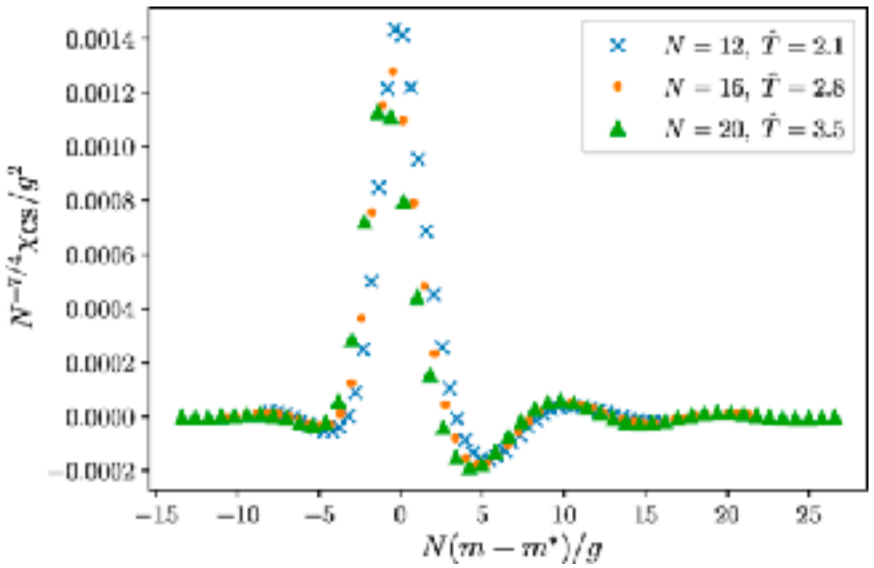
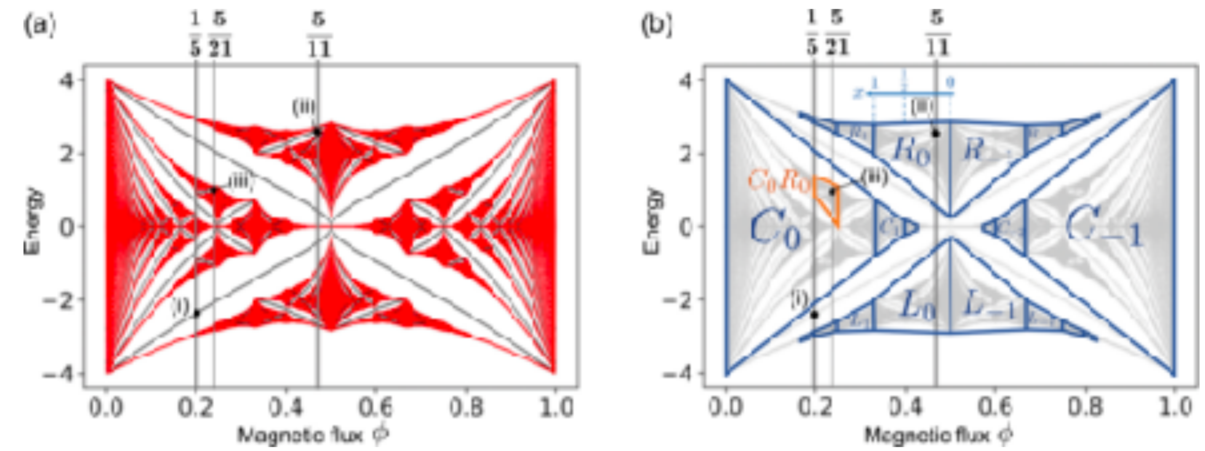
High Energy Physics

with D. Kharzeev & Y. Kikuchi, PRD (2021)



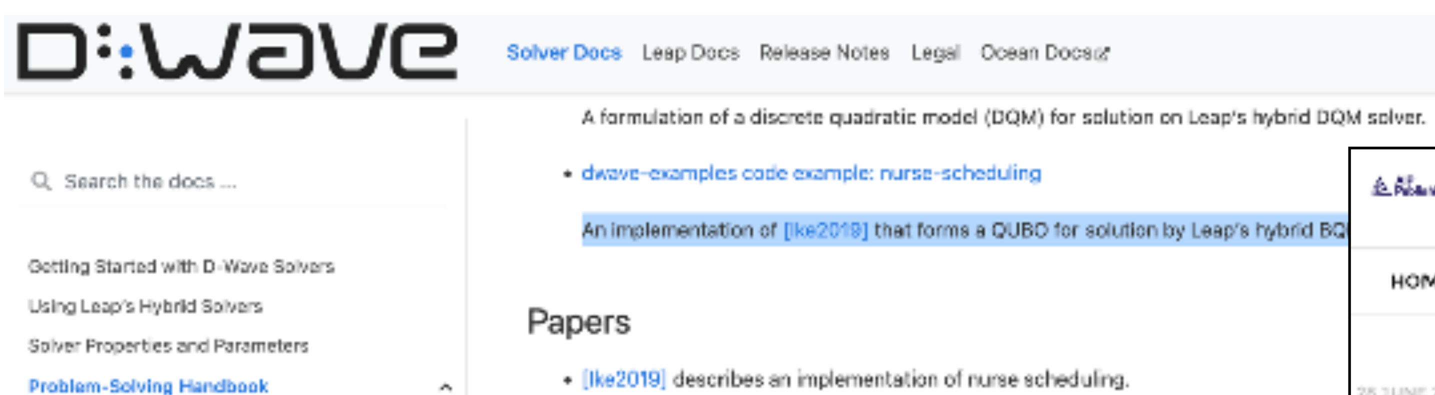
Condensed Matter

with Y. Matsuki & M. Koshino, PRB (2021)



Quantum Computation

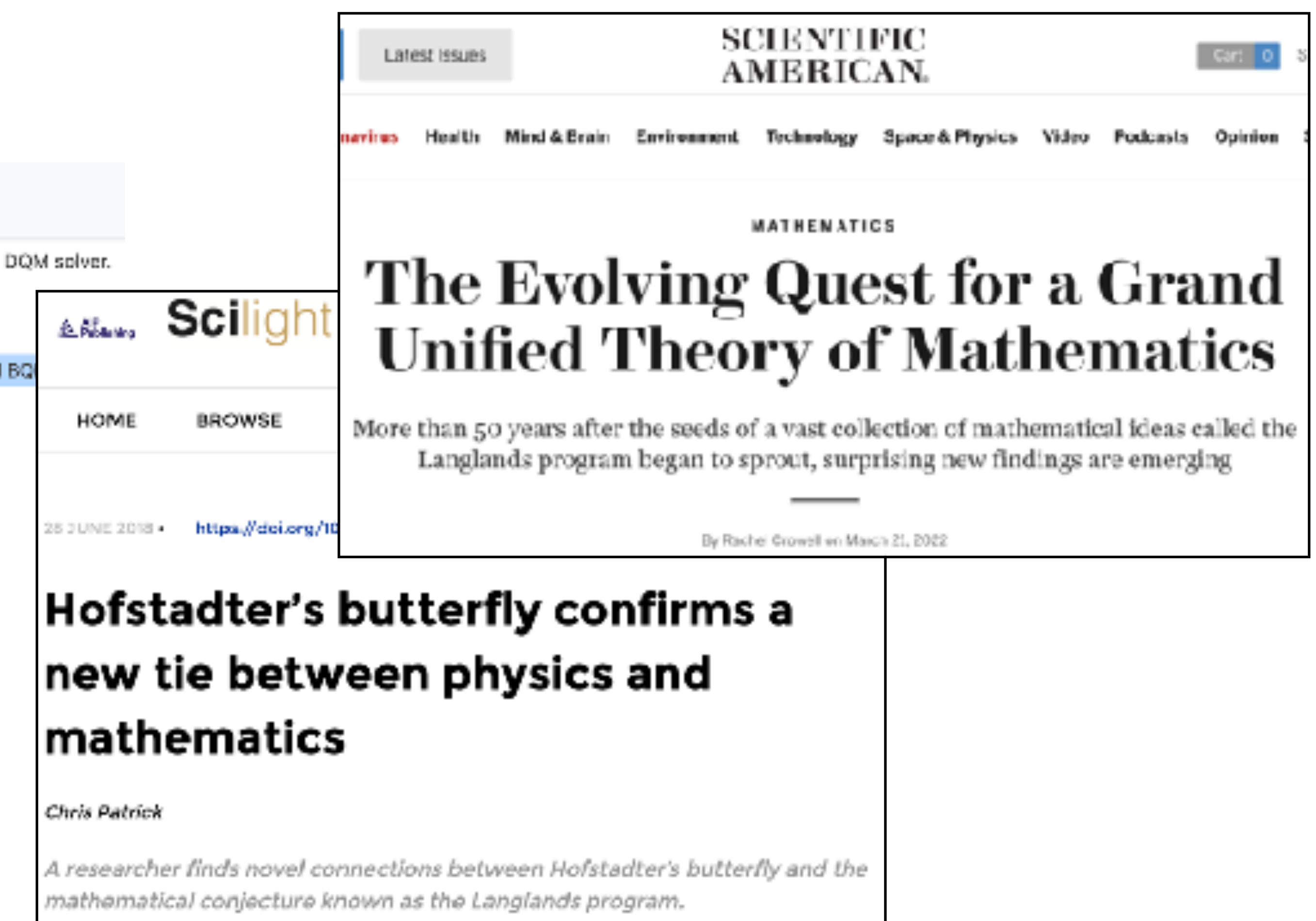
with Y. Nakamura, T. Humble, Sci Rep (2019)



Collected in the user handbook of a quantum computer company

Mathematical Physics

Ann. Phys (2018) & J. Math. Phys (2018)



Outline

1. Introduction to Quantum Computation

2. Quantum Computation for High Energy Physics

   KI, Dmitri Kharzeev (Stony Brook, BNL) & Yuta Kikuchi (BNL), Phys. Rev. D 103 (2021)

   KI and 6 other authors at Stony Brook & BNL, to appear (2022)

3. Beyond qubits: Operator Algebra

   KI, arXiv: 2210. 05133 [math.OA] (2022)

4. Summary and Outlook

History of Quantum Computation

Feynman 1981 at 1st Symposium on Physics and Computation

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical. If you want to make a simulation of nature you'd better make it quantum mechanical"

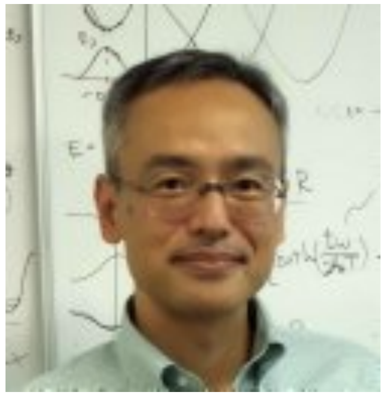
Shor 1994

Discovery of exponentially fast algorithm solving prime factorisation

The vulnerability of RSA & elliptic curve cryptography

The First Revolution of Quantum Computation

Superconducting Quantum Computer



Nakamura et al, Nature 1999

The first construction of a qubit

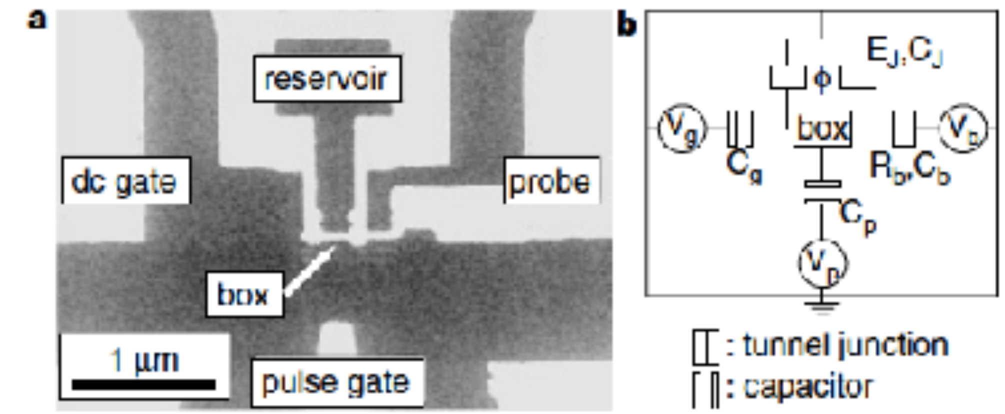
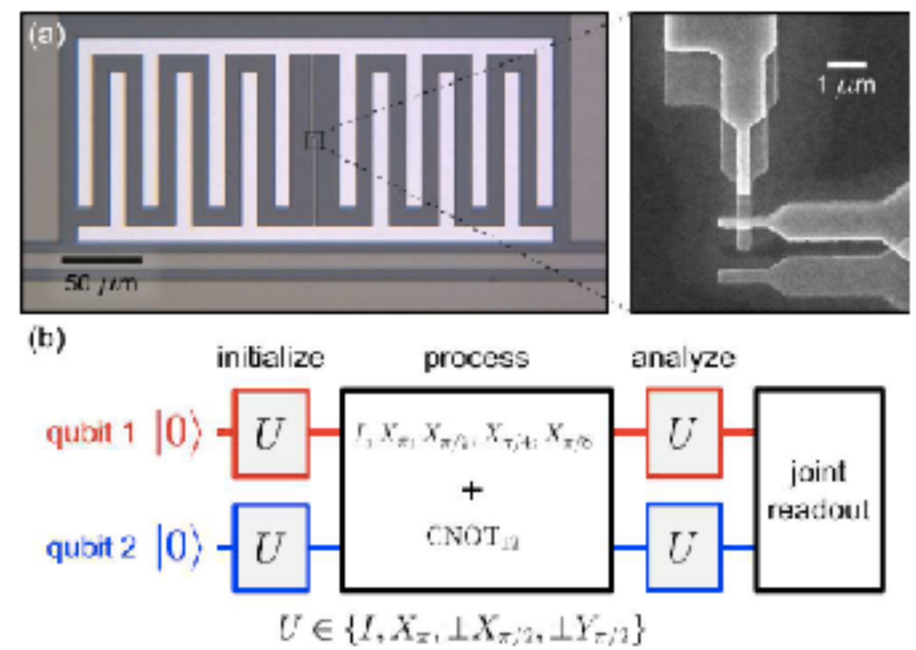


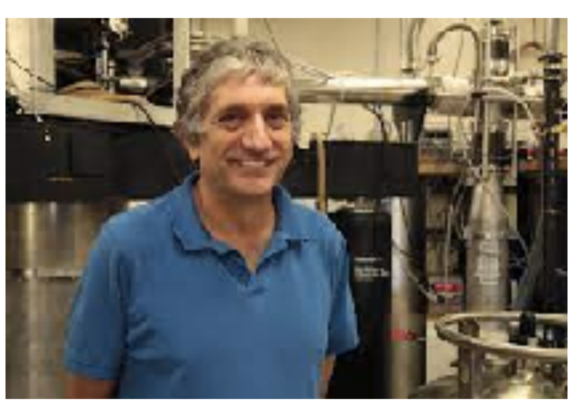
Figure 1 Single-Cooper-pair box with a probe junction. **a**, Micrograph of the

IBM Watson Research Center

Gate fidelity 95% for Universal Quantum Gate Set for two quits

Chow et al, PRL 2012





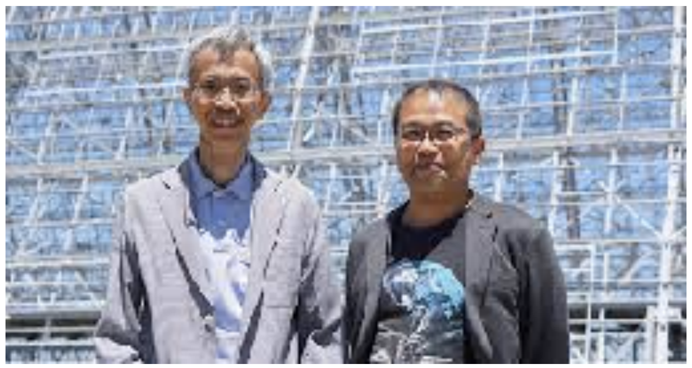
Google (Martinis group), Nature 2019

Quantum supremacy using a programable superconducting processor

3 Approaches to Quantum Computation

Quantum Annealing

Kadowaki & Nishimori, PRE 1998



- 5000 qubits are implemented by D-wave
- No known way for error correction

Topological Fault-Tolerant Quantum Computer

Kitaev's Toric code, Annals of Physics 1997

- Quantum supremacy is possible
- Requires 1000000000 qubits for practical use

Google, IBM, ect…

Topological Quantum Computer with Anyons

- Protects quantum information topologically on real hardware
- Need and control non-abelian anyons

Microsoft

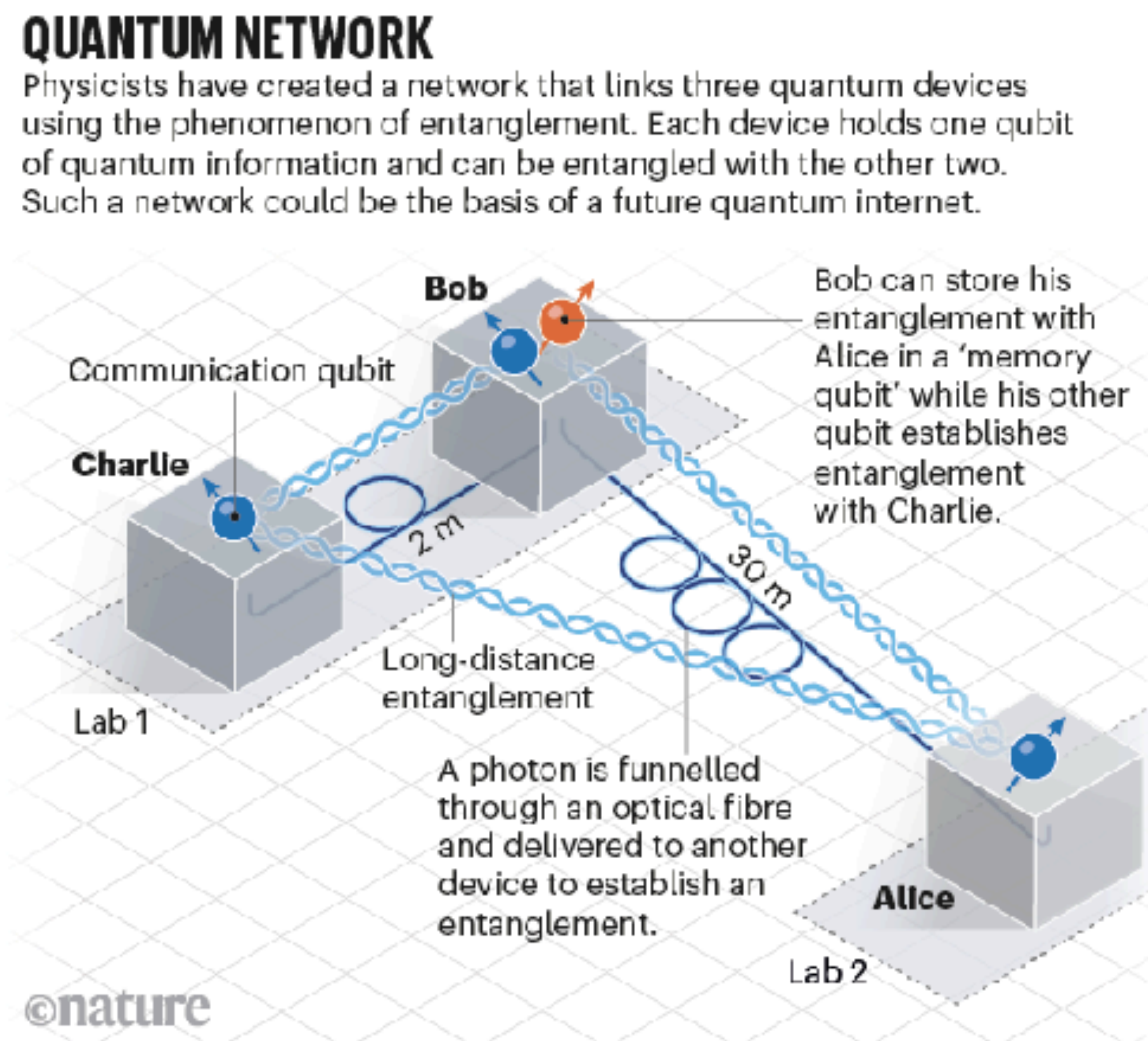
Quantum Information Processing & Communication

Kimble "Quantum Internet" Nature 2008

Connecting quantum computers in a network

UT Delft group, Nature 2018

Delivery of remote entanglement on a quantum network



Nature 590, 540-541 (2021)

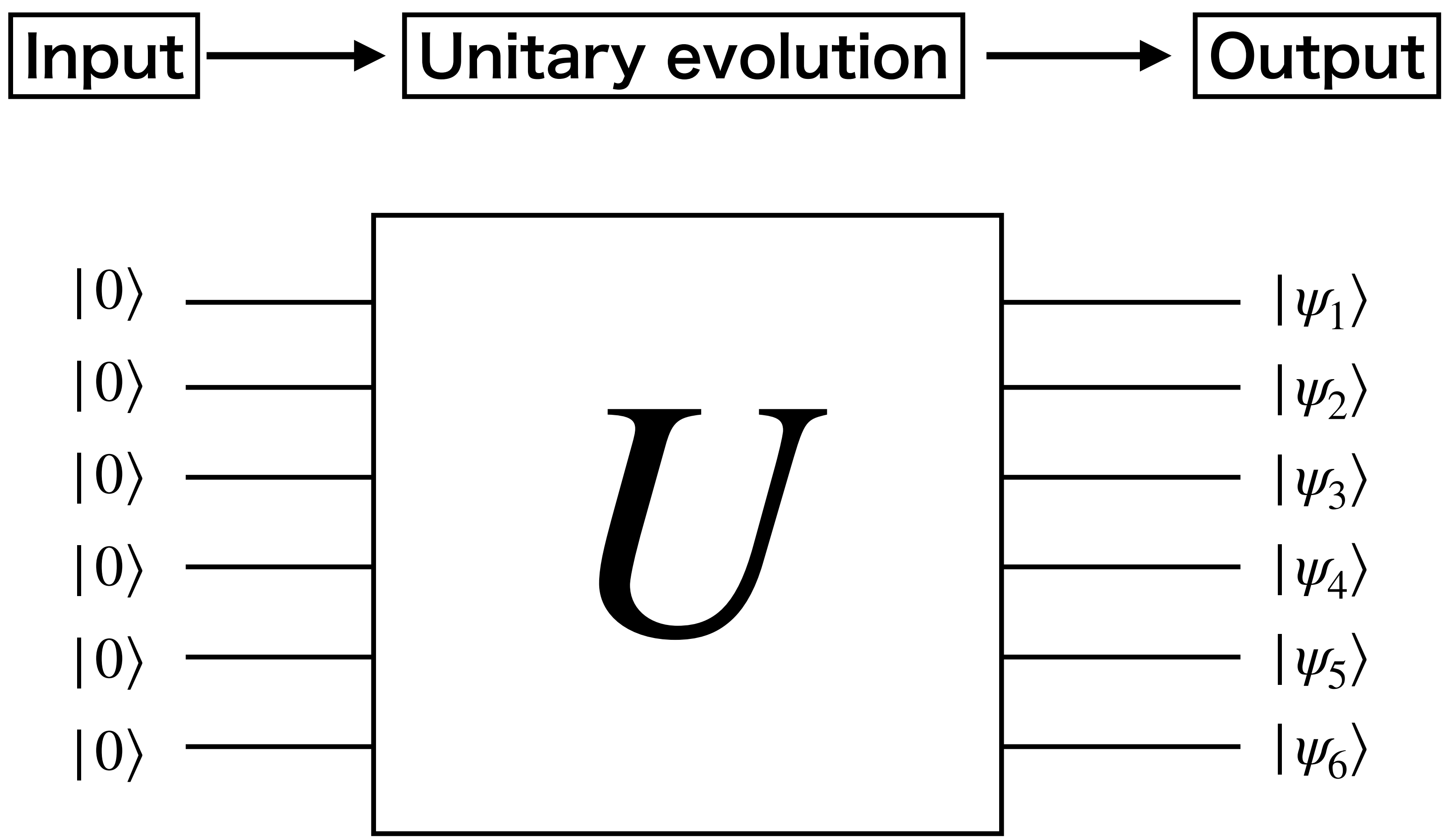
HP of TU Delft https://www.tudelft.nl/en/2019/tu-delft/kpn-and-qutech-join-forces-to-make-quantum-internet-a-reality

Basics of Quantum Computation

Basics of Quantum Computation

Qubit

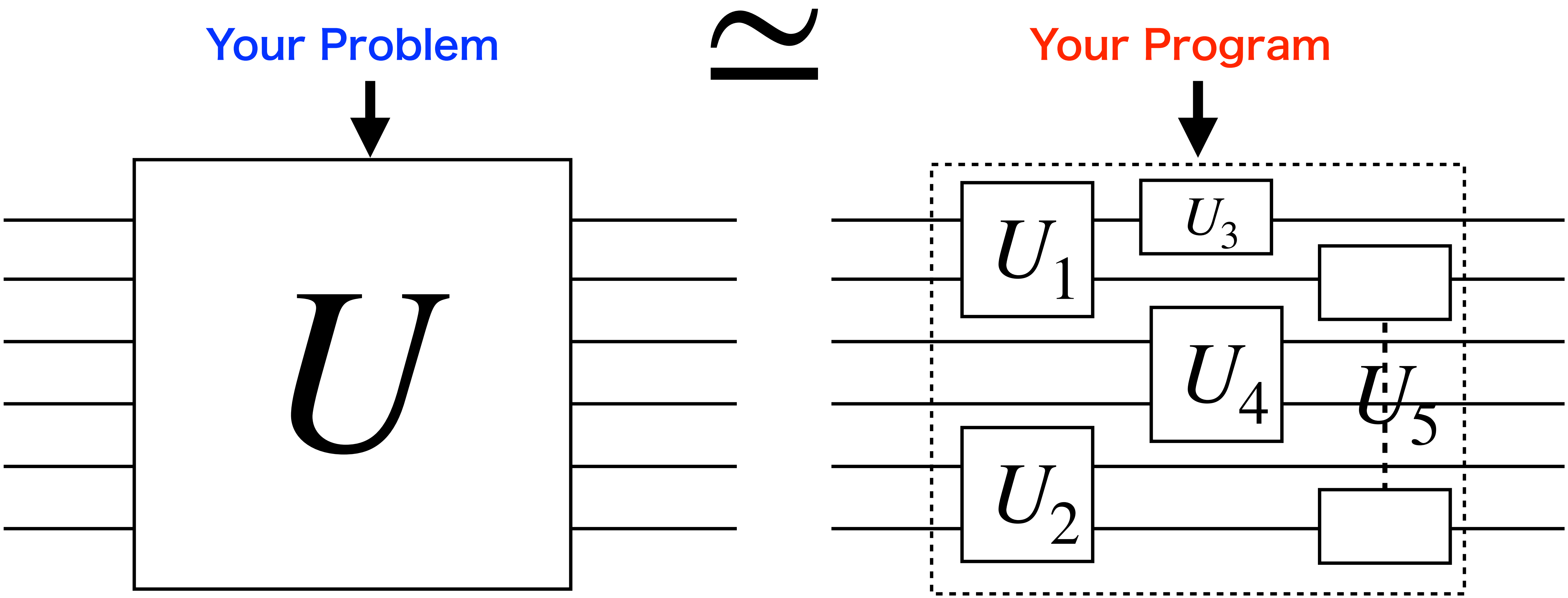
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1,\ \alpha, \beta \in \mathbb{C}$$



How to Implement Your Problem

1. Discretize your problem

2. Construct a gate set



Universal Quantum Computation

Pauli operators

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli X flips a qubit

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$X|0\rangle = |1\rangle, X|1\rangle = |0\rangle$$

CNOT operator

$$\Lambda(X) = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$$

$$|1\rangle \otimes |0\rangle \mapsto |1\rangle \otimes |1\rangle, |1\rangle \otimes |1\rangle \mapsto |1\rangle \otimes |0\rangle$$

Theorem (Dawson-Nielsen 2006)

Paulis and CNOT are enough for universal computation

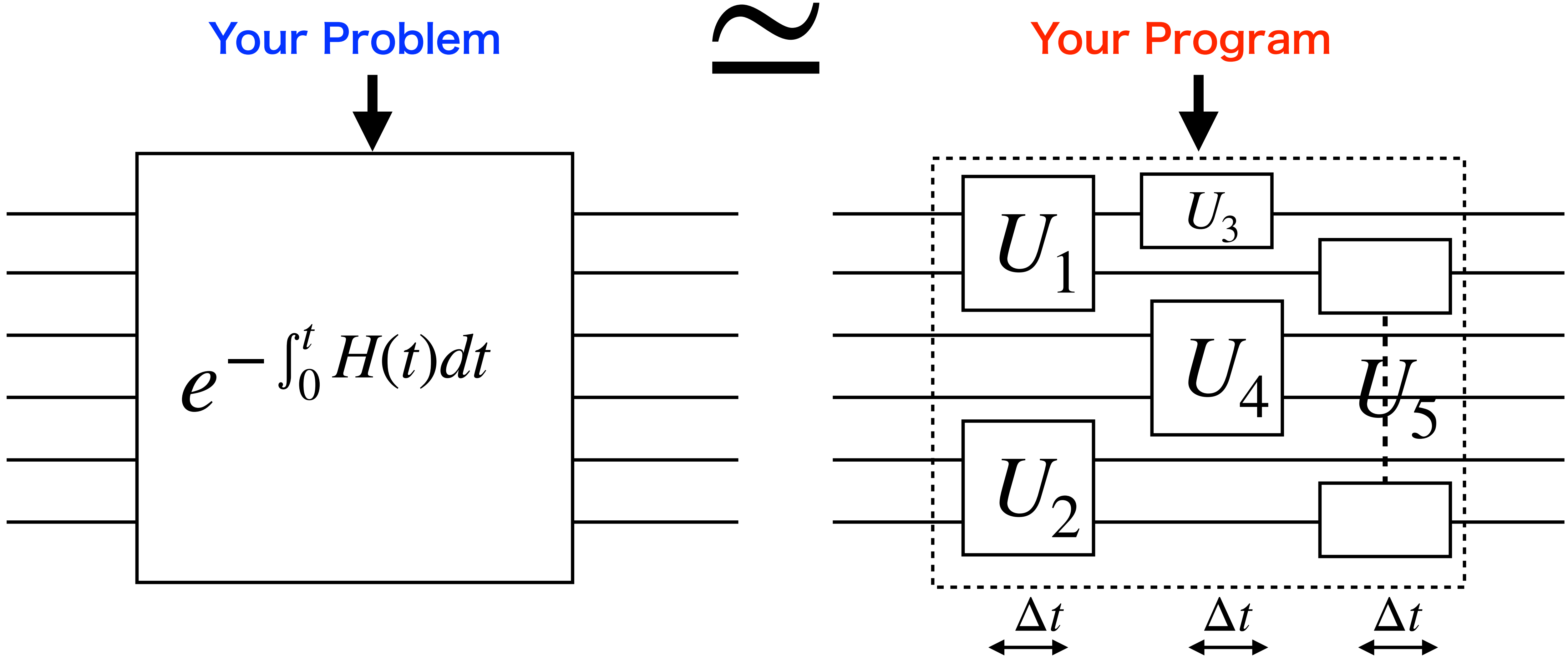
Application to QFT

How to Implement Your Problem

1. Discretize your problem

2. Write a spin Hamiltonian

$$H_{spin} = \sum_{i=1}^{N} h_i^X X_i + h_i^Y Y_i + h_i^Z Z_i + \sum_{i,j} J_{ij}^{XX} X_i X_j + J_{ij}^{XY} X_i Y_j + J_{ij}^{XZ} X_i Z_j + \cdots$$



Quantum Computation for QED

“Real-time Dynamics of Chern-Simons Fluctuations near a critical point”, PRD (2021)

with Dmitri Kharzeev & Yuta Kikuchi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Real-time dynamics of Chern-Simons fluctuations near a critical point

Kazuki Ikeda, Dmitri E. Kharzeev, and Yuta Kikuchi

Phys. Rev. D **103**, L071502 – Published 21 April 2021

Work in progress with Frenklakh, Kharzeev, Korepin, Shi (Stony Brook) & Florio, Yu (BNL)

arXiv: 2211.XXXX (2022)

Quantum Electric Dynamics in 1+1 d

$$S = \int \mathrm{d}^2 x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g\theta}{4\pi} \epsilon^{\mu\nu} F_{\mu\nu} + \bar{\psi}(\mathrm{i}\not{D} - m)\psi \right]$$

Field strength (Gauge boson) Dirac fermion

Similarities to QCD in 3+1 d

- Confinement
- Chiral symmetry breaking
- CP violation
- Vacuum decay by external magnetic field (Schwinger effect)

Schwinger Model = QED in 1+1 d

$$S = \int \mathrm{d}^2x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g\theta}{4\pi} \epsilon^{\mu\nu} F_{\mu\nu} + \bar{\psi}(\mathrm{i}\not{D} - m)\psi \right]$$

(we put θ =0 in this talk)

Construction of the Spin Hamiltonian

1. Derive the Hamiltonian on Lattice

2. Use Jordan-Winger Transformation

$$\mathrm{e}^{-\mathrm{i}H\epsilon} \approx \mathrm{e}^{-\mathrm{i}H_Z\epsilon} \mathrm{e}^{-\mathrm{i}H_{XX}\epsilon} \mathrm{e}^{-\mathrm{i}H_{YY}\epsilon} \mathrm{e}^{-\mathrm{i}H_{ZZ}\epsilon}$$

Suzuki-Trotter decomposition

Quantum circuits

Schwinger model on lattice (staggers fermion)

$$H = \frac{ag^2}{2}\sum_{n=1}^{N-1}\left[\sum_{i=1}^{n}\left(\chi_i^\dagger\chi_i - \frac{1-(-1)^i}{2}\right)\right]^2 - \frac{\mathrm{i}}{2a}\sum_{n=1}^{N-1}\left[\chi_{n+1}^\dagger\chi_n - \chi_n^\dagger\chi_{n+1}\right] + m\sum_{n=1}^{N}(-1)^n\chi_n^\dagger\chi_n$$

Electric field op satisfying the Gauss law $L_n = \sum_{i=1}^{n}\left(\chi_i^\dagger\chi_i - \frac{1-(-1)^i}{2}\right)$

Jordan-Wigner transformation

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\prod_{i=1}^{n-1}(-\mathrm{i}Z_i),$$

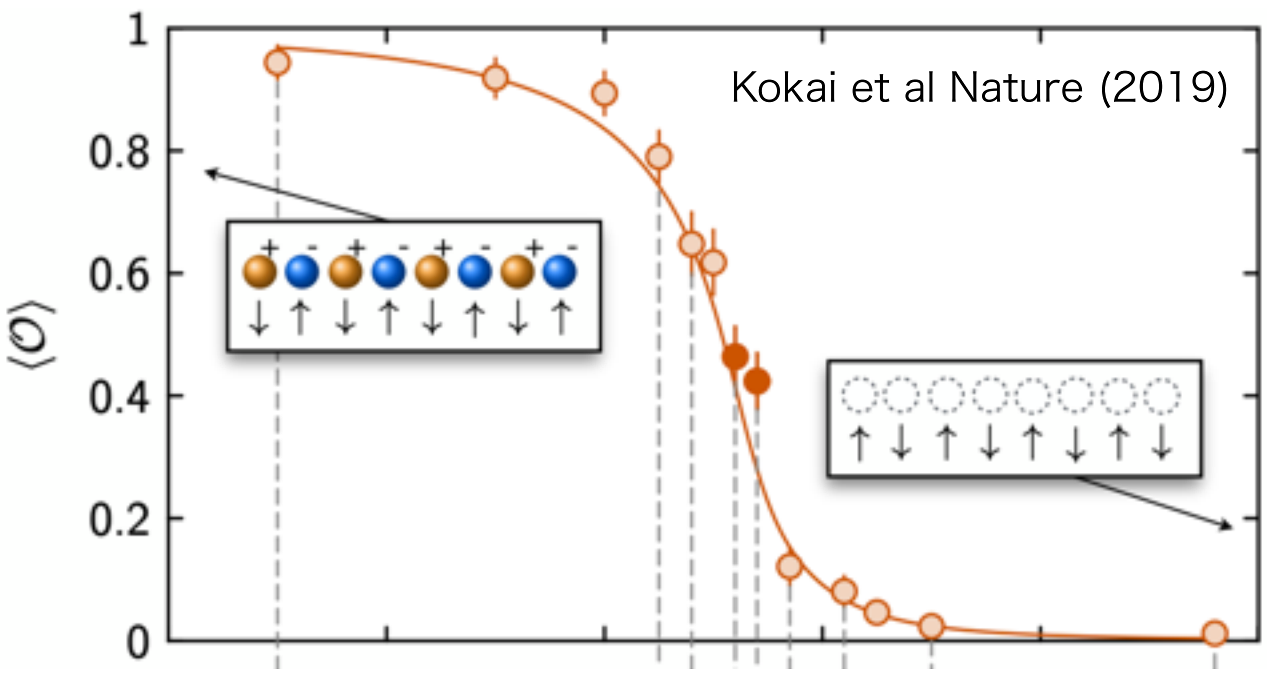
$$\chi_n^\dagger = \frac{X_n + \mathrm{i}Y_n}{2}\prod_{i=1}^{n-1}(\mathrm{i}Z_i),$$

Spin representation of Schwinger model

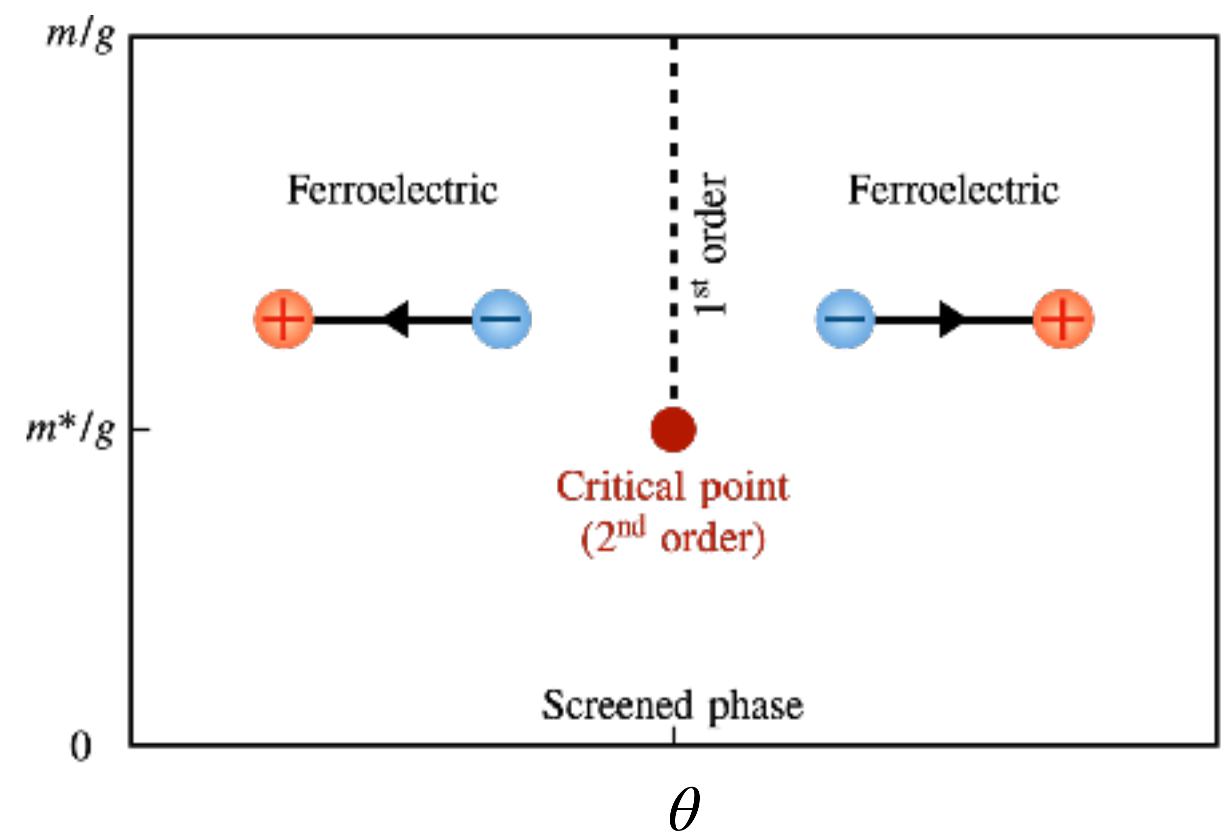
$$H = \frac{1}{4a}\sum_{n=1}^{N-1}(X_nX_{n+1} + Y_nY_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{ag^2}{2}\sum_{n=1}^{N}\left(\sum_{i=1}^{n}\frac{Z_i + (-1)^i}{2}\right)^2$$

$$L_n = \sum_{i=1}^{n}\frac{Z_i + (-1)^i}{2}$$

Ground state phase transition of the Schwinger model



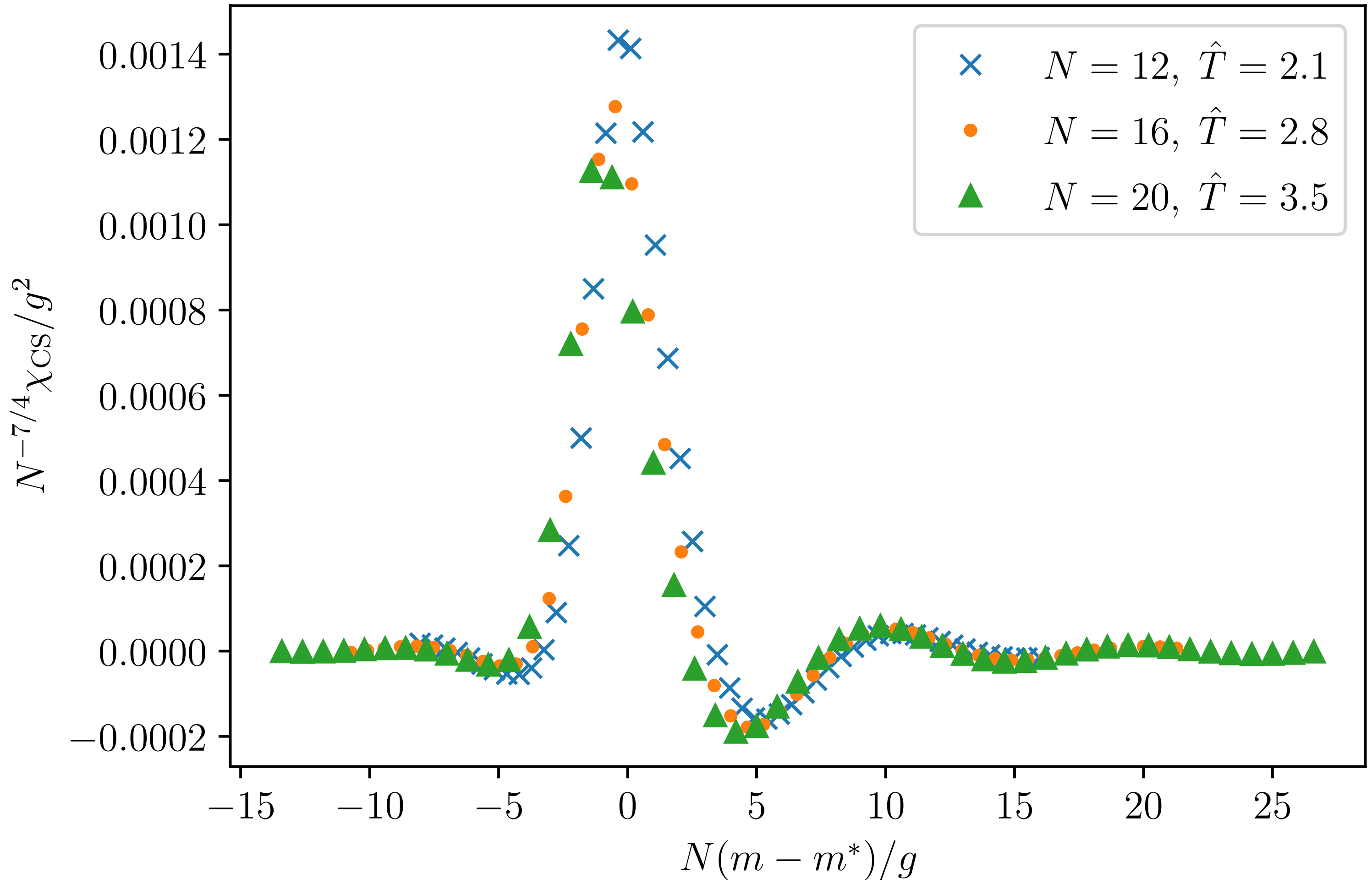
$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{2N(N-1)} \sum_{i,j>i} \langle (1 + (-1)^i \hat{\sigma}_i^z)(1 + (-1)^j \hat{\sigma}_j^z) \rangle$$



Coleman (1976) noticed that there is the 2nd order critical point, belonging to the universality class of the 1+1d transverse Ising model.

Real-time Topological Susceptibility

$$\frac{\chi_{CS}}{g^2} = \frac{N-1}{\pi^2} Re \int_0^{\hat{T}} d\hat{t}(t)(\langle \bar{L}(t)\bar{L}(0)\rangle - \langle \bar{L}(0)\rangle^2) \qquad \hat{t} := (ag^2/2)t,\ \hat{T} := (ag^2/2)T$$



Scaling rule

$m \sim m_*$

$\tilde{\chi}(m; N) \approx N^{\gamma/\nu} Q(N/\xi),$

$Q(x) \approx A(\xi_0 x)^{-\gamma/\nu}$

$\gamma = \frac{7}{4} \qquad \nu = 1.$

Beyond Qubits

arXiv > math > arXiv:2210.05133

Mathematics > Operator Algebras

(Submitted on 11 Oct 2022)

Quantum Fibrations: quantum computation on an arbitrary topological space

Kazuki Ikeda

Beyond Qubits

Feynman 1981 at 1st Symposium on Physics and Computation

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical. If you want to make a simulation of nature you'd better make it quantum mechanical"

Can we perform Feynman path integral with a quantum computer?

→ **Maybe NO, in general**

How to Implement Your Problem

1. Discretize your problem

Extremely non-trivial unless it is a discrete problem

Even if it is discrete, not easy to solve (cf: Monte Carlo)

2. Write a spin Hamiltonian

Inefficient for bosons

How to create a model of quantum computation stranger than BQP?

Freedman, Kitaev, Wan (2002)

TQFT is not stronger than BQP

What about using
- String Theory ?
- AQFT ?

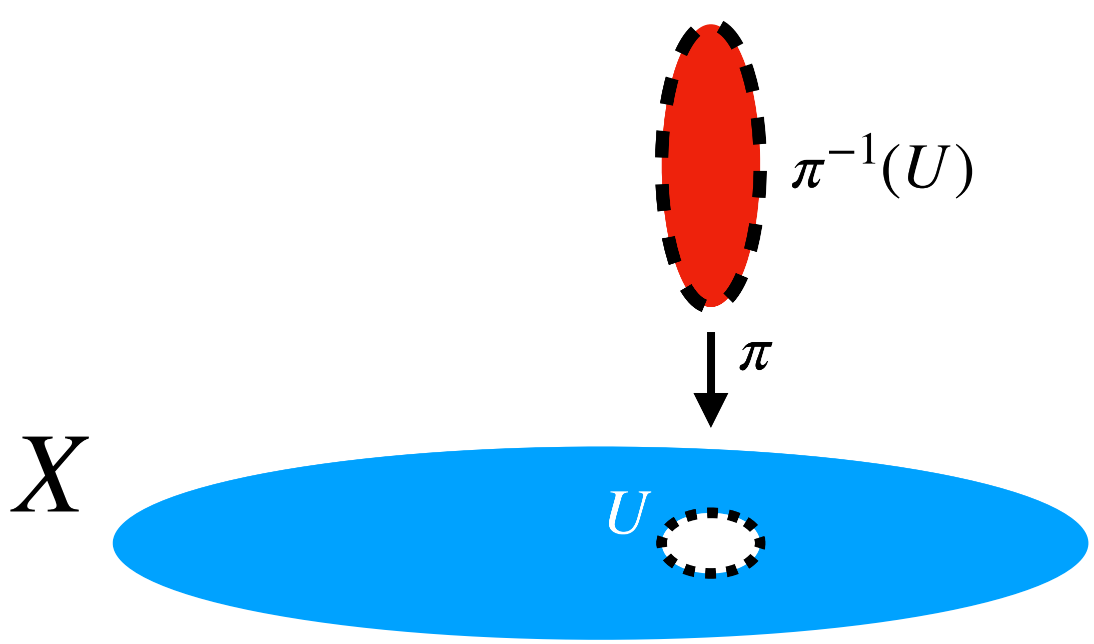
naive motivation

What could be a general mathematical framework that addresses any quantum theory?

X & $\mathscr{F}$: Topological Spaces

$\pi : \mathscr{F} \to X$: Continuous map s.t.

$\pi^{-1}(U)$ is a set of quantum states for each open U of X.

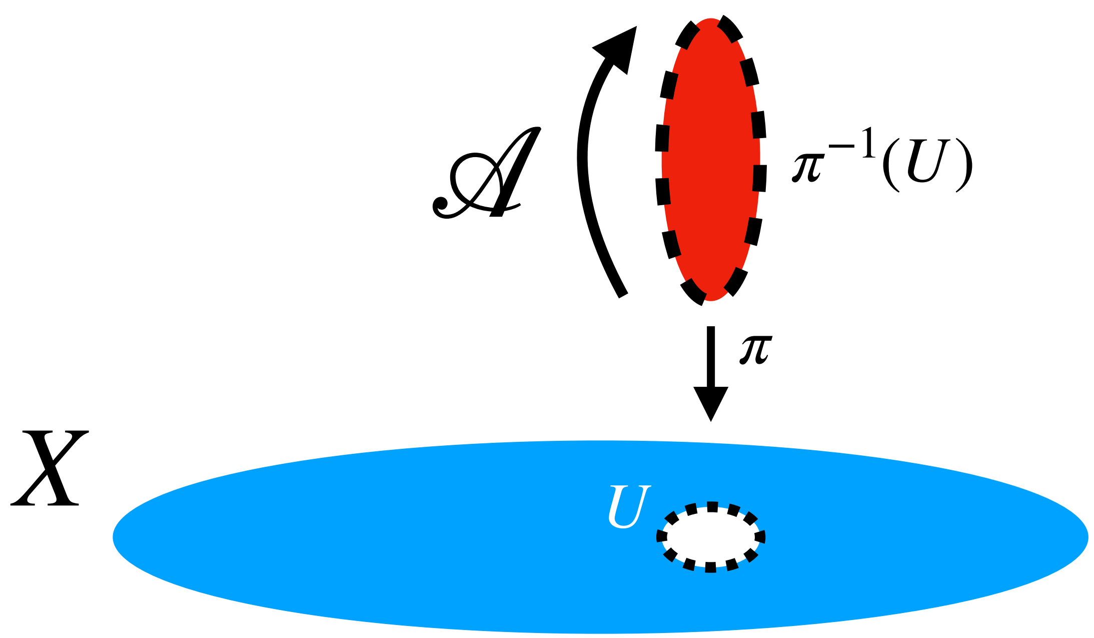


How to define operation on $(\mathscr{F}, \pi, X)$?

$B(\mathscr{H}_U)$: the set of all bounded operators on U

$\mathscr{A} \subset B(\mathscr{H}_U)$: a von Neumann algebra on U

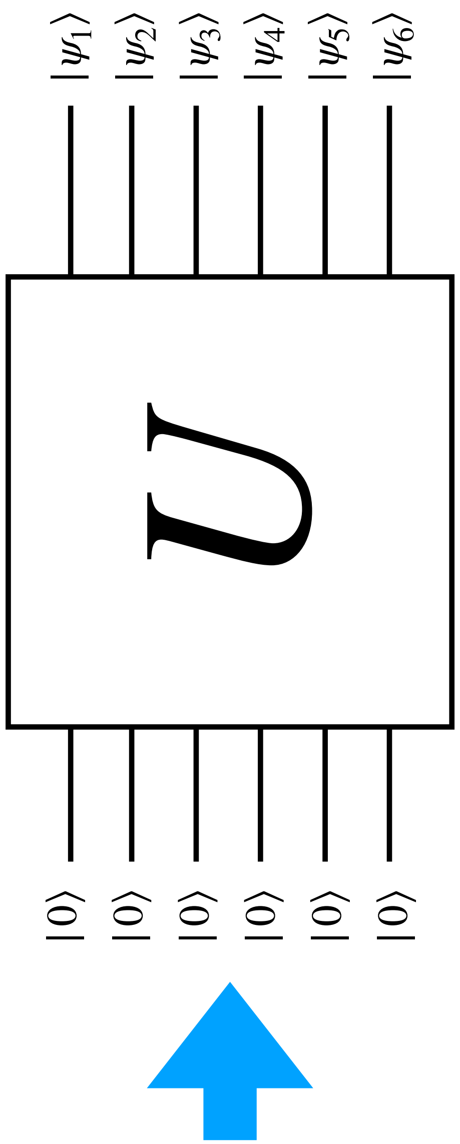
$D(\mathscr{H}_U)$: the set of all density operators on U



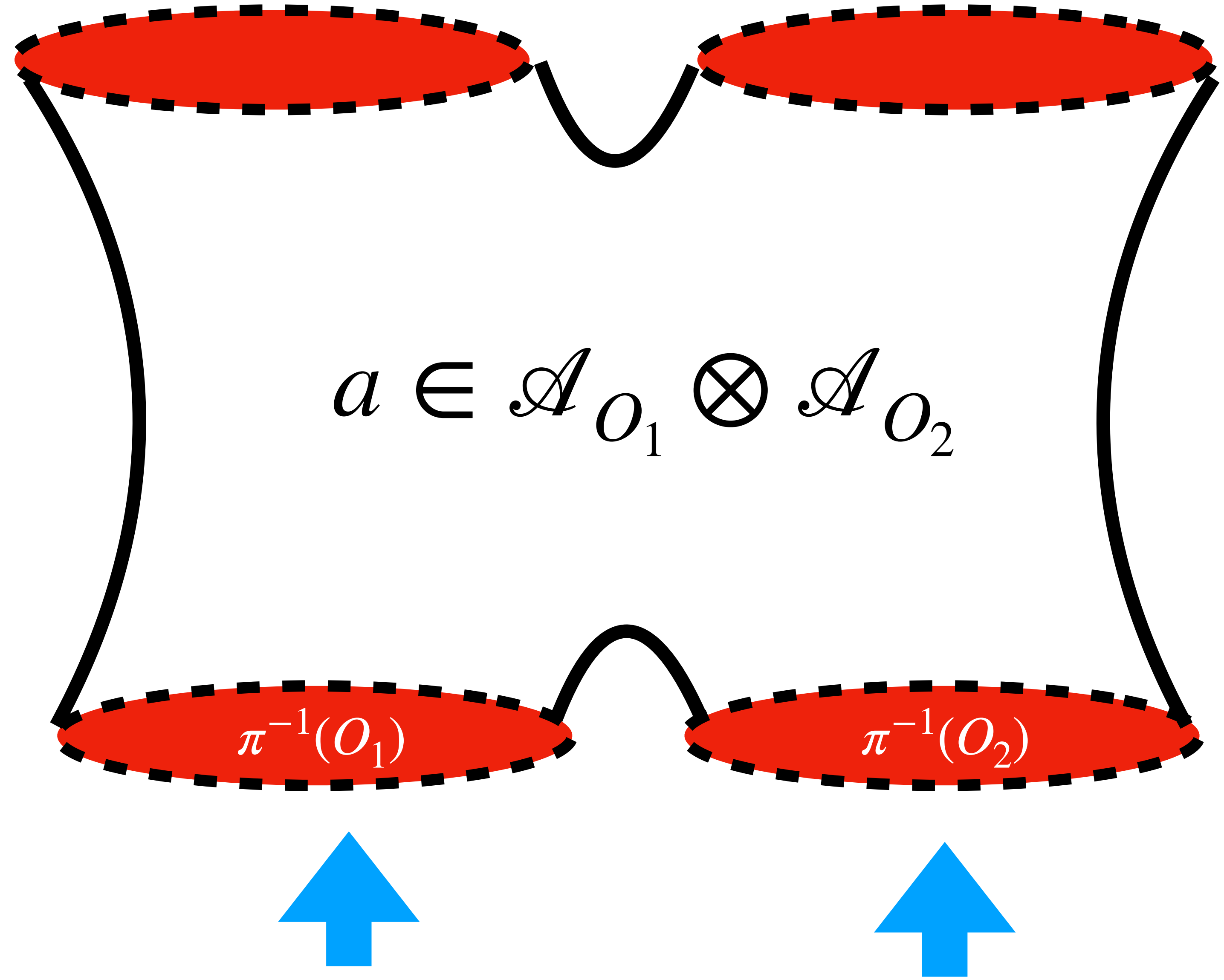
Operation of $\mathscr{A}$

$$D(\mathscr{H}_U) \ni \rho \mapsto a\rho a^\dagger \in D(\mathscr{H}_U), \quad a \in \mathscr{A}$$

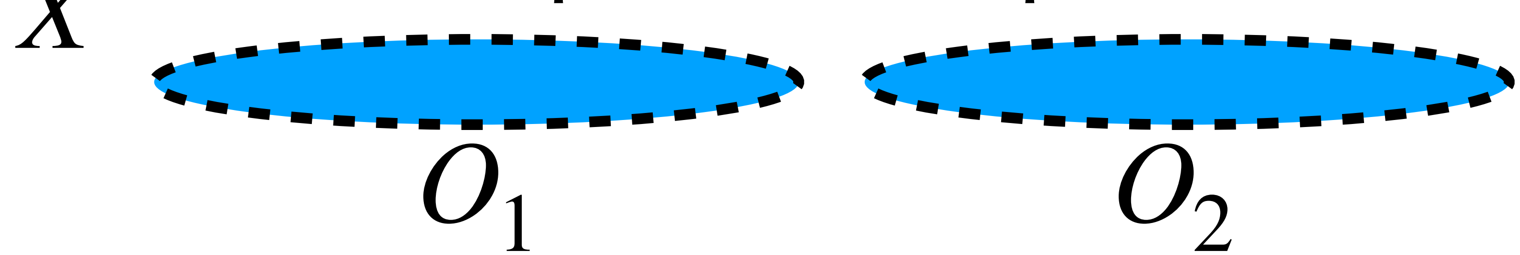
Comparison with Quantum Computation



Discrete inputs on graph



Open sets of inputs

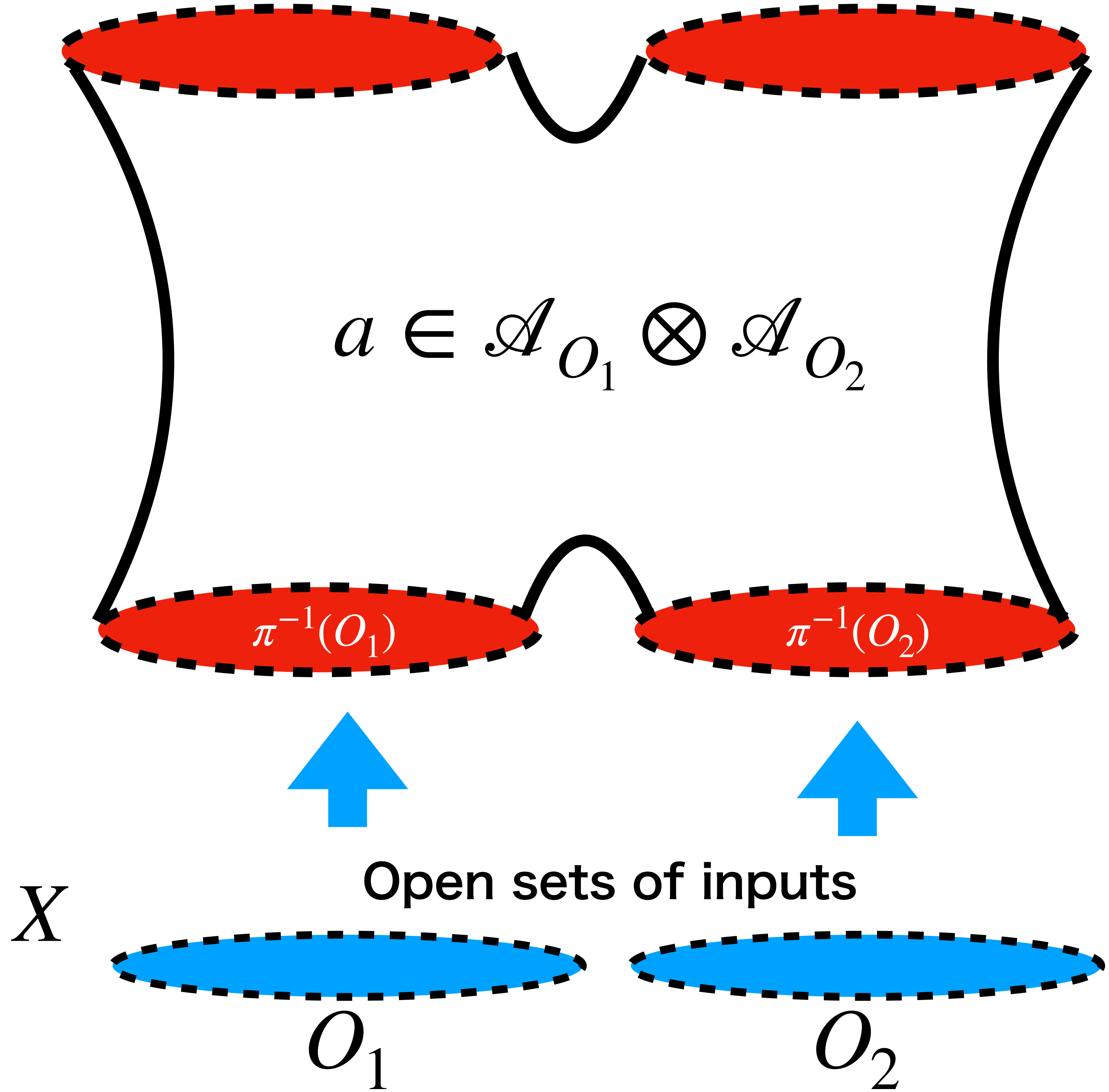


Comparison with Quantum Computation

Applicable to bounded parts

- String Theory ?
- AQFT ?

Perfectly applicable!!



Summary and Outlook

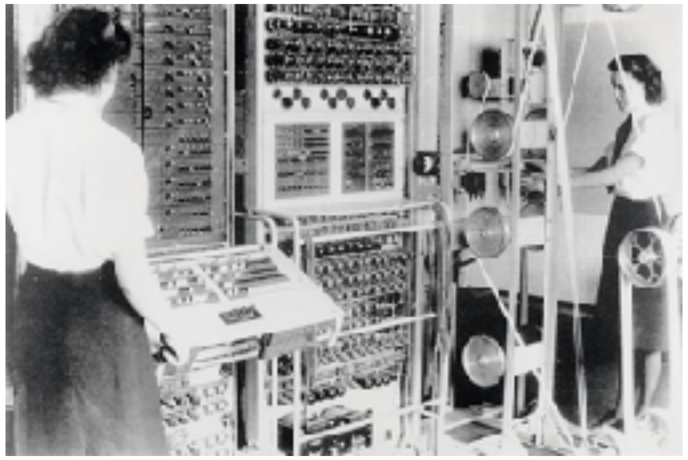
Quantum information communication has established advantages over classical communication.

For discretizable problems, quantum computation may work well.

To successfully handle general problems, quantum computation should be generalized. (I gave a general formulation)

Homework for People 100 Years Ahead:

Implement von Neumann algebras on a space of exponential memory.



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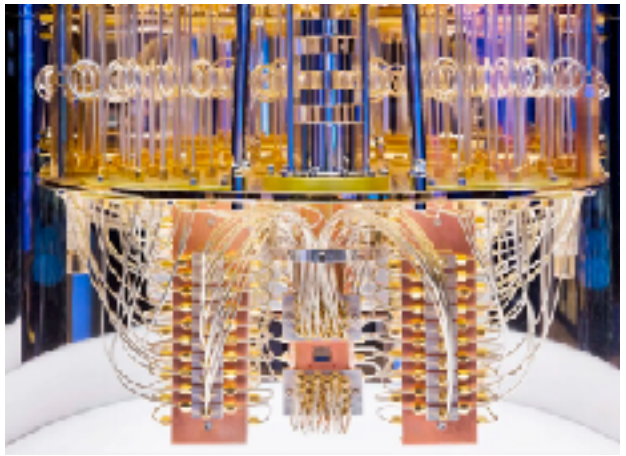

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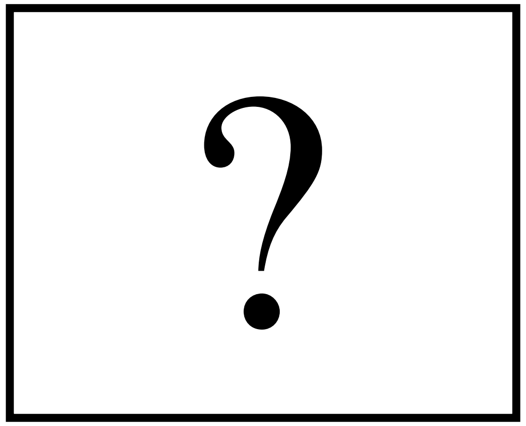

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