Azimuthal dijet correlations: a window to gluon saturation at the EIC

Farid Salazar

EICUG 2nd Detector Meeting
December 8th, 2022

Based on

(1) [2108.06347](2108.06347) [JHEP 11 (2021) 222]
(2) [2208.13872](2208.13872) [JHEP 11 (2022) 169]

+ work in progress

In collaboration with

Paul Caucal (BNL→ Nantes)
Björn Schenke (BNL)
Tomasz Stebel (Jagiellonian)
Raju Venugopalan (BNL)

Outline

- Azimuthal correlations a window to gluon saturation

- Dijet production in the CGC at NLO

  P. Caucal, FS, R. Venugopalan. *2108.06347* *[JHEP 11 (2021) 222]*

- Back-to-back limit: gluon saturation and Sudakov

  P. Caucal, FS, B. Schenke ,R. Venugopalan. *2208.13872* *[To appear in JHEP soon]*

  + very preliminary numerical results with T. Stebel

- Experimental requirements and related observables

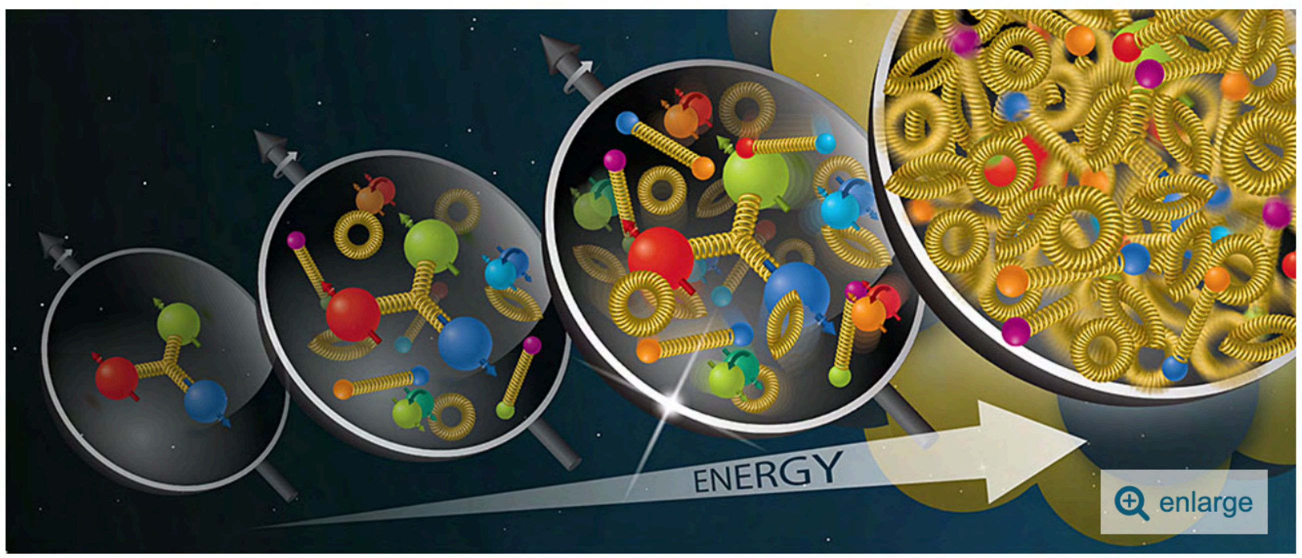
Azimuthal correlation a window to gluon saturation

Forward dihadron azimuthal correlations at RHIC

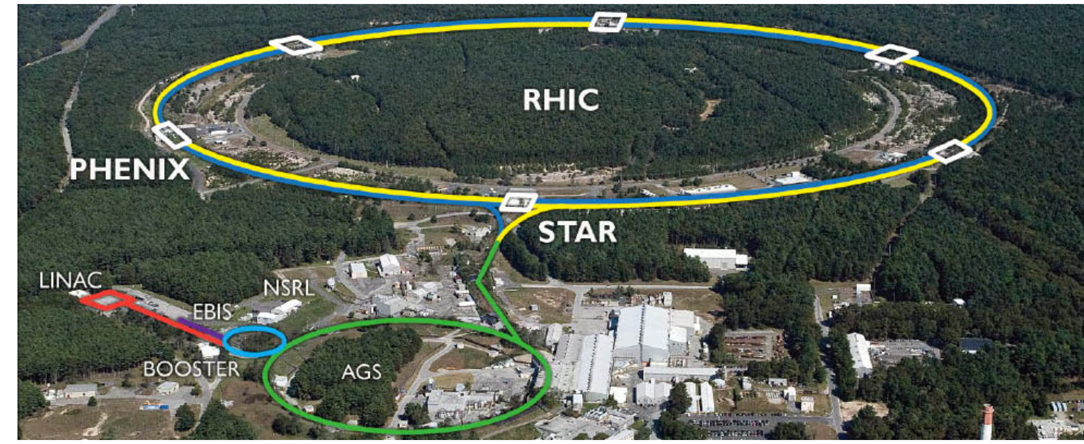
Signs of Saturation Emerge from Particle Collisions at RHIC

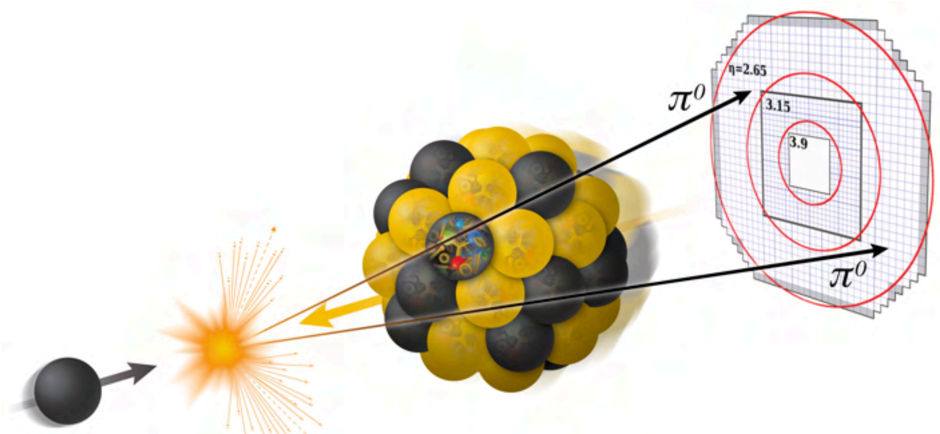
Suppression of a telltale sign of quark-gluon interactions presented as evidence of multiple scatterings and gluon recombination in dense walls of gluons

August 31, 2022



Members of the STAR collaboration report new data that indicate nuclei accelerated to very high energies at the Relativistic Heavy Ion Collider (RHIC) may be reaching a state where gluons are starting to saturate.





"We varied the species of the colliding ion beam because theorists predicted that this sign of saturation would be easier to observe in heavier nuclei," explained Brookhaven Lab physicist Xiaoxuan Chu, a member of the STAR collaboration who led the analysis. "The good thing is RHIC, the world's most flexible collider, can accelerate different species of ion beams. In our analysis, we used collisions of protons with other protons, aluminum, and gold."

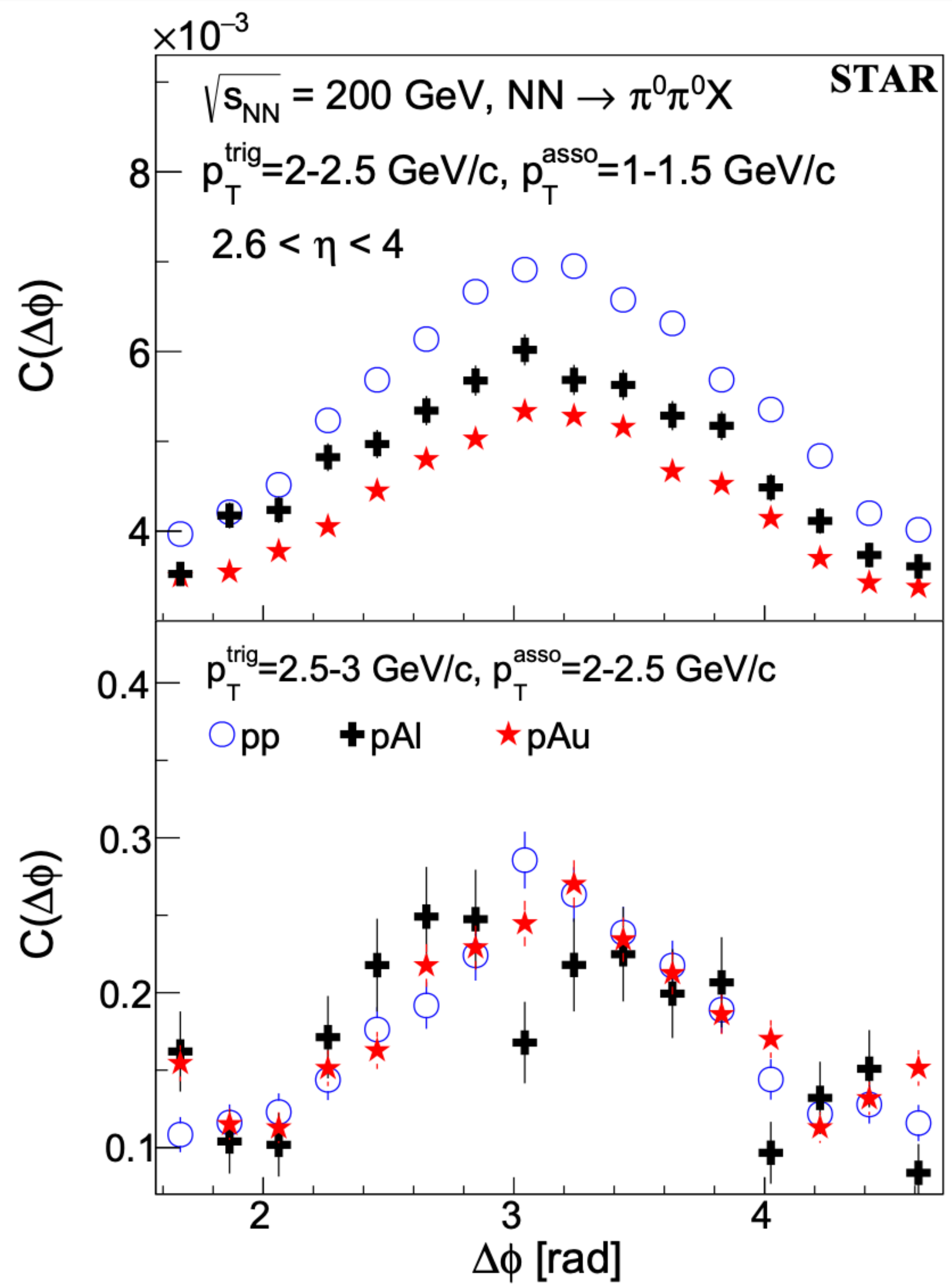
Xiaoxuan Chu and Elke Aschenauer

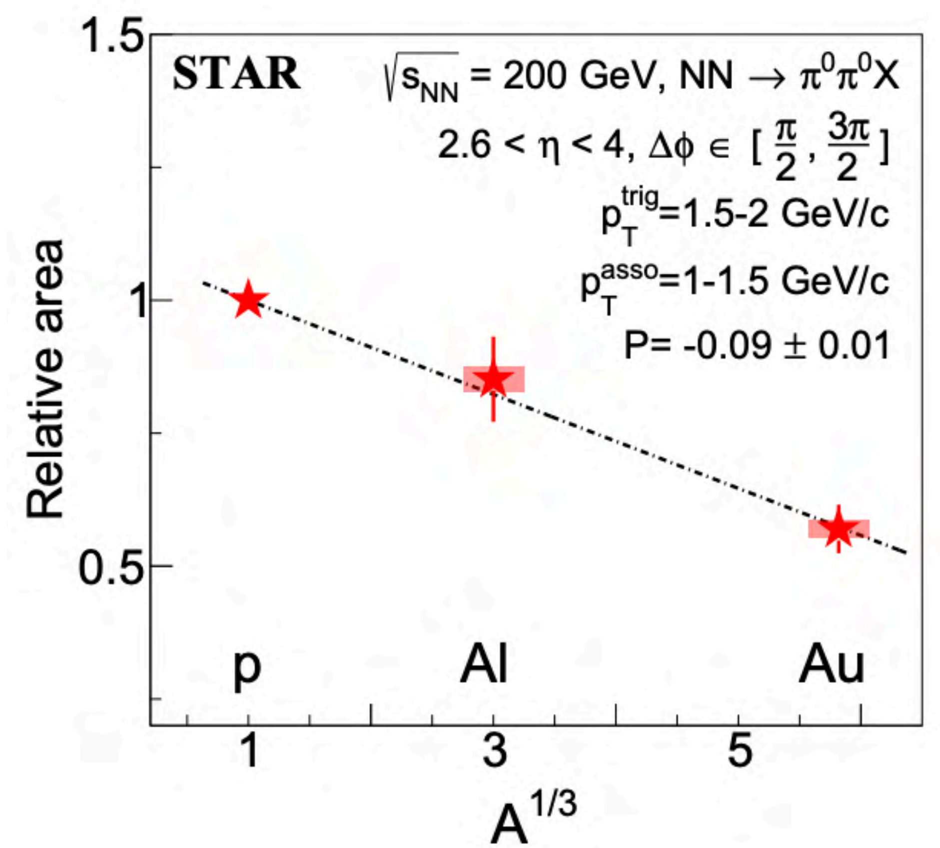
Source: BNL Newsroom

Azimuthal correlation a window to gluon saturation

Forward dihadron azimuthal correlations at RHIC

Evidence for Nonlinear Gluon Effects in QCD and Their Mass Number Dependence at STAR



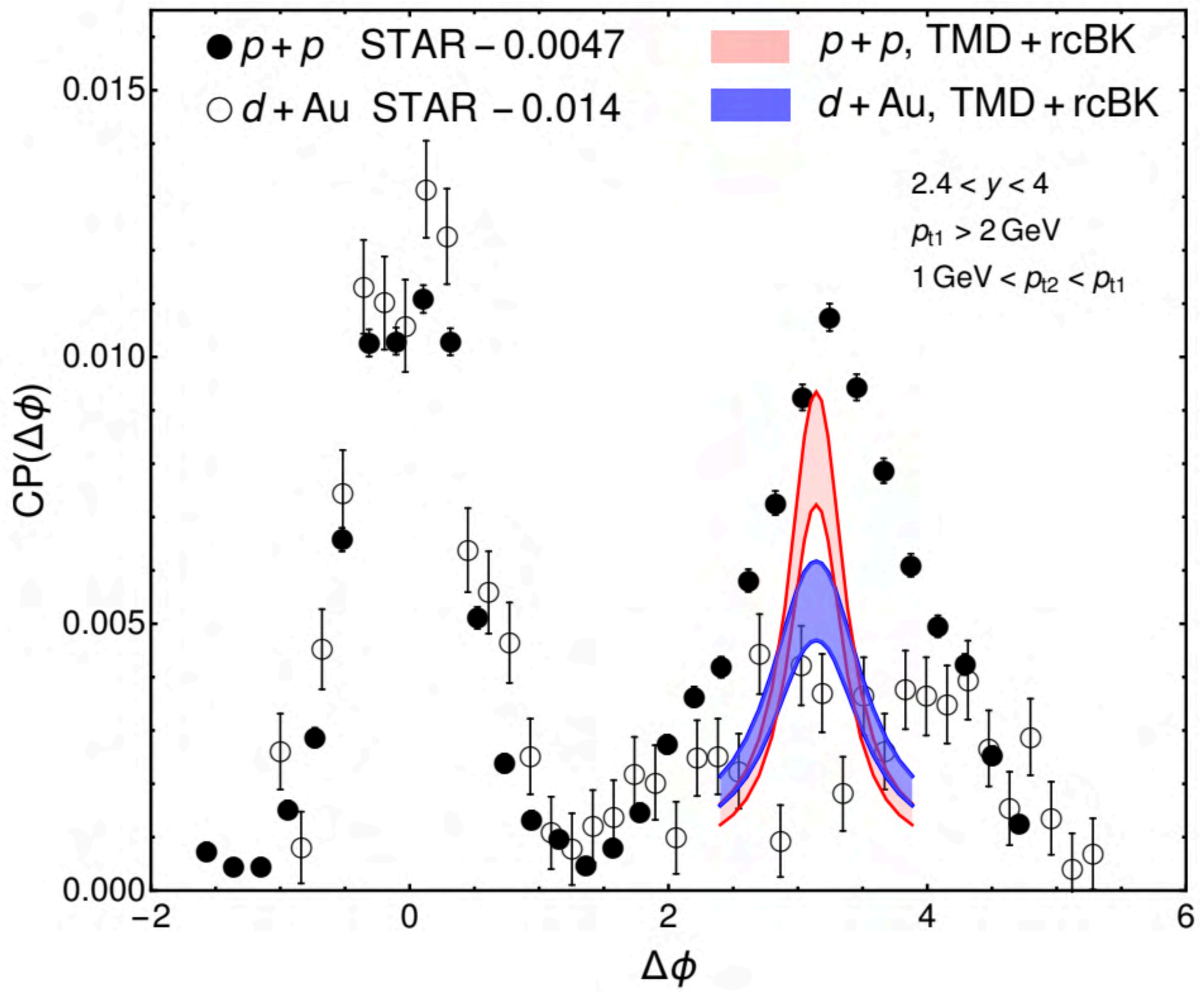


$A^{1/3}$ **suppression supports saturation picture**

STAR Collaboration *(PRL 2022)*

Azimuthal correlation a window to gluon saturation

Gluon saturation without Sudakov



Experimental data: E. Braidot [STAR Collaboration] *arXiv:1005.2378*

Theory curves: J. Albacete, G. Giacalone, C. Marquet, M. Matas *(PRD 2019)*

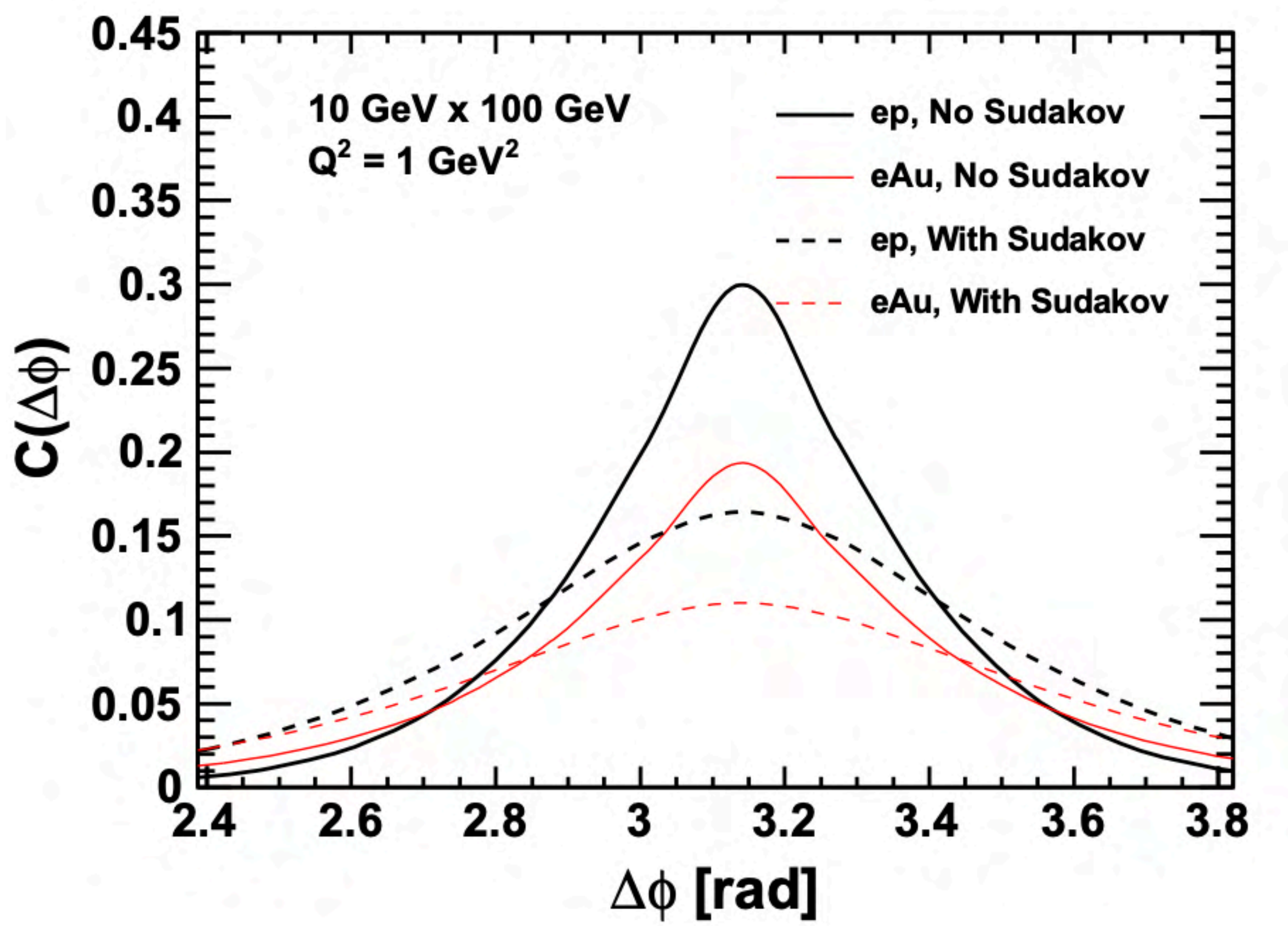
Gluon saturation alone cannot describe data

For recent phenomenology of Sudakov + Gluon Saturation see talks at DIS2022 by Marquet and Benić

See also A. Stasto, S.Y. Wei, B.W. Xiao, F. Yuan *(PLB 2018)*

Azimuthal correlation a window to gluon saturation

Forward dihadron azimuthal correlations at the EIC



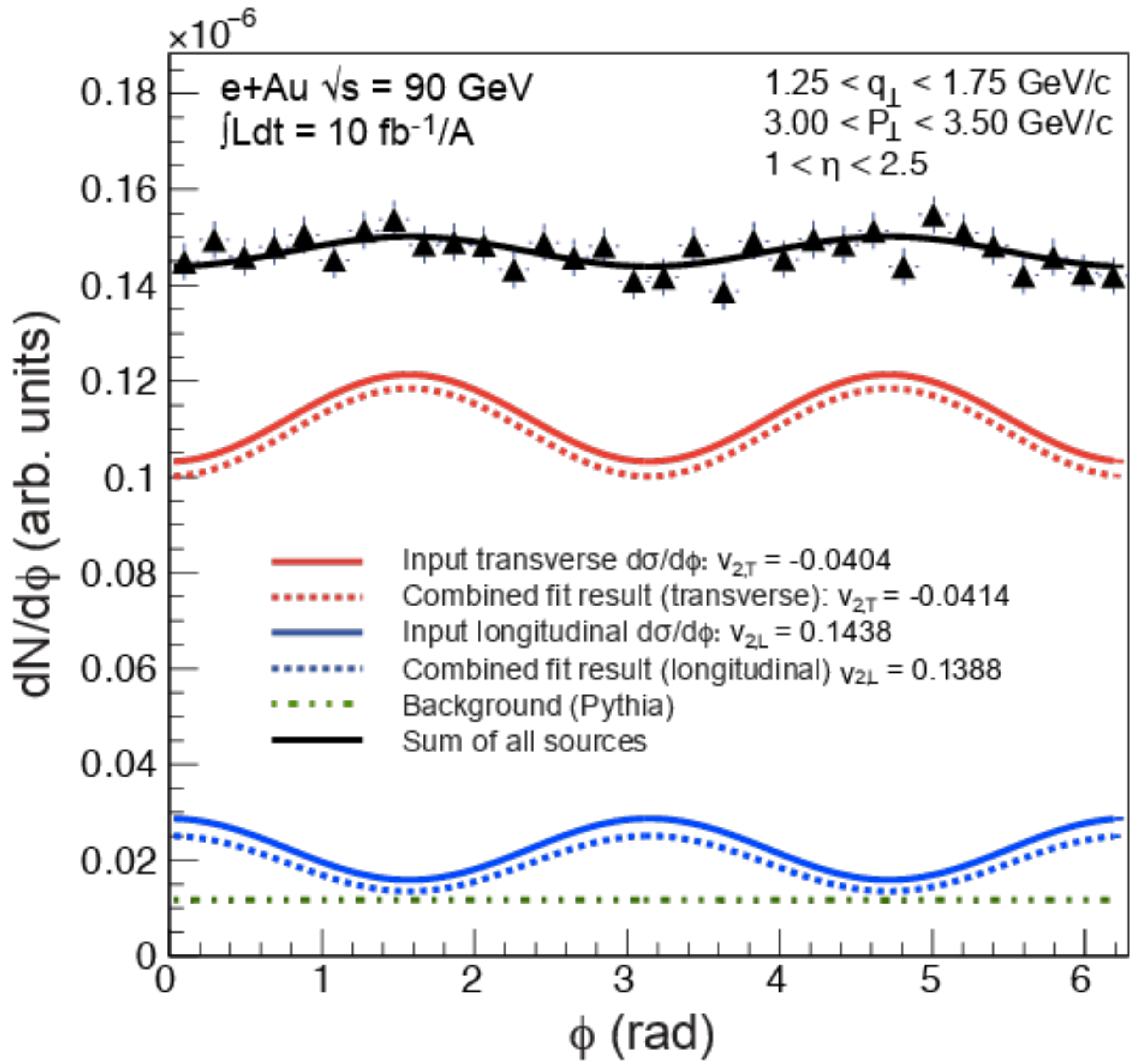
L. Zheng, E.C. Aschenauer, J.H. Lee, B.W. Xiao *(PRD 2014)*

Advantages of EIC over RHIC: better control over kinematics, no pedestal, one-channel (depends only on WW distribution)

For dijet production (at LO) see A. van Hameren, P. Kotko, K. Kotak, S. Sapeta, E. Żarów (EPJC 2021)

Azimuthal correlation a window to gluon saturation

Dihadron azimuthal correlations at the EIC



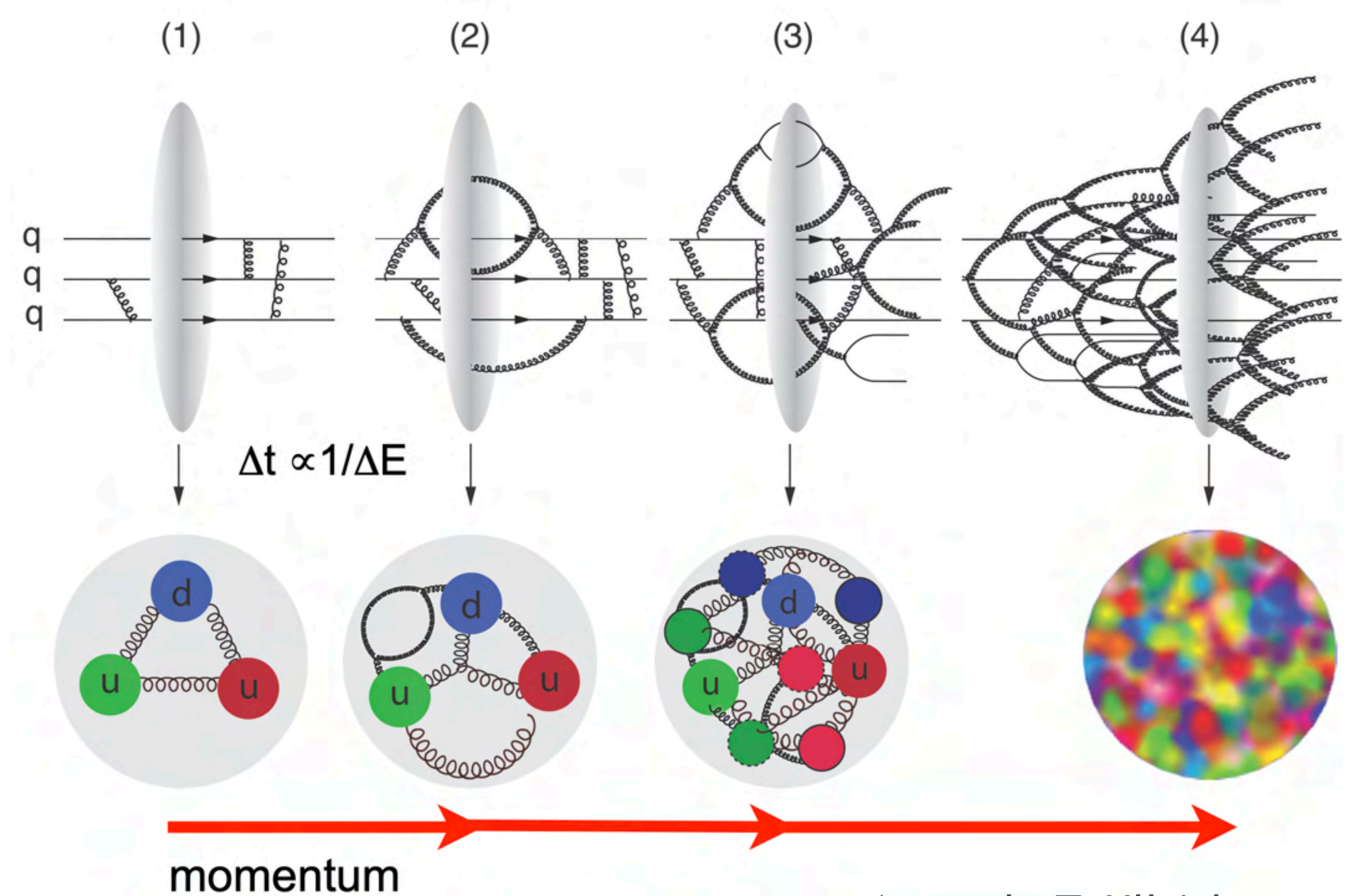
A. Dumitru, V. Skokov, T. Ullrich *(PRC 2019)*

See also A. Dumitru, T. Lappi, V. Skokov (PRL 2015)

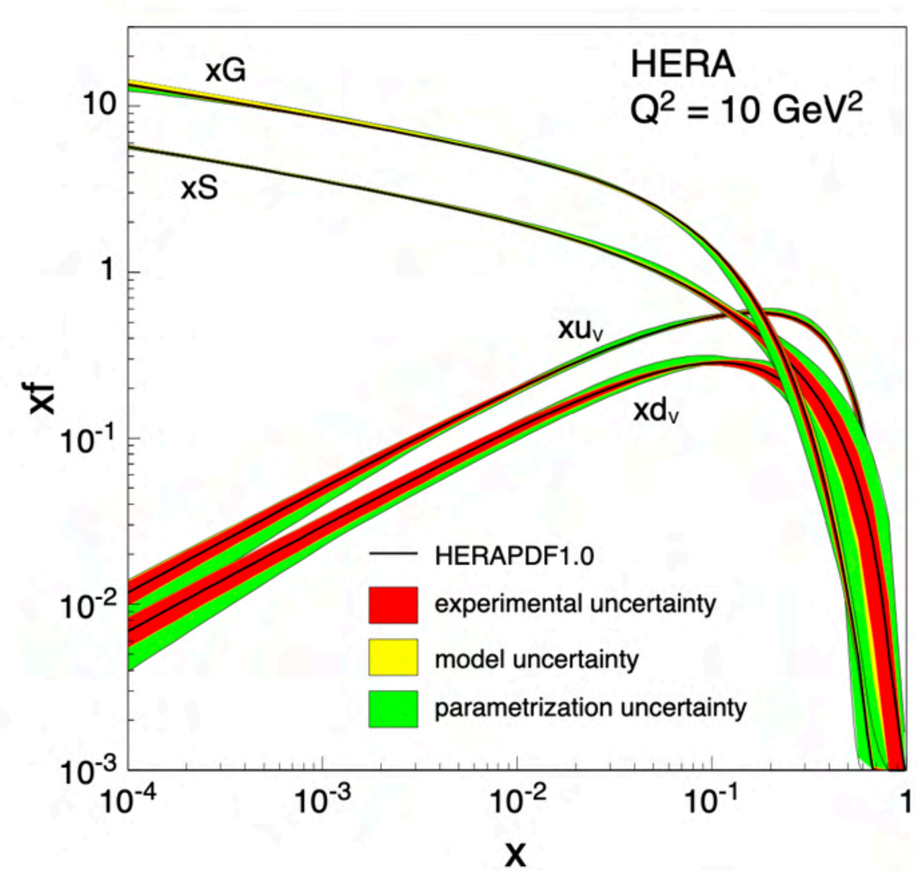
Momentum imbalance azimuthal distribution (relative to jet azimuthal angle) sensitive to linearly polarized gluons

Quarkonium production in pp and pA

The Color Glass Condensate in a Nutshell: anatomy of QCD at high-energy



Artwork: T. Ullrich



Emergence of an energy and nuclear specie dependent momentum scale (saturation scale)

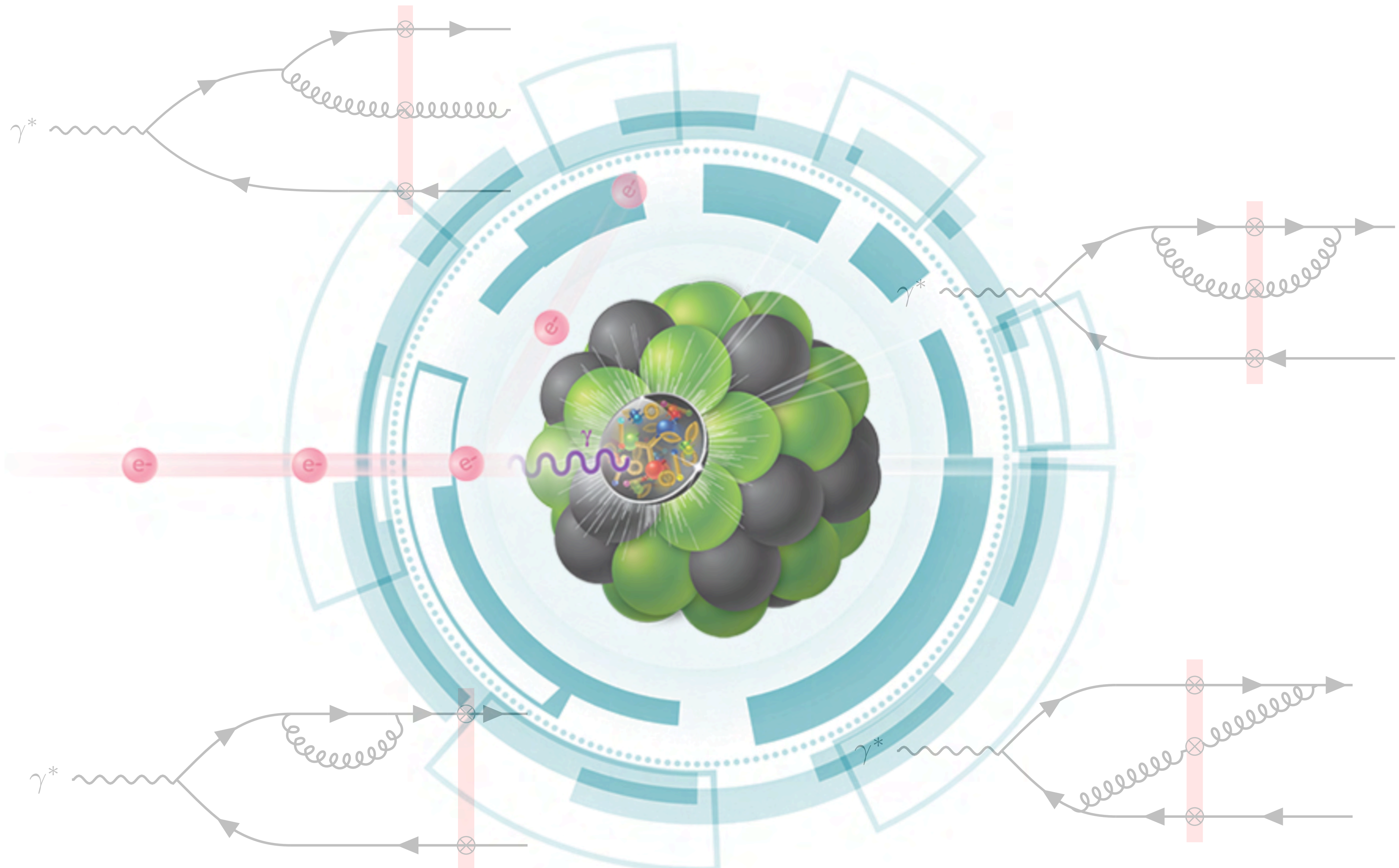
Multiple scattering (higher twist effects)

Non-linear evolution equations (BK/JIMWLK)

$$Q_s^2 \propto A^{1/3} x^{-\lambda} \qquad x \propto 1/s$$

For a review see Mining gluon saturation at colliders. FS, A. Morreale (Universe 2021)

Dijet impact factor in DIS at next-to-leading order in the Color Glass Condensate



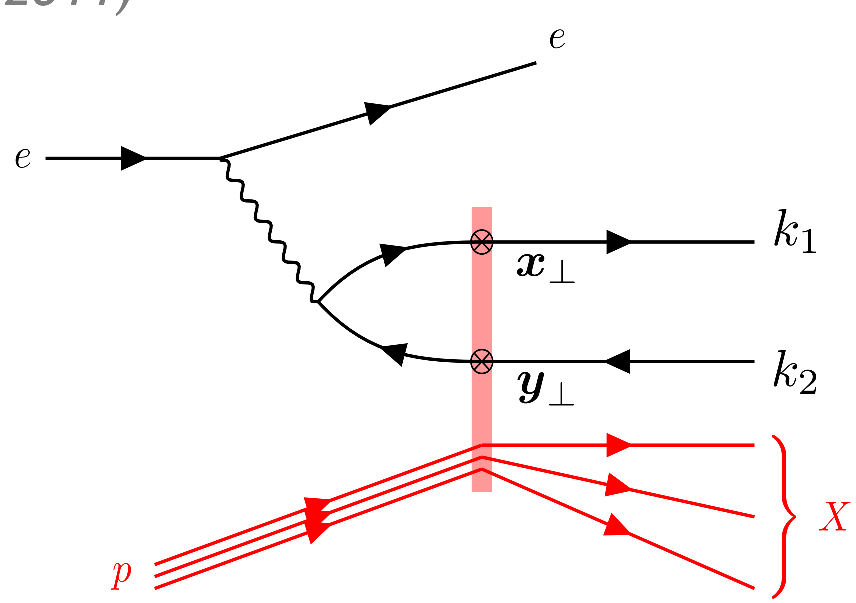
[2108.06347](https://arxiv.org/abs/2108.06347) *[JHEP 11 (2021) 222]*

In collaboration with Paul Caucal and Raju Venugopalan

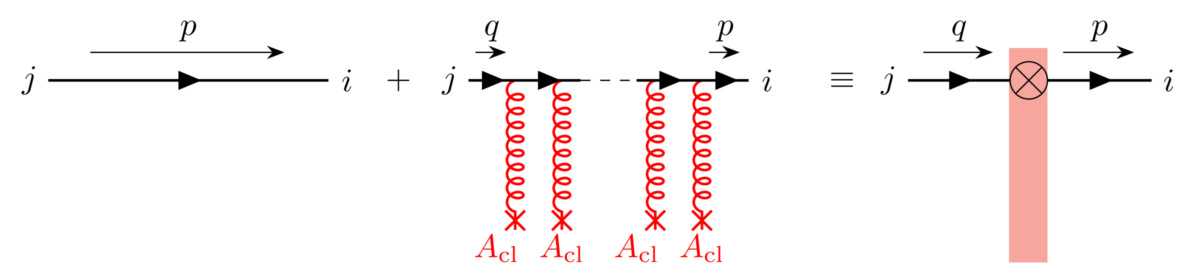
Dijet production the CGC

Leading order result

F. Dominguez, C. Marquet, B.W. Xiao, F. Yuan. *(PRD 2011)*



Dense gluon field $A_{\rm cl} \sim 1/g$ needs <u>resummation</u> of multiple gluon interactions



$$V_{ij}(\boldsymbol{x}) = P\exp\left\{ig\int dx^- A^{+,a}_{cl}(\boldsymbol{x},x^-)t^a\right\}$$

Unpolarized differential cross-section:

$$\frac{\mathrm{d}\sigma^{\gamma^*_\lambda+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} \propto \int \mathrm{d}^8\boldsymbol{X}_\perp\ e^{-i\boldsymbol{k}_{1\perp}\cdot(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)}e^{-i\boldsymbol{k}_{2\perp}\cdot(\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}$$

$$\times \boxed{\langle\Xi_{\rm LO}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp\boldsymbol{x}'_\perp)\rangle_Y} \boxed{\mathcal{R}^\lambda(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp-\boldsymbol{y}'_\perp)}$$

$q\bar{q}$ interaction with nucleus $\qquad$ γ^* splitting to $q\bar{q}$

$$\Xi_{\rm LO}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp\boldsymbol{y}'_\perp) = 1 - S^{(2)}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) - S^{(2)}(\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp) + S^{(4)}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp)$$

dipoles

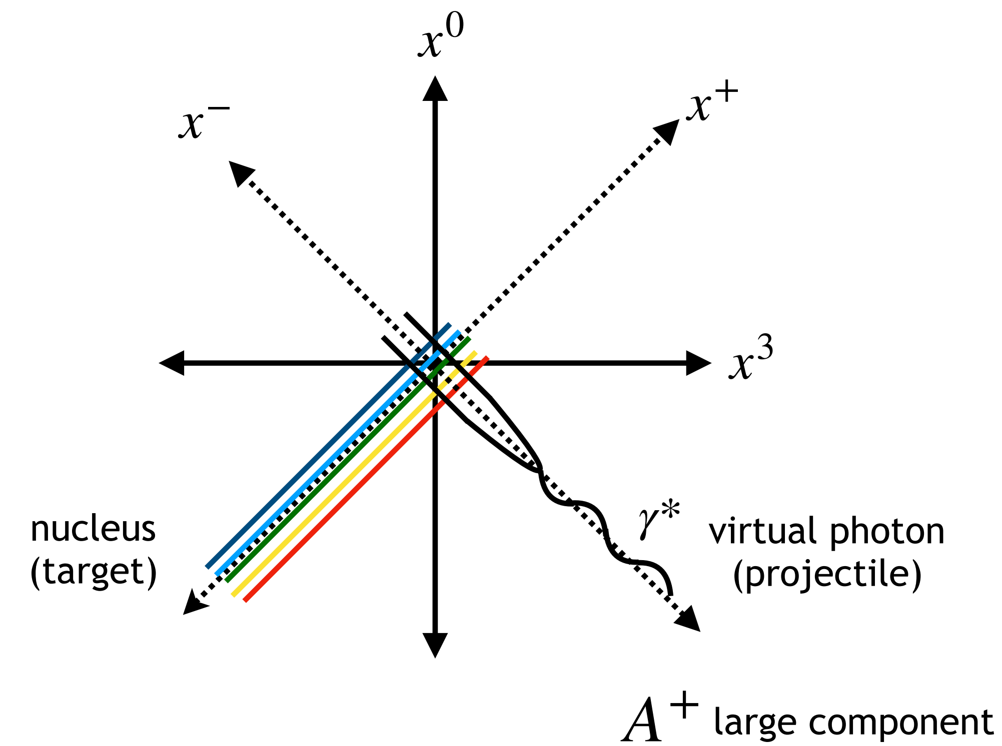
quadrupole

$$S^{(2)}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = \frac{1}{N_c}\mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]$$

Dijet production the CGC

Next-to-leading order results

P. Caucal, FS, and R. Venugopalan *(JHEP 2021)*



- Divergences: UV, soft and collinear

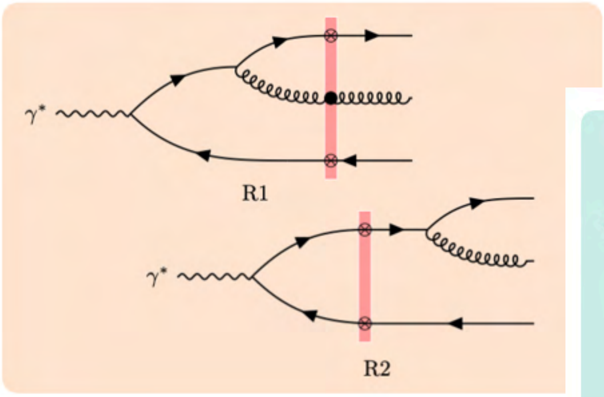
Dimensional regularization + longitudinal momentum cut-off + small-R cone algorithm

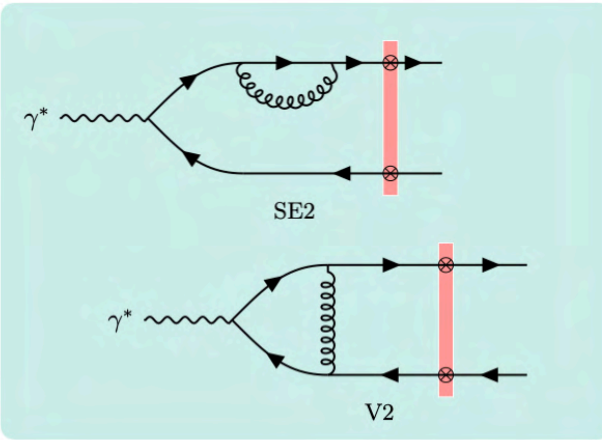
$$\int_{\Lambda^-} \frac{\mathrm{d}k_g^-}{k_g^-} \mu^\varepsilon \int \frac{\mathrm{d}^{2-\varepsilon}\boldsymbol{k_{g\perp}}}{(2\pi)^{2-\varepsilon}} f_{\Lambda^-}(k_g^-, \boldsymbol{k_{g\perp}})$$

- Large rapidity (high-energy) logs

Resummed via JIMWLK renormalization

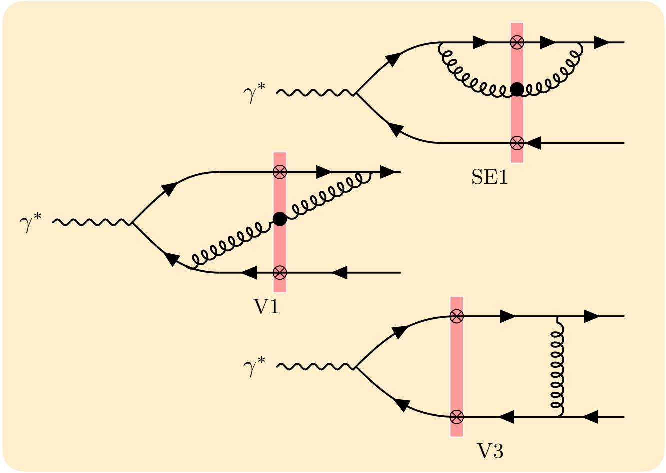
$$\left[1 + \mathcal{H}_{\mathrm{LL}} \ln\left(\Lambda_f^-/\Lambda^-\right) + \ldots\right] \langle \mathrm{d}\sigma \rangle_{\Lambda^-} = \langle \mathrm{d}\sigma \rangle_{\Lambda_f^-}$$





- Impact factor

Finite piece (free of large rapidity logs), but might contain other (potentially) large logs!



- We showed cancellation of UV, soft and collinear divergences
- Absorbed large energy/rapidity logs into JIMWLK resummation
- Isolated genuine $\mathcal{O}(\alpha_s)$ contributions (aka impact factor)

Dijet production the CGC

Next-to-leading order results: rapidity factorization

P. Caucal, FS, and R. Venugopalan *(JHEP 2021)*

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \boxed{\mathrm{d}\widetilde{\sigma}_0 \ln\left(\frac{z_f}{z_0}\right)} + \boxed{\int_0^{z} \frac{\mathrm{d}z_g}{z_g}\left[\mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}} - \mathrm{d}\widetilde{\sigma}_0 \Theta(z_f - z_g)\right]} + \mathcal{O}(z_0)$$

Leading Logarithmic in x (LLx)

impact factor

$$\left.\frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{NLO}}}{\mathrm{d}^2\boldsymbol{k_{1\perp}}\mathrm{d}\eta_1\mathrm{d}^2\boldsymbol{k_{2\perp}}\mathrm{d}\eta_2}\right|_{\mathrm{LLx}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^6}\delta(1 - z_q - z_{\bar{q}}) \int \mathrm{d}\boldsymbol{X}_\perp \mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{x'y'}) \ln\left(\Lambda_f^-/\Lambda^-\right)$$

$$\mathcal{H}_{\mathrm{LL}} \left\langle \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{x}'_\perp \boldsymbol{y}'_\perp)\right\rangle_Y$$

$$\begin{aligned}
\times \frac{\alpha_s N_c}{4\pi^2}\Bigg\langle \int \mathrm{d}^2\boldsymbol{z}_\perp \Bigg\{ &\frac{\boldsymbol{r}^2_{xy}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy}}(2D_{xy} - 2D_{xz}D_{zy} + D_{zy}Q_{y'x',xz} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \\
&+ \frac{\boldsymbol{r}^2_{x'y'}}{\boldsymbol{r}^2_{zx'}\boldsymbol{r}^2_{zy'}}(2D_{y'x'} - 2D_{y'z}D_{zx'} + D_{zx'}Q_{xy,y'z} + D_{y'z}Q_{xy,zx'} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \\
&+ \frac{\boldsymbol{r}^2_{xx'}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zx'}}(D_{zx'}Q_{xy,y'z} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\
&+ \frac{\boldsymbol{r}^2_{yy'}}{\boldsymbol{r}^2_{zy}\boldsymbol{r}^2_{zy'}}(D_{y'z}Q_{xy,zx'} + D_{zy}Q_{y'x',xz} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\
&+ \frac{\boldsymbol{r}^2_{xy'}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy'}}(D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{zx'}Q_{xy,y'z} - D_{zy}Q_{y'x',xz}) \\
&+ \frac{\boldsymbol{r}^2_{x'y}}{\boldsymbol{r}^2_{zx'}\boldsymbol{r}^2_{zy}}(D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{y'z}Q_{xy,zx'} - D_{xz}Q_{y'x',zy}) \Bigg\}\Bigg\rangle_Y
\end{aligned}$$

Small-x evolution of dipole and quadrupole!

Evolution of quadrupole can be found in Dominguez, Mueller, Munier, Xiao. Phys.Lett.B 705 (2011) 106-111

JIMWLK LL Hamiltonian acting on LO color structure → Renormalization of Wilson line operators

Dijet production the CGC

Next-to-leading order results: impact factor

P. Caucal, FS, and R. Venugopalan *(JHEP 2021)*

$$\mathrm{d}\sigma_{\mathrm{R}_2\times\mathrm{R}_2,\mathrm{sud2}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) \times C_F\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{\alpha_s}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}}\right]\ln\left(\frac{\boldsymbol{k_{1\perp}}^2\boldsymbol{r}_{xx'}^2R^2\xi^2}{c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{R}_2\times\mathrm{R}'_2,\mathrm{sud2}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) \times\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{(-\alpha_s)}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy'}}\right]\ln\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{xy'}^2\xi^2}{z_2^2c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{sud1}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\times\frac{\alpha_s}{\pi} \times\Bigg\{C_F\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_f}{z_1}\right)\ln\left(\frac{\boldsymbol{r}_{xx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_f}{z_2}\right)\ln\left(\frac{\boldsymbol{r}_{yy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right] + \Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_1}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{xy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_2}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{yx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\Bigg\}$$

$$\mathrm{d}\sigma_{\mathrm{V,no-sud,LO}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times\frac{\alpha_sC_F}{\pi}\left\{-\frac{3}{4}\ln\left(\frac{\boldsymbol{k_{1\perp}}^2\boldsymbol{k_{2\perp}}^2\boldsymbol{r}_{xy}^2\boldsymbol{r}_{x'y'}^2}{c_0^4}\right)-3\ln(R)+\frac{1}{2}\ln^2\left(\frac{z_1}{z_2}\right)+\frac{11}{2}+3\ln(2)-\frac{\pi^2}{2}\right\}$$

$$\mathrm{d}\sigma^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,LO}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}(4\alpha_sC_F)\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times\frac{e^{-i\boldsymbol{k_{g\perp}}\cdot\boldsymbol{r}_{xx'}}}{(\boldsymbol{k_{g\perp}}-\frac{z_g}{z_1}\boldsymbol{k_{1\perp}})^2}\Bigg\{8z_1z_2^3(1-z_2)^2Q^2\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)K_0(\bar{Q}_{\mathrm{R}2}r_{xy})K_0(\bar{Q}_{\mathrm{R}2}r_{x'y'})\delta_z^{(3)} -\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta(z_1-z_g)\delta_z^{(2)}\Bigg\}+(1\leftrightarrow 2)$$

$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^3\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'}) \times\frac{\alpha_s}{\pi}\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}\Bigg\{K_0(\bar{Q}_{\mathrm{V}3}r_{xy})\Bigg[\left(1-\frac{z_g}{z_1}\right)^2\left(1+\frac{z_g}{z_2}\right)(1+z_g)e^{i(\boldsymbol{P}_\perp+z_g\boldsymbol{q}_\perp)\cdot\boldsymbol{r}_{xy}}K_0(-i\Delta_{\mathrm{V}3}r_{xy}) -\left(1-\frac{z_g}{2z_1}+\frac{z_g}{2z_2}-\frac{z_g^2}{2z_1z_2}\right)e^{i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy}}\mathcal{J}_\odot\left(\boldsymbol{r}_{xy},\left(1-\frac{z_g}{z_1}\right)\boldsymbol{P}_\perp,\Delta_{\mathrm{V}3}\right)\Bigg] +K_0(\bar{Q}r_{xy})\ln\left(\frac{z_gP_\perp r_{xy}}{c_0z_1z_2}\right)+(1\leftrightarrow 2)\Bigg\}\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)+c.c.$$

$$\mathrm{d}\sigma^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c}{(2\pi)^8}\int \mathrm{d}^2\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}(-4\alpha_s)\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times\frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy'}}}{\boldsymbol{l}_\perp^2}\Bigg\{8z_1^2z_2^2(1-z_2)(1-z_1)Q^2K_0(\bar{Q}_{\mathrm{R}2}r_{xy})K_0(\bar{Q}_{\mathrm{R}2'}r_{x'y'})\left[1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right] \times e^{-i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_{xy'}}\frac{\boldsymbol{l}_\perp\cdot(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)}{(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)^2}\delta_z^{(3)}-\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta\left(\frac{c_0^2}{\boldsymbol{r}_{xy'}^2}\geq\boldsymbol{l}_\perp^2\geq\boldsymbol{K}_\perp^2\right)\Theta(z_1-z_g)\delta_z^{(2)}\Bigg\} +(1\leftrightarrow 2)$$

$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,other}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'})\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g} \times\frac{\alpha_s}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\Bigg\{\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO},1} -\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})-\Theta(z_f-z_g)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})\right]C_F\Xi_{\mathrm{LO}} -\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy}^2}\Bigg[\left(1-\frac{z_g}{z_1}\right)\left(1+\frac{z_g}{z_2}\right)\left(1-\frac{z_g}{2z_1}-\frac{z_g}{2(z_2+z_g)}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}K_0(QX_V) -\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\Bigg]\Xi_{\mathrm{NLO},1}+(1\leftrightarrow 2)\Bigg\}+c.c..$$

$$\mathrm{d}\sigma^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,other}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(3)}}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\frac{\mathrm{d}^2\boldsymbol{z}'_\perp}{\pi}e^{-i\boldsymbol{k_{g\perp}}\cdot\boldsymbol{r}_{zz'}} \alpha_s\Bigg\{-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R}2}r_{w'y'})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO},1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp) +\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R}2'}r_{w'y'})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO},1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp) +\frac{1}{2}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO},4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp) -\frac{1}{2}\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO},4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp) +(1\leftrightarrow 2)+c.c.\Bigg\}-\frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^8}\alpha_s\Theta(z_f-z_g)\times\text{“slow”}$$

Similar expressions (but more lengthy) for transversely polarized photon

Back-to-back inclusive dijets in DIS at small-*x*: Sudakov suppression and gluon saturation at NLO

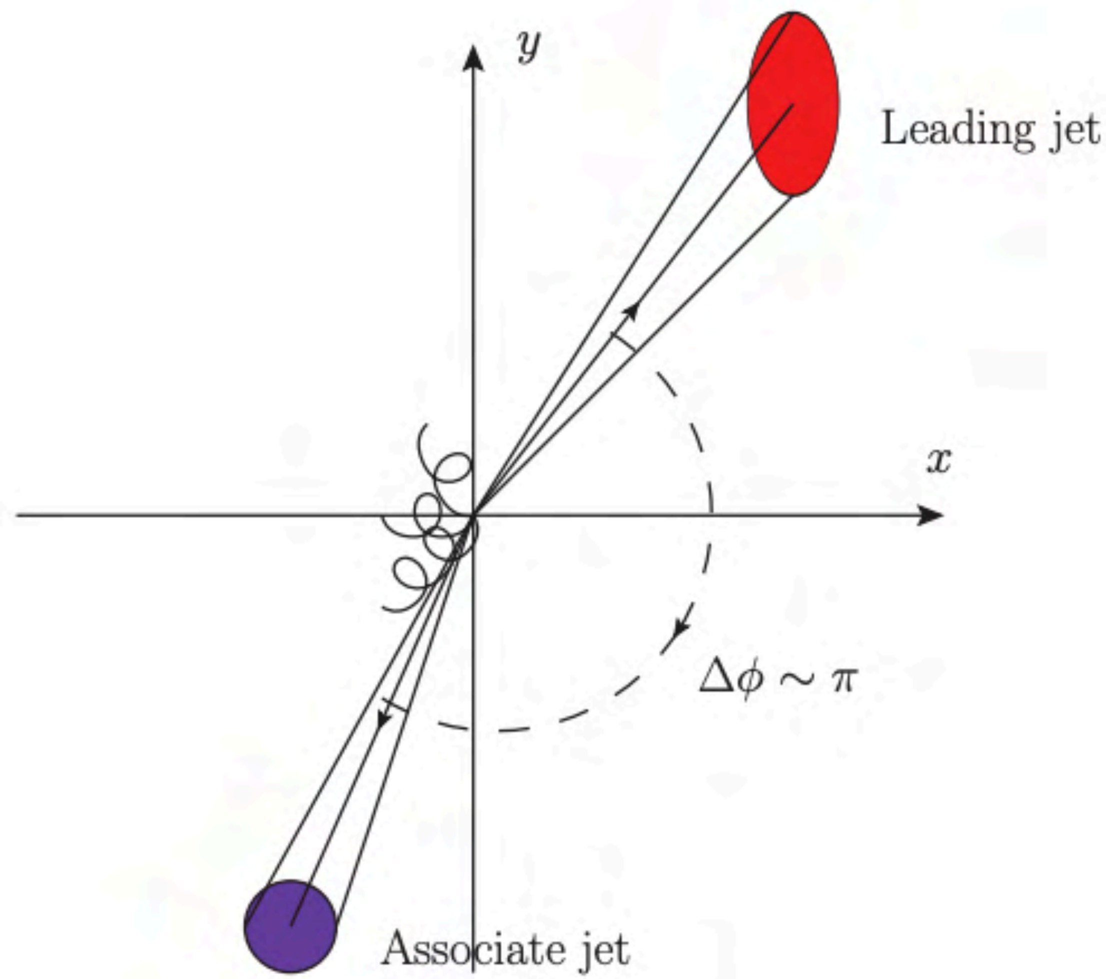


Figure from Xiao lecture notes on Sudakov

2208.13872 *[JHEP 11 (2022) 169]*

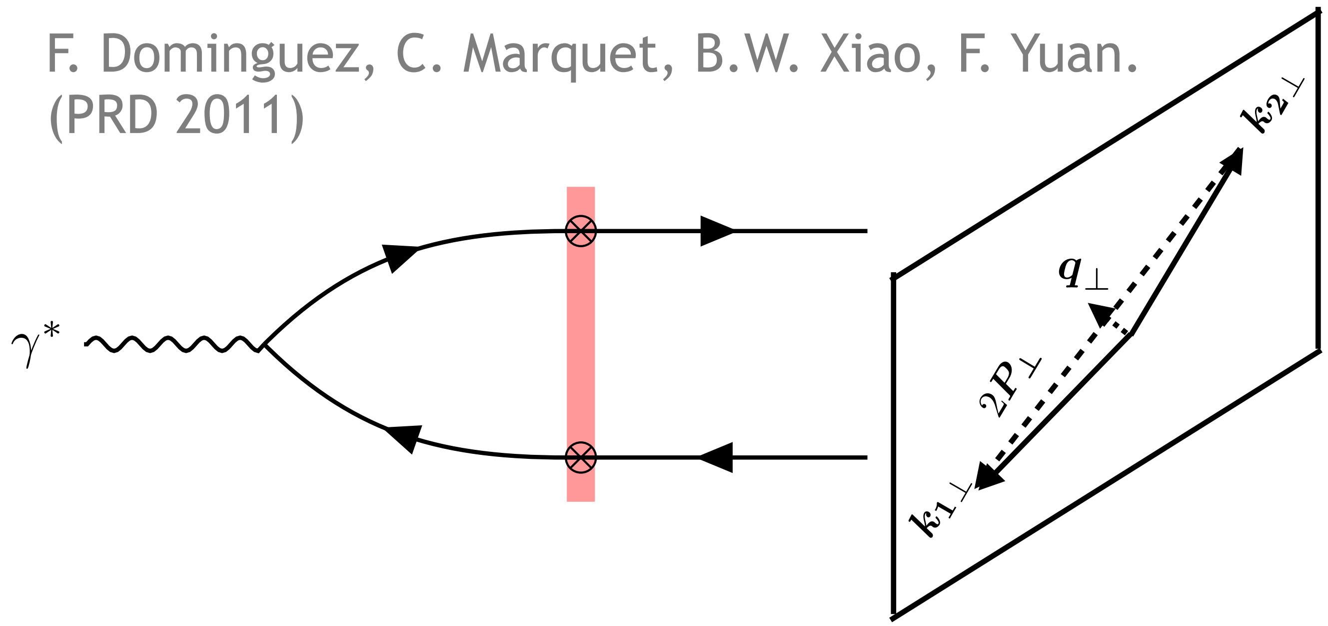
In collaboration with Paul Caucal, Björn Schenke, and Raju Venugopalan

+ work in progress with Tomasz Stebel

Back-to-back limit: gluon saturation and Sudakov

Leading order results:

F. Dominguez, C. Marquet, B.W. Xiao, F. Yuan.
(PRD 2011)



TMD valid $q_\perp, Q_s \ll k_{1\perp}, k_{2\perp}$

back-to-back hadrons/jets and transverse momenta larger than sat scale

In the back-to-back (or more precisely in the correlation limit) the CGC cross-section factorizes

$$\mathrm{d}\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X} \sim \mathcal{H}^{ij}(\boldsymbol{P}_\perp)\; \alpha_s G_Y^{ij}(\boldsymbol{q}_\perp)$$

Perturbatively calculable

WW gluon TMD

$$G_Y^{ij}(\boldsymbol{q}_\perp) = \frac{1}{2}\delta^{ij} G_Y^0(\boldsymbol{q}_\perp) + \frac{1}{2}\left(\frac{2\boldsymbol{q}_\perp^i \boldsymbol{q}_\perp^j}{\boldsymbol{q}_\perp^2} - \delta^{ij}\right) h_Y^0(\boldsymbol{q}_\perp)$$

Unpolarized — Linearly polarized

Azimuthal anisotropy

$$\langle \cos(2\phi) \rangle \sim h_Y^0(\boldsymbol{q}_\perp)/G_Y^0(\boldsymbol{q}_\perp)$$

$$\phi \equiv \phi_{\boldsymbol{q}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

F. Dominguez, J.W. Qiu, B.W. Xiao, F. Yuan.
(PRD 2012)

For an explicit quantitative comparison between TMD and CGC see

R. Boussarie, H. Mäntysaari, FS, B. Schenke.
(JHEP 2021)

Back-to-back limit: gluon saturation and Sudakov

Sudakov resummation and saturation

A.H. Mueller, B.W. Xiao, F. Yuan (PRD 2013)

“Large logarithms [Sudakov] can be generated due to incomplete real and virtual graph cancellation when the phase space for additional gluon emission is limited”

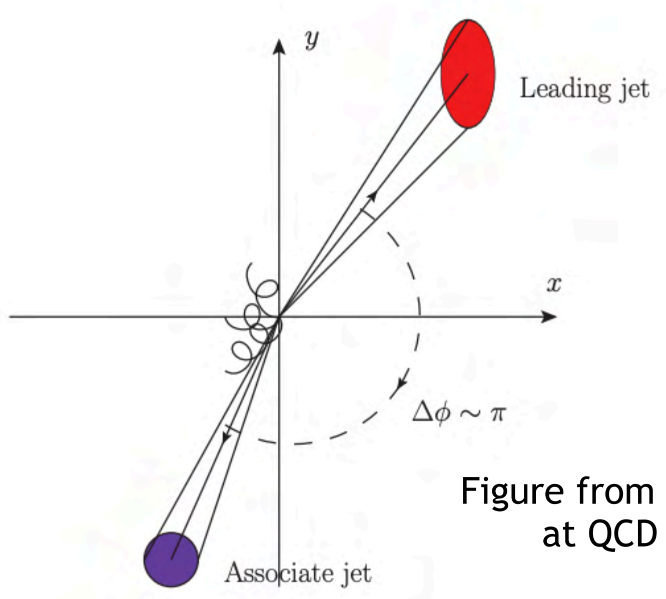


Figure from B.W. Xiao lecture at QCD master class

$$\mathrm{d}\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X} \propto \mathcal{H}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2 \boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp} \alpha_s \widetilde{G}^0_Y(\boldsymbol{b}_\perp) e^{-S_{\mathrm{Sud}}(\boldsymbol{b}_\perp, \boldsymbol{P}_\perp)}$$

Perturbative Sudakov factor:

$$S_{\mathrm{Sud}}(\boldsymbol{b}_\perp, Q) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[A \log\left(\frac{Q^2}{\bar{\mu}^2}\right) + B \right]$$

Mueller, Xiao, Yuan (PRD 2013) conclusion:

Sudakov resummation as in collinear factorization. TMD needs to include BFKL/BK/JIMWLK evolution

Joint Sudakov+small-x resummation

MXY assume TMD factorization in the first place

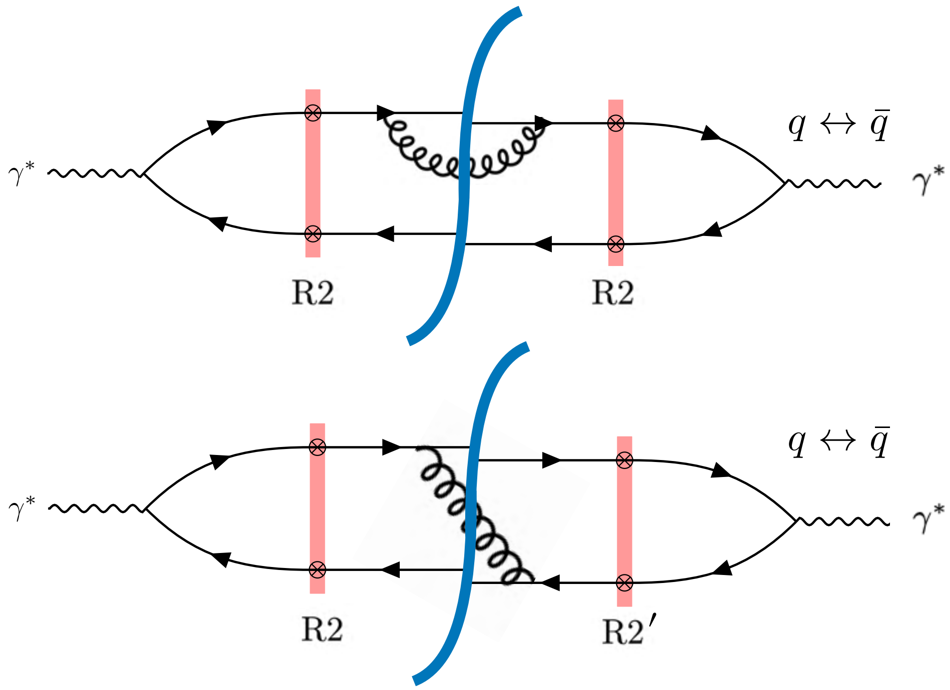
Can we derive these results more rigorously? Can we obtain finite NLO pieces?

Back-to-back limit: gluon saturation and Sudakov

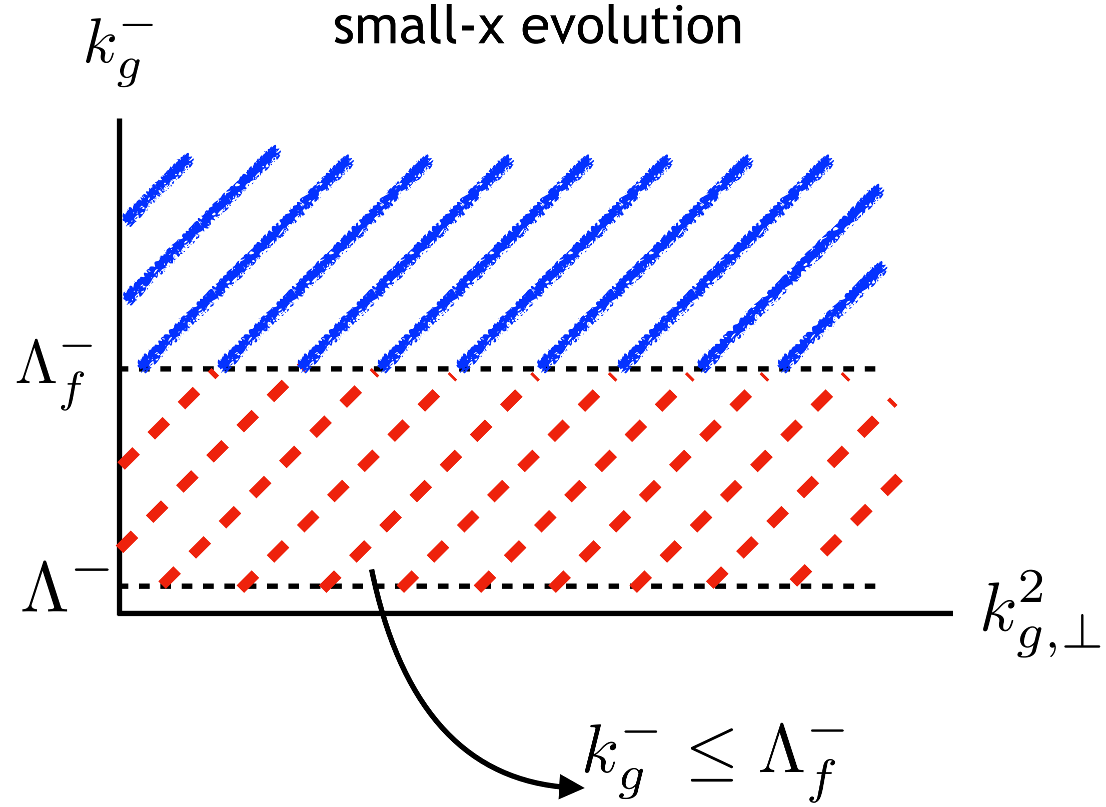
Where is the Sudakov factor?

P. Caucal, FS, B. Schenke, and R. Venugopalan *(JHEP 2022)*

- Take back-to-back limit of impact factor (soft gluon emission in real diagrams)



Phase space for naive separation small-x evolution



$$\mathrm{d}\sigma^{\gamma_\lambda^* + A \to q\bar{q} + X} \propto \mathcal{H}(\boldsymbol{P}_\perp) \int \mathrm{d}^2 \boldsymbol{b}_\perp e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}$$

$$\left[1 \boxed{+ \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{b}_\perp^2}{c_0^2}\right)} + \cdots + \alpha_s \ln\left(\Lambda_f^- / \Lambda^-\right) \mathcal{K}_{LL} \otimes\right] \alpha_s \widetilde{G}_Y(\boldsymbol{b}_\perp)$$

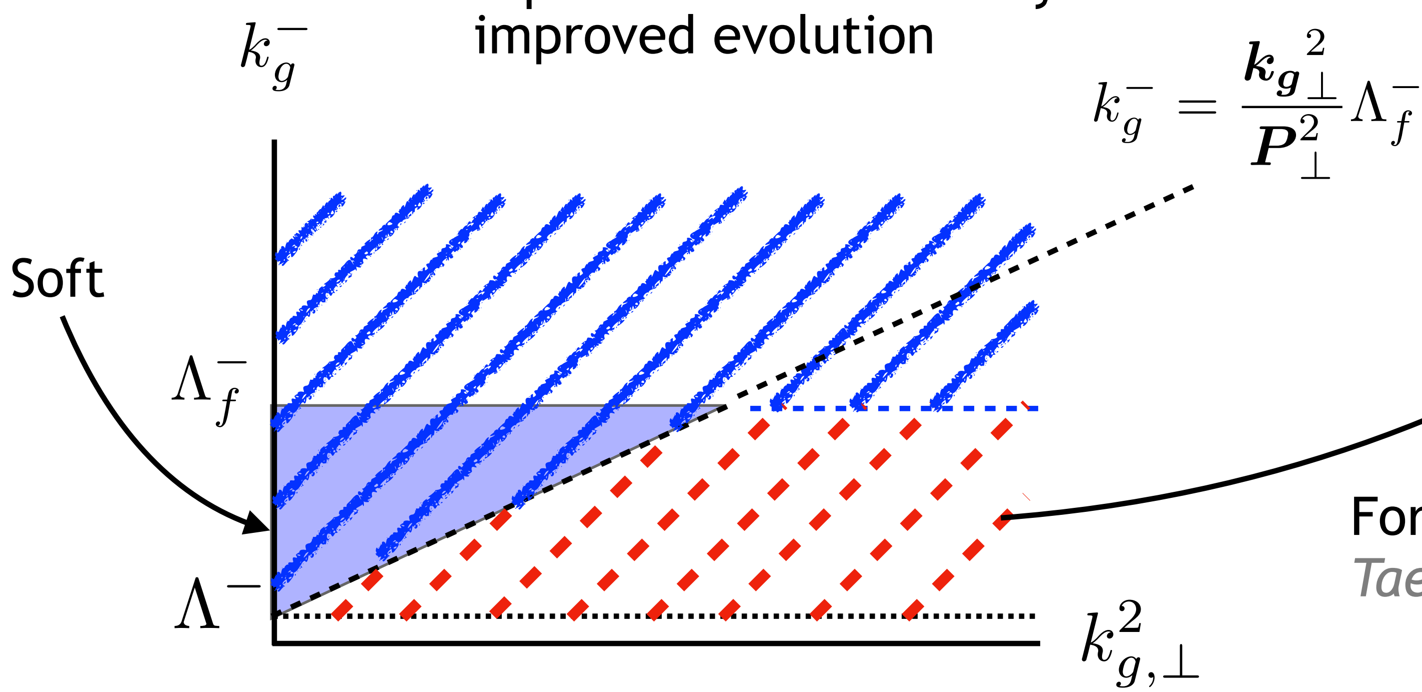
Sudakov double log but with positive sign!

Back-to-back limit: gluon saturation and Sudakov

Sudakov resummation and kinematically improved small-x evolution

P. Caucal, FS, B. Schenke, and R. Venugopalan *(JHEP 2022)*

Phase space for kinematically improved evolution



Kinematic constraint: ordering in + and - momenta

$$k_g^+ \geq \Lambda_f^+ \ \ \& \ \ k_g^- \leq \Lambda_f^-$$

For photo-production see *Taels, Altinoluk, Marquet, Beuf* (JHEP 2022)

$$\mathrm{d}\sigma^{\gamma_\lambda^* + A \to q\bar{q} + X} \propto \mathcal{H}(\boldsymbol{P}_\perp) \int \mathrm{d}^2\boldsymbol{b}_\perp e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}$$

$$\left[1 \boxed{- \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{b}_\perp^2}{c_0^2}\right)} + \cdots + \alpha_s \ln\left(\Lambda_f^-/\Lambda^-\right) \boxed{\mathcal{K}_{LL}} \otimes \right] \alpha_s \widetilde{G}_Y(\boldsymbol{b}_\perp)$$

Correct Sudakov double log

Kinematically improved small-x evolution

Back-to-back limit: gluon saturation and Sudakov

Sudakov resummation at single log accuracy

P. Caucal, FS, B. Schenke, and R. Venugopalan *(JHEP 2022)*

$$\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} \propto \mathcal{H}(\boldsymbol{P}_\perp)\int \frac{\mathrm{d}^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}\alpha_s \widetilde{G}_Y^0(\boldsymbol{b}_\perp)e^{-S_{\mathrm{Sud}}(\boldsymbol{b}_\perp,\boldsymbol{P}_\perp)}$$

$$S_{\mathrm{Sud}}(\boldsymbol{b}_\perp, Q) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A\log\left(\frac{Q^2}{\bar{\mu}^2}\right) + B\right]$$

$$\boxed{A = \frac{\alpha(\mu^2)N_c}{2\pi}} \qquad B = \boxed{\frac{\alpha(\mu^2)C_F}{2\pi}\left[\ln\left(\frac{2(1+\cosh(\Delta Y_{12}))}{R^2}\right)\right.} + \boxed{\ln\left(\frac{x_{Bj}}{z_1 z_2 c_0^2 x_f}\right)}\Bigg]$$

- Terms in blue are in agreement with Mueller, Xiao, Yuan (PRD 2013) and Y. Hatta, B.W. Xiao, F. Yuan, J. Zhou *(PRD 2021)*

- Term in red is rapidity factorization scale dependent single log is new. Suggest a natural choice for the factorization scale

$$x_f = \frac{x_{\mathrm{Bj}}}{z_1 z_2 c_0^2}$$

Back-to-back limit: gluon saturation and Sudakov

Genuine $\mathcal{O}(\alpha_s)$ pieces (impact factor)

P. Caucal, FS, B. Schenke, and R. Venugopalan *(JHEP 2022)*

Azimuthally averaged cross-section

$$
\begin{aligned}
\mathrm{d}\sigma^{(0),\lambda=\mathrm{L}} =\; & \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{L}}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
& \times \left\{1 + \frac{\alpha_s C_F}{\pi}\left[\frac{3}{2}\ln(c_0^2) - 3\ln(R) + \frac{1}{2}\ln^2\left(\frac{z_1^2}{z_2^2}\right) + \frac{11}{2} + 3\ln(2) - \frac{\pi^2}{2}\right]\right. \\
& \left. + \frac{\alpha_s N_c}{2\pi}\left[\ln\left(\frac{z_f^2}{z_1 z_2}\right)\ln(c_0^2) - \ln^2\left(\frac{Q_f^2 c_0^2}{\boldsymbol{P}_\perp^2}\right)\right]\right\} \\
& + \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{L}}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{h}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
& \times \frac{\alpha_s N_c}{2\pi}\left\{1 + \frac{2C_F}{N_c}\ln(R^2) - \frac{1}{N_c^2}\ln(z_1 z_2)\right\} \\
& + \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \left[\frac{1}{2}\mathcal{H}_{\mathrm{NLO},1}^{\lambda=\mathrm{L},ii}(\boldsymbol{P}_\perp)\right] \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
& \times \frac{\alpha_s N_c}{2\pi}\left[\frac{1}{2}\ln\left(\frac{z_1 z_2}{z_f^2}\right) - \frac{3C_F}{2N_c}\right] \\
& + \left\{\alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \left[\frac{1}{2}\mathcal{H}_{\mathrm{NLO},2}^{\lambda=\mathrm{L},ii}(\boldsymbol{P}_\perp)\right] \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2\right. \\
& \left. \times \left(\frac{-\alpha_s}{2\pi N_c}\right) + c.c.\right\} + \mathrm{d}\sigma_{\mathrm{other}}^{(0),\lambda=\mathrm{L}} .
\end{aligned}
$$

Elliptic anisotropy

$$
\begin{aligned}
\mathrm{d}\sigma^{(2),\lambda=\mathrm{L}} =\; & \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{L}}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \frac{\cos(2\theta)}{2} \hat{h}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
& \times \left\{1 + \frac{\alpha_s C_F}{\pi}\left[\frac{3}{2}\ln(c_0^2) - 4\ln(R) + \frac{1}{2}\ln^2\left(\frac{z_1^2}{z_2^2}\right) + \frac{11}{2} + 3\ln(2) - \frac{\pi^2}{2}\right]\right. \\
& \left. + \frac{\alpha_s N_c}{2\pi}\left[-\frac{5}{4} + \ln\left(\frac{z_f^2}{z_1 z_2}\right)\ln(c_0^2) - \ln^2\left(\frac{Q_f^2 c_0^2}{\boldsymbol{P}_\perp^2}\right)\right] + \frac{\alpha_s}{4\pi N_c}\ln(z_1 z_2)\right\} \\
& + \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{L}}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \frac{\cos(2\theta)}{2} \hat{G}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
& \times \frac{\alpha_s N_c}{\pi}\left\{1 + \frac{2C_F}{N_c}\ln(R^2) - \frac{1}{N_c^2}\ln(z_1 z_2)\right\} \\
& + \alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \left[\frac{1}{2}\mathcal{H}_{\mathrm{NLO},1}^{\lambda=\mathrm{L},ii}(\boldsymbol{P}_\perp)\right] \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \frac{\cos(2\theta)}{2} \hat{h}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \\
& \times \frac{\alpha_s N_c}{2\pi}\left[\frac{1}{2}\ln\left(\frac{z_1 z_2}{z_f^2}\right) - \frac{3C_F}{2N_c}\right] \\
& + \left\{\alpha_{\mathrm{em}}\alpha_s e_f^2 \delta_z^{(2)} \left[\frac{1}{2}\mathcal{H}_{\mathrm{NLO},2}^{\lambda=\mathrm{L},ii}(\boldsymbol{P}_\perp)\right] \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \frac{\cos(2\theta)}{2} \hat{h}^0_{Y_f}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2)\right. \\
& \left. \times \left(\frac{-\alpha_s}{2\pi N_c}\right) + c.c.\right\} + \mathrm{d}\sigma_{\mathrm{other}}^{(2),\lambda=\mathrm{L}} .
\end{aligned}
$$

- Only blue terms had been computed before by Hatta, Xiao, Yuan, Zhou (PRD 2021)
- Red terms break TMD factorization at NLO and grow with Q_s^2

Bonus: we computed all even (2n) harmonics!

Back-to-back limit: gluon saturation and Sudakov

TMD factorization breaking at NLO

P. Caucal, FS, B. Schenke, and R. Venugopalan *(JHEP 2022)*

$$\alpha_s \widetilde{G}^{j}_{Y,\mathrm{NLO},1}(\boldsymbol{b}_\perp, \boldsymbol{z}_\perp) \qquad \alpha_s \widetilde{G}^{j}_{Y,\mathrm{NLO},2}(\boldsymbol{b}_\perp, \boldsymbol{z}_\perp) \qquad \alpha_s \widetilde{G}_{Y,\mathrm{NLO},4}(\boldsymbol{b}_\perp, \boldsymbol{z}_\perp)$$

Featured in

- the JIMWLK evolution of the WW gluon TMD

breaks TMD factorization since evolution equation is not closed (even at large Nc)

In the dilute limit (small Q_s^2), "new" correlators collapse to WW TMD $\alpha_s \widetilde{G}^{ij}_{Y}(\boldsymbol{b}_\perp)$

Dominguez, Mueller, Munier, Xiao. PLB (2011)

recover BFKL evolution

- Impact factor

apparently, breaks TMD factorization due to the appearance of correlators beyond the WW

See also *Taels, Altinoluk, Marquet, Beuf* (JHEP 2022)

Back-to-back inclusive dijets in DIS at NLO

In collaboration with Paul Caucal, Björn Schenke, Tomasz Stebel, and Raju Venugopalan

Preliminary results

- Possible to show that apparently factorization breaking terms in impact factor reduce to contributions proportional to WW and power surpassed terms $Q_s/P_\perp$ or $q_\perp/P_\perp$

$$\mathrm{d}\sigma^{\gamma_\lambda^* + A \to \mathrm{dijet} + X} \propto \boxed{\mathcal{H}^{\lambda}_{\mathrm{NLO}}(Q, \boldsymbol{P}_\perp; R)} \int \frac{\mathrm{d}^2 \boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp} \boxed{\alpha_s \widetilde{G}^0_Y(\boldsymbol{b}_\perp)} \boxed{e^{-S_{\mathrm{Sud}}(\boldsymbol{b}_\perp, \boldsymbol{P}_\perp)}}$$

fully analytic result

evolution via kinematically constrained JIMWLK

Sudakov factor

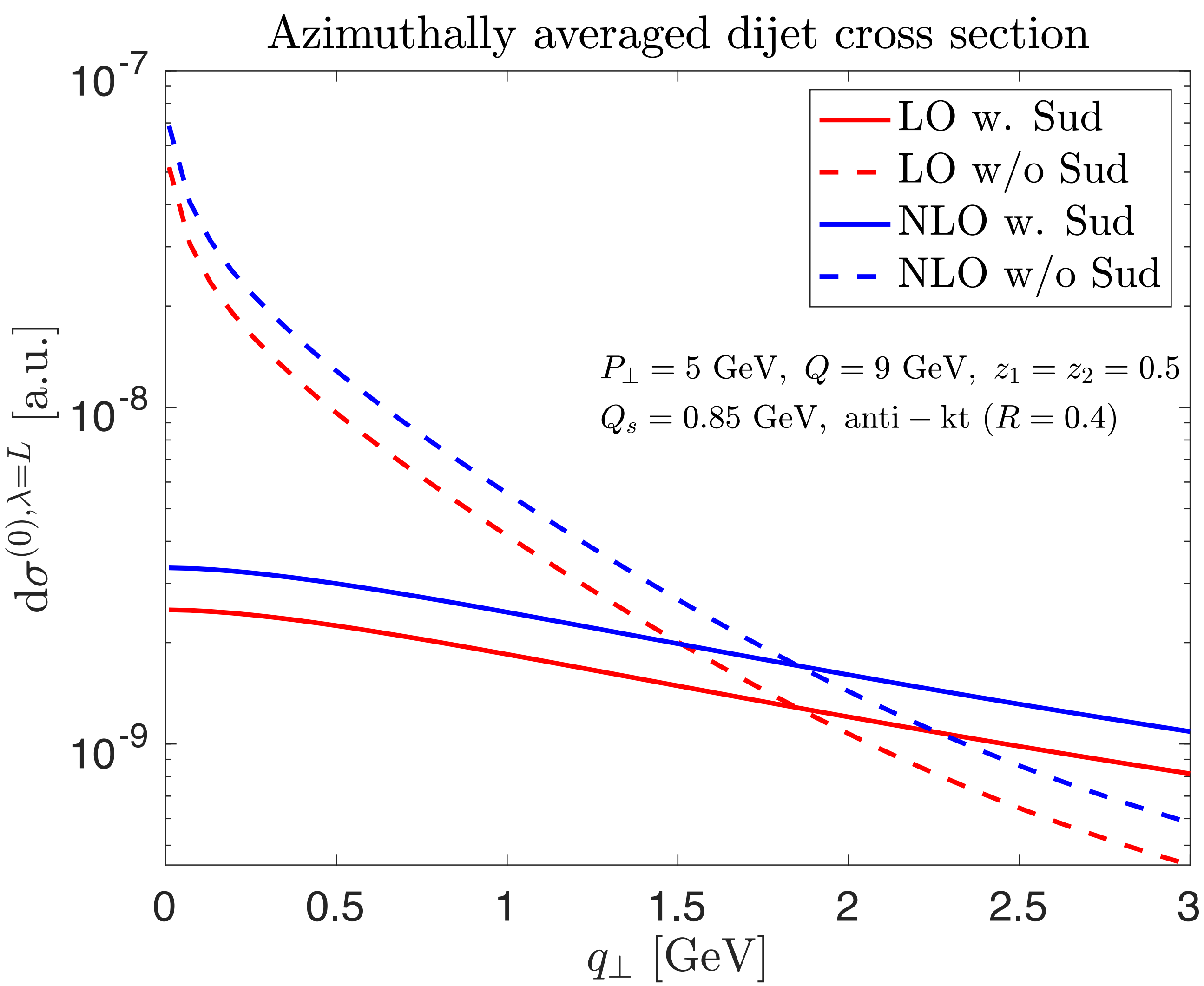
- The first proof of TMD factorization at NLO in small-x kinematics (modulo the non-linear evolution of the WW)?

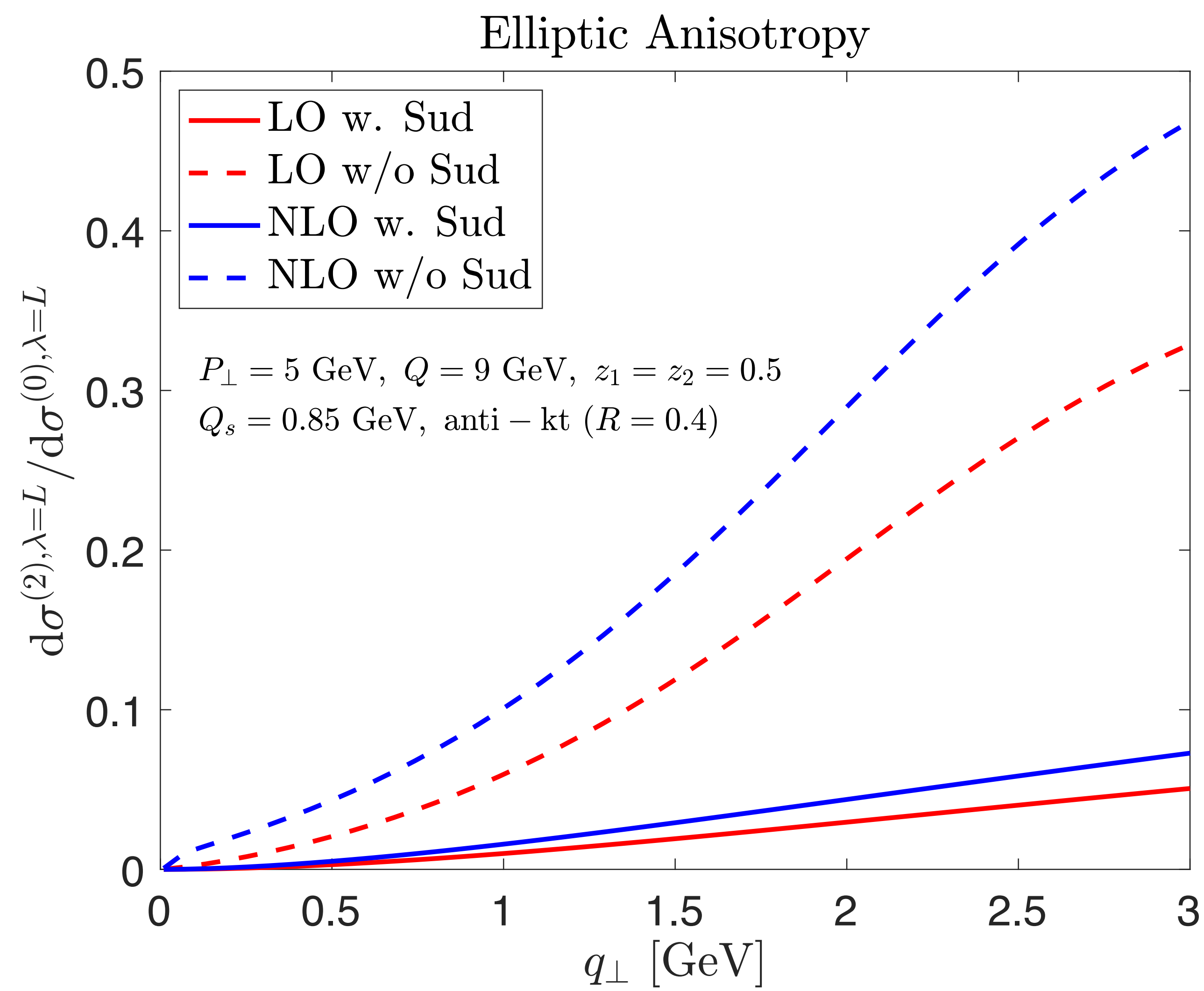
- Fully analytic hard factor at NLO in back-to-back kinematics, suitable for suitable numerical implementation of NLO cross-section

Back-to-back inclusive dijets in DIS at NLO

In collaboration with Paul Caucal, Björn Schenke, Tomasz Stebel, and Raju Venugopalan

Preliminary results

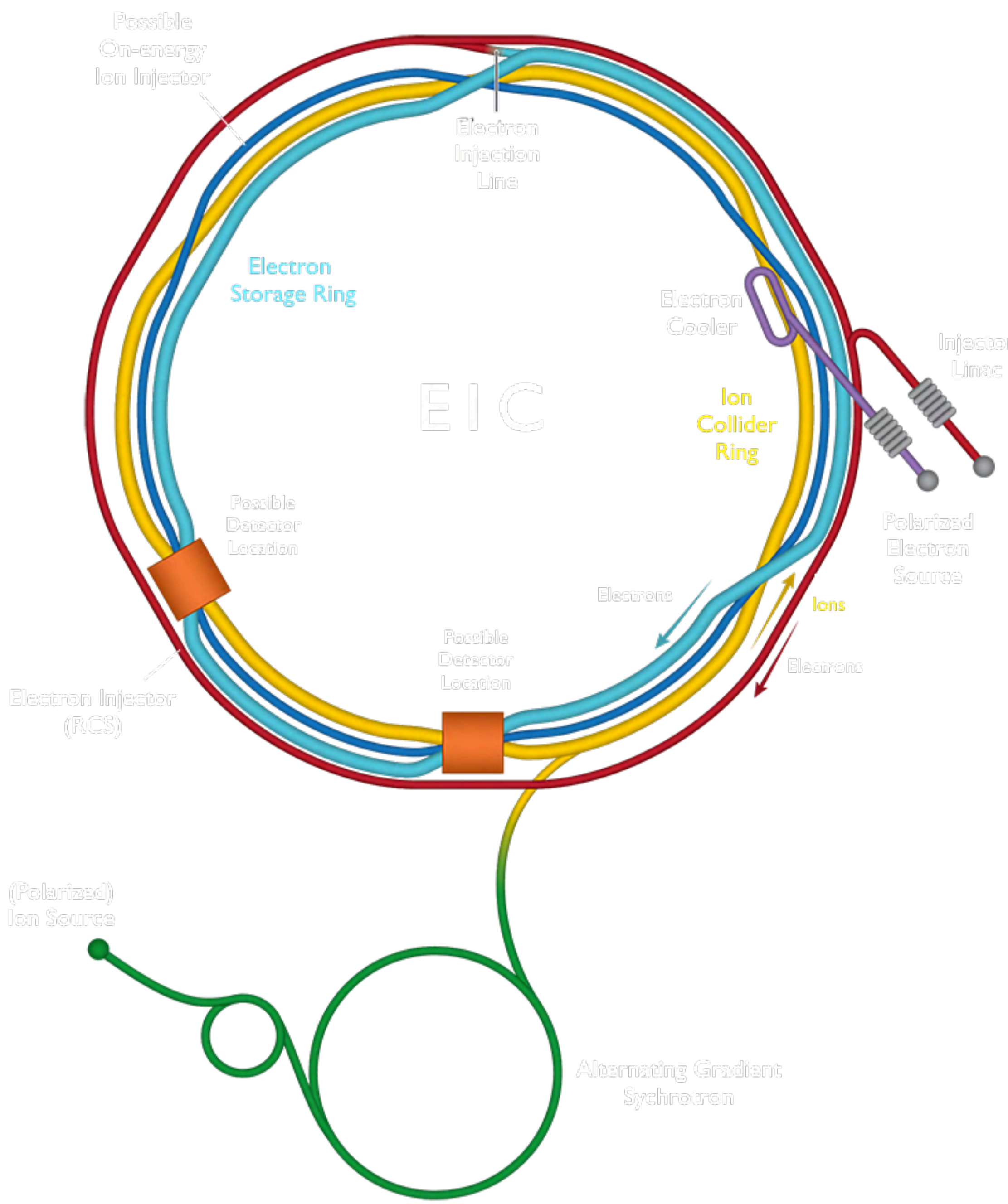


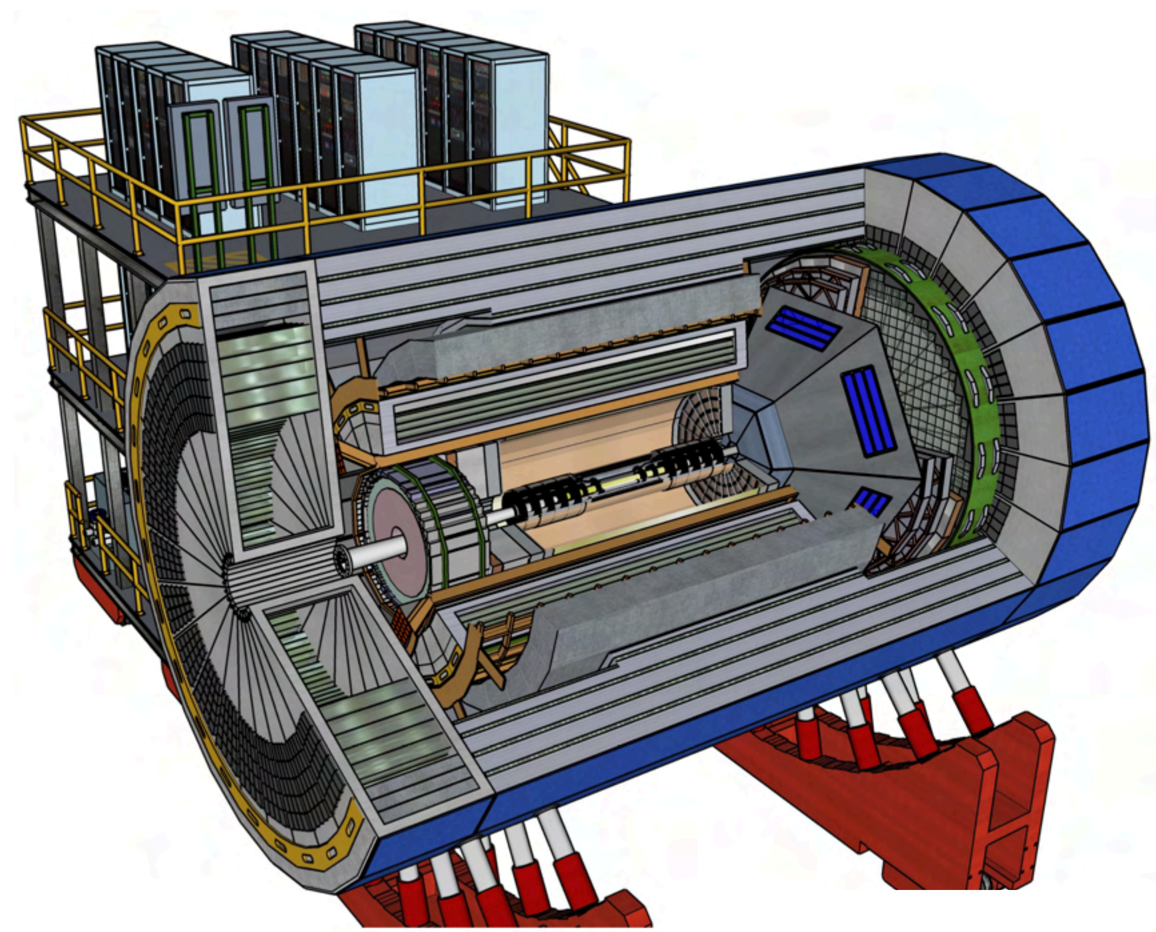


- Include:
  Sudakov double and single logs at finite Nc, but with fixed coupling
  All finite NLO pieces proportional to WW gluon TMD

- Does not include:
  Proper small-x evolution.
  NLO hard factors apparently factorization breaking terms

Experimental requirements and related observables





Experimental requirements

- Good reconstruction of scattered electron (at small-x) to pin down DIS kinematics

- Inelasticity close to 1, in order to maximize the center of mass energy of the photon-nucleus system (thus maximal sensitivity to small-x gluons)

- Reconstruct small pT jets in the backward region (electron/virtual photon going direction) with the ability to resolve their azimuthal angle distribution

  Alternatively: measure dihadrons instead of dijets

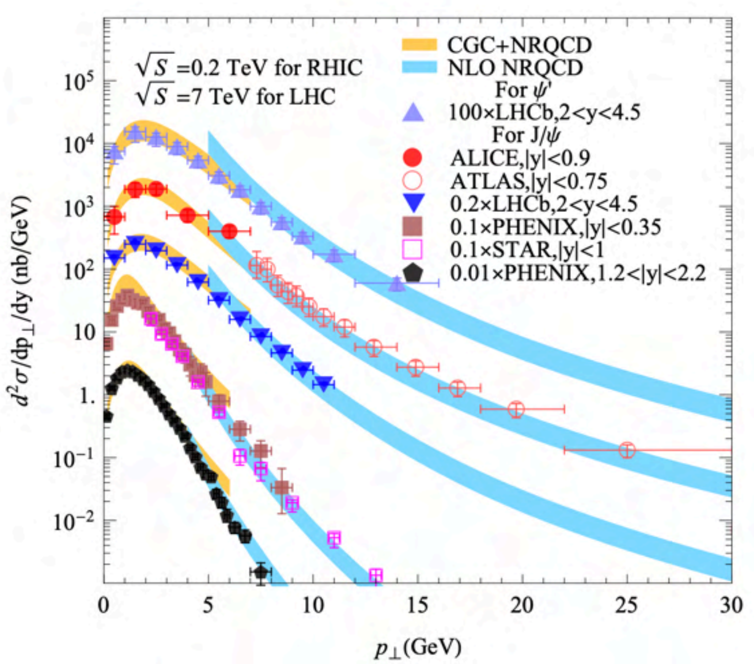
Progress at NLO see e.g.

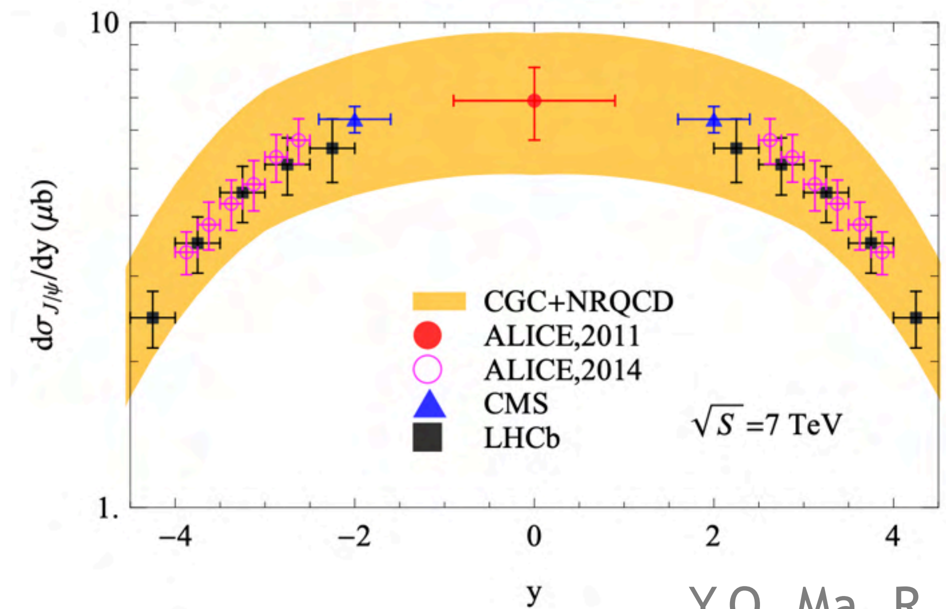
F. Bergabo, J. Jalilian-Marian (PRD 2022)
E. Iancu, Y. Mulian (arXiv 2022)

Need to extract Sudakov, and compute the impact factor for dihadrons…
Possible to apply the techniques developed for dijets to compute dihadrons at back-to-back kinematics.

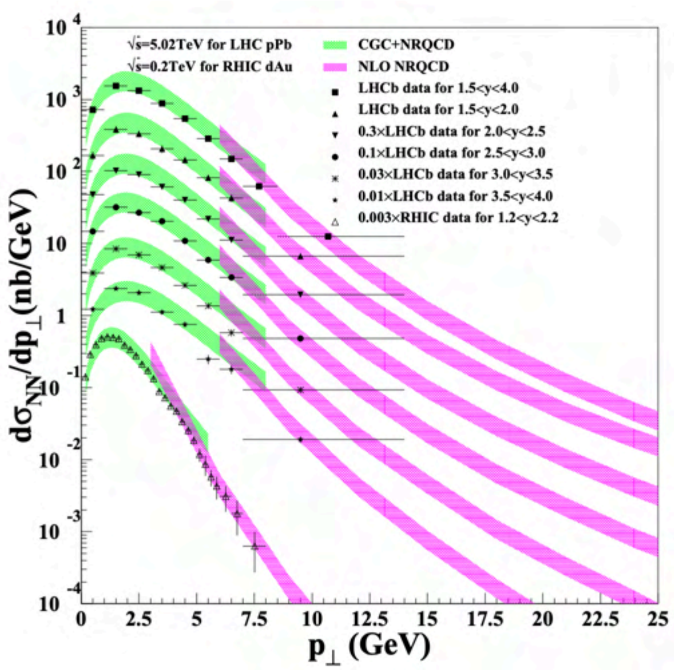
Related observable: inclusive J/ψ production in DIS

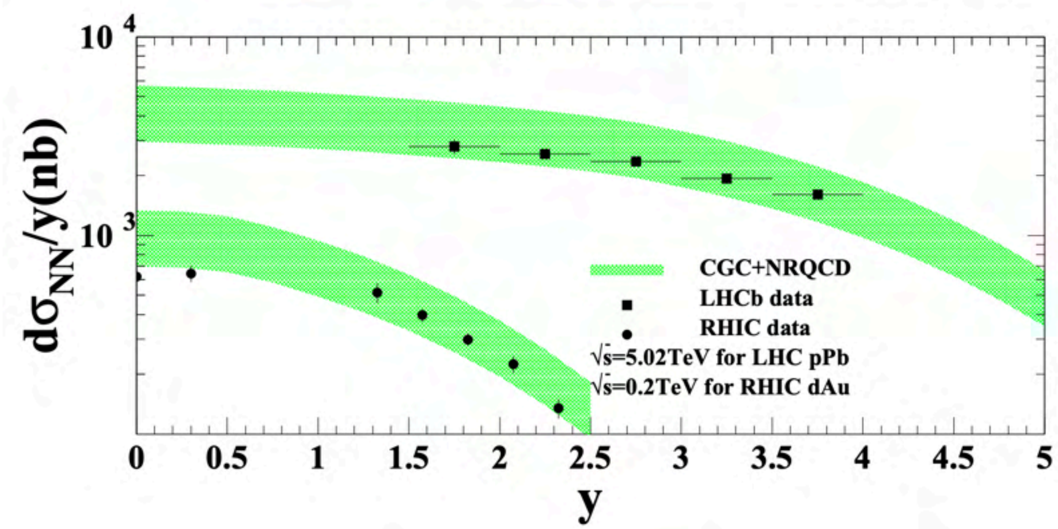
Motivation: Very successful phenomenology in pp and pA collisions at RHIC and LHC





Y.Q. Ma, R. Venugopalan (PRL 2014)





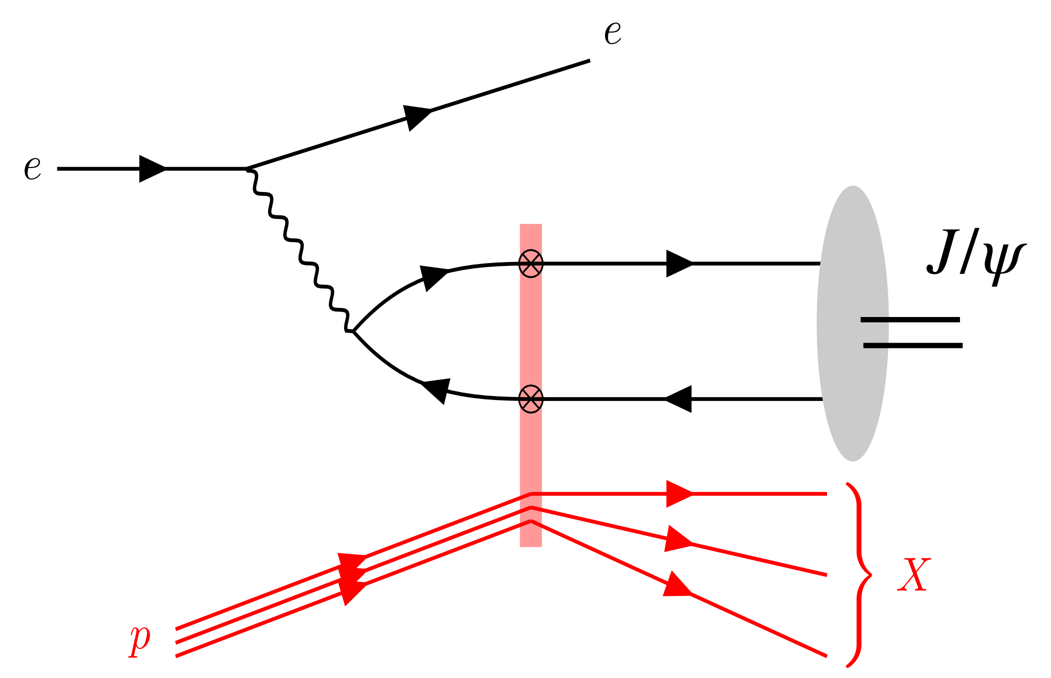
Y.Q. Ma, R. Venugopalan, H.F. Zhang (PRD 2015)

See also: FS, A. Soto-Ontoso, B. Schenke (PLB 2022)

Surprisingly no results for inclusive J/ψ production in DIS within saturation physics (not even at leading order!)

Related observable: inclusive J/ψ production in DIS

- Studies of quarkonium production in DIS at small-x have focused mostly on diffraction, and employ a non-perturbative model for the light-cone wavefunction of quarkonium
- Our goal: Employ CGC + NRQCD to study the nuclear modification factor of quarkonium production in DIS



In progress Vincent Cheung, Zhongbo Kang, FS, and Ramona Vogt

Numerical results at LO coming soon!

How about NLO?

Employ our expertise to study J/ψ production at NLO in the limit ($p_\perp \lesssim M_{J/\psi}$) where Sudakov resummation is necessary, and one might be able to obtain analytic NLO impact factors, suitable for numerical implementation

Summary

Motivation:

2-particle azimuthal correlations → powerful observables to search for saturation

Results:

full NLO calculation →

small-x and soft gluon resummation at finite Nc

computed all even azimuthal harmonics

Identified potential TMD factorization breaking terms

In progress:

Numerical results for dijets at NLO

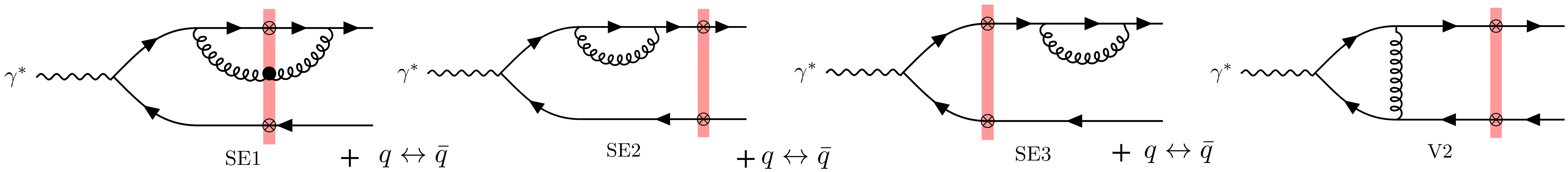
Study related observable: inclusive J/ψ production in DIS

Back-up Slides

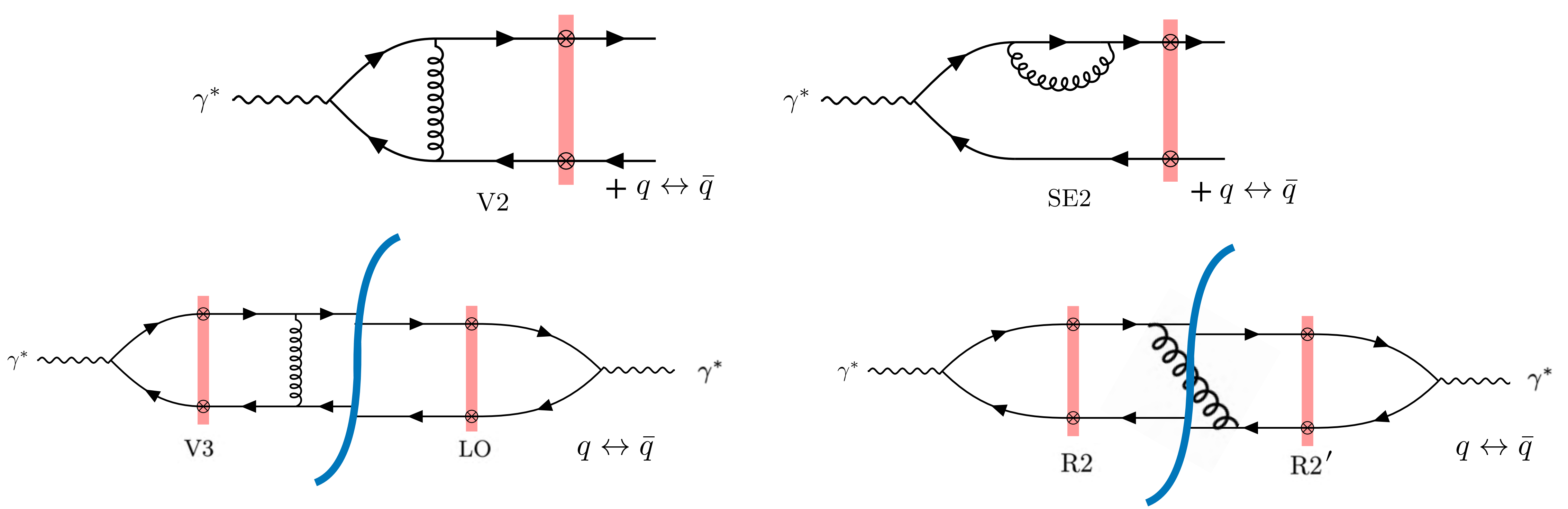
One-loop corrections

Cancellation of divergences: UV and soft

- UV divergences cancel among virtual diagrams: cancellations of poles $1/\epsilon$



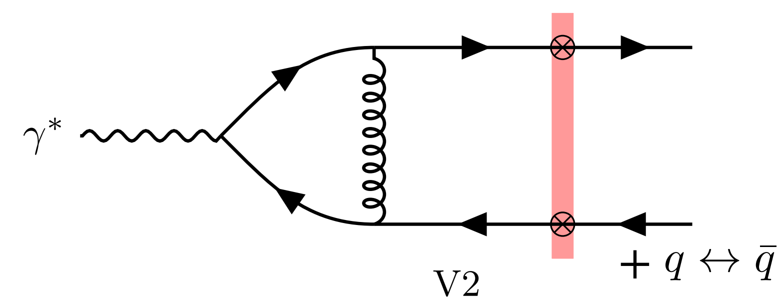
- Soft divergences in real and virtual diagrams: cancellation of double logs/poles



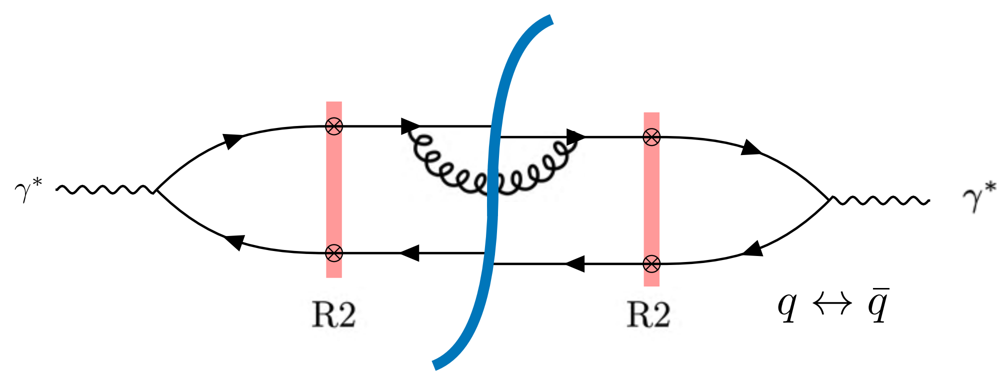
One-loop corrections

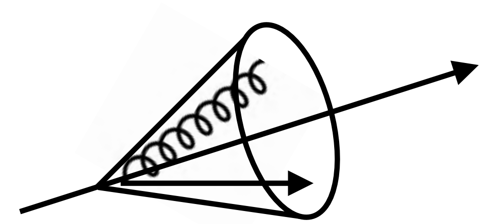
Cancellation of divergences: IR and collinear

- Remaining IR divergence in V2 manifesting as $1/\epsilon$ pole



- Need to define IRC safe observable $\rightarrow$ jets





Collinearity variable:

$$\mathcal{C}_{qg,\perp} = \frac{z_q}{z_j}\left(\boldsymbol{k}_{\boldsymbol{g}\perp} - \frac{z_g}{z_q}\boldsymbol{k}_{\perp}\right)$$

Small-cone condition:

$$\mathcal{C}^2_{qg,\perp} \leq \mathcal{C}^2_{qg,\perp}\big|_{\max} = R^2 \boldsymbol{p}_j^2 \min\left(\frac{z_g^2}{z_j^2}, \frac{(z_j - z_g)^2}{z_j^2}\right)$$

- Collinear divergence contains $1/\epsilon$ pole cancels IR pole

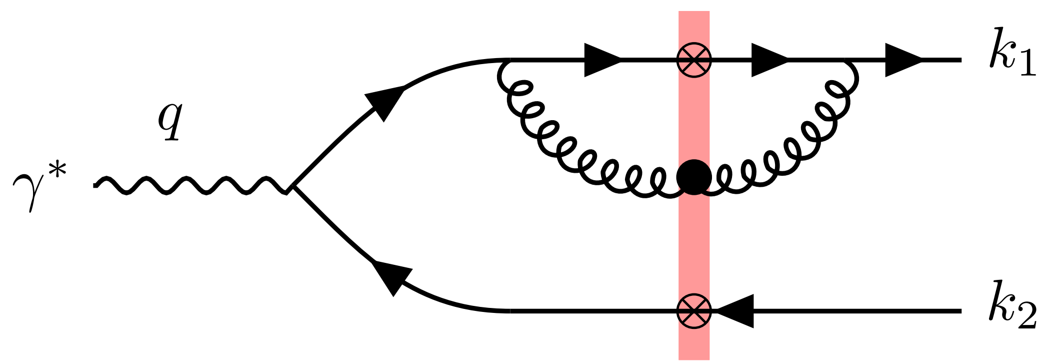
- Exclude slow gluon divergence phase space (must go into evolution)

$$\int_{z_f}^{z_j} \frac{\mathrm{d}z_g}{z_g}\mu^{\varepsilon} \int \frac{\mathrm{d}^{2-\varepsilon}\mathcal{C}_{qg,\perp}}{(2\pi)^{2-\varepsilon}} \frac{1}{\mathcal{C}^2_{qg,\perp}}$$

- Other jet algorithms can be implemented $\rightarrow$ modify impact factor (finite piece)

One-loop corrections

Example: Self energy with gluon crossing SW (others in back-up)



$$\mathcal{C}_{\mathrm{SE1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp) = \left[t^a V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{z}_\perp)t_a V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{y}_\perp) - C_F\right]_{ij}$$

$$\mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = C_F\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

UV finite piece

$$\mathcal{M}^{\lambda}_{\mathrm{SE1,UVfinite},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\left[\mathcal{C}_{\mathrm{SE1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp)\mathcal{N}^{\lambda}_{\mathrm{SE1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) - \mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{N}^{\lambda}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx})\right]$$

$$\mathcal{N}^{\lambda=0,\sigma\sigma'}_{\mathrm{SE1}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = -\frac{\alpha_s}{\pi^2}\int_{z_0}^{z_1}\frac{\mathrm{d}z_g}{z_g}\frac{1}{2}\left[1+\left(1-\frac{z_g}{z_1}\right)^2\right]\frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}}{\boldsymbol{r}_{zx}^2}2(z_1z_2)^{3/2}QK_0\left(QX_{\mathrm{V}}\right)\delta_{\sigma_1,-\sigma_2}$$

$$\mathcal{N}^{\lambda=0}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = -\frac{\alpha_s}{\pi^2}\int_{z_0}^{z_q}\frac{\mathrm{d}z_g}{z_g}\frac{e^{-\frac{\boldsymbol{r}_{zx}^2}{2\xi}}}{\boldsymbol{r}_{zx}^2}2(z_1z_2)^{3/2}QK_0\left(Q\sqrt{z_1z_2}r_{xy}\right)\delta_{\sigma_1,-\sigma_2}$$

$$X_{\mathrm{V}}^2 = z_2(z_1-z_g)\boldsymbol{r}_{xy}^2 + z_g(z_1-z_g)\boldsymbol{r}_{zx}^2 + z_2z_g\boldsymbol{r}_{zy}^2$$

UV divergent piece

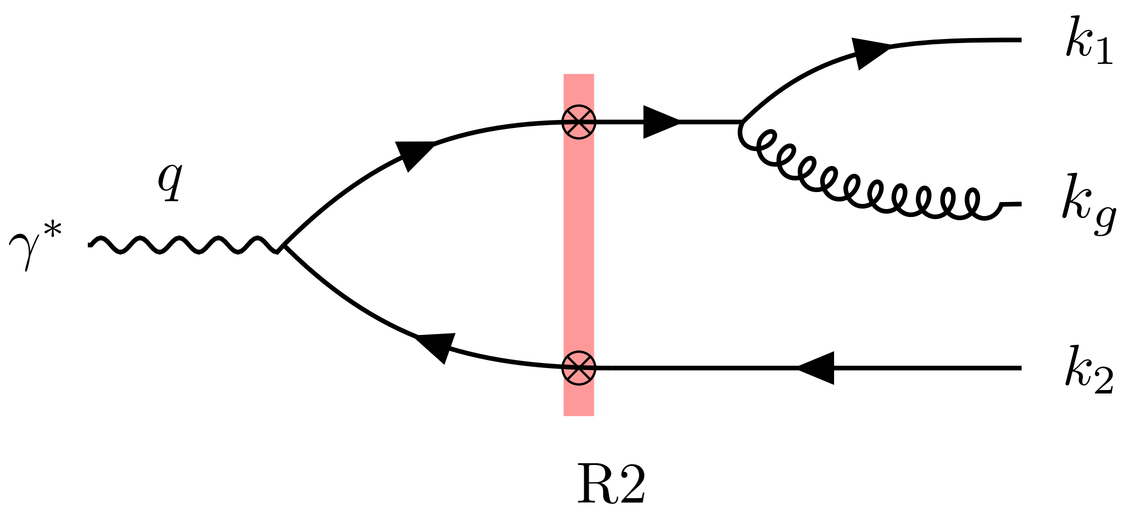
$$\mathcal{M}^{\lambda}_{\mathrm{SE1,UV},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\,\mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{N}^{\lambda}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx})$$

$$\mathcal{N}^{\lambda}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = \frac{\alpha_s}{2\pi}\left\{\left(2\ln\left(\frac{z_1}{z_0}\right)-\frac{3}{2}\right)\left(\frac{2}{\varepsilon}+\ln(2\pi\mu^2\xi)\right)-\frac{1}{2}+\mathcal{O}(\varepsilon)\right\}\mathcal{N}^{\lambda}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

UV pole

One-loop corrections

Real gluon emission after SW



$$\mathcal{C}_{\mathrm{R2},ija}(\boldsymbol{w}_\perp, \boldsymbol{y}_\perp) = \left[t_a V(\boldsymbol{w}_\perp) V^\dagger(\boldsymbol{y}_\perp) - t_a\right]_{ij}$$

$$\mathcal{M}^{\lambda\bar{\lambda}}_{\mathrm{R2},ija,\sigma_1\sigma_2} = \frac{e e_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{z}_\perp} e^{-i(\boldsymbol{k_{1\perp}} \cdot \boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}} \cdot \boldsymbol{y}_\perp + \boldsymbol{k_{g\perp}} \cdot \boldsymbol{z}_\perp)} \mathcal{C}_{\mathrm{R2},ija}(\boldsymbol{w}_\perp, \boldsymbol{y}_\perp) \mathcal{N}^{\lambda\bar{\lambda}}_{\mathrm{R2},\sigma_1\sigma_2}(\boldsymbol{r}_{wy}, \boldsymbol{r}_{zx})$$

Perturbative factor:

$$\mathcal{N}^{\lambda=0,\bar{\lambda}}_{\mathrm{R2},\sigma_1\sigma_2}(\boldsymbol{r}_{wy}, \boldsymbol{r}_{zx}) = 2(z_1 z_2)^{3/2} Q K_0(Q X_{wy}) \delta_{\sigma_1,-\sigma_2} \frac{ig}{\pi} \frac{\boldsymbol{r}_{zx} \cdot \boldsymbol{\epsilon}^{\bar{\lambda}*}_\perp}{\boldsymbol{r}^2_{zx}} \frac{\left[z_1 \delta^{\bar{\lambda}}_{\sigma_1} + (z_1 + z_g) \delta^{\bar{\lambda}}_{\sigma_2}\right]}{z_1}$$

$$\mathcal{N}^{\lambda=\pm1,\bar{\lambda}}_{\mathrm{R2},\sigma_1\sigma_2}(\boldsymbol{r}_{wy}, \boldsymbol{r}_{zx}) = -2(z_1 z_2)^{3/2} \left[z_2 \delta^\lambda_{\sigma_1} - z_1 \delta^\lambda_{\sigma_2}\right] \frac{iQ \boldsymbol{r}_{wx} \cdot \boldsymbol{\epsilon}^\lambda_\perp}{X_{wx}} K_1(Q X_{wx}) \delta_{\sigma_1,-\sigma_2} \frac{ig}{\pi} \frac{\boldsymbol{r}_{zx} \cdot \boldsymbol{\epsilon}^{\bar{\lambda}*}_\perp}{\boldsymbol{r}^2_{zx}} \frac{\left[z_1 \delta^{\bar{\lambda}}_{\sigma_1} + (z_1 + z_g) \delta^{\bar{\lambda}}_{\sigma_2}\right]}{z_1}$$

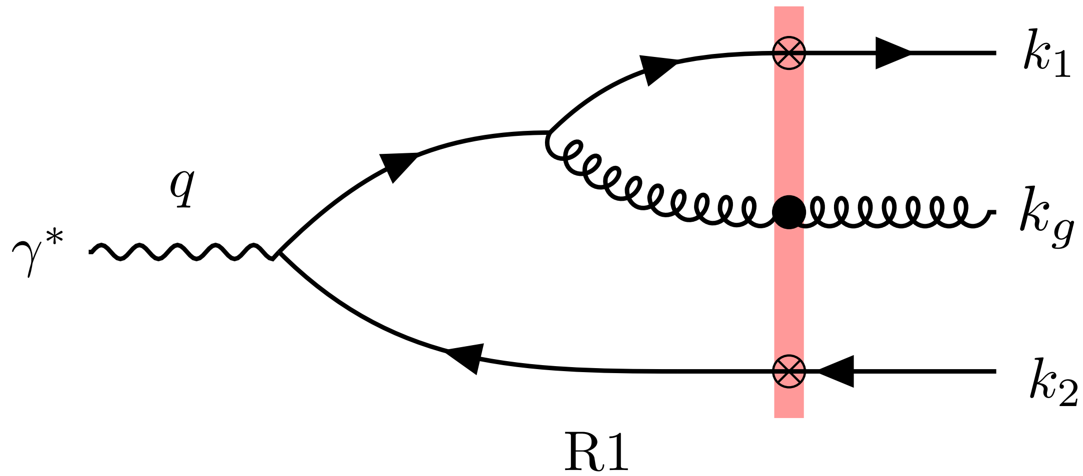
$$\boldsymbol{w}_\perp = \frac{z_1 \boldsymbol{x}_\perp + z_g \boldsymbol{z}_\perp}{z_1 + z_g}$$

$$X^2_{wy} = z_2 (z_1 + z_g) r^2_{wy}$$

Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans (2017)
with spinor-helicity techniques

One-loop corrections

Real gluon emission before SW



$$\mathcal{C}_{\mathrm{R1},ija}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp) = \left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{z}_\perp)t_a V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{y}_\perp) - t_a\right]_{ij}$$

$$\mathcal{M}^{\lambda\bar{\lambda}}_{\mathrm{R1},ija,\sigma_1\sigma_2} = \frac{e e_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp\boldsymbol{z}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp+\boldsymbol{k_{g\perp}}\cdot\boldsymbol{z}_\perp)}\mathcal{C}_{\mathrm{R1},ija}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp)\mathcal{N}^{\lambda\bar{\lambda}}_{\mathrm{R1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx})$$

Perturbative factor:

$$\mathcal{N}^{\lambda=0,\bar{\lambda}}_{\mathrm{R1,reg},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = -2(z_1z_2)^{3/2}QK_0(QX_\mathrm{R})\delta_{\sigma_1,-\sigma_2}\frac{ig}{\pi}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{\epsilon}^{\bar{\lambda}*}_\perp}{\boldsymbol{r}^2_{zx}}\frac{[z_1\delta^{\bar{\lambda}}_{\sigma_1}+(z_1+z_g)\delta^{\bar{\lambda}}_{\sigma_2}]}{z_1}$$

$$\mathcal{N}^{\lambda=\pm1,\bar{\lambda}}_{\mathrm{R1,reg},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = 2(z_1z_2)^{3/2}[z_2\delta^{\lambda}_{\sigma_1}-(z_1+z_g)\delta^{\lambda}_{\sigma_2}]\frac{iQ\boldsymbol{R}_\mathrm{R}\cdot\boldsymbol{\epsilon}^{\lambda}_\perp}{X_\mathrm{R}}K_1(QX_\mathrm{R})\delta_{\sigma_1,-\sigma_2}\frac{ig}{\pi}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{\epsilon}^{\bar{\lambda}*}_\perp}{\boldsymbol{r}^2_{zx}}\frac{[z_1\delta^{\bar{\lambda}}_{\sigma_1}+(z_1+z_g)\delta^{\bar{\lambda}}_{\sigma_2}]}{z_1}$$

$$\mathcal{N}^{\lambda=\pm1,\bar{\lambda}}_{\mathrm{R1},q\mathrm{ins},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = \frac{g}{\pi}z_g\frac{(z_1z_2)^{3/2}}{(z_1+z_g)}\frac{QK_1(QX_\mathrm{R})}{X_\mathrm{R}}\delta_{\sigma_1,-\sigma_2}\delta^{\lambda}_{\sigma_1}\delta^{\lambda,\bar{\lambda}}$$

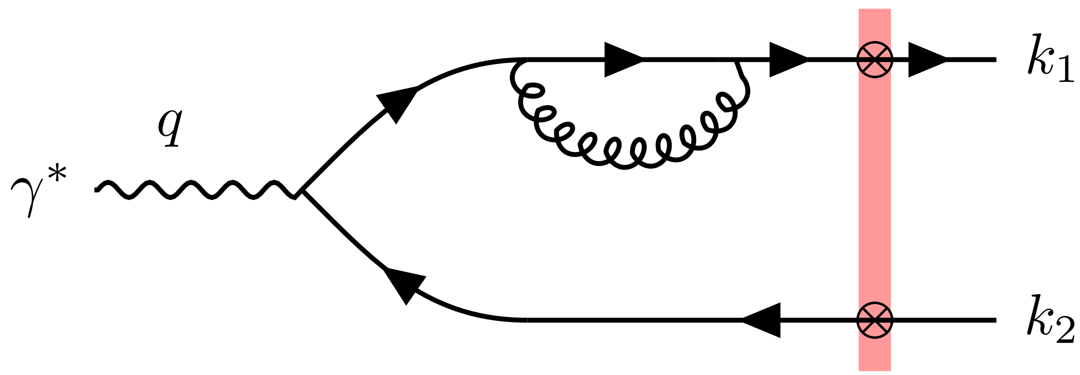
$$\boldsymbol{R}_\mathrm{R} = \boldsymbol{r}_{xy}+\frac{z_g}{z_g+z_1}\boldsymbol{r}_{zx}$$

$$X^2_\mathrm{R} = z_1z_2\boldsymbol{r}^2_{xy}+z_1z_g\boldsymbol{r}^2_{zx}+z_2z_g\boldsymbol{r}^2_{zy}$$

Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans (2017)
with spinor-helicity techniques

One-loop corrections

Self energy with gluon before SW



SE2

$$\mathcal{C}_{\mathrm{SE2},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = C_F\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{SE2},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\,\mathcal{C}_{\mathrm{SE2},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\,\mathcal{N}^{\lambda}_{\mathrm{SE2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

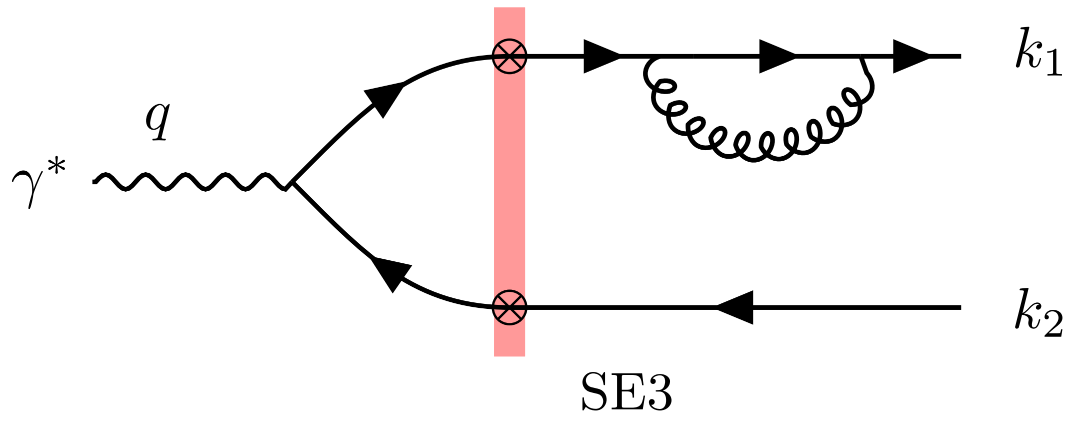
Perturbative factor:

$$\mathcal{N}^{\lambda=0}_{\mathrm{SE2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = \frac{\alpha_s}{2\pi}\left\{\left(-2\ln\left(\frac{z_1}{z_0}\right)+\frac{3}{2}\right)\left(\underbrace{\boxed{\frac{2}{\varepsilon}}}_{\text{UV pole}}+\frac{1}{2}\ln\left(\frac{z_1 z_2 Q^2\boldsymbol{r}^2_{xy}}{4}\right)+\gamma_E-\ln\left(\frac{z_1Q^2}{\tilde{\mu}^2}\right)\right)\right.$$
$$\left.+\left(\frac{1}{2}+3-\frac{\pi^2}{3}-\underbrace{\boxed{\ln^2\left(\frac{z_1}{z_0}\right)}}_{\text{double log}}\right)+\mathcal{O}(\varepsilon)\right\}\mathcal{N}^{\lambda=0}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}$$

Beuf (2016,2017), Hänninen, T. Lappi, and R. Paatelainen (2017)
Loop corrections light-cone photon wave-function

One-loop corrections

Self energy with gluon after SW



$$\mathcal{C}_{\mathrm{SE3},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = C_F\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)-\mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{SE2},ij,\sigma_1\sigma_2} = \frac{e e_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k}_{1\perp}\cdot\boldsymbol{x}_\perp+\boldsymbol{k}_{2\perp}\cdot\boldsymbol{y}_\perp)}\,\mathcal{C}_{\mathrm{SE3},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\,\mathcal{N}^{\lambda}_{\mathrm{SE3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

Perturbative factor:

UV pole

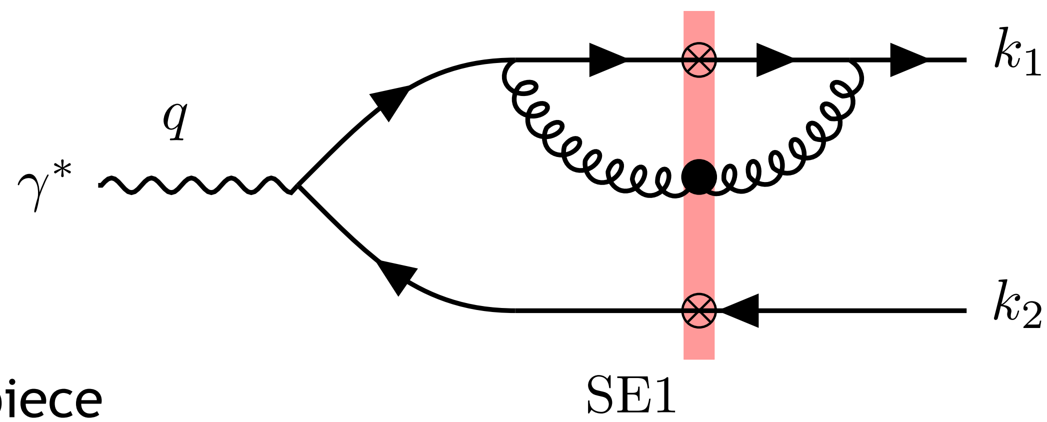
$$\mathcal{N}^{\lambda}_{\mathrm{SE3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = -\frac{\alpha_s}{2\pi}\mathcal{N}^{\lambda}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})\left(\frac{2}{\varepsilon_{\mathrm{UV}}}-\frac{2}{\varepsilon_{\mathrm{IR}}}\right)\left\{2\ln\left(\frac{z_q}{z_0}\right)-\frac{3}{2}\right\}$$

IR pole

Self-energy contribution vanishes exactly in dim reg (IR and UV pole cancel each other out)
turns UV divergences into IR (massless quarks)

One-loop corrections

Self energy with gluon crossing SW



$$\mathcal{C}_{\mathrm{SE1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp) = \left[t^a V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{z}_\perp)t_a V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{y}_\perp) - C_F\right]_{ij}$$

$$\mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = C_F\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

UV finite piece

$$\mathcal{M}^\lambda_{\mathrm{SE1,UVfinite},ij,\sigma_1\sigma_2} = \frac{e e_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\left[\mathcal{C}_{\mathrm{SE1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp)\mathcal{N}^\lambda_{\mathrm{SE1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) - \mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{N}^\lambda_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx})\right]$$

$$\mathcal{N}^{\lambda=0,\sigma\sigma'}_{\mathrm{SE1}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = -\frac{\alpha_s}{\pi^2}\int_{z_0}^{z_1}\frac{\mathrm{d}z_g}{z_g}\frac{1}{2}\left[1+\left(1-\frac{z_g}{z_1}\right)^2\right]\frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}}{\boldsymbol{r}_{zx}^2}2(z_1z_2)^{3/2}QK_0\left(QX_{\mathrm{V}}\right)\delta_{\sigma_1,-\sigma_2}$$

$$\mathcal{N}^{\lambda=0}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = -\frac{\alpha_s}{\pi^2}\int_{z_0}^{z_q}\frac{\mathrm{d}z_g}{z_g}\frac{e^{-\frac{\boldsymbol{r}_{zx}^2}{2\xi}}}{\boldsymbol{r}_{zx}^2}2(z_1z_2)^{3/2}QK_0\left(Q\sqrt{z_1z_2}r_{xy}\right)\delta_{\sigma_1,-\sigma_2}$$

Hänninen, Lappi, Paatelainen (Annals Phys. 2017)

$$X_{\mathrm{V}}^2 = z_2(z_1-z_g)\boldsymbol{r}_{xy}^2 + z_g(z_1-z_g)\boldsymbol{r}_{zx}^2 + z_2z_g\boldsymbol{r}_{zy}^2$$

UV divergent piece

$$\mathcal{M}^\lambda_{\mathrm{SE1,UV},ij,\sigma_1\sigma_2} = \frac{e e_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\,\mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{N}^\lambda_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx})$$

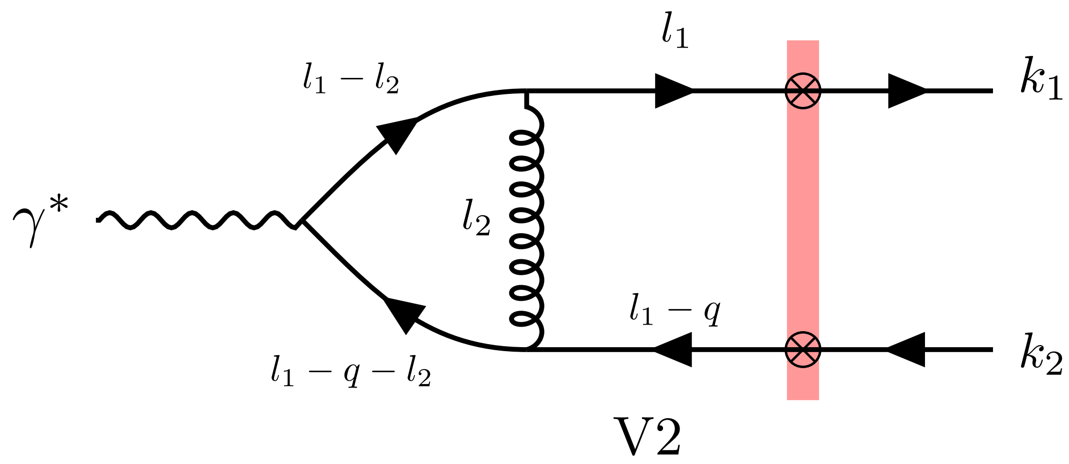
$$\mathcal{N}^\lambda_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = \frac{\alpha_s}{2\pi}\left\{\left(2\ln\left(\frac{z_1}{z_0}\right)-\frac{3}{2}\right)\left(\frac{2}{\varepsilon}+\ln(2\pi\mu^2\xi)\right)-\frac{1}{2}+\mathcal{O}(\varepsilon)\right\}\mathcal{N}^\lambda_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

UV pole

Caucal, FS, Venugopalan (JHEP 2021)

One-loop corrections

Connection to LCPT: an example



$$\mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = C_F \left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{V2},ij,\sigma_1\sigma_2} = \frac{e e_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)} \boxed{\mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)} \boxed{\mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})}$$

Perturbative factor

$$\boxed{\mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})} = g^2 \int_{l_1} \int_{l_2} \frac{(2q^-)\delta(k^- - l_1^-) \boxed{N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1, l_2)} e^{i\boldsymbol{l_{1\perp}}\cdot\boldsymbol{r}_{xy}}}{[l_1^2 + i\epsilon]\,[(l_1 - l_2)^2 + i\epsilon]\,[(l_1 - l_2 - q)^2 + i\epsilon]\,[(l_1 - q)^2 + i\epsilon]\,[l_2^2 + i\epsilon]}$$

Dirac-Lorentz structure

$$\boxed{N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1, l_2)} = \frac{1}{(2q^-)^2} \left[\bar{u}(k_1, \sigma_1)\gamma^- \not{l}_1 \gamma^\mu (\not{l}_1 - \not{l}_2) \not{\epsilon}(q, \lambda)(\not{l}_1 - \not{l}_2 - \not{q})\gamma^\nu (\not{l}_1 - \not{q})\gamma^- v(k_2, \sigma_2)\right] \Pi_{\mu\nu}(l_2)$$

One-loop corrections

Connection to LCPT: an example

Perturbative factor

$$\boxed{\mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})} = g^2 \int_{l_1}\int_{l_2} \frac{(2q^-)\delta(k^- - l_1^-)\boxed{N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1,l_2)}e^{i\boldsymbol{l}_{1\perp}\cdot\boldsymbol{r}_{xy}}}{[l_1^2+i\epsilon]\,[(l_1-l_2)^2+i\epsilon]\,[(l_1-l_2-q)^2+i\epsilon]\,[(l_1-q)^2+i\epsilon]\,[l_2^2+i\epsilon]}$$

Dirac-Lorentz structure (DLS)

$$\boxed{N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1,l_2)} = \frac{1}{(2q^-)^2}\left[\bar{u}(k_1,\sigma_1)\gamma^- \not{l}_1\gamma^{\mu}(\not{l}_1-\not{l}_2)\not{\epsilon}(q,\lambda)(\not{l}_1-\not{l}_2-\not{q})\gamma^{\nu}(\not{l}_1-\not{q})\gamma^- v(k_2,\sigma_2)\right]\Pi_{\mu\nu}(l_2)$$

Useful decomposition (dissecting Dirac-Lorentz structure)

$$\boxed{N_{\mathrm{V2}}} = N_{\mathrm{V2,reg}} + l_2^2 N_{\mathrm{V2},g\mathrm{inst}} + (l_1-l_2)^2 N_{\mathrm{V2},q\mathrm{inst}} + (l_1-l_2-q)^2 N_{\mathrm{V2},\bar{q}\mathrm{inst}}$$

$$\boxed{\mathcal{N}_{\mathrm{V2}}} = \boxed{\mathcal{N}_{\mathrm{V2,reg}}} + \boxed{\mathcal{N}_{\mathrm{V2},g\mathrm{inst}}} + \boxed{\mathcal{N}_{\mathrm{V2},q\mathrm{inst}}} + \boxed{\mathcal{N}_{\mathrm{V2},\bar{q}\mathrm{inst}}}$$

After contour integration l_1^+ and l_2^+ one obtains light-cone energy denominators in LCPT

Diagram A'

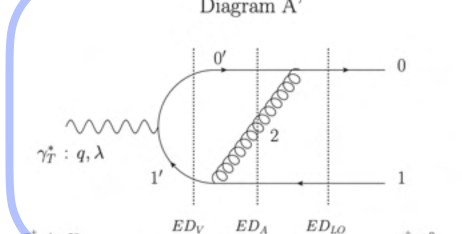
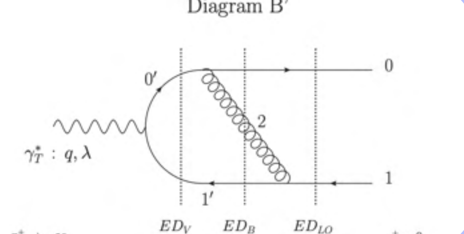


Diagram B'



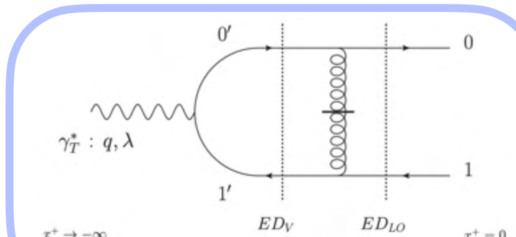


Diagram 3

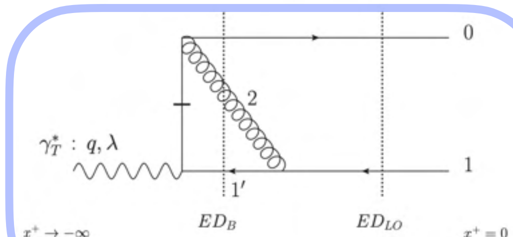


Diagram 2'

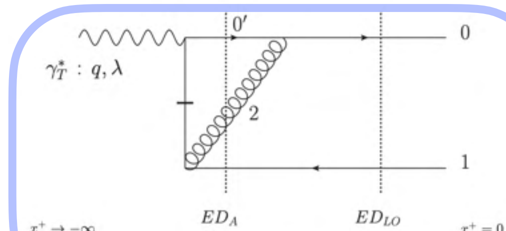


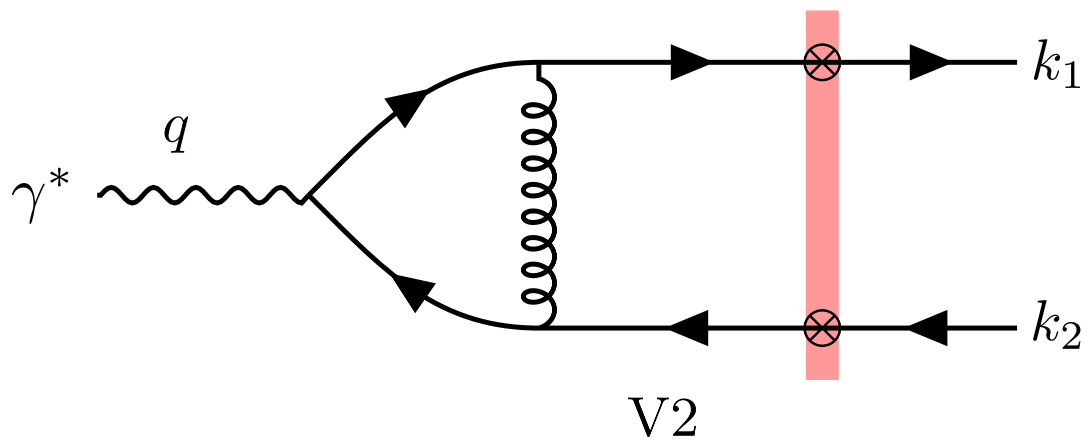
Diagram 1'

Diagrams from Beuf (2016)

These perturbative factors have been computed in Beuf (2016,2017), Hänninen, Lappi, and Paatelainen (2017)

One-loop corrections

Vertex with gluon before SW



$$\mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = C_F \left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{V2},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\, \mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)\, \mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

Perturbative factor*:

UV pole

double logs

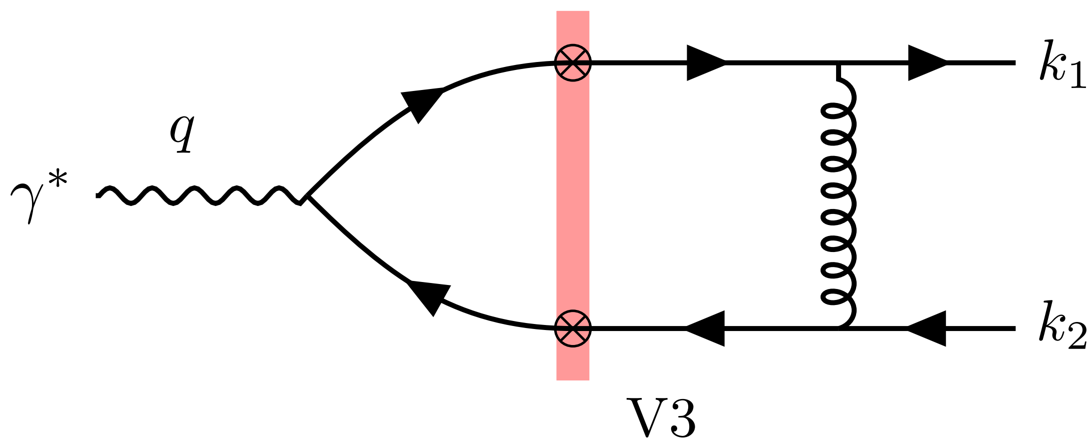
$$\mathcal{N}^{\lambda=0}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = \frac{\alpha_s}{2\pi}\left\{\left(\frac{2}{\varepsilon} + \ln\left(\frac{\tilde{\mu}^2}{z_1 z_2 Q^2}\right)\right)\left[\ln\left(\frac{z_1}{z_0}\right) + \ln\left(\frac{z_2}{z_0}\right) - \frac{3}{2}\right] + \boxed{\ln^2\left(\frac{z_1}{z_0}\right) + \ln^2\left(\frac{z_2}{z_0}\right)} + \frac{1}{2}\ln^2\left(\frac{z_1}{z_2}\right) + \frac{\pi^2}{2} \right.$$
$$\left. + \left(2\ln\left(\frac{z_2}{z_0}\right) - \frac{3}{2}\right)\ln(z_1) + \left(2\ln\left(\frac{z_1}{z_0}\right) - \frac{3}{2}\right)\ln(z_2) - \frac{7}{2} - \frac{1}{2} + \mathcal{O}(\varepsilon)\right\} \mathcal{N}^{\lambda=0}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

*includes both regular and gluon instantaneous contribution

Beuf (2016,2017), Hänninen, T. Lappi, and R. Paatelainen (2017)
Loop corrections light-cone photon wave-function

One-loop corrections

Vertex with gluon after SW



$$\begin{aligned}\mathcal{C}_{\mathrm{V3},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \\ = C_F \left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}\end{aligned}$$

$$\mathcal{M}^\lambda_{\mathrm{V3},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi} \int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)} \boxed{\mathcal{C}_{\mathrm{V3},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)} \boxed{\mathcal{N}^\lambda_{\mathrm{V3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})}$$

Perturbative factor*:

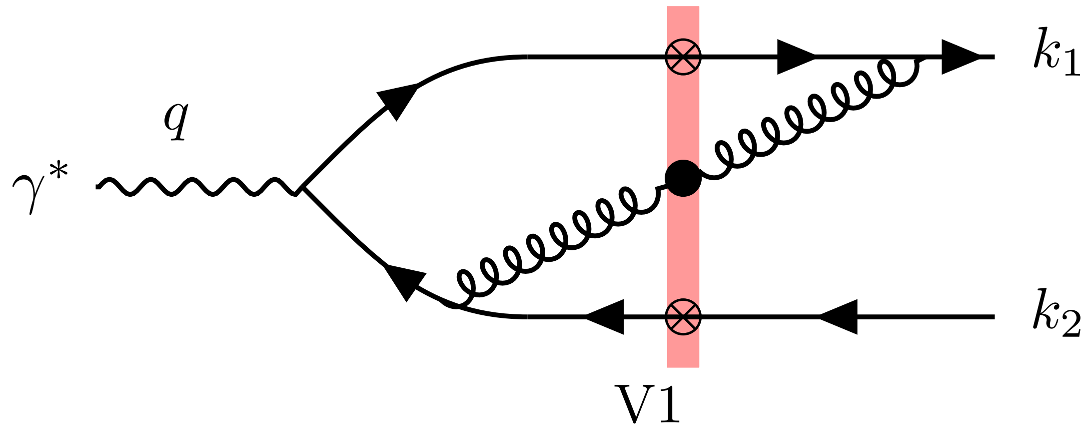
$$\begin{aligned}\mathcal{N}^{\lambda=0}_{\mathrm{V3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = &-\frac{\alpha_s}{\pi}\int_0^{z_q} \frac{\mathrm{d}z_g}{z_g} 2(z_1 z_2)^{1/2}(z_1 - z_g)(z_2 + z_g) Q K_0\left(Q\sqrt{(z_1 - z_g)(z_2 + z_g)}\, r_{xy}\right)\delta_{\sigma_1,-\sigma_2} \\ &\times \left\{\left[(1+z_g)\left(1 - \frac{z_g}{z_1}\right)\right] e^{i(\boldsymbol{P}_\perp + z_g(\boldsymbol{k_{1\perp}} + \boldsymbol{k_{2\perp}}))\cdot \boldsymbol{r}_{xy}} K_0(-i\Delta_{\mathrm{V3}} r_{xy})\right. \\ &- \left[1 - \frac{z_g}{2z_1} + \frac{z_g}{2z_2} - \frac{z_g^2}{2z_1 z_2}\right] e^{i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy}} \boxed{\mathcal{J}_\odot\left(\boldsymbol{r}_{xy}, \left(1 - \frac{z_g}{z_1}\right)\boldsymbol{P}_\perp, \Delta_{\mathrm{V3}}\right)} \\ &\left. + \sigma\left[\frac{z_g}{z_1} - \frac{z_g}{z_2} + \frac{z_g^2}{z_1 z_2}\right] e^{i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy}} \mathcal{J}_\otimes\left(\boldsymbol{r}_{xy}, \left(1 - \frac{z_g}{z_1}\right)\boldsymbol{P}_\perp, \Delta_{\mathrm{V3}}\right)\right\} + (q \leftrightarrow \bar{q})\end{aligned}$$

Contains a double log
$\ln^2(z_0)$

*includes both regular and gluon instantaneous contribution

One-loop corrections

Vertex with gluon crossing SW



$$\mathcal{C}_{\mathrm{V1},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp \boldsymbol{z}_\perp) = \left[t^a V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{z}_\perp)t_a V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{y}_\perp) - C_F\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{V1},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\mathcal{C}_{\mathrm{V1},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{z}_\perp)\mathcal{N}^{\lambda}_{\mathrm{V1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{zy})$$

Perturbative factor:

$$\mathcal{N}^{\lambda=0}_{\mathrm{V1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{zy}) = \frac{\alpha_s}{\pi^2}\int_{z_0}^{z_1}\frac{\mathrm{d}z_g}{z_g} e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}} 2(z_1 z_2)^{3/2} Q K_0\left(QX_V\right)\delta_{\sigma_1,-\sigma_2}$$
$$\times\left\{\left(1-\frac{z_g}{z_1}\right)\left(1+\frac{z_g}{z_2}\right)\left[1-\frac{z_g}{2z_1}-\frac{z_g}{2(z_2+z_g)}\right]\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zy}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy}} + \ldots\right\}$$

This contribution is UV finite!

$$X^2_{\mathrm{V}} = z_2(z_1 - z_g)\boldsymbol{r}^2_{xy} + z_g(z_1 - z_g)\boldsymbol{r}^2_{zx} + z_g z_2 \boldsymbol{r}^2_{zy}$$

Caucal, FS, Venugopalan (JHEP 2021)

Back-to-back limit: gluon saturation and Sudakov

TMD factorization breaking at NLO: correlators beyond WW

P. Caucal, FS, B. Schenke, and R. Venugopalan. *JHEP 11 (2022) 169*

- Color correlators at NLO

$\Xi_{\rm LO}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$	$\langle 1 - D_{xy} - D_{y'x'} + Q_{xy,y'x'}\rangle$
$\Xi_{\rm NLO,1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$	$\frac{N_c}{2}\langle 1 - D_{y'x'} + Q_{zy,y'x'}D_{xz} - D_{xz}D_{zy}\rangle - \frac{1}{2N_c}\Xi_{\rm LO}$
$\Xi_{\rm NLO,2}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$	$\frac{N_c}{2}\langle 1 - D_{y'x'} + Q_{xz,y'x'}D_{zy} - D_{xz}D_{zy}\rangle - \frac{1}{2N_c}\Xi_{\rm LO}$
$\Xi_{\rm NLO,3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$	$\frac{N_c}{2}\langle 1 - D_{xy} - D_{y'x'} + D_{xy}D_{y'x'}\rangle - \frac{1}{2N_c}\Xi_{\rm LO}$
$\Xi_{\rm NLO,4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$	$\frac{N_c}{2}\langle 1 - D_{xz}D_{zy} - D_{y'z}D_{zx'} + Q_{xz,z'x'}Q_{y'z',zy}\rangle - \frac{1}{2N_c}\Xi_{\rm LO}$

- Blue correlators collapse to the WW gluon TMD, red correlators result in other TMDs. e.g.

$$\Xi_{\rm LO} \approx \boldsymbol{u}_\perp^i \boldsymbol{u}_\perp'^j \frac{1}{2N_c}(-2)\underbrace{\left\langle {\rm Tr}\left[\left(V(\boldsymbol{b}_\perp)\partial^i V^\dagger(\boldsymbol{b}_\perp)\right)\left(V(\boldsymbol{0}_\perp)\partial^j V^\dagger(\boldsymbol{0}_\perp)\right)\right]\right\rangle_Y}_{\alpha_s \widetilde{G}_Y^{ij}(\boldsymbol{b}_\perp)}$$

$$\Xi_{\rm NLO,1} \approx -\boldsymbol{u}_\perp'^j \frac{1}{2N_c}(-2)\underbrace{\left\langle {\rm Tr}\left[V(\boldsymbol{b}_\perp)V^\dagger(\boldsymbol{z}_\perp)\right]{\rm Tr}\left[V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{b}_\perp)\partial^j V(\boldsymbol{0}_\perp)V^\dagger(\boldsymbol{0}_\perp)\right]\right\rangle_Y}_{\alpha_s \widetilde{G}^j_{Y,\rm NLO,1}(\boldsymbol{b}_\perp,\boldsymbol{z}_\perp)}$$

breaks TMD factorization!