EICUG Theory WG: radiative corrections at the EIC

Tuesday Nov 15, 2022, 10:00 AM → 11:15 AM US/Eastern

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Radiative Corrections vs. QED Contribution to Hard Probes at the EIC

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In collaboration with: Tianbo Liu, Wally Melnitchouk, Nobuo Sato, Kazuhro Watanabe, Zhite Yu, …

Phys.Rev.D 104 (2021) 9, 094033; *JHEP* 11 (2021) 157; … …

Jefferson Lab

TMD Collaboration

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JSA

Outline of my talk

- **QCD at a Fermi-Scale – Nuclear Femtography**

  => Need a lepton-hadron facility & Electron-Ion Collider (EIC)

  Collision induced QED radiation is a part of hard probes at the EIC

- **Inclusive ep deep inelastic scattering (DIS)**

  => Inclusive production of single high-PT electron in ep collision

  Collinear QED and QCD factorization

- **Single hadron (or jet) photoproduction in ep collision**

  => Inclusive production of single high-PT hadron (or jet) in ep collision

  Collinear QED and QCD factorization

- **Lepton-hadron (ep) Semi-inclusive DIS (SIDIS)**

  => Inclusive production of a pair of high-PT lepton and hadron in ep-collision

  Hybrid (collinear QED) and (TMD QCD) factorization

- **Summary and outlook**

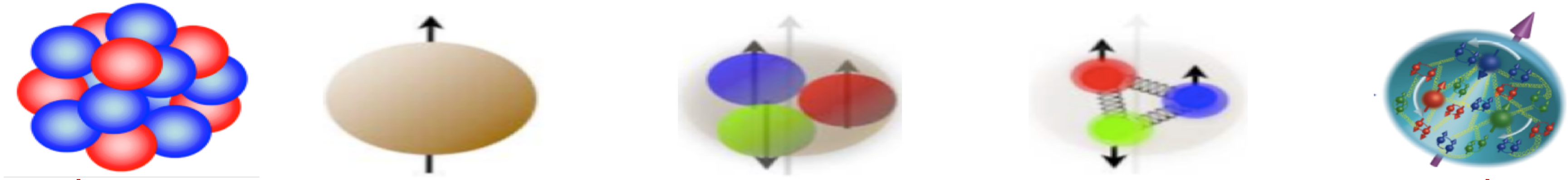
QCD Landscape of Nucleons and Nuclei

Color Confinement — Asymptotic freedom

Q (GeV)

20 MeV (10 fm) — 200 MeV (1 fm) — 2 GeV (1/10 fm)

Probing scale



QCD at the Fermi Scale: Femto-science (0.1-10 fm)

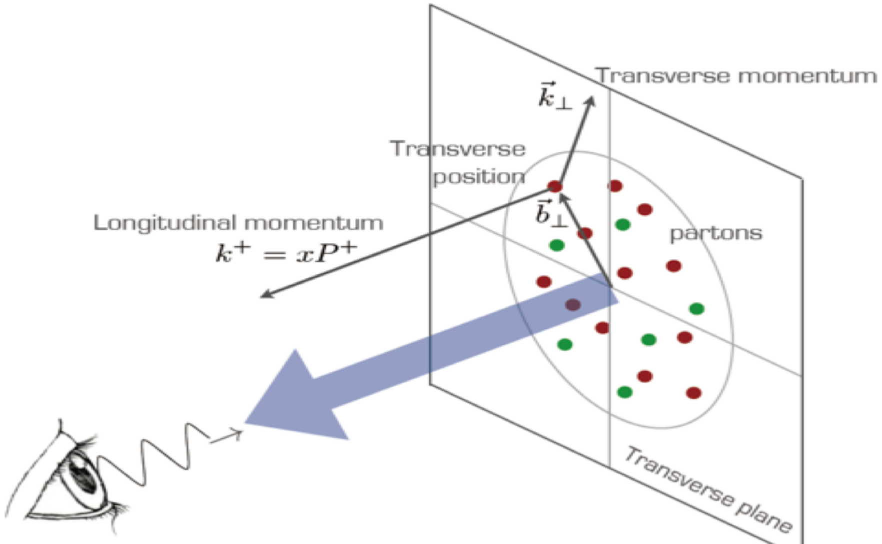
Asymptotic regime PQCD works beautifully!

- *The most interesting, rich, and complex regime of the theory!*
- *All emergent phenomena depend on the scale at which we probe them!*

❑ **Need new observables with two distinctive: scales:**

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

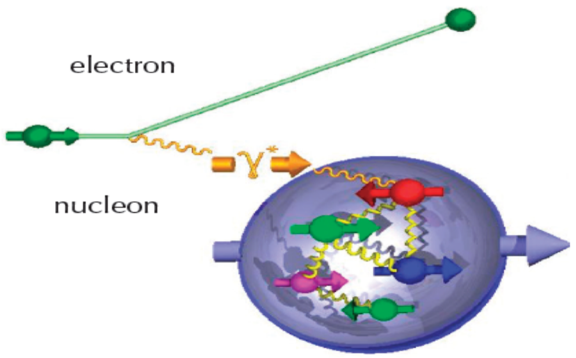
- **Hard scale:** Q_1 **to localize the probe to see the particle nature of quarks/gluons**
- **"Soft" scale:** Q_2 **could be more sensitive to the hadron structure ~ 1/fm**

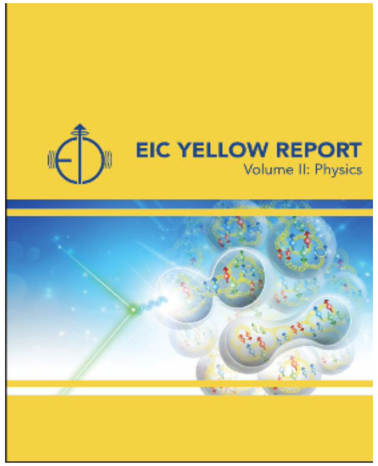


Lepton-Hadron Facility – Many Complementary Probes at One Facility

❑ **The new generation of "Rutherford" experiments:**

arXiv:2103.05419





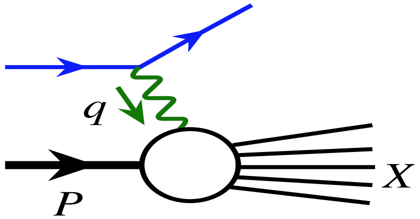
- ✧ **A controlled "hard probe" – virtual photon**
- ✧ **Can either break or not break the hadron**

Two-scale observables are natural at lepton-hadron facility!

✧ **Inclusive events**: e+p/A → e'+X

Detect only the scattered lepton in the detector

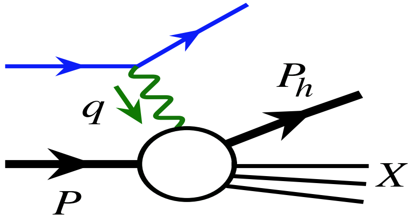
(Modern Rutherford experiment!)



✧ **Semi-Inclusive events**: e+p/A → e'+h(p,K,p,jet)+X

Detect the scattered lepton in coincidence with identified hadrons/jets

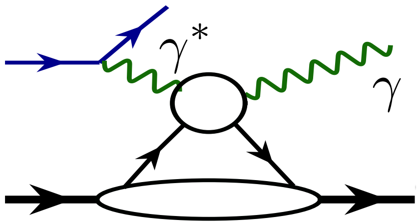
(Initial hadron is broken – confined motion! – cleaner than h-h collisions)



✧ **Exclusive events:** e+p/A → e'+ p'/A'+ h(p,K,p,jet)

Detect every things including scattered proton/nucleus (or its fragments)

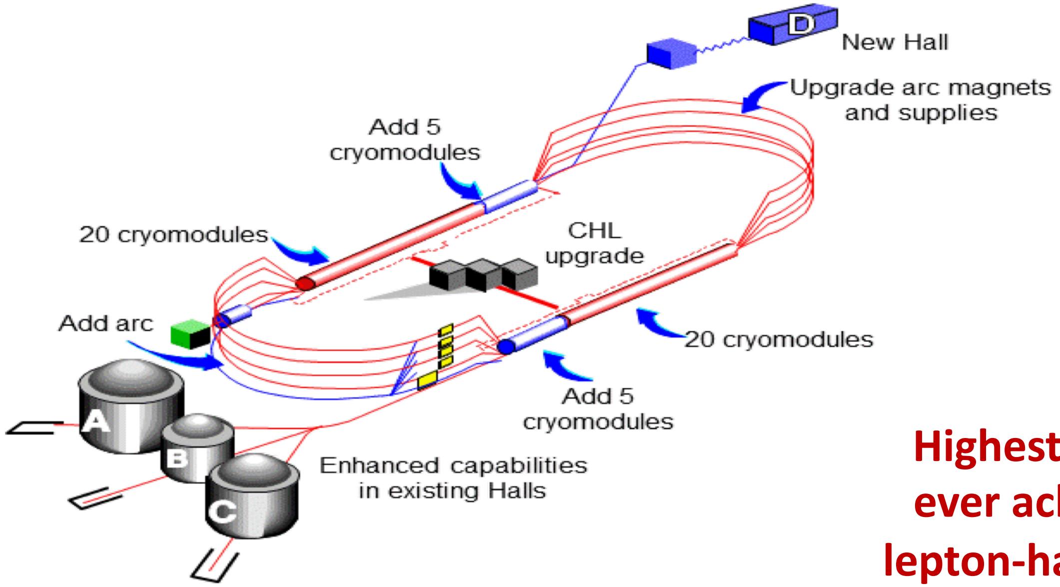
(Initial hadron is NOT broken – tomography! – almost impossible for h-h collisions)



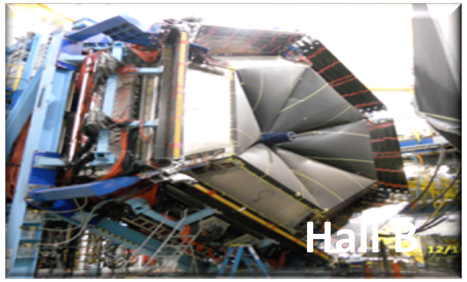
CEBAF at 12 GeV @ Jefferson Lab

❑ Lepton-hadron facility in the US now:

12 GeV CEBAF Upgrade Project was just completed, on-time and on-budget!



Highest luminosity ever achieved by a lepton-hadron facility
10^{38} ($cm^{-2}\,s^{-1}$)
up to 10^{39} ($cm^{-2}\,s^{-1}$)



- Search for exotic hadrons, …
- Explore for the 3D hadron structure, …
- Search for dark matter, photon, …
- Advance accelerator technology, …
- …

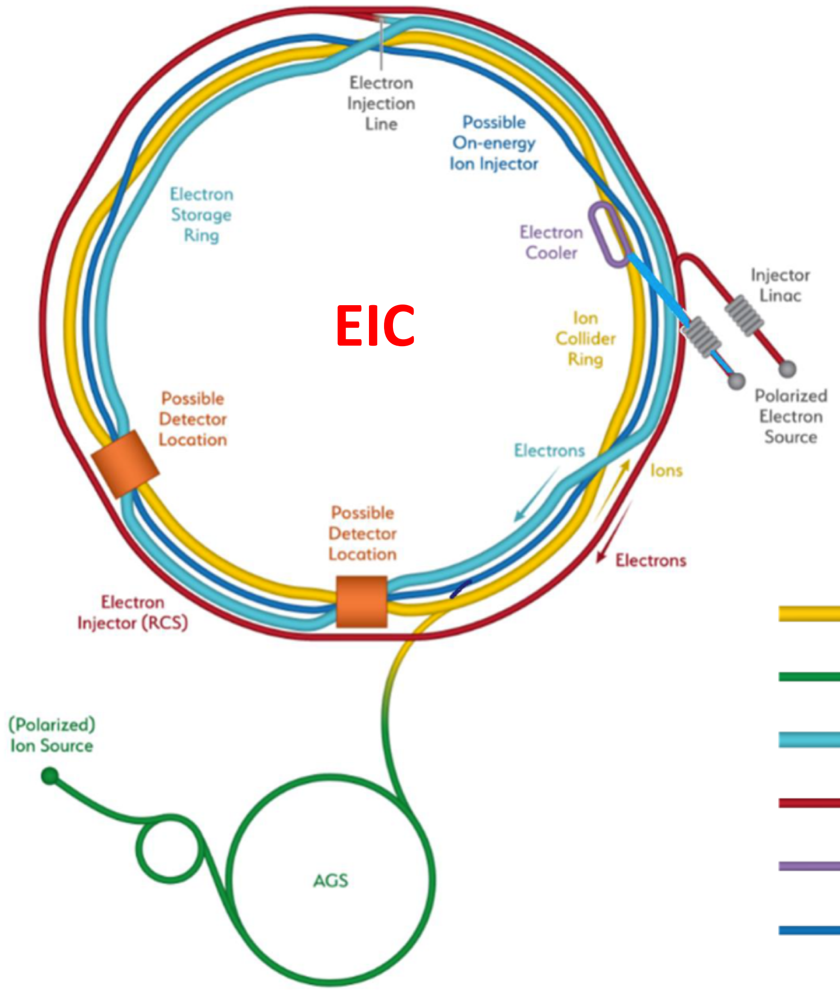
Future: MOLLER
SoLID
Detector

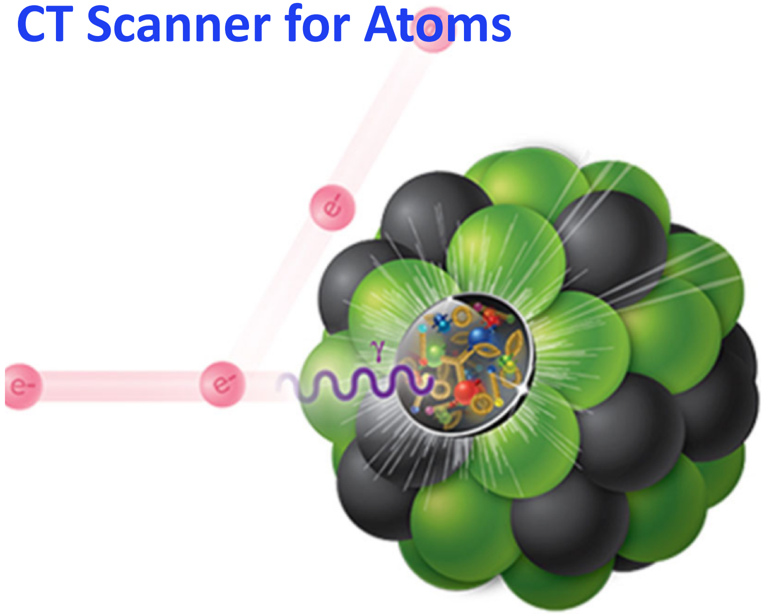
U.S. - based Electron-Ion Collider (EIC)

https://www.bnl.gov/eic/

A machine that will unlock the secrets of the strongest force in Nature

Like a CT Scanner for Atoms



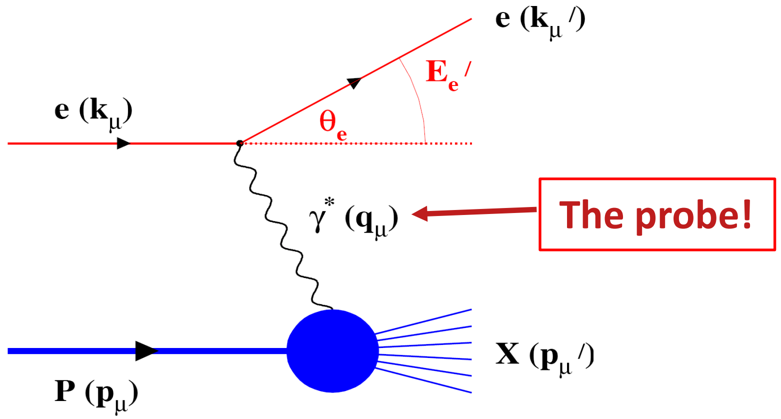


Basic Tech Requirements

- **Center of Mass Energies:**
  20 GeV – 141 GeV
- **Required Luminosity:**
  10^{33} - 10^{34} $cm^{-2}s^{-1}$
- **Hadron Beam Polarization:**
  80%
- **Electron Beam Polarization:**
  80%
- **Ion Species Range:**
  p to Uranium
- **Number of interaction regions:**
  up to two

Lepton-hadron inclusive deep inelastic scattering (DIS)

❑ Approximation of one-photon exchange:



Q^2 = - (k-k')^2 → Measure of the resolution

y = P.(k-k')/P.k → Measure of inelasticity

x_B = Q^2/2P.(k-k') → Measure of momentum fraction of the struck quark in a proton

$Q^2 = S\, x_B\, y$

$$E'\frac{d\sigma}{d^3k'} = \frac{2\alpha_{\rm EM}^2}{s}\frac{1}{Q^4}L^{\mu\nu}(k,k;q)W_{\mu\nu}(q,P)$$

$$L^{\mu\nu}(k,k;q) = 2(k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k' g^{\mu\nu}) + \text{spin...}$$

❑ Deep inelastic scattering (DIS) structure functions:

$$W_{\mu\nu}(q,P) = \frac{1}{4\pi}\sum_X (2\pi)^4\delta^4(P+q-X)\,\langle P|J_\mu(0)|X\rangle\,\langle X|J_\nu(0)|P\rangle + \text{spin...}$$

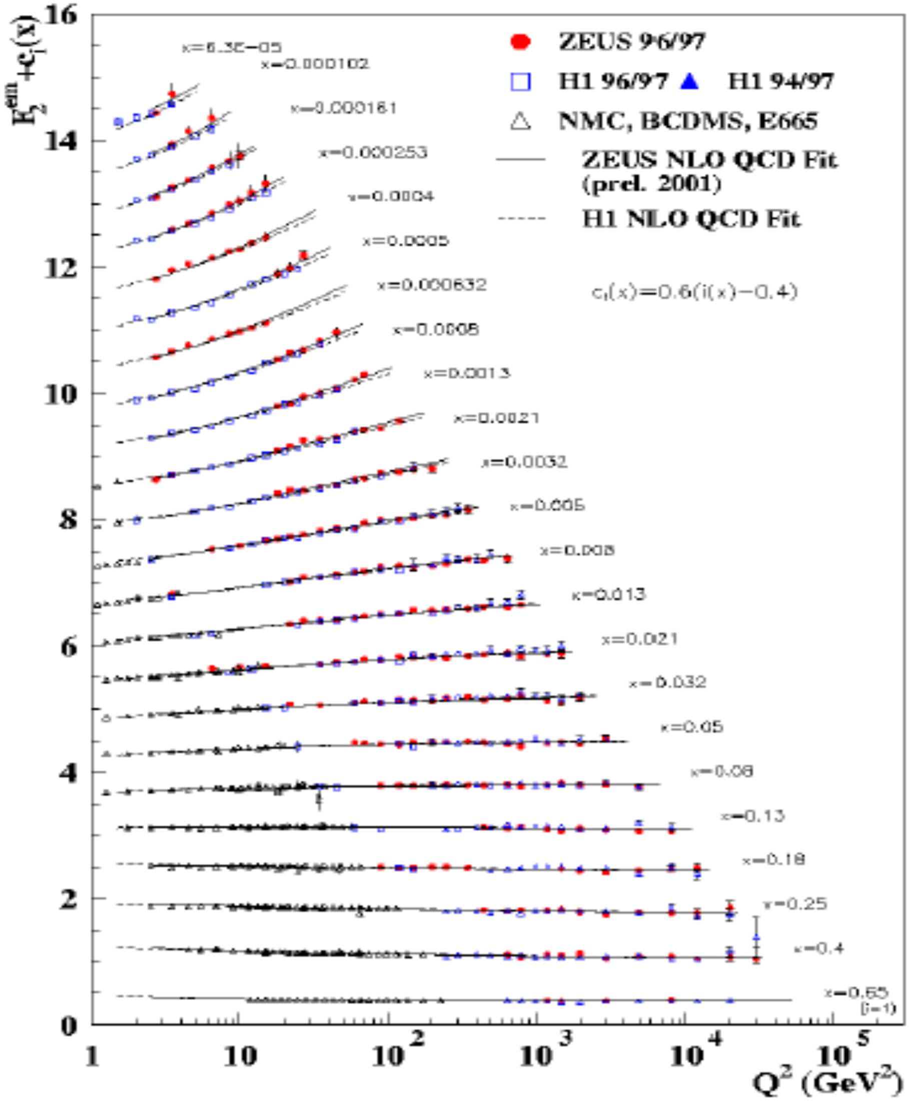
$$= -\widetilde{g}_{\mu\nu}F_1(x_B,Q^2) + \frac{\widetilde{P}_\mu\widetilde{P}_\nu}{P\cdot q}F_2(x_B,Q^2) + \text{spin...}$$

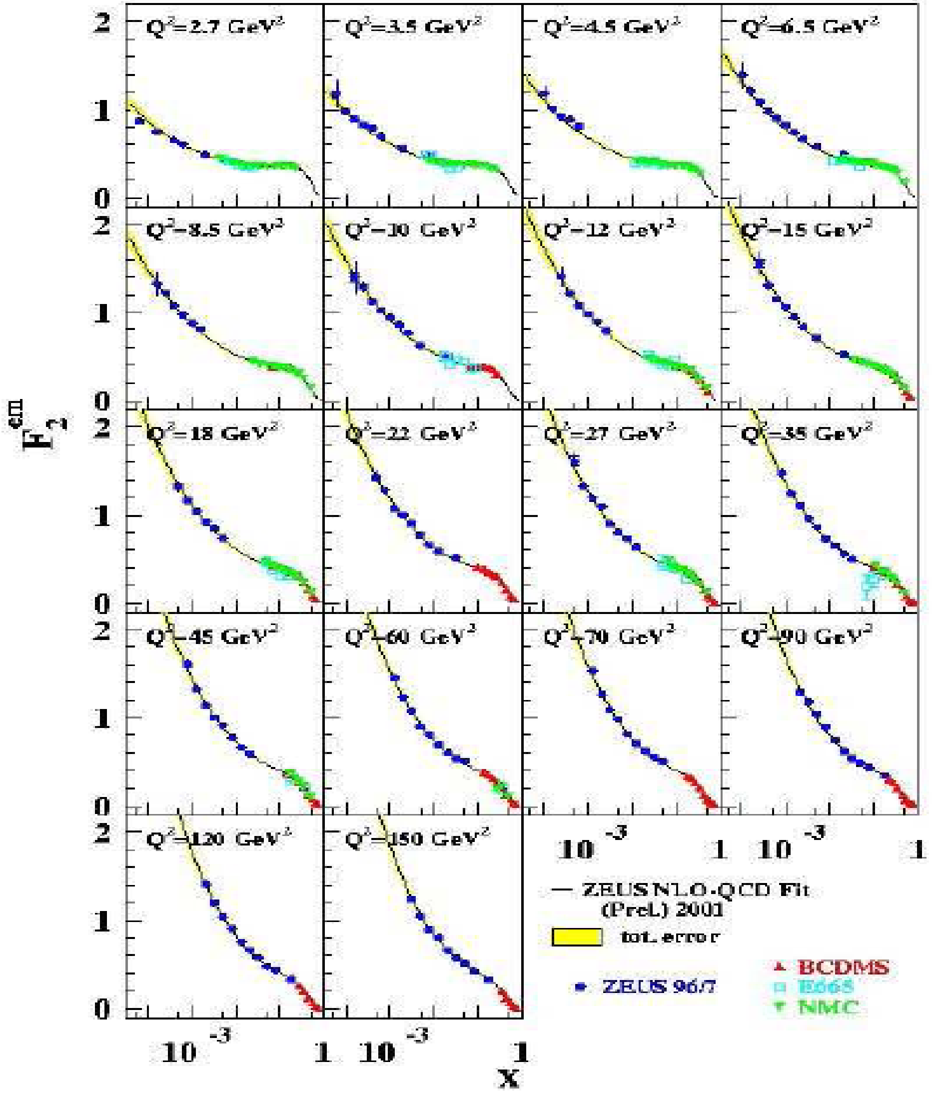
$$\widetilde{g}_{\mu\nu} = -g_{\mu\nu} + q_\mu q_\nu/q^2 \qquad \widetilde{P}_\mu = \widetilde{g}_{\mu\nu}P^\nu$$

QCD Factorization - Approximation

$$F_i(x_B,Q^2) \approx \sum_f C_{if}(x_B,Q^2;x,\mu^2)\otimes f(x,\mu^2) + \mathcal{O}(1/Q^2)$$

Lepton-hadron inclusive deep inelastic scattering (DIS)





A very successful story of QCD, QCD Factorization, and QCD evolution!

But, this success depends on how precise we were able to treat collision-induced QED radiation!

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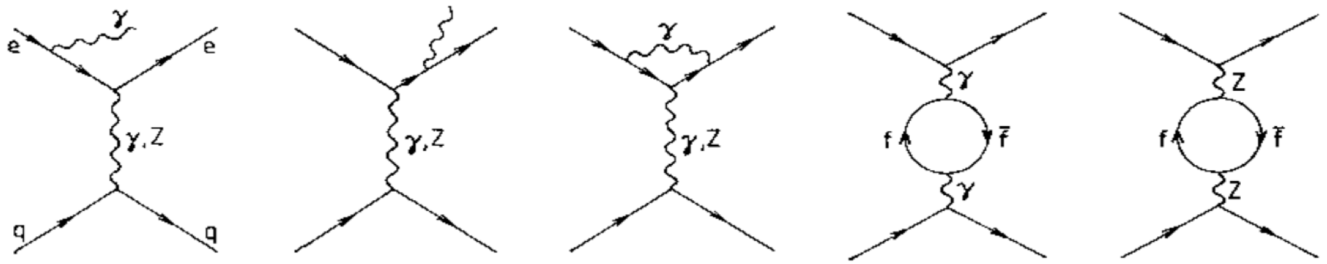
Collision with a large momentum transfer induces strong QED radiation

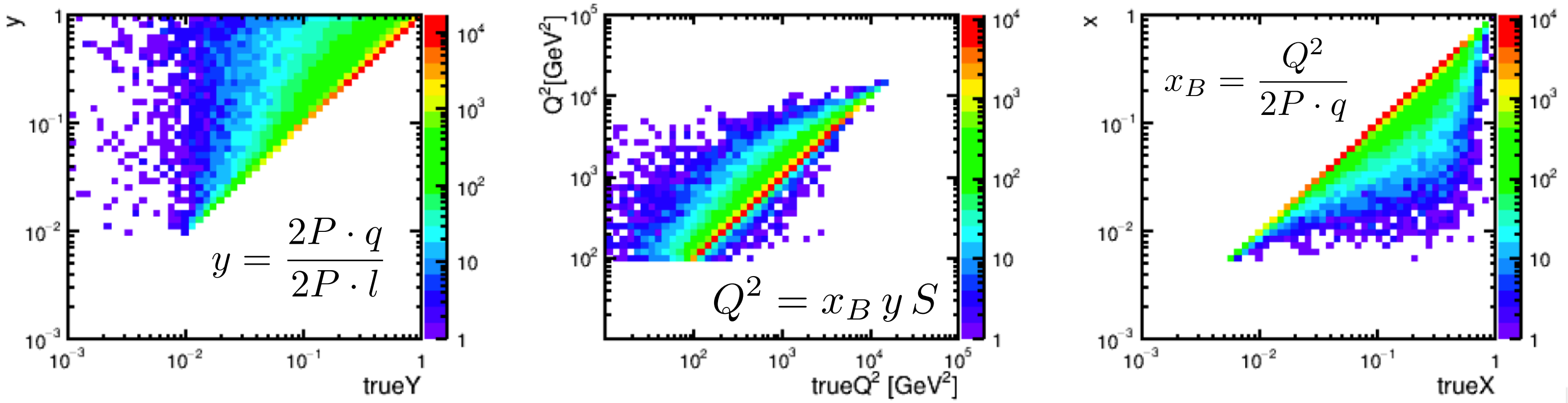
❑ The “Probe” for the hadron is smeared by the induced QED radiation:

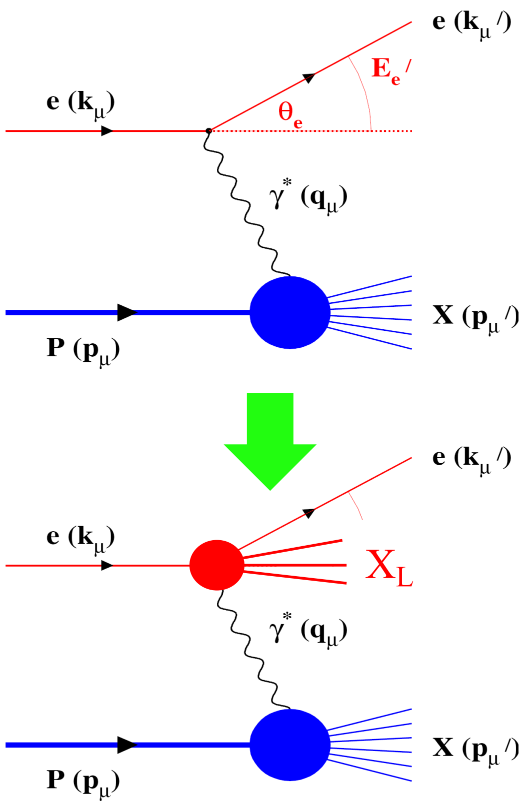
Data sample : Int L = 10 fb^{-1}, Kinematics settings: 0.01<y<0.95, 10^2 GeV^2 <Q^2<10^5 GeV^2



initial **final** **vacuum** **loops**

See Xiaoxuan Chu @2nd EIC YR workshop





Instead of a straight line – linear correlation,

the kinematic variables, y, Q^2 , x$_B$, from the leptons are smeared so much to make them different from what the scattered “quark” experienced!

$$q_\mu \rightarrow \hat{q}_\mu$$

$$Q^2 = -q^2 \rightarrow \widehat{Q}^2 = -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \rightarrow \hat{x}_B = \frac{\widehat{Q}^2}{2P\cdot \hat{q}}$$

Ill-defined “photon-hadron” frame?!

QED Radiative Corrections

See B. Badelek et al.
Z Phys C 66 (1995) 591

- Radiative correction factor is too big for comfort:

$$\eta(x,y) = \frac{\sigma_{1\gamma}}{\sigma_{\text{meas}}}$$

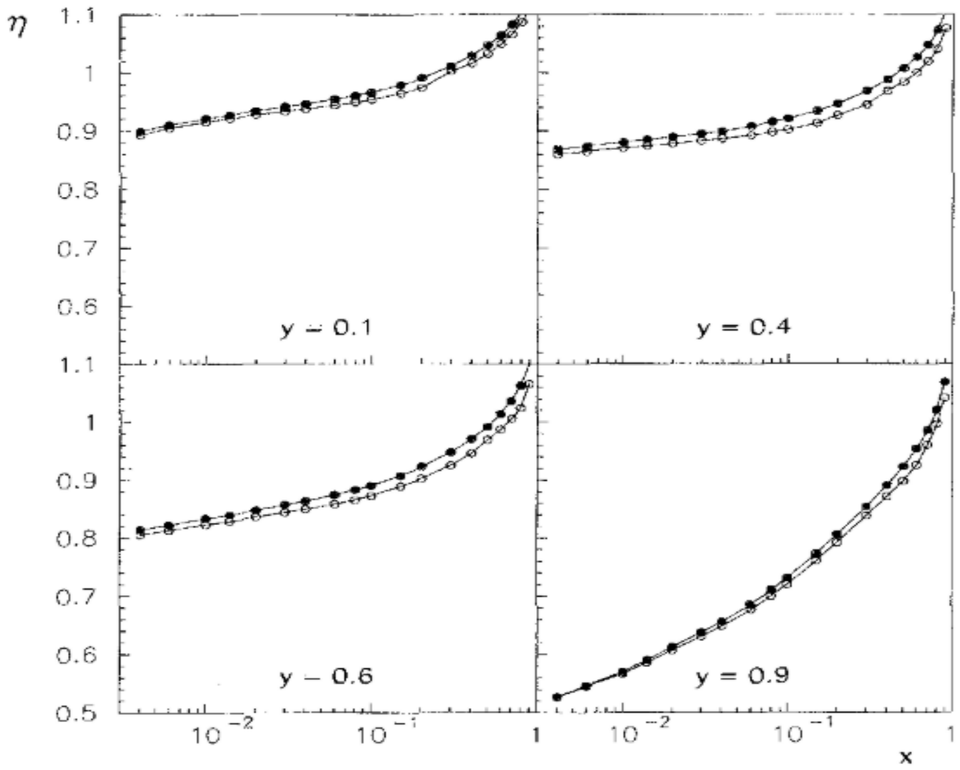


Fig. 5. Radiative correction factor η calculated in FERRAD35 (open symbols) and TERAD86 (closed symbols) for the muon – proton scattering at 280 GeV

Larger phase-space for shower – smaller x_B

Larger momentum transfer – larger y (or Q)

Larger radiative corrections!

Fits to EIC kinematics?!

Radiative corrections are very large, exceeding 50% at low x and high y region!

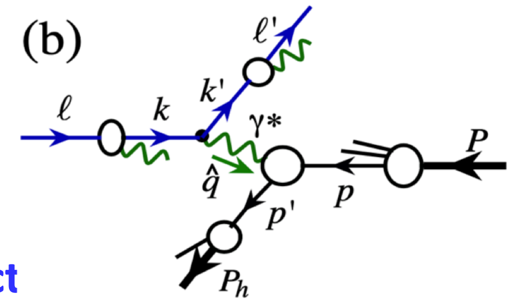
Recall: $y = \frac{2P \cdot q}{2P \cdot l}$ $x_B = \frac{Q^2}{2P \cdot q}$ $Q^2 = x_B\, y\, S$

No simple radiative correction for SIDIS

Radiative correction = keeping the Born kinematics:

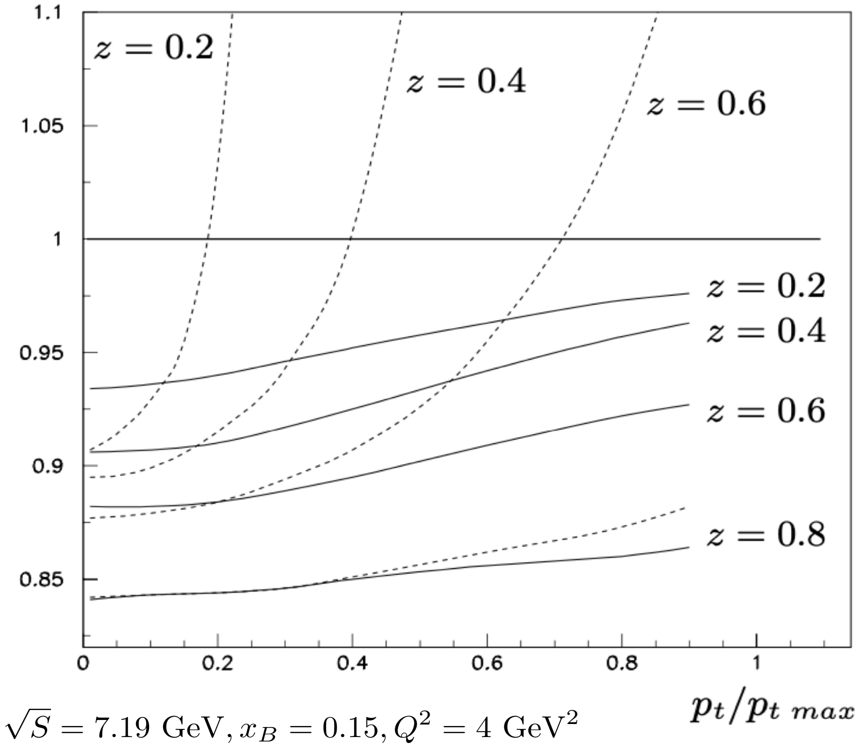
$$\sigma_{\text{Measured}} \equiv \text{RC} \otimes \sigma_{\text{No QED Radiation}}$$

Necessary requirement: **RC – Radiative correction factor does not depend on the hadronic physics that we want to extract**



Impact of QED radiation to SIDIS – order of α_{EM}:

$$\bar{\delta} = \sigma_{\text{Measured}} / \sigma_{\text{No QED Radiation}}$$



$\sqrt{S} = 7.19$ GeV, $x_B = 0.15$, $Q^2 = 4$ GeV^2

$$e(l) + N(P) \rightarrow e'(l') + \gamma(k) + h(P_h) + X$$

I. Akushevich et al. EPJ C10 (1999) 681

Dashed line:

Gaussian pT-dependence

$$b\,\exp(-b\,p_t^2) \qquad \text{where} \quad b = R^2/z^2$$

Solid line:

Power pT-dependence

$$\left[\frac{1}{a + b\,z + p_t^2}\right]^{c+d\,z} \qquad \text{parameters:} \quad R, a, b, c, d$$

$\bar{\delta}$ **depends on physics we want to extract!**

NO simple RC for SIDIS!

QED radiative corrections vs. QED radiative contributions

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

❑ QED radiative corrections – historical approach:

$$\sigma_{\rm obs}(x_B, Q^2) \overset{?}{=} R_{\rm QED}(x_B, Q^2; x_{B,{\rm true}}, Q^2_{\rm true}) \times \sigma_{\rm Born}(x_{B,{\rm true}}, Q^2_{\rm true}) + \sigma_X(x_B, Q^2)$$

- The correction factors R_{QED} and σ_x should not depend on the hadron structure that we wish to extract, and they can be systematically calculated in QED to high precision (not satisfied);
- The effective scale Q^2_{true} for the Born cross section σ_{Born} should be large enough to keep the "true" scattering within the DIS regime (questionable);
- Extraction of $\sigma_{\rm Born}$ is an inverse problem

❑ QED radiative contributions – our proposed solution:

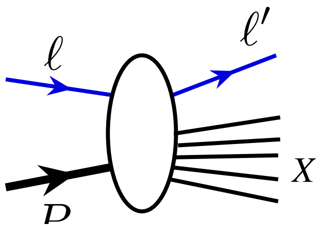
$$\sigma_{\rm obs}(x_B, Q^2) = \sigma^{\rm univ}_{\rm lep}(\mu^2; m_e^2) \otimes \sigma^{\rm univ}_{\rm had}(\mu^2; \Lambda^2_{\rm QCD}) \otimes \widehat{\sigma}_{\rm IR\text{-}safe}(\hat{x}_B, \widehat{Q}^2, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}, \frac{m_e^2}{Q^2}\right)$$

- Infrared sensitive QED contributions – divergent as $m_e/Q \to 0$, are absorbed to universal LDFs and LFFs
- Infrared safe QED contributions – finite as $m_e/Q \to 0$, are calculated order-by-order in power of α
- Power suppressed contributions as $m_e/Q \to 0$, are neglected

Predictive power: Universality of LDFs and LFFs, their evolution, calculable hard parts
Neglect power corrections

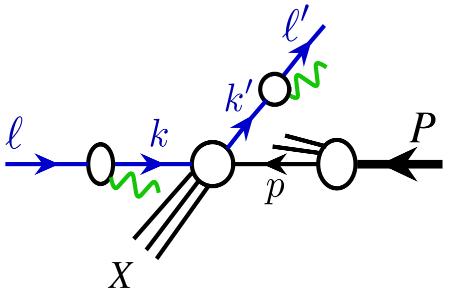
Inclusive lepton-hadron deep inelastic scattering (DIS)

❑ Inclusive production of single high p_T lepton in lepton-hadron collision:



$$e(\ell,\lambda_\ell) + N(P,S) \to e(\ell') + X$$

$$d\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X} = \frac{1}{2s}\left|M_{\ell(\lambda_\ell)P(S)\to\ell' X}\right|^2 d\text{PS}$$



$$E'\frac{d\sigma_{\ell P\to\ell' X}}{d^3\ell'} \approx \frac{1}{2s}\sum_{ija}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$

$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi\ell,xP,\ell/\zeta,\mu^2) + \cdots$$

Collinear QED & QCD factorization

Lepton distribution functions (LDFs): $f_{i/e}(\xi,\mu^2)$

Lepton fragmentation functions (LFFs): $D_{e/j}(\zeta,\mu^2)$ $\quad i.j = e,\gamma,\bar{e},...,q,g,...$

Parton distribution functions (PDFs): $f_{a/N}(x,\mu^2)$ $\quad a = q,g,\bar{q},e,\gamma,\bar{e},...$

Short-distance hard coefficients: $\widehat{H}_{ia\to jX}(\xi\ell,xP,\ell/\zeta,\mu^2)$

$$\approx \widehat{H}^{(m,n)}_{ia\to jX}(\xi\ell,xP,\ell/\zeta,\mu^2) \approx \mathcal{O}(\alpha^m\alpha_s^n)$$

Photon is charge neutral
QED factorization works

- **No DIS "Structure Functions"!**

  Concept of one-photon exchange

- **QED & QCD contribution are factorized at the same scale: μ**

  $$(x_B,Q^2) \;\to\; (y,\ell'_T)$$

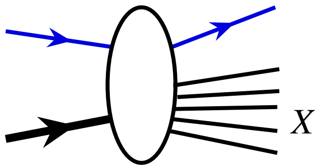
- **Corrections suppressed by power**

  $$(1/\ell'_T)^\alpha$$

Nayak, Qiu, Sterman, Phys.Rev.D 72 (2005) 114012

Inclusive lepton-hadron deep inelastic scattering (DIS)

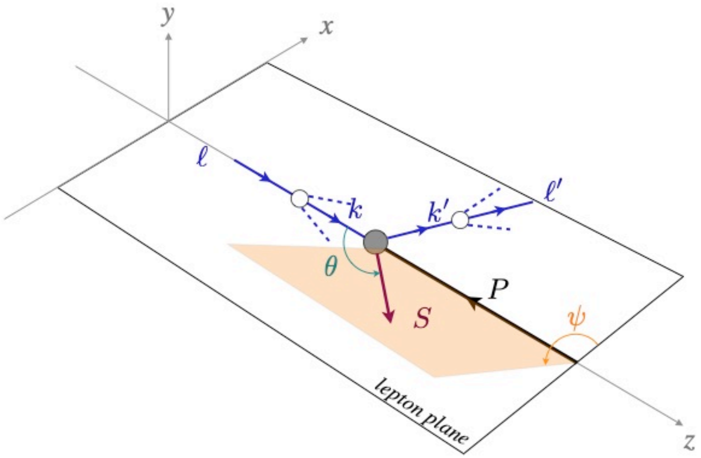
- **Inclusive production of single high p_T lepton in lepton-hadron collision:**



$$e(\ell,\lambda_\ell)+N(P,S)\to e(\ell')+X$$

$$d\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X}=\frac{1}{2s}\left|M_{\ell(\lambda_\ell)P(S)\to\ell' X}\right|^2 d\mathrm{PS}$$

- **Recover the concept of structure functions – one-photon approximation?**



$$E_{\ell'}\frac{d^3\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X}}{d^3\ell'}\approx\sum_{\lambda_k}\int_{\zeta_{\min}}^1\frac{d\zeta}{\zeta^2}D_{e/e}(\zeta,\mu^2)\int_{\xi_{\min}}^1 d\xi\, f_{e(\lambda_k)/e(\lambda_\ell)}(\xi,\mu^2)$$
$$\times\left[E_{k'}\frac{d^3\hat\sigma_{k(\lambda_k)P(S)\to k' X}}{d^3k'}\right]_{k=\xi\ell,\,k'=\ell'/\zeta},$$

$$E_{k'}\frac{d^3\hat\sigma_{k(\lambda_k)P(S)\to k' X}}{d^3k'}\approx\frac{2\alpha^2}{\hat s\,\widehat{Q}^4}L^{(0)}_{\mu\nu}(k,k',\lambda_k)\,W^{\mu\nu}(\hat q,P,S)$$

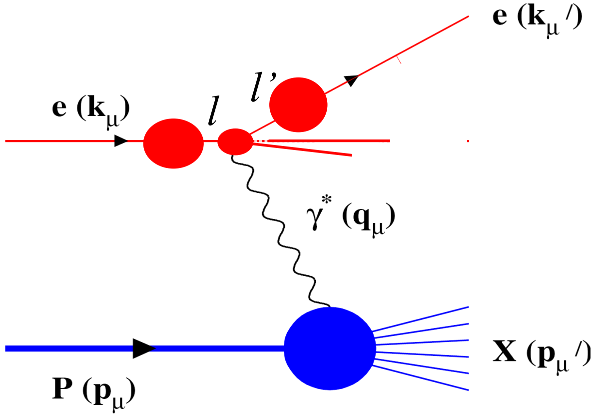
$$W^{\mu\nu}(\hat q,P,S)=-\widetilde{g}^{\mu\nu}(\hat q)\,F_1(\hat x_B,\widehat{Q}^2)+\frac{1}{P\cdot\hat q}\widetilde{P}^{\mu}(\hat q)\widetilde{P}^{\nu}(\hat q)F_2(\hat x_B,\widehat{Q}^2)+\ldots$$

Structure functions are evaluated at $(\hat x_B,\widehat{Q}^2)$ instead of (x_B,Q^2)!

Inclusive lepton-hadron deep inelastic scattering (DIS)

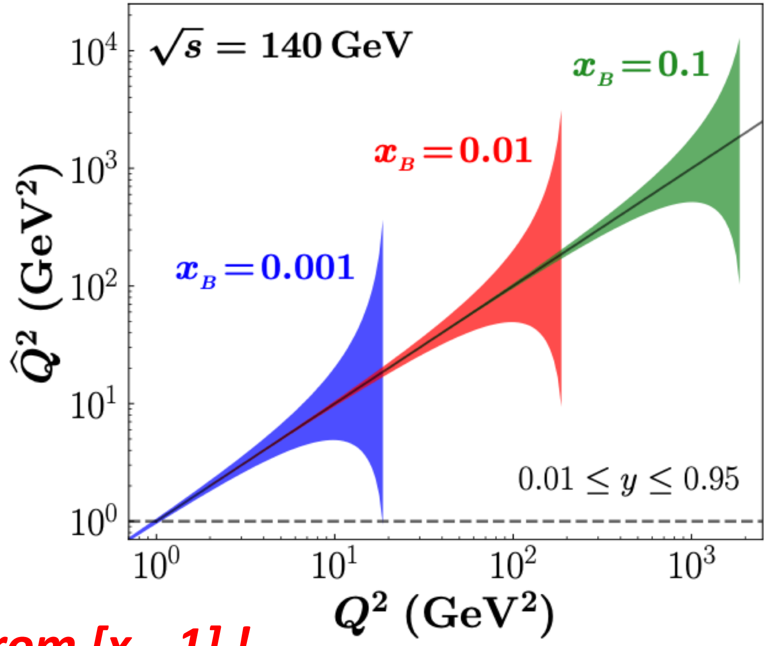
- Collinear factorization with the "one-photon" approximation:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B dy} \approx \int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1} d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2)\left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right]$$
$$\times\frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2F_1(\hat{x}_B,\widehat{Q}^2)+\left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

- QED radiation prevents a well-defined "photon-hadron" frame
- Radiation is CO sensitive as $m_e/Q \to 0$, factorized into LDFs & LFFs
- Hadron is probed by $(x_B, Q^2) \to (\hat{x}_B, \widehat{Q}^2)$



$x_B \to \hat{x}_B \in [x_B, 1]$ $\qquad$ $\widehat{Q}^2_{\min} = Q^2\,\frac{(1-y)}{(1-x_B\,y)}$ $\qquad$ $\widehat{Q}^2_{\max} = Q^2\,\frac{1}{(1-y+x_B\,y)}$

A simple RC factor at x_B is necessarily sensitive to hadronic information from [x_B, 1] !

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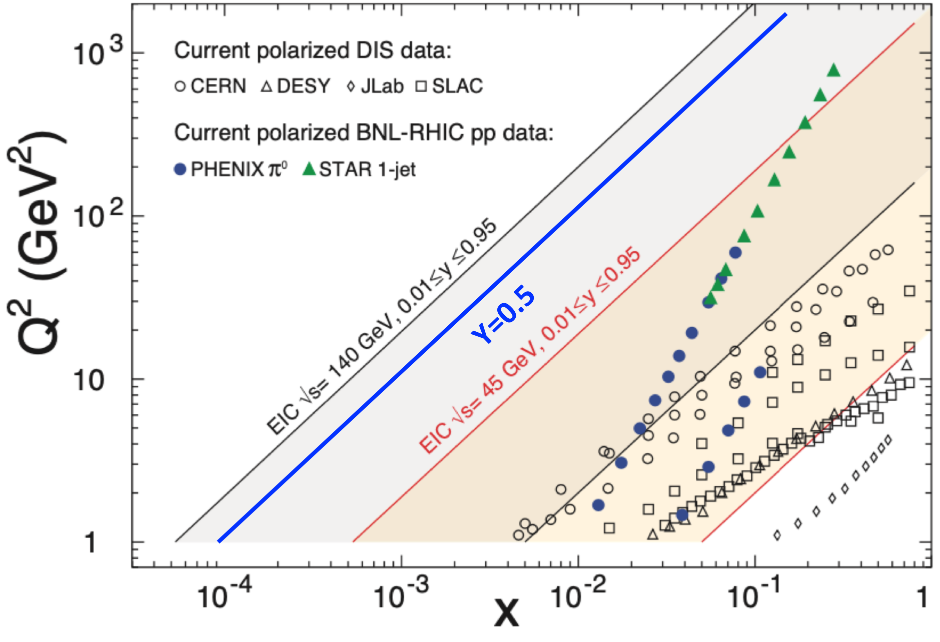
Inclusive lepton-hadron deep inelastic scattering (DIS)

- **Numerical impact of QED contribution at EIC ($\sqrt{S} = 140$ GeV):**

$$\frac{\sigma_{\rm noRC}}{\sigma_{\rm RC}} \leftrightarrow \frac{\sigma_{1\gamma}}{\sigma_{\rm measured}} = \eta(x_B, y)$$

B. Badelek et al.
Z Phys C 66 (1995) 591



At $\sqrt{S} = 140$ GeV

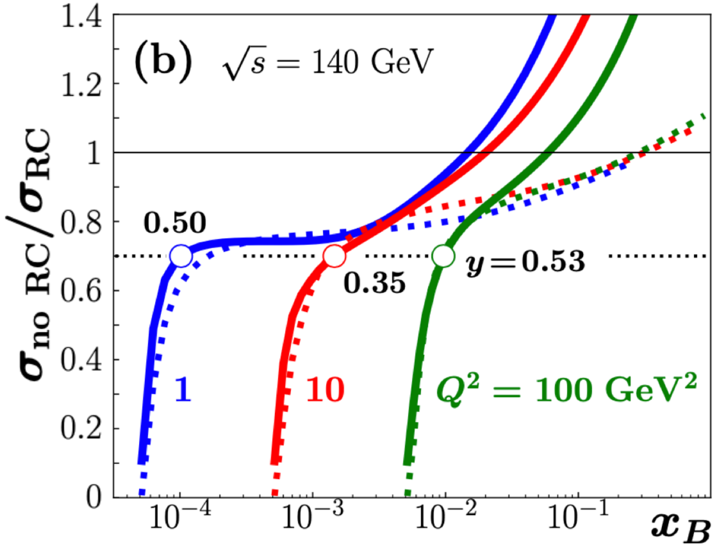
$Q^2 = 1$ GeV^2

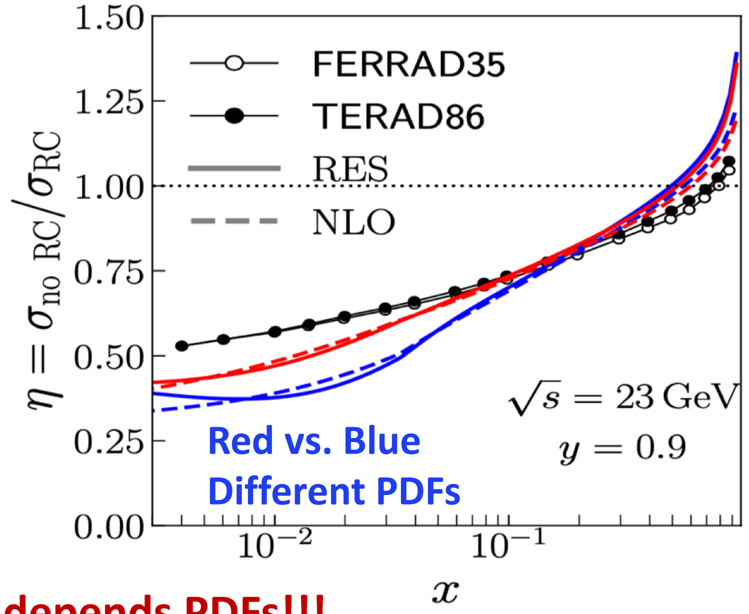
$y = 0.95$

EIC eP could reach:

$x_{\rm min} \sim 5 \times 10^{-5}$

$Q^2 = x_B\, y\, S$





If we do not have confidence for y > 0.5, due to QED radiation,

EIC's eP reach to small-x could be reduced to $x_{\rm min} \sim 1 \times 10^{-4}$

or effectively, $\sqrt{S} = 140$ GeV $\rightarrow 102$ GeV at $y = 0.95$

"RC" depends PDFs!!!

Inclusive lepton-hadron deep inelastic scattering (DIS)

❑ Numerical impact of QED contribution at EIC ($\sqrt{S} = 140$ GeV):

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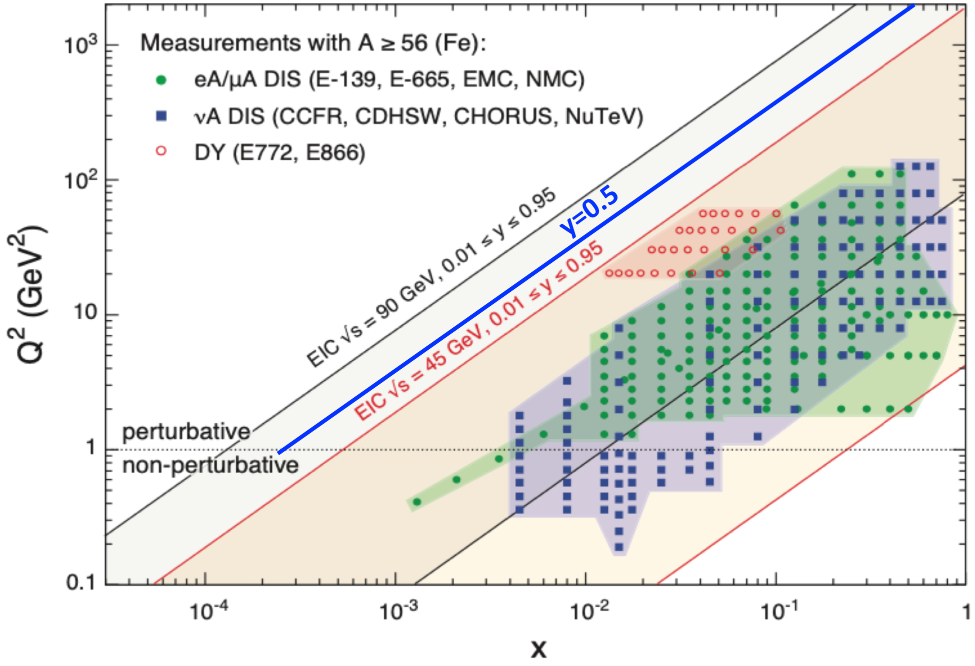
B. Badelek et al.
Z Phys C 66 (1995) 591



At $\sqrt{S} = 100$ GeV

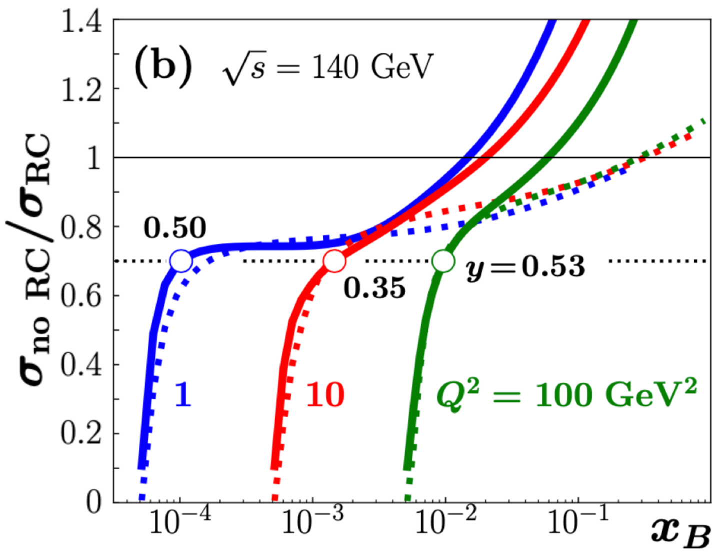
$Q^2 = 1$ GeV^2

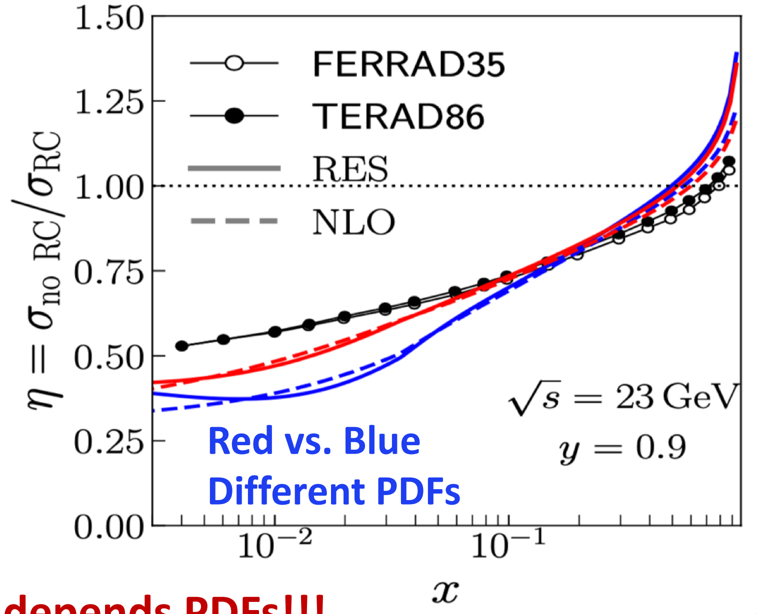
$y = 0.95$

EIC eA could reach:

$x_{\rm min} \sim 1 \times 10^{-4}$

$Q^2 = x_B\, y\, S$





If we do not have confidence for y > 0.5, due to QED radiation,

EIC's eA reach to small-x could be reduced to $x_{\rm min} \sim 2 \times 10^{-4}$

or effectively, $\sqrt{S} = 100$ GeV $\rightarrow 73$ GeV at $y = 0.95$

"RC" depends PDFs!!!

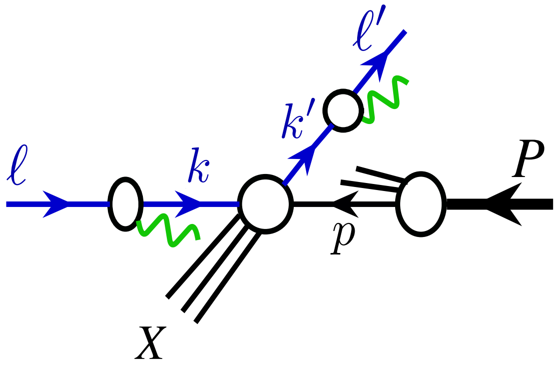
Jefferson Lab

Inclusive lepton-hadron deep inelastic scattering (DIS)

- **Without the "one-photon" approximation:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

~ Inclusive single lepton production at high transverse momentum

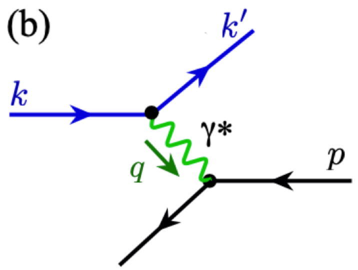


$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2) + \cdots$$

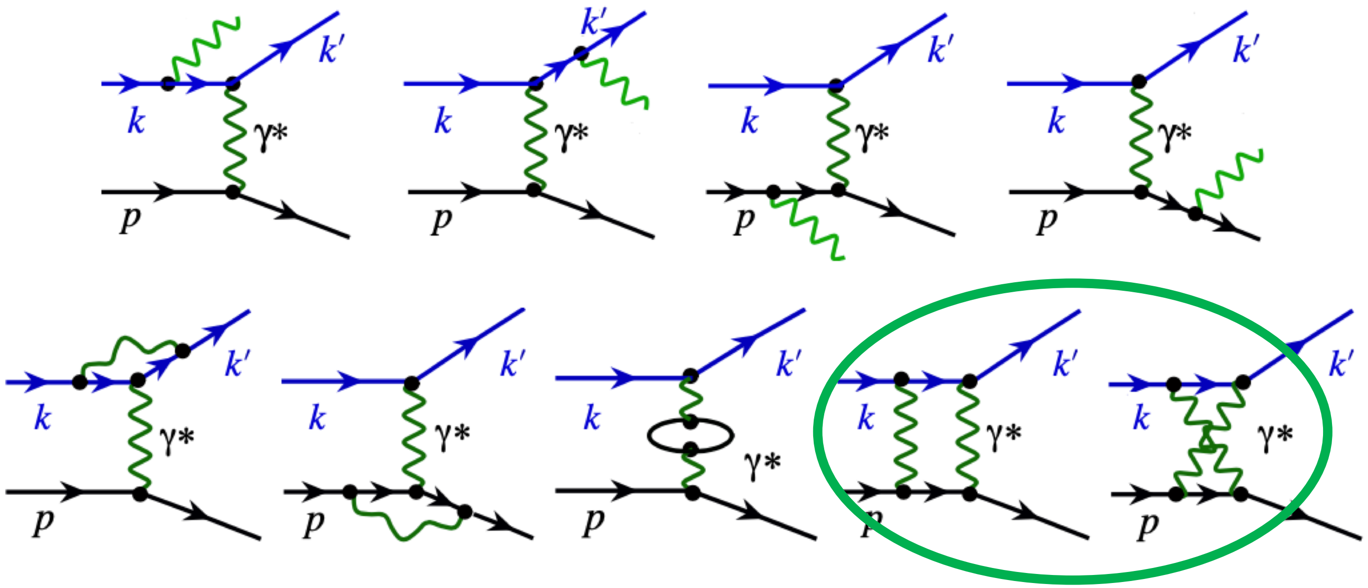
No structure functions, but have PDFs, LDFs, LFFs, …

- **Calculated hard parts in power of** $\alpha^m\alpha_s^n$ **:**

LO



NLO:



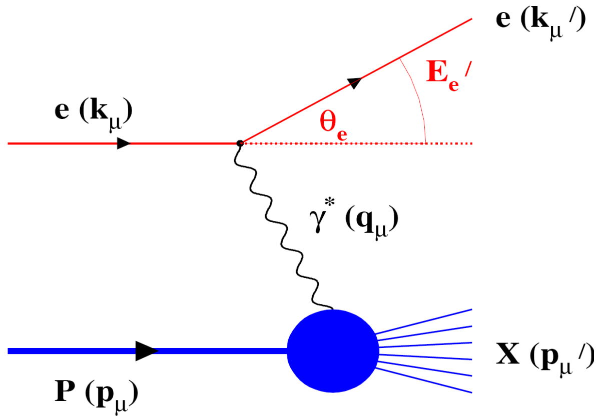
More systematic for PVDIS!

Beyond one-photon exchange

Jefferson Lab

Single hadron (or jet) "photo"-production in lepton-hadron collision

❑ Photoproduction in ep collision is sensitive to how the "photon" is defined:



- Real or quasi-photon is defined by

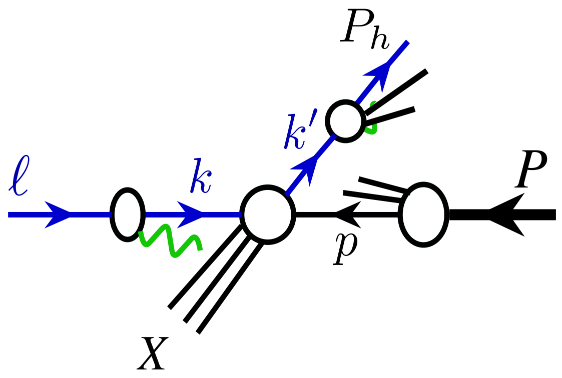
$$k'_T \le k_{T_{\rm cut}} \quad \text{or} \quad \theta_e \le \theta_{\rm cut}$$

- Photon flux is derived by

  Evaluating the photon shower with above "cut"
  Weizsaecker-Williams photon distribution, …

Kang, Meta, Qiu, Zhou, PRD 2011
Hinderer, Schlegel, Vogelsang, PRD 2015, 2016
Abelof, Boughezal, Liu, Petriello, PLB, 2016
Qiu, Wang, Xing, CPL, 2021
Qiu, Watanabe, in preparation

❑ Inclusive single hadron (jet) production in ep collision:



***Without** measuring the scattered electron!*

Single hard scale, collinear factorization

$$E_h \frac{d\sigma_{\ell P\to P_h x}}{d^3P_h} = \frac{1}{2s}\sum_{i,a,b}\int_{z_{\rm min}}^1 \frac{dz}{z^2}\int_{\xi_{\rm min}}^1 \frac{d\xi}{\xi} D_{h/b}(z,\mu^2) f_{i/e}(\xi,\mu^2) \times \int_{x_{\rm min}}^1 \frac{dx}{x} f_{a/N}(x,\mu^2)\, \widehat{H}_{ia\to bX}(\xi\ell, xP, P_h/z, \mu^2) + \ldots$$

- Universal lepton distribution functions (LDFs)
- No artificial cut to define the "photon"
- Single factorization scale: μ

Jefferson Lab

Single hadron (or jet) "photo"-production in lepton-hadron collision

❑ Evolution of lepton distribution functions (LDFs):

D. Florian, et al. EPJ C, 2016
Qiu, Watanabe, in preparation

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix} = \left(\begin{array}{ccc|ccc} P_{ee}^{(1,0)} & P_{e\bar{e}}^{(2,0)} & P_{e\gamma}^{(1,0)} & P_{eq}^{(2,0)} & P_{e\bar{q}}^{(2,0)} & P_{eg}^{(2,1)} \\ P_{\bar{e}e}^{(2,0)} & P_{\bar{e}\bar{e}}^{(1,0)} & P_{\bar{e}\gamma}^{(1,0)} & P_{\bar{e}q}^{(2,0)} & P_{\bar{e}\bar{q}}^{(2,0)} & P_{\bar{e}g}^{(2,1)} \\ P_{\gamma e}^{(1,0)} & P_{\gamma\bar{e}}^{(1,0)} & P_{\gamma\gamma}^{(1,0)} & P_{\gamma q}^{(1,0)} & P_{\gamma\bar{q}}^{(1,0)} & P_{\gamma g}^{(1,1)} \\ \hline P_{qe}^{(2,0)} & P_{q\bar{e}}^{(2,0)} & P_{q\gamma}^{(1,0)} & P_{qq}^{(0,1)} & P_{q\bar{q}}^{(0,2)} & P_{qg}^{(0,1)} \\ P_{\bar{q}e}^{(2,0)} & P_{\bar{q}\bar{e}}^{(2,0)} & P_{\bar{q}\gamma}^{(1,0)} & P_{\bar{q}q}^{(0,2)} & P_{\bar{q}\bar{q}}^{(0,1)} & P_{\bar{q}g}^{(0,1)} \\ P_{ge}^{(2,1)} & P_{g\bar{e}}^{(2,1)} & P_{g\gamma}^{(1,1)} & P_{gq}^{(0,1)} & P_{g\bar{q}}^{(0,1)} & P_{gg}^{(0,1)} \end{array}\right) \otimes \begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix}$$

- **Factorization scale:**

  $\mu^2 \sim m_c^2$

- **Input LDFs at μ^2:**
  - **Perturbatively generated by solving QED evolution from lepton mass threshold**
  - **With perturbatively calculated fixed-order MSbar LDFs**
  - **Test the size of non-perturbative hadronic contribution**
  - **…**

Evolution kernels in both QCD and QED:

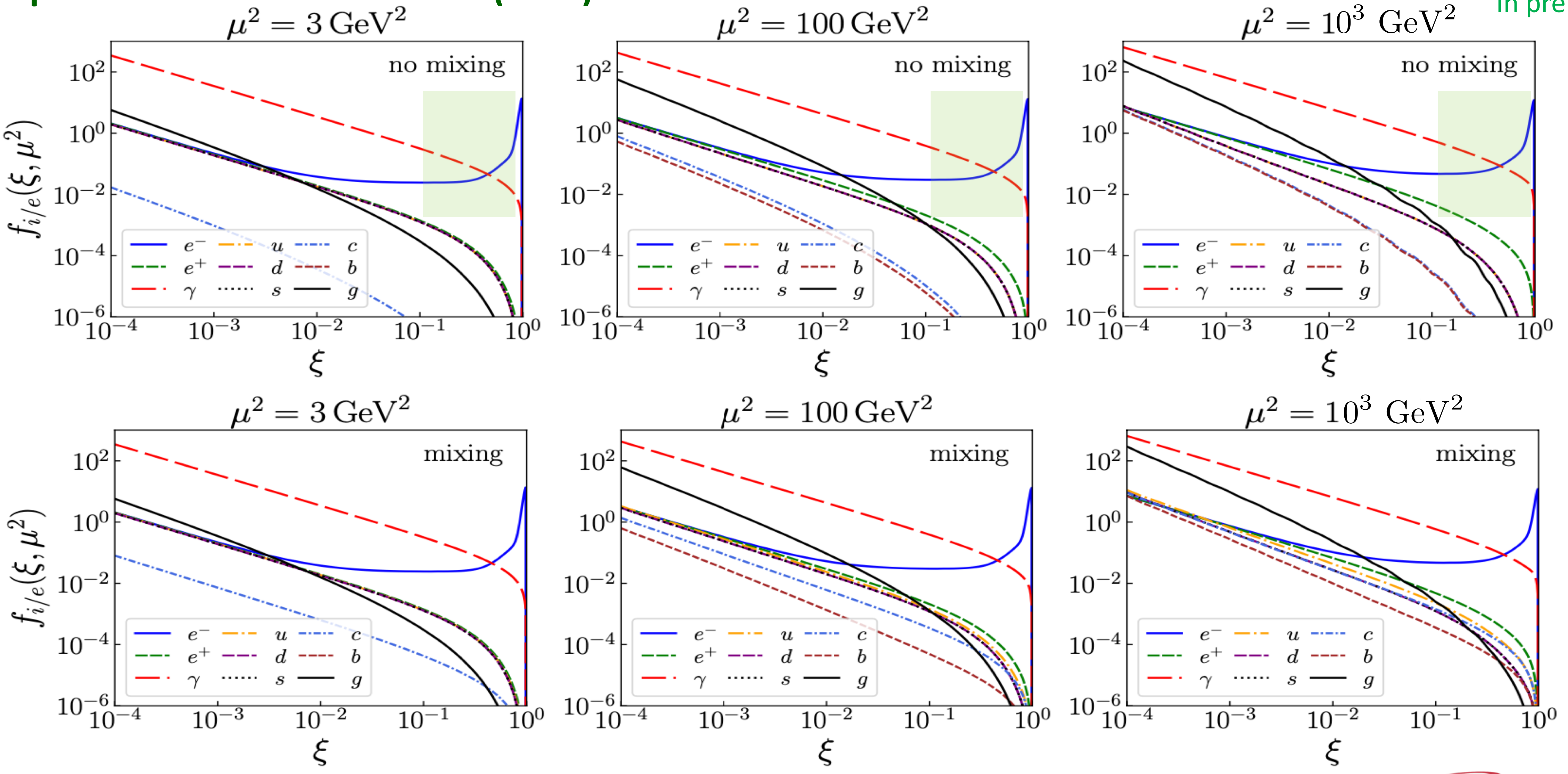
$$P_{ij}(\xi,\mu^2) = \sum_{n,m=0}^{\infty}\left(\frac{\alpha_{em}(\mu^2)}{2\pi}\right)^n\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^m \hat{P}_{ij}^{(n,m)}(\xi) = \sum_{n,m=0}^{\infty} P_{ij}^{(n,m)}(\xi,\mu^2)$$

with $P_{ij}^{(0,0)} = 0\,,\quad N_F\,,\quad N_l$

Single hadron (or jet) "photo"-production in lepton-hadron collision

❑ Lepton distribution functions (LDFs):

Qiu, Watanabe
In preparation

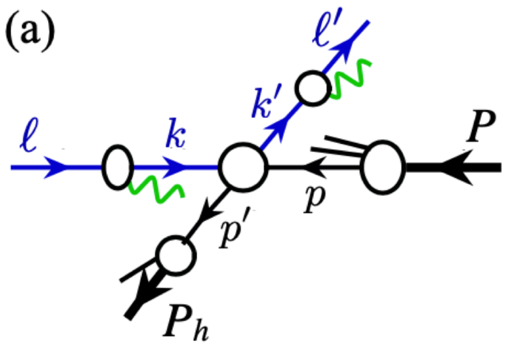


With LDFs, we calculated single hadron production, including J/ψ production at the EIC

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

- **Inclusive production of a lepton and a hadron:**



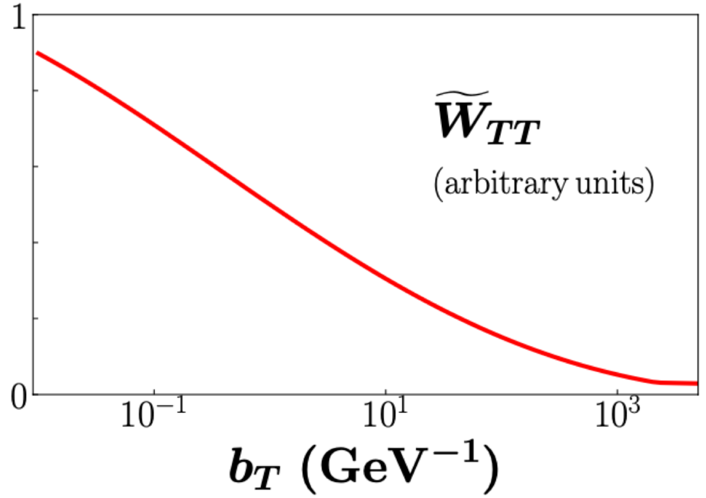
$$e(\ell) + N(P) \to e(\ell') + h(P_h) + X$$

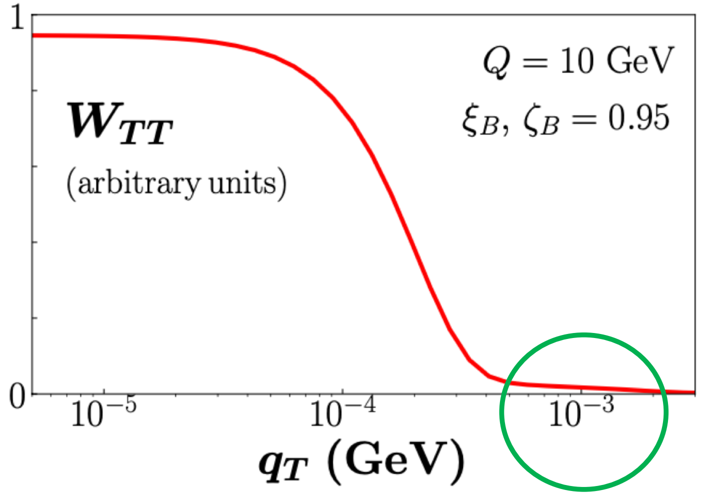
Momentum imbalance between the lepton and the hadron could be sensitive to both parton TMDs and lepton TMDs

Typical parton transverse momentum: $k_T^2 \sim \Lambda_{\rm QCD}^2 + \langle k_T^2 \rangle_{\rm generated\ by\ QCD\ shower}$

- **Estimate of lepton transverse momentum generated by QED shower:**

Resummation to lepton TMD





TMDQED broadening for lepton is much smaller than typical parton kT!

Collinear factorization for high order QED contributions

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

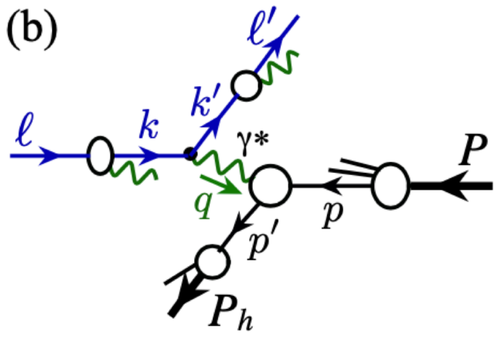
- **QED factorization of collision-induced radiation – collinear:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$E_{\ell'} E_{P_h} \frac{d^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{d^3\ell'\, d^3P_h} \approx \sum_{ij\lambda_k} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} D_{e/j}(\zeta) \int_{\xi_{\min}}^1 d\xi\, f_{i(\lambda_k)/e(\lambda_\ell)}(\xi) \left[E_{k'} E_{P_h} \frac{d^6\hat{\sigma}_{k(\lambda_k)P(S)\to k' P_h X}}{d^3k'\, d^3P_h} \right]_{k=\xi\ell, k'=\ell'/\zeta} + \mathcal{O}\left(\frac{m_e^n}{Q^n}\right)$$

- Leading power IR sensitive contribution is universal, as $m_e/Q \to 0$, factorized into LDFs and LFFs
- IR safe contributions are calculated order-by-order in powers of α
- Neglect m_e/Q power suppressed contributions
- Collinear QED factorization for both inclusive DIS and SIDIS, or e^+e^-, … [global fits of LDFs, LFFs]

- **"One photon"-approximation** ⇒ Hybrid factorization: CO for QED and TMD for QCD!



$$\frac{d^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{dx_B dy\, d\psi\, dz_h\, d\phi_h dP_{hT}^2} = \sum_{ij\lambda_k} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^1 \frac{d\xi}{\xi} f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\, D_{e/j}(\zeta)$$

$$\times \frac{\hat{x}_B}{x_B \xi \zeta} \left[\frac{\alpha^2}{\hat{x}_B\, \hat{y}\, \hat{Q}^2} \frac{\hat{y}^2}{2(1-\hat{\varepsilon})} \left(1 + \frac{\hat{\gamma}^2}{2\hat{x}_B}\right) \sum_n \hat{w}_n F_n^h(\hat{x}_B, \hat{Q}^2, \hat{z}_h, \hat{P}_{hT}^2) \right]$$

Evaluated in a "virtual photon-hadron" frame

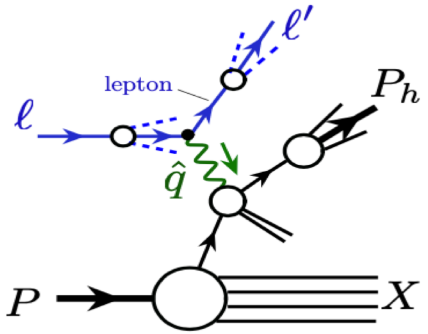
Apply a (ξ,ζ)-dependent Lorentz transformation:

$$\{\hat{q}, P, \hat{P}_h\} \xrightarrow{(\xi,\zeta)} \{q, P, P_h\}$$

In a frame to compare with exp. measurements

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

Two-step approach to SIDIS:



One-photon approximation

1) In "virtual-photon" frame, defined by $\hat{q}(\xi,\zeta) - p$

- TMD factorization when $\widehat{P}_T^2 \ll \widehat{Q}^2$
- CO factorization when $\widehat{P}_T^2 \sim \widehat{Q}^2$
- Matching to get the $\widehat{P}_T$-distribution

2) Lorentz transformation from the "virtual-photon" frame to any experimentally defined frame
– lepton-hadron Lab frame, Breit frame (x_B,Q^2), …

QED contribution (not correction) can be systematically improved order-by-order in power α!

Case study F_{UU}:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \;& \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{\boxed{F_{UU,T}} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\, F_{UU}^{\cos\phi_h} \\
& + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h\, F_{LU}^{\sin\phi_h} \\
& + S_\parallel\Big[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h}\Big] \\
& + S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h\, F_{LL}^{\cos\phi_h}\Big] \\
& + |\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) \\
& \quad + \varepsilon\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad + \sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S\, F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
& + |\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S\, F_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

- Case study F_{UU}:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$\frac{d\sigma^h_{\rm SIDIS}}{dx_B dy\, dz\, dP^2_{hT}} = \int_{\zeta_{\min}}^1 d\zeta \int_{\xi_{\min(\zeta)}}^1 d\xi\, D_{e/e}(\zeta)\, f_{e/e}(\xi) \times \left[\frac{\hat{x}_B}{x_B\,\xi\zeta}\right] \left[\frac{(2\pi)^2\,\alpha}{\hat{x}_B \hat{y}\,\widehat{Q}^2} \frac{\hat{y}^2}{2(1-\hat{\varepsilon})} F^h_{UU}(\hat{x}_B, \widehat{Q}^2, \hat{z}, \widehat{P}_{hT})\right]$$

Evaluated in a "virtual photon-hadron" frame

Unpolarized structure function:

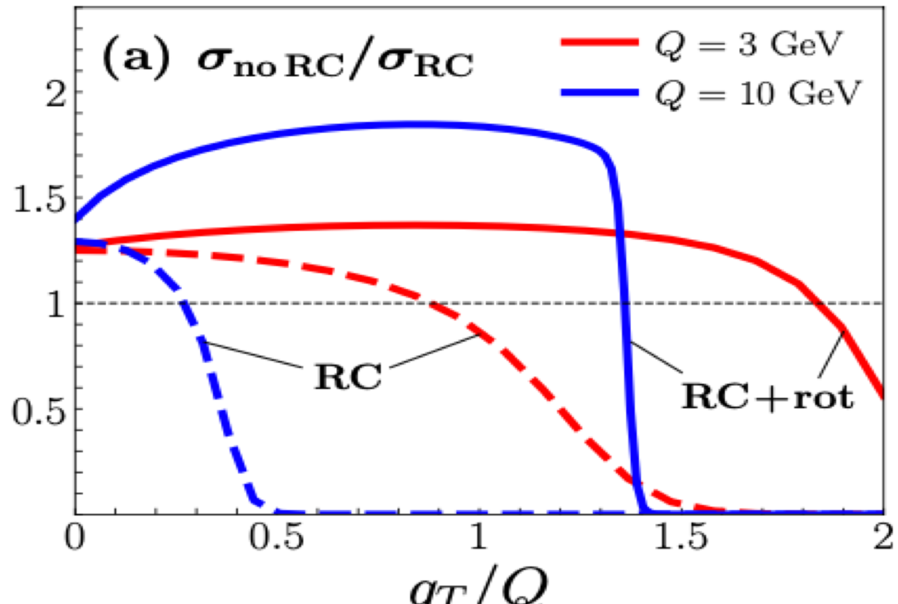
$$F^h_{UU} = x_B \sum_q e_q^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{q}_T) \times f_{q/N}(x_B, \boldsymbol{p}_T^2)\, D_{h/q}(z, \boldsymbol{k}_T^2) \qquad \boldsymbol{q}_T = \boldsymbol{P}_{hT}/z$$

(ξ,ζ) - Dependent Lorentz transformation

Effectively, a rotation in hadron-rest frame

Solid – with Lorentz transformation
Dashed – without Lorentz transformation

Impact of $q \to \hat{q}(\xi,\zeta)$!



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Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

- **Case study – single transverse spin asymmetry:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h}$$

$$+ \varepsilon\cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}$$

$$+ S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h) F_{UL}^{\sin 2\phi_h}\right]$$

$$+ S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right]$$

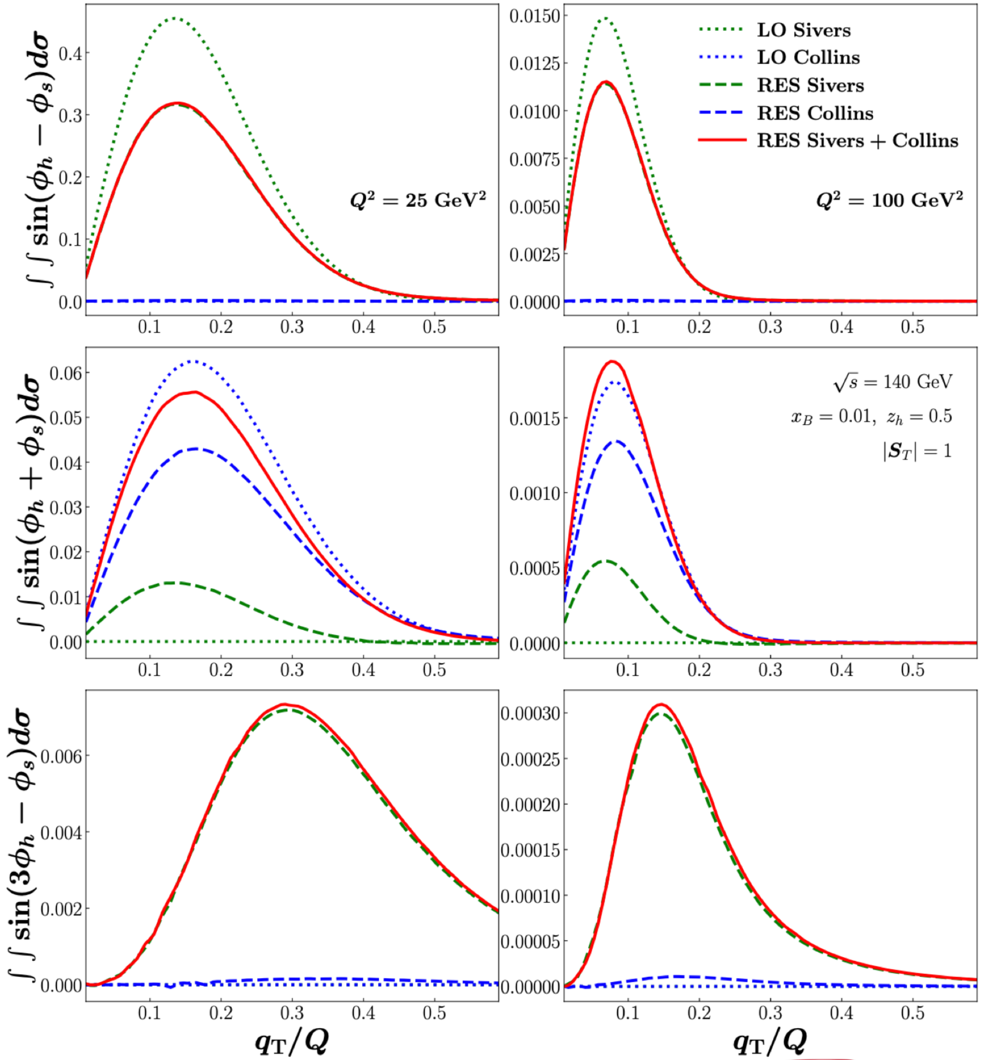
$$+ |\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+ \varepsilon\sin(\phi_h+\phi_S) F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S) F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+ \sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S) F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+ |\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S) F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$

$$+ \sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S) F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$



Impact of $q \to \hat{q}(\xi,\zeta)$ *!*

Jefferson Lab

Summary and Outlook

- ❑ **Collision induced QED radiation is an integral part of the lepton-hadron collision**
  - **Radiative correction approach is difficult for a consistent treatment beyond the inclusive DIS**
  - **No well-defined photon-hadron frame, if we cannot recover all QED radiation**
  - **Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC**
- ❑ **Factorization approach to include both QCD and QED radiative contributions provides a consistent and controllable approximation**
  - **QED radiation is a part of production cross sections, treated in the same way as QCD radiation from quarks and gluons**
  - **No artificial and/or process dependent scale(s) introduced for treating QED radiation, other than the standard factorization scale, universal lepton distribution and fragmentation functions**
  - **All perturbatively calculable hard parts are IR safe for both QCD and QED**
  - **All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)**

Thank you!

More work are needed!

Special thanks to experimental colleagues at JLab for helpful discussions!

We believe we have the right Theory, …

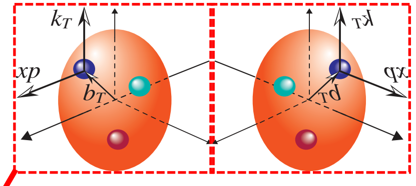
- **QCD – A theory of quarks & gluons:**

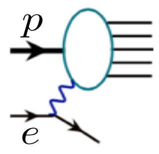
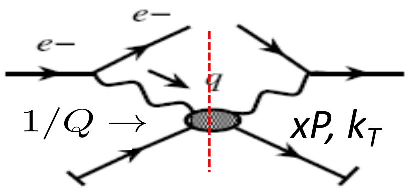
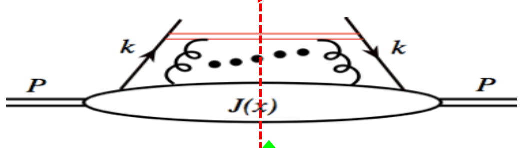
$$\mathcal{L}_{QCD}(\psi,A)=\sum_f \overline{\psi}_i^f\left[(i\partial_\mu\delta_{ij}-gA_{\mu,a}(t_a)_{ij})\gamma^\mu-m_f\delta_{ij}\right]\psi_j^f-\frac{1}{4}\left[\partial_\mu A_{\nu,a}-\partial_\nu A_{\mu,a}-gC_{abc}A_{\mu,b}A_{\nu,c}\right]^2$$

But, we saw none of them directly !!!

- **Try to "see" quarks & gluons indirectly – QCD Factorization:**

Approximation at Feynman diagram level



$\sigma_{\rm DIS}(x,Q^2) = \Big|$  $\Big|^2 \approx$  $\otimes$  $+\, O\left(\frac{1}{QR}\right)$

Factorization Theory	Hard-part Probe	Parton-distribution "Structure"	Power corrections Approximation

- **Effective field theory (EFT) – *Approximation at the Lagrangian level*:**

  Soft-collinear effective theory (SCET), Non-relativistic QCD (NRQCD), Heavy quark EFT, chiral EFT(s), …

- **Lattice QCD – *Approximation mainly due to computer power*:**

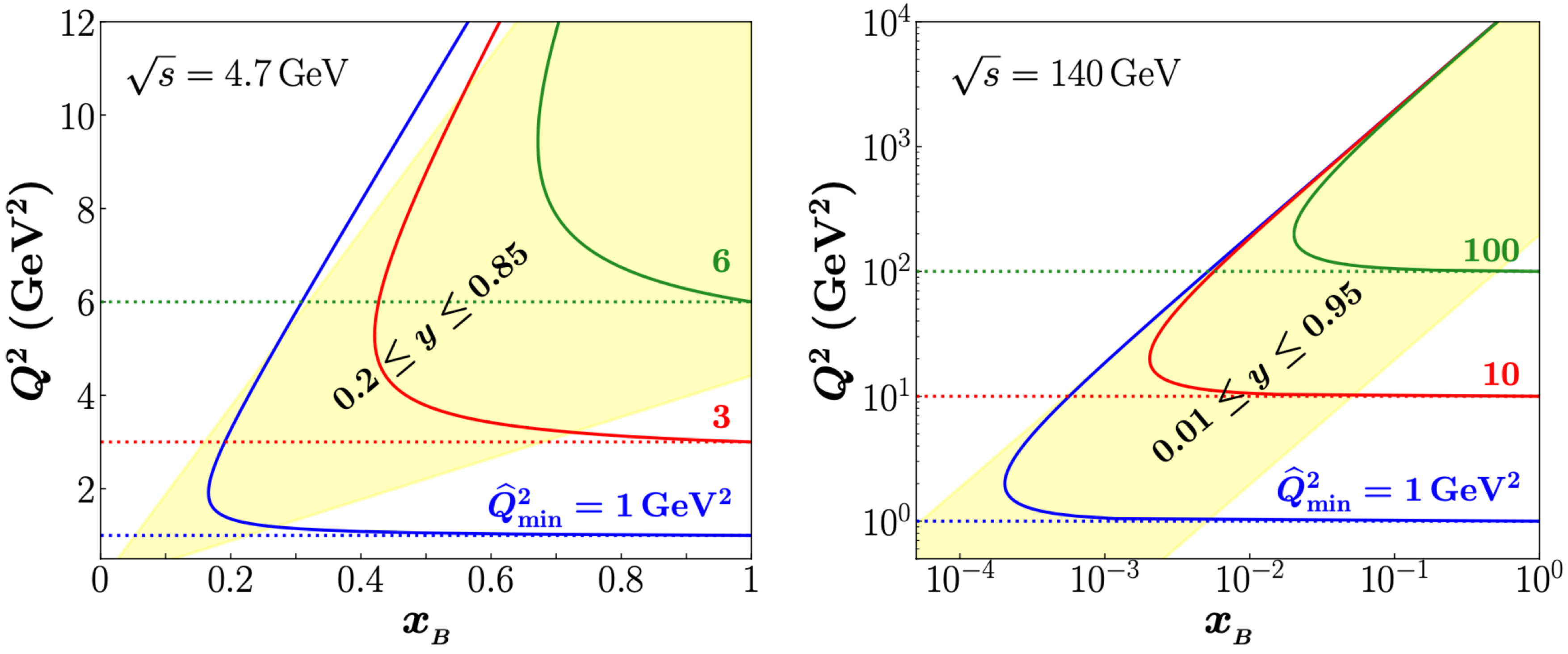
  Hadron spectroscopy, phase shift, nuclear structure, *hadron structure (with pQCD factorization)*, …

Inclusive lepton-hadron deep inelastic scattering (DIS)

- **QED radiation effectively reduces the reach of the "hard" probe:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$x_B \to \hat{x}_B \in [x_B, 1] \qquad \widehat{Q}^2_{\rm min} = Q^2\,\frac{(1-y)}{(1-x_B\,y)} \qquad \widehat{Q}^2_{\rm max} = Q^2\,\frac{1}{(1-y+x_B\,y)}$$



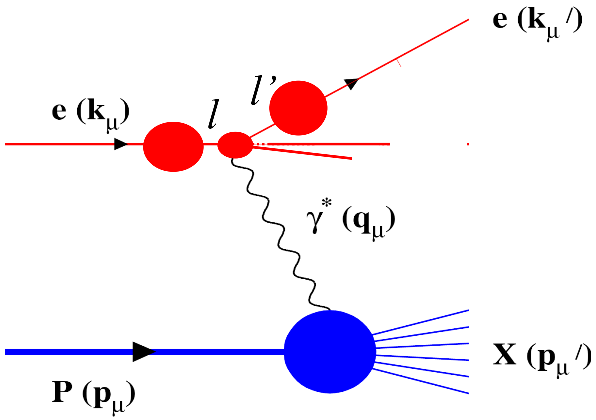
For example, for $Q^2 > 3$ GeV^2, amount of the reach to the small-x regime is significant (red curves)!

Smaller x, more phase space for radiation, both QCD and QED!

Inclusive lepton-hadron deep inelastic scattering (DIS)

❑ Collinear factorization with the "one-photon" approximation:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

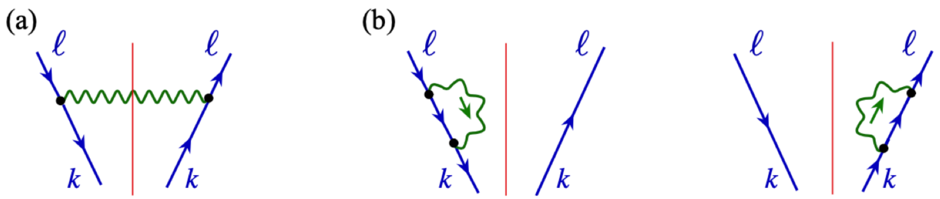


$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B dy} \approx \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^{1} d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right] \times \frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2 F_1(\hat{x}_B,\widehat{Q}^2) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

Without QED radiation: $\xi = \zeta = 1$

$$f_{e/e}(\xi) \approx f^{(0)}_{e/e}(\xi) = \delta(\xi-1) \quad \text{and} \quad D_{e/e}(\zeta) \approx D^{(0)}_{e/e}(\zeta) = \delta(\zeta-1)$$

With QED radiation:



$$f^{(1)}_{e/e}(\xi,\mu^2) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$$

+ nonperturbative contributions …

Lepton evolution – e.g., valence:

$$\mu^2\frac{\mathrm{d}}{\mathrm{d}\mu^2} f_{e/e}(\xi,\mu^2) = \int_\xi^1 \frac{\mathrm{d}\xi'}{\xi'} P_{ee}\left(\frac{\xi}{\xi'},\alpha\right) f_{e/e}(\xi',\mu^2)$$

Lepton fragmentation function:

$$D^{(1)}_{e/e}(\zeta,\mu) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+$$

+ nonperturbative contributions …