New $^{16}\text{O}$ Evaluation Based on R-Matrix Analysis of the $^{17}\text{O}$ System

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T-2

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Outline

• Reminder of R-matrix properties, EDA code
• Status of the $^{17}$O system analysis and $^{16}$O evaluation
  – Low-energy scattering cross sections
  – $^{13}$C($\alpha$,n) and $^{16}$O(n,$\alpha_0$) cross sections
  – Fits, data renormalizations, etc.
  – Extension of the evaluation to higher energies
• Summary and conclusions
R-matrix Formalism

\[ H + \mathcal{L}_B \]

compact, hermitian operator with real, discrete spectrum; eigenfunctions in Hilbert space

\[ \psi^+ = (H + \mathcal{L}_B - E)^{-1} \mathcal{L}_B \psi^+ \]

INTERIOR (Many-Body) REGION
(Microscopic Calculations)

ASYMPTOTIC REGION
(S-matrix, phase shifts, etc.)

\[ \langle r_c' | \psi^+_c \rangle = -I_c' (r_c') \delta_{c'c} + O_c' (r_c') S_{c'c} \]
or equivalently,

\[ \langle r_c' | \psi^+_c \rangle = F_{c'} (r_c') \delta_{c'c} + O_{c'} (r_c') T_{c'c} \]

Measurements

\[ \mathcal{L}_B = \sum_c c \left( \frac{\partial}{\partial r_c} - B_c \right) \]

\[ (r_c | c) = \frac{\hbar}{\sqrt{2 \mu_c a_c}} \frac{\delta(r_c - a_c)}{r_c} \left[ (\phi_{\mu_1}^{\mu_2} \otimes \phi_{\mu_2}^{\mu_1})^\mu_v \otimes Y^m_l (\hat{r}_c) \right]^M_J \]

\[ R_{c'c} = (c' | (H + \mathcal{L}_B - E)^{-1} | c) = \sum_{\lambda} \frac{(c' | \lambda)(\lambda | c)}{E_\lambda - E} \]
R-Matrix Theory Enforces Basic Properties of Scattering Theory

1) Unitarity \((SS^\dagger = S^\dagger S = 1)\): enforced by \(R_B\) being real and symmetric \((H + L_B^\dagger\text{ hermitian})\).

2) Reciprocity (TRI): enforced by the symmetry of \(R_B\) and all asymptotic matrices (such as \(S\)) derived from it.

3) Causality: no poles of \(S\) in upper-half \(k\)-plane. Poles of \(R_L\) are all in the lower half-plane, at \(k = k_0\) and \(-k_0^*\).

Note that the MLBW approximation violates all of these basic principles.
Scheme and Properties of the EDA Code

Energy Dependent Analysis Code

- Accommodates general (spins, masses, charges) two-body channels
- Uses relativistic kinematics and R-matrix formulation
- Calculates general scattering observables for $2 \rightarrow 2$ processes
- Has rather general data-handling capabilities (but not as general as, e.g., SAMMY)
- Uses modified variable-metric algorithm that gives parameter covariances at a solution

R-matrix:
$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{c\lambda} \gamma_{c'\lambda}}{E_{\lambda} - E}$$

T- (or S-) matrix elements

Scattering observables using Wolfenstein trace formalism.

Data-related parameters: normalizations energy shifts

compare ($\chi^2$)

Experimental data for all reactions

Adjust parameters for minimum $\chi^2$
# R-Matrix Analysis of Reactions in the $^{17}$O System

<table>
<thead>
<tr>
<th>channel</th>
<th>$a_c$ (fm)</th>
<th>$l_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n + ^{16}$O</td>
<td>4.4</td>
<td>4</td>
</tr>
<tr>
<td>$\alpha + ^{13}$C</td>
<td>5.4</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Energies (MeV)</th>
<th># data points</th>
<th>Data types</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O$(n,n)^{16}$O</td>
<td>$E_n = 0 – 7$</td>
<td>2540</td>
<td>$\sigma_T$, $\sigma(\theta)$, $P_n(\theta)$</td>
</tr>
<tr>
<td>$^{16}$O$(n,\alpha)^{13}$C</td>
<td>$E_n = 2.35 – 5$</td>
<td>672</td>
<td>$\sigma_{\text{int}}$, $\sigma(\theta)$, $A_n(\theta)$</td>
</tr>
<tr>
<td>$^{13}$C$(\alpha,n)^{16}$O</td>
<td>$E_\alpha = 0 – 5.4$</td>
<td>870</td>
<td>$\sigma_{\text{int}}$</td>
</tr>
<tr>
<td>$^{13}$C$(\alpha,\alpha)^{13}$C</td>
<td>$E_\alpha = 2 – 5.7$</td>
<td>1168</td>
<td>$\sigma(\theta)$</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td>5250</td>
<td>8</td>
</tr>
</tbody>
</table>

$\chi^2$ per degree of freedom = 1.68
## Total Cross Section Data

<table>
<thead>
<tr>
<th>Authors (n,n):</th>
<th>Energy Range</th>
<th>Energy Shift</th>
<th>Normalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schneider</td>
<td>0.0253 eV</td>
<td>0</td>
<td>1.0 (fixed)</td>
</tr>
<tr>
<td>Dilg,Koester,Block</td>
<td>0.13 – 23.5 keV</td>
<td>0</td>
<td>1.0 (fixed)</td>
</tr>
<tr>
<td>Ohkubo (corr. for H)</td>
<td>0.8 – 935 keV</td>
<td>0</td>
<td>0.9989</td>
</tr>
<tr>
<td>Johnson &amp; Fowler (including LOX)</td>
<td>49 – 3139 keV</td>
<td>0</td>
<td>0.9799</td>
</tr>
<tr>
<td>Cierjacks et al.</td>
<td>3.143 – 7.0 MeV</td>
<td>0</td>
<td>1.0378</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Authors (α,n):</th>
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<th>Energy Shift</th>
<th>Normalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drotleff et al.</td>
<td>346 – 1389 keV</td>
<td>0</td>
<td>1.0 (fixed)</td>
</tr>
<tr>
<td>Heil et al.</td>
<td>416–899 keV</td>
<td>0</td>
<td>1.0 (fixed)</td>
</tr>
<tr>
<td>Kellogg</td>
<td>445–1045 keV</td>
<td>0</td>
<td>1.506</td>
</tr>
<tr>
<td>Bair and Haas</td>
<td>0.997–5.402 MeV</td>
<td>-4 keV</td>
<td>0.9410</td>
</tr>
</tbody>
</table>
Integrated (total) Cross Sections

\[ \sigma_T (b) \]

\[ E_n (MeV) \]

\[ E_\alpha (MeV) \]

\[ ^{13}C(\alpha,n) \]
n$^{16}$O Total Cross Section

$\sigma_T (\text{b})$

$E_n \ (\text{MeV})$

- total_exp
- total
- $1/2^+$
- $3/2^-$
- $1/2^-$
- $5/2^+$
- $3/2^+$
- $7/2^-$
- $5/2^-$
- $9/2^+$
- $7/2^+$
n$^+$16O Elastic Scattering Cross Section

The graph shows the elastic scattering cross section ($\sigma_{el}$) in barns (b) as a function of energy ($E_n$) in electron volts (eV). The recommended value of Kopecky and Plompen is indicated by a shaded area. Various data sets are represented by different symbols and lines:

- ENDF/B-VII.1
- Cielo 7/14
- Cielo 3/16
- Kopecky-Plompen
- Schneider `76
- Dilg `71
- Koester `90
- Block `75
- Ohkubo
- Johnson X 0.98

The graph includes a linear scale for energy from 0.01 to 10$^5$ eV and a logarithmic scale for the cross section from 3.6 to 3.9 barns.
Giorginis’ Analysis of (α,n) Measurements

• Considered two measurements, Bair and Haas (B&H73) and Harissopulos et al. (Har05).

• Determined a preliminary cross-section scale for B&H73 based on the integral of the thick-target yield over the narrow resonance at 1.056 MeV that agrees with the published scale of Har05.

• Then applied a correction common to both data sets related to characterization of the $^{13}$C target that gives the cross-section scales $0.95 \times \text{B&H73}$ and $\sim 1.42 \times \text{Har05}$.

• Considers the relative shape of the B&H73 measurement to be the most accurate since it had the thinnest target.
$^{13}\text{C}(\alpha,n)^{16}\text{O}$ Cross Section

\[ \sigma_{\alpha,n}(b) \]

\[ E_{\alpha} \text{ (MeV)} \]

- calc
- B&H73 x .94
- Har05 x 1.42
$^{16}\text{O}(n,\alpha_0)^{13}\text{C}$ Cross Section

$\sigma_{n,\alpha}(b)$ vs $E_n$ (MeV) graph showing:
- Cielo 3-16
- ENDFVII.1
- IRMM07

R-matrix calculation matched to IRMM data at ~6.3 MeV.
n$^{+16}$O Elastic Cross Section

\begin{figure}
\centering
\includegraphics[width=\textwidth]{n+_16_O_cross_section.pdf}
\end{figure}
n$^{16}$O Total Cross Section

![Graph showing n$^{16}$O Total Cross Section](image)
Capture Cross Section

\[ \sigma_{n,\gamma}(b) \]

\[ E_n \text{ (MeV)} \]

+ Cielo 3/16

--- ENDF/B VII.1
Summary and Conclusions

- R-matrix descriptions are constrained by fundamental properties (unitarity, causality, TRI) of nuclear reaction theory.
- EDA analysis of the $^{17}$O system includes data from all possible reactions, giving results that are highly constrained by the properties above (especially unitarity).
- The low-energy $n+^{16}$O scattering cross sections are now in better agreement with high-precision measurements, and the $(n,\alpha_0)$ cross section agrees with the data of B&H73, IRMM07 (Giorginis).
- The evaluated $^{16}$O file Cielo 3/16 extends to 150 MeV, and is the same as ENDF/B VII.1 above 9 MeV (except for capture).