Center for Frontiers in Nuclear Science

EICUG 2nd Detector Meeting

December 6-8, 2022

TMDs Multiparticle Final States

The outline of my talk:

- EIC: Lepton-hadron collisions and their advantages and challenges
- TMDs: Collision effect vs. the true information on the 3D hadron structure
- Ideas for overcoming the challenges
- Summary and outlook

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Jefferson Lab

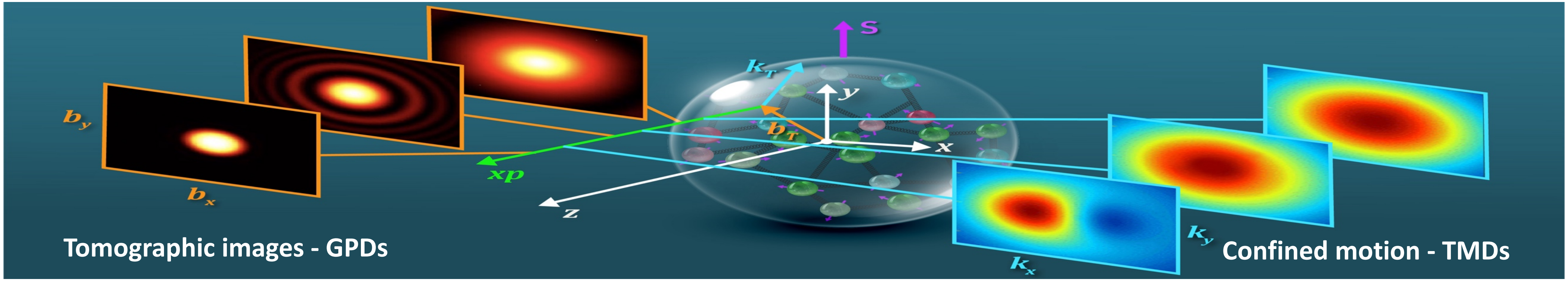
TMD Collaboration

U.S. Department of Energy | Office of Science

JSA

TMDs: 3D imaging in momentum space

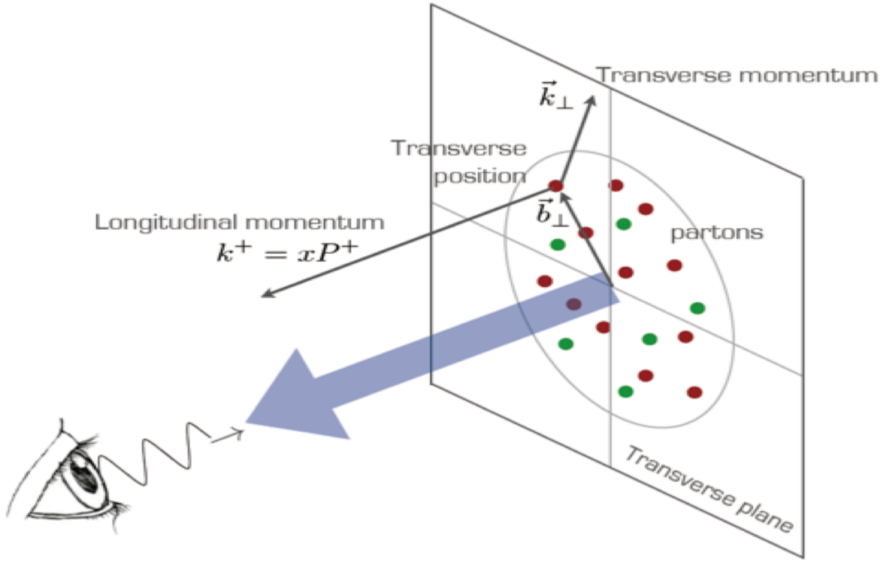
NO quarks and gluons can be seen in isolation!



- **Need new observables with two distinctive scales:**

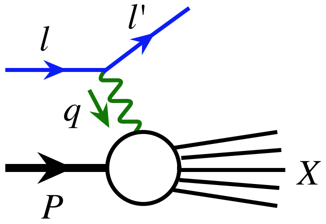
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

  - **Hard scale:** Q_1 **to localize the probe to see the particle nature of quarks/gluons**
  - **"Soft" scale:** Q_2 **could be more sensitive to the hadron structure ~ 1/fm**

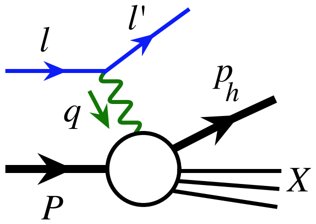


- **SIDIS:** $e(\ell) + N(p) \to e(\ell') + h(p_h) + X$ with $Q^2 = -(\ell - \ell')^2$
  - **Provide a natural two-scale process:** $Q^2 \gg p_{hT}^2$ **in photon-hadron frame**

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

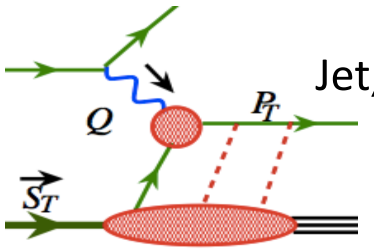


Scale: Q^2 **- PDFs**



$Q^2 \gg P_{h_T}^2$

In photon-hadron frame!

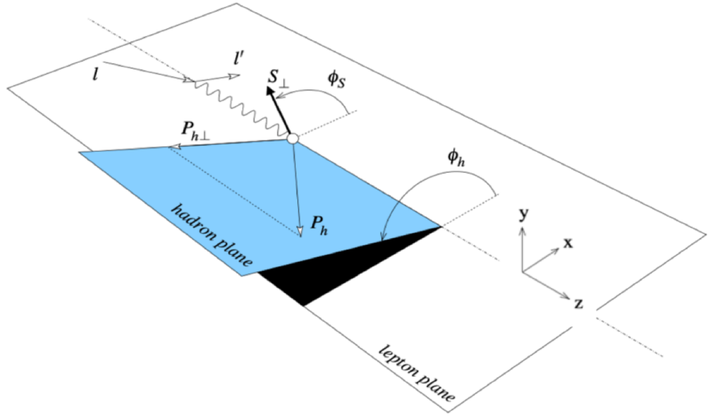


Jet, π, J/ψ, …

$f(x, k_T, Q)$ **- TMDs**

Parton's confined motion, …

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&\quad + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\quad + S_\parallel\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big] \\
&\quad + S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big] \\
&\quad + |\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) \\
&\qquad + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&\quad + |\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$



18 SIDIS
Structure Functions

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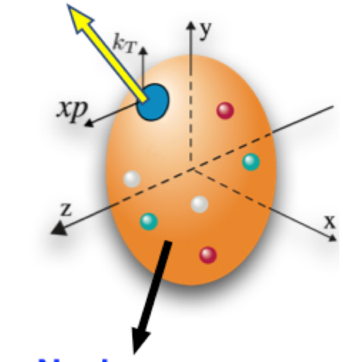
Transverse momentum dependent PDFs (TMDs)

❑ Quark TMDs with polarization:

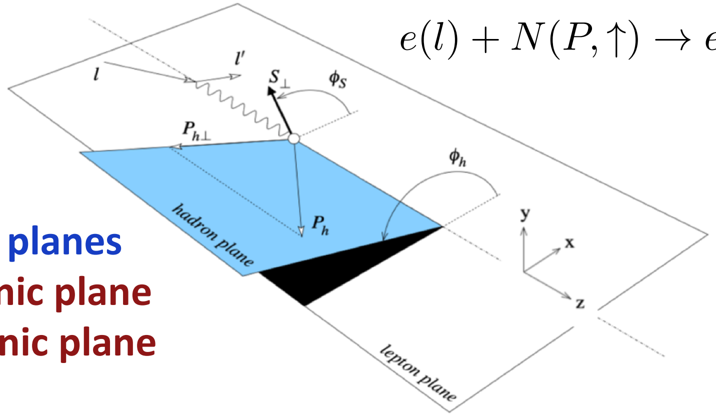
Nucleon Polarization		Quark Polarization: Unpolarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
	U	$f_1(x,k_T^2)$		$h_1^\perp(x,k_T^2)$ - *Boer-Mulders*
	L		$g_1(x,k_T^2)$ - *Helicity*	$h_{1L}^\perp(x,k_T^2)$ - *Long-Transversity*
	T	$f_{1T}^\perp(x,k_T^2)$ - *Sivers*	$g_{1T}(x,k_T^2)$ - *Trans-Helicity*	$h_1(x,k_T^2)$ - *Transversity* $h_{1T}^\perp(x,k_T^2)$ - *Pretzelosity*



Analogous tables for:
- **Gluons** $f_1 \rightarrow f_1^g$ etc
- **Fragmentation functions**
- **Nuclear targets** $S \neq \frac{1}{2}$

❑ Polarized SIDIS:

$$e(l) + N(P,\uparrow) \rightarrow e(l') + h(P_h) + X$$



Two planes
Leptonic plane
Hadronic plane

Single Transverse-Spin Asymmetry

$$A_{UT} = \frac{1}{P}\frac{\sigma_{lN(\uparrow)} - \sigma_{lN(\downarrow)}}{\sigma_{lN(\uparrow)} + \sigma_{lN(\downarrow)}}$$

In photon-hadron frame:

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^\perp$$

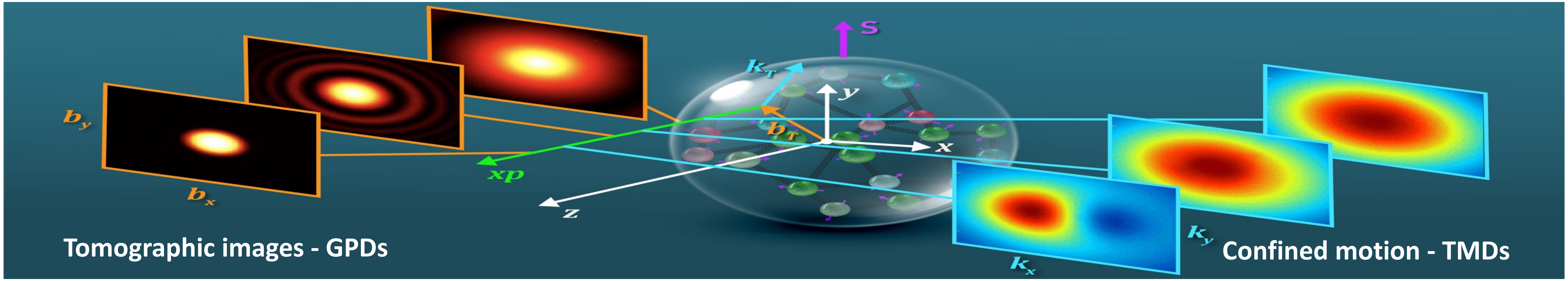
$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$

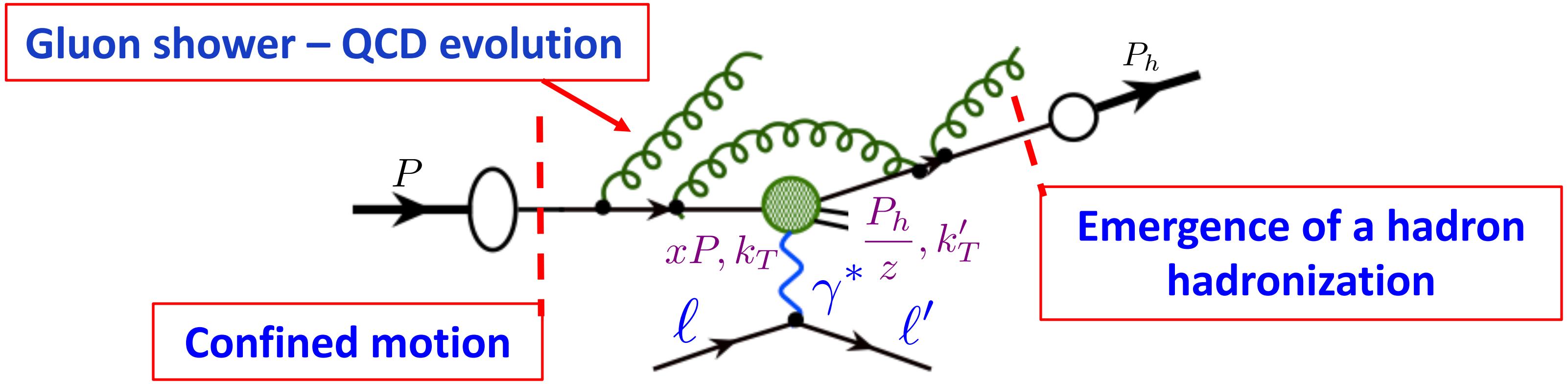
Angular modulation provides the best way to separate TMDs

TMDs: 3D imaging in momentum space

NO quarks and gluons can be seen in isolation!



If the proton is broken, e.g., in SIDIS, …



Transverse momentum broadening:

$$\Delta k_T^2 \propto \Lambda_{\rm QCD}^2 \left.\begin{array}{l} \times\ \alpha_s(C_F, C_A) \\ \times\ \log(Q^2/\Lambda_{\rm QCD}^2) \\ \times\ \log(s/Q^2) \end{array}\right\} \gtrsim 1$$

Structure information is diluted by the collision induced shower!

- *Measured k_T is NOT the same as k_T of the confined motion!*
- *Structure information vs. collision effects – The Challenge One!*

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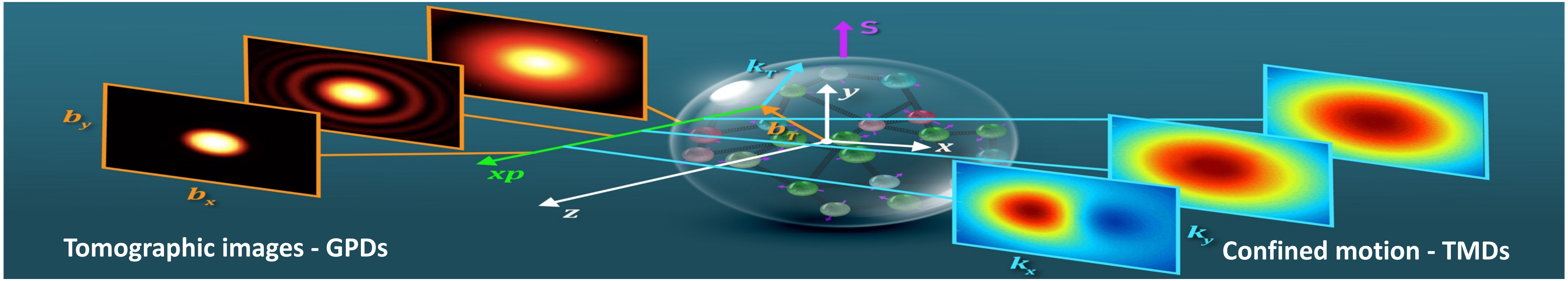
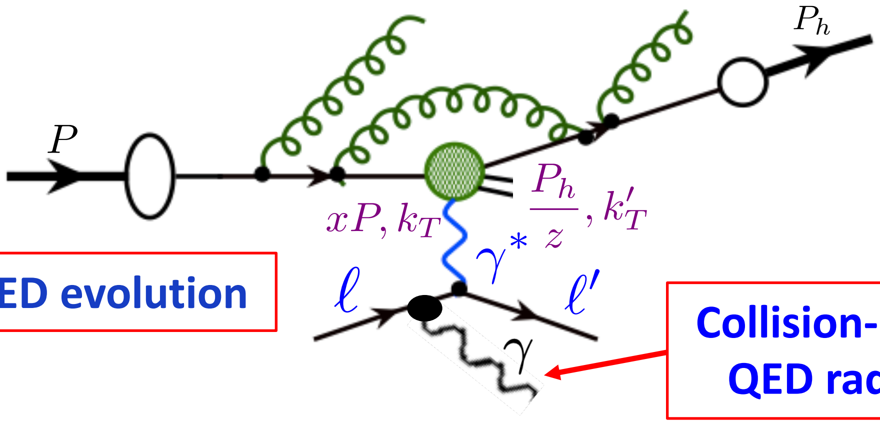
TMDs: 3D imaging in momentum space

NO quarks and gluons can be seen in isolation!



If the proton is broken, e.g., in SIDIS, with a large momentum transfer …



Photon shower – QED evolution

Collision-induced QED radiation

- *True kinematics of the probe – The Challenge Two!*

Without measuring radiated photon:

$$q_\mu \to \hat{q}_\mu$$

$$Q^2 = -q^2 \to \widehat{Q}^2 = -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P \cdot q} \to \hat{x}_B = \frac{\widehat{Q}^2}{2P \cdot \hat{q}}$$

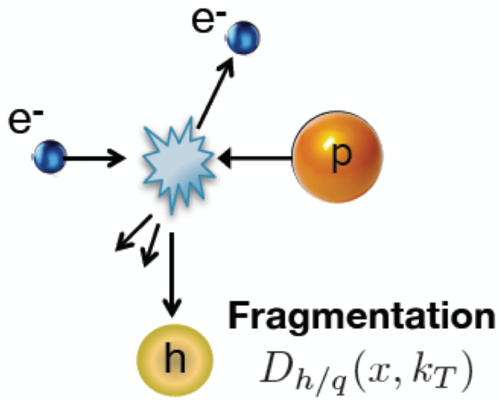
Ill-defined "photon-hadron" frame?!

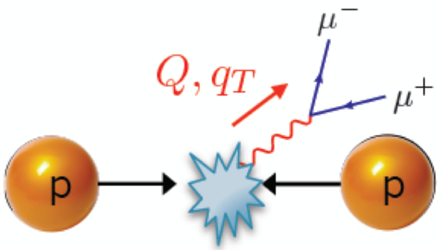
$$p_{hT} \leftarrow \hat{p}_{hT}$$

The challenge one: Gluon shower – QCD evolution

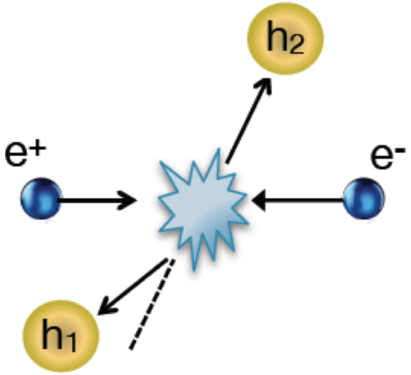
❑ The classical two-scale observables for TMDs:

Semi-Inclusive DIS	Drell-Yan	Dihadron in e+e-
$\sigma \sim f_{q/P}(x,k_T) D_{h/q}(x,k_T)$	$\sigma \sim f_{q/P}(x,k_T) f_{q/P}(x,k_T)$	$\sigma \sim D_{h_1/q}(x,k_T) D_{h_2/q}(x,k_T)$



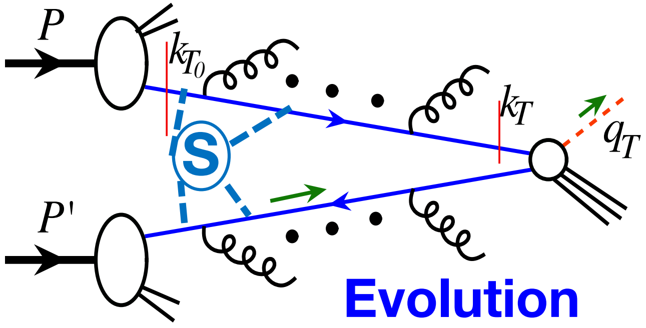


$q_T \ll Q$



Need all of them to separate TMD PDFs from TMD FFs!

❑ Lessens learned from Drell-Yan from a wide range of Q^2:

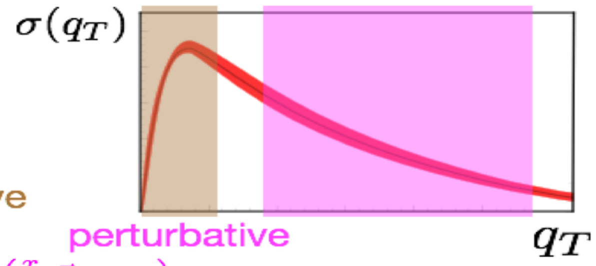


$$\sigma(q_T) = H(Q,\mu) \int d^2\vec{b}_T \, e^{i\vec{q}_T\cdot\vec{b}_T} \, f_q(x_a, \vec{b}_T, \mu, \zeta_a) \, f_q(x_b, \vec{b}_T, \mu, \zeta_b) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

Hard virtual corrections

$\updownarrow$ FT

$f_q(x_a, \vec{k}_T, \mu, \zeta_a)$



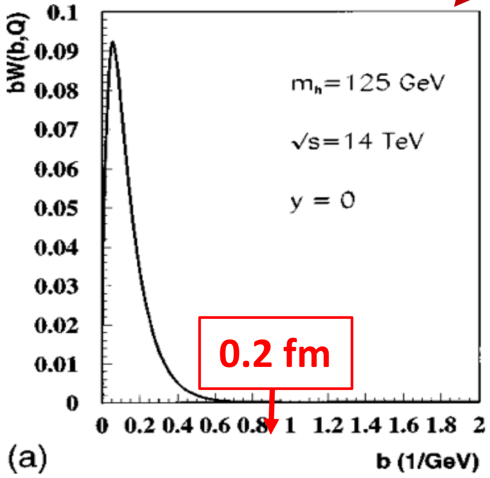
- $k_T \sim b_T^{-1} \sim \Lambda_{\rm QCD}$ $\quad f_q(x, \vec{k}_T)$ nonperturbative
- $k_T \sim b_T^{-1} \gg \Lambda_{\rm QCD}$ $\quad f_q(x, \vec{k}_T, \mu, \zeta) = \sum_i \int \frac{dy}{y} C_{qi}\left(\frac{x}{y}, \vec{k}_T, \mu, \zeta\right) f_i(x,\mu)$

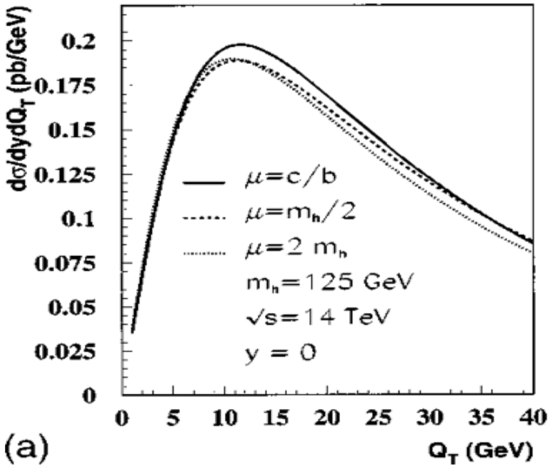
PDF

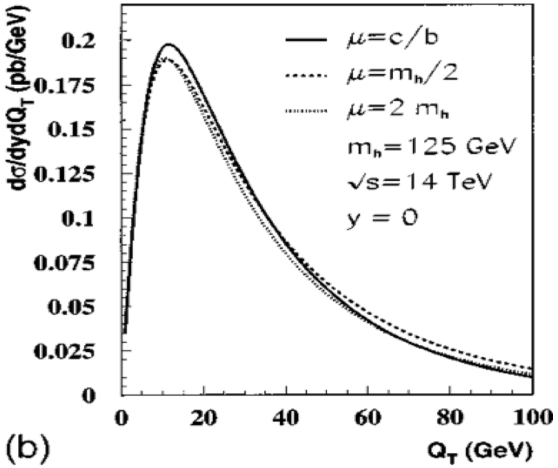
The challenge one: Gluon shower – QCD evolution

- **Lessens learned from Drell-Yan at a large Q^2:** *e.g., Higgs production at the LHC*

$$\frac{d\sigma(Q,q_T)}{dy\,dq_T} = \int_0^{\infty} \frac{db_T}{2\pi} \underbrace{b_T J_0(q_T b_T) H(Q,\mu) f(x_a,b_T,\mu,\zeta_a) f(x_b,b_T,\mu,\zeta_b)}_{} + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$



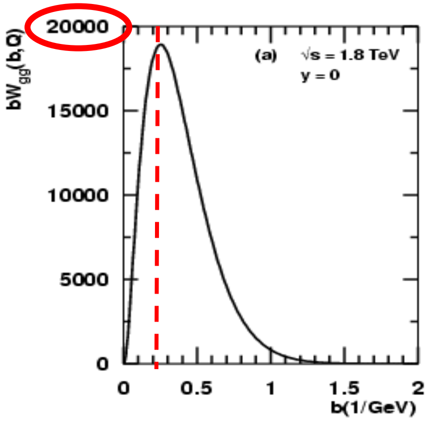


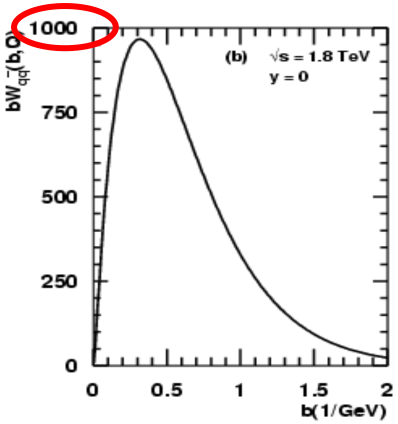


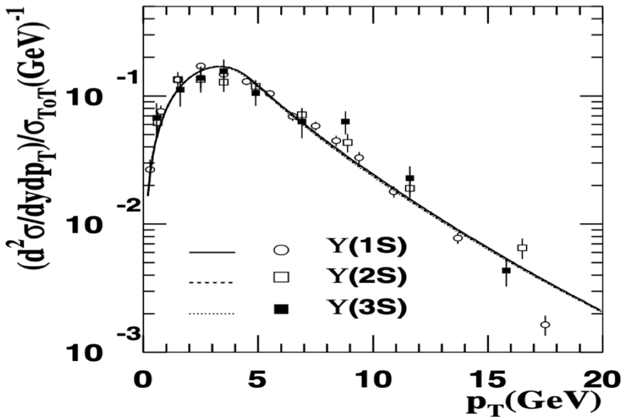
Berger, Qiu PRD67 (2003) 034026

Not sensitive to the structure other than PDFs!

- **Lessens learned from Drell-Yan at a lower Q^2:** *e.g., Upsilon production at the Tevatron*







Gluon shower generates k_T

Not much sensitive to the structure other than PDFs!

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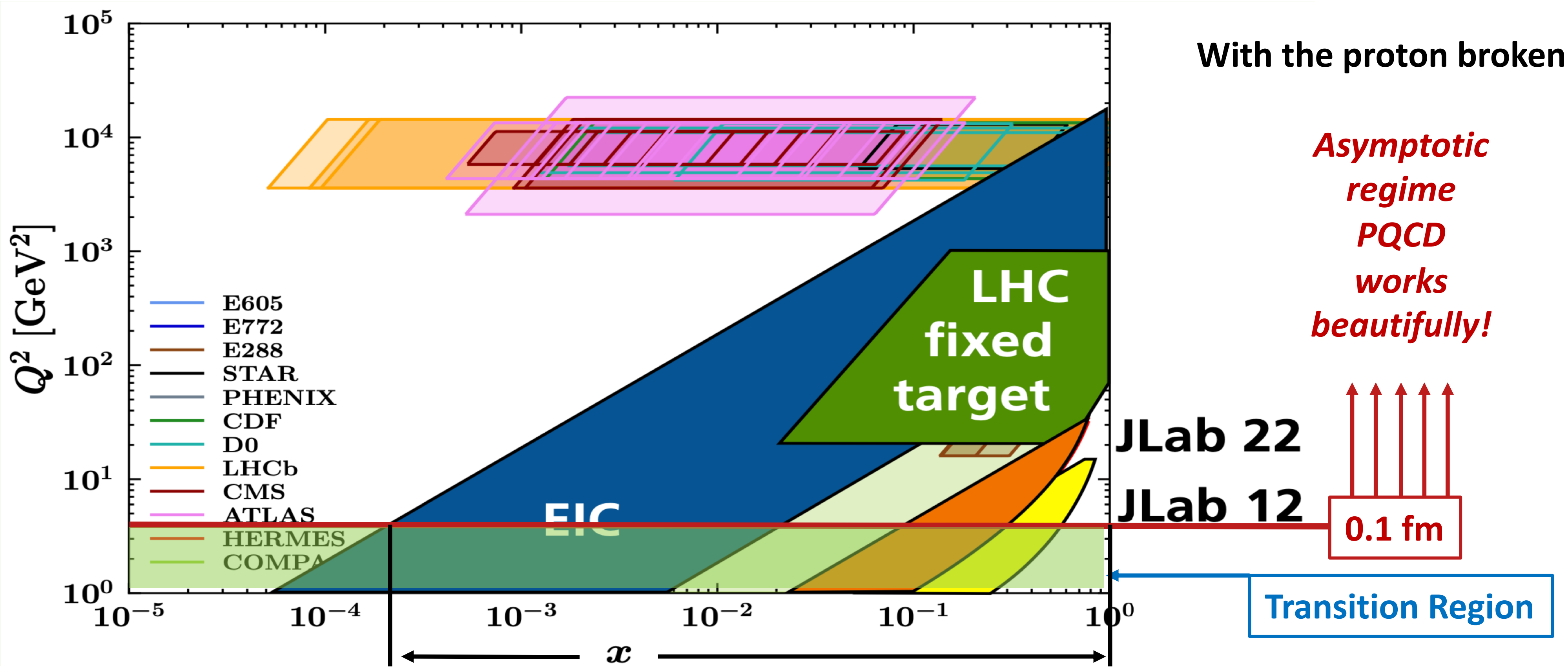
The challenge one: Gluon shower – QCD evolution

- Extracting hadron structure needs data with sufficiently large Q^2, but, not too large S

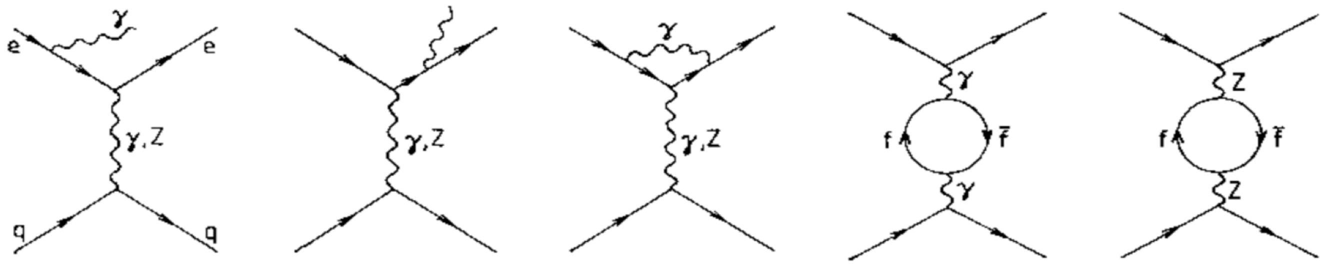


- Factorization is proved for a given Q^2, which is sufficiently large to neglect the power corrections
- Q^2-evolution is a perturbative QCD prediction and improvement
  – physically measured cross section does not depend on how it is factorized by the "theorists"

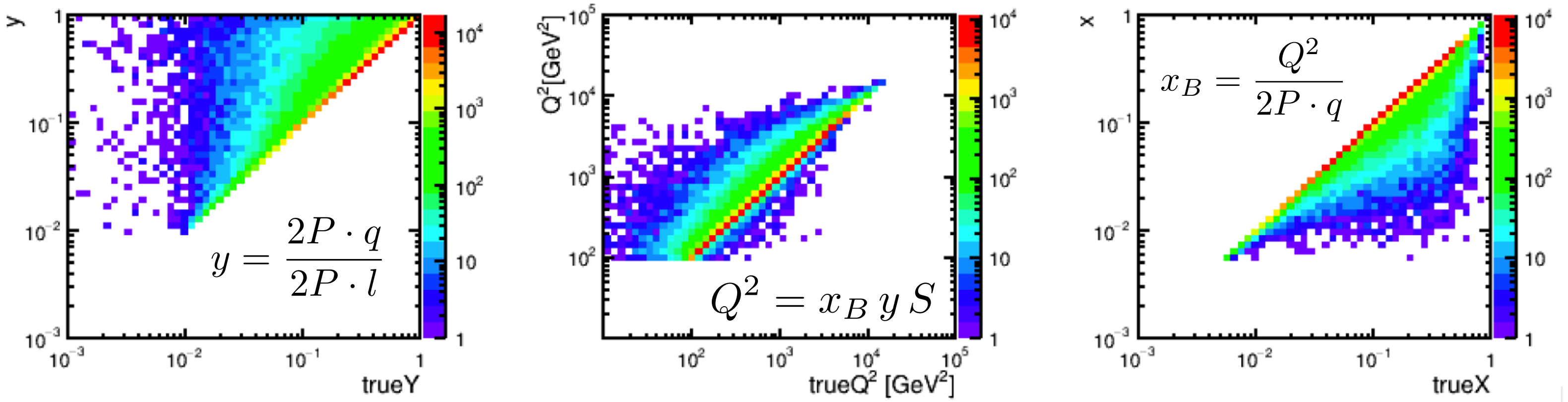
The challenge two: Photon shower – QED evolution

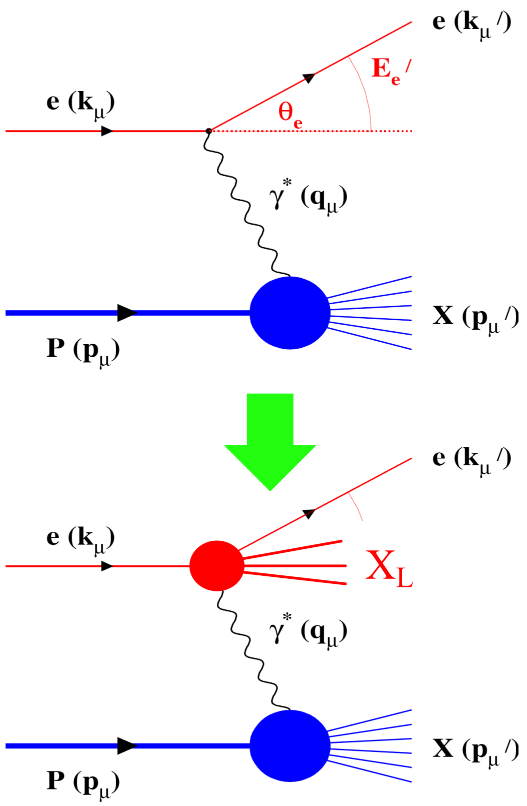
- **The "Probe" for the hadron is smeared by the induced QED radiation:**

Data sample : Int L = 10 fb^{-1}, Kinematics settings: 0.01<y<0.95, 10^2 GeV^2 <Q^2<10^5 GeV^2



initial **final** **vacuum** **loops**





Instead of a straight line – linear correlation,

the kinematic variables, y, Q^2 , x_B, from the leptons are smeared so much to make them different from what the scattered "quark" experienced!

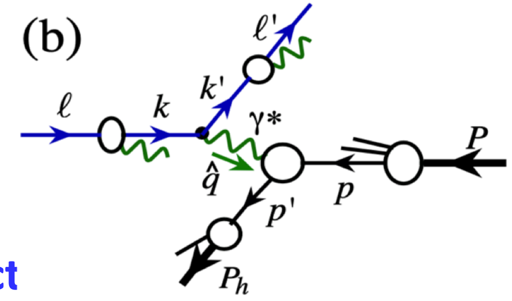
$$q_\mu \;\rightarrow\; \hat{q}_\mu$$

$$Q^2 = -q^2 \;\rightarrow\; \widehat{Q}^2 = -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \;\rightarrow\; \hat{x}_B = \frac{\widehat{Q}^2}{2P\cdot \hat{q}}$$

Ill-defined "photon-hadron" frame?!

No simple radiative correction for SIDIS



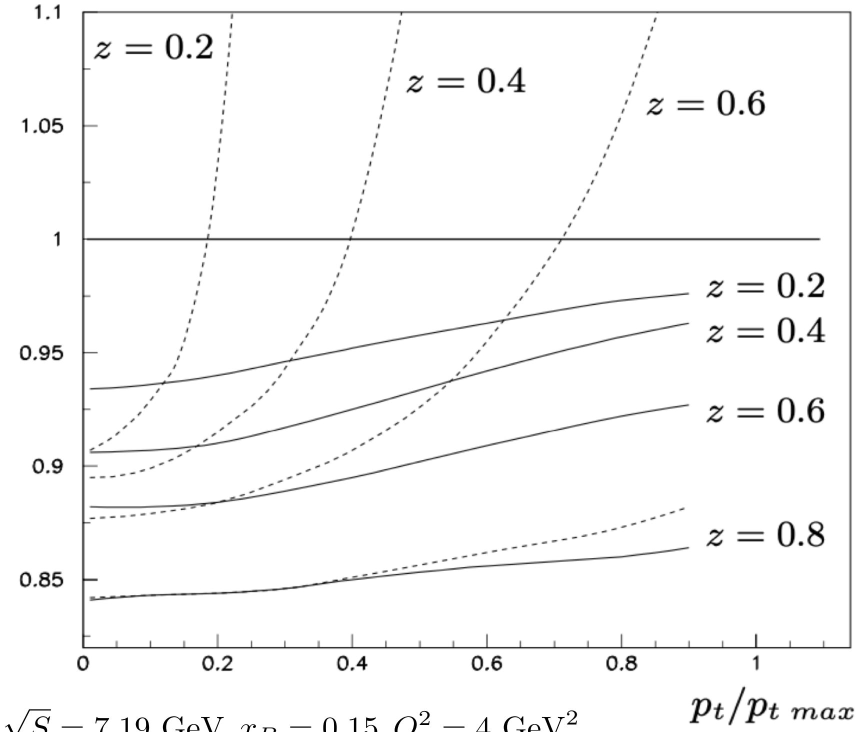
Radiative correction = keeping the Born kinematics:

$$\sigma_{\text{Measured}} \equiv \text{RC} \otimes \sigma_{\text{No QED Radiation}}$$

Necessary requirement: **RC – Radiative correction factor does not depend on the hadronic physics that we want to extract**

Impact of QED radiation to SIDIS – order of α_{EM}:

$$\bar{\delta} = \sigma_{\text{Measured}} / \sigma_{\text{No QED Radiation}}$$



$\sqrt{S} = 7.19$ GeV, $x_B = 0.15$, $Q^2 = 4$ GeV^2

$$e(l) + N(P) \to e'(l') + \gamma(k) + h(P_h) + X$$

I. Akushevich et al.
EPJ C10 (1999) 681

Dashed line:

Gaussian pT-dependence

$$b \exp(-b\, p_t^2) \qquad \text{where} \quad b = R^2/z^2$$

Solid line:

Power pT-dependence

$$\left[\frac{1}{a + b\,z + p_t^2}\right]^{c+d\,z} \qquad \text{parameters:} \quad R, a, b, c, d$$

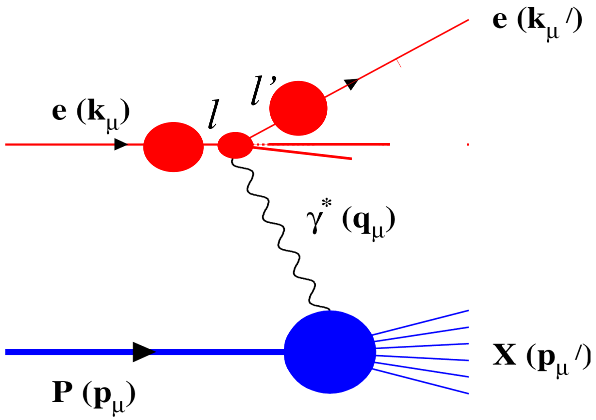
$\bar{\delta}$ **depends on physics we want to extract!**

NO simple RC for SIDIS!

Inclusive lepton-hadron deep inelastic scattering (DIS)

❑ Collinear factorization with the "one-photon" approximation:

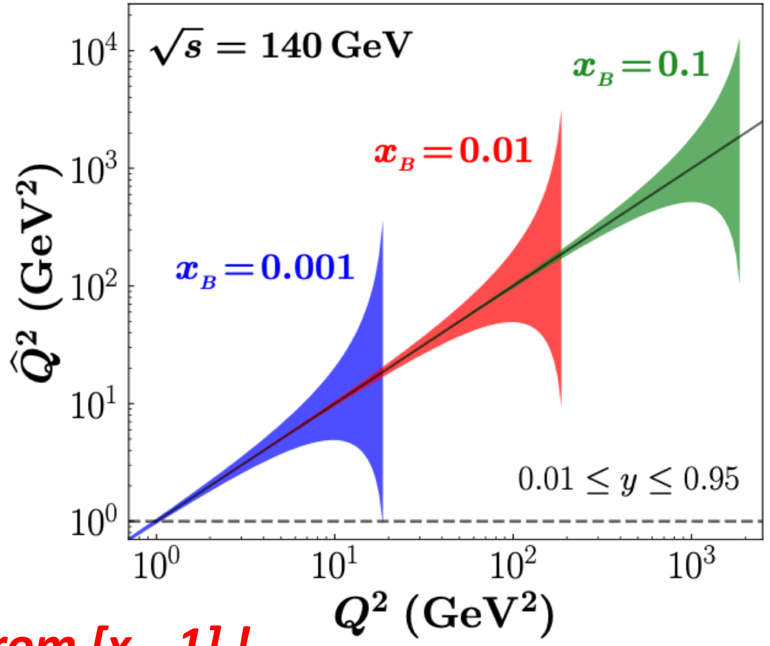
Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B dy} \approx \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^{1} d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right]$$

$$\times \frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2 F_1(\hat{x}_B,\widehat{Q}^2) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right) F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

- QED radiation prevents a well-defined "photon-hadron" frame
- Radiation is CO sensitive as $m_e/Q \to 0$, factorized into LDFs & LFFs
- Hadron is probed by $(x_B, Q^2) \to (\hat{x}_B, \widehat{Q}^2)$



$x_B \to \hat{x}_B \in [x_B, 1]$ $\qquad \widehat{Q}^2_{\min} = Q^2\,\dfrac{(1-y)}{(1-x_B\,y)}$ $\qquad \widehat{Q}^2_{\max} = Q^2\,\dfrac{1}{(1-y+x_B\,y)}$

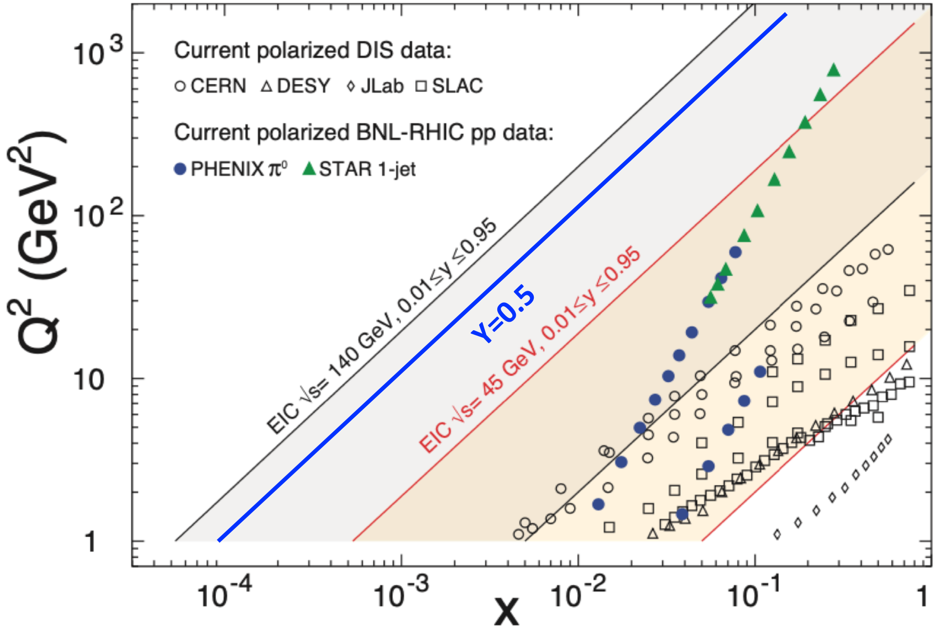
A simple RC factor at x_B is necessarily sensitive to hadronic information from [x_B, 1] !

Inclusive lepton-hadron deep inelastic scattering (DIS)

❑ Numerical impact of QED contribution at EIC ($\sqrt{S} = 140\ \text{GeV}$):

$$\frac{\sigma_{\text{noRC}}}{\sigma_{\text{RC}}} \leftrightarrow \frac{\sigma_{1\gamma}}{\sigma_{\text{measured}}} = \eta(x_B, y)$$

B. Badelek et al.
Z Phys C 66 (1995) 591



At $\sqrt{S} = 140\ \text{GeV}$

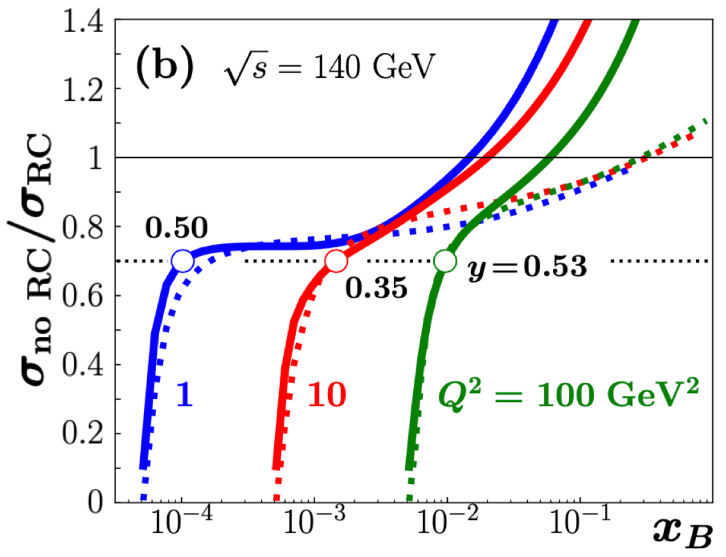
$$Q^2 = 1\ \text{GeV}^2$$

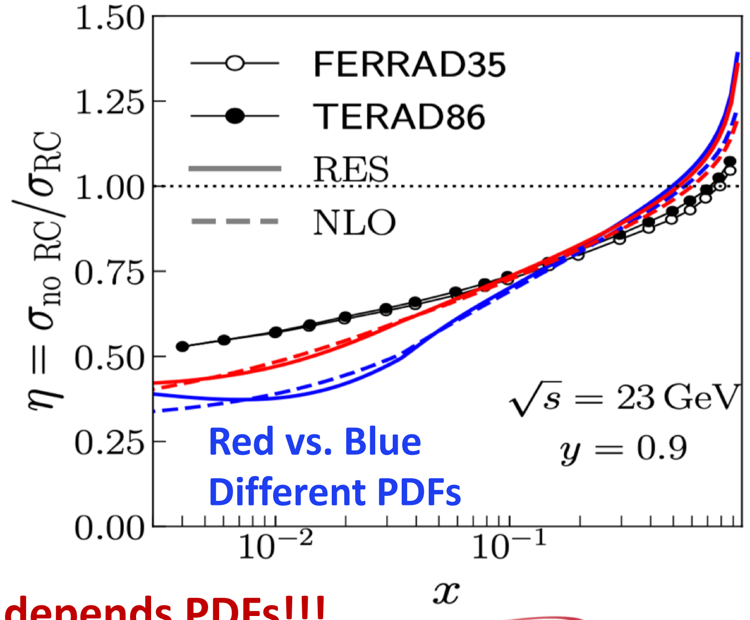
$$y = 0.95$$

EIC eP could reach:

$$x_{\min} \sim 5 \times 10^{-5}$$

$$Q^2 = x_B\, y\, S$$





If we do not have confidence for y > 0.5, due to QED radiation,

EIC's eP reach to small-x could be reduced to $x_{\min} \sim 1 \times 10^{-4}$

or effectively, $\boxed{\sqrt{S} = 140\ \text{GeV} \rightarrow 102\ \text{GeV at } y = 0.95}$

"RC" depends PDFs!!!

Inclusive lepton-hadron deep inelastic scattering (DIS)

❑ Numerical impact of QED contribution at EIC ($\sqrt{S} = 140\ \text{GeV}$):

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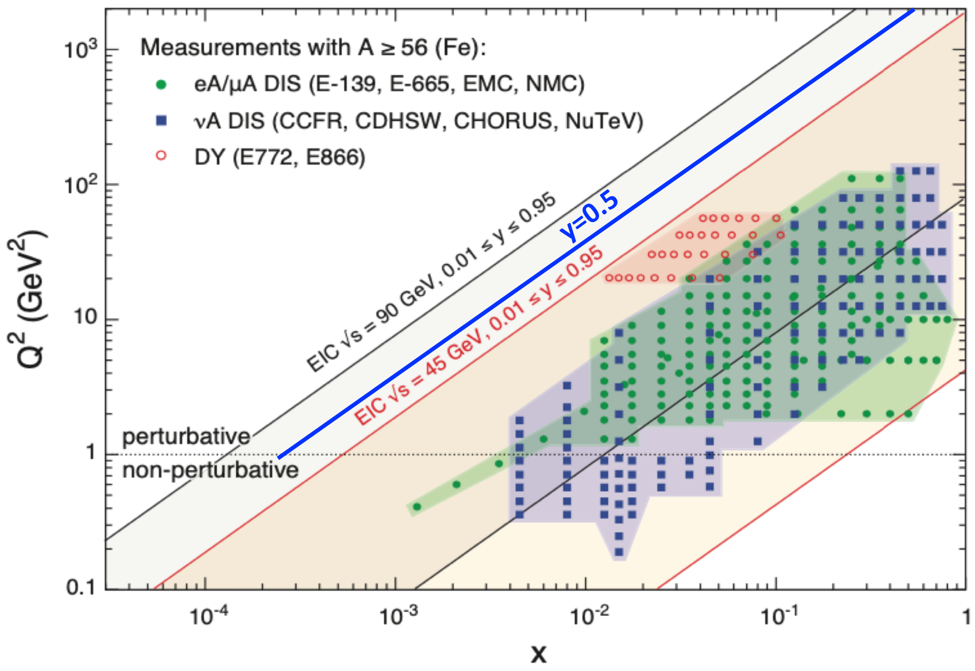


At $\sqrt{S} = 100\ \text{GeV}$

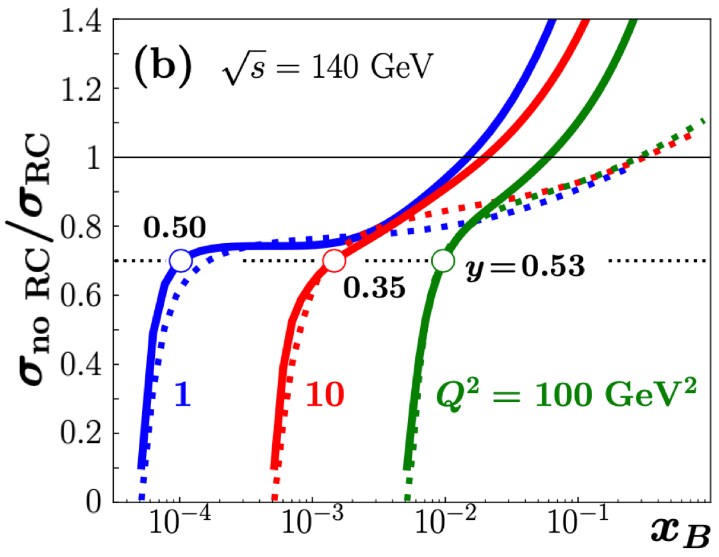
$Q^2 = 1\ \text{GeV}^2$

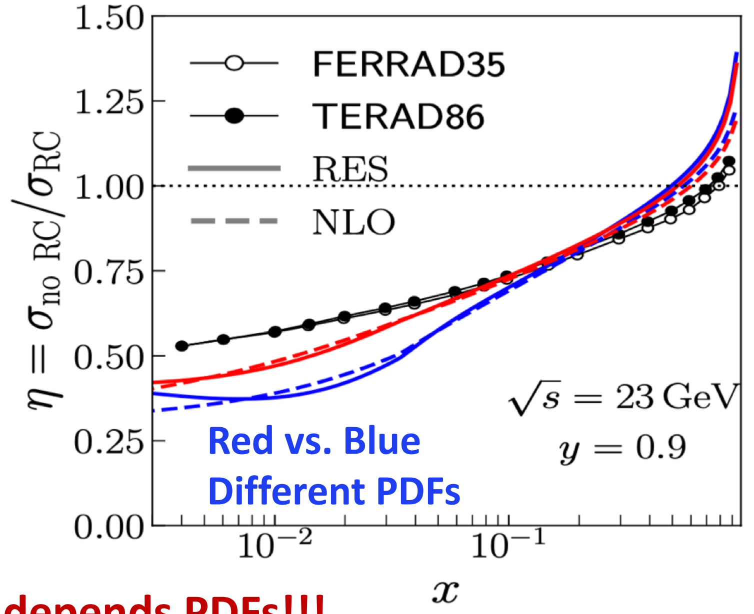
$y = 0.95$

EIC eA could reach:

$x_{\min} \sim 1 \times 10^{-4}$

$Q^2 = x_B\, y\, S$





If we do not have confidence for y > 0.5, due to QED radiation,

EIC's eA reach to small-x could be reduced to $x_{\min} \sim 2 \times 10^{-4}$

or effectively, $\boxed{\sqrt{S} = 100\ \text{GeV} \rightarrow 73\ \text{GeV at } y = 0.95}$

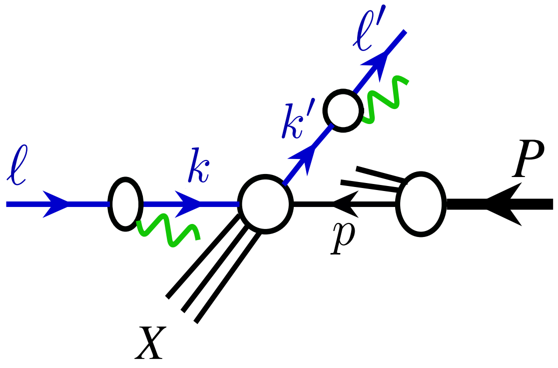
"RC" depends PDFs!!!

Collinear Factorization for QED Radiative Conntribution

- **Without the "one-photon" approximation:**

  ~ **Inclusive single lepton production at high transverse momentum**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

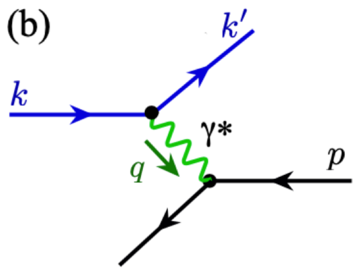


$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2)\;+\;\cdots$$

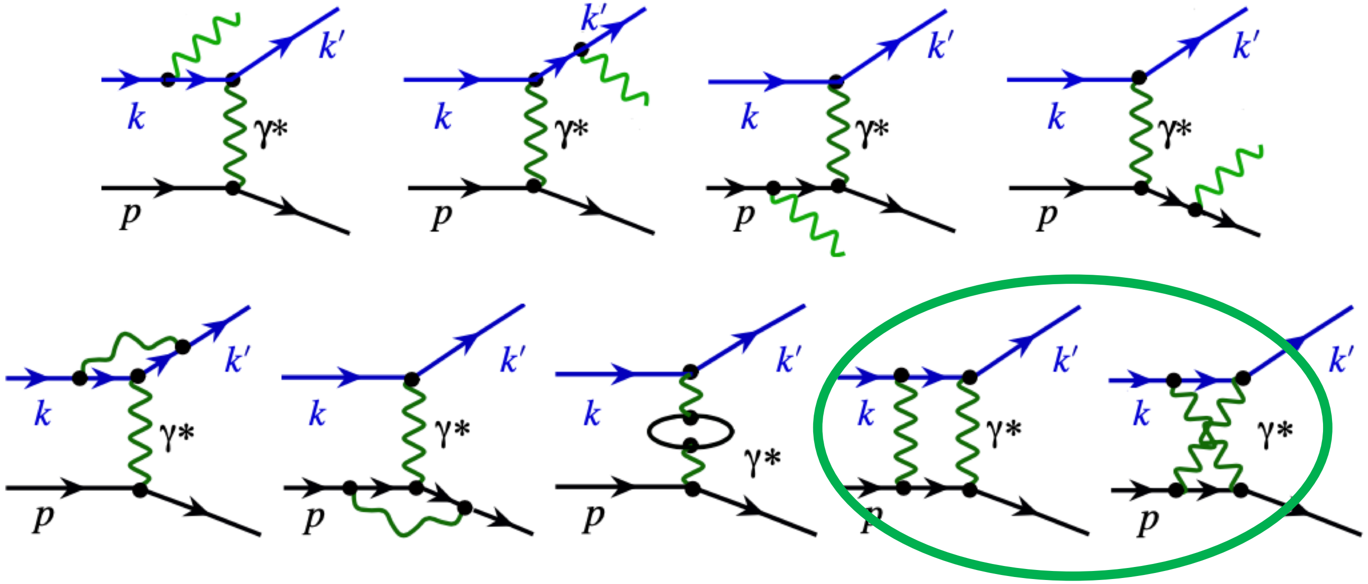
No structure functions, but have PDFs, LDFs, LFFs, …

- **Calculated hard parts in power of $\alpha^m\alpha_s^n$:**

LO



NLO:

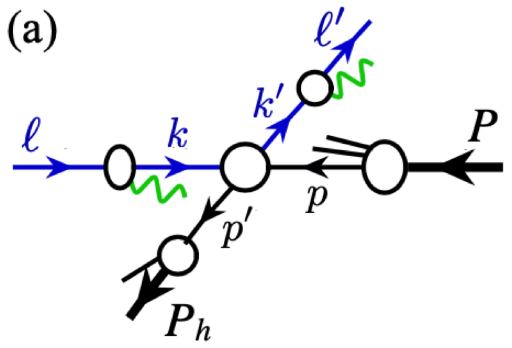


Beyond one-photon exchange

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

❑ Inclusive production of a lepton and a hadron:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



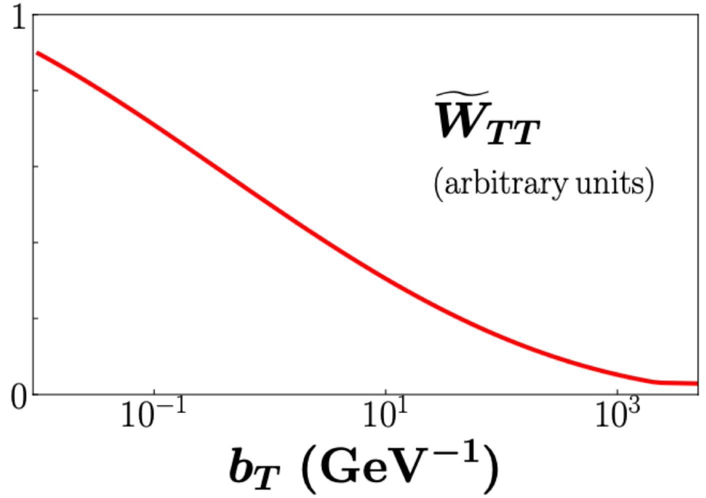
$$e(\ell) + N(P) \to e(\ell') + h(P_h) + X$$

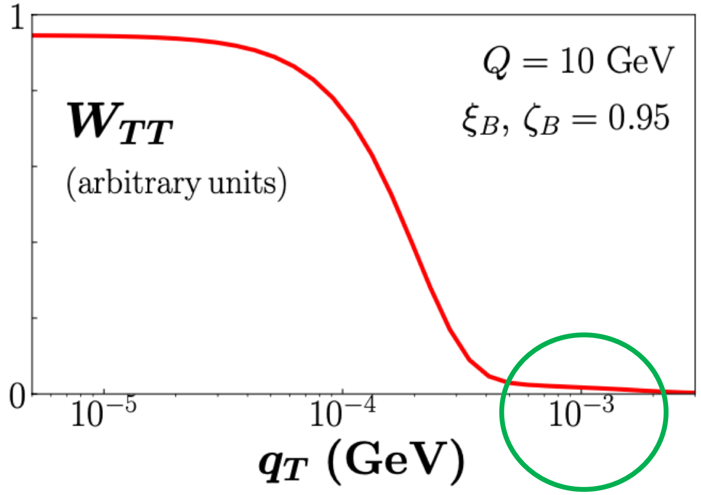
Momentum imbalance between the lepton and the hadron could be sensitive to both parton TMDs and lepton TMDs

Typical parton transverse momentum: $k_T^2 \sim \Lambda_{\mathrm{QCD}}^2 + \langle k_T^2 \rangle_{\text{generated by QCD shower}}$

❑ Estimate of lepton transverse momentum generated by QED shower:

Resummation to lepton TMD





TMDQED broadening for lepton is much smaller than typical parton kT!

Collinear factorization for high order QED contributions

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

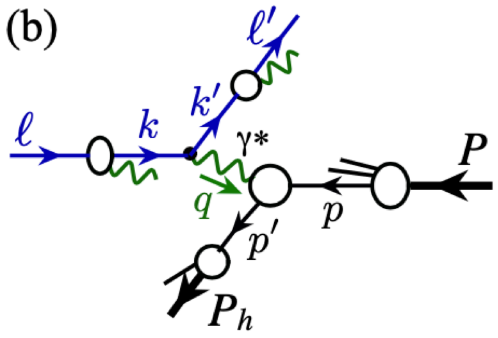
- **QED factorization of collision-induced radiation – collinear:**

Liu, Melnitchouk, Qiu, Sato 2008.02895, 2108.13371

$$E_{\ell'}E_{P_h}\frac{d^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{d^3\ell'\, d^3P_h} \approx \sum_{ij\lambda_k}\int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} D_{e/j}(\zeta)\int_{\xi_{\min}}^1 d\xi\, f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\left[E_{k'}E_{P_h}\frac{d^6\hat\sigma_{k(\lambda_k)P(S)\to k' P_h X}}{d^3k'\, d^3P_h}\right]_{k=\xi\ell, k'=\ell'/\zeta} + \mathcal{O}\left(\frac{m_e^n}{Q^n}\right)$$

  - Leading power IR sensitive contribution is universal, as $m_e/Q \to 0$, factorized into LDFs and LFFs
  - IR safe contributions are calculated order-by-order in powers of α
  - Neglect m_e/Q power suppressed contributions
  - Collinear QED factorization for both inclusive DIS and SIDIS, or e^+e^-, … [global fits of LDFs, LFFs]

- **"One photon"-approximation** ⟹ **Hybrid factorization: CO for QED and TMD for QCD!**



$$\frac{d^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, dP_{hT}^2} = \sum_{ij\lambda_k}\int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^1 \frac{d\xi}{\xi} f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\, D_{e/j}(\zeta)$$

$$\times \frac{\hat x_B}{x_B\xi\zeta}\underbrace{\left[\frac{\alpha^2}{\hat x_B\, \hat y\, \widehat{Q}^2}\frac{\hat y^2}{2(1-\hat\varepsilon)}\left(1+\frac{\hat\gamma^2}{2\hat x_B}\right)\sum_n \hat w_n F_n^h(\hat x_B, \widehat{Q}^2, \hat z_h, \widehat{P}_{hT}^2)\right]}_{\text{Evaluated in a "virtual photon-hadron" frame}}$$

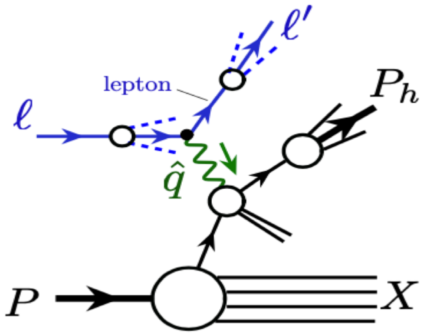
Apply a (ξ,ζ)-dependent Lorentz transformation:

$$\{\hat q, P, \widehat{P}_h\} \xrightarrow[(\xi,\zeta)]{} \{q, P, P_h\}$$

In a frame to compare with exp. measurements

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

❑ Two-step approach to SIDIS:



One-photon approximation

1) In "virtual-photon" frame, defined by $\hat{q}(\xi,\zeta) - p$

- TMD factorization when $\widehat{P}_T^2 \ll \widehat{Q}^2$
- CO factorization when $\widehat{P}_T^2 \sim \widehat{Q}^2$
- Matching to get the $\widehat{P}_T$-distribution

2) Lorentz transformation from the "virtual-photon" frame to any experimentally defined frame
– lepton-hadron Lab frame, Breit frame (x_B,Q^2), …

QED contribution (not correction) can be systematically improved order-by-order in power α!

❑ Case study F_{UU}:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} &= \\
\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{ & \boxed{F_{UU,T}} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\, F_{UU}^{\cos\phi_h} \\
& + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h\, F_{LU}^{\sin\phi_h} \\
& + S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h}\right] \\
& + S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h\, F_{LL}^{\cos\phi_h}\right] \\
& + |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
& \quad + \varepsilon\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad + \sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S\, F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
& + |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S\, F_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

Jefferson Lab

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

- **Case study F_{UU}:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$\frac{d\sigma^h_{\rm SIDIS}}{dx_B dy\, dz\, dP^2_{hT}} = \int_{\zeta_{\rm min}}^1 d\zeta \int_{\xi_{\rm min}(\zeta)}^1 d\xi\, D_{e/e}(\zeta)\, f_{e/e}(\xi) \times \underbrace{\left[\frac{\hat{x}_B}{x_B\,\xi\zeta}\right]\left[\frac{(2\pi)^2\,\alpha}{\hat{x}_B \hat{y}\,\widehat{Q}^2}\,\frac{\hat{y}^2}{2(1-\hat{\varepsilon})} F^h_{UU}(\hat{x}_B, \widehat{Q}^2, \hat{z}, \widehat{P}_{hT})\right]}_{\text{Evaluated in a “virtual photon-hadron” frame}}$$

Unpolarized structure function:

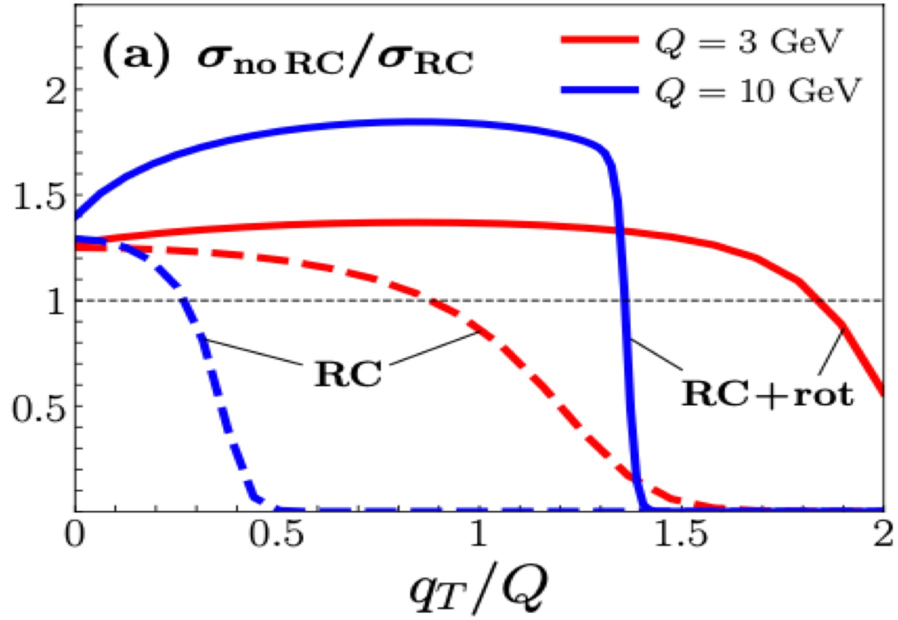
$$F^h_{UU} = x_B \sum_q e_q^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{q}_T) \times f_{q/N}(x_B, \boldsymbol{p}_T^2)\, D_{h/q}(z, \boldsymbol{k}_T^2) \qquad \boldsymbol{q}_T = \boldsymbol{P}_{hT}/z$$

(ξ, ζ) - Dependent Lorentz transformation

Effectively, a rotation in hadron-rest frame

Solid – with Lorentz transformation
Dashed – without Lorentz transformation

Impact of $q \to \hat{q}(\xi, \zeta)$!



Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

- **Case study – single transverse spin asymmetry:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =
\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}
$$

$$
+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}
$$

$$
+\,S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]
$$

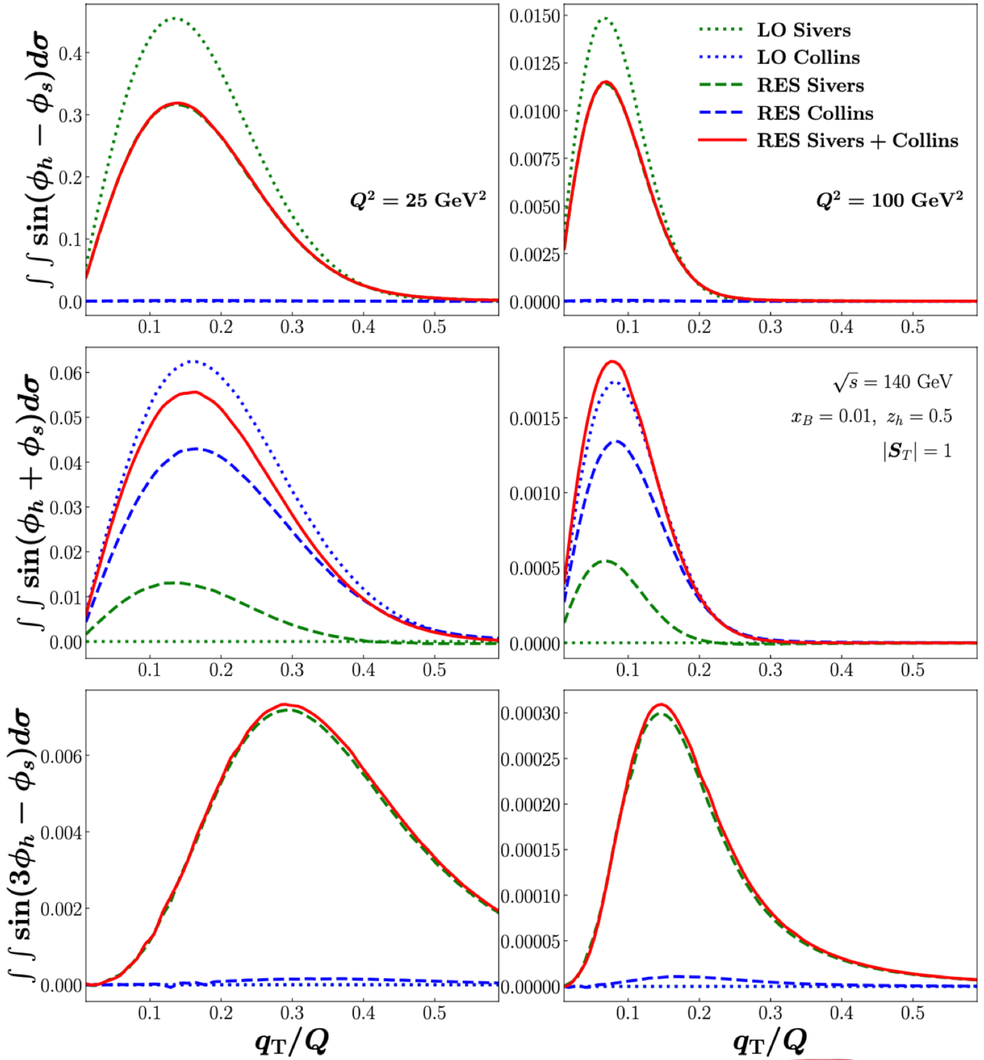
$$
+\,S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]
$$

$$
\boxed{+\,|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)
+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}}
$$

$$
+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]
$$

$$
+\,|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}
$$

$$
+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Bigg\}
$$



Impact of $q \to \hat{q}(\xi,\zeta)$ **!**

Summary and Outlook

- The lepton-hadron facility, like JLab and the future EIC, is a natural place and ideal for studying TMDs

  Two scale observables, angular modulation between the leptonic and hadronic plane, …

- TMDs provide nonperturbative, but, universal information on the correlations between a hadron and the properties (motion, polarization, flavor, and etc) of an active parton within it

- Once a hadron is broken in a high energy collision, large Q, the collision-induced QCD radiation could significantly dilute such correlations if there is a large phase space for the radiation, large S (or small x)

  The challenge One

- The hard collision in lepton-hadron scattering, e.g., at the EIC, also induces QED radiation, which makes the "photon-hadron" frame, where the TMDs are to be extracted, ill-defined

  The challenge Two

- The challenge One: Need good data from region where Q^2 & S are sufficient, but, not too large!

- The challenge Two: Treat QCD and QED calculations/factorization on an equal footing!

Thank you!

Jefferson Lab