



I. COLOR CONFINEMENT in 3+1 dim CHROMOSTATICS

Gauss' Law boundaries

Yang-Mills Maxwell equations

GAUGE CONNECTION gradients

Domain walls - Topological
 $\{ E^a_i \tilde{B}^a_i \neq 0 \}$

The Strong Conjecture

* The confinement mechanism for QCD involves a domain wall of topological (CP-odd) charge separating the interior volume of hadrons from an exterior volume.

This conjecture provides a specific starting point for the study of color confinement.

There is a \$50K prize for disproving
the conjecture

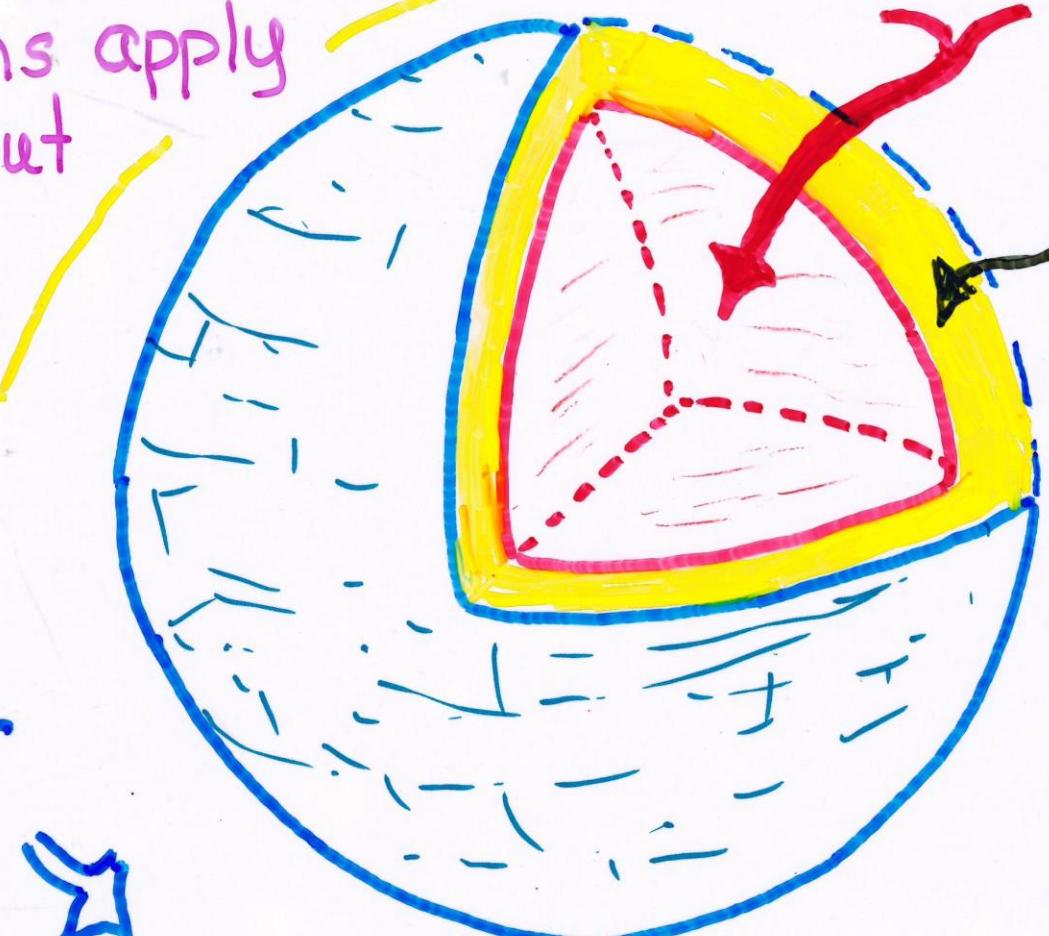
Yang-Mills Maxwell
equations apply
throughout
these
regions

Interior vol. $r < R_0 - \Delta$

Transition
volume
 $R_0 - \Delta \leq r \leq R_0 + \Delta$



Exterior
Volume
 $r > R_0 + \Delta$



$2R_0$

2Δ

2Δ

TOPOLOGICAL CHARGE

- CP-odd condensate $E_i^a B_i^a(r, t)$
- forms in regions with different values of constant $a(r, t)$ as a consequence of Yang-Mills Maxwell equations $-E_L + \frac{\partial}{\partial r}(a r E_a) + \frac{\partial}{\partial t}(a r B_a) = q^2 r^2 E_i^a B_i^a$



- not a conserved charge
Disappears after EW hadronic decays (ex. $\pi^0 \rightarrow \gamma\gamma$)

Instantons & Merons ..

Classification of condensates in SU(2) chromostatics with spherical symmetry

$\alpha(r) = \pm 1$ $E_i^a E_i^a \neq 0$ $B_i^a B_i^a = 0$ color electric

$\alpha(r) = \pm 1$ $E_i^a E_i^a = 0$ $B_i^a B_i^a \neq 0$ color magnetic

$\alpha(r) = \pm 1$ $E_i^a E_i^a \neq 0$ $B_i^a B_i^a \neq 0$ color glass

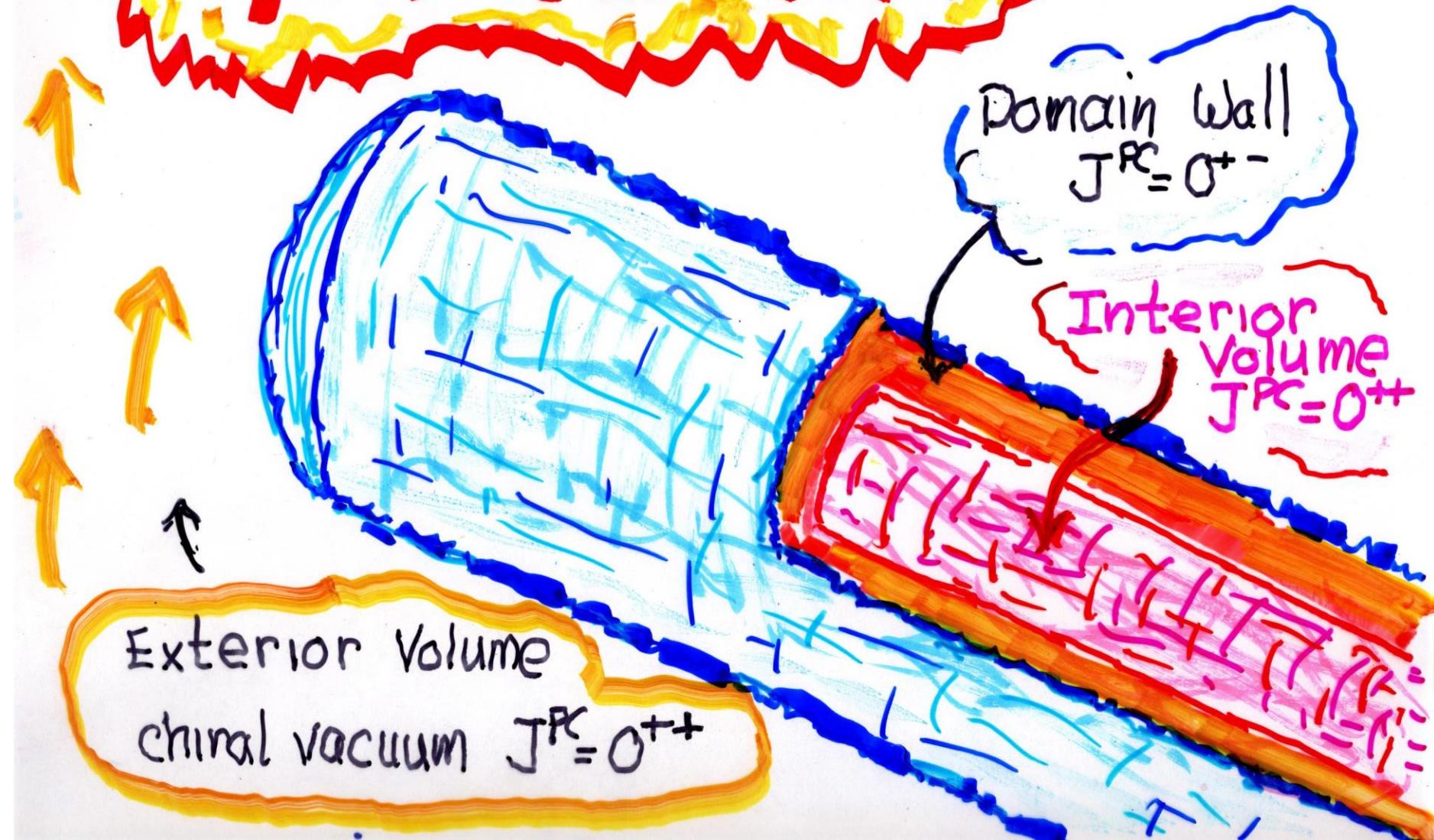
$\alpha(r) = \pm 1$ $E_i^a E_i^a = 0$ $B_i^a B_i^a = 0$ sterile vacuum

$\alpha(r) = 0$ $E_i^a E_i^a = 0$ $B_L B_L = (\pm) \frac{1}{r} + \frac{1}{r}$ 't Hooft Polyakov

$\alpha(r) = c \neq \pm 1, 0$ $E_i^a E_i^a \neq 0$ $B_i^a B_i^a \neq 0$ topological
or dyonic

A domain wall is a region where $\alpha(r) \neq 0$
that separates other condensates
and also carries topological charge

Cylindrical SU(2)

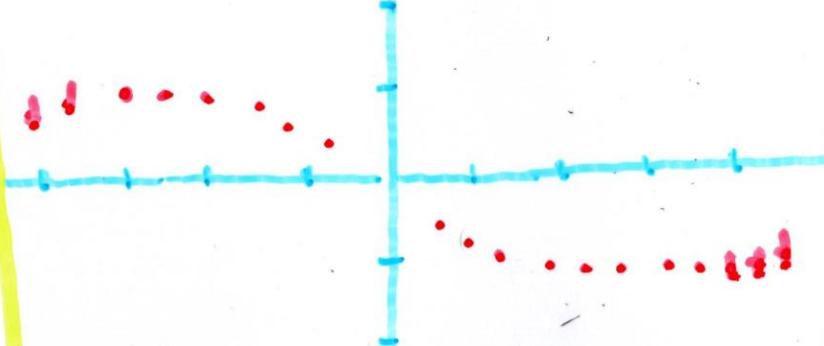


M. Engelhardt, B. Musch, J. Negele, A. Schäfer

Strong
Evidence
of large-scale

Chiral Structure

3P_0 $\bar{q}q$ pairs

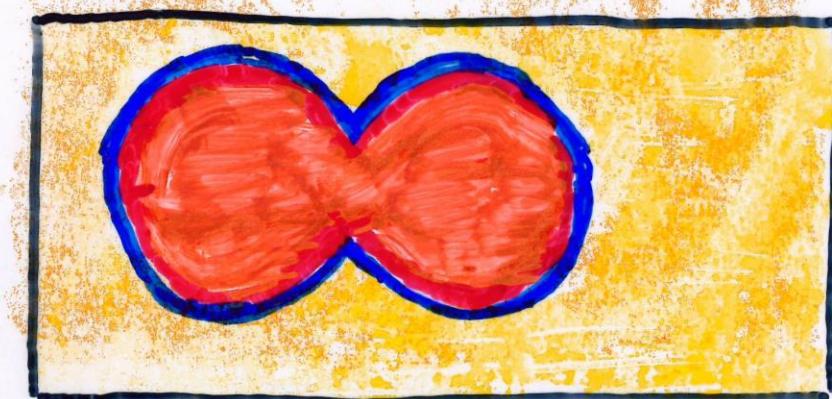


u-quark Boer-Mulders spin-directed momentum shift in a pion

$$\langle 2\Sigma \vec{L} \cdot \vec{s} \rangle = -0.65 \pm 0.10$$

M. Engelhardt
QCD Evolution 2021

Domain Zones

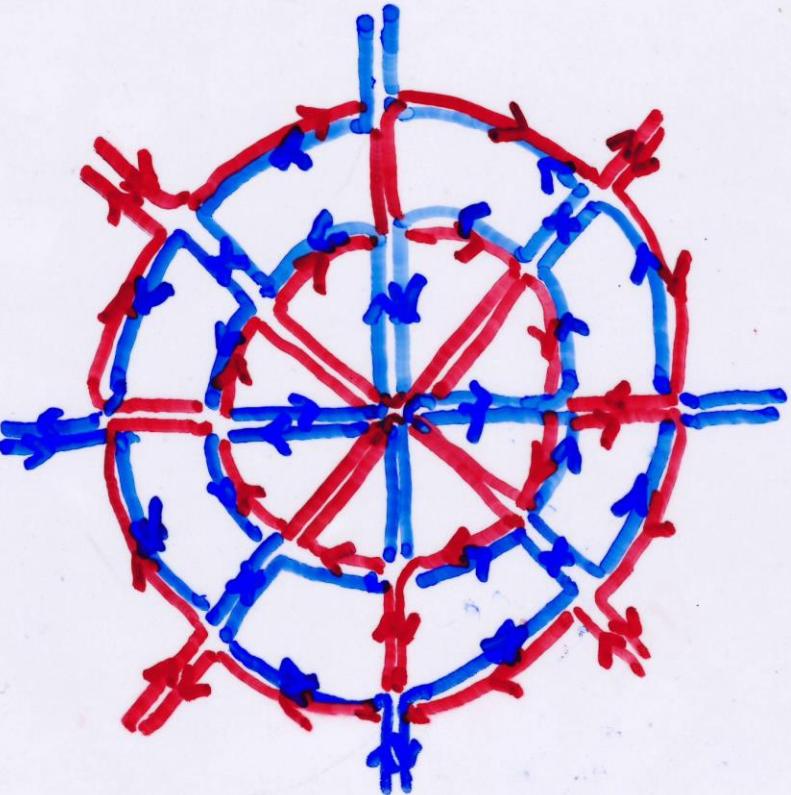


■ interior volume

■ exterior volume

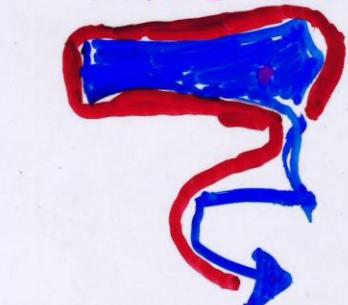
acr) R_0 \xrightarrow{r}
topological
charge

Here is the sketch showing the chiral nature covariant Derivatives generate



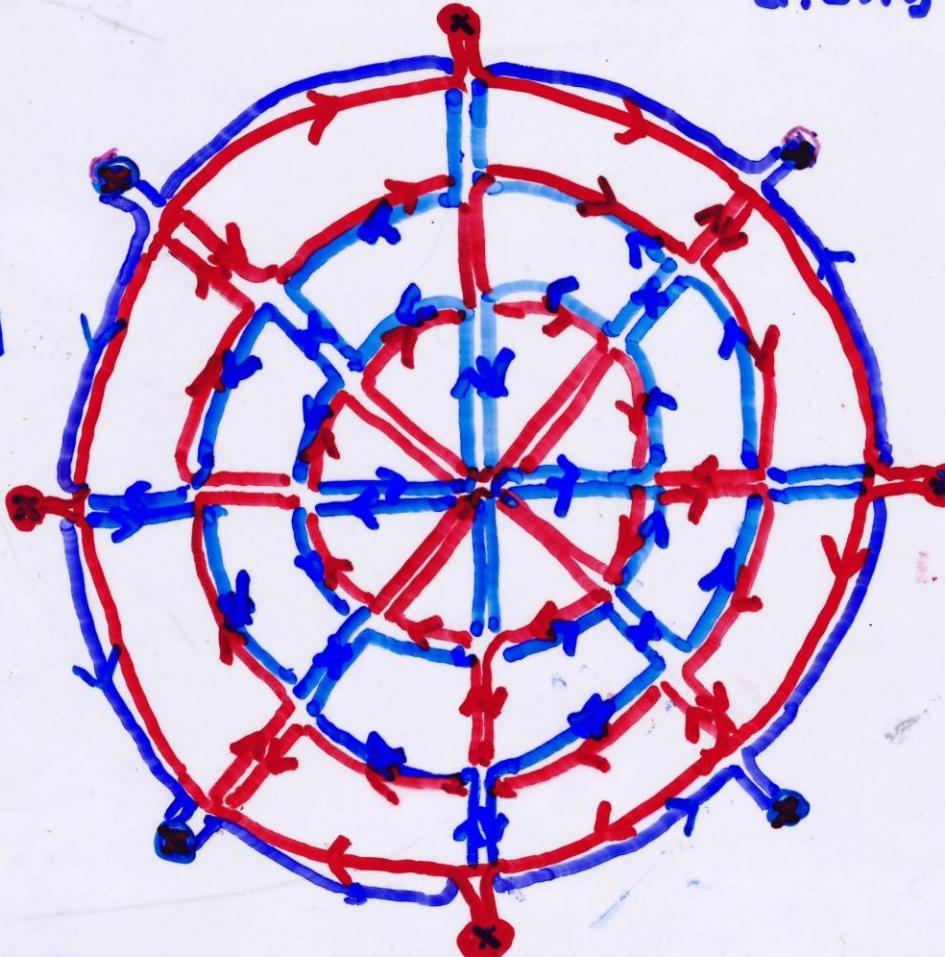
Let's compare putting on another layer with $a=1$ with two alternatives

Here is another layer with $\alpha(r)=1$ with
ends tied off by a distributed $SU(2)$ color source
along each diagonal



a standard
nontopological
sol'n
to
Yang-Mills
Maxw

$\alpha=1$



the exterior
volume is
a sterile
vacuum
condensate
with the
same chirality
 $\alpha=1$ as interior

Leinweber & Collaborators
dynamical fermions increase
density of color vortices in vacuum

pure gauge 3277 ± 156
full qcd 5923 ± 259

Improves fit for confining
potential & Landau gauge propagator



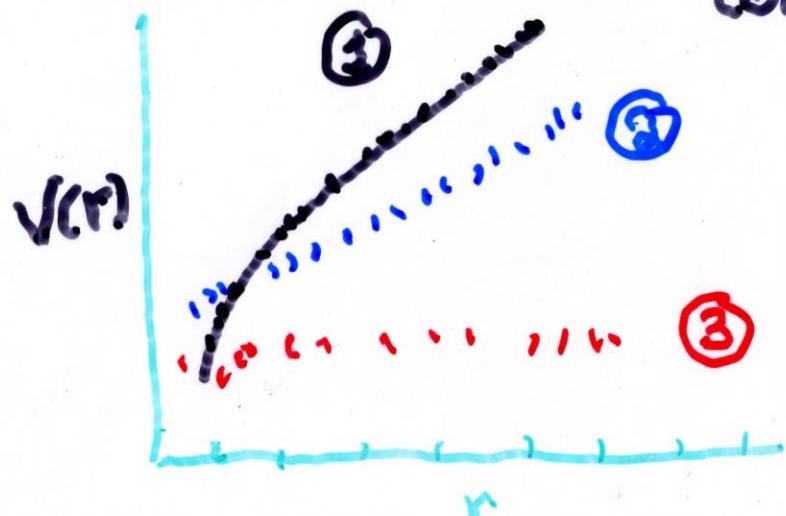
3P_0 $q\bar{q}$ $J^{PC} = 0^{++}$
pairs



Chromostatics
Interior Volume of hadron
one chiral adiabatic
vortex $J^{PC} = 0^{++}$

Using the "maximal center gauge" it is possible to generate monte carlo gauge configurations with or without color vortices

- ① regular
- ② vortices only
- ③ vortices removed



$$\text{cornell Potential } V(r) = -\frac{q}{r} + \alpha_r + \Gamma r$$

Bowman et al
PR D84 034501 (2011)

Correlations (in)

transverse-spin asymmetries

Connecting the SIDIS

Target and Current Fragmentation

Regions

Gary Goldstein
Simone Liuti
Dennis Sivers

$e^+ p \rightarrow e^+ p \pi^\pm$



QUANTUM ENTANGLEMENT

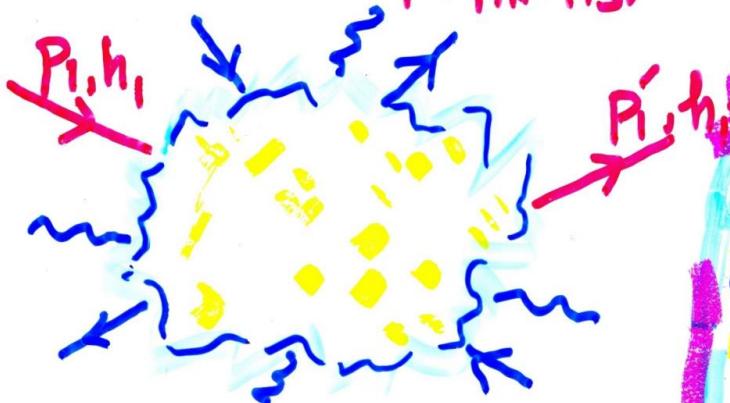
IN TRANSVERSE SPIN ASYMMETRY

I. Introduction to KPR factorization

Kane, Pumplin & Repko PRL 41, 1689 (1978)

$$A_N | q\bar{q} \uparrow \rightarrow q X = \left[\frac{d\sigma(q\bar{q}\uparrow \rightarrow q) - d\sigma(q\bar{q}\downarrow \rightarrow q)}{d\sigma(q\bar{q}\uparrow \rightarrow q) + d\sigma(q\bar{q}\downarrow \rightarrow q)} \right] = \alpha_s(Q^2) \frac{m_q}{T_S} f(\theta_{cm}) (1 + \dots)$$

where $f(\theta_{cm}) = \frac{P_{TN}}{(M^2 + P_T^2 + P_S^2)^{1/2}} F(\theta_{cm})$ vanishes at $\theta_{cm} = 0$



quark helicity presented
 $O(m_q/v_{FS})$

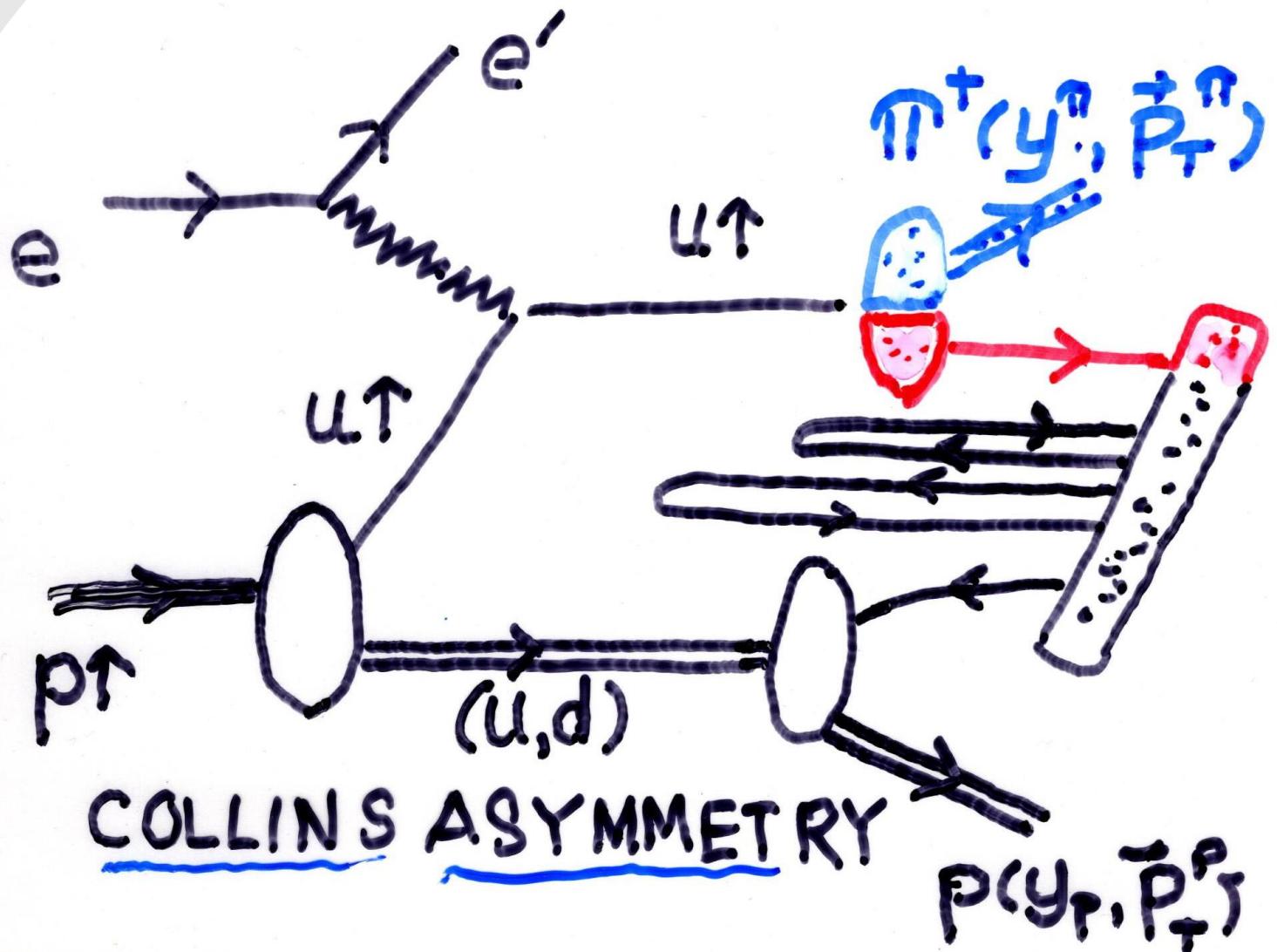
quark helicity conservation in QCD pert. theory implies an additional symmetry in pqcd that is broken in the full theory

Lab frame scattering planes $\gamma^* p \rightarrow \pi^+ p X$

target polarization
 $\pm y$ direction

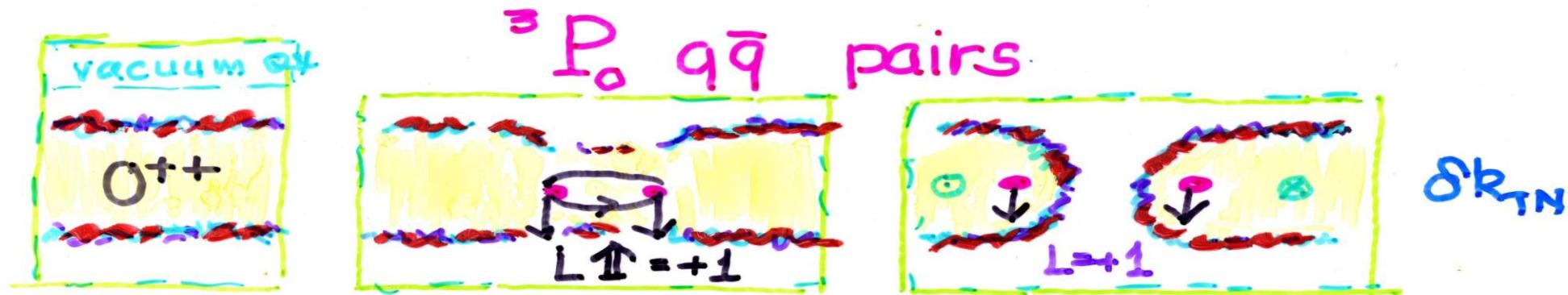


assume proton & π^+ product of rank-1 fragmentation functions



SPIN-DIRECTED momentum transfers

δk_{TN} generated in fragmentation process

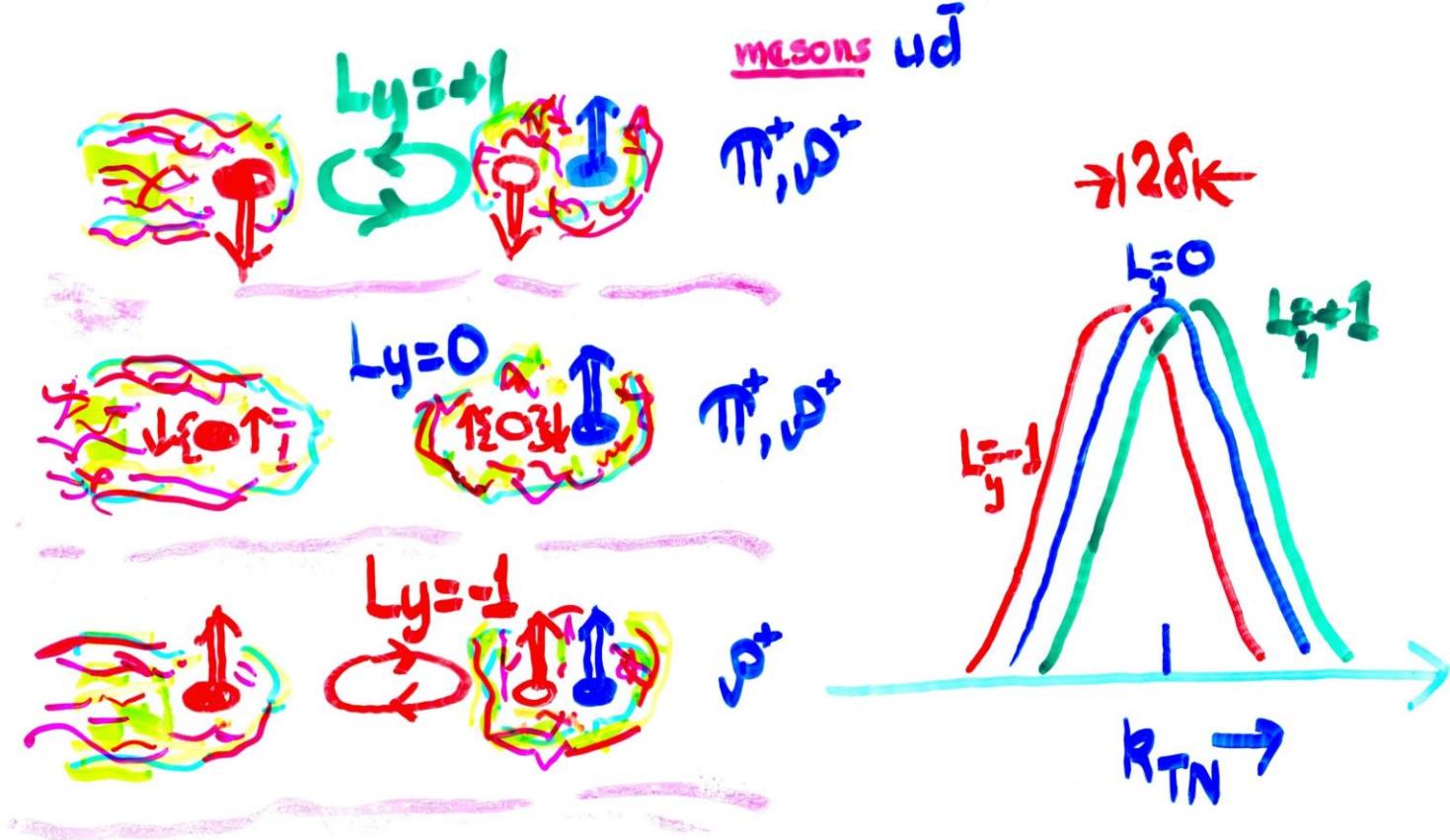


combine with flux rupture to produce
Collins and polarizing fragmentation functions

2-particle correlations short-range in rapidity

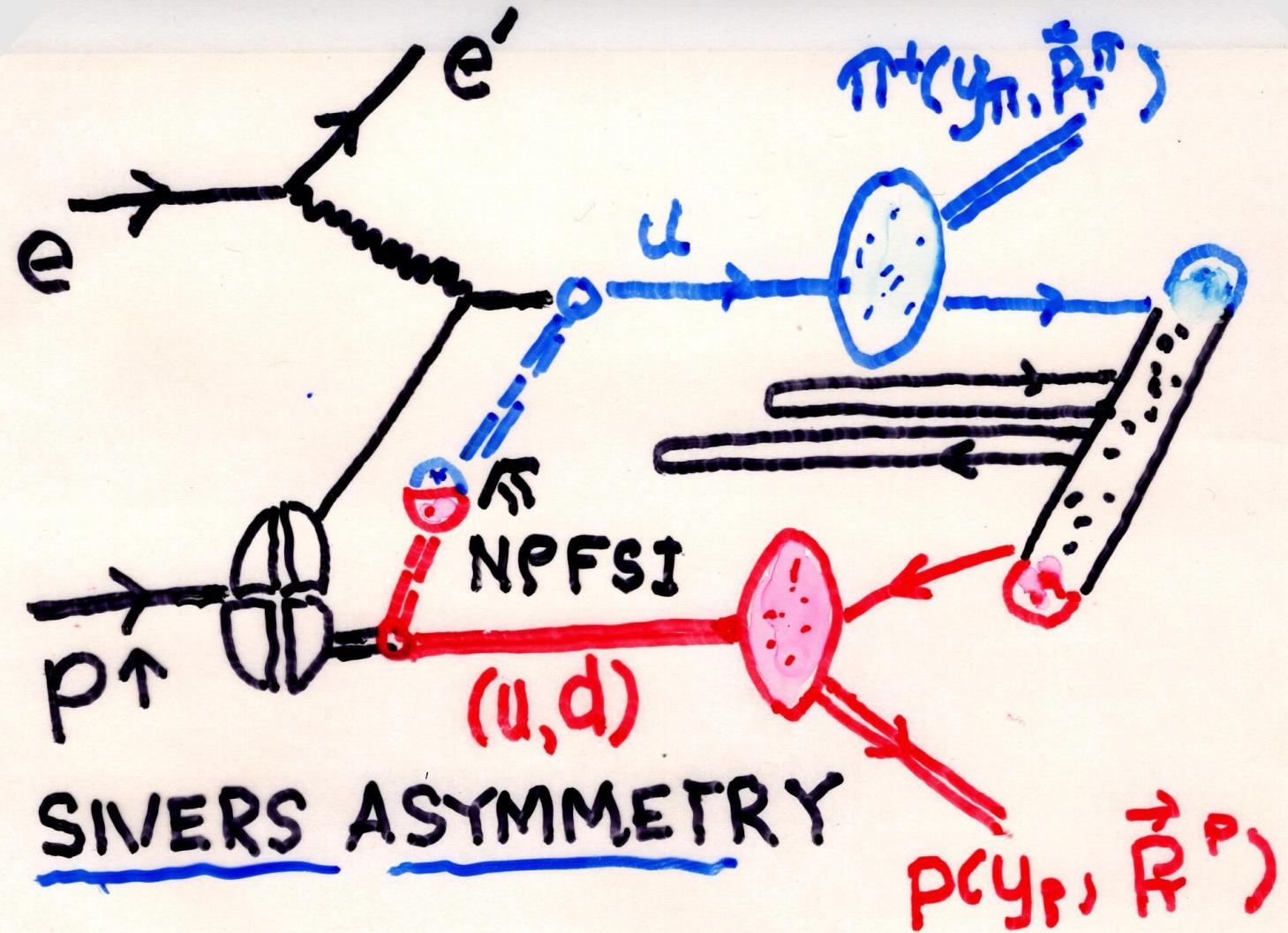
Jaffe, Bacchetta, Radici, D'Alesio ...

2-particle fragmentation fcns



Partial Wave Expansion Separates Spin-Directed
 Momentum transfer in fragmentation Process

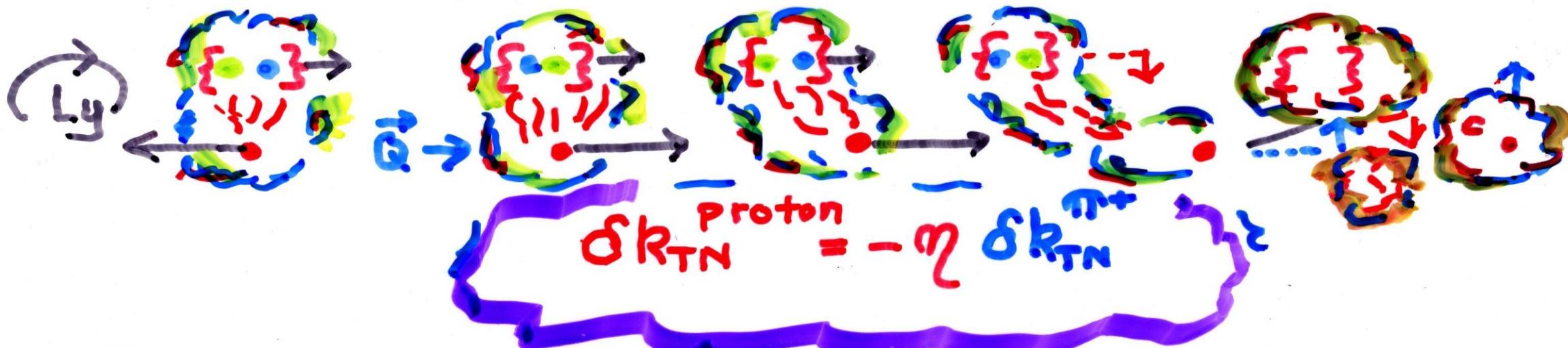
$$\delta = 0.2 \text{ GeV/c}$$



Non-Perturbative Final-State Interactions

orbital angular momentum $\langle \vec{Q} \vec{L} \cdot \vec{s} \rangle = -0.65 \pm 0.1$ (Engelhardt)

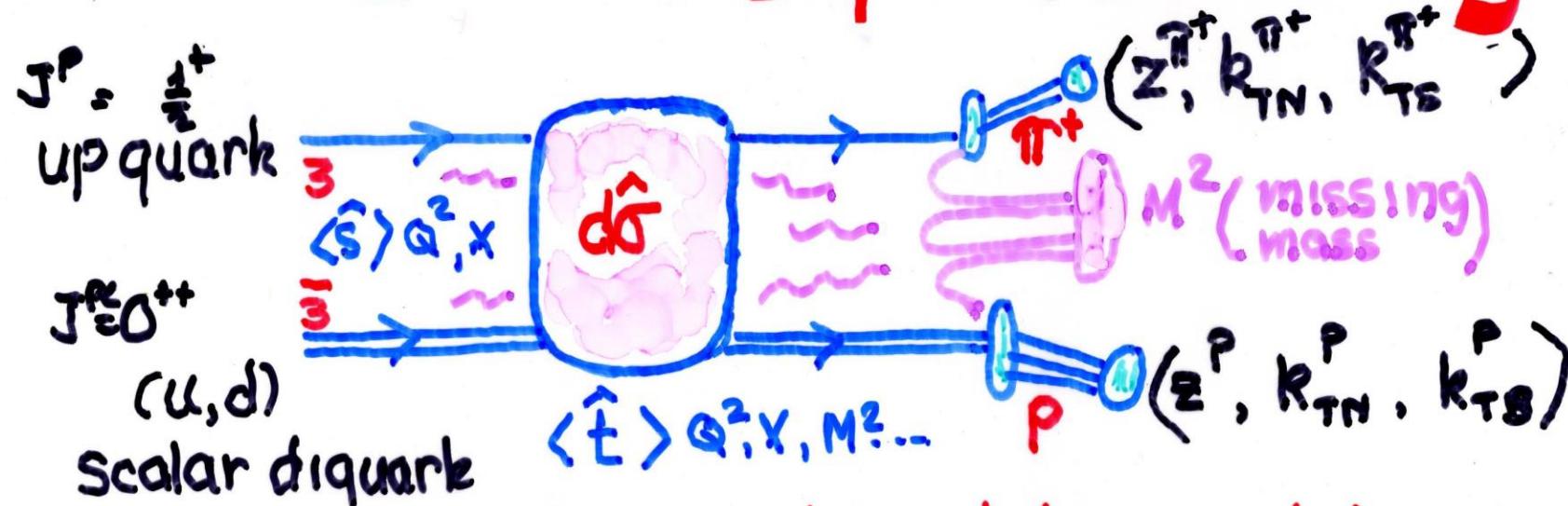
SIDIS for proton polarized + \hat{y} direction



ordinary entanglement

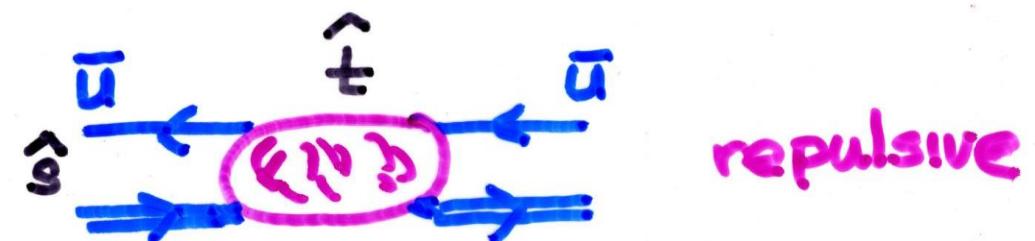
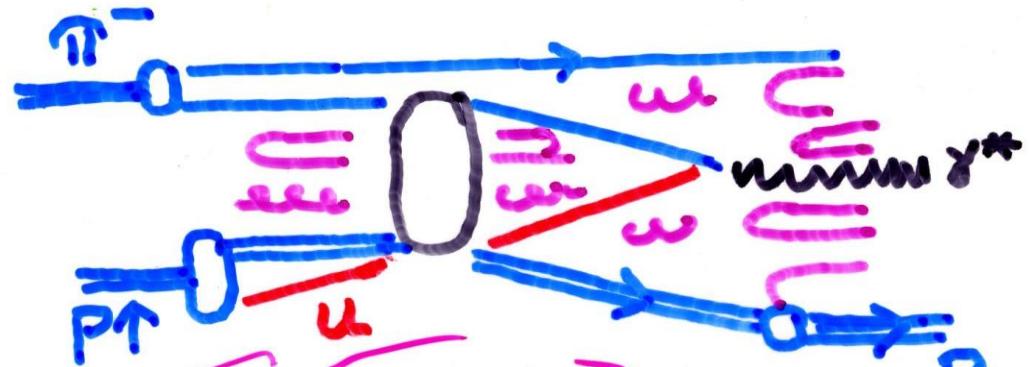
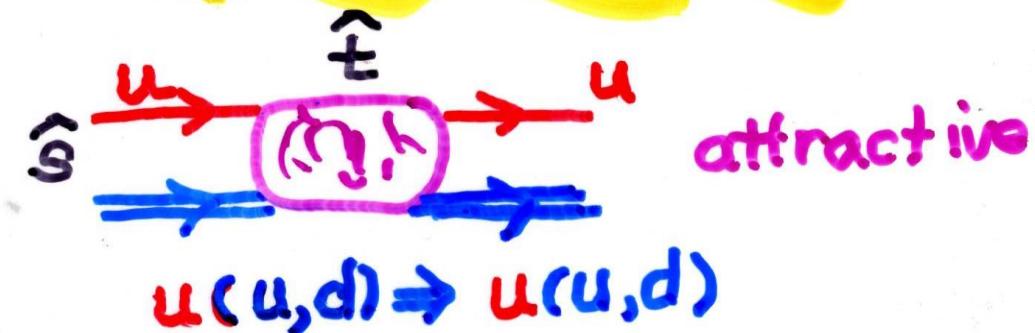
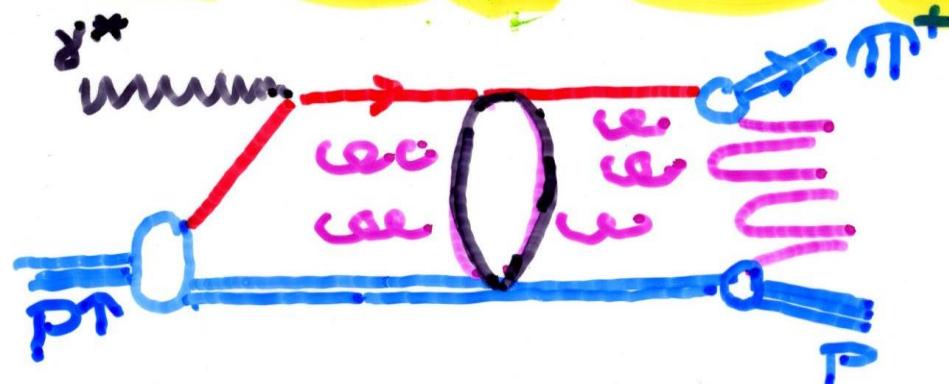
proton in target fragmentation
 π^+ in current fragmentation

Examine non-perturbative Quark Diquark Scattering



exchange kinematics determined by
measuring $\delta k_{TN}^{\pi^+}$ and δk_{TN}^P involved
in spin asymmetry

COLLINS CONJUGATION



u quark not involved
"not deflected"

\bar{u} quark from π^-
"deflected"

Two Hadron TMD fracture functions in SIDIS

IV extend the fracture function formalism

Trentadue
Veneziano

$$M_{(h_1, h_2) / X, Q^2}^q (y^{h_1}, \vec{k}_T^{h_1}) (y^{h_2}, \vec{k}_T^{h_2})$$

1-jet

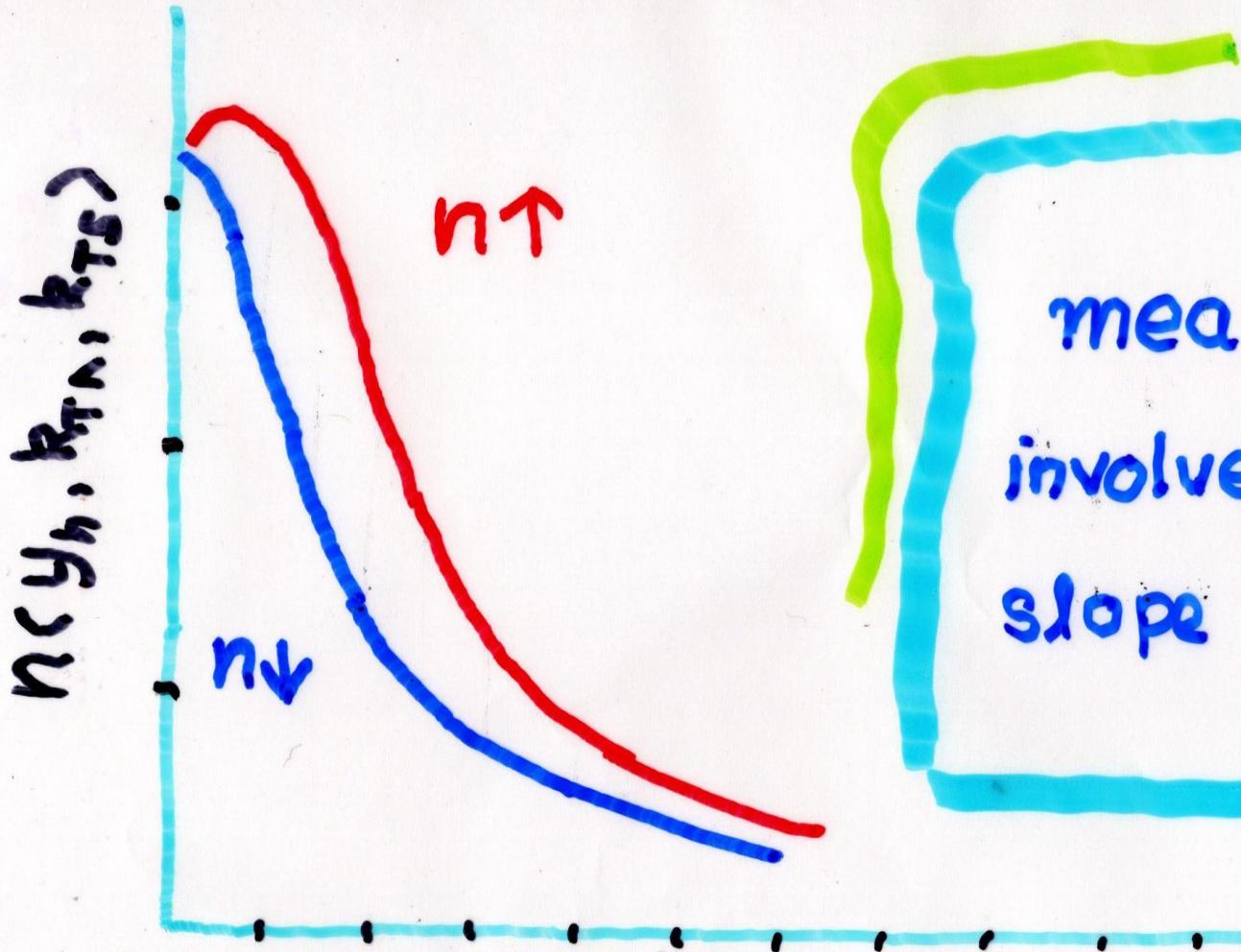
conjoint probabilities

$$\Delta^N M_{(\pi^+, p) / p\pi}^u = M_{(\pi^+, p) / p\pi}^u - M_{(\pi^+, p) / p\pi}^u$$

$$\Delta^N M_{(\pi^+, p) / p\pi}^u (x, Q^2; (y^{\pi^+}, R_{TN}^{\pi^+}, R_{TS}^{\pi^+}); (y^\rho, R_{TN}^\rho, R_{TS}^\rho))$$

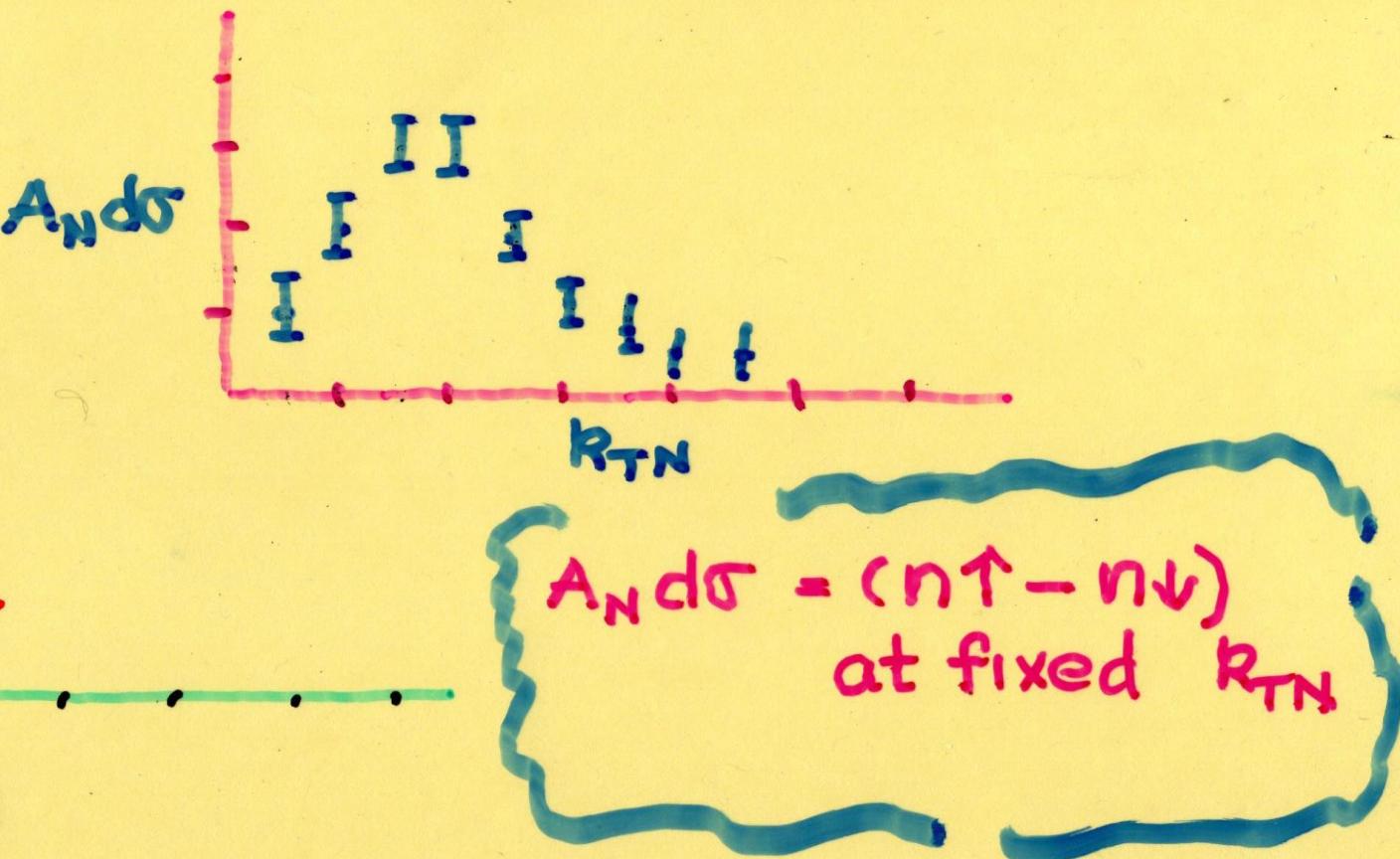
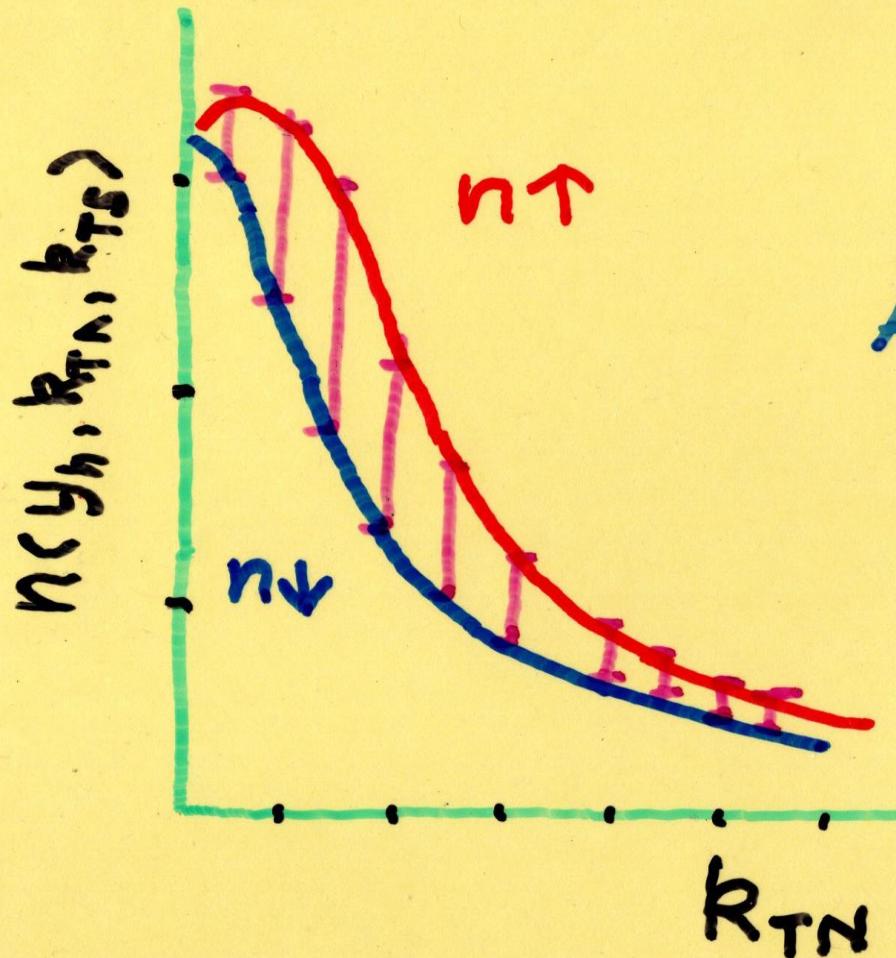
TRANSVERSE SPIN ASYMMETRY

III. HOW to MEASURE δk_{TN}

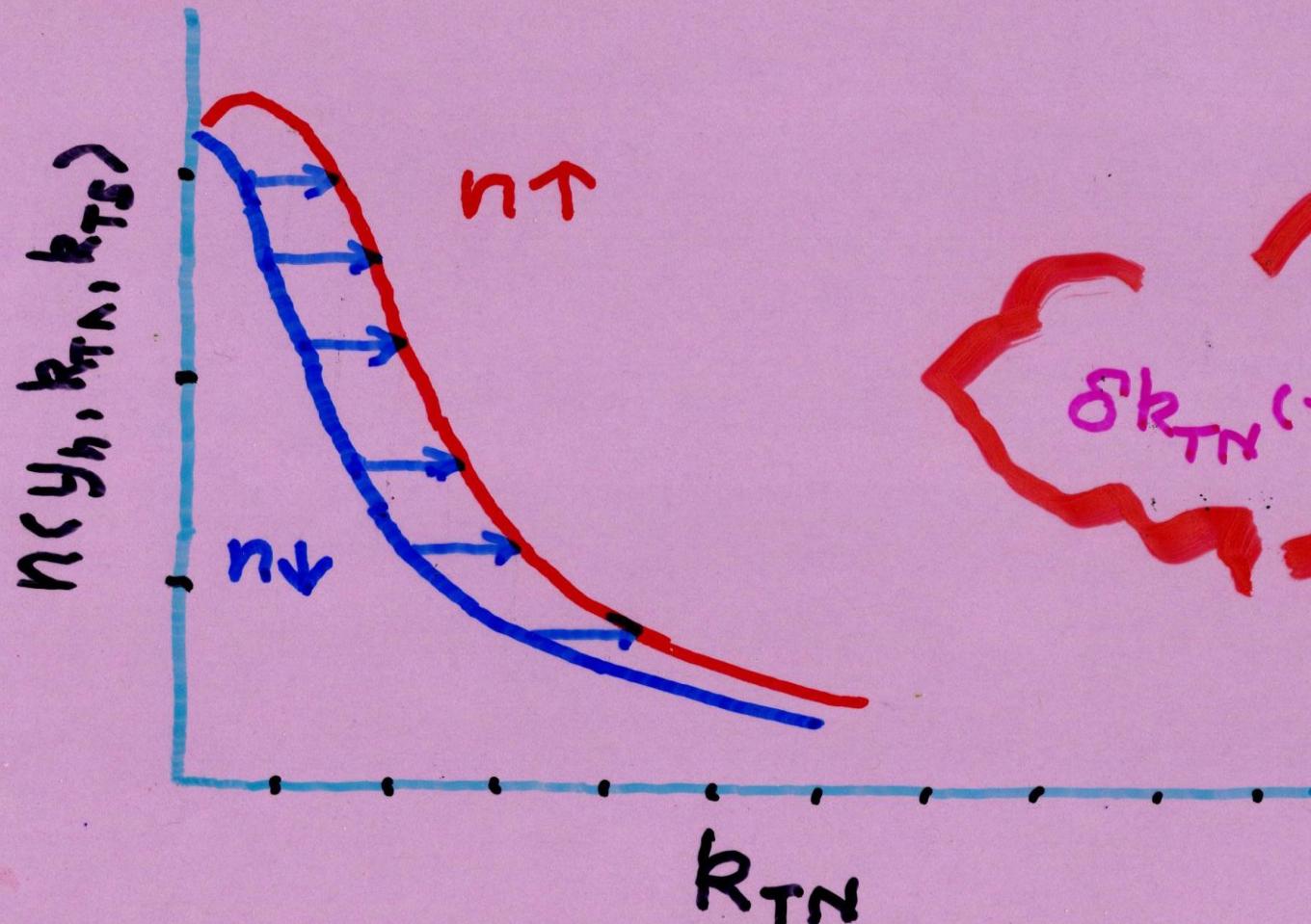


measuring δk_{TN}
involves A_N and the
slope $\frac{d}{\delta k_{TN}} d\sigma$

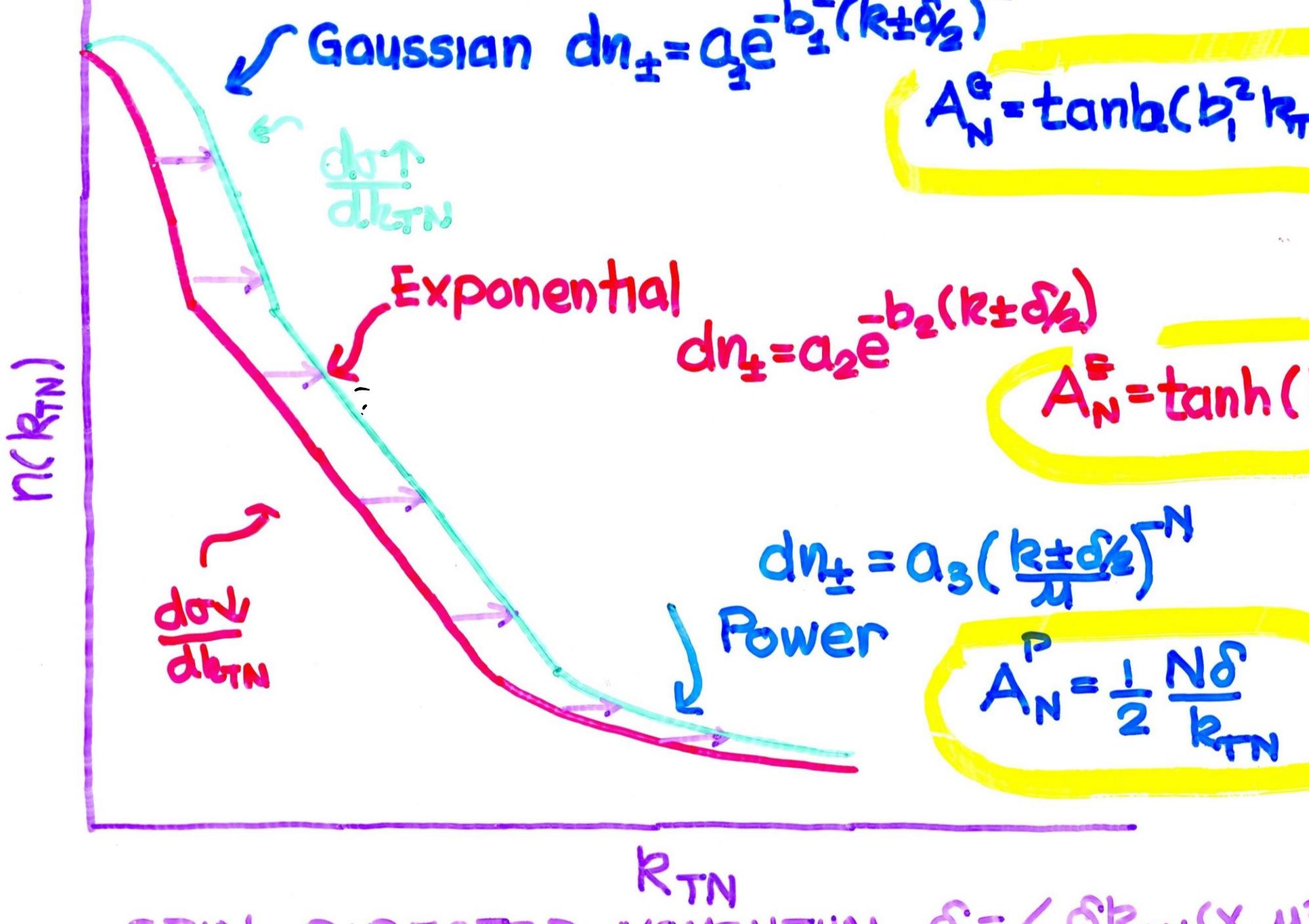
III. HOW to MEASURE δR_{TN}



III. HOW to MEASURE δk_{TN}



$\delta k_{TN}(--) = A_N d\sigma / \partial k_{TN}$
at fixed n



Experimental Evidence of $e\bar{p}^{\dagger} = e\bar{n}^{\dagger} p$
transverse spin asymmetry correlations
should already exist in CLAS-12
data from Jefferson Lab.

A range of similar correlation
measurements should be a part of
any program to study  QCD at EIC

Two-Hadron TMD Fracture Functions in SIDIS

extend the fracture function formalism Trentadue Veneziano
conjoint probabilities instead of number densities

$$M_{(h_1, h_2)/p}^q [x, Q^2; (z^{h_1}, \vec{R}_T^{h_1}), (z^{h_2}, \vec{k}_T^{h_2})] \quad \text{1-jet fracture function}$$

transverse-spin asymmetry

$$\Delta^N M_{(\pi^\pm, p)/p\uparrow}^u \equiv M_{(\pi^\pm, p)/p\uparrow}^u - M_{(\pi^\pm, p)/p\downarrow}^u$$

$$= \Delta^N M_{(\pi^\pm, p)/p\uparrow}^u [x, Q^2; (\underline{z}^{\pi^\pm}, \underline{k}_{TN}^{\pi^\pm}, \underline{k}_{TS}^{\pi^\pm}) (\underline{z}^p, \underline{k}_{TN}^p, \underline{k}_{TS}^p)]$$

assume π^\pm, p 1^{st.}-rank fragmentation products

Diquarks and Diquark Fragmentation

$[q, \bar{q}]$ $J^P = 0^+$ 3 color

$[u, \bar{d}]$ $[d, \bar{s}]$ $[s, \bar{u}]$ 3 flavor SU(3)

$\{q, \bar{q}\}^\uparrow$ $J^P = 1^+$ 3 color

$\{\bar{u}, u\}$ $\{\bar{d}, d\}$ $\{\bar{s}, s\}$

$\{\bar{u}, s\}$ $\{\bar{d}, d\}$

$\{\bar{s}, s\}$

6 flavor SU(3)

gluonic radiation

$[q, \bar{q}] \Rightarrow [q, \bar{q}]_6^- G \quad \{q, \bar{q}\} \Rightarrow \{q, \bar{q}\}_6^-$

changes parity and color of diquarks but not flavor or symmetry

changes of flavor or symmetry require mesonic degrees of freedom

$\{q_i, \bar{q}_j\}^\uparrow \Rightarrow [q_i, \bar{q}_j](\bar{q}_k q_j)$

$[q_i, \bar{q}_j](\bar{q}_k \bar{q}_l) \Rightarrow \{q_i, \bar{q}_k\}^\uparrow (q_j \bar{q}_l)$

that are incorporated into

fragmentation process



FINITE SYMMETRIES

	\mathcal{T}	C	P	(CPT)	Θ	A_x	A_y	A_c
Σ_x	-	+	-	+	-	+	-	+
Σ_y	+	+	+	+	-	-	-	-
Σ_z	+	-	-	+	-	+	+	-

$(\star P)^*$ OP $A_x T$ $A_y C$

* HODGE DUAL OPERATOR

$$\begin{aligned} P: (\nabla, A) &= (-\nabla, A) \\ \star: (\nabla, A) &= (\tilde{A}, \tilde{\nabla}) \\ P^*: (\nabla, A) &= (\tilde{A}, -\tilde{\nabla}) \\ \star P^*: (\nabla, A) &= (\nabla, -A) \end{aligned}$$

$\Theta = \star P^*$ "Snake Operator"

Changes sign of spins
without changing momenta

$$A_\gamma: (\hat{P}, \hat{\sigma}) = (-\hat{P}, -\hat{\sigma})$$

$\Theta = PA_\gamma = -$ for all
single-spin observables

QUANTUM ENTANGLEMENT IN TRANSVERSE SPIN ASYMMETRY

Transverse-Spin Asymmetries

delineate the boundary

The diagram illustrates the boundary between two regions: **PQCD** (left) and **NPQCD** (right). A blue circle labeled "PQCD" is positioned to the left of a green circle labeled "NPQCD". A curved green line, representing the boundary, starts from the top of the PQCD circle and descends towards the right, eventually enclosing the NPQCD circle. The region between the two circles is shaded light yellow.

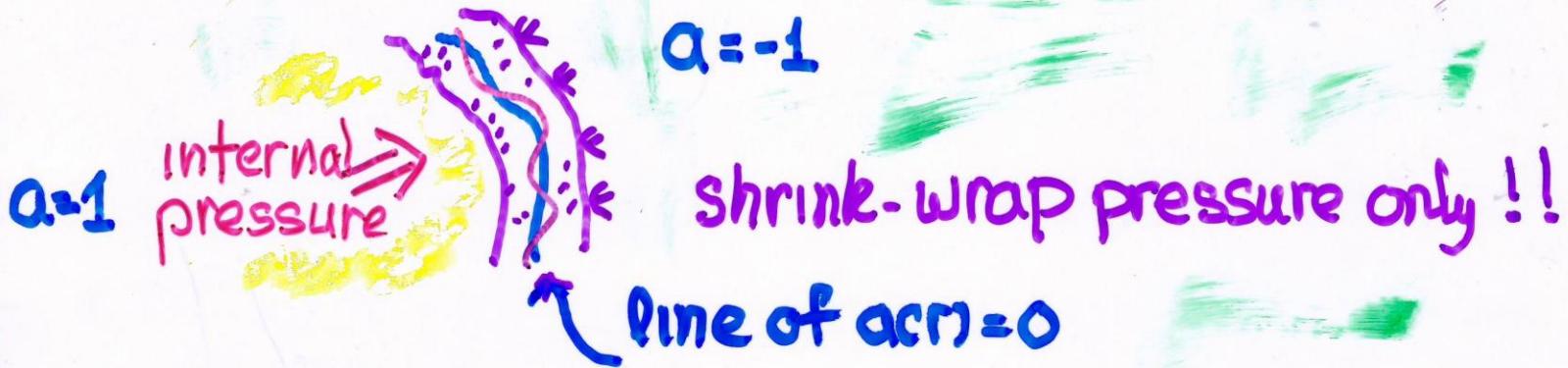
in pqcd

$\text{Im } \frac{q}{m} \frac{+ \cancel{q}}{\cancel{q}} = 0 \quad \text{Im } m_q > 0$

However there are nonperturbative dynamical mechanisms in the full theory that combine orbital angular momentum and hadronic-scale coherence

3P_0 $q\bar{q}$ pairs

The color-glass/sterile vacuum "kink" soliton with $\Delta a = \pm 2$ is also a topologically stable solution to $SU(N)$ field equations



Confines Abelian Gluons - adjoint charge

replaces dual superconducting "QCD vacuum" condensate with a dual insulator

Translates to cylindrical symmetry - QCD jets & Collins functions

not yet
studied
extensively

II. Lattice Studies of Color Vortices

Lattice QCD most important tool
for understanding color confinement

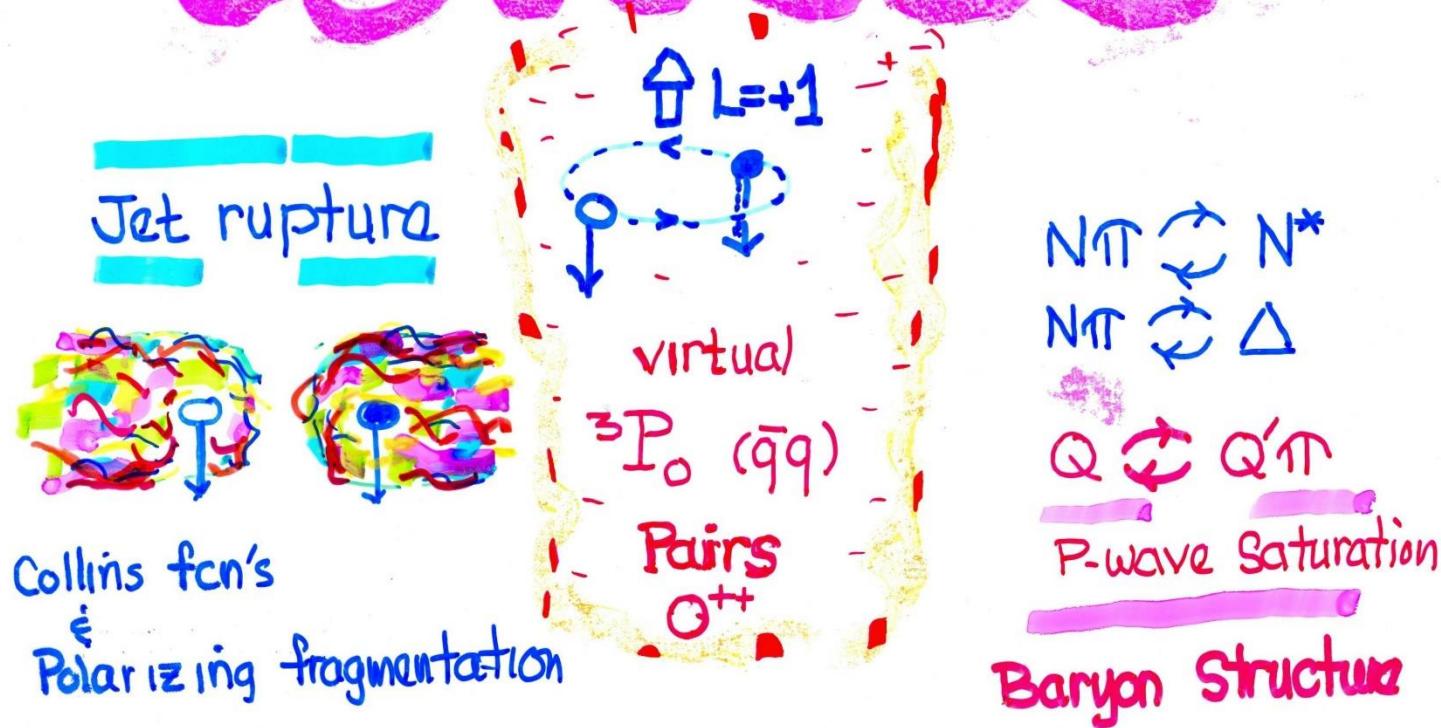
Jeff Greensite An Intro. to the Confinement Problem

Detailed Calculations Hadron spectra &
parton distributions

Engelhardt & Collaborators
Leinweber & Collaborators

Role of Color Vortices in the Confinement
mechanism Emergent Structures

The Straw that Stirs the Drink



Streater Wightman Axioms

Gauge theories &
Cluster Decomposition

non-Abelian gauge theories accomodate:

- (A) area law for Wilson loops
- (B) emergent structures such as topological domain walls

local fields \Rightarrow non-local correlations

ORBITAL CHROMODYNAMICS

and the

PION TORNADO

$$LL \rightarrow d \downarrow \pi^+$$

$$P \uparrow \rightarrow n \downarrow \pi^+$$

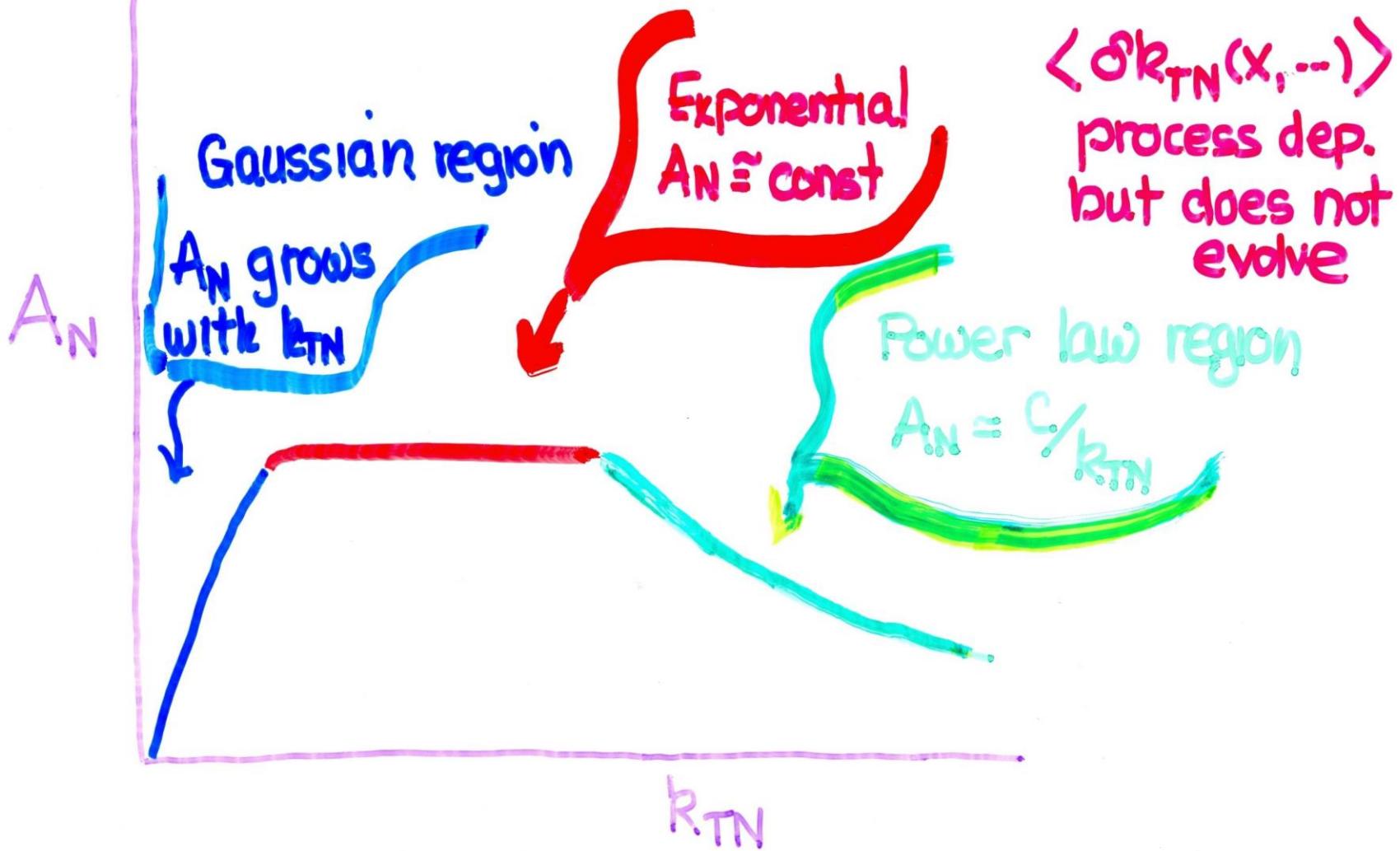
$$\langle L \rangle = 1$$

$$J(J+1) = L(L+1) + S(S+1)$$
$$+ 2L \cdot S$$

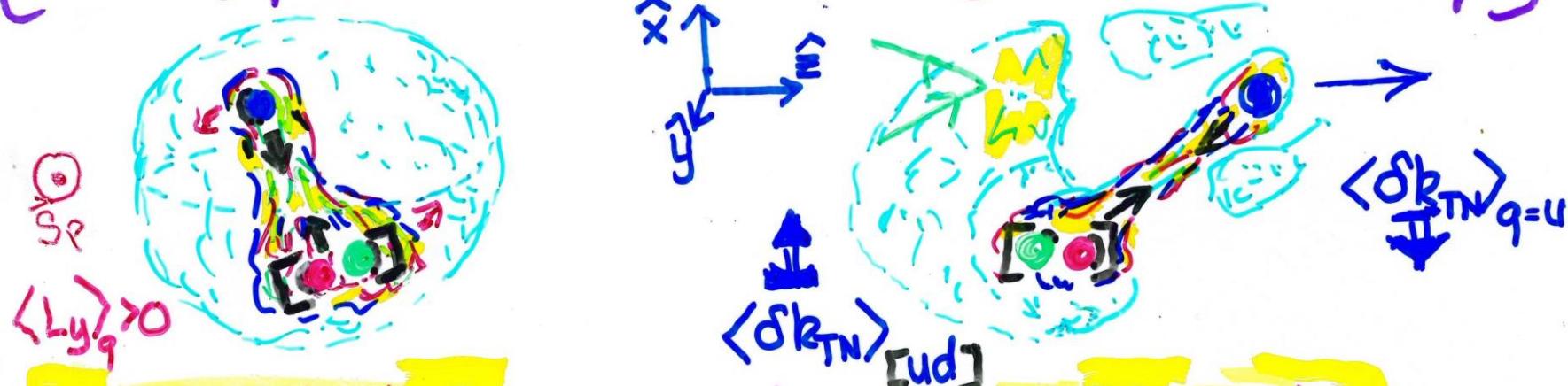
den sivers

$$J = L + S$$

A_N changes dramatically in response to QCD evolution in shape of $d\sigma/dk_{TN}$



{ In valence SIDIS diquark participates both
 in $\langle L_y \rangle_q$ and in the FSI that generates $\langle \delta k_{TN} \rangle_q$ }



This leads to $\langle \delta k_{TN}(x; \mu^2) \rangle_{[ud]} = -\eta(x) \langle \delta k_{TN}(x; \mu^2) \rangle_u$
 with ($\eta < 1$)

and the expectation

$$A_N(e p \uparrow \rightarrow e' p X) \approx A_N(e p \uparrow \rightarrow e' n X) \approx A_N(e p \uparrow \rightarrow e' \Lambda X)$$

$$\approx -e^{-\sigma} \hat{A}_N(e p \uparrow \rightarrow e' \pi^+ X)$$

Opposite sign

Diquarks and Diquark Fragmentation

$[q,\bar{q}] \ J^P = 0^+$ 3 color $[u,d] \ [d,s] \ [s,u]$ 3 flavor SU(3)

$\{\bar{q},\bar{q}\} \uparrow \ J^P = 1^+$ 3 color $\{\bar{u},\bar{u}\} \ \{\bar{u},\bar{d}\} \ \{\bar{d},\bar{d}\}$
 $\{\bar{u},\bar{s}\} \ \{\bar{d},\bar{s}\}$ 6 flavor SU(6)
 $\{\bar{s},\bar{s}\}$

gluonic radiation $[q,\bar{q}] \Rightarrow [\bar{q},\bar{q}]_6^- G \quad \{\bar{q},\bar{q}\} \Rightarrow \{\bar{q},\bar{q}\}_6^-$

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that are incorporated into fragmentation process

Two-Hadron TMD Fracture Functions in SIDIS

extend the fracture function formalism Trentadue Veneziano
conjoint probabilities instead of number densities

$$M_{(h_1, h_2)/p}^q [x, Q^2; (z^{h_1}, R_T^{h_1}), (z^{h_2}, \tilde{R}_T^{h_2})] \quad \text{1-jet fracture function}$$

transverse-spin asymmetry

$$\Delta^N M_{(\pi^\pm, p)/p\uparrow}^u \equiv M_{(\pi^\pm, p)/p\uparrow}^u - M_{(\pi^\pm, p)/p\downarrow}^u$$

$$= \Delta^N M_{(\pi^\pm, p)/p\uparrow}^u [x, Q^2; (\Sigma^{\pi^\pm}, k_{TN}^{\pi^\pm}, k_{TS}^{\pi^\pm})(\Sigma^p, k_{TN}^p, k_{TS}^p)]$$

assume π^\pm, p 1^{st.}-rank fragmentation products

constituent Quark

$J^P = \frac{1}{2}^+$ not Dirac Fermion

$G \neq 0$ (spin-orbit structure)



$\bar{3}$ color



$\bar{3}$ color



$\bar{3}$ color

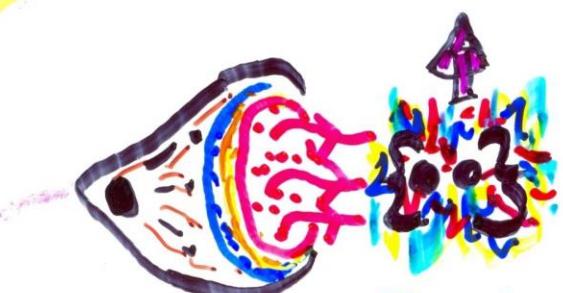
Scalar diquark

$\{\}$ antisymmetric in flavor

$J^P = 0^+$

0 dirac
antifermion
(hole)

• dirac
fermion



Axial vector diquark

$\{\}$ symmetric in flavor

$J^P = 1^+$ spin-orbit structure

Fundamental charge
 $3, \bar{3}$

OUTLINE

I. KPR Factorization & nonperturbative QCD

II. spin-directed momentum transfers at $m_q \neq 0$

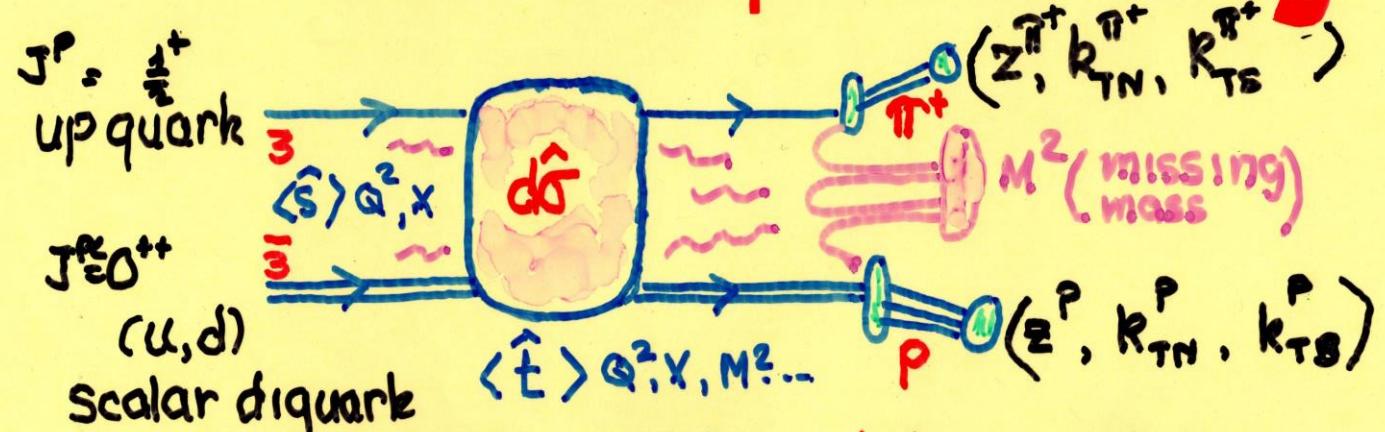
a) 3P_0 pairs & flux rupture.

b) nonperturbative final-state interactions

III. How to measure δR_{TN}

IV. Transverse-spin asymmetries & 2-hadron
fracture functions in $e p \uparrow \rightarrow e p \pi^\pm X$

Examine non-perturbative Quark Diquark Scattering



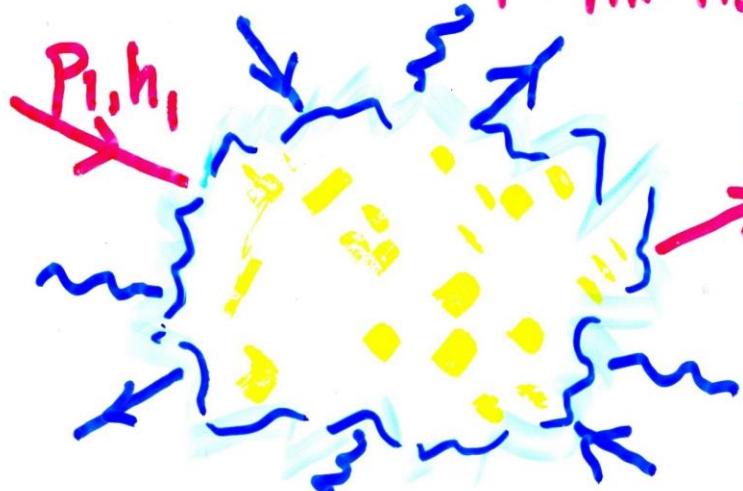
exchange kinematics determined by
measuring $\delta k_{TN}^{\pi^+}$ and δk_{TN}^Σ involved
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where $f(\theta_{cm}) = \frac{P_T N}{(\mu^2 + P_T^2 + P_S^2)^{1/2}} F(\theta_{cm})$ vanishes at $\theta_{cm} = 0$



quark helicity preserved
 $O(m_q v_{FS})$

quark helicity conservation in QCD pert. theory implies an additional symmetry in pqcd that is broken in the full theory



FRACTURED

FOOTY

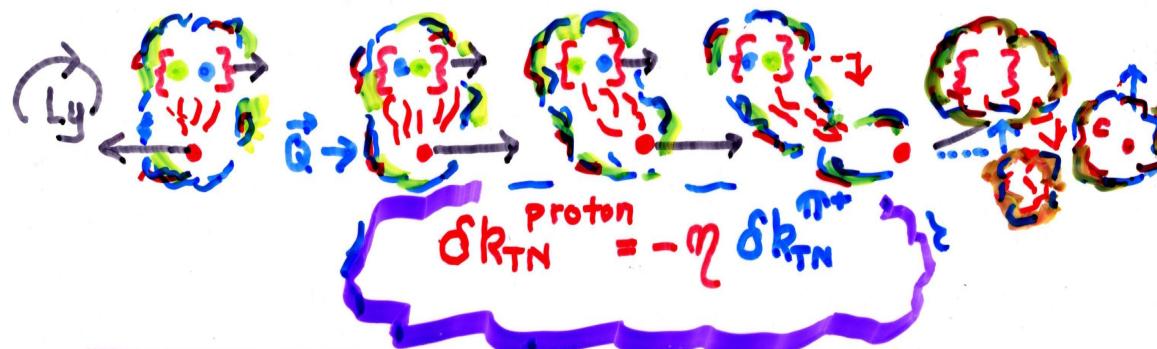
COLLINS
HEPPELMAN
LADINSKY



Non-Perturbative Final-State Interactions

orbital angular momentum $\langle \mathbf{Q} \cdot \hat{\mathbf{s}} \rangle = -0.65 \pm 0.1$ (Engelhardt)

SIDIS for proton polarized +ŷ direction



ordinary entanglement

proton in target fragmentation
 π^+ in current fragmentation

Diquarks and Diquark Fragmentation

$[q, \bar{q}]$ $J^P = 0^+$ 3 color

$[u, \bar{d}]$ $[d, \bar{s}]$ $[s, \bar{u}]$ 3 flavor SU(3)

$\{q, \bar{q}\}^\uparrow$ $J^P = 1^+$ 3 color

$\{\bar{u}, u\}$ $\{\bar{d}, d\}$ $\{\bar{s}, s\}$

$\{\bar{u}, s\}$ $\{\bar{d}, \bar{s}\}$

$\{\bar{s}, \bar{u}\}$

6 flavor SU(3)

gluonic radiation

$[q, \bar{q}] \Rightarrow [q, \bar{q}]_6^- G$

$\{q, \bar{q}\} \Rightarrow \{q, \bar{q}\}_6^-$

changes parity and color of diquarks but not flavor or symmetry

changes of flavor or symmetry require mesonic degrees of freedom

$\{q_i, \bar{q}_j\}^\uparrow \Rightarrow [q_i, \bar{q}_j](\bar{q}_k q_j)$

$[q_i, \bar{q}_j](\bar{q}_k \bar{q}_l) \Rightarrow \{q_i, \bar{q}_k\}^\uparrow (q_j \bar{q}_l)$

that are incorporated into

fragmentation process

Transverse-Spin Asymmetries

delineate the boundary

in pqcd

$$\text{Im } \Sigma^q = 0 \quad \text{Im } m_q > 0$$

pqcd
npqcd

However there are non perturbative dynamical mechanisms in the full theory that combine orbital angular momentum and hadronic-scale coherence

3P_0 $q\bar{q}$ pairs

QUANTUM ORBITAL DYNAMICS & SPIN-DIRECTED MOMENTUM TRANSFERS

$\langle \delta k_{Tn}(x, \mu^2) \rangle$



$\langle \delta p_{Tn}(z, \mu^2) \rangle$



from **CONFINED**

non-Abelian
SYSTEMS

Group Hadronic Physics
2014 - Baltimore

d-sivers

The Isospin dependence of
 $A_N(ep\uparrow \Rightarrow e'\Sigma^+x)$: $A_N(ep\uparrow \Rightarrow e'\Xi^0x)$
should prove to be very interesting



The fractured Boer-Mulders effect presents
a unique opportunity to confirm that a polarization
asymmetry can be generated by dynamical effects
within proton

$$P_{BM}(ep \Rightarrow e'\Lambda^0 x) \approx 0$$

$P_{BM}(ep \Rightarrow e'\Sigma^{+\prime} x)$ large and
negative !!

$$\left. \begin{array}{l} P_{BM}(ep \Rightarrow e' p x) \\ P_{BM}(ep \Rightarrow e' n x) \end{array} \right\}$$

can be measured
by rescattering on a
carbon polarimeter



The orbital dst's for quarks measured in SIDIS require both $\langle L \rangle \neq 0$ and final-state interactions involving the target remnants

The fraction of the spin-directed momentum transfer of the struck quark that is transmitted to the appropriate diquark

$$\langle \delta k_{TN}(x, \mu^2) \rangle_{\{ud\}}^{(c)} + \langle \delta k_{TN}(x, \mu^2) \rangle_{\{ud\}}^{(d)} = -2 \eta_u(x) \langle \delta k_{TN}(x, \mu^2) \rangle_u^{(c)}$$

$$\langle \delta k_{TN}(x, \mu^2) \rangle_{\{u\}}^{(d)} = -\eta_d(x) \langle \delta k_{TN}(x, \mu^2) \rangle_d^{(c)}$$

$$0 < \eta_u, \eta_d < 1$$

measures the binding of the quark-diquark system to the remaining constituents

The experimental evidence for HTT processes in

$$\frac{d\sigma}{dp} (pp \rightarrow \pi X)$$

$$\frac{d\sigma}{dp} (pp \rightarrow \pi X) = \frac{1}{\pi} \sum_{abcd} \int dx_a dx_b dz_c G_{q\bar{q}}(x_a; \mu^2) G_{b/\bar{p}}(x_b; \mu^2) D_{\pi X}(z_c; \mu^2) [\hat{S} \frac{d\sigma}{dt} (ab \rightarrow cd)] \delta(\hat{S} + \hat{t} + \hat{u} + \mu^2)$$

should include contributions from

$Gq = \pi q$ $\pi q \rightarrow Gq$ $\Theta\pi \rightarrow q\bar{q}$ $q\bar{q} = G\pi$ $\pi q \rightarrow \pi q$ in addition to the leading-order QCD processes

This observation has implications for

$$A_N \frac{d\sigma}{dp} (pp \rightarrow \pi X) = \frac{1}{\pi} \sum_{abcd} \int dx_a d^2 k_a dx_b d^2 k_b dz_c d^2 p_c \Delta^N G_{q\bar{q}p\bar{p}}(x_a, k_{Tb}^a, k_{Tc}^a; \mu^2) G_{b/\bar{p}}(x_b, k_{Tb}^b; \mu^2) D_{\pi X}(z_c, p_c^2; \mu^2) [\hat{S} \frac{d\sigma}{dt} (ab \rightarrow cd)] \delta(\hat{S} + \hat{t} + \hat{u} + \mu^2)$$

(also for the Collins, Heppelman, Ladinsky contribution)

~~Correlations~~ ⁱⁿ transverse-spin asymmetries

Connecting the SIDIS
[Target and Current Fragmentation Regions]

Gary Goldstein
Simonetta Liuti
Dennis Sivers

$e^+ p \rightarrow e^+ p$
 $e^- p \rightarrow e^- p$

OUTLINE

I. KPR Factorization & nonperturbative QCD

II. spin-directed momentum transfers at $m_q \rightarrow 0$

a) 3P_0 pairs & flux rupture

b) nonperturbative final-state interactions

III. How to measure δR_{TN}

IV. Transverse-spin asymmetries & 2-hadron
fracture functions in $e p \uparrow \rightarrow e p \pi^\pm X$

To appreciate the value of KPR factorization it is appropriate to incorporate the power of superselection rules and idempotent projection operators in QFT and quantum mechanics

all single-spin asymmetries

$$A(\vec{\sigma}) = [N(\vec{\sigma}) - N(-\vec{\sigma})] / [N(\vec{\sigma}) + N(-\vec{\sigma})]$$

are odd under an operator Θ

$$\Theta \{ \vec{k}_i; \vec{\sigma}_j \} \Theta^{-1} = \{ \vec{k}_i; -\vec{\sigma}_j \}$$

\vec{k}_i = 3 vectors
 $\vec{\sigma}_j$ = axial 3 vectors

Θ serves as a 3-D Hodge dual of the parity operator

$$P \{ \vec{k}_i; \vec{\sigma}_j \} P' = \{ -\vec{k}_i; \vec{\sigma}_j \}$$

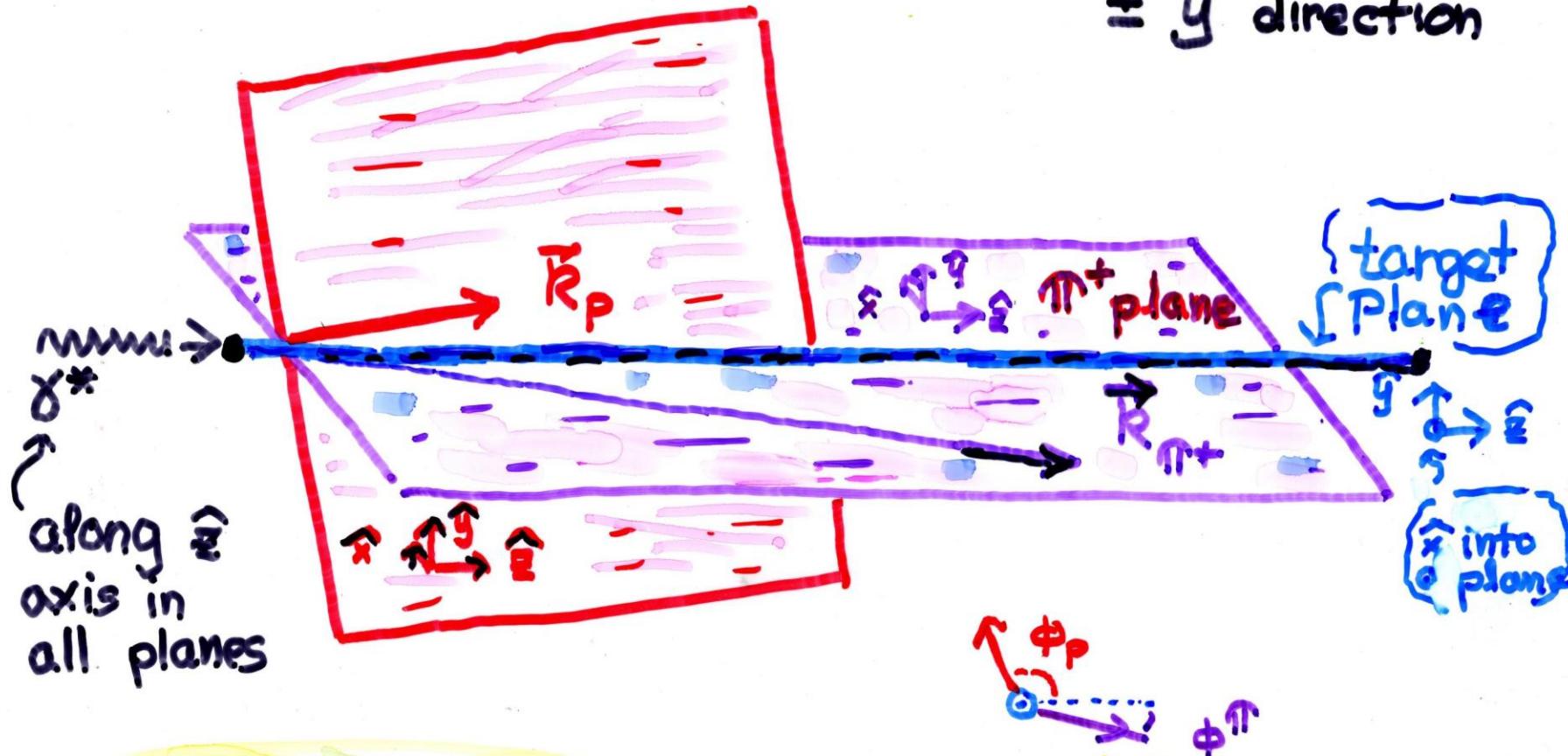
the product $A_\Sigma = P \Theta$ has the action

$$A_\Sigma \{ \vec{k}_i; \vec{\sigma}_j \} A_\Sigma^{-1} = \{ -\vec{k}_i; -\vec{\sigma}_j \}$$

A_Σ "naive time reflection" (Jaffe, 1994 ; Sivers, 1994)

Lab frame scattering planes $\gamma^* p \rightarrow \pi^+ p X$

target polarization
 $\pm y$ direction



assume proton & π^+ product of rank-1 fragmentation functions

Only one class of spherically-symmetric chromostatic condensates is compatible with the interior of a confined Lorentz-invariant structure - a color-glass condensate with

$$\alpha(r) = \pm 1 \text{ for } r < R_0 - \Delta$$

$$L(O^+) = E_i^a E_i^a - B_i^a B_i^a \geq 0 \quad L(O^-) = 2 E_i^a B_i^a = 0$$

for $\alpha(r) = +1$ there are two solitons to consider

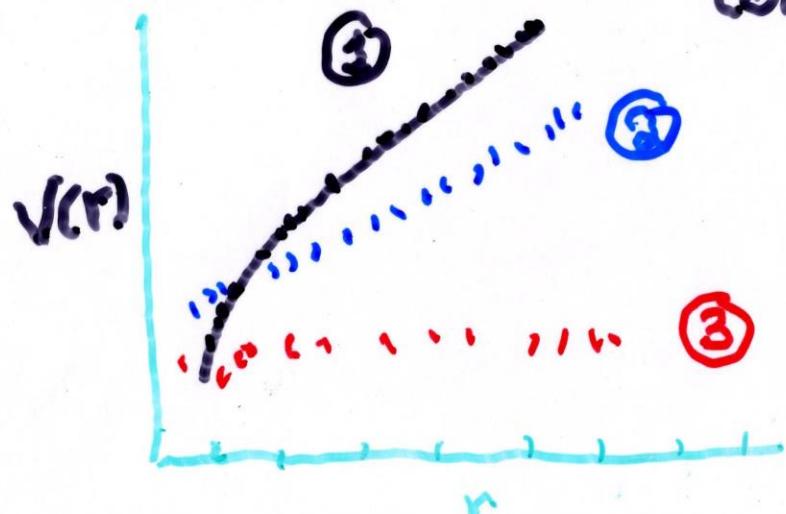
't Hooft-Polyakov $\alpha(r) = \alpha_{tp}(z) = \frac{1}{2} [1 - \tanh(\frac{\kappa z}{2})]$

chiral transition $\alpha(r) = \alpha_{ct}(z) = -\tanh(\kappa z)$

where $z = \left(\frac{r-R_0}{\Delta}\right)$ $\frac{\kappa}{\Delta} \gg 1$

Using the "maximal center gauge" it is possible to generate monte carlo gauge configurations with or without color vortices

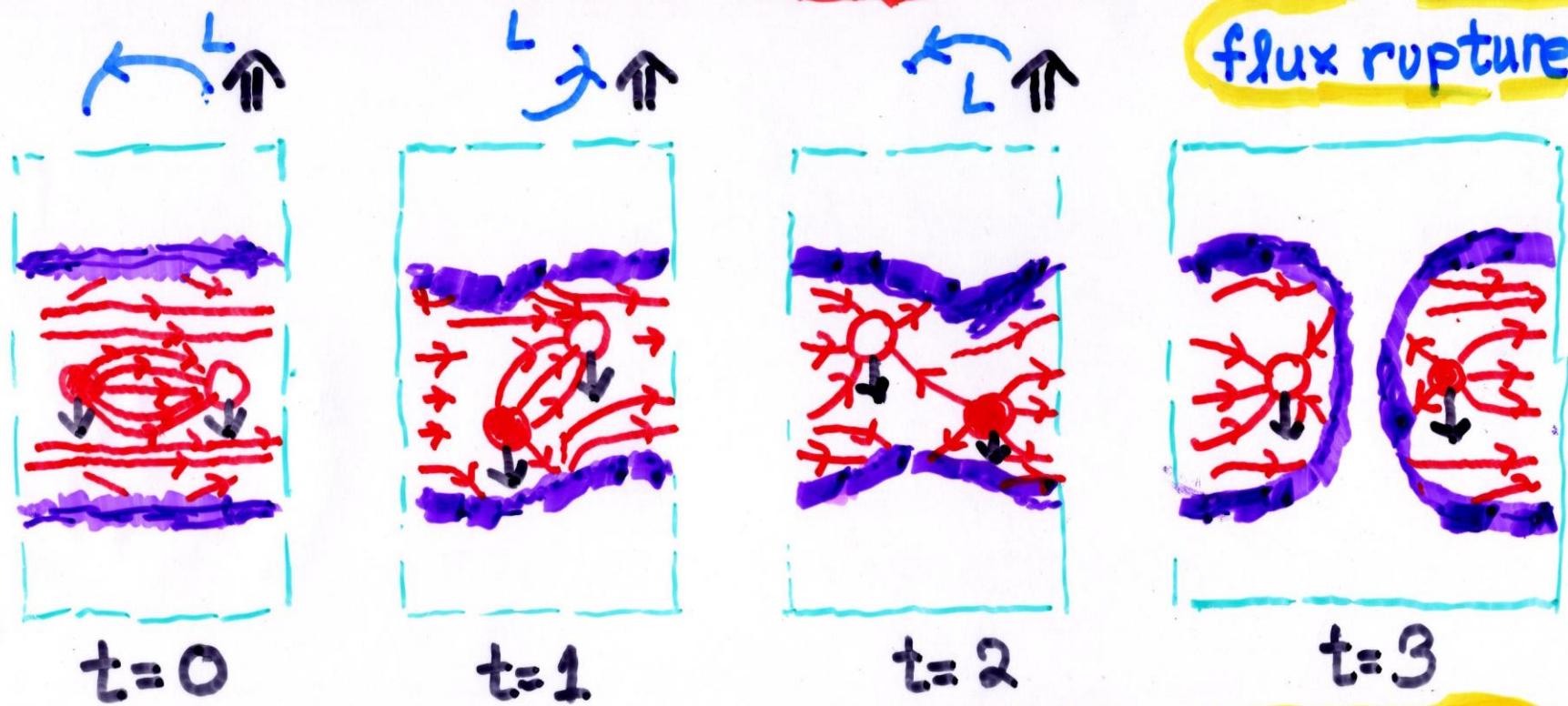
- ① regular
- ② vortices only
- ③ vortices removed



$$\text{Cornell Potential } V(r) = -\frac{q}{r} + \alpha_c + \beta r^2$$

Bowman et al
PR D84 034501 (2011)

3P_0 , $J^{PC} = 0^{++}$ $q\bar{q}$ -Pairs



generate flux rupture with
spin-oriented momentum transfer

δk_{TN}

Extension to SU(3)

The dimensional compression shown here is not unique to the gauge group SU(2) but can be extended to any SU(N). One possible extension to SU(3)

$$|\hat{r}_a\rangle^{(3)} = |\hat{z}_3, \hat{z}_8\rangle \quad \text{Abelian projections}$$

$$\frac{a(r,t)}{r} e_{ia}^S(\omega^3(r,t), w^8(r,t)) = \partial_i^{ab} |\hat{r}_b\rangle$$

$$\frac{a(r,t)}{r} e_{ia}^A(\omega^3(r,t), w^8(r,t)) = -i [|\hat{r}_b\rangle, \partial_i^{bc} |\hat{r}_c\rangle]_a$$

$i = 1, 2, 3 \quad a, b, c = 1 \dots 8$

Transverse Fields Carry SU(3) color
(1,2 4,5 , 6,7)

Matthias Burkardt

Nucl Phys (2004)
Phys Rev (2005)

non perturbative fsi deflects scattered
quark - attractive confining force ($\delta k_{TN}, \delta y$)

two particle fracture functions provide
access to internal dynamical mechanisms

beyond static structures



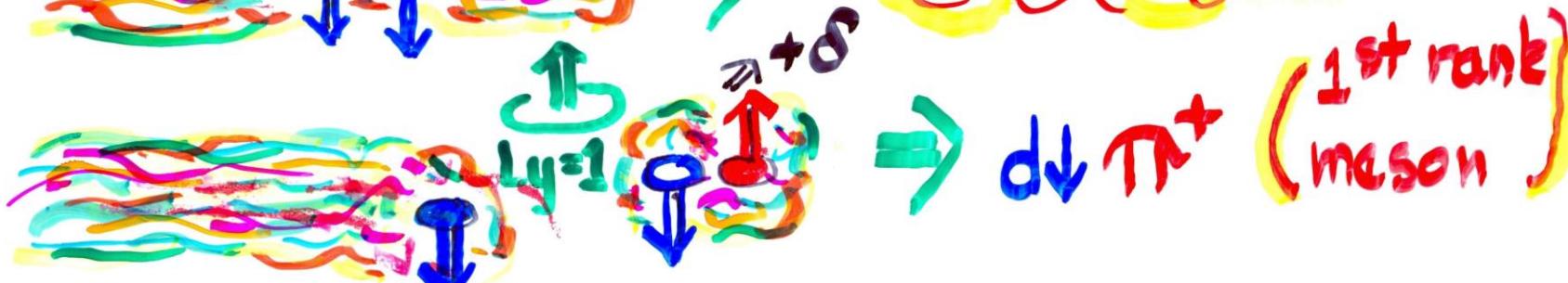
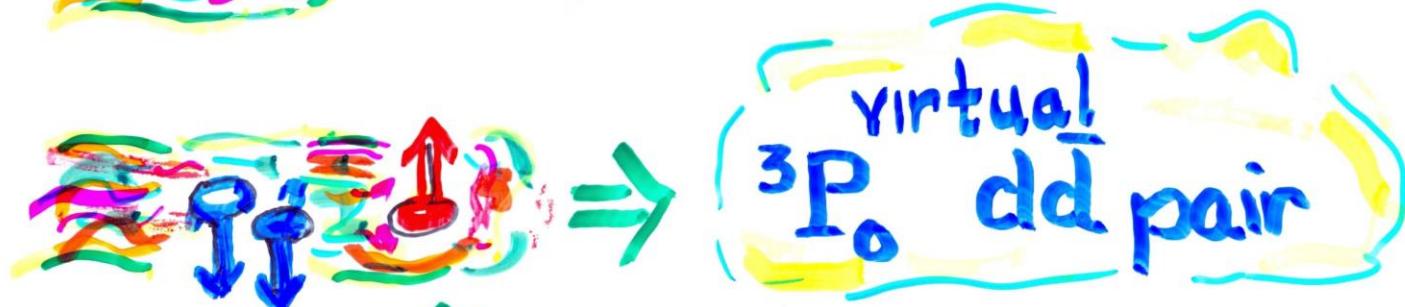
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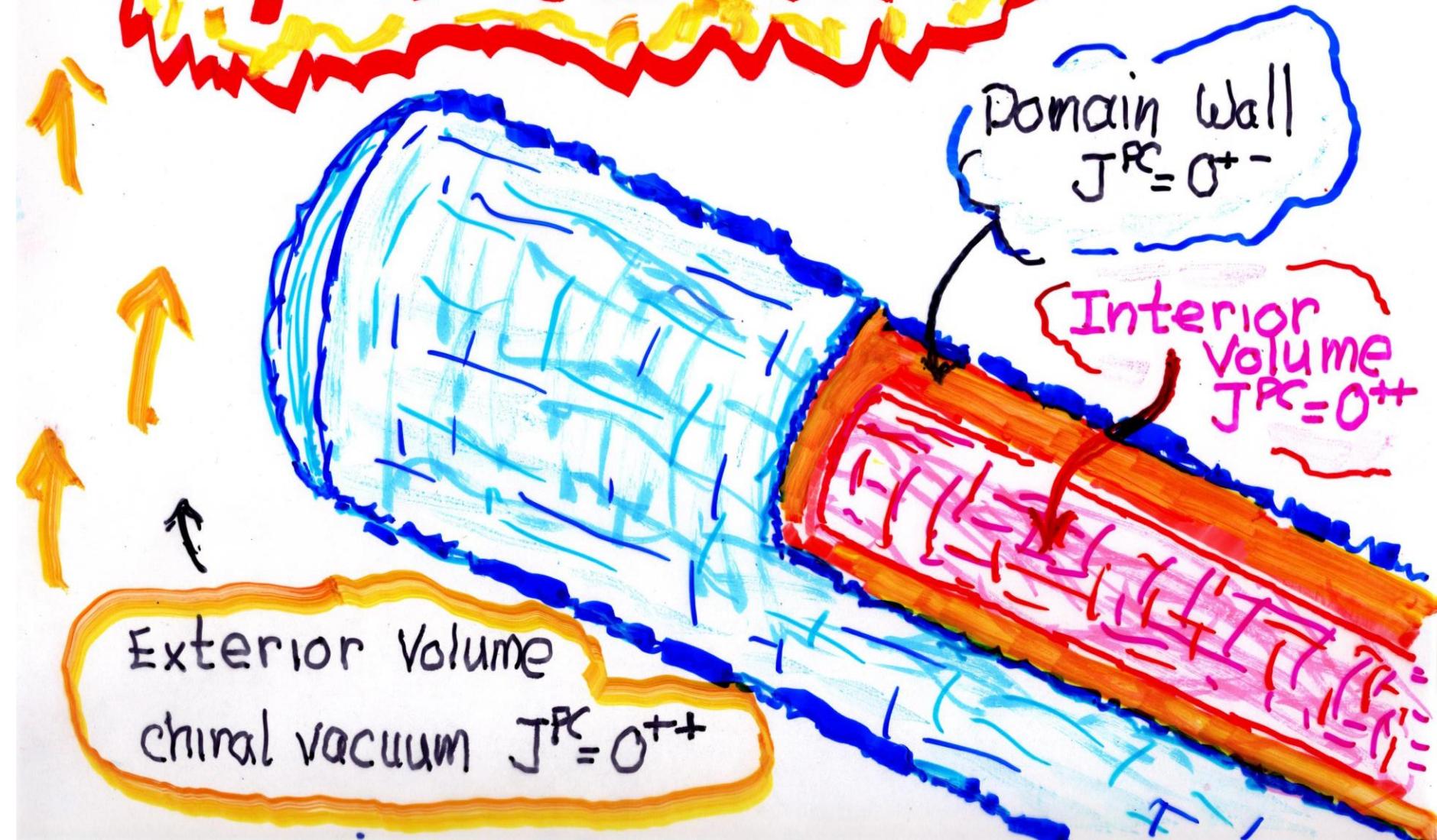
scattering amplitudes



"String" tension $\sim \kappa \Delta L \equiv 2\delta$ $\langle \delta \rangle \approx 0.2 \pm 0.03 \text{ GeV}$

Energy stored in flux \rightarrow transverse spin/directed momentum

Cylindrical SU(2)



Classification of condensates in $SU(2)$ chromostatics with spherical symmetry

$\alpha(r) = \pm 1$ $E_i^a E_i^a \neq 0$ $B_i^a B_i^a = 0$ color electric

$\alpha(r) = \pm 1$ $E_i^a E_i^a = 0$ $B_i^a B_i^a \neq 0$ color magnetic

$\alpha(r) = \pm 1$ $E_i^a E_i^a \neq 0$ $B_i^a B_i^a \neq 0$ color glass

$\alpha(r) = \pm 1$ $E_i^a E_i^a = 0$ $B_i^a B_i^a = 0$ sterile vacuum

$\alpha(r) = 0$ $E_i^a E_i^a = 0$ $B_L B_L = \pm \frac{1}{r} + \frac{1}{r}$ 't Hooft Polyakov

$\alpha(r) = c \neq \pm 1, 0$ $E_i^a E_i^a \neq 0$ $B_i^a B_i^a \neq 0$ topological
or dyonic

A domain wall is a region where $\alpha'(r) \neq 0$
that separates other condensates
and also carries topological charge

Experimental Evidence of $e\bar{p}^{\dagger} = e\bar{n}^{\dagger} p$
transverse spin asymmetry correlations
should already exist in CLAS-12
data from Jefferson Lab.

A range of similar correlation
measurements should be a part of
any program to study  QCD at EIC

