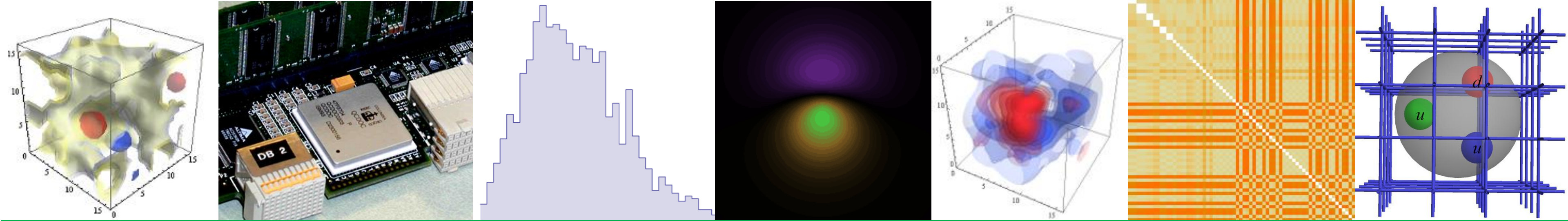


PDFs/GPDs on Lattice

Huey-Wen Lin

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405
and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

Outline

§ Consumer's Guide to Lattice Structure Calculations

- **Nucleon** structure with controlled systematics in the physical limit ($m_\pi \to m_\pi^{\text{phys}}, a \to 0, L \to \infty$)
- Physical-continuum lattice charges/moments

§ *x*-dependent Hadron Structure

- Recent Lattice PDFs Progress
- Applications to Generalized Parton Distributions
- Future Prospects and Challenges

Too much recent results to cover in 25 mins
Biased selected/highlighted results

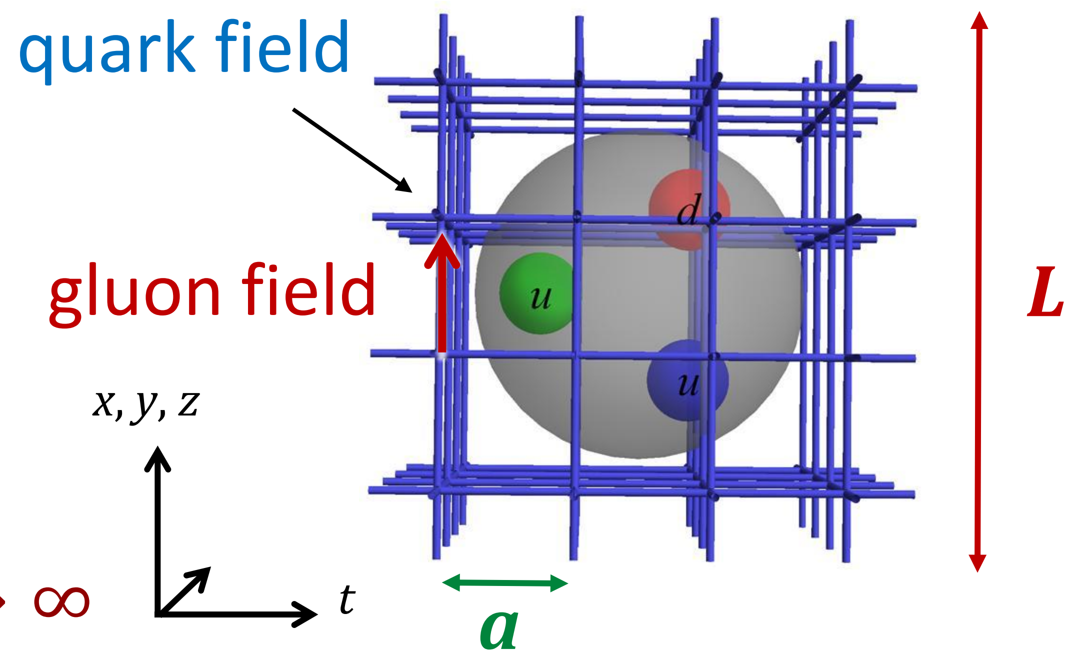
Lattice QCD in a Nutshell

- Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories
- Physical observables are calculated from the path integral

$$\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}A\, \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)} O(\bar{\psi}, \psi, A)$$

  in **Euclidean** space

  - Quark mass parameter (described by m_π)
  - Impose a UV cutoff
    discretize spacetime
  - Impose an infrared cutoff
    finite volume

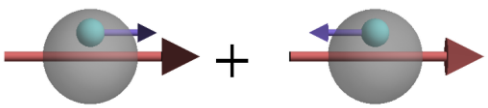


- Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

Moments of PDFs

- First moments are most commonly done

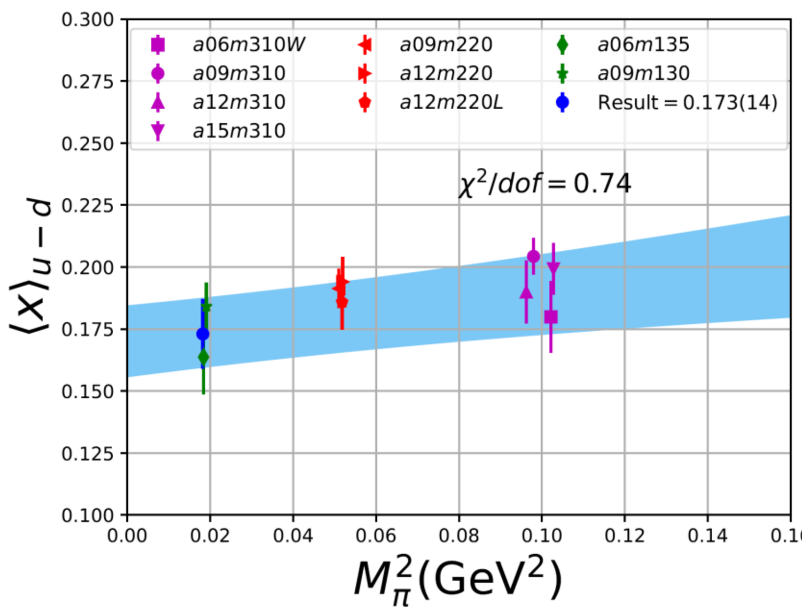


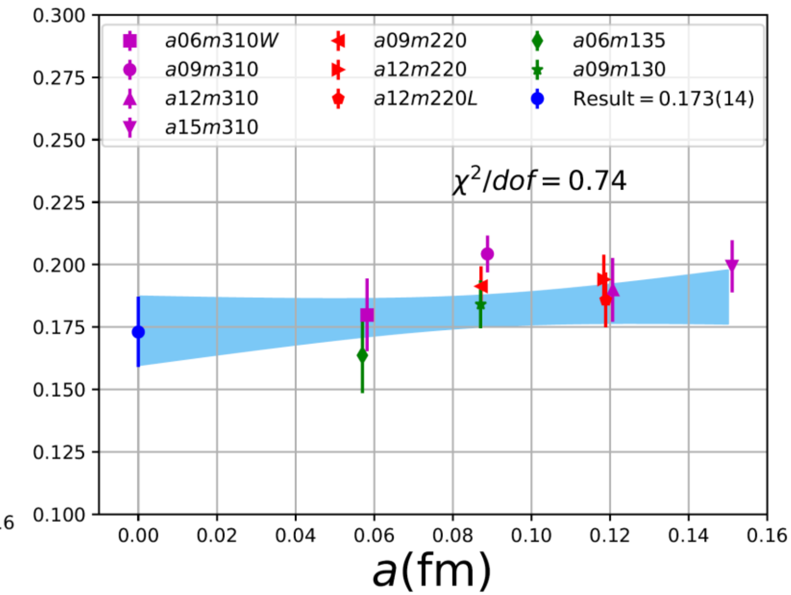
- State-of-the art example

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

  - Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779





- Usually more than one LQCD calculation
  - Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$

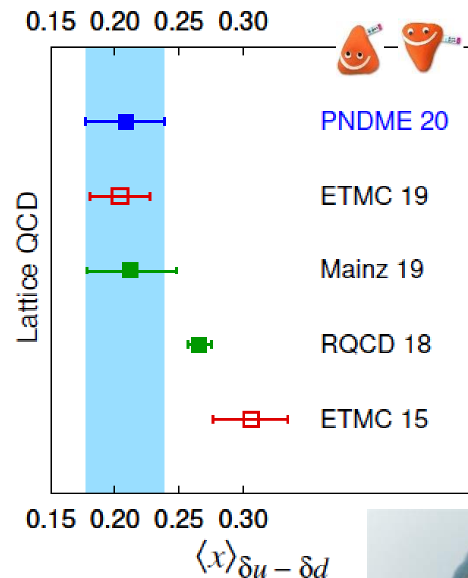
LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit	
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)		
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)		
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)		
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)		
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	0.10 — 1.1	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)		
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)		
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)		
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)		
$\langle 1\rangle_{\delta u^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)		
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	-0.14 — 0.91	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)		
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)		
$\langle 1\rangle_{\delta d^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)		
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	-0.97 — 0.47	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)		
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)		
$\langle 1\rangle_{\delta s^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)		
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	N/A	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)		
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)		

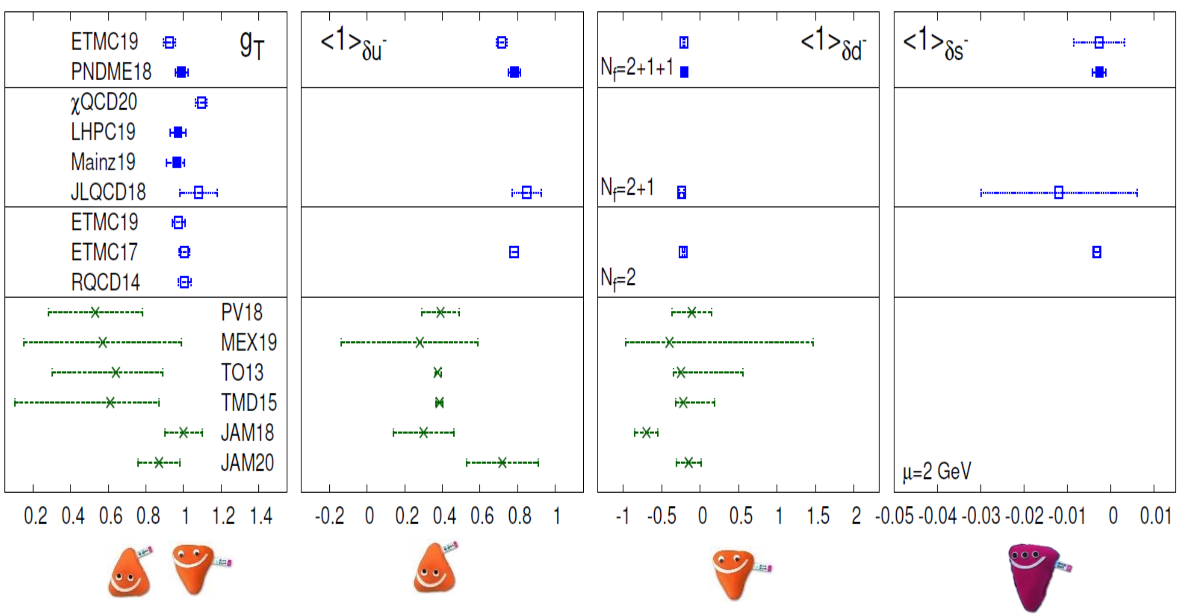
Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

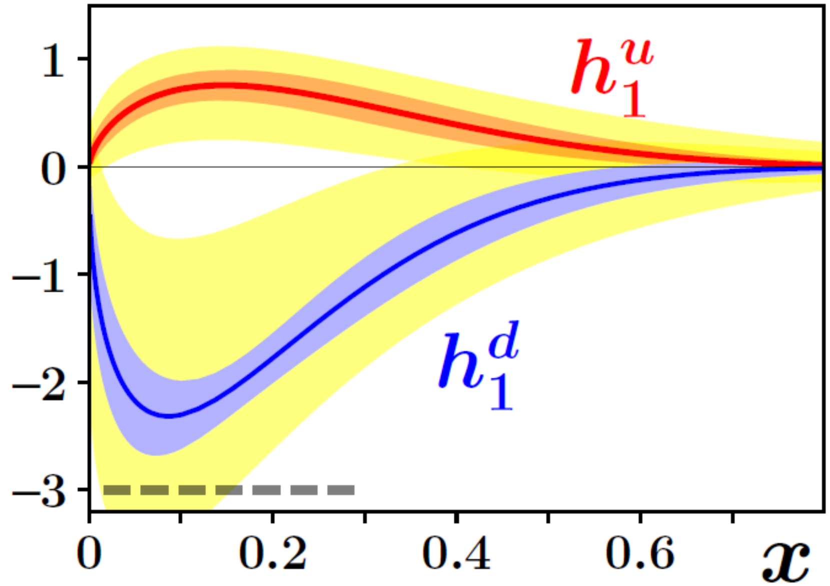


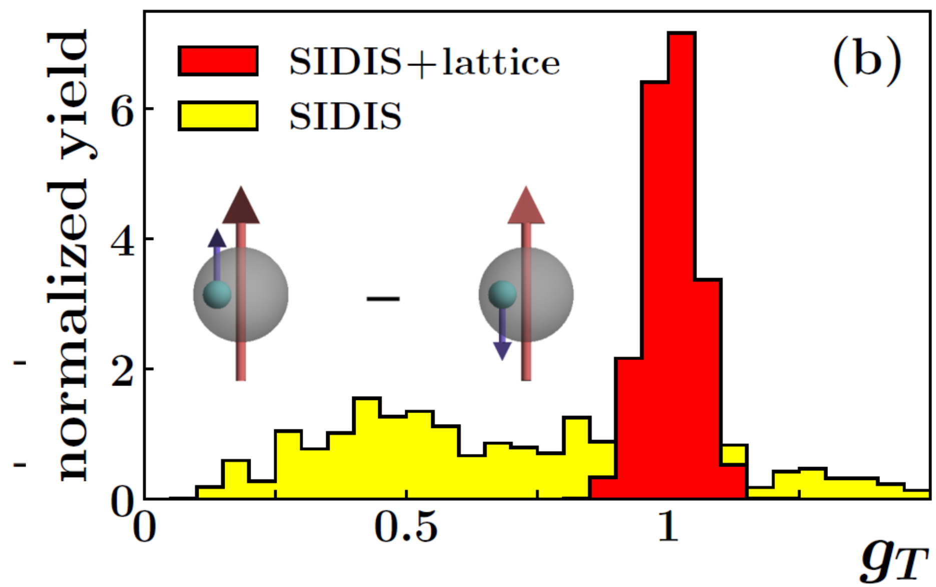
S. Mondal et al 2005.13779

From Charges to PDFs

§ Improved transversity distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global-analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets



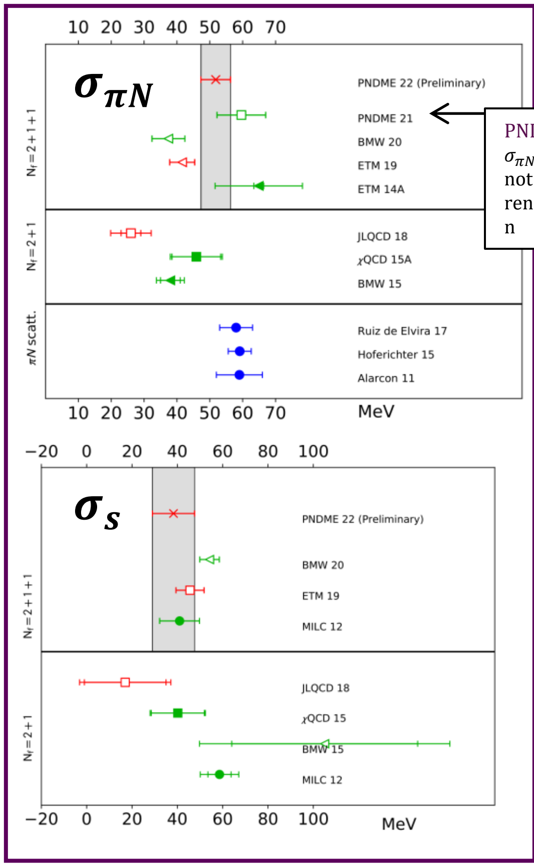


Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

Nucleon Flavor Diagonal Charges

Comparison with FLAG 2021 results

Nucleon sigma terms (Scalar charges)

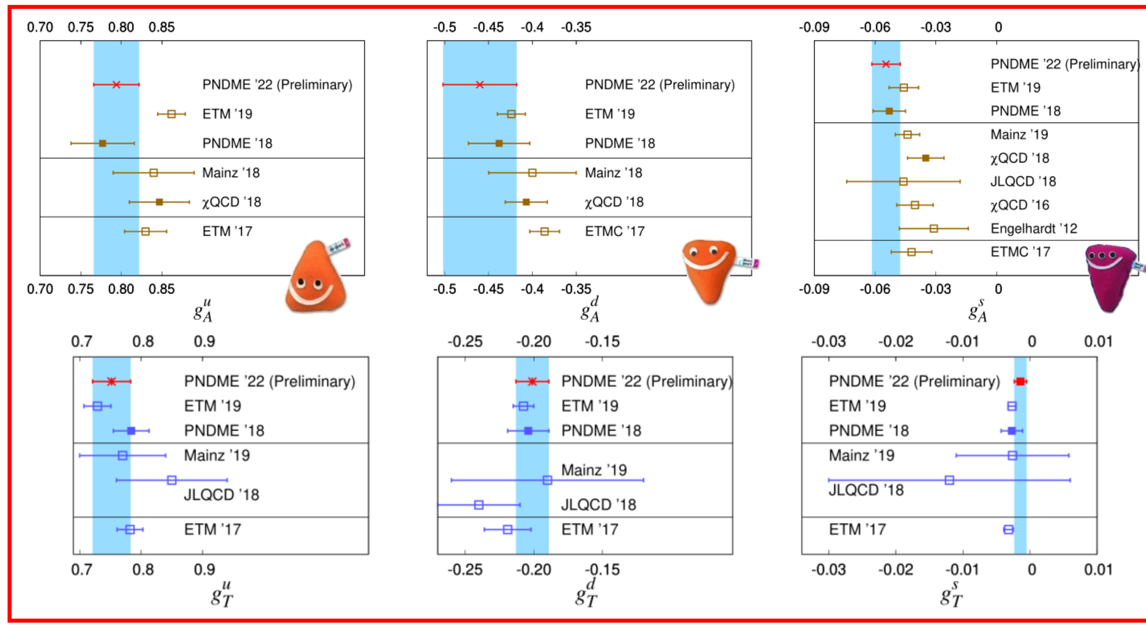


PNDME (2021) $\sigma_{\pi N}$ which does not require renormalizatio n

[PNDME collab., Lattice 2022 update, **preliminary**]

- Clover fermion on $N_f = 2 + 1 + 1$ HISQ ensembles
- Flavor mixing calculated nonperturbatively
- Chiral-Continuum extrapolation including a data at $M_\pi^{\rm Phys}$

Axial and Tensor charges



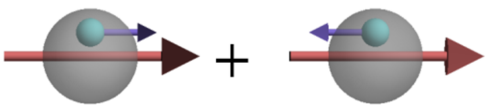
Plots by Sungwoo Park

Lots of new Lattice form factor results that I did not have time to cover

Structure on the Lattice

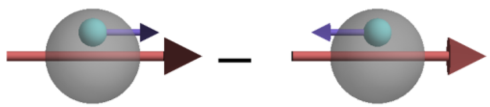
§ Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



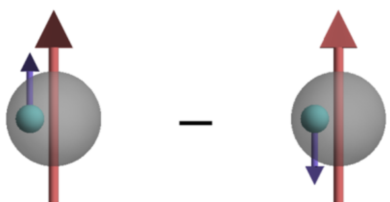
spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



spin-dependent
transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

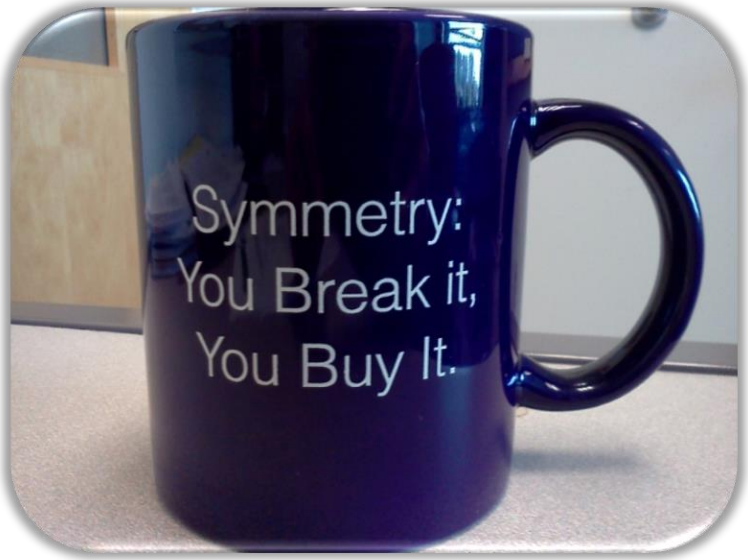
§ True distribution can only be recovered with all moments

PDFs on the Lattice

- § Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - Novel proposals to overcome this problem
- § Relative error grows in higher moments
  - Calculation would be costly
  - Hard to separate valence contrib. from sea

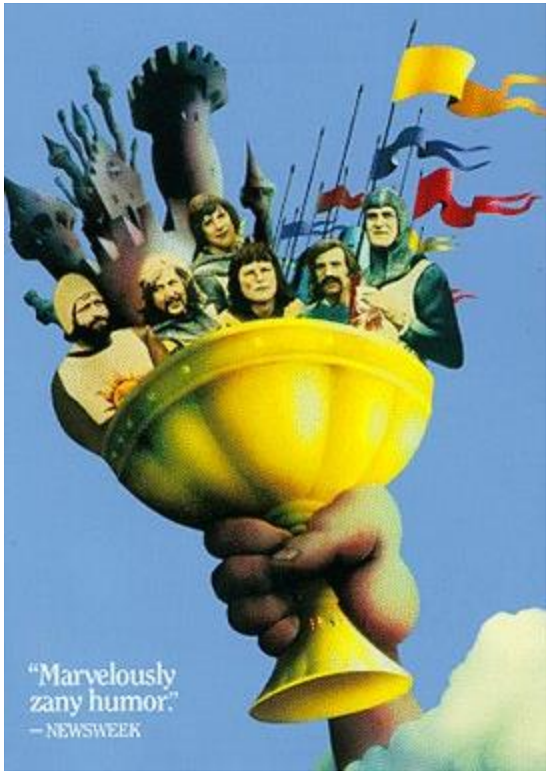
W. Detmold and C. Lin, Phys. Rev. D73 (2006) 014501

Z. Davoudi and M. J. Savage, Phys. Rev. D86 (2012) 054505



Beyond Traditional Moments?

- Longstanding obstacle!
- Holy grail of structure calculations
- Applies to many structure quantities:
  - Generalized parton distributions (GPDs)
  - Transverse-momentum distributions (TMD)
  - Meson distribution amplitudes…
  - Wigner distribution



A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

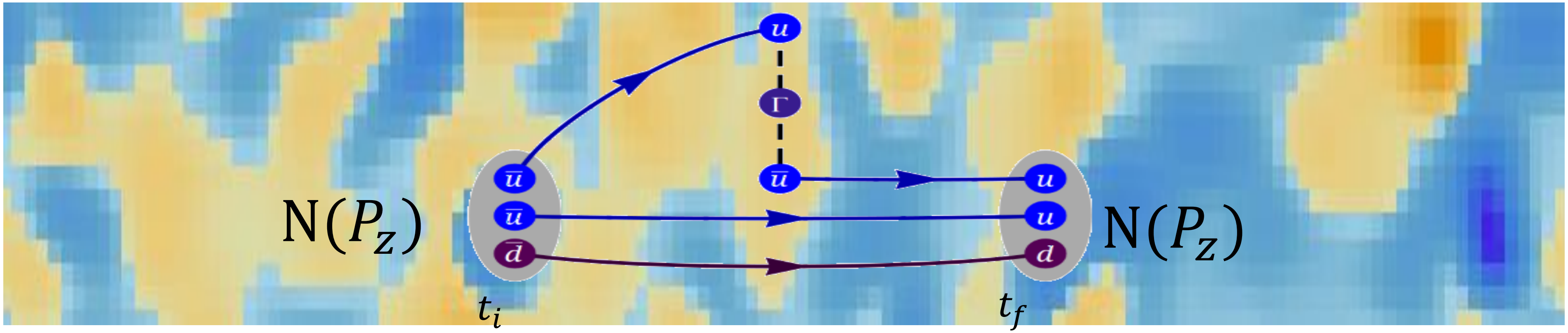
They carry secret plans to build the most powerful

Bjorken-x Dependent Hadron Structure

Biased selected/highlighted results

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF
(X. Ji, 2013; See 2004.03543 for review)



§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

§ Recover true distribution (take Pz → ∞ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{M_N^2}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(xP_z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{((1-x)P_z)^2}\right)$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Lattice Parton Method

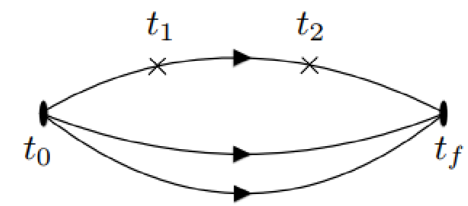
§ Short-distance factorization (SDF)

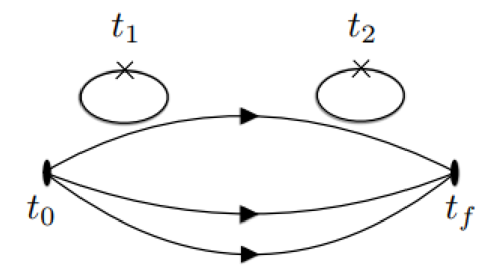
- **pseudo-PDF** method (A. Radyushkin, 2017)
- Hadronic tensor currents

  (Liu et al., hep-ph/9806491, … 1603.07352)
- Lattice cross-section method (LCS)

  (Y Ma and J. Qiu, 2014, 2017)
- Euclidean correlation functions

  (RQCD, 1709.04325)
- Compton amplitude approach (QCDSF, 1703.01153)



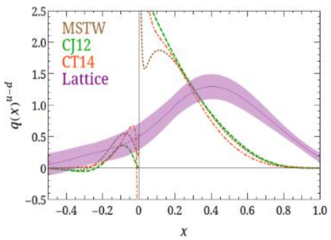


Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

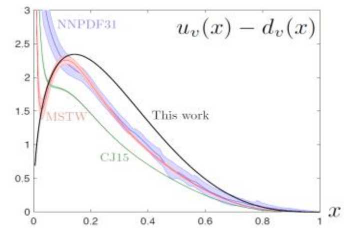
Lattice Parton Calculations

§ Rapid developments!

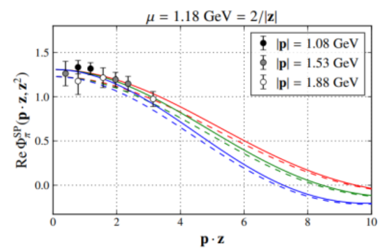
First unpol. PDF lattice calculation



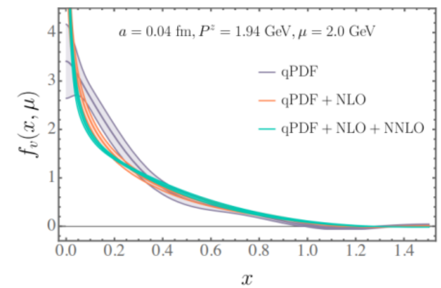
First lattice pseudo-PDFs



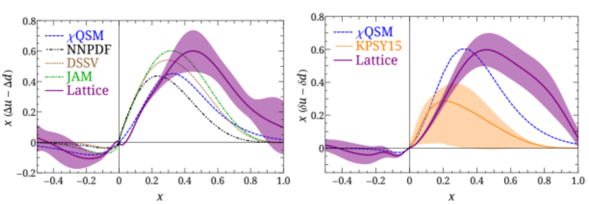
Euclidean correlation functions



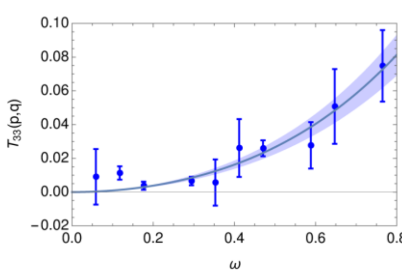
1st NNLO PDF



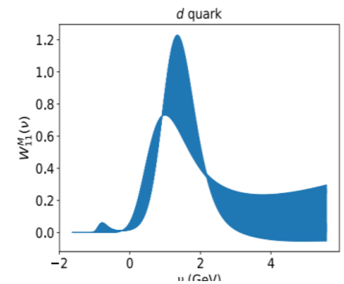
Pol. PDFs and mass corrections



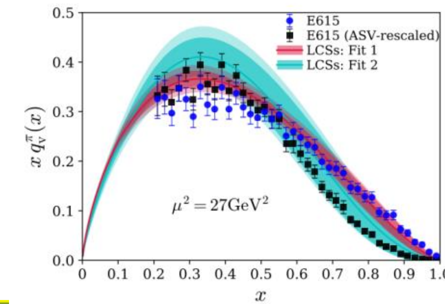
Compton amplitude



Hadronic tensor



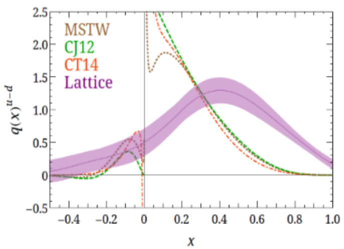
LCS



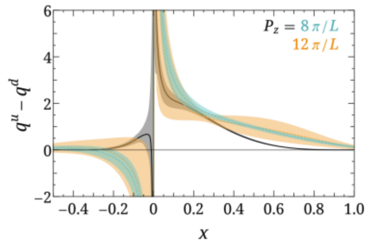
Lattice Parton Calculations

§ Physics quantity milestones

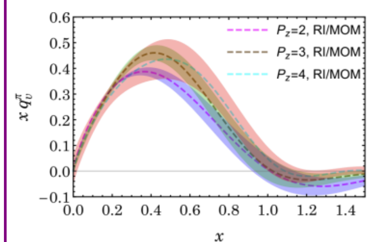
First unpol. lattice PDF



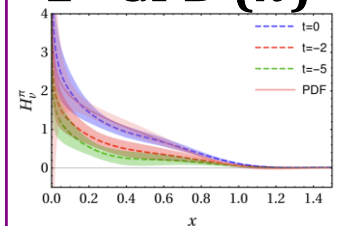
First PDFs at M_π^{phys}



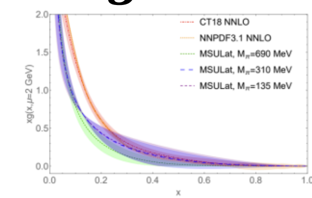
Pion v-PDF



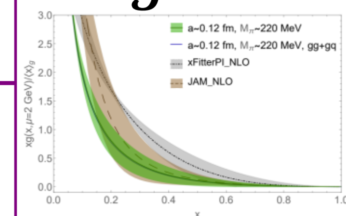
1st GPD (π)



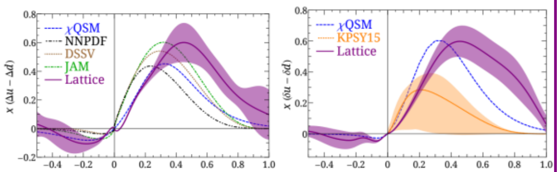
N g-PDF



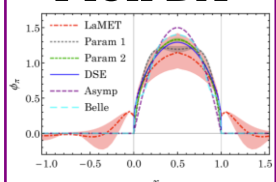
π g-PDF



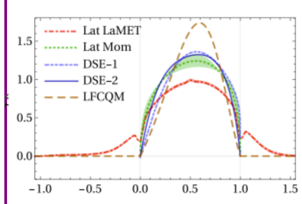
Pol. PDFs and mass corrections



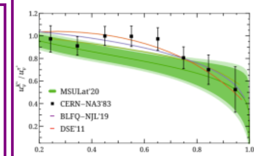
Pion DA



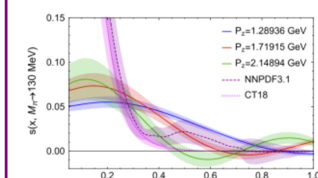
Kaon DA



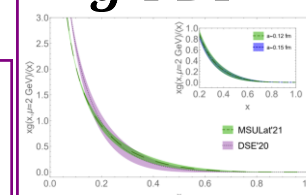
K PDF



s,c PDF



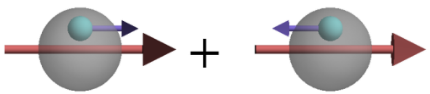
Kaon g-PDF



Lattice Example Results

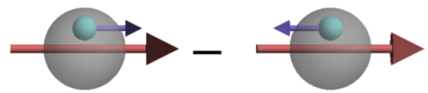
§ Summary of physical pion mass PDFs results

unpolarized



$u(x) - d(x)$

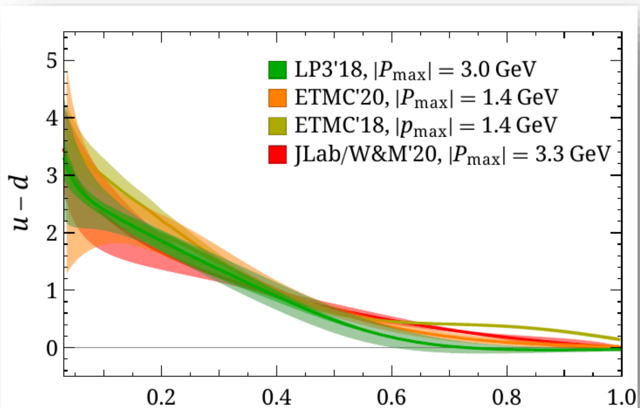
longitudinally polarized

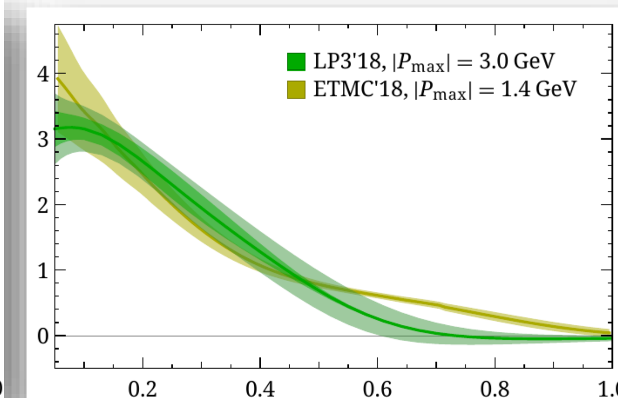


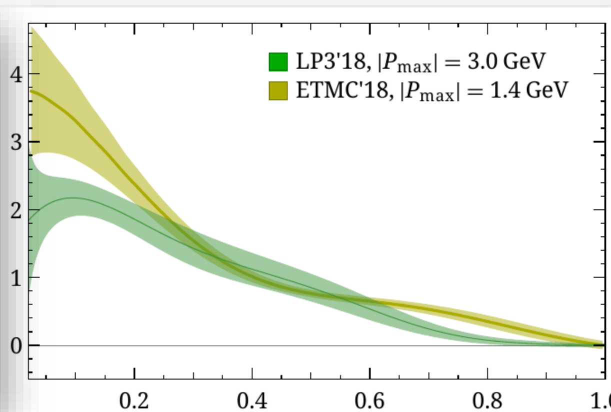
$\Delta u(x) - \Delta d(x)$

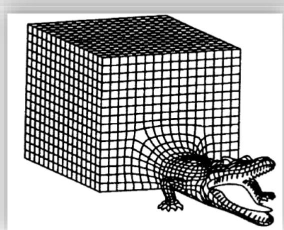
transversely polarized

$\delta u(x) - \delta d(x)$









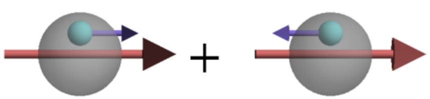
Finite volume,
Discretization,
…

2006.08636 (PDFLattice2019)

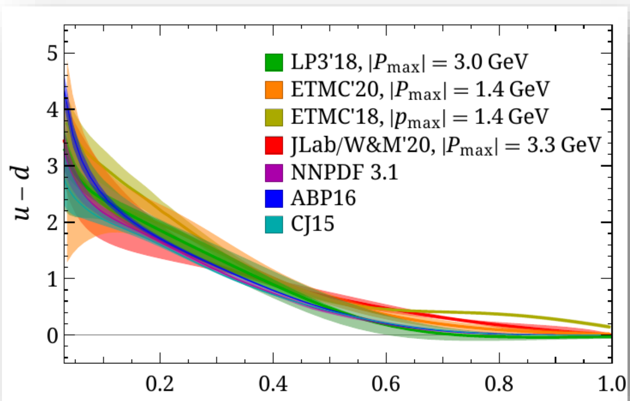
Lattice Example Results

§ Summary of physical pion mass PDFs results

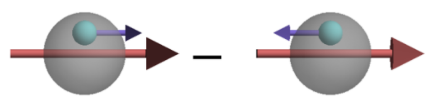
unpolarized



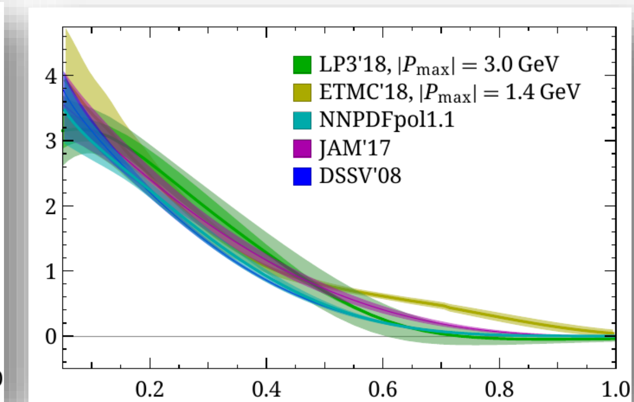
$u(x) - d(x)$



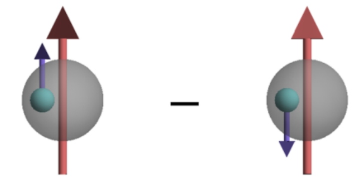
longitudinally polarized



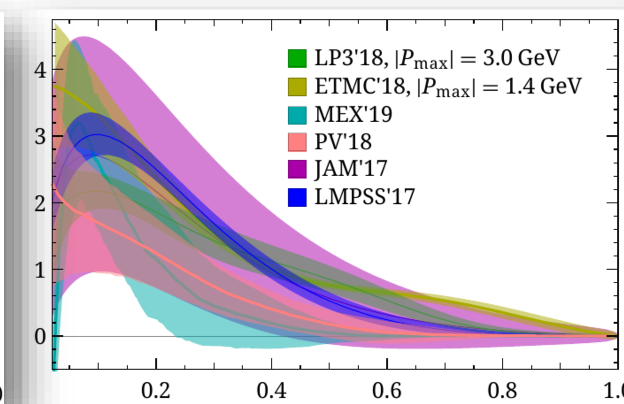
$\Delta u(x) - \Delta d(x)$

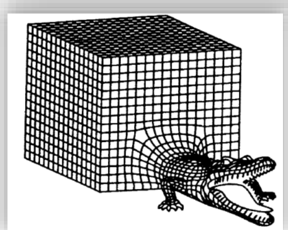


transversely polarized



$\delta u(x) - \delta d(x)$



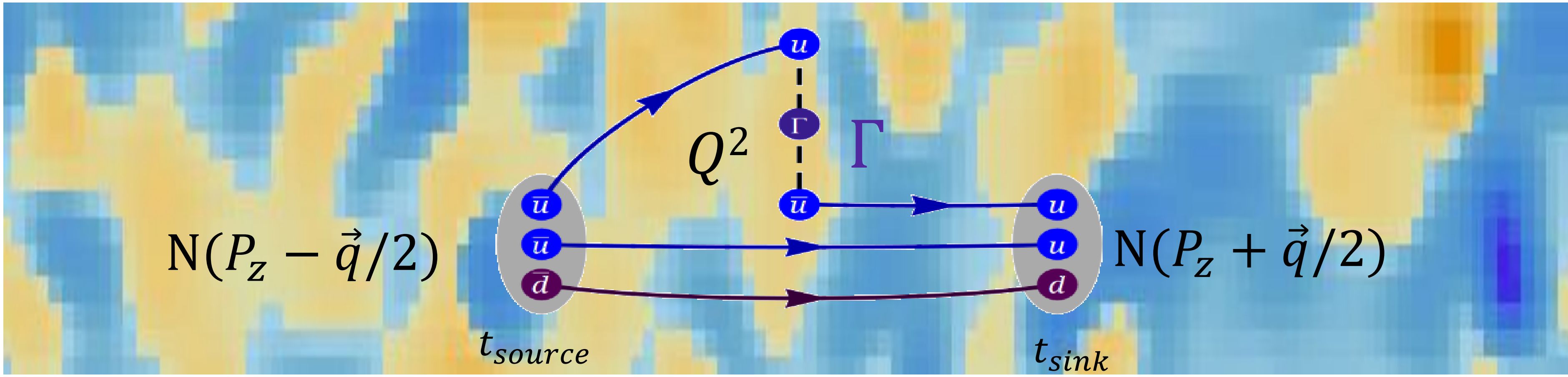


Finite volume,
Discretization,
…

2006.08636 (PDFLattice2019)

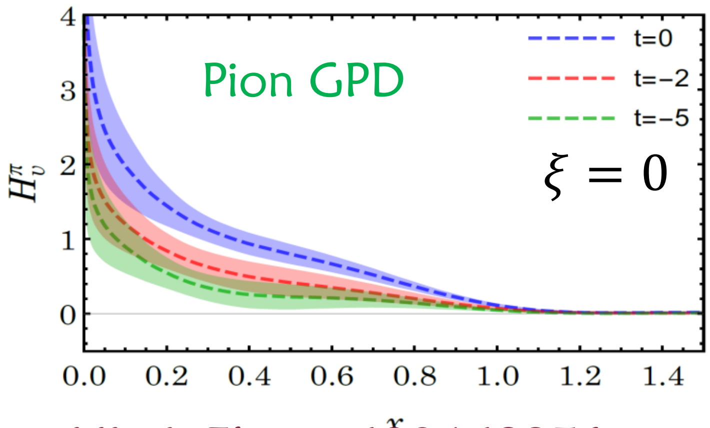
Generalized Parton Distributions

§ On the lattice, one needs to calculate the following

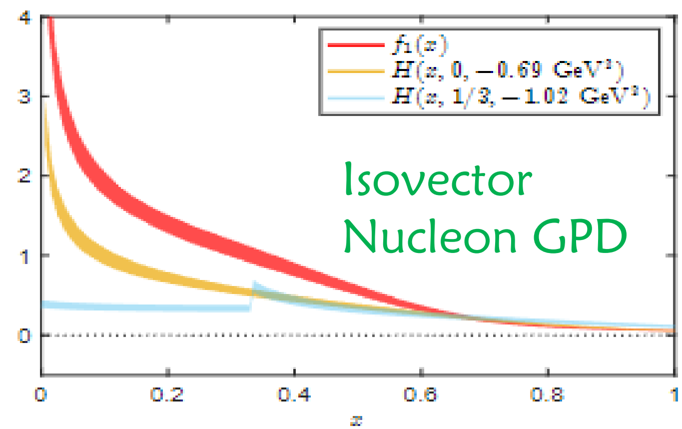


See 2209.05373 [hep-lat] by BNL/ETMC for new setup for GPD calculations

§ Heavy pion-mass results



J. Chen, HL, J. Zhang, 1904.12376

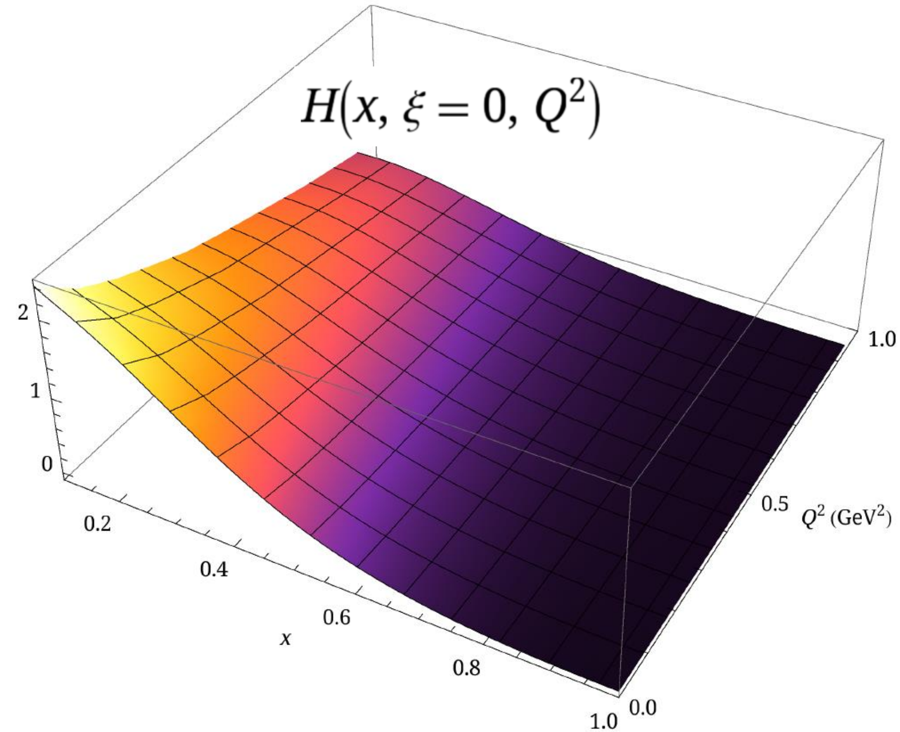


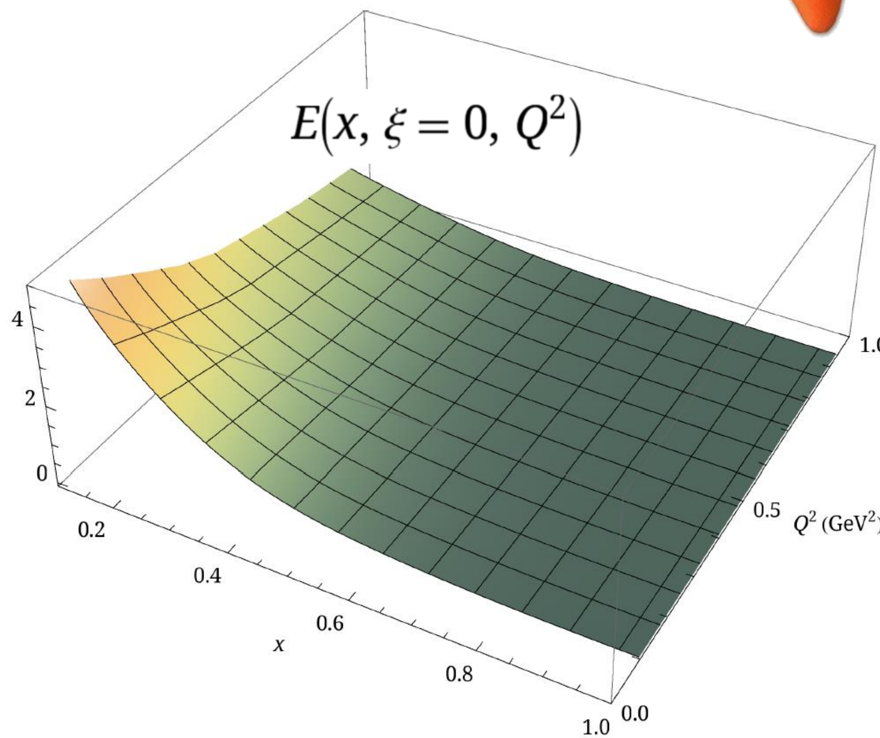
C. Alexandrou et al, 2008.10573

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results





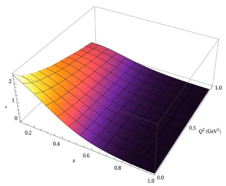
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Isovector Nucleon GPDs

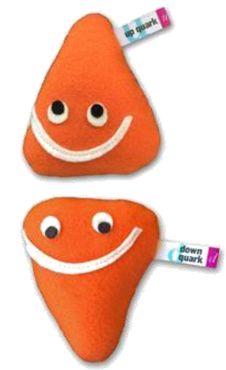
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

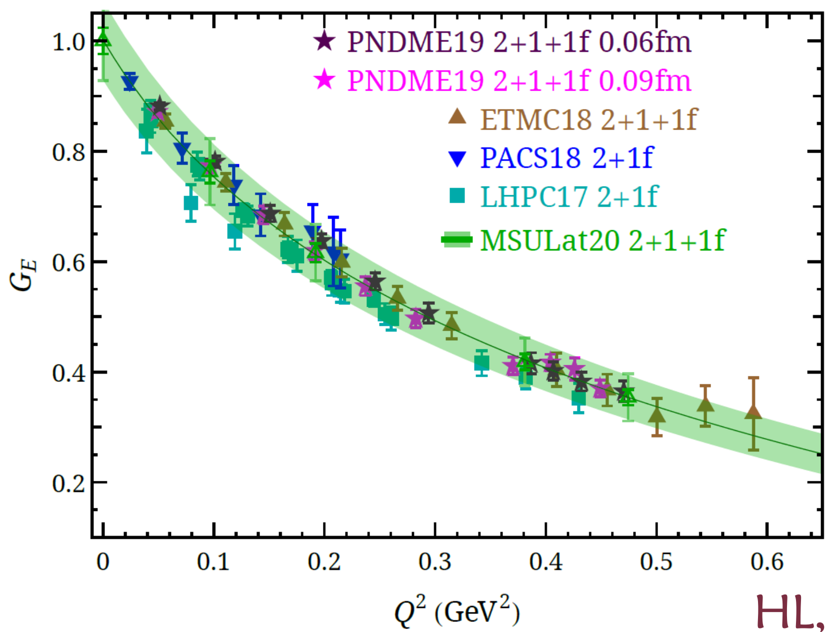
$$\int_{-1}^{+1} dx\, x^{n-1}$$



$$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A_{ni}^q(t) + (-2\xi)^n C_{n0}^q(t)\Big|_{n\ \text{even}}$$



$n = 1$

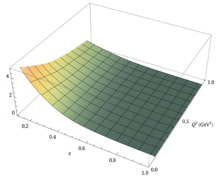


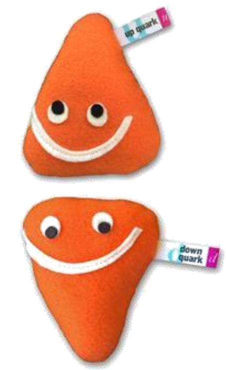
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon GPDs

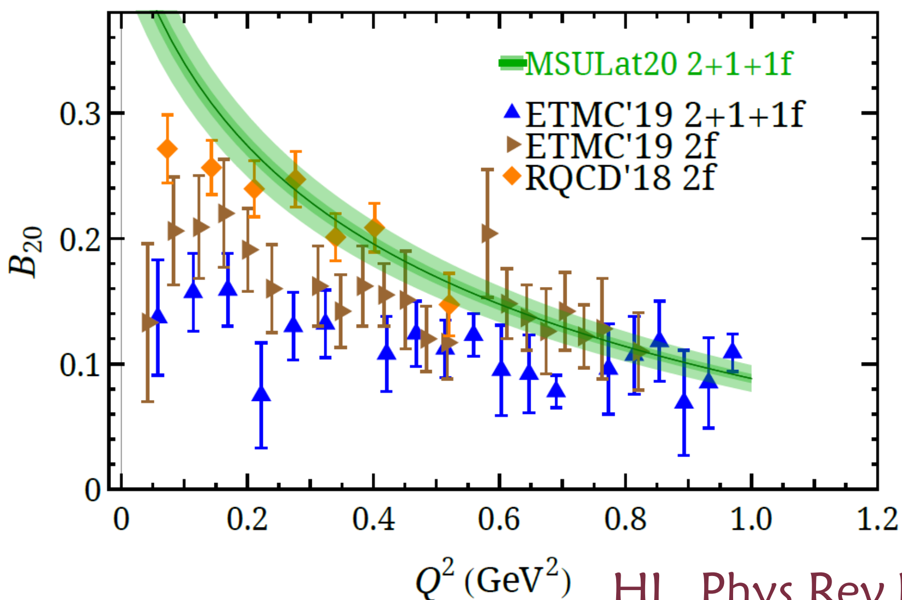
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$\int_{-1}^{+1} dx\, x^{n-1}$$  $$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B_{ni}^q(t) - (-2\xi)^n C_{n0}^q(t)\Big|_{n\ \text{even}}$$



$n = 2$

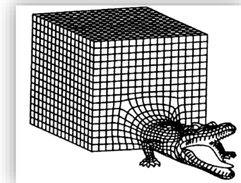


HL, Phys.Rev.Lett. 127 (2021) 18, 182001

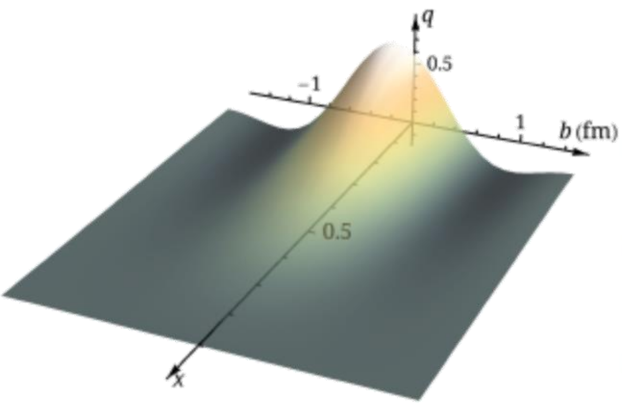
Nucleon Tomography

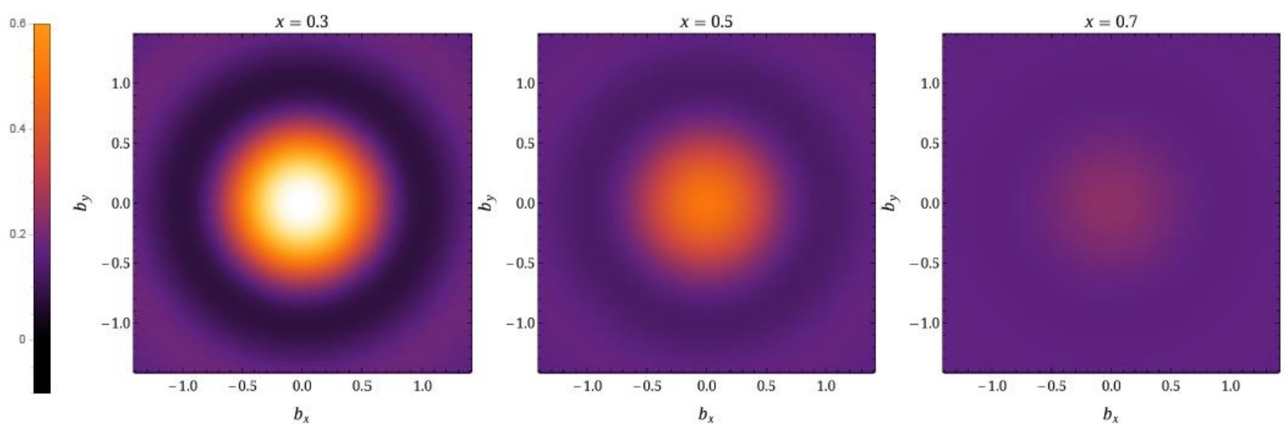
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results



$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x,\xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$





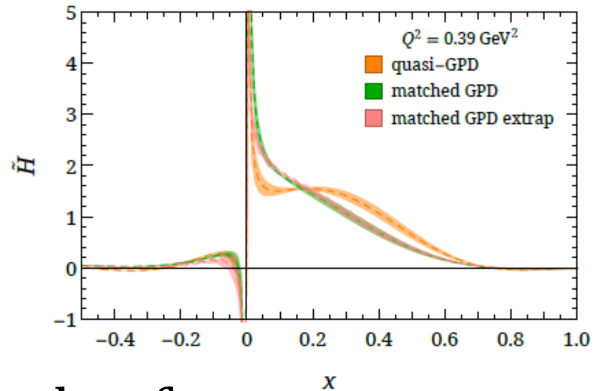
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

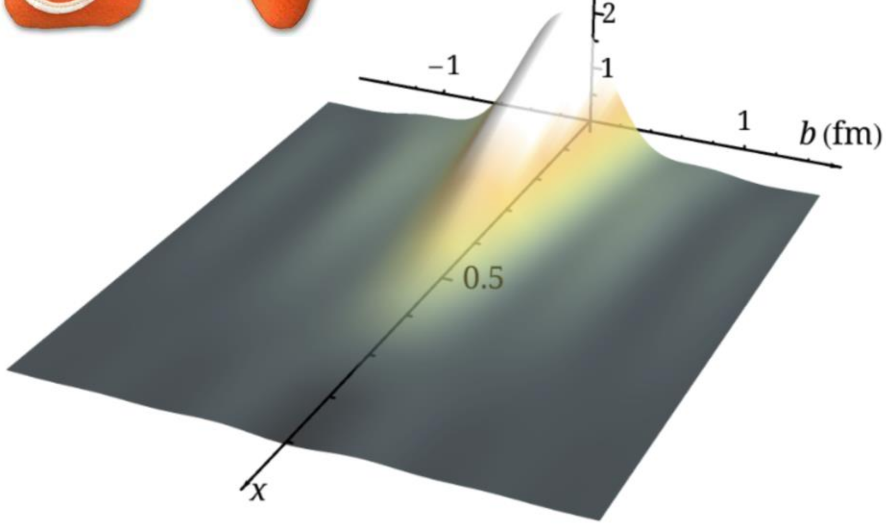
Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

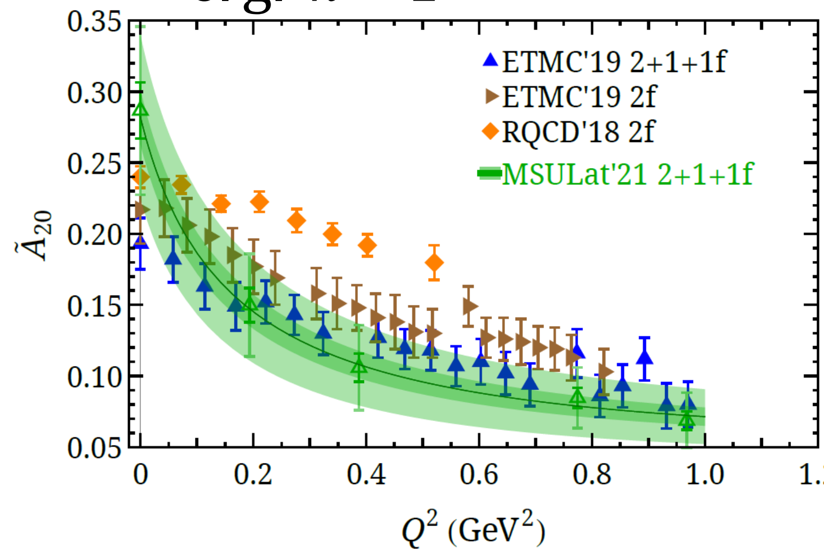
- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon (quasi-)GPD results

HL (MSULat), Phys.Lett.B 824 (2022) 136821



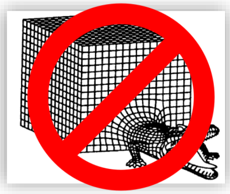


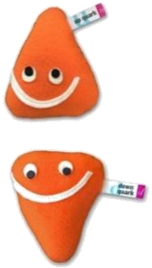
- Take the integral to form moments;
  e. g. $n = 2$



First Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit



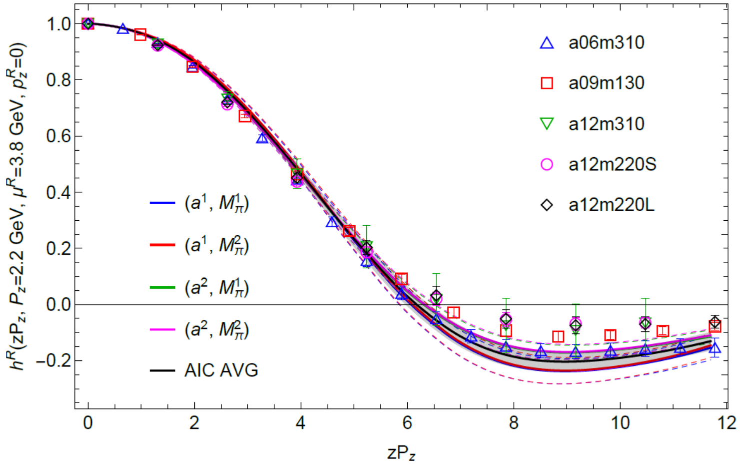


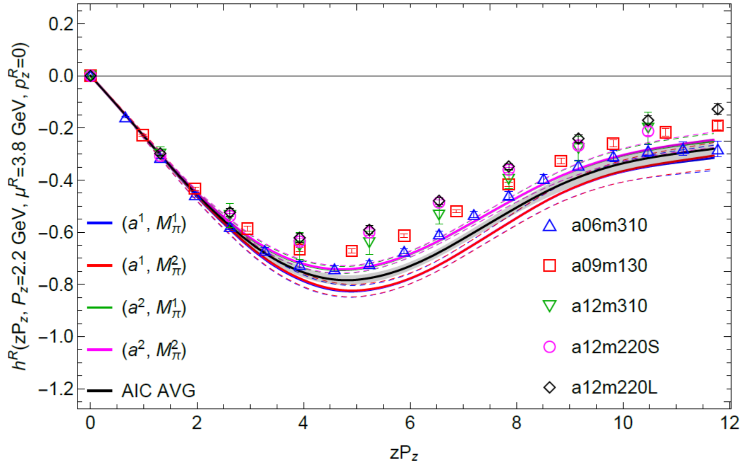
- Lattice details: clover/2+1+1 HISQ (MSULat)
  - $a \approx \{0.06, 0.09, 0.12\}$ fm,
  - $M_\pi \in \{135, 220, 310\}$-MeV pion,
  - $M_\pi L \in \{3.3, 5.5\}$.
  - $P_z \approx 2$ GeV

2011.14971, HL et al (MSULat)

- Naïve extrapolation to physical-continuum limit

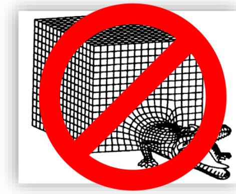
Quantities that can be calculated on the lattice





First Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit



- Lattice details: clover/2+1+1 HISQ (MSULat)
  - $a \approx \{0.06, 0.09, 0.12\}$ fm,
  - $M_\pi \in \{135, 220, 310\}$-MeV pion,
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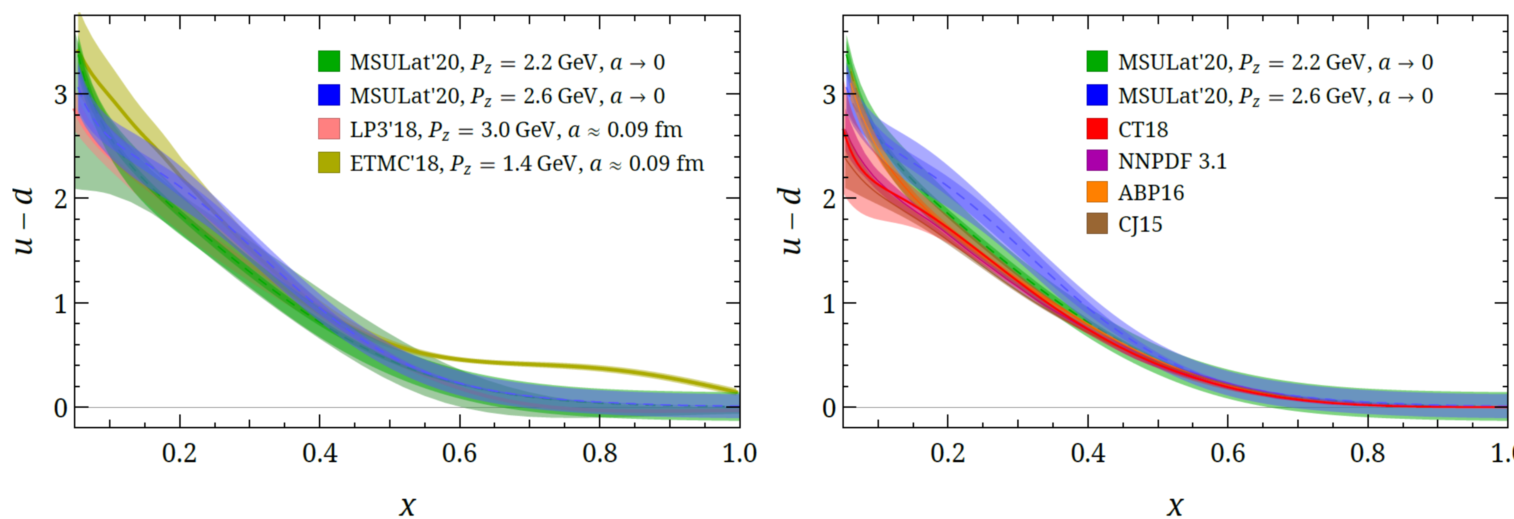
  - $P_z \approx 2$ GeV

2011.14971, HL et al (MSULat)

- Naïve extrapolation to physical-continuum limit

Wanted PDFs, GPDs, etc…

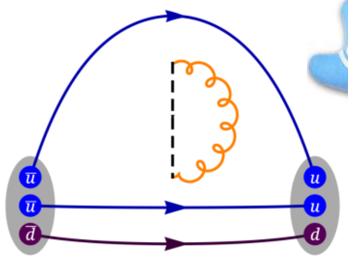


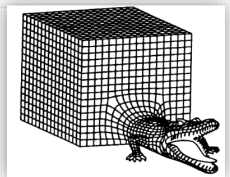
First Gluon PDF in Nucleon

§ Gluon PDF using pseudo-PDF

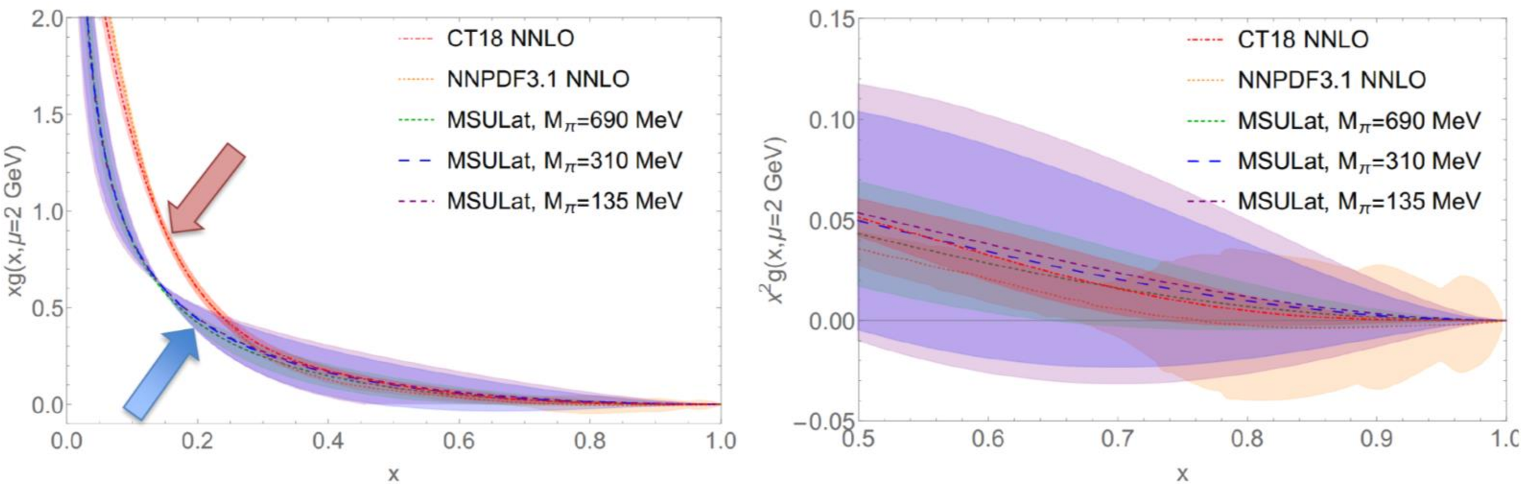
- Lattice details: clover/2+1+1 HISQ 0.12 fm, 310-MeV sea pion
- Study strange/light-quark

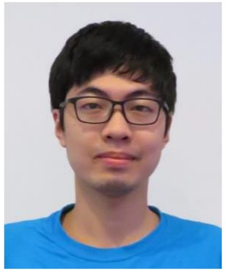
Z. Fan. et al (MSULat), 2007.16113





The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{\mathrm{MS}}$ scheme.





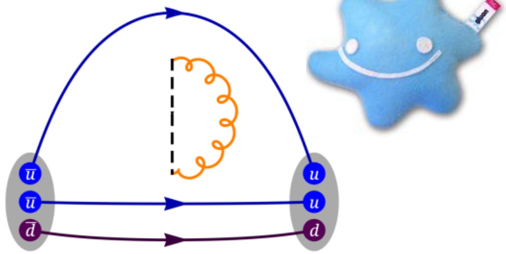
Zhouyou Fan (MSU)

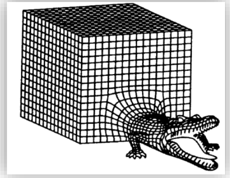
Slide by Zhouyou Fan@DNP2020

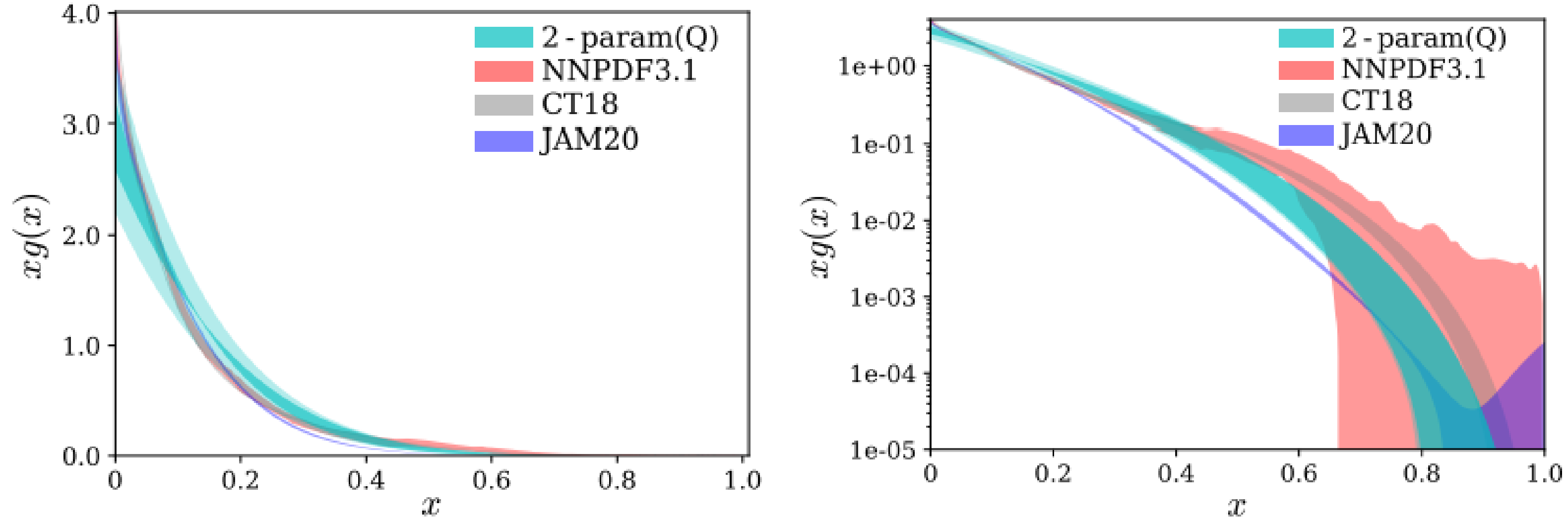
Gluon PDF in Nucleon

§ Gluon PDF using pseudo-PDF

- Lattice details: 2+1 clover, 0.09 fm, 360-MeV sea pion   T. Khan et al. (HadStruc), 2107.08960
- Use many nucleon Interpolating operators to improve signal with larger boosted momentum state



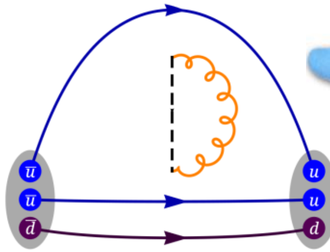




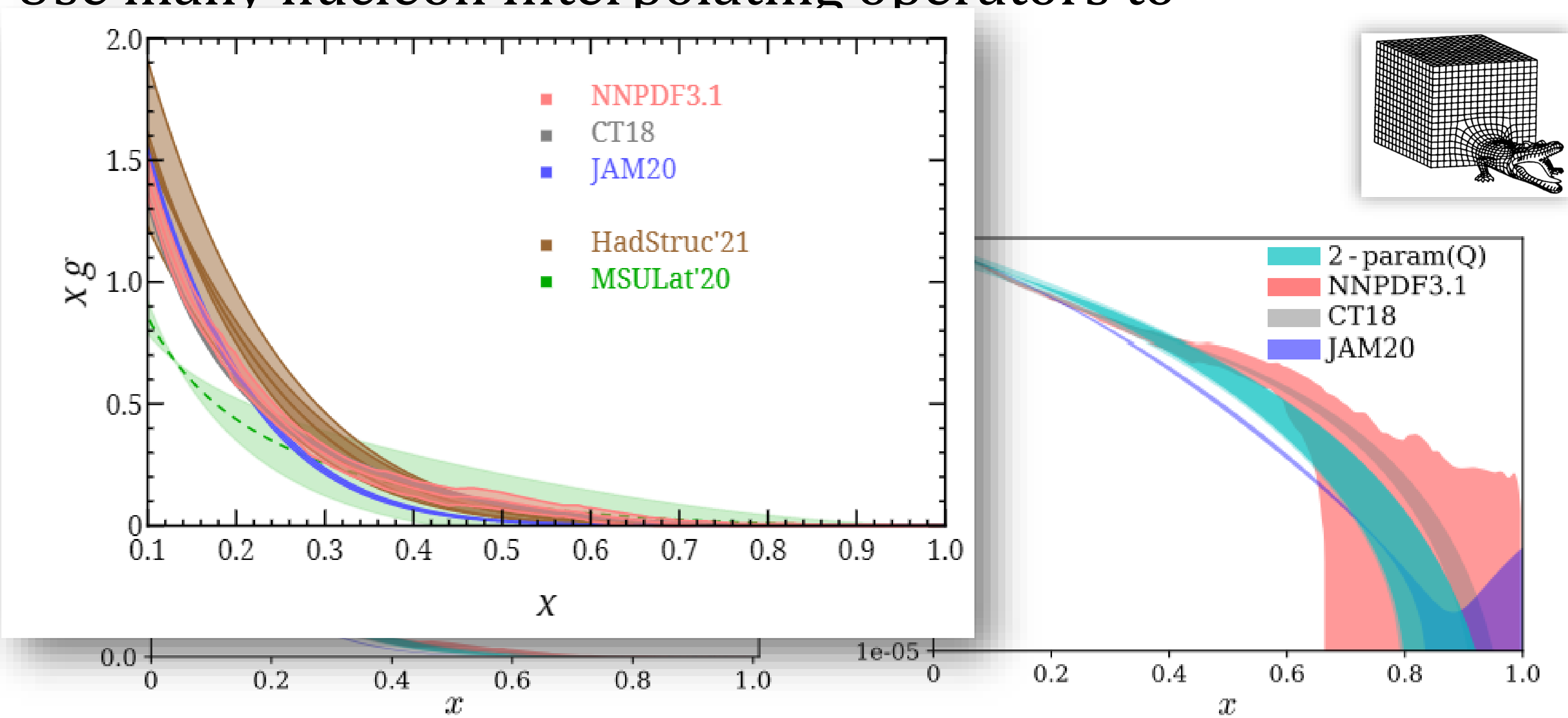
T. Khan et al. (HadStruc), 2107.08960

Gluon PDF in Nucleon

§ Gluon PDF using pseudo-PDF

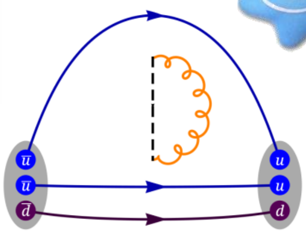


- Lattice details: 2+1 clover, 0.09 fm, 360-MeV sea pion   T. Khan et al. (HadStruc), 2107.08960
- Use many nucleon Interpolating operators to



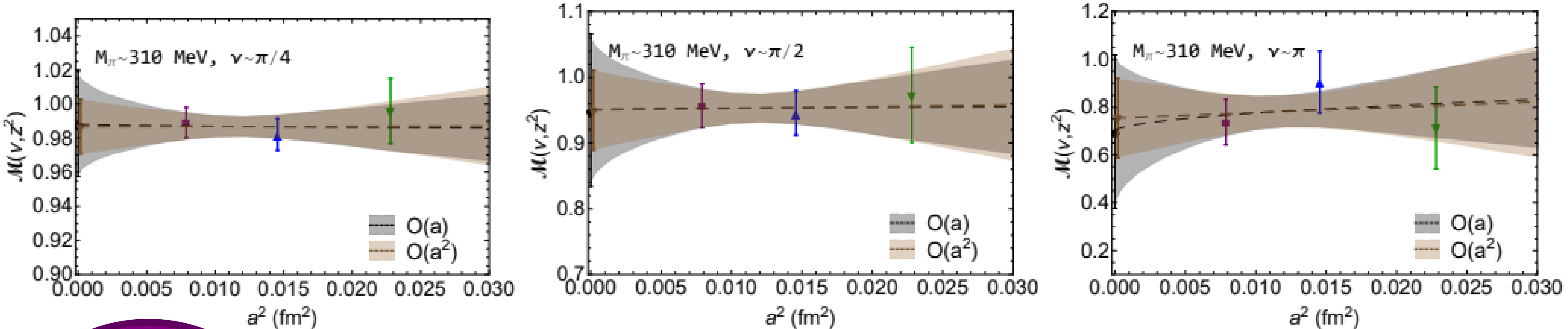
M. Begel et al. (Snowmass Whitepaper), 2209.14872

§ Continuum Gluon PDF w/ pseudo-PDF



- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics

Z. Fan, W. Good, HL (MSULat), [2210.09985]

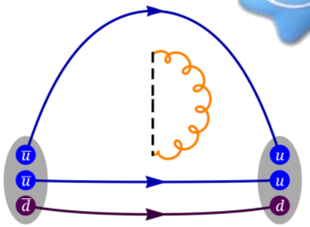


Quantities that can be calculated on the lattice

G: Bill Good (CEU21)

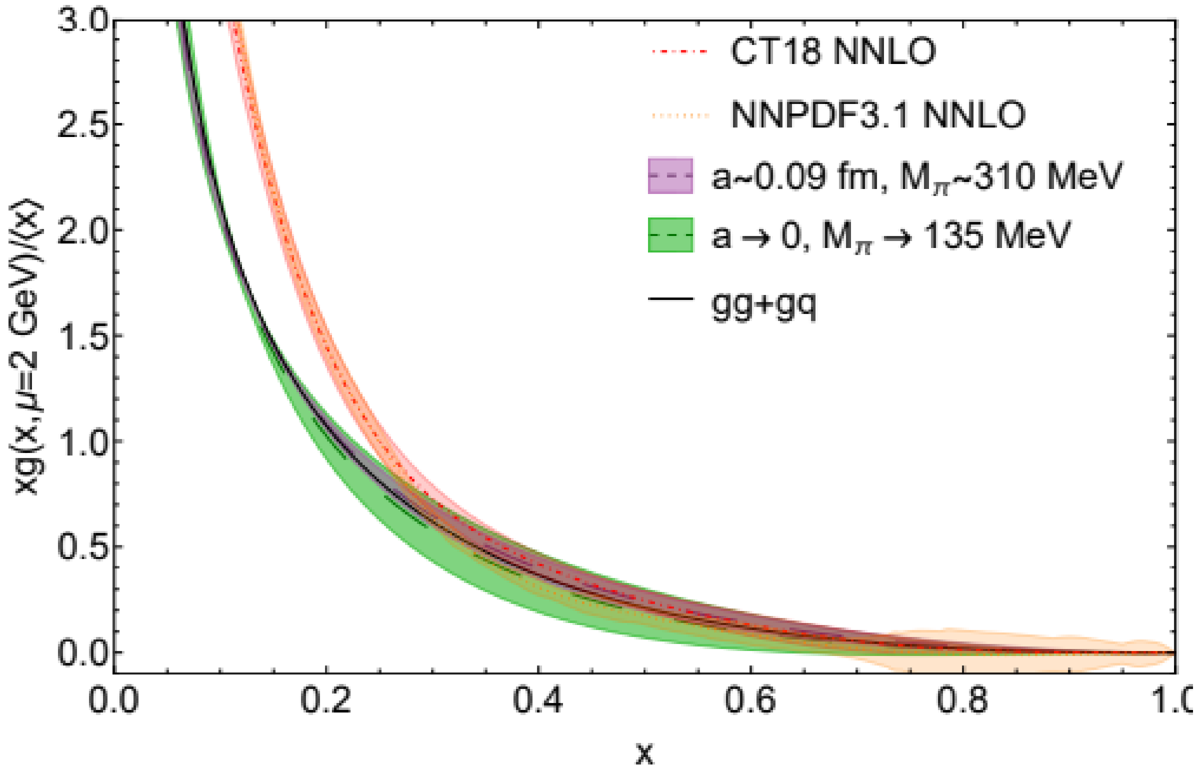
Gluon PDF in Nucleon

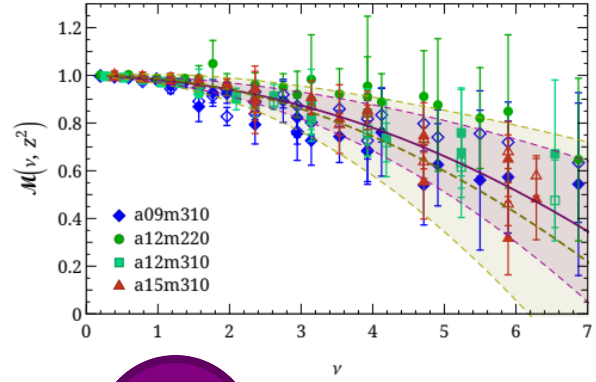
§ Continuum Gluon PDF w/ pseudo-PDF



- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics

Z. Fan, W. Good, HL (MSULat), [2210.09985](https://arxiv.org/abs/2210.09985)





Quantities that can be calculated on the lattice

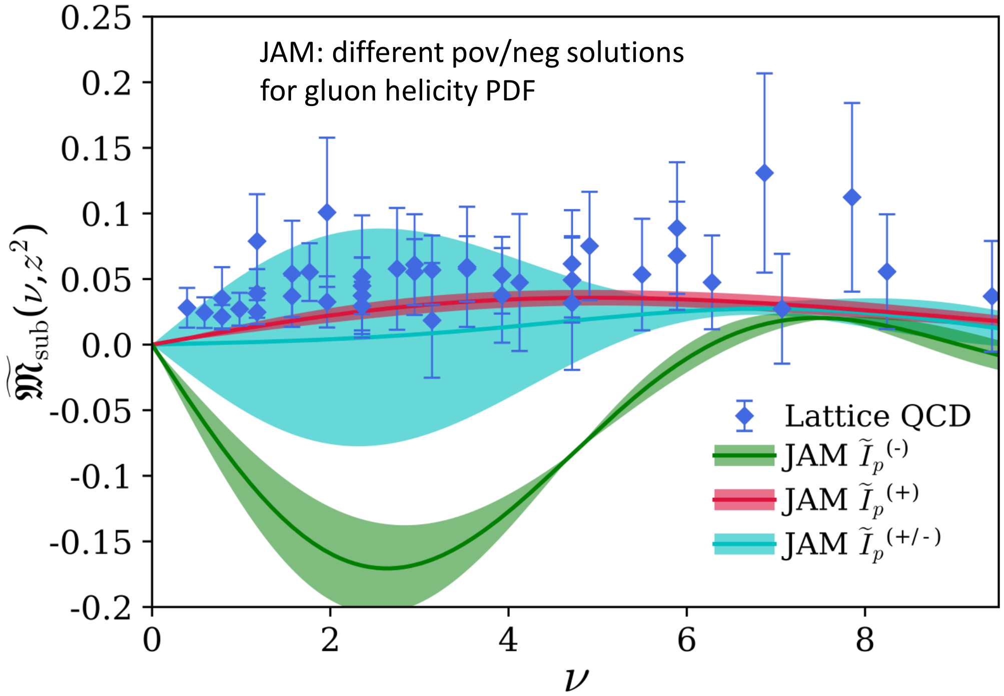
G: Bill Good (CEU21)

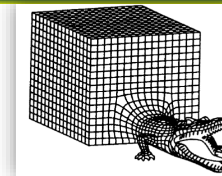
First Gluon Helicity Distribution

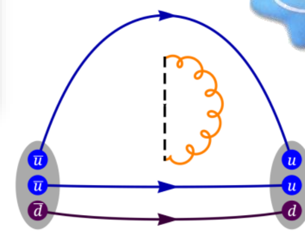
§ Helicity Gluon PDF w/ pseudo-PDF

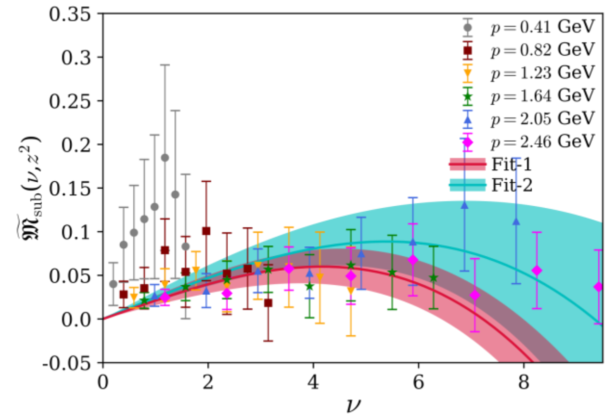
- 2+1 clover 0.09 fm, 360-MeV pion
- Lattice QCD hints at a positive gluon helicity distrib nucleon

C. Egerer el al (HadStruc), 2207.08733







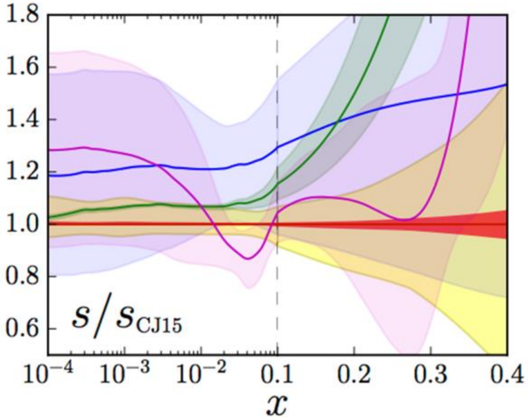


Quantities that can be calculated on the lattice

Plots by Raza Sabbir Sufian

Strange PDF

§ Large uncertainties in global PDFs

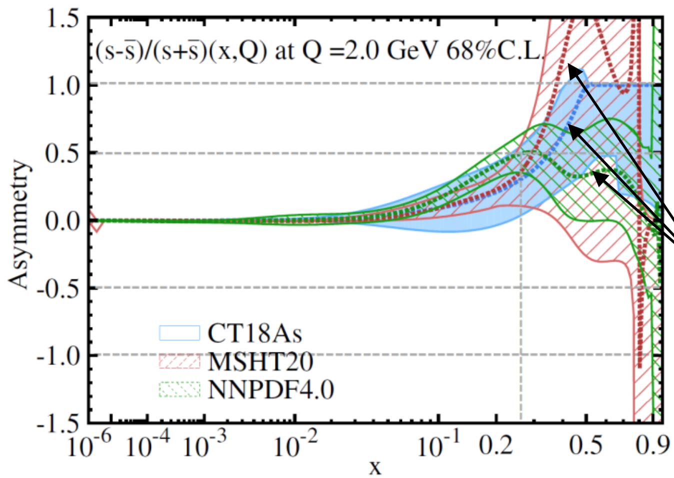


❧ Assumptions imposed due to lack of precision data

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

CTEQ-JLAB https://www.jlab.org/theory/cj/

§ The strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure (left), but can be predicted in lattice QCD (right)

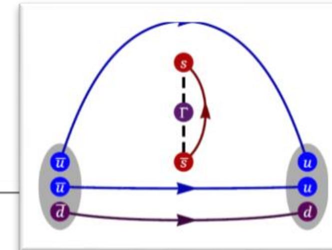


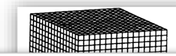
differences reflect the pulls of LHC and other experiments

First Lattice Strange PDF

§ Results by MSULat/quasi-PDF method

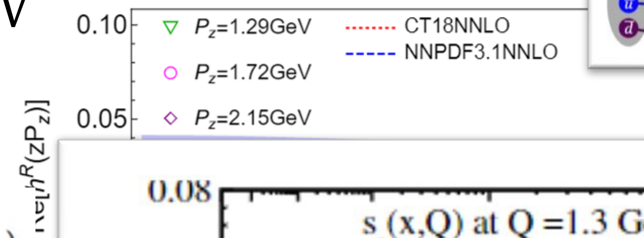
- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV





2005.12015, R. Zhang et al (MSULat)

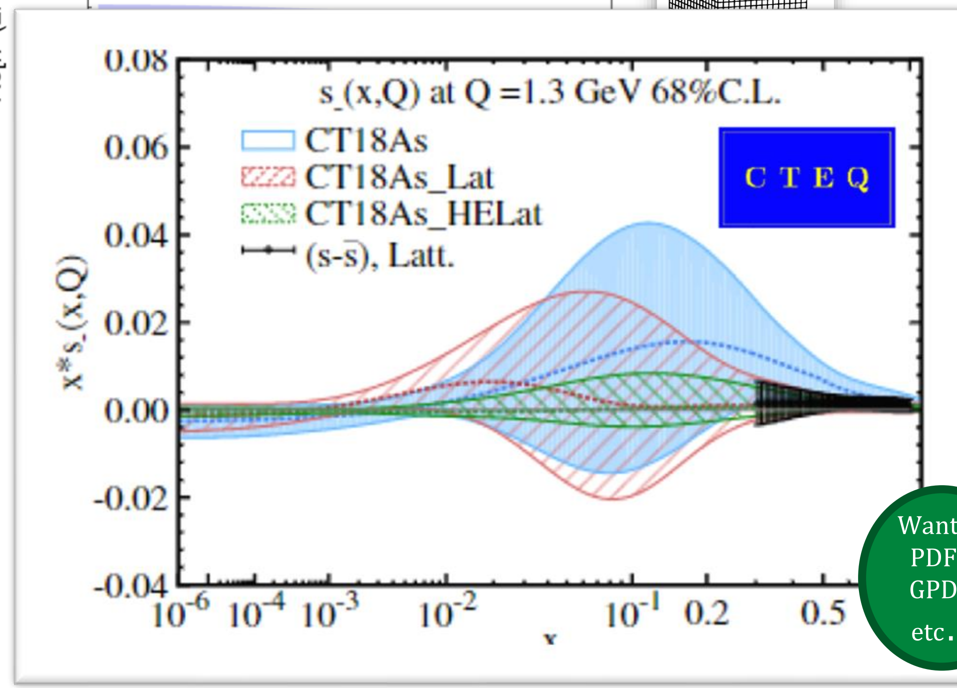
$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x)-\bar{s}(x))\cos(xzP_z)$$



§ From quasi-PDF to PDF

$$\tilde{f}_q(x,P_z) = \int_{-1}^{1} \frac{dy}{|y|} f_q(y) C_{q/q}(x,y,P_z,\mu) + O\left(\frac{\Lambda_{\rm QCD}^2}{x^2P_z^2}, \frac{\Lambda_{\rm QCD}^2}{(1-x)^2P_z^2}\right)$$

T. Hou, HL, M. Yan, C. Yuan, 2204.07944



Wanted PDFs, GPDs, etc…

§ The strangeness asymmetry $s(x,Q)-\bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

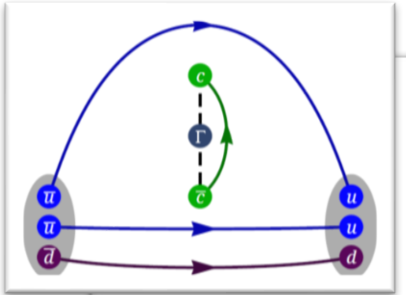
First Lattice Charm PDF

§ Large uncertainties in global PDFs

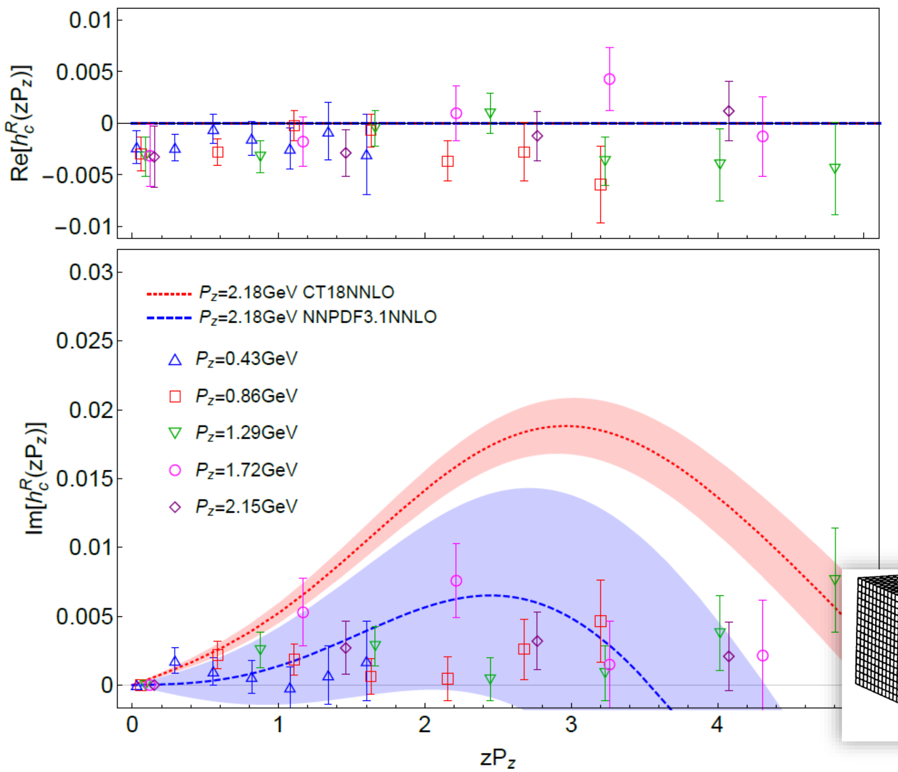
§ Results by MSULat/quasi-PDF method

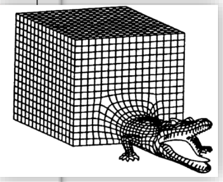
- Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

2005.12015, R. Zhang et al (MSULat)



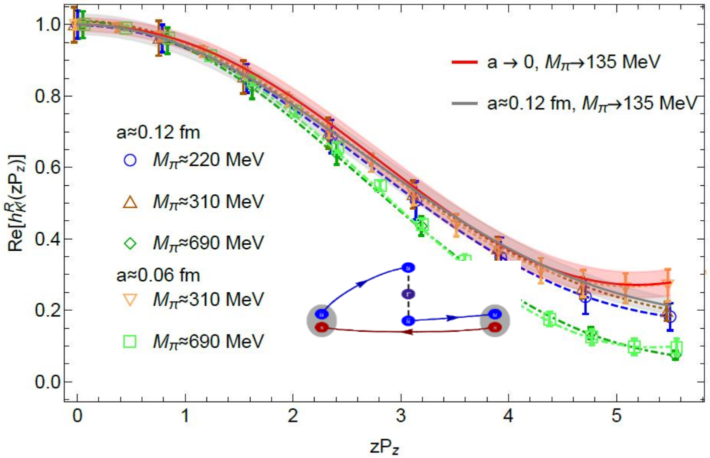
- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF

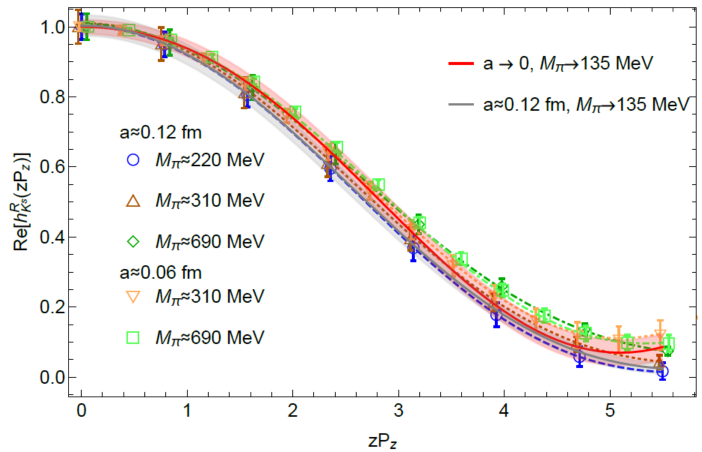




Meson Valence-quark PDFs

§ Pion/Kaon PDFs using quasi-PDF in the continuum limit

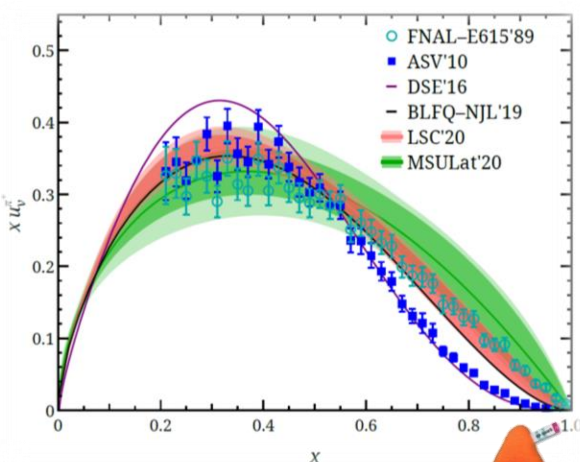


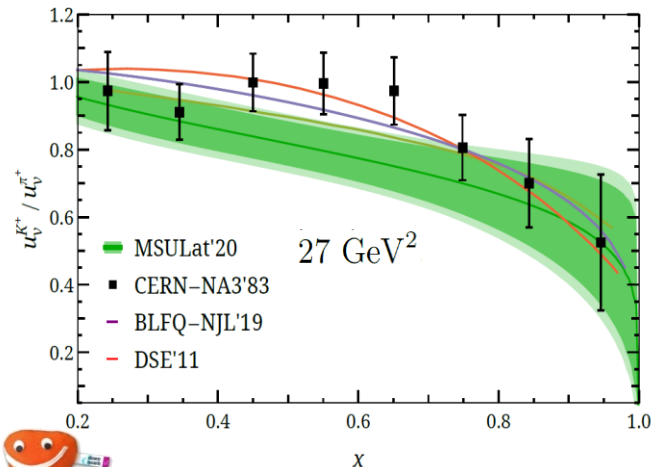


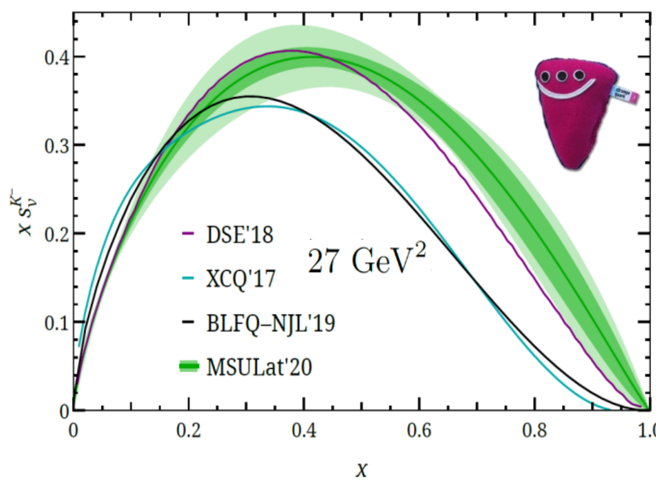
Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

MSULat, 2003.14128

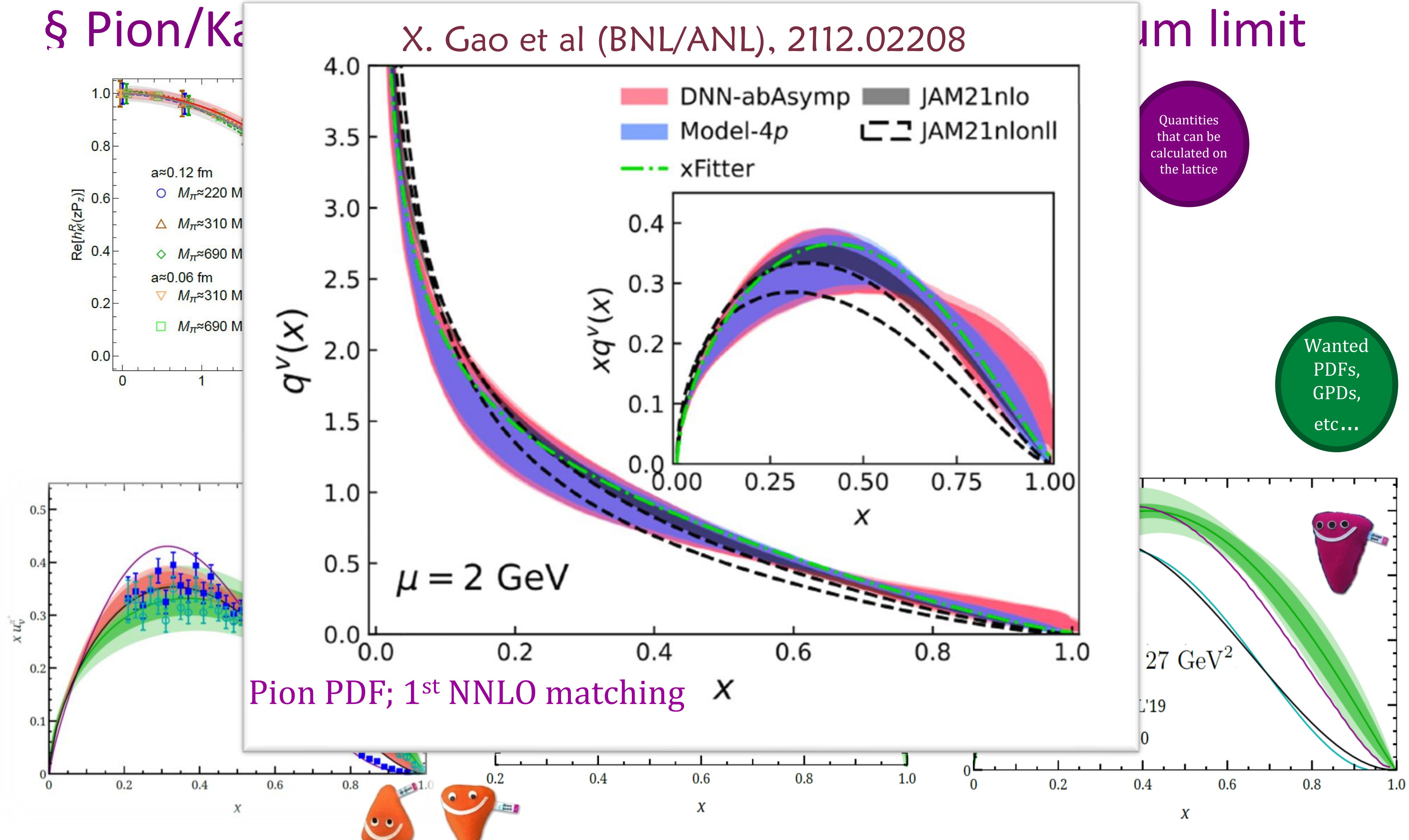






Meson Valence-quark PDFs

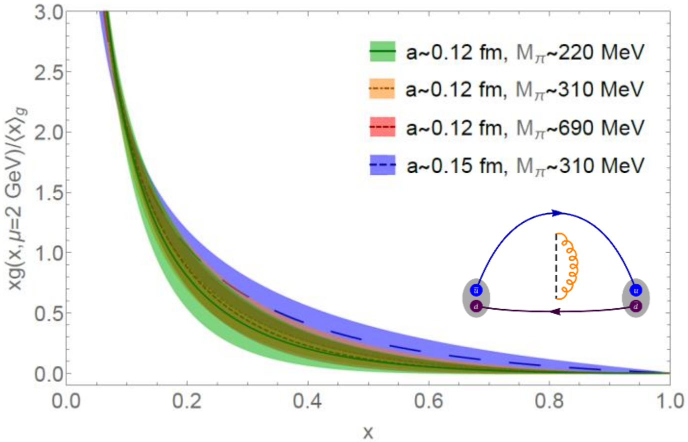
§ Pion/Ka... ...um limit

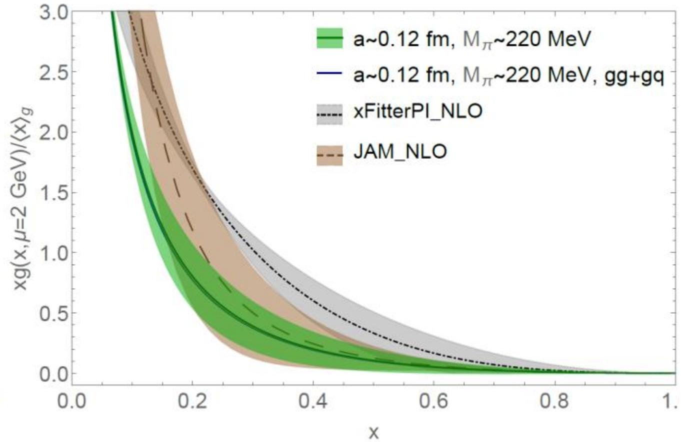


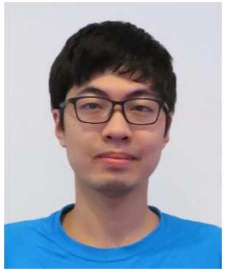
Pion PDF; 1st NNLO matching

Meson Gluon PDFs

§ First pion and kaon gluon PDFs using pseudo-PDF

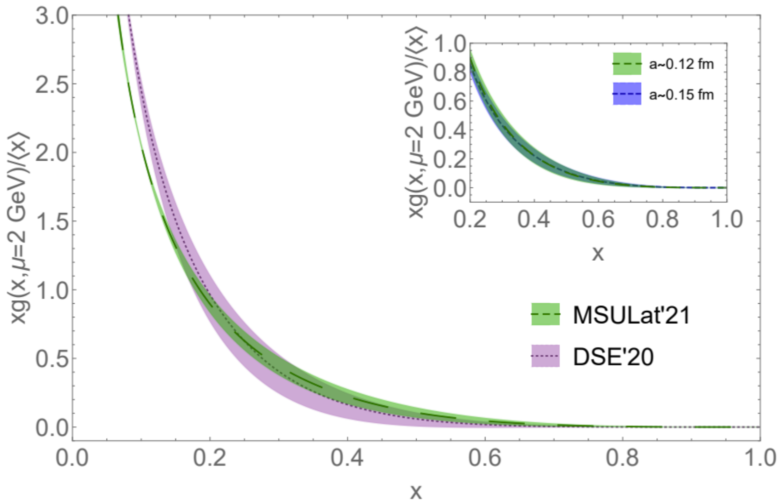






Zhouyou Fan (MSU)

2104.06372, Fan et al (MSULat)

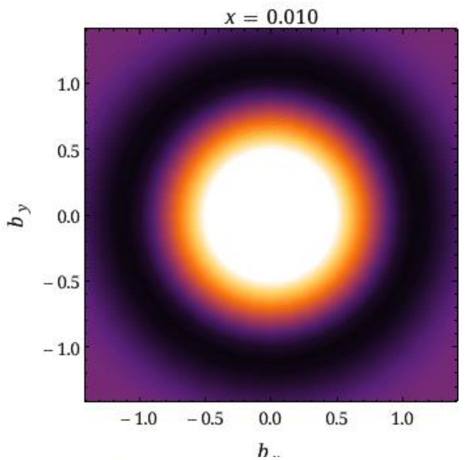


Alejandro Salas-Chavira (MSU)

2112.03124 , Salas-Chavira et al (MSULat)

Summary and Outlook

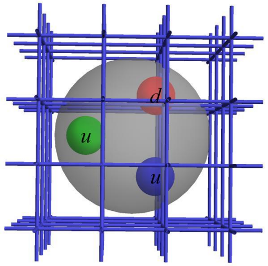
- § Exciting era using LQCD to study hadron structure
- § Overcoming longstanding limitations of moment method
  - Bjorken-x dependence of parton distributions are widely studied
  - More study of systematics planned for the near future



- § Precision and progress are limited on resources
  - Challenges = new opportunities quantities
- § In the future

Theory Input

Exp't Input +



Global Analysis of PDFs/GPDs

See talks in this workshop by Yaohang Li & Brandon Kriesten

The work of HL is sponsored by NSF CAREER Award under grant PHY 1653405 & RCSA Cottrell Scholar Award

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources

Lattice Progress & Challenges

- Exploratory study on charm and gluon PDFs
- Many approaches are moving to the NNLO level
  - Expect to see more improved lattice calculations
- Beyond the standard twist-2 collinear PDFs
  - Generalized parton distributions (GPDs) for the pion and unpolarized/polarized nucleon
  - Transverse-momentum– dependent distributions (TMDs)
    - Collins-Soper kernel, soft function and wavefunctions
  - Twist-3 PDFs and GPDs

For more details and references, refer to 2202.07193

- Challenges ahead for precision PDFs
  - Need large boost mom., better signal-to-noise, inverse problems in PDF extraction in SDF, more computational resources, etc.

DIS23 Advertisement

30th International Workshops on

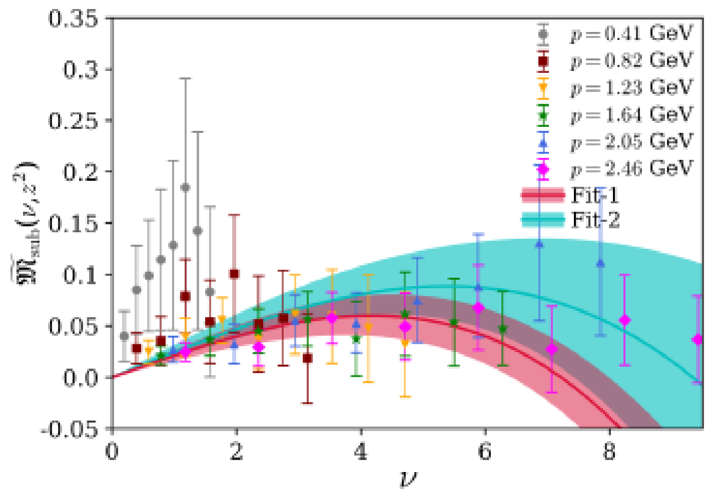
Deep-Inelastic Scattering (DIS) and Related Subjects

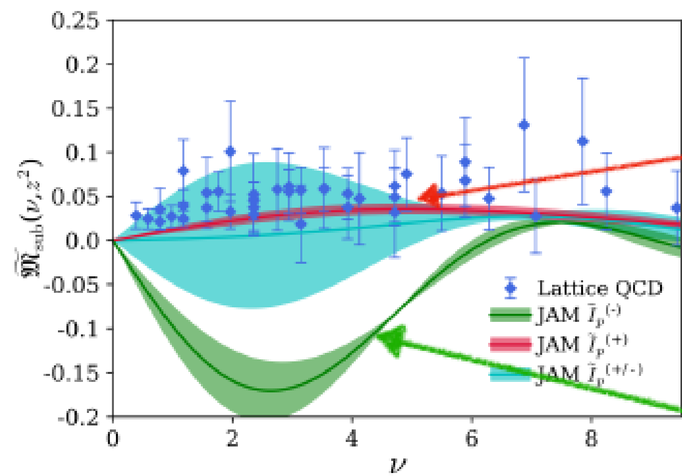
Backup Slides

From Raza

Gluon helicity Ioffe-time distribution from LQCD data

- Lattice QCD determination of gluon helicity pseudo-distribution
- Fit-1,2 are extrapolated results from fit to lattice data at different nucleon boost
- Data points are from direct lattice calculation





JAM: different soluti
for gluon helicity PD

positive solution

negative solution

- Lattice QCD hints at a positive gluon helicity distribution i

Sufian, Khan, Karthik, et al (HadStruc Collaboration) : PRD 2022

From Raza

Gluon helicity Ioffe time pseudo-distribution

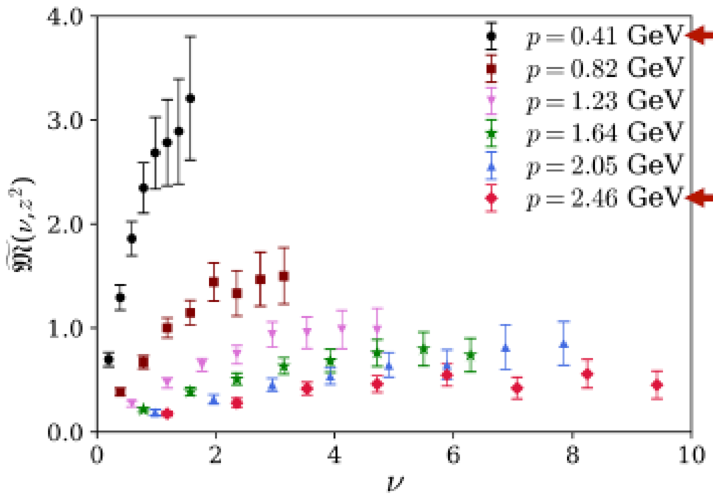
- Light-cone Ioffe-time distribution associated with gluon helicity

$$\widetilde{\mathcal{I}}_p(\nu) \equiv i\,[\widetilde{\mathcal{M}}^{(+)}_{ps}(\nu) - \nu\widetilde{\mathcal{M}}_{pp}(\nu)]$$

- Contamination term present in LQCD matrix element at finite momenta do

$$\widetilde{\mathfrak{M}}(\nu,z^2) = [\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu,z^2) - (1 + m_p^2/p_z^2)\nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2)]$$

Balitsky et al [JHEP 2022]



$m_p^2/p^2 \sim 7.3$

$m_p^2/p^2 \sim 0.2$

grows with ν

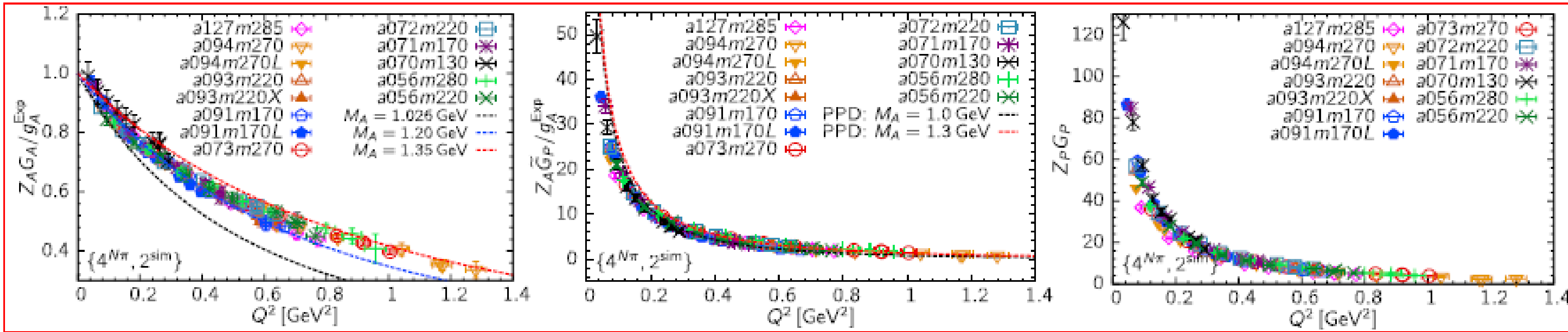
2/14

From Sungwoo Park

Nucleon Isovector Form Factors

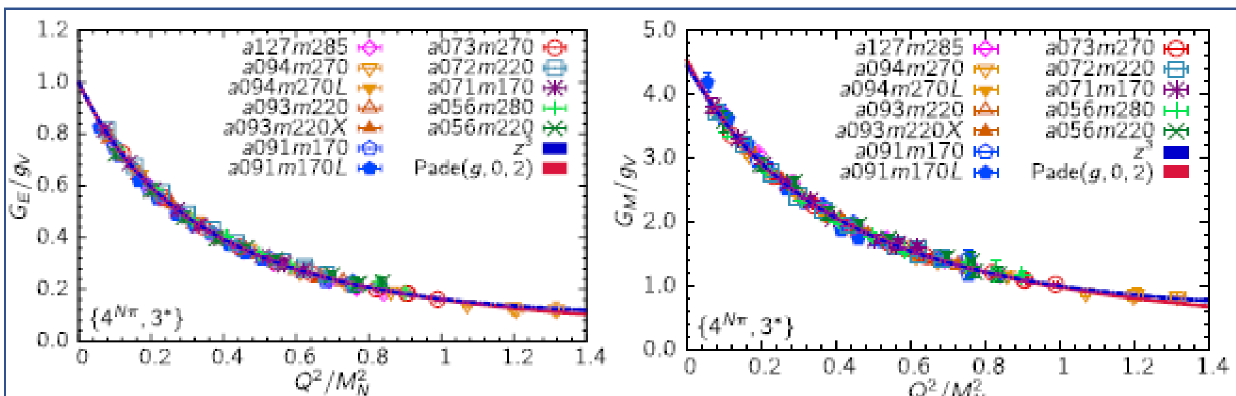
[NME collab., Lattice 2022 update, all preliminary]

- Clover fermion on $N_f = 2 + 1$ clover ensembles



Axial form factors

- $N\pi$ excited state needed to satisfy PCAC relation. Impact on FF is large



Electric & Magnetic form factors

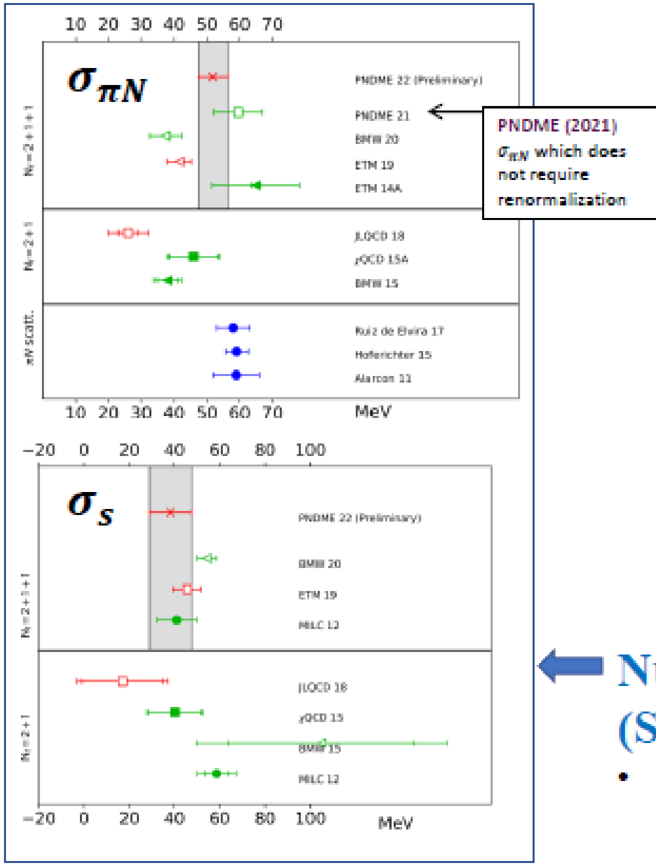
- Less sensitive to the details of the excited states
- Good agreement with the Kelly curve

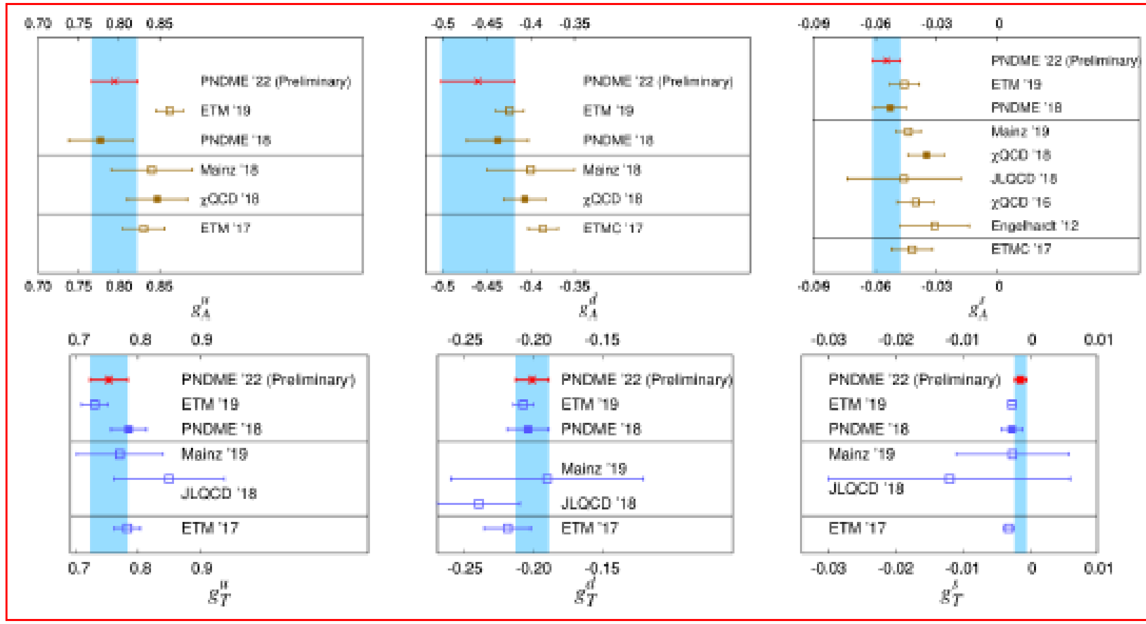
Nucleon Flavor Diagonal Charges
: Comparison with FLAG 2021 results

[PNDME collab., Lattice 2022 update, preliminary]

Clover fermion on $N_f = 2 + 1 + 1$ HISQ ensembles

- Flavor mixing calculated nonperturbatively
- Chiral-Continuum extrapolation including a data at M_π^{Phys}





Nucleon sigma terms (Scalar charges)

- $\sigma_{N\pi}$: Excited-state effects are large and results very sensitive to $N\pi$ / $N\pi\pi$ states

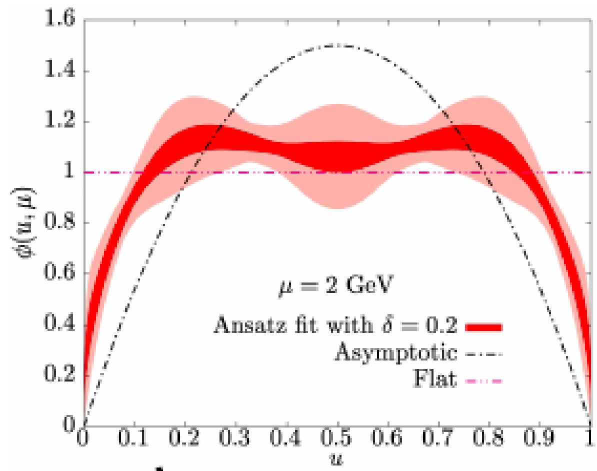
Axial and Tensor charges

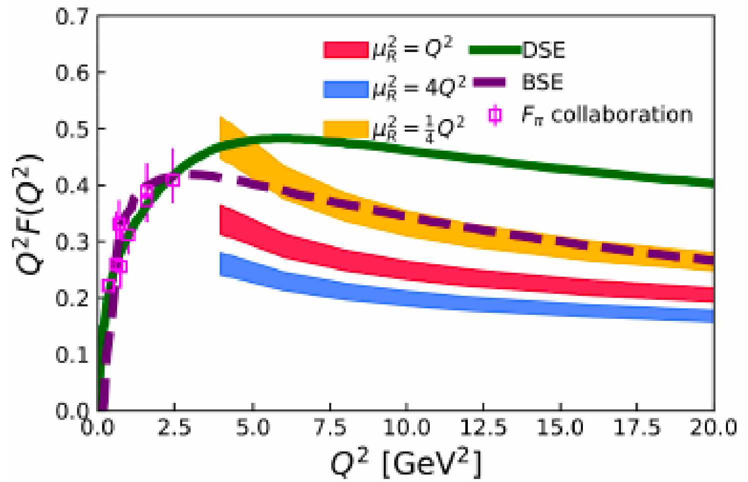
- Less sensitive to the details of the excited states

From Swagato Mukherjee

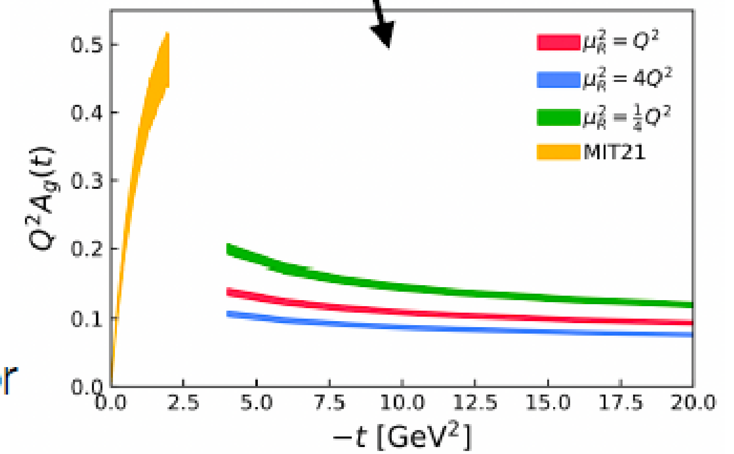
distribution amplitude of physical pion

Nikhil Karthik *et al.*, Phys. Rev. D 106, 074505 (2022)





asymptotic pion EM form factor



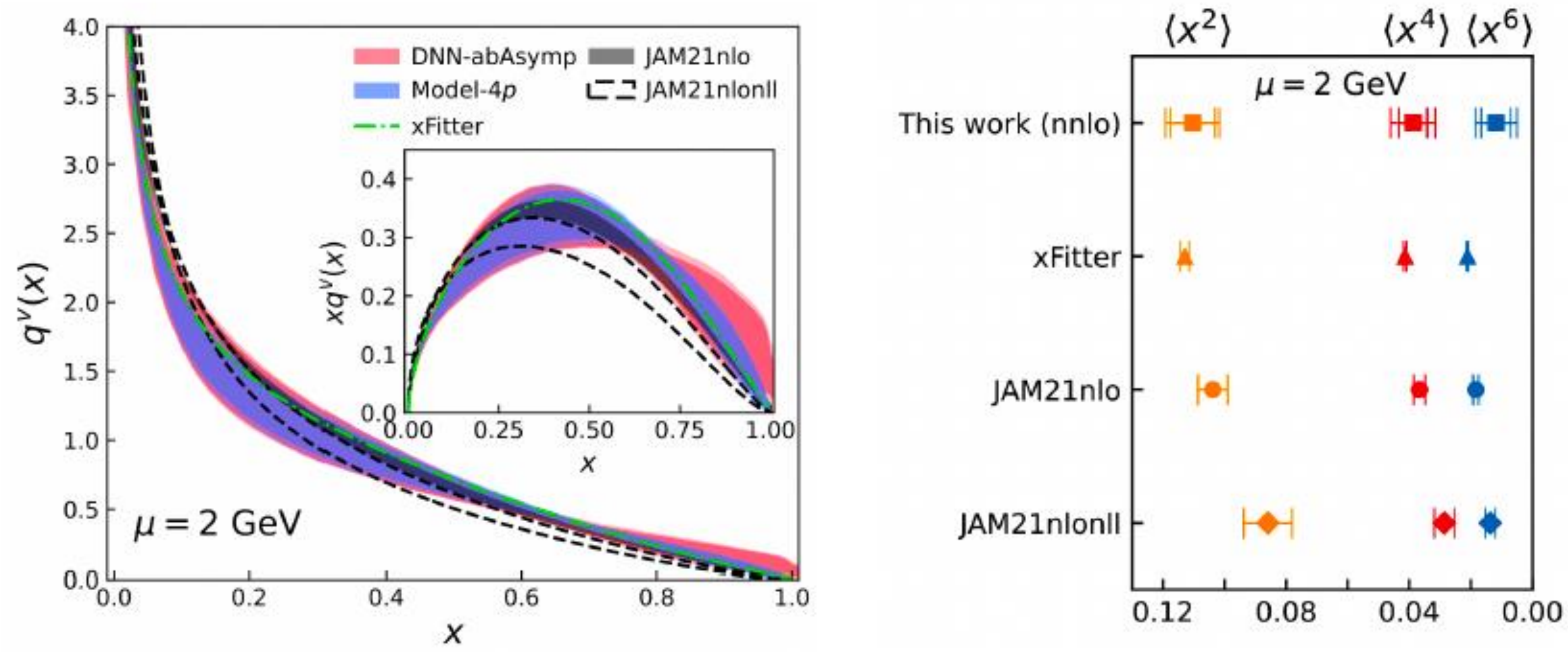
asymptotic pion gravitational orm factor

1

From Swagato Mukherjee

NNLO valance PDF of physical pion

Xiang Gao *et al.*, arXiv:2208.02297 [hep-lat]



2

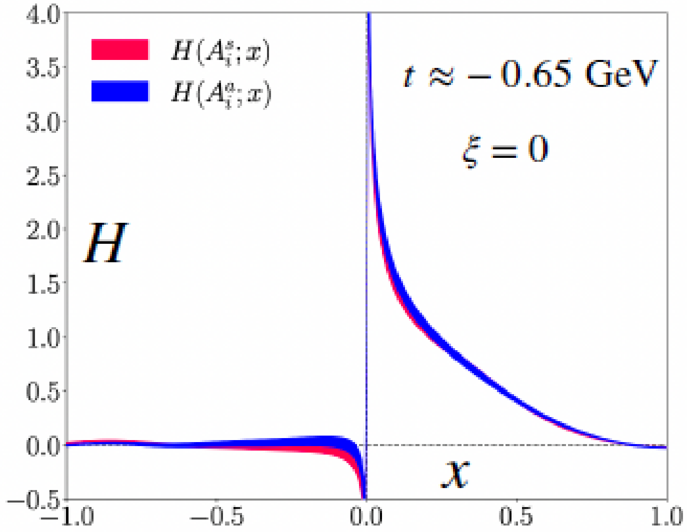
From Swagato Mukherjee

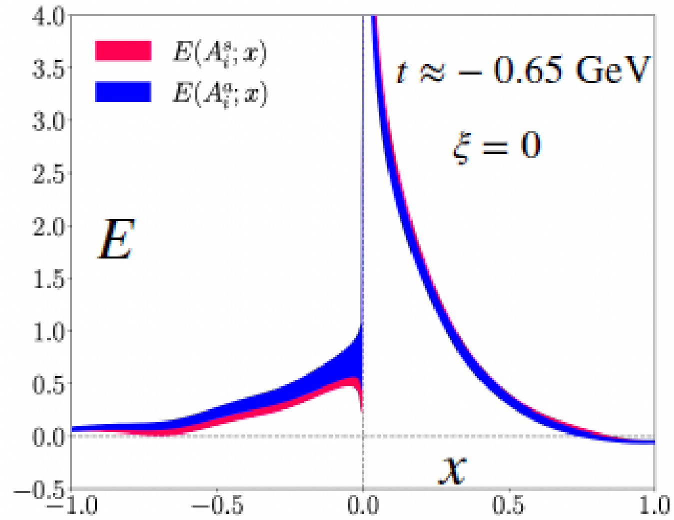
proton generalized parton distribution functions

new novel Lorentz invariant formalism for lattice QCD computations using asymmetric momentum transfers:

- ➡ significantly increases access to t-dependence
- ➡ significantly reduces power corrections

Shohini Bhattacharya *et al.*, arXiv:2209.05373 [hep-lat]



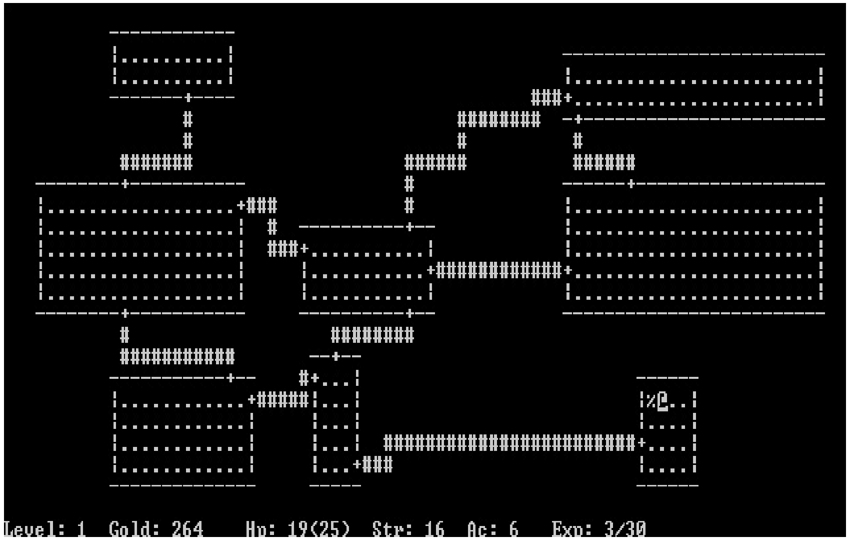


3

Are We There Yet?

- § Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven't we solved QCD yet?
- § Progress is limited by computational resources

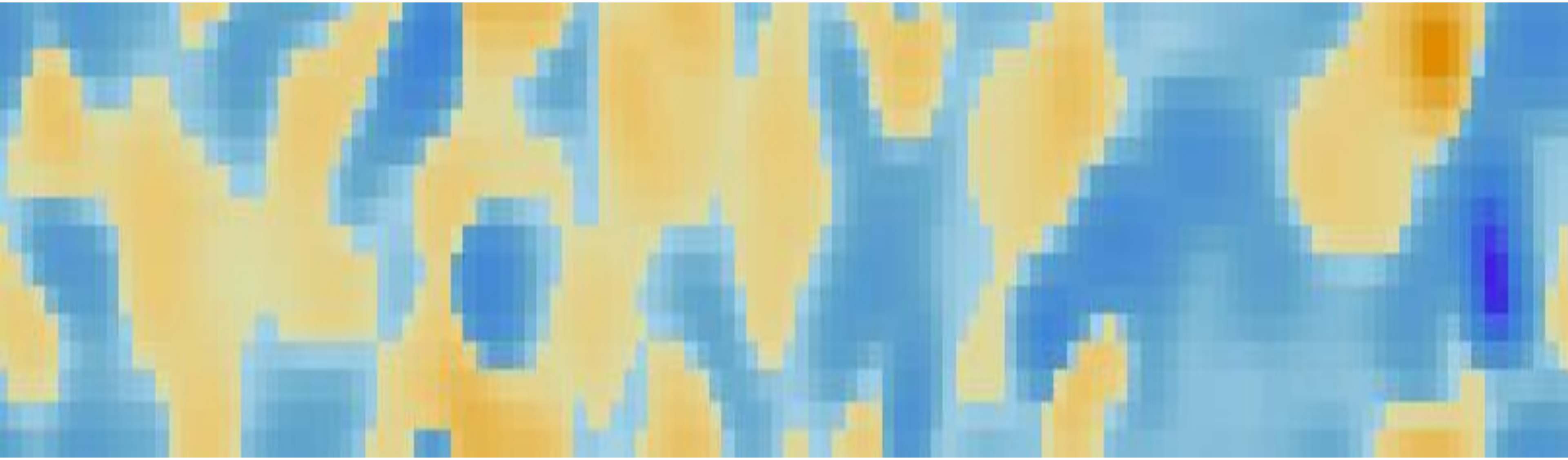
1980s



Today

- § Greatly assisted by advances in algorithms
  - Physical pion-mass ensembles are not uncommon!

Nucleon Matrix Elements

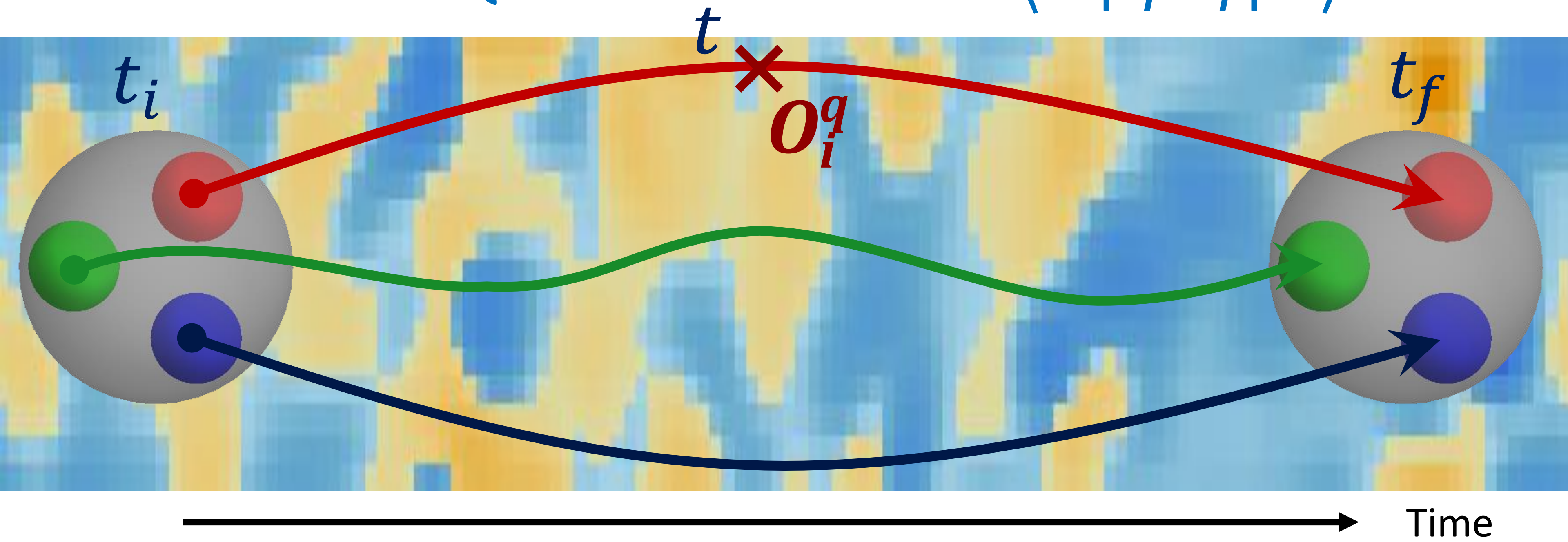


§ Pick a QCD vacuum

- Gauge/fermion actions, flavor (2, 2+1, 2+1+1), m_π, a, L, …

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



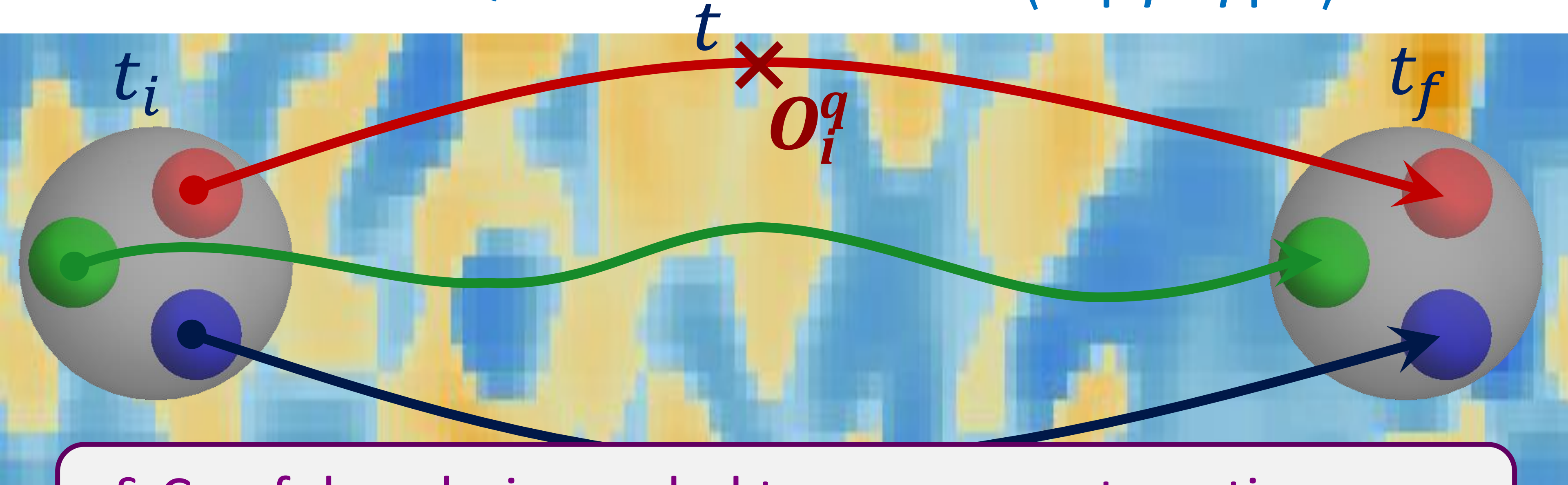
§ Construct correlators (hadronic observables)

- Requires "quark propagator"

  Invert Dirac-operator matrix (rank $O(10^{12})$)

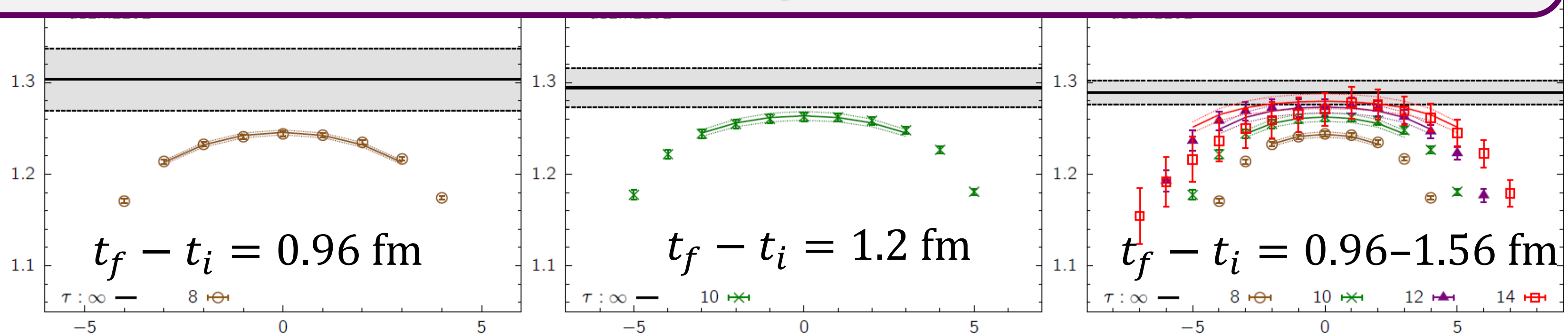
Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



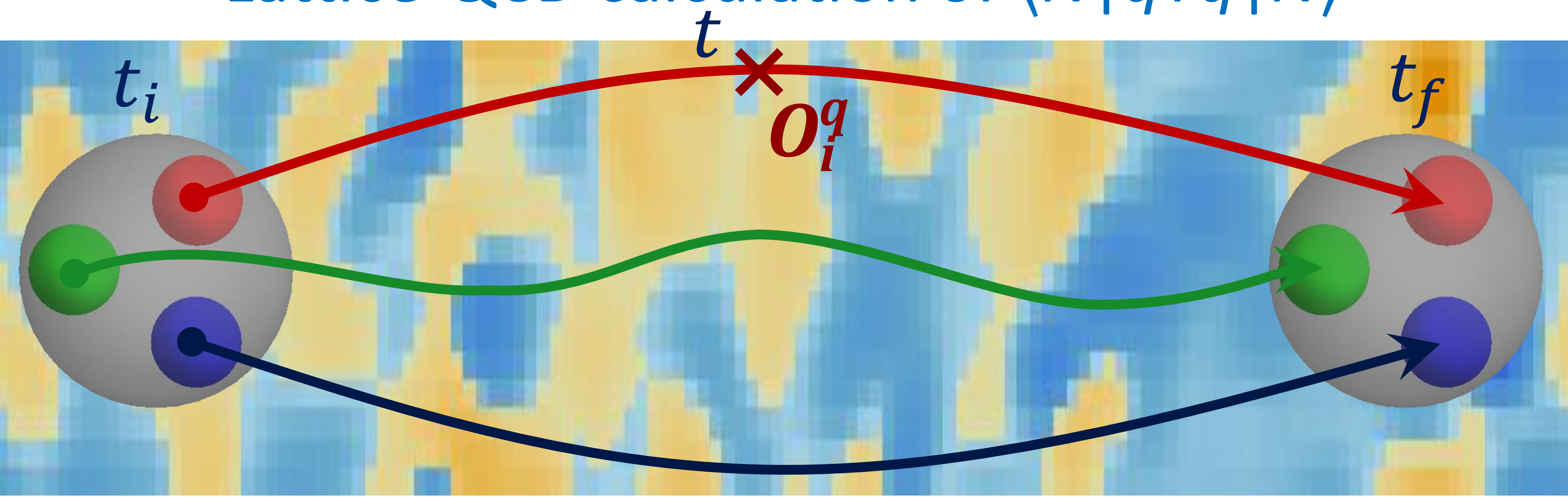
§ Careful analysis needed to remove systematics

- Wrong results if **excited-state systematic** is not under control



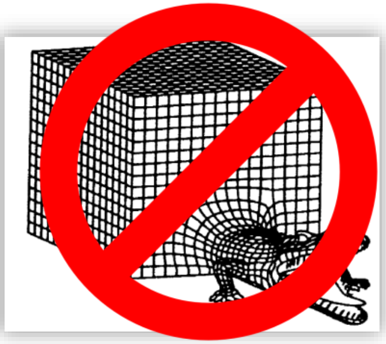
Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



§ Systematic uncertainty (nonzero *a*, finite *L*, etc.)

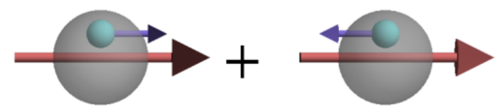
- Nonperturbative renormalization
  e.g. RI/SMOM scheme in $\overline{\text{MS}}$ at 2 GeV
- Extrapolation to the continuum limit
  $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboraton	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
$\langle x\rangle_{u^+-d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.171(18)	
	PNDME 20	(Mondal *et al.*, 2020)	2+1+1	★	★	★	★	★		0.173(14)(07)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.180(25)(^{+14}_{-6})$	0.161(18)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.151(28)(29)	
	RQCD 18	(Bali *et al.*, 2019b)	2	★	★	○	★	★		0.195(07)(15)	
$\langle x\rangle_{u^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.359(30)	0.353(12)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.307(30)(18)	
$\langle x\rangle_{d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.188(19)	0.192(6)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.160(27)(40)	
$\langle x\rangle_{s^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.052(12)	0.037(3)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.051(26)(5)	
$\langle x\rangle_g$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.427(92)	
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.482(69)(48)	0.411(8)
	χQCD 18a	(Yang *et al.*, 2018a)	2+1	■	★	★	★	■		0.47(4)(11)	

** No quenching effects are seen.

Moments of PDFs

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- LatticePDF Workshop
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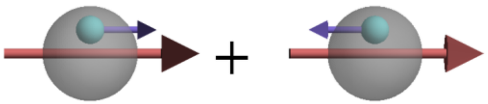
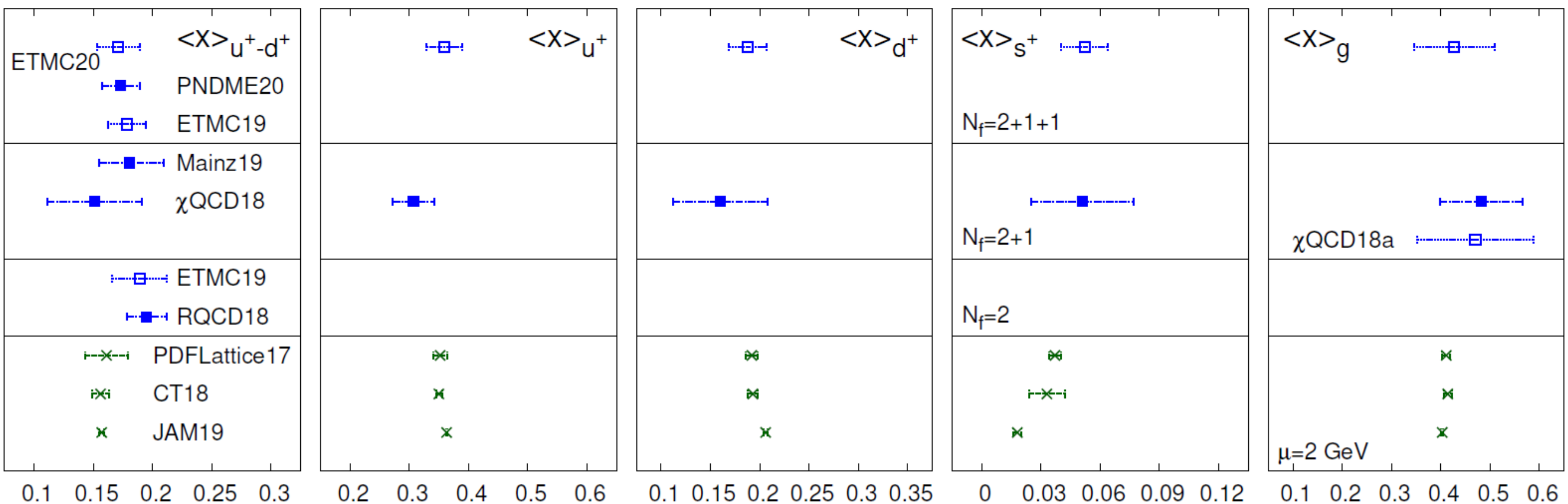
- Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



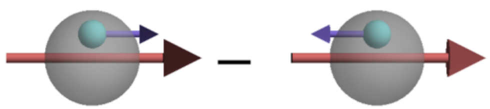
LatticePDF Report, 1711.07916, 2006.08636



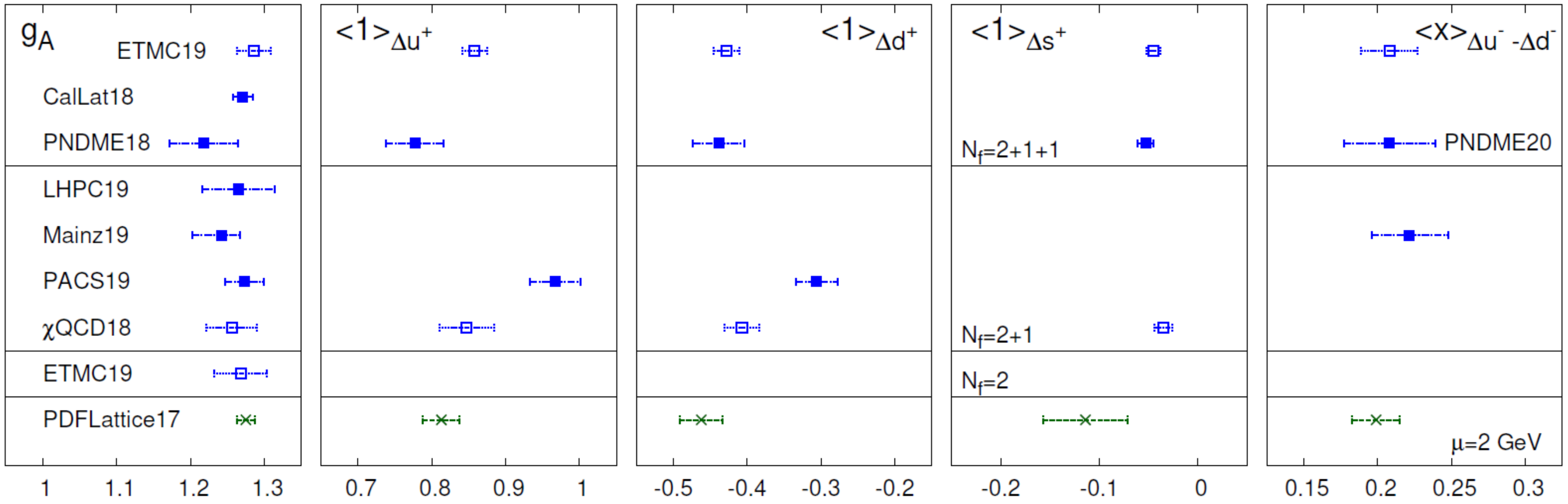
Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



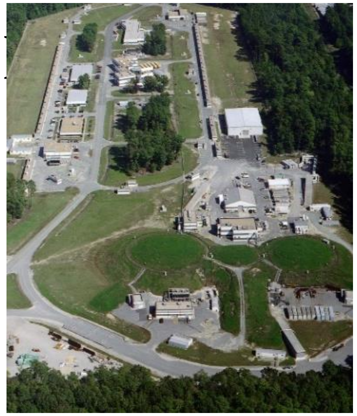
LatticePDF Report, 1711.07916,2006.08636



Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

- Many ongoing/planned experiments
  (BNL, JLab, J-PARC, COMPASS, GSI, EIC, AMBER, …

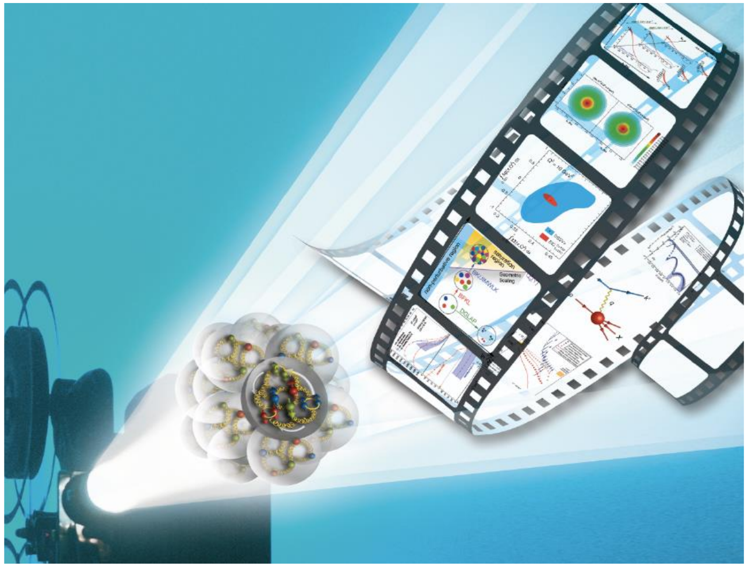


**Electron Ion Collider:
The Next QCD Frontier**

Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701



Global Analysis

§ Experiments cover diverse kinematics of parton variables

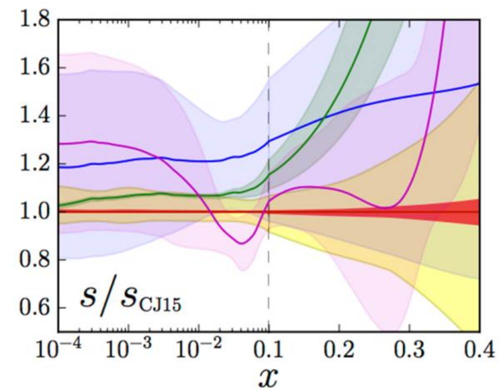
- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$$



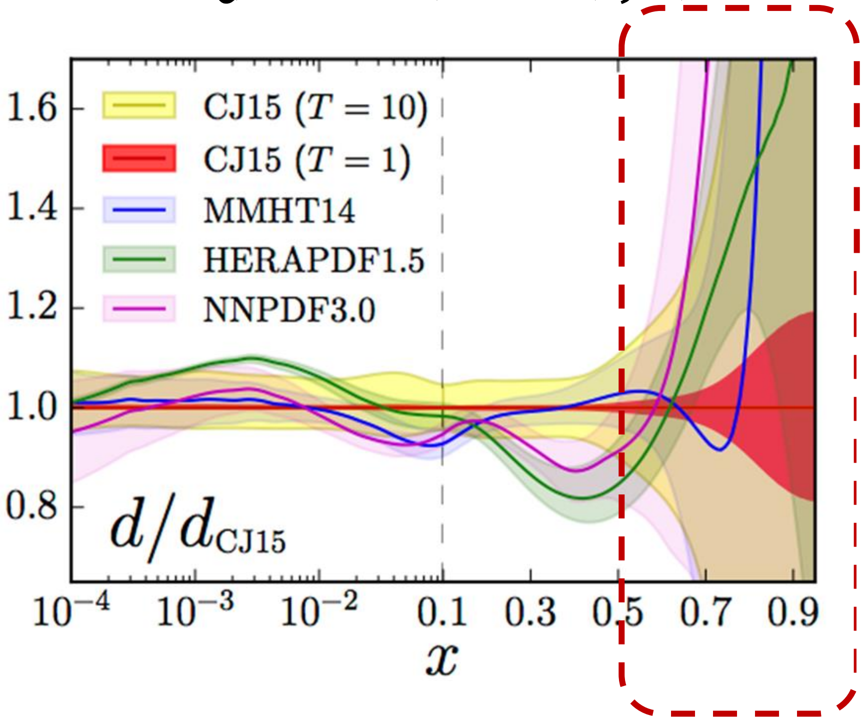
CTEQ-JLAB

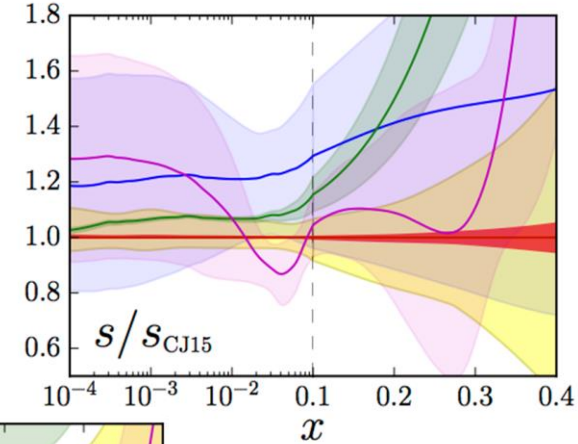
- Assumptions imposed

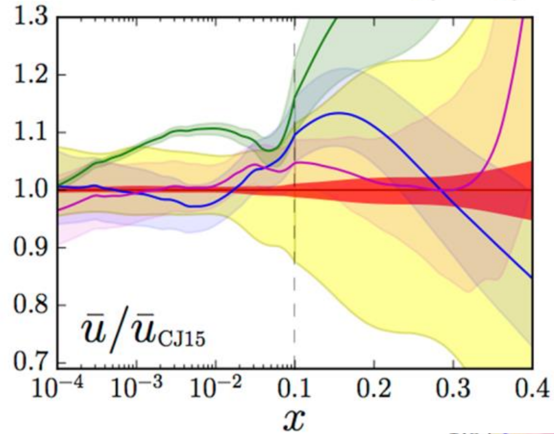
  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

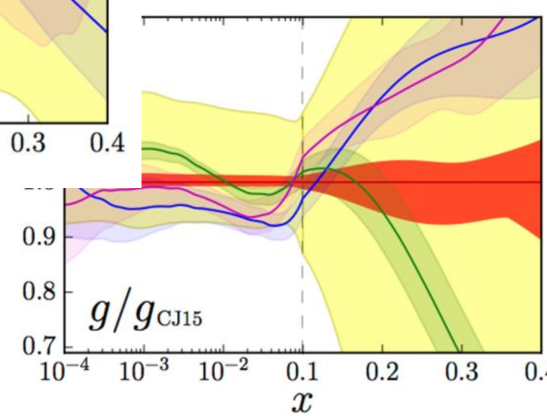
  $s = \bar{s} = \kappa(\bar{u} + \bar{d})$

- § Discrepancies appear when data is scarce
- § Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.









CTEQ-JLAB
https://www.jlab.org/theory/cj/

Consumer's Guide

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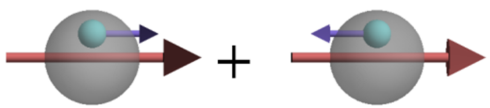


LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboraton	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
$\langle x\rangle_{u^+-d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.171(18)	0.161(18)
	PNDME 20	(Mondal *et al.*, 2020)	2+1+1	★	★	★	★	★		0.173(14)(07)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.180(25)(^{+14}_{-6})$	
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.151(28)(29)	
	RQCD 18	(Bali *et al.*, 2019b)	2	★	★	○	★	★		0.195(07)(15)	
$\langle x\rangle_{u^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.359(30)	0.353(12)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.307(30)(18)	
$\langle x\rangle_{d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.188(19)	0.192(6)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.160(27)(40)	
$\langle x\rangle_{s^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.052(12)	0.037(3)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.051(26)(5)	
$\langle x\rangle_g$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.427(92)	0.411(8)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.482(69)(48)	
	χQCD 18a	(Yang *et al.*, 2018a)	2+1	■	★	★	★	■		0.47(4)(11)	

** No quenching effects are seen.

Direct x-Dependent Structure

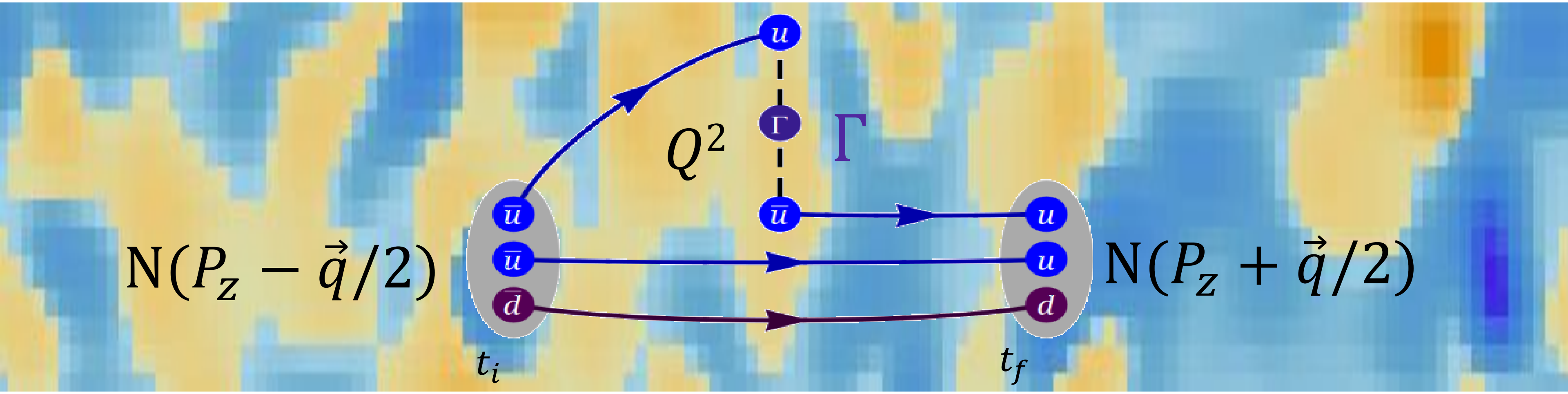
§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today $= \sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)
- …

Generalized Parton Distributions

§ On the lattice, one needs to calculate the following (nucleon example)



$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_Z) = \frac{\bar{P}_Z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_Z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(\tilde{H}(x,\tilde{\xi},t,\bar{P}_Z)\gamma^0 + \tilde{E}(x,\tilde{\xi},t,\bar{P}_Z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

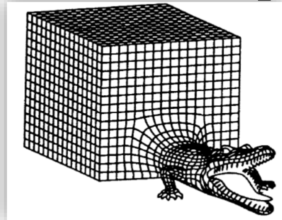
First Lattice GPDs

§ Pioneering first glimpse into pion GPD using LaMET

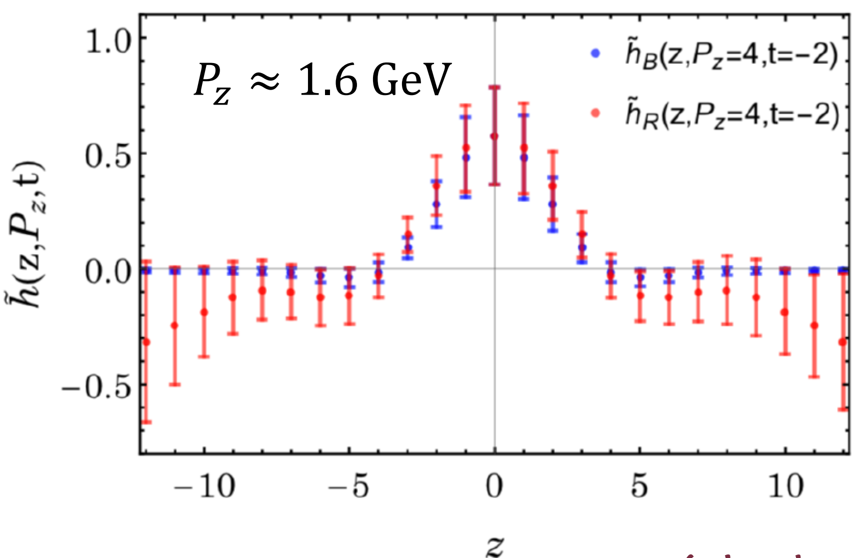
- Lattice details: clover/HISQ, 0.12fm, **310-MeV** pion mass

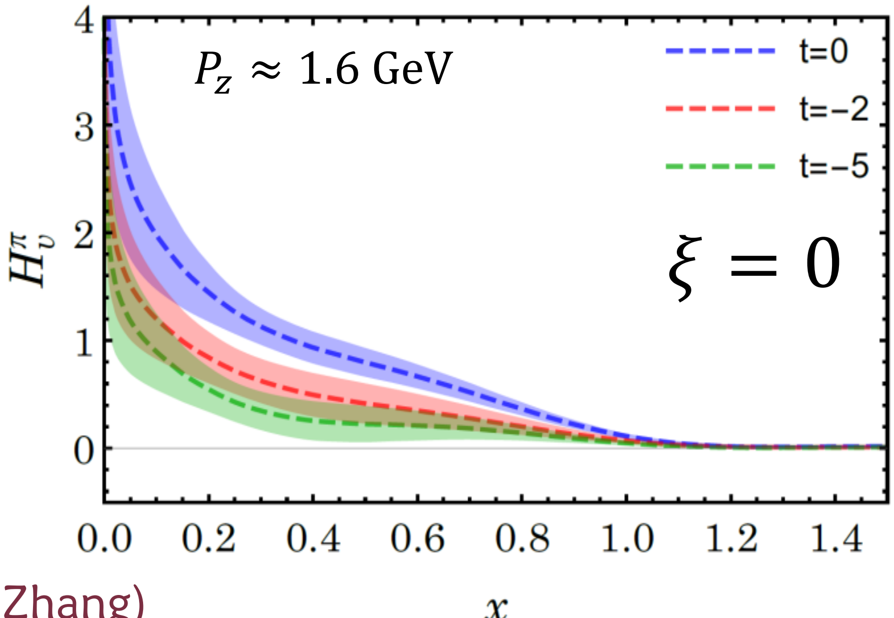
$P_z \approx 1.3, 1.6$ GeV

J. Chen, HL, J. Zhang, 1904.12376



$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right) \gamma^+ \Gamma\left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$



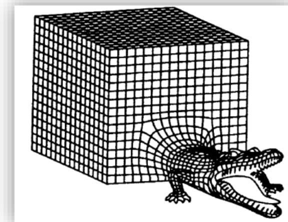


(plot by J. Zhang)

Isovector Nucleon GPDs

§ Pioneering first glimpse into nucleon GPD using quasi-PDFs

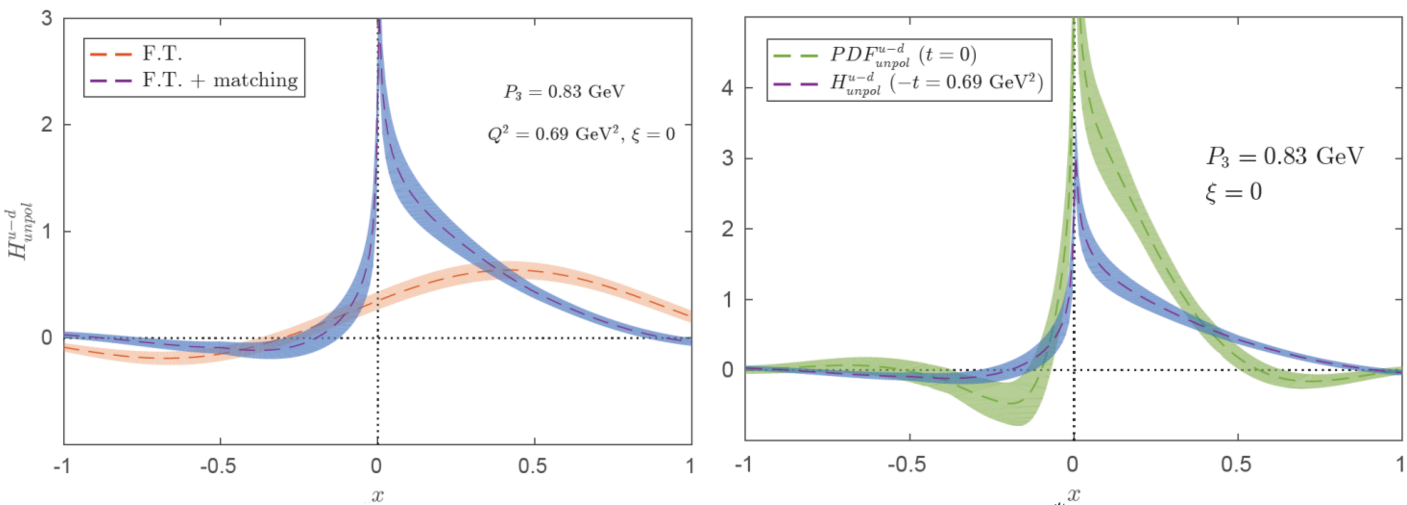
- Lattice details: twisted-mass fermions, 0.09fm, **270-MeV** pion mass, $P_z \approx 0.83$ GeV



$$F(x,\xi,t)=\int \frac{d\zeta^-}{4\pi} e^{-ix\bar{P}^+\zeta^-}\langle P'|O_{\gamma^+}(\zeta^-)|P\rangle = \frac{1}{2\bar{P}^+}\bar{u}(P')\left\{\boxed{H(x,\xi,t)}\gamma^+ + E(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}\right\}u(P)$$

nucleon $\xi = 0$ isovector results

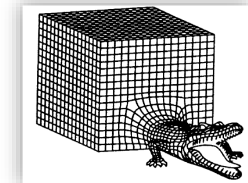
C. Alexandrou, (**ETMC**), 1910.13229 (Lattice 2019 Proceeding)



Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

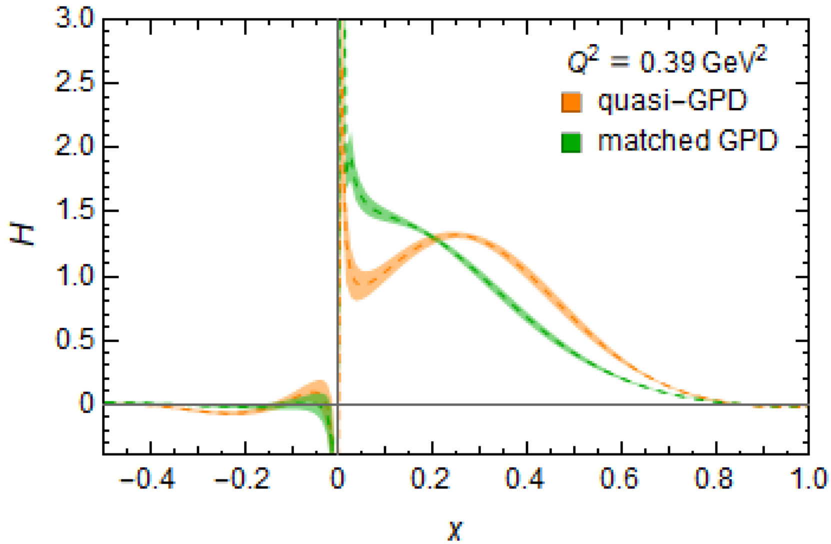
- Lattice details: clover/2+1+1 HISQ
  0.09fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

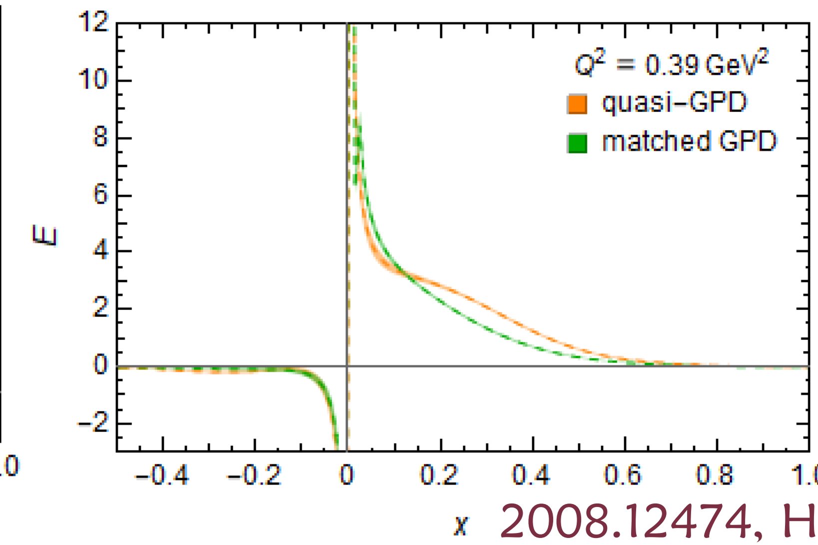


finite-volume, discretization, …

$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(\tilde{H}(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + \tilde{E}(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

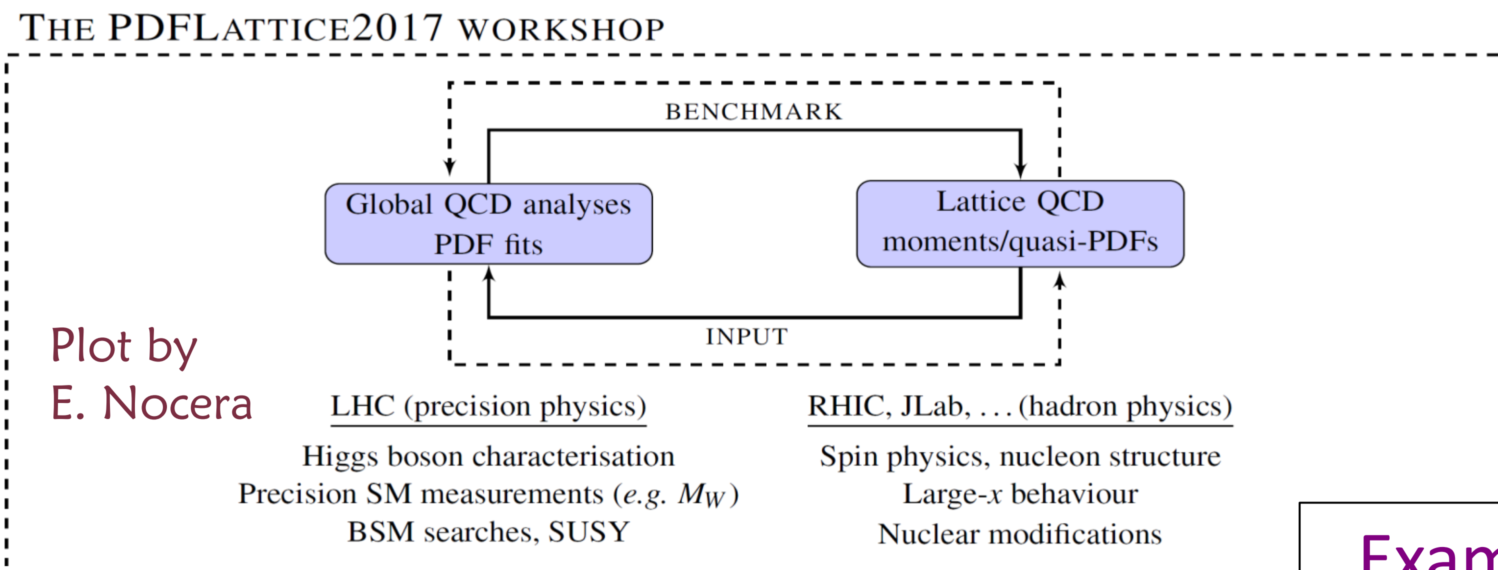
$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$





2008.12474, HL (MSULat)

How Can Lattice Help?

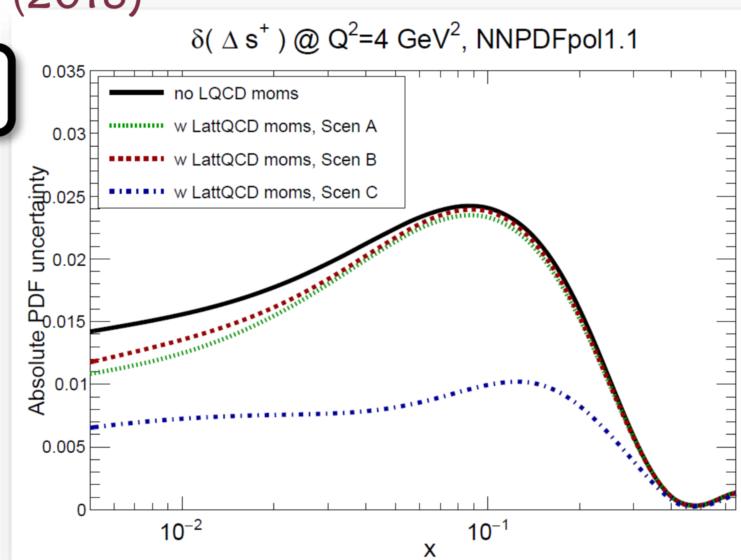


Plot by
E. Nocera

Whitepaper , Progress in Par. and Nuc. Phy. 100, 106 (2018)

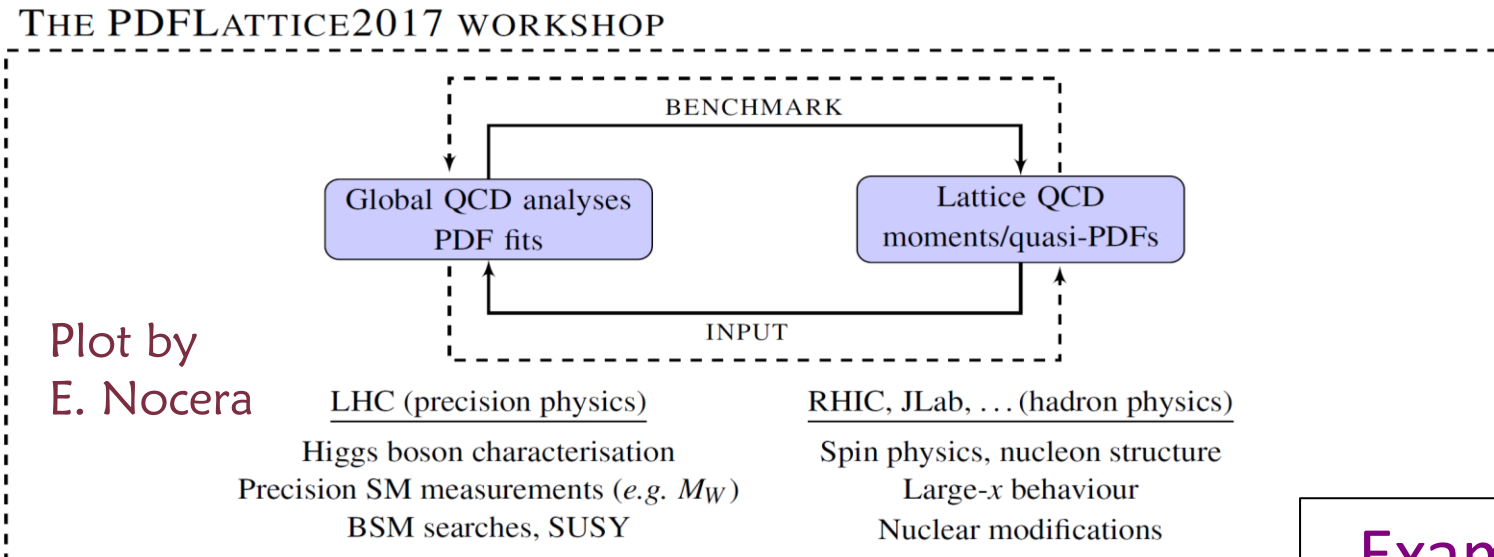
Example study

A: 70% B: 50% C: 20%



- Is there one quantity for which LQCD can achieve a precision at which it can make a significant difference?

How Can Lattice Help?



Plot by
E. Nocera

Whitepaper , Progress in Par. and Nuc. Phy. 100, 106 (2018)

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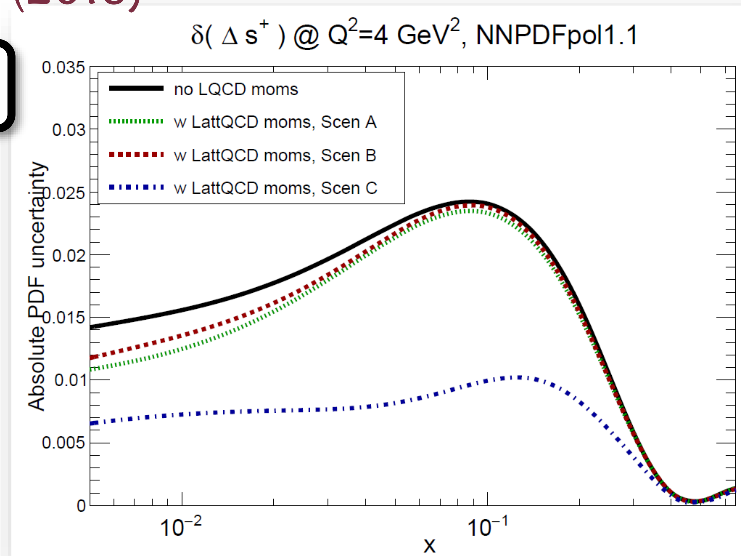
Example study



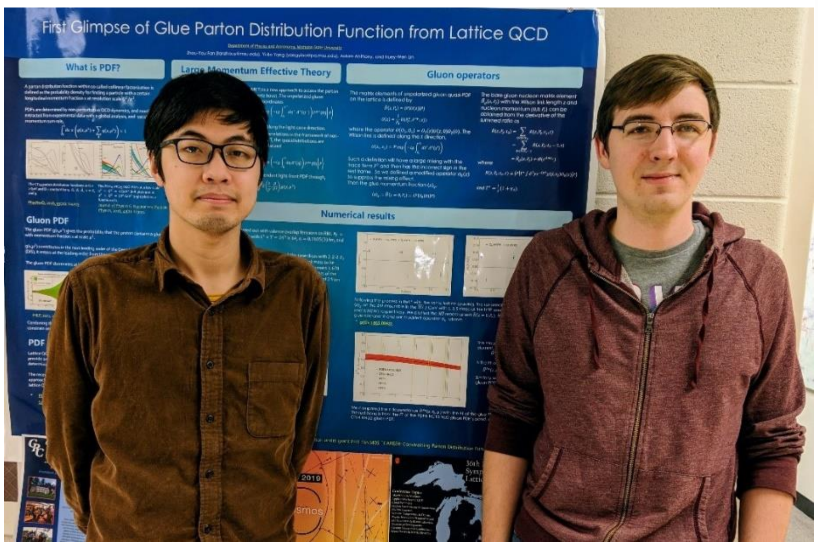
- Is there one quantity for which LQCD can achieve a precision at which it can make a significant difference?

Gluon PDF

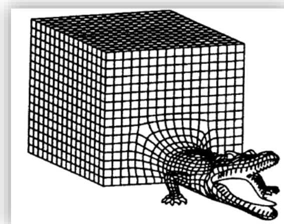
§ Pioneering first glimpse into gluon PDF using LaMET

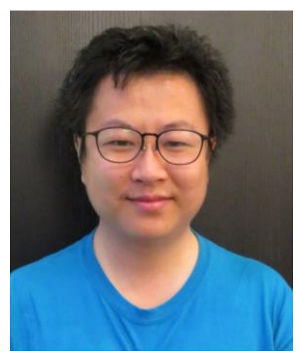
- Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- Promising results using coordinate-space comparison, but signal does not go far in z
- Hard numerical problem to be solved

Fan et al, Phys.Rev.Lett. 121, 242001 (2018)

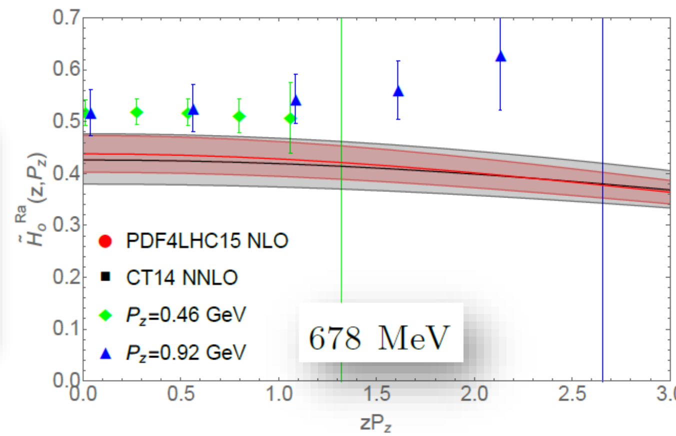


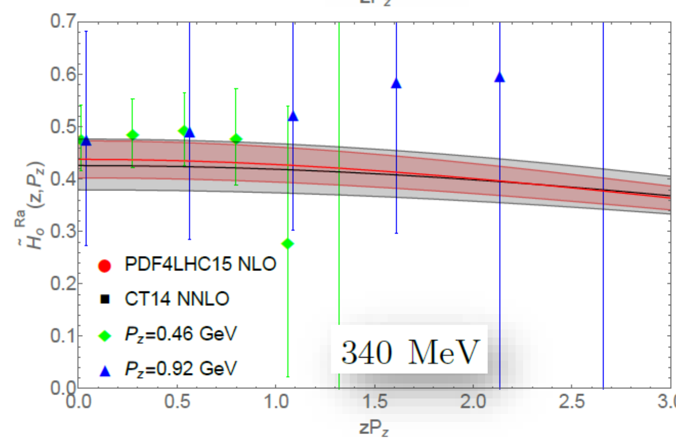
G: Zhouyou Fan G: Adam Antony





P: Yi-Bo Yang





iCER@MSU is crucial for earlier code development and completion of this work