 96, 016022; 1704.06968 (2017)
Jia, Vary, PRC 99, 035206; 1811.08512 (2019)

Light-front eigenvalue equation: $H_{\mathrm{LC}}|\psi\rangle = M^2|\psi\rangle$

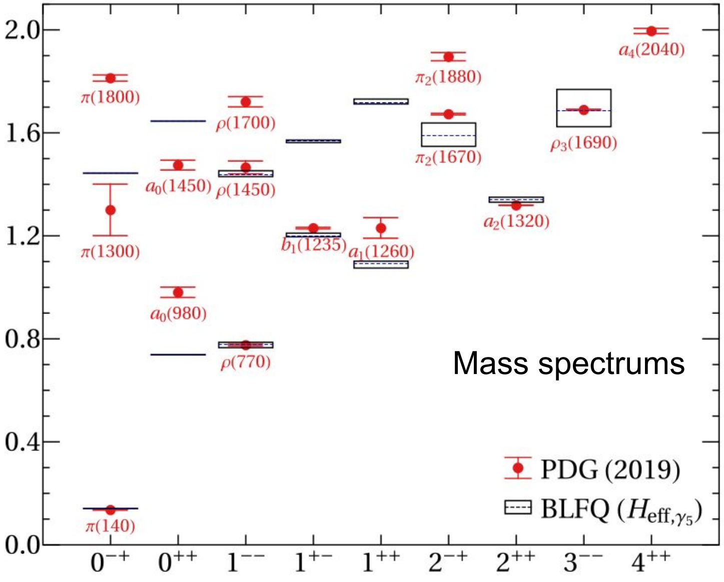
Light-front wave function (LFWF):

$$\psi_{s\bar{s}}^{m_j}(\boldsymbol{k}_\perp, x) = \sum_{nml} \tilde{\psi}_{s\bar{s}}^{m_j}(n,m,l)\phi_{nm}\left(\frac{\boldsymbol{k}_\perp}{\sqrt{x(1-x)}}\right)\chi_l(x)$$

2D harmonic oscillator *Jacobi polynomial*

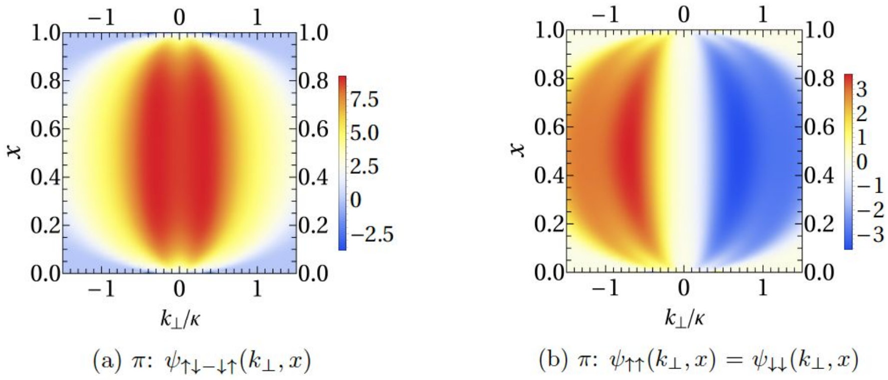
$$2n + |m| + 1 \le N_{\max} \qquad l \le L_{\max}$$

Light mesons

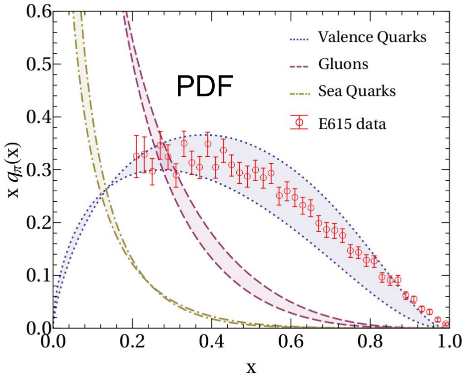


Mass spectrums

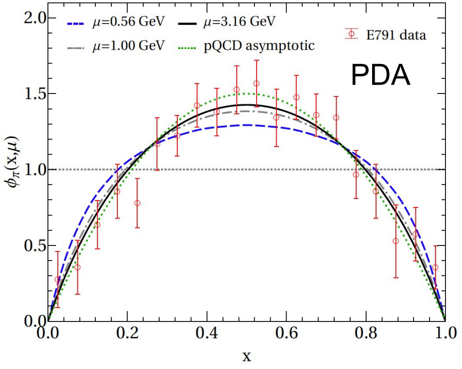
LFWFs



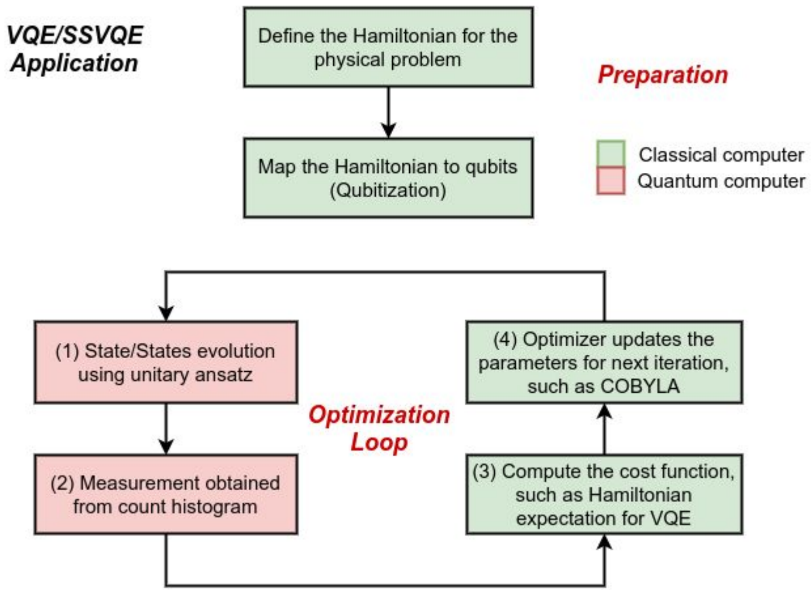
PDF



PDA



Variational quantum eigensolvers (VQE)



Directly inspired by the variational principle.

VQE uses **parameterized unitary ansatz** (educated guess) to obtain the lowest eigenvalue through continuous optimizations.

VQE families:
SSVQE: Subspace-search VQE
VQD: Variational quantum deflation
…

Peruzzo, McClean, Shadbolt, Yung, Zhou, Love, Aspuru-Guzik, O'Brien, Nature Comm 5:4213; 1304.3061 (2014)
Nakanishi, Mitarai, Fujii, PRR 1, 033062; 1810.09434 (2019)
Higgott, Wang, Brierley, Quantum 3, 156; 1805.08138 (2019)

Adapting physical Hamiltonian to quantum computer

For an initial application on quantum computers (or simulators), we truncate the basis functions

	N_f	$\alpha_s(0)$	κ (MeV)	m_q (MeV)	N_{max}	L_{max}	Matrix dimension
$H_{eff}^{(1,1)}$			560 ± 10	300 ± 10	1	1	4 by 4
$H_{eff}^{(4,1)}$	3	0.89	560 ± 10	380 ± 10	4	1	16 by 16
$H_{eff}^{(4,3)}$			560 ± 10	400 ± 10	4	3	32 by 32

$$(N, N) = (2^n, 2^n)$$

- Qubit Encoding: **Jordan-Wigner** encoding $\mathcal{O}(N)$, **compact** encoding $\mathcal{O}(\log N)$ $\rightarrow H_q = \sum_\alpha c_\alpha P_\alpha$

  Logarithmic scaling

- Variational ansatzes: **Unitary Coupled Cluster (UCC) ansatz** and **hardware efficient ansatzes (HEA, ALT, TEN)**
- Algorithms: **VQE** for the ground state. **SSVQE** for the full spectroscopy.
- Observables: With LFWFs encoded by qubits, physical observables are **directly** computed on the circuits.

Hamiltonian & basis encoding example

Example of Nmax = Lmax = 1 (smallest Hamiltonian matrix) where matrix element corresponds to $(n, m, l, s, \bar{s})$ basis state $(\text{All units in MeV}^2)$

$$H_{\text{eff}}^{(1,1)} = \begin{pmatrix} 568487 & 0 & 25428 & 0 \\ 0 & 1700976 & 0 & -15767 \\ 25428 & 0 & 568487 & 0 \\ 0 & -15767 & 0 & 1700976 \end{pmatrix}$$

	n	m	l	s	$\bar{s}$	Direct encoding	Compact encoding
①	0	0	0	1/2	-1/2	$\|0001\rangle$	$\|00\rangle$
②	0	0	0	-1/2	1/2	$\|0010\rangle$	$\|01\rangle$
③	0	0	1	1/2	-1/2	$\|0100\rangle$	$\|10\rangle$
④	0	0	1	-1/2	1/2	$\|1000\rangle$	$\|11\rangle$

From second quantization, the Hamiltonian can be written in terms of creation and annihilation operators,

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \cdots = \sum_{ij} h_{ij} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{4} \sum_{ijkl} h_{ijkl} \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l + \cdots$$

We focus only on the single-body interactions and identify h_{ij} as the Hamiltonian matrix elements.

Qubit encoding

Jordan, Wigner, Zeitschriftfür Physik, 47, 631 (1928)
Seeley, Richard, Love, J.Chem.Phys 137, 224109; 1208.5986 (2012)
Kreshchuk, Kirby, Goldstein, Beauchemin, Love, PRA 105, 032418; 2002.04016 (2020)
Nielsen, Chuang, Quantum Computation & Quantum Information (2000)

Suppose H of dimension $(N,N)=(2^n,2^n)\rightarrow H_q=\sum_\alpha c_\alpha P_\alpha$

- **Direct encoding**: Jordan-Wigner (JW) encoding, map directly from the Pauli matrices $\sigma_k\in\{I_k,X_k,Y_k,Z_k\}$

$$\hat{a}_j^\dagger=\bigotimes_{i=1}^{j-1}Z_i\otimes\frac{X_j-iY_j}{2}$$

$$\hat{a}_j=\bigotimes_{i=1}^{j-1}Z_i\otimes\frac{X_j+iY_j}{2}$$

$$\begin{aligned}H^{(1,1)}_{\text{direct}}&=2269462\,\text{IIII}-284243\,(\text{ZIII}+\text{IIZI})\\&-850488\,(\text{IZII}+\text{IIIZ})+12714\,(\text{XZXI}+\text{YZYI})\\&-7883\,(\text{IXZX}+\text{IYZY}),\end{aligned}$$

$\mathcal{O}(N)$

- **Compact encoding**: utilize orthogonal basis formed by Pauli strings $P_\alpha=\otimes_{k=1}^n\sigma_k$ under trace to further reduce the Hamiltonian (Hilbert-Schmidt inner product space)

$$H_q=\frac{1}{N}\sum_{\alpha=1}^{N^2}\text{Tr}(P_\alpha H)\cdot P_\alpha$$

$$\text{Tr}(P_jP_k)=2^n\delta_{j,k}=N\delta_{j,k}$$

$$\begin{aligned}H^{(1,1)}_{\text{compact}}&=1134731\,\text{II}-566245\,\text{IZ}\\&+4831\,\text{XI}+20598\,\text{XZ}\end{aligned}$$

$\mathcal{O}(\log N)$

Variational ansatzes

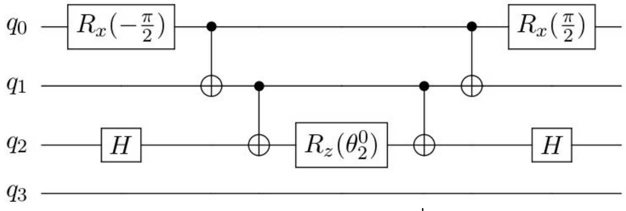
Barkoutsos et al., PRA 98, 022322; 1805.04340 (2018)
Romero, Babbush, McClean, Hempel, Love, Aspuru-Guzik, Quant Sci. Tech 4, 014008; 1701.02691 (2017)

Educated guess of the unitary circuit with parameters to be optimized in each iteration.

1. **<u>Unitary coupled cluster (UCC)</u>** ansatz
   - physically motivated by coupled cluster method

$$\hat{U}(\vec{\theta}) = e^{\hat{T}(\vec{\theta}) - \hat{T}^\dagger(\vec{\theta})}, \quad \hat{T}(\vec{\theta}) = \sum_{\substack{r \in \text{occ} \\ p \in \text{virt}}} \theta_p^r \hat{a}_p^\dagger \hat{a}_r$$

$$\hat{U}(\vec{\theta}) = e^{i \sum_\alpha c_\alpha P_\alpha}$$

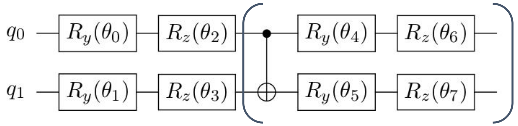


UCC(Single) circuit for $e^{i\, \theta_3^0\, \hat{a}_3^\dagger\, \hat{a}_0}$

2. **<u>Hardware efficient</u>** ansatz: HEA, ALT, TEN, etc
   - heuristic ansatz
   - consists of alternating single-qubit rotations and entangling blocks (repetition layers)
   - proven to work for general problems

Kandala, Mezzacapo, Temme, Takita, Brink, Chow, Gambetta, Nature 549, 242; 1704.05018 (2017)

rep=1



One possible HEA with 1 repetition layer using Qiskit EfficientSU2 library

Algorithms

VQE: Peruzzo, McClean, Shadbolt, Yung, Zhou, Love, Aspuru-Guzik, O'Brien, Nature Comm 5:4213; 1304.3061 (2014)
SSVQE: Nakanishi, Mitarai, Fujii, PRR 1, 033062; 1810.09434 (2019)

1. **Variational Quantum Eigensolver** (VQE) algorithm finds ground state. In one iteration:

Initial state $|\psi\rangle$ → Unitary evolution $|\psi(\vec{\theta})\rangle = \hat{U}(\vec{\theta})\,|\psi\rangle$ → Measure & compute cost function $C(\vec{\theta}) = \langle\psi(\vec{\theta})|\,\hat{H}\,|\psi(\vec{\theta})\rangle$

2. **Subspace-search VQE (SSVQE)** algorithm finds excited states. In particular, Weighted SSVQE for all k excited states.

Set of orthogonal states $\{|\psi_i\rangle\}_{i=0}^{k}$ → Evolution for each state $|\psi_i(\vec{\theta})\rangle = \hat{U}(\vec{\theta})\,|\psi_i\rangle$ → Measure & compute cost function $C_{\vec{\omega}}(\vec{\theta}) = \vec{\omega}\cdot\vec{E}$

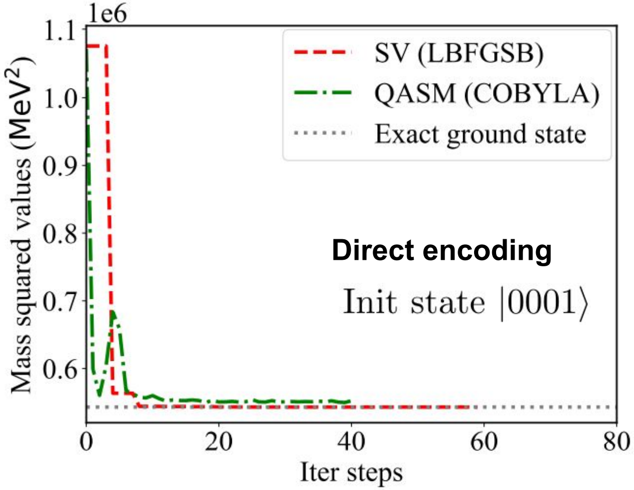
$$\vec{E} = (E_0, E_1, \cdots, E_k) \qquad E_i = \langle\psi_i(\vec{\theta})|\hat{H}|\psi_i(\vec{\theta})\rangle$$

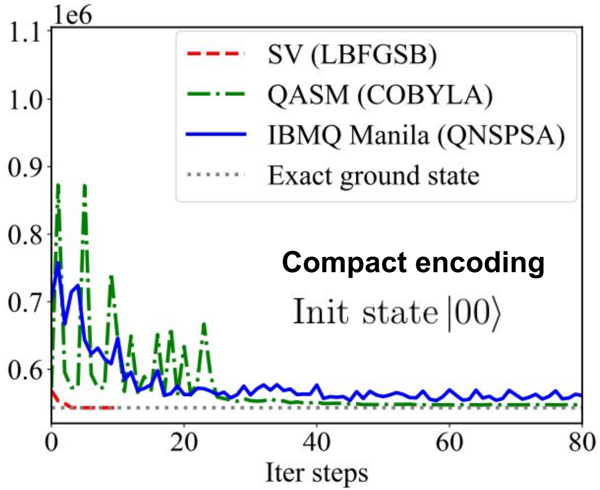
Strictly decreasing weight vector $\vec{\omega}$ prioritizing lower-lying states, example:

$$\vec{\omega} = (1, 0.5, 0.25)$$

$$C_{\vec{\omega}}(\vec{\theta}) = E_0 + 0.5E_1 + 0.25E_2$$

Results: VQE $(N_{\max}, L_{\max}) = (1, 1)$

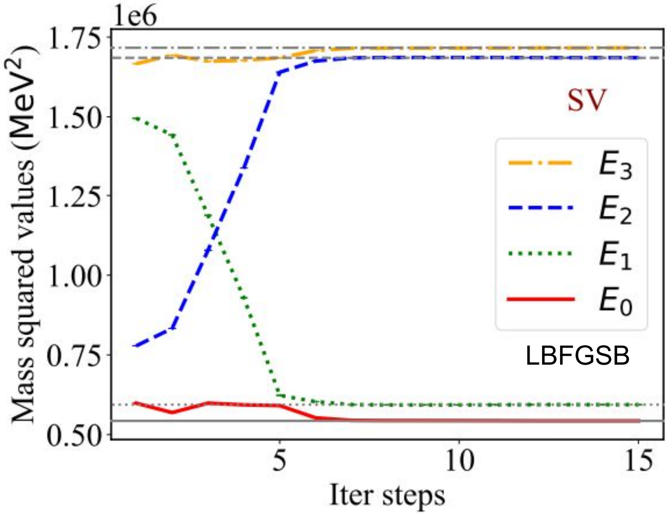


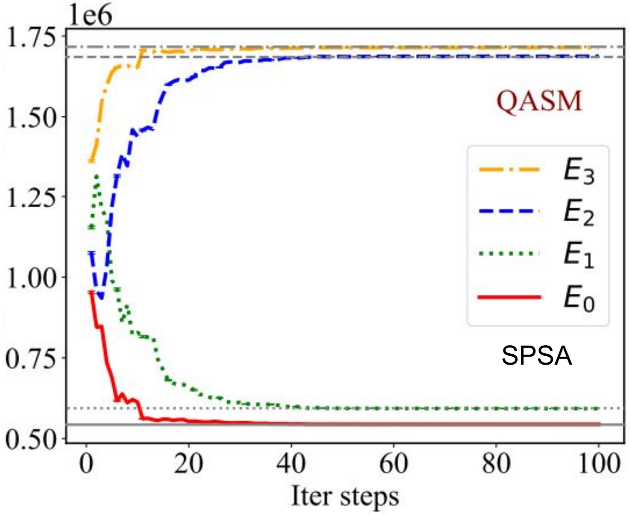


- Ansatz: UCCS ansatz, 50 gates HEA ansatz, 9 gates (1 layer)

- IBM backends:
- Statevector (SV) simulator (noise-free exact simulation)
- QASM simulator (sampling noise from 8192 shots per measurement)
- IBMQ manila (5 Qubits, 32 QV, 2.8K CLOPS, 2e-2% readout error, 8192 shots per measurement)

Results: SSVQE $(N_{\max}, L_{\max}) = (1, 1)$



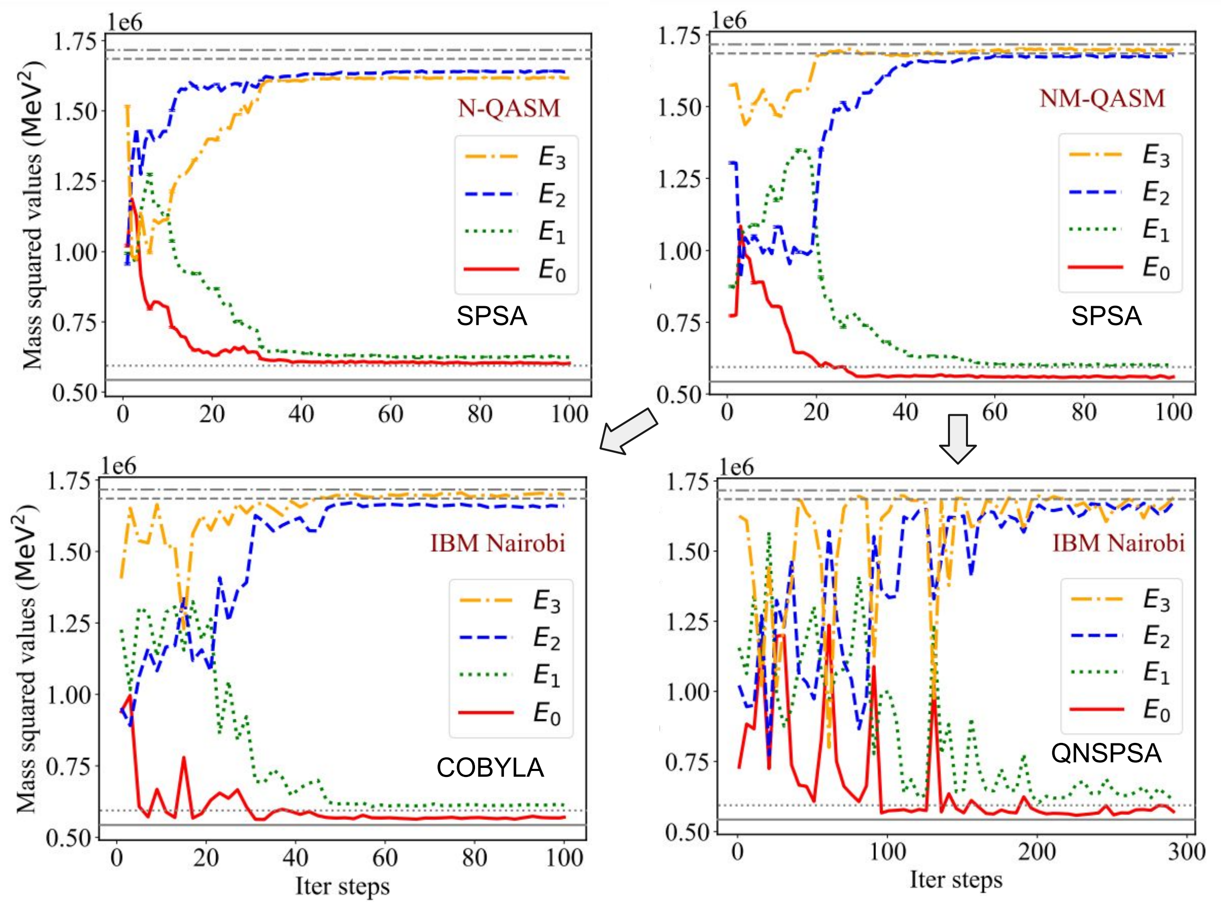


- Both use compact encoding with HEA (2 repetition layers, 12 params)
- Cost function:

$$1.0 \cdot E_{|00\rangle} + 0.5 \cdot E_{|01\rangle} + 0.25 \cdot E_{|10\rangle} + 0.125 \cdot E_{|11\rangle}$$

$E_{|00\rangle} \to E_0$ $\quad E_{|10\rangle} \to E_2$

$E_{|01\rangle} \to E_1$ $\quad E_{|11\rangle} \to E_3$



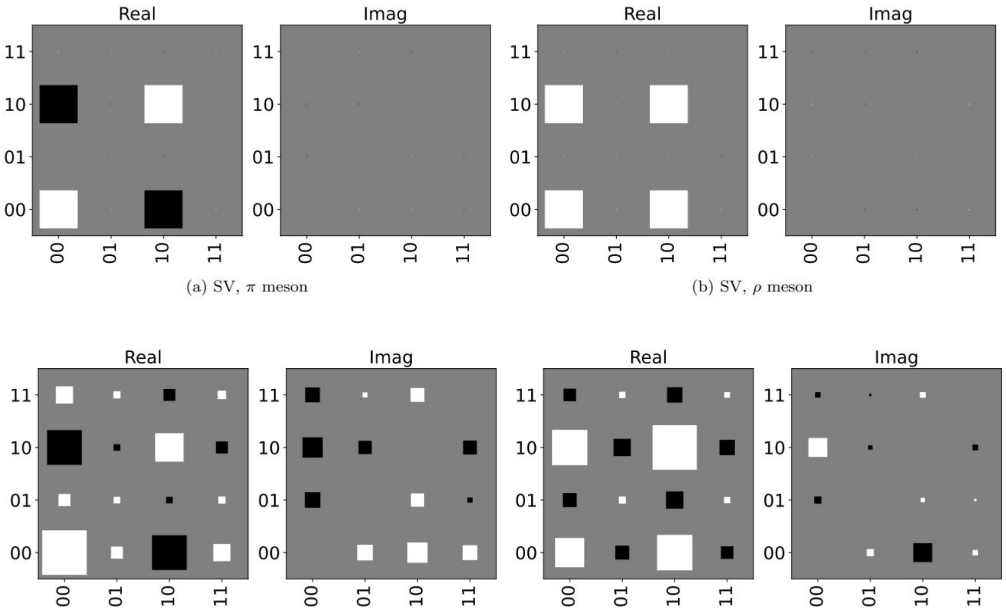
Noisy-QASM (N-QASM)

Noise-mitigated QASM (NM-QASM)

useful for predicting simulation outcome

IBM quantum computer Nairobi (7 qubit, 32 QV)

Results: LFWFs as quantum states



(a) SV, π meson (b) SV, ρ meson

(e) Nairobi, π meson (f) Nairobi, ρ meson

Density matrix $D_{ij} = |\psi_i\rangle \langle\psi_j|$ visualized in Hinton diagrams.

Orthonormality and trace are both preserved

Results: decay constants

Decay constants are defined as vacuum-to-hadron matrix element of the quark current operator. In BLFQ basis, we write the decay constants as:

$$f_{\mathrm{P,V}} = 2\sqrt{2N_c}\int_0^1 \frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int \frac{\mathrm{d}^2\boldsymbol{k}_\perp}{(2\pi)^3}\psi^{(m_j=0)}_{\uparrow\downarrow\mp\downarrow\uparrow}(x,\boldsymbol{k}_\perp)$$
$$\equiv \frac{\kappa\sqrt{N_c}}{\pi}\sum_{nl}(-1)^n C_l(m_q,\kappa)\left(\tilde{\psi}^{(m_j=0)}_{\uparrow\downarrow}(n,0,l)\mp\tilde{\psi}^{(m_j=0)}_{\downarrow\uparrow}(n,0,l)\right)$$

Li, Maris, Vary, PRD 96, 016022; 1704.06968 (2017)

In the VQE/SSVQE, the light-front wave function (LFWF) is encoded on the qubits. One can directly compute observables such as decay constants on the quantum circuit:

$$f_{\mathrm{P,V}} \propto \quad |\langle \nu_{\mathrm{P,V}}|\psi(\vec{\theta})\rangle| = \sqrt{\langle\psi(\vec{\theta})|(|\nu_{P,V}\rangle\langle\nu_{P,V}|)|\psi(\vec{\theta})\rangle}$$

Example:

$$\nu_{\mathrm{P}}^{(1,1)} = (1,-1,0,0)$$
$$\nu_{\mathrm{V}}^{(1,1)} = (1,1,0,0)$$

$\longrightarrow$

$$|\nu^{(1,1)}_{P,V}\rangle\langle\nu^{(1,1)}_{P,V}|_q = 0.5\,(II \mp IX + ZI \pm ZX)$$

Results: decay constants

Summary of decay constants for the lowest two states (π and ρ mesons). Experimental decay constants are around 130 MeV and 216 MeV, respectively. Uncertainties are from measurements of 20,000 shots.

	$N_{\max}$	$L_{\max}$	Exact result	SV	QASM	NM-QASM	IBM Nairobi
f_π	1	1	178.18	178.18	177.11 ± 4.94	174.64 ± 6.61	164.20 ± 8.51
f_ρ			178.18	178.18	177.17 ± 4.88	174.55 ± 6.65	167.76 ± 8.21
f_π	4	1	199.36	200.61	200.32 ± 11.99	196.02 ± 12.23	
f_ρ			227.63	230.08	228.13 ± 10.10	224.80 ± 10.55	
f_π	4	3	199.34	199.57	201.90 ± 10.72	186.15 ± 11.01	
f_ρ			229.25	230.04	228.58 ± 9.58	203.04 ± 10.58	

Results: parton distribution functions

Parton distribution functions (PDFs) is the probability of finding a particle with longitudinal momentum fraction x under some factorization scale related to experimental conditions,

$$q(x;\mu) = \frac{1}{x(1-x)} \sum_{s\bar{s}} \int \frac{\mathrm{d}^2 \boldsymbol{k}_\perp}{2(2\pi)^3} |\psi_{s\bar{s}}^{(m_j=0)}(x, \boldsymbol{k}_\perp)|^2$$

$$\equiv \frac{1}{4\pi} \sum_{s\bar{s}} \sum_{nm} \sum_{l\bar{l}} \tilde{\psi}_{s\bar{s}}^{*(m_j=0)}(n,m,\bar{l}) \tilde{\psi}_{s\bar{s}}^{(m_j=0)}(n,m,l) \chi_l(x) \chi_{\bar{l}}(x)$$

Li, Maris, Vary, PRD 96, 016022; 1704.06968 (2017)

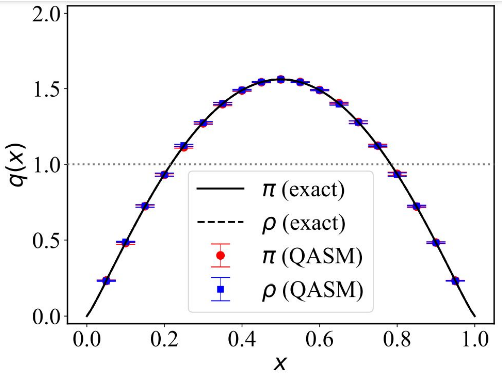
Using projection operators, one can directly compute the PDF on quantum computers as well.

$$q(x) = \sum_{s\bar{s}} \sum_{nm} \sum_{l\bar{l}} \langle \psi(\vec{\theta}) | \hat{O}_{\mathrm{pdf}}(x) | \psi(\vec{\theta}) \rangle \qquad \hat{O}_{\mathrm{pdf}}(x) = \hat{U}_{\mathrm{p}}(s,\bar{s},n,m,\bar{l})^\dagger \hat{U}_{\mathrm{p}}(s,\bar{s},n,m,l) \chi_l(x) \chi_{\bar{l}}(x)/4\pi$$

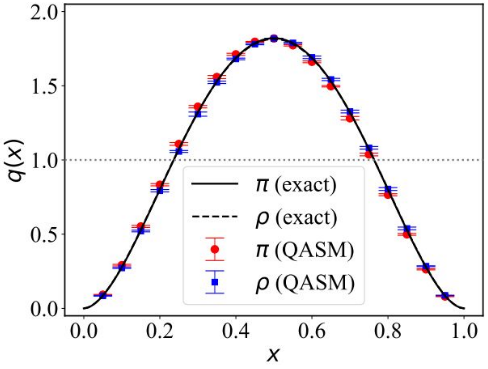
Example:

$$\hat{O}_{\mathrm{pdf}}^{(1,1)}(0.5)_q = 1.30\, II - 1.29\, IX - 0.18\, IZ, \qquad \hat{O}_{\mathrm{pdf}}^{(1,1)}(0.25)_q = 0.78\left(II + IZ\right).$$

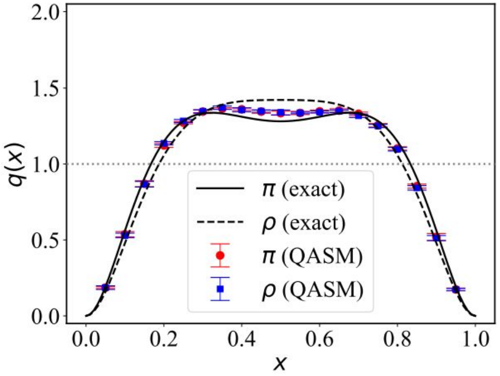
Results: parton distribution functions



(a) PDFs at $N_{\max} = L_{\max} = 1$



(b) PDFs at $N_{\max} = 4, L_{\max} = 1$



(c) PDFs at $N_{\max} = 4, L_{\max} = 3$

Results with increasing basis truncations (i.e. qubits). Difficulty in obtain accurate results at 5 qubits.

Transition amplitudes

The electromagnetic transition between two hadron states is governed by the hadron matrix element, which is key to calculating the transition form factor & decay width (probe the internal structure of QCD bound state):

$$I^{\mu}_{m_j',m_j} = \langle \psi_B(p', j', m_j') | J^{\mu}(x) | \psi_A(p, j, m_j) \rangle$$

M Li, Y Li, Maris, Vary, PRD 98, 034024; 1803.11519 (2018)

The SSVQE approach is particularly useful, as it allows to compute any transition amplitude by using a superposition of the incoming and outgoing meson states:

$$A = \langle \psi_i(\vec{\theta}) | \hat{A} | \psi_j(\vec{\theta}) \rangle = \langle \psi_i | U^{\dagger}(\vec{\theta}) \hat{A} U(\vec{\theta}) | \psi_j \rangle$$

$$\mathrm{Re}(A) = \langle \psi_{ij}^{+x} | U^{\dagger}(\vec{\theta}) \hat{A} U(\vec{\theta}) | \psi_{ij}^{+x} \rangle - \frac{1}{2}\Big(\langle \psi_i | U^{\dagger}(\vec{\theta}) \hat{A} U(\vec{\theta}) | \psi_i \rangle + \langle \psi_j | U^{\dagger}(\vec{\theta}) \hat{A} U(\vec{\theta}) | \psi_j \rangle \Big)$$

$$|\psi_{ij}^{+x}\rangle = \frac{1}{\sqrt{2}}(|\psi_i\rangle + |\psi_j\rangle)$$

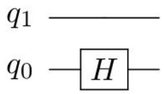
$$\mathrm{Im}(A) = \langle \psi_{ij}^{+y} | U^{\dagger}(\vec{\theta}) \hat{A} U(\vec{\theta}) | \psi_{ij}^{+y} \rangle - \frac{1}{2}\Big(\langle \psi_i | U^{\dagger}(\vec{\theta}) \hat{A} U(\vec{\theta}) | \psi_i \rangle + \langle \psi_j | U^{\dagger}(\vec{\theta}) \hat{A} U(\vec{\theta}) | \psi_j \rangle \Big)$$

$$|\psi_{ij}^{+y}\rangle = \frac{1}{\sqrt{2}}(|\psi_i\rangle + i\,|\psi_j\rangle)$$

Example

$$\frac{1}{\sqrt{2}}(\psi_0 + \psi_1)$$

$$|0+\rangle$$



Summary

We present a quantum simulation framework to simulate medium induced jet broadening on a quantum computer. It provides an opportunity to study effects beyond eikonal limit and evolution in more realistic media.

We show that variational quantum eigensolvers can be used to find meson spectroscopy and observables to study hadron structures of the bound states. In particular, we take advantage of the SSVQE algorithm to study the excited states.

Thank you very much!