Input Information for the Present and Future Radiative Corrections Programs

Barbara Badelek
University of Warsaw

Radiative Corrections at EIC

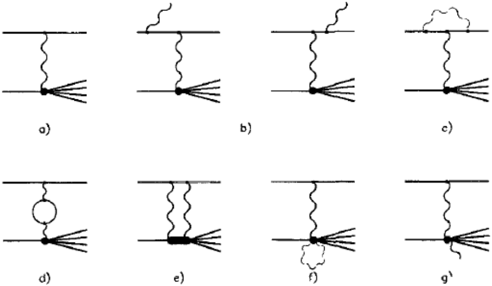
EICUG theory WG meeting, 15 XI 2022

Outline

1. Mo & Tsai and Dubna schemes
2. DJANGOH event generator
3. Extension of $F_2(x, Q^2)$ down to $Q^2 = 0$ (to be updated !)
   - Data at low Q^2
   - JKBB
   - Martin-Ryskin-Stasto
   - (Modified) saturation model
   - ALLM97
   - ZEUS Regge fit
4. Extension of $F_L(x, Q^2)$ down to $Q^2 = 0$ (updated recently)
5. Extension of $g_1(x, Q^2)$ down to $Q^2 = 0$ (to be updated !)
6. Outlook

Lowest order radiative processes

BB, Bardin, Kurek, Scholz, Z.Phys. C66 (1995) 591

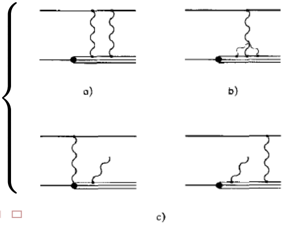


Mo and Tsai scheme: b) – d)

L.W. Mo, Y.S. Tsai, Rev.Mod.Phys.41 (1969) 205; SLAC-PUB-848 (1971)

Dubna scheme b) – g) but replaces e) – g) by:

A.A. Akhundov et al., Fortschr. Phys. 44(1996) 373



Mo and Tsai scheme: FERRAD, model indep., non-covariant

Measured cross section:

$$\frac{d^2\sigma_{\text{meas}}}{d\nu d\Omega} = e^{-\delta_R(\Delta)} F(Q^2) \frac{d^2\sigma_{1\gamma}}{d\nu d\Omega} + \frac{d^2\sigma_{\text{tails}}}{d\nu d\Omega},$$

where

$$\delta_R(\Delta) = \frac{\alpha}{\pi}\left(\ln\frac{E_s}{\Delta} + \ln\frac{E_p}{\Delta}\right)\left(\ln\frac{Q^2}{m^2} - 1\right)$$

$$F(Q^2) = 1 + \delta^e_{\text{vac}} + \delta^\mu_{\text{vac}} + \delta_{vtx} + \delta_s$$

$\delta_R(\Delta)$ is a residue of cancellation of IR divergent terms

σ_{tails} processes where real photons of $E_\gamma > \Delta$ are emitted

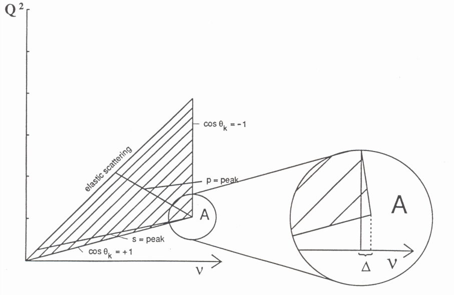
Mo and Tsai scheme: FERRAD...cont'd

Here:

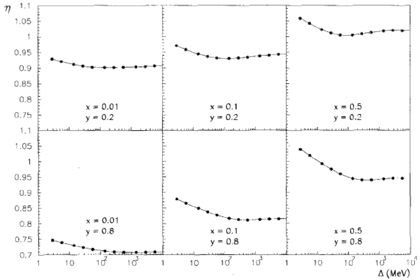
$$\sigma_{\mathbf{meas}} = \mathbf{v}\sigma_{\mathbf{1}\gamma} + \sigma_{\mathbf{tails}} = \mathbf{v}\sigma_{\mathbf{1}\gamma} + \sigma_{\mathbf{inel}} + \sigma_{\mathbf{el}} + \sigma_{\mathbf{qel}}$$

v – virtual corrections + soft photon emission.

Range of kinematical variables from which the radiative tails contribute to the cross section measured at the point $A(Q^2, \nu)$; parallel lines: W = const



Even if we measure at DIS, information on F_L, F_2 (or R, F_2) needed down to Q^2 =0!



Weak dependence of $\eta(x, y) = \sigma_{1\gamma}/\sigma_{meas}$ on Δ (here E = 280 GeV). We took Δ = 0.1% E

BB, Bardin, Kurek, Scholz, Z. Phys. C66 (1995) 591

Dubna scheme: TERAD (also: POLRAD), QPM, covariant

Measured cross section:

$$\frac{d^2\sigma_{meas}}{dQ^2dx} = \frac{d^2\sigma^B}{dQ^2dx}\left\{e^{-\delta_R(x,Q^2)} + \delta^{VR}(x,Q^2)\right\}$$

Born × { resummed collinear γ
+ remnant of exponentiation
+ remnant of subtraction in σ_{in}
(vertex) }

$$+ \frac{d^2\sigma_{in.tail}}{dQ^2dx} - \frac{d^2\sigma^{IR}}{dQ^2dx}$$

inelastic radiative tail
and regularization
$\Longrightarrow$ Dubna scheme Δ indep.

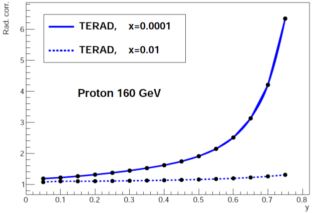
$$+ \frac{2\pi\alpha^2}{Q^4}\sum_{B=\gamma,I,Z}\sum_{b=i,q}\sum_{Q,\bar{Q}} c_b K(B,p)[V(B,p)R_b^V(B)$$

QPM calculations
of RC for hadron current

$$+ pA(B,p)R_b^A(B)] + \frac{d^2\sigma_{el.tails}}{dQ^2dx}.$$

elastic radiative tails
as in MT but covariant

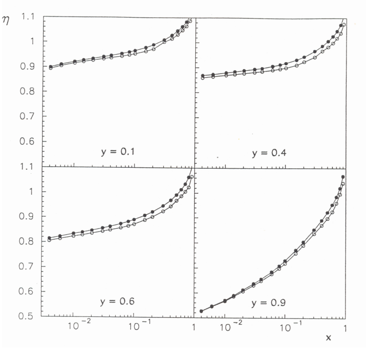
- "Vacuum polarisation" through running of $\alpha(Q^2)$
- $O(\alpha^2)$ in amplitude implemented
- Weak loop correction also present

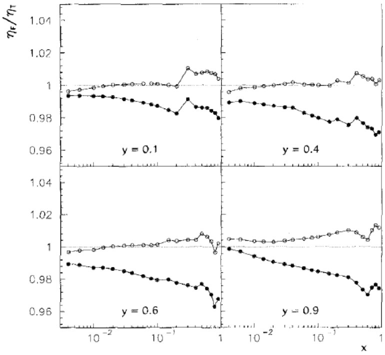


FERRAD vs TERAD (μp, 280 GeV)

$\eta(x,y) = \sigma_{1\gamma}/\sigma_{meas}$
open symbols = FERRAD
closed symbols = TERAD

η_F/η_T
open symbols = FERRAD without $\tau\bar{\tau}$, $q\bar{q}$
closed symbols = full FERRAD





BB, Bardin, Kurek, Scholz, Z. Phys. C66 (1995) 591

Practicalities

- At (unpolarised) DIS: $\frac{\mathrm{d}^2\sigma_{meas}}{\mathrm{d}Q^2\mathrm{d}x} = f\left[F_{el}(Q^2), F_{qel}(Q^2), F_L(x,Q^2), F_2(x,Q^2)\right]$

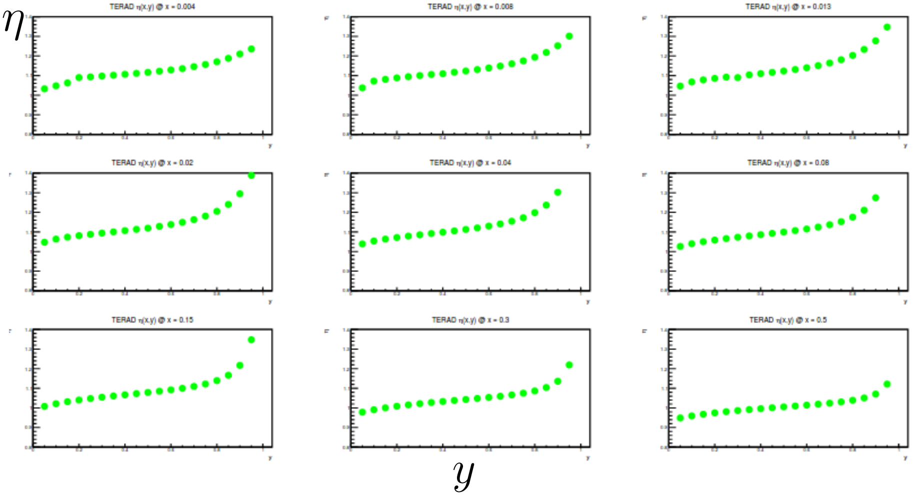
  Here:

  $F_{el}(Q^2)$ – target elastic form factor
  $F_{qel}(Q^2)$ – target quasi-elastic form factor
  $F_L(x,Q^2), F_2(x,Q^2)$ – DIS structure functions
  must be known for $x_{meas} < x < 1, \quad 0 < Q^2 < Q^2_{max}$

- In case of polarised DIS, also $g_1(x,Q^2), g_2(x,Q^2)$ must be included in the x-section $\mathrm{d}^2\sigma_{meas}/(\mathrm{d}Q^2\mathrm{d}x)$
- TERAD provides only inclusive radiative corrections (cross sections)!!!!
- Measurements are corrected event-by-event by a radcorr factor $\eta(x,Q^2) = \sigma 1\gamma/\sigma_{meas}$
- Different input functions were collected for p, d and nuclear targets
- Attention! Elastic radiative tail which fakes inelastic one!
- TERAD used in NMC, SMC, COMPASS;

  exact calculations $\Longrightarrow$ tables $\Longrightarrow$ 2-D interpolation

TERAD η vs y in bins of x; μp @ 160 GeV



COMPASS Note 2017-3

TERAD used in NMC, SMC, COMPASS,...

Ref: COMPASS Note 2015-6 Date: 1 March 2016

TERAD15 user guide, version 1.0

From/De : Barbara Badelek and Barbara Latacz
To/à : COMPASS Collaboration
Subject/Sujet : TERAD15 user guide

This note explains and summarizes basic information related to TERAD15 structure and usage. TERAD15 is a 2105 version of TERAD which is written in a simple FORTRAN without using PATCHY and it is user-friendly. **OBSERVE THAT THIS NOTE IS BEING UPDATED (programs are tested) and new versions will be released!** Please contact Barbara Badelek if you notice any error or encounter a problem.

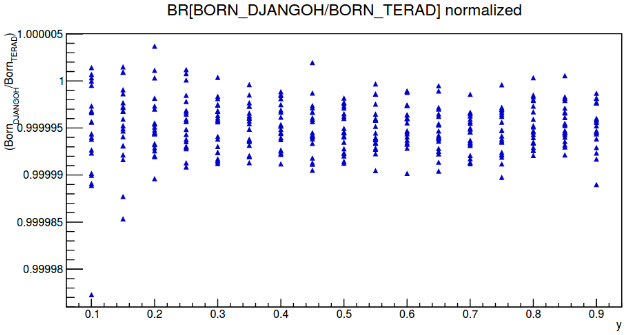
Also available at **http://wwwcompass.cern.ch/compass/notes/2015-6/2015-6.html** .

Contents

Outline

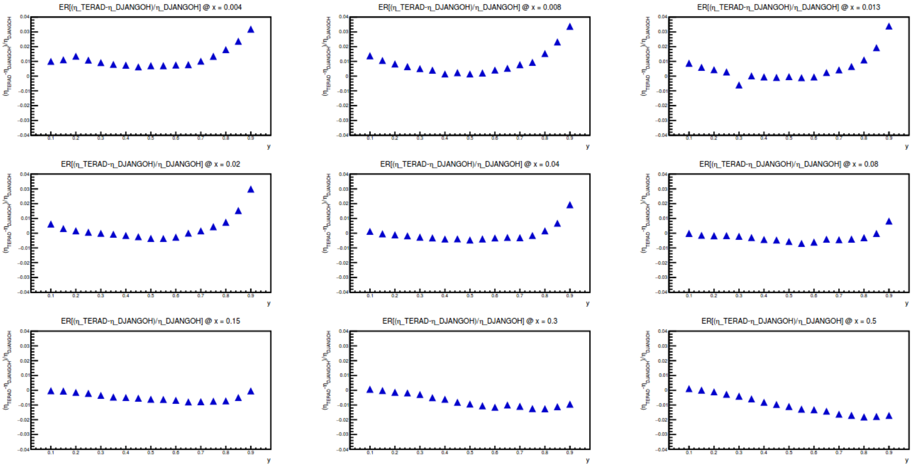
DJANGOH H. Spiesberger, MZ-TH/05-15

- Created for the ep interactions at HERA
- DJANGOH is an event generator
- Includes both QED and QCD radiative events
- Comprises: single photon emission from the lepton and the quark line, self-energy corrections, complete set of one-loop weak corrections.
- Radiation from quark lines is permanently disabled in DJANGOH.



COMPASS Note 2017-3

Comparison of RCs from TERAD and DJANGOH (at the same input)



Differences at most 3%, understood

COMPASS Note 2017-3

Input information for polarised/unpolarised and inclusive/semiinclusive RC calculations

The items below should be known for
$x_{meas} < x < 1$ and $0 < Q^2 < Q^2_{max}$

- Spin independent structure function $F_2(x, Q^2)$ (nucleon, nuclei)
- Spin independent structure function $F_L(x, Q^2)$
- Spin dependent structure function $g_1(x, Q^2)$
- Quasielastic suppression factors(Q^2) (nuclei)
- Elastic form factors(Q^2) (nucleon, nuclei)

All the input information is collected in a COMPASS note 2015-6, for the moment not accessible for outsiders but this may be changed

Outline

$F_2(x, Q^2)$ and $F_L(x, Q^2)$ in the low Q^2 region

(see e.g. BB, Kwieciński, Rev. Mod. Phys. 68 (1996) 445)

F_2 and F_L needed at: $x_{meas} < x < 1$ and $0 < Q^2 < Q^2_{max}$

- They are either physics motivated fits or models of dynamic origin and
- have to have a proper asymptotic behaviour:
  at $Q^2 \to 0$ fulfilling the conditions for arbitrary ν

$$F_2 = O(Q^2), \qquad \frac{F_1}{M} + \frac{F_2}{M}\frac{pq}{q^2} = O(Q^2).$$

and $\qquad F_L = O(Q^4) \qquad$ for fixed $2pq$

or $\quad R(x, Q^2) = \frac{\sigma_L}{\sigma_T} = \frac{(1 + 4M^2x^2/Q^2)F_2}{2xF_1} - 1 = \frac{F_L}{2xF_1} \longrightarrow 0 \ \text{ at } \ Q^2 \to 0$

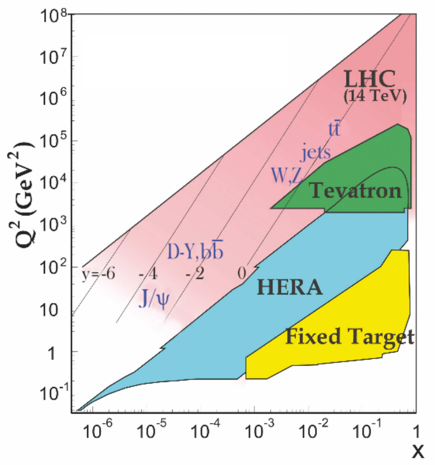
and at $Q^2 \to \infty$ joining the QCD improved parton model expressions.

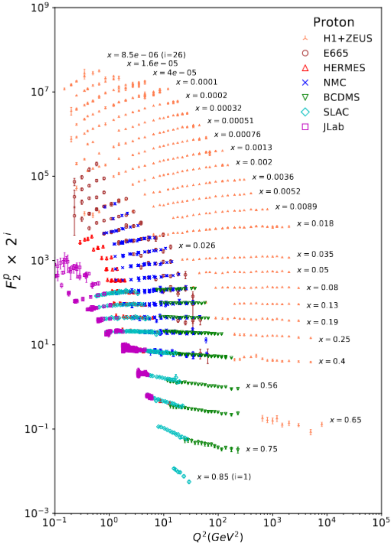
Observe that:

- Growth of F_2 with decreasing x is slower at low Q^2
- $F_L(x, Q^2)$ essentially independent of x in the low x, low Q^2 region.

Outline

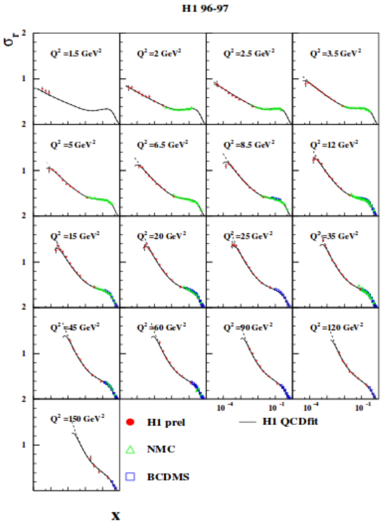
What do the F_2 data show around Q^2 = 1 GeV^2?





PDG 2022

What do the F_2 data show around Q^2 = 1 GeV^2?... cont'd



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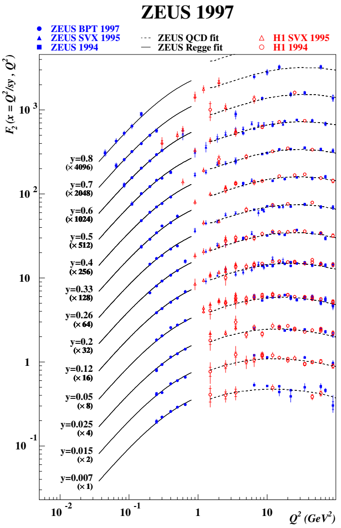
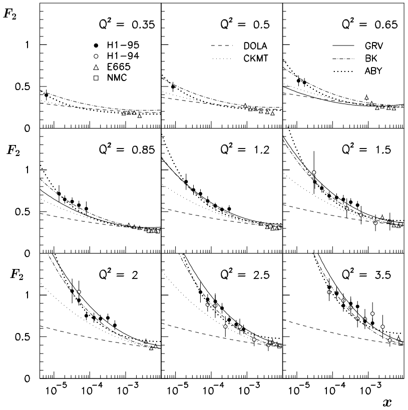


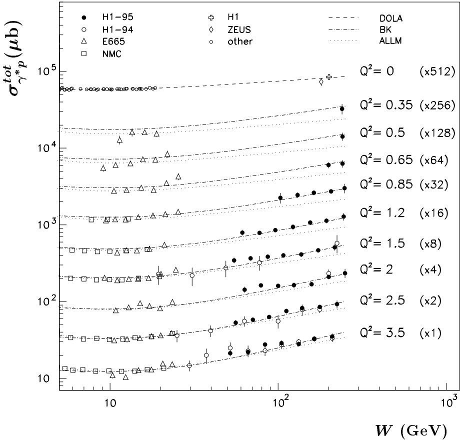
ZEUS, Eur. Phys. J. C7 (1999) 609

Outline

F_2^p in the low Q^2, low x region

JKBB (Kwieciński, BB, Z. Phys. C43 (1989) 43; Phys. Lett. B295 (1992) 263)



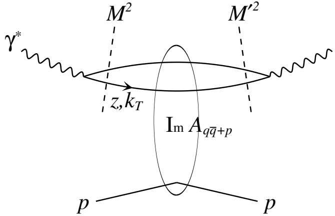


H1 Collaboration, Nucl.Phys. B497 (1997) 3

Outline

F_2^p in the low Q^2, low x region

Martin-Ryskin-Stasto; (Martin, Ryskin, Stasto, Eur. Phys. J. C7 (1999) 643)



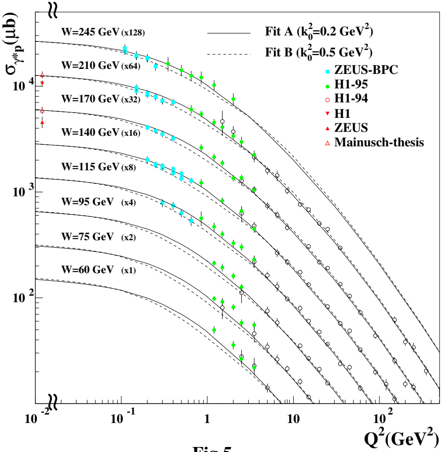
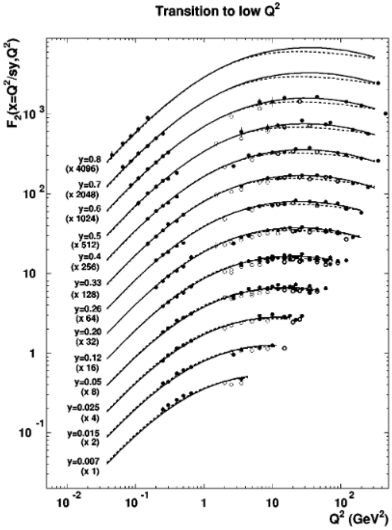


Fig.5

Outline

F_2^p in the low Q^2, low x region

(Modified saturation model, Bartels, Golec-Biernat, Kowalski, Phys. Rev. D66 (2002) 014001)



- Original saturation model of Golec-Biernat and Wüsthoff modified by DGLAP
- 5 parameters fitted to E665, H1 and ZEUS data, $x <$ 0.01, 0.1$< Q^2 <$500 GeV^2 (claimed to be valid down to Q^2 = 10^{-5} GeV^2).

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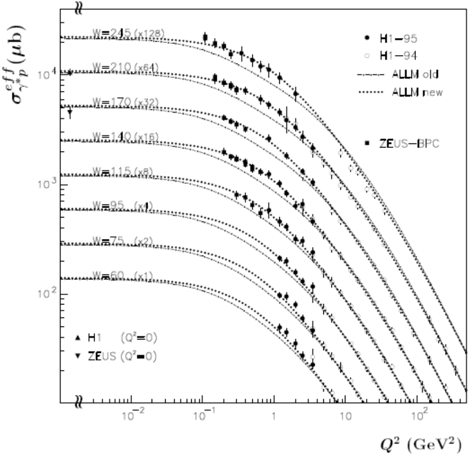
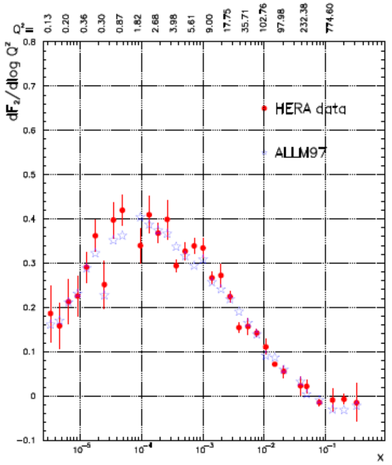
6. Outlook

F_2^p in the low Q^2, low x region

ALLM97; (Abramowicz, Levy, hep-ph/9712415)

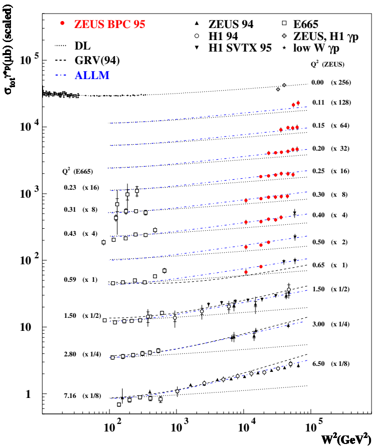


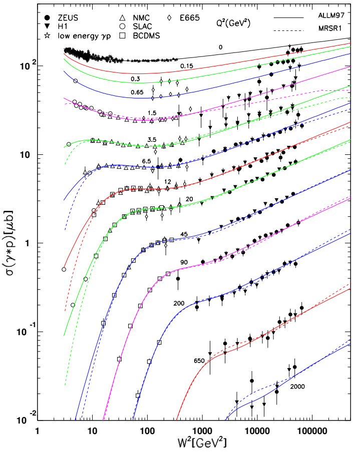


$(\sigma^{eff}_{\gamma^* p} \approx \sigma^{tot}_{\gamma^* p}$ at HERA energies)

F_2^p in the low Q^2, low x region

ALLM97...cont'd





Outline

F_2^p in the low Q^2, low x region

ZEUS Regge fit (ZEUS, Eur. Phys. J. C7 (1999) 609)

Combines the Q^2 dependence of the VMD with the energy dependence from the Regge model:

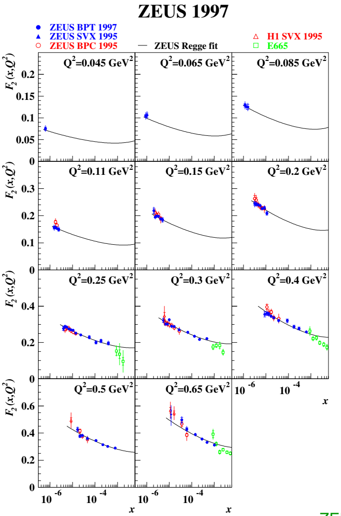
$$F_2(x, Q^2) = \left(\frac{Q^2}{4\pi^2\alpha}\right) \cdot \left(\frac{M_0^2}{M^2 + Q^2}\right) \cdot \left[A_R \cdot (W^2)^{\alpha_R - 1} + A_P \cdot (W^2)^{\alpha_P - 1}\right]$$

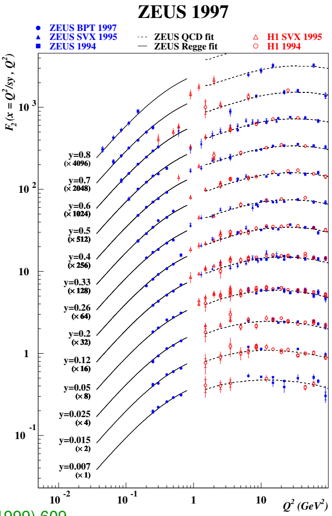
where A_R, A_P, M_0 are constants; α_R, α_P are reggeon and pomeron intercepts. Fixed: M_0^2 = 0.53 GeV^2, α_R = 0.53.
Remaining 3 parameters fitted to Q^2 =0 data at W^2 >3 GeV^2.

Result: α_P = 1.097 $\pm$ 0.002.

F_2^p in the low Q^2, low x region

ZEUS Regge fit...cont'd

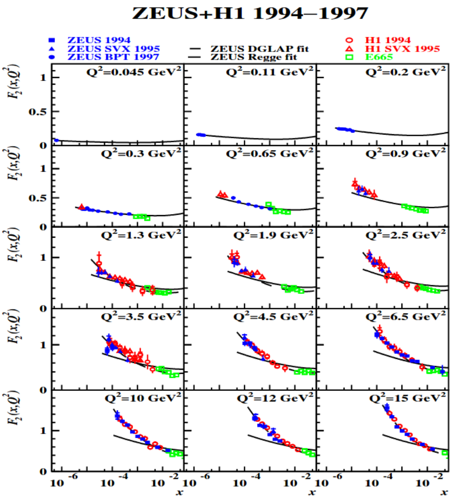




ZEUS, Eur. Phys. J. C7 (1999) 609

F_2^p in the low Q^2, low x region

ZEUS Regge fit...cont'd



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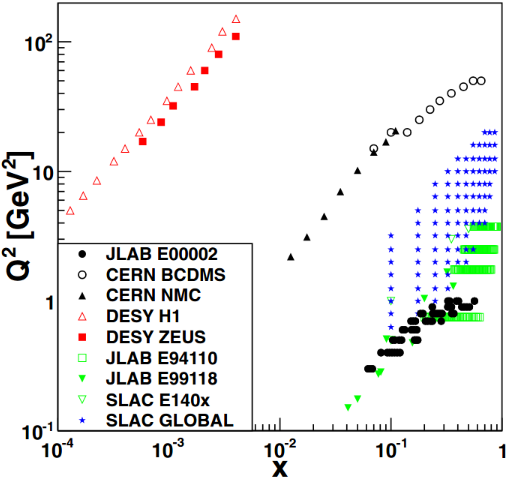
6. Outlook

Phase space (x, Q^2) for F_L measurements limited!...

...especially at low Q^2

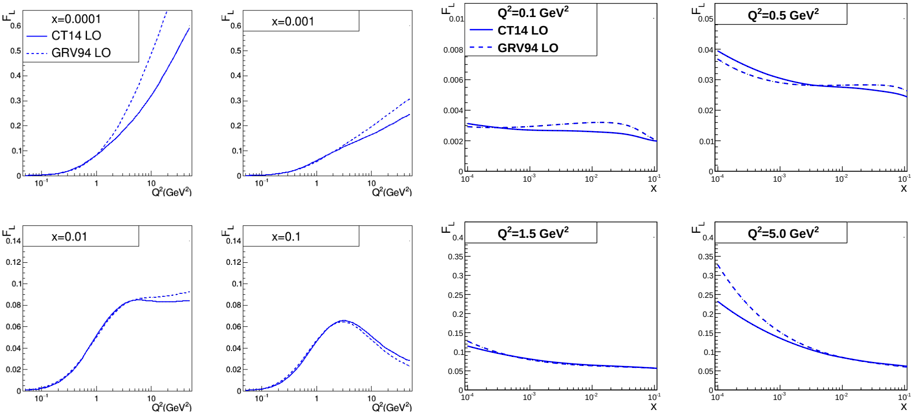


JLab, E00002, V. Tvaskis et al. Phys. Rev. C 97 (2018) 045204

F_L in the low Q^2, low x region: results

(BB, Stasto, Phys. Lett. B829 (2022) 137086)

Observe different vertical scales in different panels!

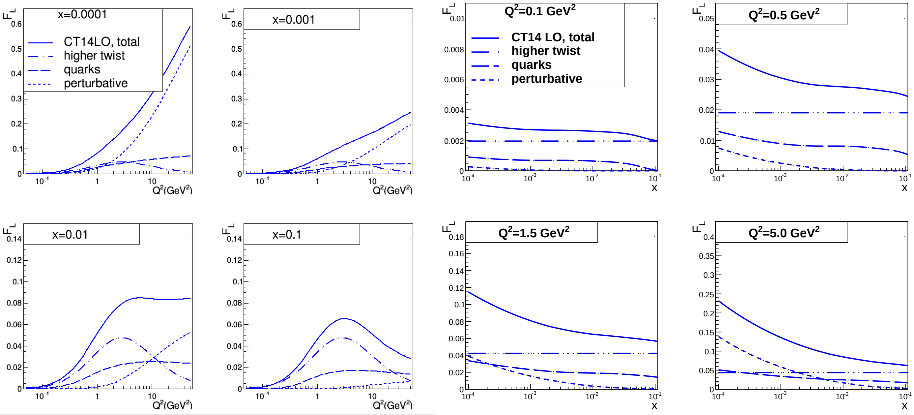


Only weak dependence on the PDFs in the region of interest; same for LO/NLO dependence

F_L in the low Q^2, low x region: results... cont'd

(BB, Stasto, Phys. Lett. B829 (2022) 137086)

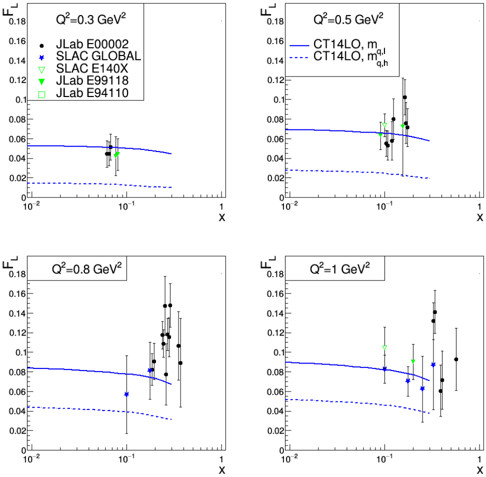
Observe different vertical scales in different panels!



Interplay of different mechanisms building F_L; perturbative one contributes little at low Q^2

F_L in the low Q^2, low x region: results... cont'd

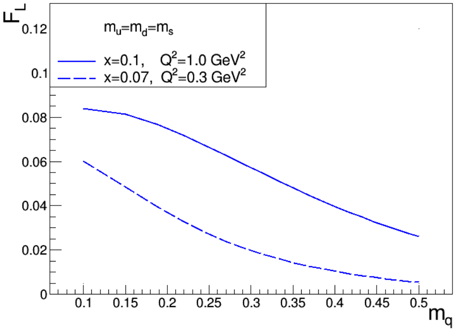
(BB, Stasto, Phys. Lett. B829 (2022) 137086)

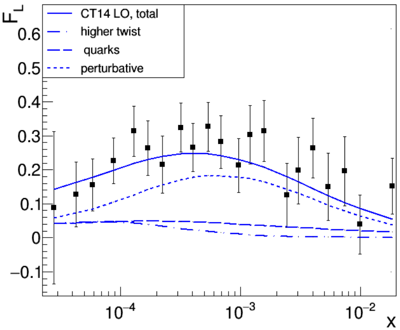


- model underestimates the JLab and SLAC data...
- ...unless light quark masses lowered to 0.14 GeV...
- ...as in the dipole model of Golec-Biernat and Wusthoff

F_L in the low Q^2, low x region: results...cont'd

(BB, Stasto, Phys. Lett. B829 (2022) 137086)





Data: at lowest x, $Q^2 \approx 1.5$ GeV^2

The model reproduces well the HERA/H1 data in the perturbative region, $Q^2 \gtrsim 1.5$ GeV^2

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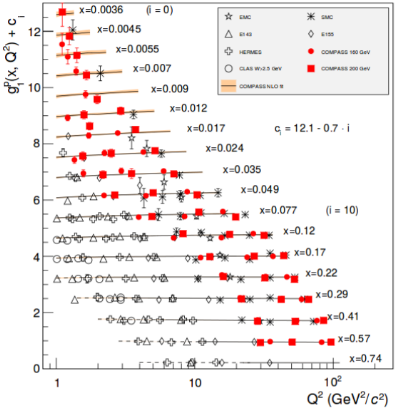
6. Outlook

Measurements of $g_1^p(x)$ for proton

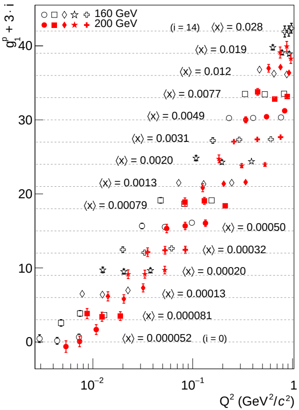
$Q^2 > 1$ (GeV/c)^2

COMPASS NLO QCD at $W^2 > 10$ (GeV/c^2)^2
dashed line: extrapolation to $W^2 < 10$ (GeV/c^2)^2



COMPASS, PLB753 (2016) 18

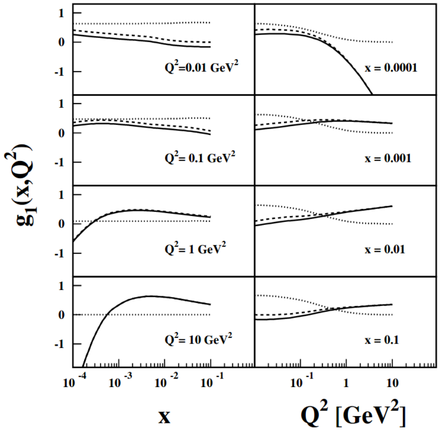
$Q^2 < 1$ (GeV/c)^2



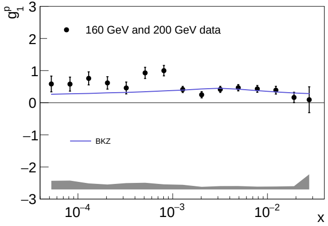
COMPASS, PLB781 (2018) 464

Model for g_1^p at low Q^2: results

(BB, Kwiecinski, Ziaja, Eur. Phys. J. C26 (2002)45)

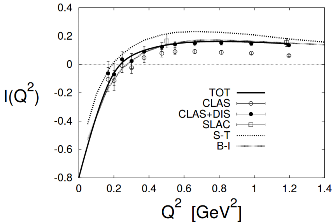


broken lines – g_1^{AS}, dotted – g_1^{L}, continuous – total g_1



COMPASS data at $Q^2 < 1$ GeV^2

COMPASS, PL781 (2018) 464



BB, Kwiecinski, Ziaja, Eur. Phys. J. C26 (2002) 45.

Outline

1. Mo & Tsai and Dubna schemes
2. DJANGOH event generator
3. Extension of $F_2(x, Q^2)$ down to $Q^2 = 0$ (to be updated !)
   - Data at low Q^2
   - JKBB
   - Martin-Ryskin-Stasto
   - (Modified) saturation model
   - ALLM97
   - ZEUS Regge fit
4. Extension of $F_L(x, Q^2)$ down to $Q^2 = 0$ (updated recently)
5. Extension of $g_1(x, Q^2)$ down to $Q^2 = 0$ (to be updated !)
6. Outlook

Outlook

In the precision RC calculations a part of systematic uncertainties come from a choice of the input information.

We have a collection of expressions for $Q^2 \to 0$ extrapolations for:

- $F_2^p(x, Q^2) \implies$ to be updated
- $F_L^p(x, Q^2) \implies$ updated recently
- $g_1^p\,(x, Q^2) \implies$ to be updated
- form factors, suppression factors $\implies$ to be updated (not discussed here)

These expressions are valid at low x, appropriate for the EIC

$\implies$ update of F_2^p and g_1^p to be done “soon”

SPARES

Parametrizations of F_2 in the low Q^2, low x region

JKBB (Kwieciński, BB, Z. Phys. C43 (1989) 43; Phys. Lett. B295 (1992) 263)

Starting point: Generalised Vector Meson Dominance (GVMD) representation of $F_2(x, Q^2)$:

$$\begin{aligned} F_2[x = Q^2/(s + Q^2 - M^2), Q^2] &= \frac{Q^2}{4\pi} \sum_v \frac{M_v^4 \sigma_v(s)}{\gamma_v^2 (Q^2 + M_v^2)^2} + Q^2 \int_{Q_0^2}^{\infty} dQ'^2 \frac{\Phi(Q'^2, s)}{(Q'^2 + Q^2)^2} \\ &\equiv F_2^{(v)}(x, Q^2) + F_2^{(p)}(x, Q^2) \end{aligned}$$

where

$$\Phi(Q'^2, s) = -\frac{1}{\pi} Im \int^{-Q'^2} \frac{dQ''^2}{Q''^2} F_2^{AS}(x', Q''^2)$$

- Asymptotic structure function $F_2^{AS}(x, Q^2)$ assumed to be given.
- By construction, $F_2(x, Q^2) \to F_2^{AS}(x, Q^2)$ for large Q^2.
- First term in F_2 corresponds to the low mass vector meson dominance.
- Second term (integral) in F_2 covers contributions of vector mesons heavier then Q_0.
- This integral can be looked upon as the extrapolation of the (QCD improved) parton model for arbitrary Q^2 (including Q^2 =0).
- The above representation of F_2 is written for fixed s and valid at $s \gg Q^2$, i.e. at low x but for arbitrary Q^2 – and above the resonances.

F_2^p in the low Q^2, low x region...cont'd

JKBB...cont'd

- Choosing the parameter $Q_0^2 > (M_v^2)_{max}$ where $(M_v)_{max}$ is the mass of the heaviest vector meson included in the sum one explicitly avoids double counting when adding two separate contributions to F_2.
- Q_0 should be smaller than the mass of the lightest vector meson not included in the sum.
- Representation for the partonic part $F_2^{(p)}(x, Q^2)$ may be simplified as follows:

$$F_2^{(p)}(x, Q^2) = \frac{Q^2}{(Q^2 + Q_0^2)} F_2^{AS}(\bar{x}, Q^2 + Q_0^2)$$

where

$$\bar{x} = \frac{Q^2 + Q_0^2}{s + Q^2 - M^2 + Q_0^2} \equiv \frac{Q^2 + Q_0^2}{2M\nu + Q_0^2}$$

- Simplified connection of $F_2^{(p)}(x, Q^2)$ to F_2^{AS} by an appropriate change of the arguments posseses all the main properties of the second term in the GVMD representation of F_2.

Apart from Q_0^2, constrained by physical requirements, the GVMD representation does not contain any other free parameters except those which are implicitly present in F_2^{AS}.

We took Q^2=1.2 GeV^2

F_2^p in the low Q^2, low x region

Martin-Ryskin-Stasto (Martin, Ryskin, Stasto, Eur. Phys. J. C7 (1999) 643)

Exploits further the idea of BBJK.

- Perturbative and non-perturbative QCD contributions separated by the distance configurations of the $q\bar{q}$ pair in the $\gamma^* \to q\bar{q}$:
- small distance configurations ($k_T^2 > k_0^2$) given by pQCD (unified equations, DGLAP + BFKL, unintegrated gluon distribution);
- large distance configurations ($k_T^2 < k_0^2$) given by VMD (for low $q\bar{q}$ fluctuation masses, $M^2 < Q_0^2$), and additive quark model (for high $q\bar{q}$ masses, $M^2 > Q_0^2$).
- Excellent description of the data throughout the whole Q^2 region, including Q^2 =0.
- Fitted (at x <0.05) are 3 parameters of the gluon distribution; scales k_0^2 and Q_0^2 chosen as: k_0^2 =0.2 GeV^2 (crucial) and Q_0^2 =1.5 GeV^2. Choice of k_0^2 yields physically sensible g and F_L.
- Interference between states of different $q\bar{q}$ masses is crucial for description of the data.

- Importance of the perturbative contribution in the non-perturbative domain.

F_2^p in the low Q^2, low x region

ALLM97 (Abramowicz, Levy, hep-ph/9712415)

- Parametrization of the $\sigma_{tot}(\gamma^* p)$ at $W^2 \gtrsim$ 3 GeV^2 (above resonances).
- Valid everywhere in x and Q^2 (including photoproduction).
- Based on Regge–type approach; extention to large Q^2 compatible with QCD.
- Observe that it is a fit of 23 parameters to all the data
- Fit contains contributions of the pomeron (P) and reggeon (R):

$$F_2(x,Q^2) = \frac{Q^2}{Q^2+m_0^2}\left[F_2^P(x,Q^2) + F_2^R(x,Q^2)\right]$$

of the form

$$F_2^P(x,Q^2) = c_P(t) x_P^{\alpha(t)} (1-x)^{b_P(t)}, \qquad F_2^R(x,Q^2) = c_R(t) x_R^{\alpha(t)} (1-x)^{b_R(t)}$$

where

$$t = \ln\left(\frac{\ln Q^2 + Q_0^2}{\Lambda^2} \; / \; \ln\frac{Q_0^2}{\Lambda^2}\right)$$

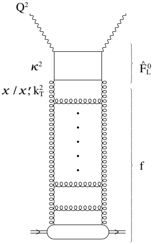
and

$$\frac{1}{x_P} = 1 + \frac{W^2 - M^2}{Q^2 + m_P^2}, \qquad \frac{1}{x_R} = 1 + \frac{W^2 - M^2}{Q^2 + m_R^2}$$

Here M is the proton mass; $m_0^2, m_P^2, m_R^2, Q_0^2$ allow a smooth transition to photoproduction. For $Q^2 \gg m_P^2, Q^2 \gg m_R^2$, $x_P \to x, \;\; x_R \to x$;
$c_R, a_R, b_R, b_P \nearrow Q^2 \nearrow; \quad c_P, a_P \searrow Q^2 \nearrow$.

F_L in the low Q^2, low x region: the (updated) model

BB, Kwiecinski, Stasto, Z. Phys. C74 (1997) 297; update: BB, Stasto, Phys. Lett. B829 (2022) 137086



- A model for F_L, valid at low x and low Q^2; based on the photon–gluon fusion and extended to low Q^2.
- The model embodies the constraint $F_L \sim Q^4$ at $Q^2 \to 0$.
- k_T factorisation with off-shell gluons

$$F_L = \int_x^1 \frac{\mathrm{d}x'}{x'} \int \frac{\mathrm{d}k_T^0}{k_T^0} F_L^0(x', Q^2, k_T^0) f\left(\frac{x}{x'}, k_T^2\right)$$

where F_L^0 comes from $\gamma^* g$ fusion, is a longitudinal structure function of the off-shell gluon of virtuality k_T^2 and is calculated perturbatively; f is an unintegrated gluon distribution related to the "ordinary" $g(y, \mu^2)$ by:

$$yg(y, \mu^2) = \int^{\mu^2} \frac{\mathrm{d}k_T^2}{k_T^2} f(y, k_T^2)$$

Its evolution is controlled by (approximate) BFKL.

- To extrapolate F_L to low Q^2 and to $Q^2 = 0$, evolution of $g(y, Q^2)$ and argument of $\alpha_s(Q^2)$ was frozen *via* $Q^2 \to Q^2 + 4m_q^2$.
- HT contribution needed at moderate Q^2, i.e. terms vanishing as $1/Q^2$ for $Q^2 \to \infty$. They were assumed to originate from low quark transverse momenta, $\kappa^2 < \kappa_0^2$ and interpreted as coming from soft pomeron exchange (intercept = 1). Such HT has a proper behaviour both at $Q^2 \to \infty$ and $Q^2 \to 0$.

F_L in the low Q^2, low x region: input to the (updated) model

(In blue marked are values taken in the final calculations)

- Models for unintegrated gluon density:
  - small x linear BFKL
  - DGLAP motivated: Kimber-Martin-Ryskin formalism
- On- and off-shell gluons
- Different standard PDF sets: GRV94LO/NLO, CT14LO/NLO, CT18NLO
- Quark transverse momenta cutoff κ_0^2 variation: 0.5 –1.2 GeV^2 (0.8 GeV^2)
- Masses for quarks (u,d,s,c): 0.35, 0.35, 0.5, 1.2–1.5 GeV
  later lowered to 0.14 GeV (for all q's)

Model for g_1 at low Q^2 BB, Kwiecinski, Ziaja Eur. Phys. J. C26 (2002) 45

Assumed the following representation of g_1, valid for fixed $W^2 \gg Q^2$, i.e. small x:

$$g_1(x,Q^2) = g_1^L(x,Q^2) + g_1^H(x,Q^2) = \frac{M\nu}{4\pi}\sum_V \frac{M_V^4 \Delta\sigma_V(W^2)}{\gamma_V^2(Q^2+M_V^2)^2} + g_1^{AS}(\bar{x}, Q^2+Q_0^2).$$

First term, $g_1^L(x,Q^2)$: contributions from light vector mesons, $M_V < Q_0$, $Q_0^2 \sim 1$ GeV^2.
Unknown $\Delta\sigma_V$: from combinations of nonperturbative parton distributions, evaluated at fixed Q_0^2.

Second term, $g_1^H(x,Q^2)$: contributions from heavy ($M_V > Q_0$) VMs.
Can be treated as extrapolation of QCD improved parton model structure function, $g_1^{AS}(x,Q^2)$, to arbitrary values of Q^2: $g_1^H(x,Q^2) = g_1^{AS}(\bar{x}, Q^2+Q_0^2)$. Here x is replaced by $\bar{x} = (Q^2+Q_0^2)/(Q^2+Q_0^2+W^2-M^2)$. At large Q^2, $g_1^H(x,Q^2) \to g_1^{AS}(x,Q^2)$.

Thus:

$$\begin{aligned} g_1(x,Q^2) &= C\left[\frac{4}{9}(\Delta u_{val}^0(x) + 2\Delta\bar{u}^0(x)) + \frac{1}{9}(\Delta d_{val}^0(x) + 2\Delta\bar{d}^0(x))\right]\frac{M_\rho^4}{(Q^2+M_\rho^2)^2} \\ &+ C\left[\frac{1}{9}(2\Delta\bar{s}^0(x))\right]\frac{M_\phi^4}{(Q^2+M_\phi^2)^2} + g_1^{AS}(\bar{x}, Q^2+Q_0^2). \end{aligned}$$

Constant C is fixed in the photoproduction limit via DHGHY sum rule

Model for g_1 at low Q^2,...cont'd

The $\gamma^* p$ scattering amplitude fulfills the dispersion relation:

$$S_1(\nu, q^2) = 4 \int_{-q^2/2M}^{\infty} \nu' \mathrm{d}\nu' \frac{G_1(\nu', q^2)}{(\nu')^2 - \nu^2}$$

where

$$G_1(\nu, q^2) = \frac{M}{\nu} g_1(x, Q^2)$$

in the $Q^2, \nu \to \infty$ limit.

As a result of Low's theorem: $S_1(0,0) = -\kappa^2_{p(n)}$, G_1 in the $Q^2 \to 0$ limit fulfills

the DHGHY sum rule:

$$\int_0^{\infty} \frac{\mathrm{d}\nu}{\nu} G_1(\nu, 0) = -\frac{1}{4} \kappa^2_{p(n)}.$$

Model for g_1 at low Q^2,...cont'd

But the $S_1(\nu, q^2)$, at ν →0 is:

$$S_1(0, q^2) = 4M \int_{Q^2/2M}^{\infty} \frac{\mathrm{d}\nu}{\nu^2} g_1(x(\nu), Q^2).$$

Now we define the DHGHY moment, $I(Q^2)$ as:

$$I(Q^2) = S_1(0, q^2)/4 = M \int_{Q^2/2M}^{\infty} \frac{\mathrm{d}\nu}{\nu^2} g_1(x(\nu), Q^2).$$

Before taking the $Q^2 \to 0$ limit of $S_1(0, q^2)$, observe that it is valid only down to some threshold value of W, $W_{th} \lesssim 2$ GeV (above resonances). Requirement $W > W_{th}$ gives the lower limit for integration over ν in $S_1(0, q^2)$, where $\nu_t(Q^2) = (W_t^2 + Q^2 - M^2)/2M$:

$$I(Q^2) = I_{res}(Q^2) + M \int_{\nu_t(Q^2)}^{\infty} \frac{d\nu}{\nu^2} g_1 \left(x(\nu), Q^2\right).$$

Here I_{res} is a contribution of resonances. The DHGHY sum rule now implies:

$$I(0) = I_{res}(0) + M \int_{\nu_t(0)}^{\infty} \frac{d\nu}{\nu^2} g_1 \left(x(\nu), 0\right) = -\kappa^2_{p(n)}/4.$$

Model for g_1 at low Q^2,...cont'd

Action plan for extracting C:

substitute $g_1(x(\nu), 0)$ from g_1 representation into $I(0)$;
C is the only free parameter, provided $I_{res}(0)$ is known from measurements

Taking:

- $I_{res}(0)$ from photoproduction, W_t=1.8 GeV GDH, Nucl. Phys. 105 (2002) 113,
- g_1^{AS} prametrized by NLO GRSV2000 Phys.Rev. D63 (2001) 094005
- nonperturbative $\Delta p_j^{(0)}(x)$ at $Q^2 = Q_0^2 = 1.2$ GeV^2 from
  1. GRSV2000 $\Longrightarrow$ $\mathrm{C} = -0.30$
  2. "flat" $\Delta p_j^{(0)}(x) = N_i(1-x)^{\eta_i}$ $\Longrightarrow$ $\mathrm{C} = -0.24$.