Spin decomposition in Charmonium

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Outline

① **Background and introduction**

② **Lattice calculation of quark spin and gluon total angular momentum**

③ **Summary**

Spin decomposition in QCD

- **Jaffe-Manohar decomposition** R. Jaffe and A. Manohar, NPB 337, 509 (1990)

Gauge dependent

$$J = \frac{1}{2}\sum_q \Delta_q + \boxed{\sum_q \mathscr{L}_q + \Delta_G + L_G}$$

quark spin

quark orbital angular momentum (OAM)

Gluon spin

Gluon OAM

Gauge indenpendent gluon spin

X. Chen, X. Lv, W. Sun, F. Wang and T. Goldman PRL 100, (2008)
X. Ji, J. Zhang and Y. Zhao, PRL, 111 (2013)
Y. Yang, et. al (χQCD Collaboration), PRL. 118 (2017)

- **Ji's decomposition** X. Ji, Phys. Rev. Lett. 78, 610 (1997)

$$J = \frac{1}{2}\sum_q \Delta_q + \sum_q L_q + J_g$$

Gauge invariant

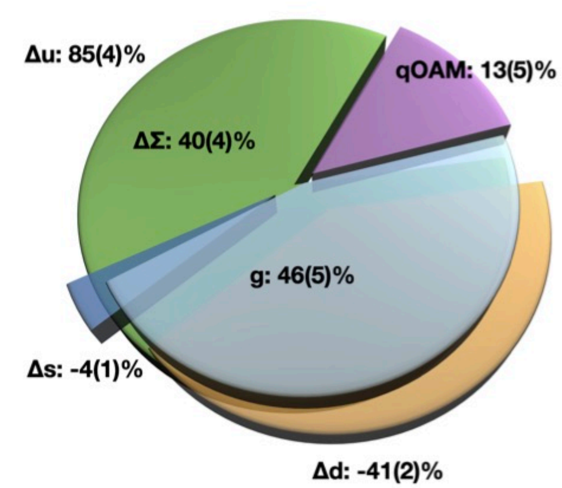
quark spin

quark orbital angular momentum (OAM)

Gluon spin+OAM

Nucleon spin decomposition

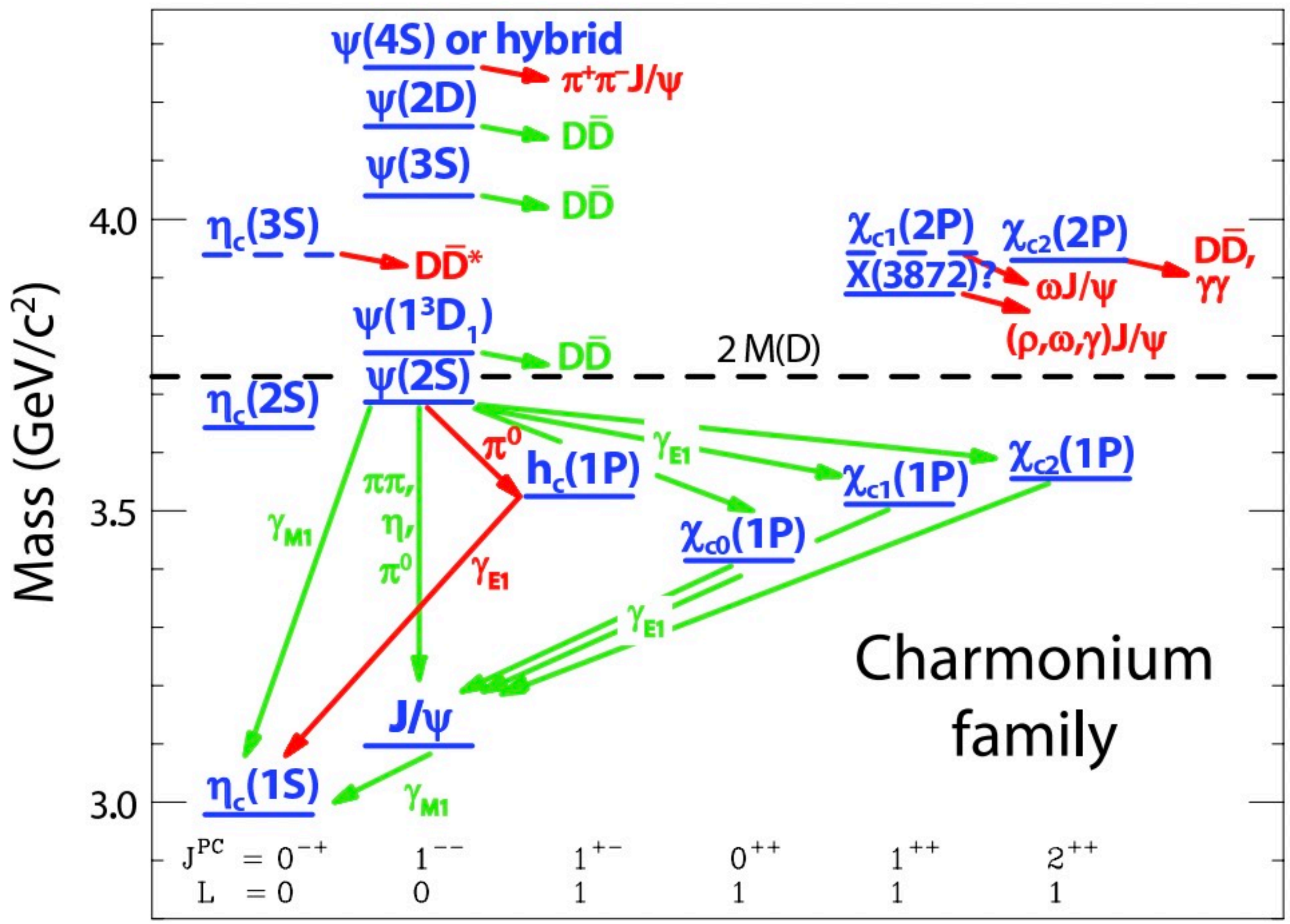
C. Alexandrou, et. al ,PRL. 119 (2017)
G. Wang, et. al, (χQCD Collaboration), *PRD* 106 (2022)



Charmonium

Known charmonium states and candidates

Rev.Mod.Phys. 80 (2008)



Naive spin decomposition of charmonium

- **The quantum number of charmonium (J^{PC})**

				exotic state
J=1	1^{--}	1^{+-}	1^{++}	1^{-+}
J=2	2^{--}	2^{-+}	2^{++}	2^{+-}

$\bar{Q}Q$

$P = (-1)^{L+1}$

$C = (-1)^{L+S}$

- **The naive spin decomposition in quark model**

1^{--}	S=1	L=0
1^{+-}	S=0	L=1
1^{++}	S=1	L=1

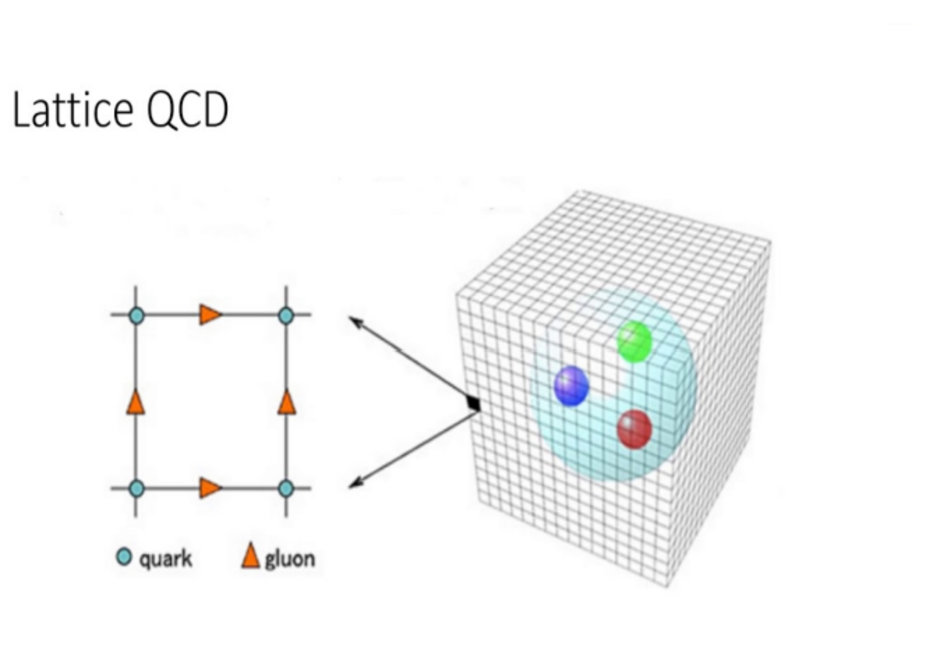
1.Are the predictions of quark model comparable with QCD?

2.How about the contribution of gluon?

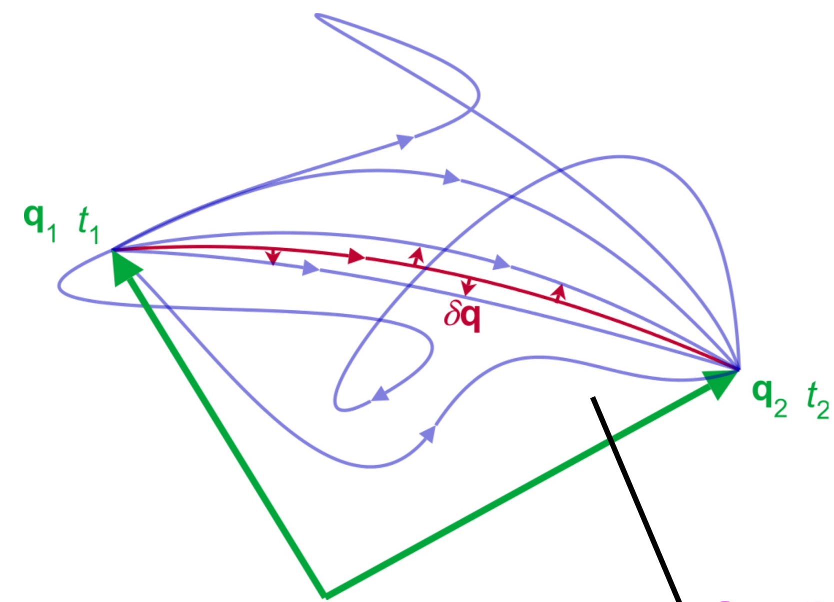
3.What is the spin structure of exotic states?

Lattice QCD

- **In lattice QCD method, the correlation functions are non-perturbatively calculated using path integral.**



Discretization the QCD action in Euclidean space



Configurations (specific path)

$$\langle O \rangle = \frac{\int D[U] e^{-S[U]} O[U]}{\int D[U] e^{-S[U]}} \approx \frac{1}{N} \sum_i O[U_i]$$

The configurations are distributed according to $\dfrac{e^{-S[U]}}{\int D[U] e^{-S[U]}}$

Outline

① **Background and introduction**

② **Lattice calculation of quark spin and gluon total angular momentum**

ensemble	$L^3 \times T$	a (fm)	m_π (MeV)	$m_c a$	$N_{\rm cfg}$
32I	$32^3 \times 64$	0.0828(3)	300	0.493	305

③ **Summary**

Quark spin in charmonium (spin one)

- **The quark spin operator sandwiched by different hadron external state.**

Quark spin operator $O_{\Sigma_q} = \sum_x \bar{q}\gamma_z\gamma_5 q(x)$

Spin basis

$$V_+ = \frac{1}{\sqrt{2}}(V_x + iV_y)$$

$$V_0 = V_z$$

$$V_- = \frac{1}{\sqrt{2}}(V_x - iV_y)$$

$$\langle V_+ | O_{\Sigma_q} | V_+ \rangle = -\langle V_- | O_{\Sigma_q} | V_- \rangle \neq 0$$

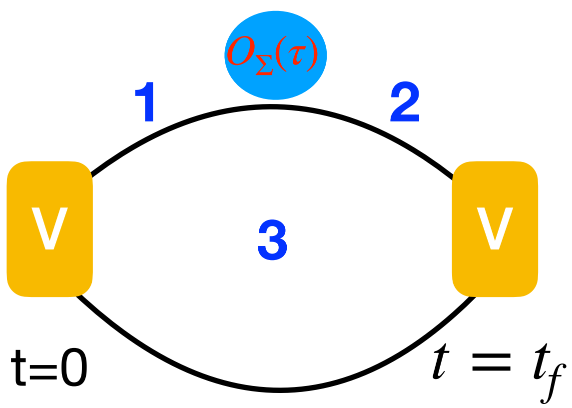
$$\langle V_0 | O_{\Sigma_q} | V_0 \rangle = 0$$

$$\boxed{\begin{gathered} \underline{\langle V_x | \bar{q}\gamma_z\gamma_5 q | V_y \rangle} = -\underline{\langle V_y | \bar{q}\gamma_z\gamma_5 q | V_x \rangle} \neq 0 \\ \underline{\langle V_z | \bar{q}\gamma_z\gamma_5 q | V_z \rangle} = 0 \end{gathered}}$$

The matrix elements we calculate from Lattice QCD

Lattice results

- **The ratio of three point correlation function to two point correlation function**



$$\frac{\langle V_y(t_f) O_\Sigma(\tau) V_x(0)\rangle}{\langle V_x(t_f) V_x(0)\rangle} = \frac{\sum_{n,m} \langle V_y(0) \,|\, n\rangle \langle n \,|\, O_\Sigma(0) \,|\, m\rangle \langle m \,|\, V_x(0)\rangle e^{-E_n(t_f-\tau)-E_m\tau}}{\sum_n \langle V_x(0) \,|\, n\rangle \langle n \,|\, V_x(0)\rangle e^{-E_n t_f}}$$

$$= \boxed{\langle V \,|\, O_\Sigma \,|\, V\rangle} + \boxed{Ae^{-\delta m(t_f-\tau)} + Be^{-\delta m\,\tau} + De^{-\delta_m t_f} + \ldots}$$

$$\delta_m = E_1 - E_0$$

Matrix element of ground state

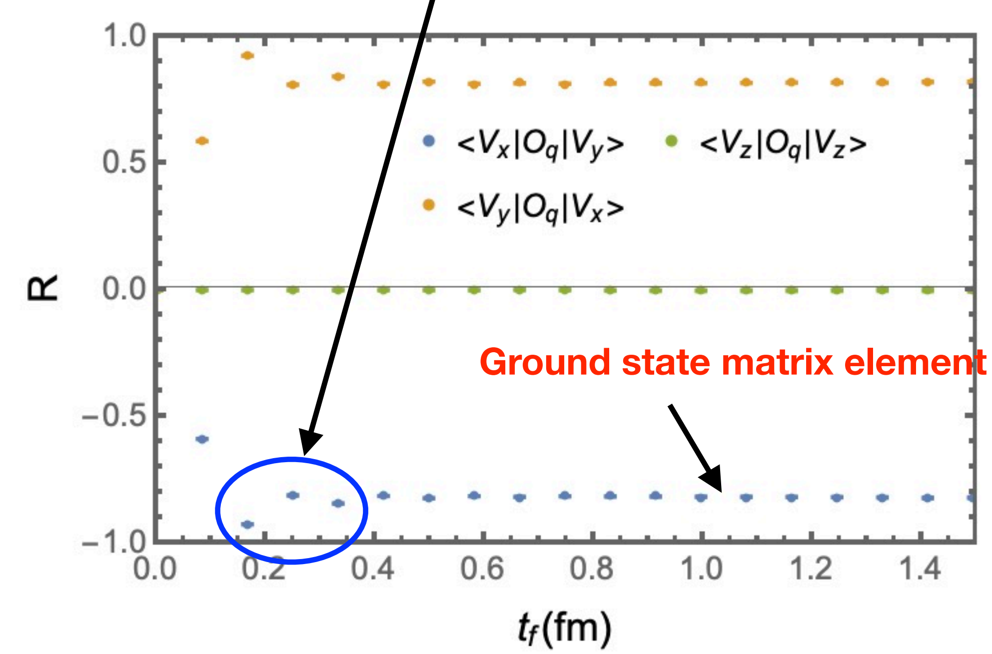
The contamination of excited state

Sum the results of timeslices from 0 to t_f

$$\sum_{0<\tau<t_f} \frac{\langle V_y(t_f) O_\Sigma(\tau) V_x(0)\rangle}{\langle V_x(t_f) V_x(0)\rangle} = (t_f - 1) \times \langle V \,|\, O_\Sigma \,|\, V\rangle \; + \; O(e^{-\delta m\, t_f}) + C$$

The contamination of excited state are suppressed at large t_f

$$R = \sum_{0<t<t_f} \frac{\langle V_y(t_f) O_\Sigma(t) V_x(0)\rangle}{\langle V_x(t_f) V_x(0)\rangle} - \sum_{0<t<t_f-1} \frac{\langle V_y(t_f-1) O_\Sigma(t) V_x(0)\rangle}{\langle V_x(t_f-1) V_x(0)\rangle} \approx \langle V \,|\, O_\Sigma(0) \,|\, V\rangle$$



Quark spin in charmonium (spin two)

- **The irreducible representation of of spin-2 charmonium can be converted to the spin basis** $S_M^{J=2} = \langle 1, m_1; 1, m_2 | J, M \rangle \bar{\psi} \gamma_{m_1} D_{m_2} \psi$

Irreducible representation (2^{++})

J. Dudek, et. al, PRD 77 (2008) 034501

T_2^x	$\lvert \epsilon_{1jk} \rvert \gamma^j D^k / \sqrt{2}$
T_2^y	$\lvert \epsilon_{2jk} \rvert \gamma^j D^k / \sqrt{2}$
T_2^z	$\lvert \epsilon_{3jk} \rvert \gamma^j D^k / \sqrt{2}$
E^x	$Q_{1jk} \gamma^j D^k$
E^y	$Q_{2jk} \gamma^j D^k$

$$\mathbb{Q}_{111} = \frac{1}{\sqrt{2}}; \quad \mathbb{Q}_{122} = -\frac{1}{\sqrt{2}}; \quad \mathbb{Q}_{211} = -\frac{1}{\sqrt{6}};$$

$$\mathbb{Q}_{222} = -\frac{1}{\sqrt{6}}; \quad \mathbb{Q}_{233} = \frac{2}{\sqrt{3}}.$$

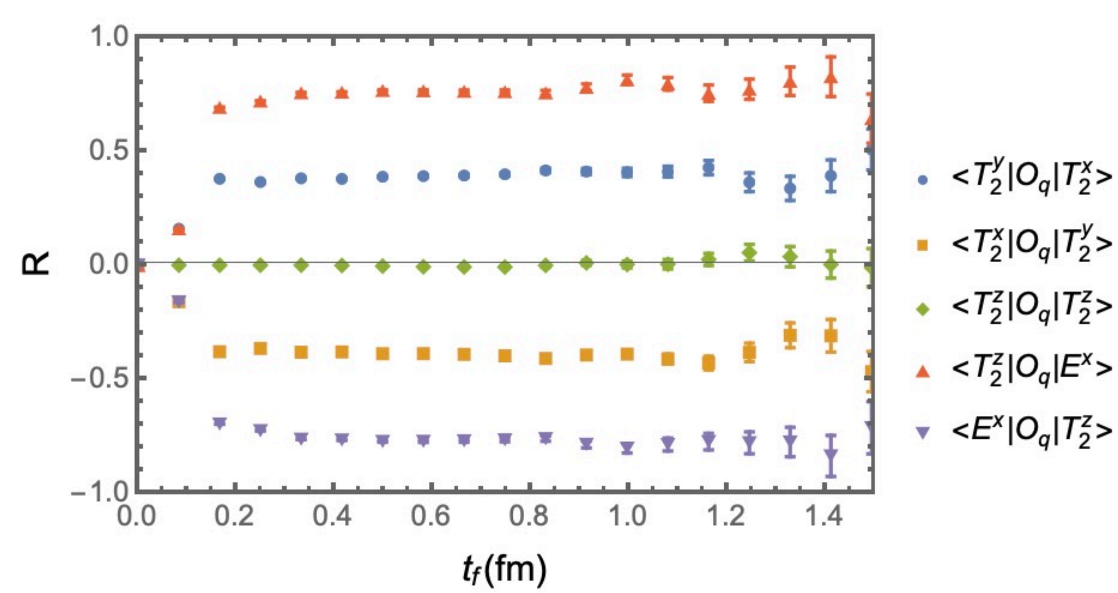
Spin basis

V_{2+}	$(iT_2^z + E^x)/\sqrt{2}$
V_{1+}	$(iT_2^y + T_2^x)/\sqrt{2}$
V_0	E^y
V_{1-}	$(-iT_2^y + T_2^x)/\sqrt{2}$
V_{2-}	$(-iT_2^z + E^x)/\sqrt{2}$

$$\langle V_{2+} | O_{\Sigma_q} | V_{2+} \rangle = 2 \langle V_{1+} | O_{\Sigma_q} | V_{1+} \rangle$$

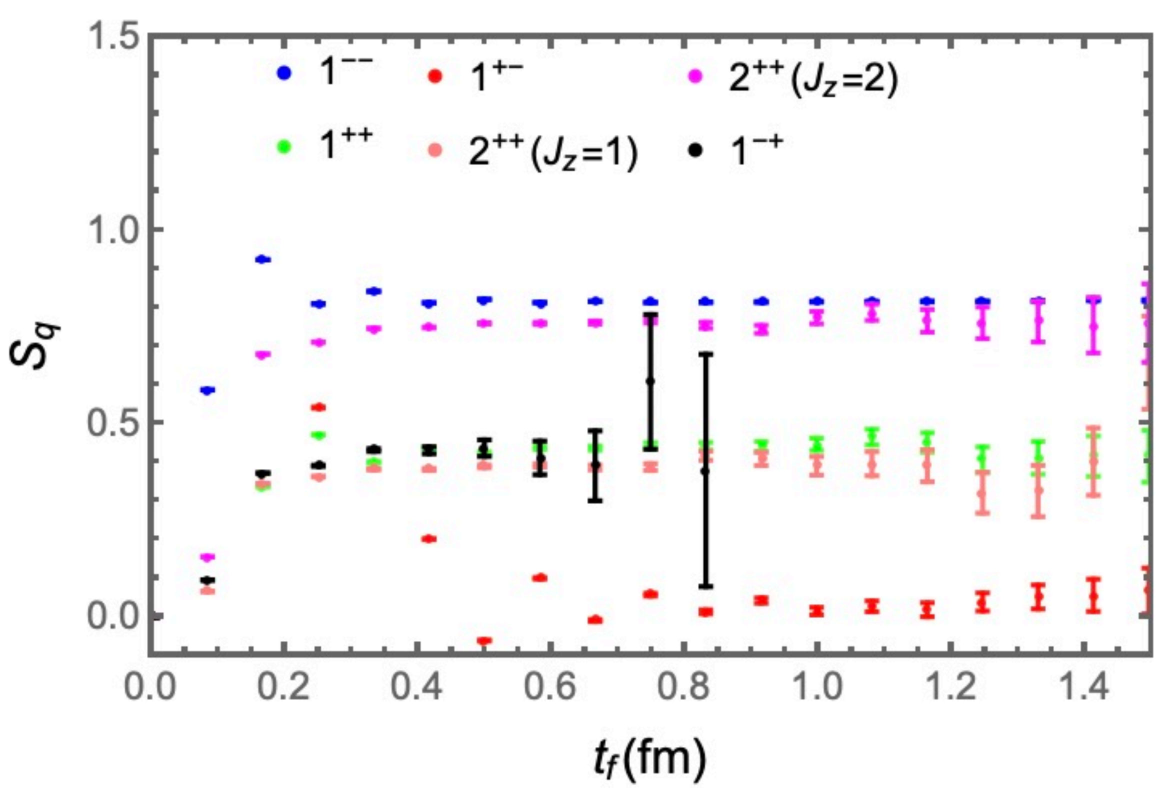
$$\boxed{\langle E^x | O_{\Sigma_q} | T_2^z \rangle = 2 \langle T_2^x | O_{\Sigma_q} | T_2^y \rangle}$$

Matrix elements of 2^{++} channel



Quark spin in different charmonium

- **The contribution of quark spin to the different charmonium spin**



The plateau at large t_f corresponds to the ground state matrix elements

Comparison with the prediction of quark model

1. The contribution of quark spin is very small in 1^{+-} (p wave), but dominantly contributes to the spin of 1^{--} (s wave) .

2. The quark spin in 1^{++} and $2^{++}(J_z = 1)$ is very close since their spin structure same (L=1, S=1).

3. The quark spin in 1^{--} and $2^{++}(J_z = 2)$ is also similar.

4. The quark spin contributes half spin of 1^{-+} exotic state.

Total angular momentum of gluon

- **Gravitational form factor (GFFs) of vector meson (moving frame)**

$$T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4} g^{\mu\nu} F^{a,\kappa\eta} F^{a,}{}_{\kappa\eta} \,.$$

M. Polyakov and B. Sun, PRD 100 (2019)

$$\begin{aligned}
\langle p', \sigma' | \hat{T}^a_{\mu\nu}(x) | p, \sigma \rangle = & \Big[2P_\mu P_\nu \Big(-\epsilon'^* \cdot \epsilon \, A_0^a(t) + \frac{\epsilon'^* \cdot P \, \epsilon \cdot P}{m^2} A_1^a(t) \Big) \\
& + 2 \left[P_\mu (\epsilon'^*_\nu \, \epsilon \cdot P + \epsilon_\nu \, \epsilon'^* \cdot P) + P_\nu (\epsilon'^*_\mu \, \epsilon \cdot P + \epsilon_\mu \, \epsilon'^* \cdot P) \right] \, J^a(t) \\
& + \frac{1}{2} (\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2) \Big(\epsilon'^* \cdot \epsilon \, D_0^a(t) + \frac{\epsilon'^* \cdot P \, \epsilon \cdot P}{m^2} D_1^a(t) \Big) \\
& + \Big[\frac{1}{2} (\epsilon_\mu \epsilon'^*_\nu + \epsilon'^*_\mu \epsilon_\nu) \Delta^2 - (\epsilon'^*_\mu \Delta_\nu + \epsilon'^*_\nu \Delta_\mu) \, \epsilon \cdot P \\
& + (\epsilon_\mu \Delta_\nu + \epsilon_\nu \Delta_\mu) \, \epsilon'^* \cdot P - 4 g_{\mu\nu} \, \epsilon'^* \cdot P \, \epsilon \cdot P \Big] \, E^a(t) \\
& + \Big(\epsilon_\mu \epsilon'^*_\nu + \epsilon'^*_\mu \epsilon_\nu - \frac{\epsilon'^* \cdot \epsilon}{2} g_{\mu\nu} \Big) \, m^2 \, \bar{f}^a(t) \\
& + g_{\mu\nu} \Big(\epsilon'^* \cdot \epsilon \, m^2 \, \bar{c}_0^a(t) \, + \, \epsilon'^* \cdot P \, \epsilon \cdot P \, \bar{c}_1^a(t) \Big) \Big] \, e^{i(p'-p)x} \;,
\end{aligned}$$

$$\Delta = p' - p$$

$$P = \frac{p + p'}{2}$$

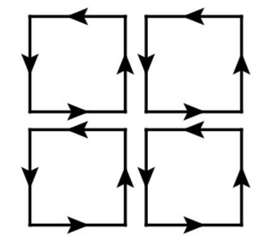
The relation between angular momentum and GFFs

- $$J^i = \epsilon^{ijk} \int d^3x T^{0k}(x) x^j$$

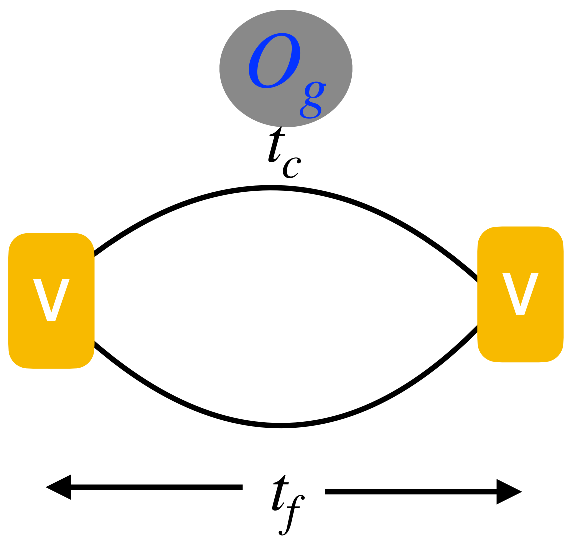
Total angular momentum = $J^a(0) + \dfrac{\bar{f}^a(0)}{2}$

Calculation of the gluon total angular momentum (AM)

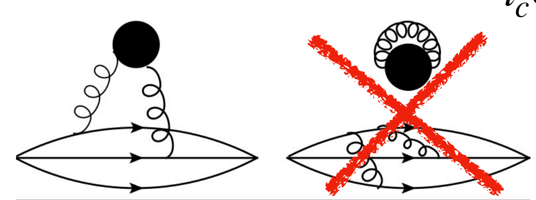
- **For the gluon AM, the 3pt correlation function can be described as**

$F^{\mu\nu} \propto$ 

1 hyp smearing

$C_3(t_f, t) =$ 

$$= \left(C_2(t_f) - \langle C_2(t_f)\rangle\right)\left(\sum_{t_c \in (o,t_f)} O_g(t_c) - \langle O_g(t_c)\rangle\right)$$



t_c: timeslice of O_g

- **We can extract the matrix element of gluon condensate at the ground state through difference between the ratio at adjacent time slice**

$$\tilde{R}(t_f, \tilde{O}) = \frac{\sum_{t_f > t > 0}\langle C_3(t_f, t, \tilde{O})\rangle}{\langle C_2(t_f)\rangle} - \frac{\sum_{t_f - 1 > t > 0}\langle C_3(t_f - 1, \tilde{O})\rangle}{\langle C_2(t_f - 1)\rangle} = \underline{\langle H|\tilde{O}|H\rangle} + \underline{\mathcal{O}(e^{-\delta m t_f})},$$

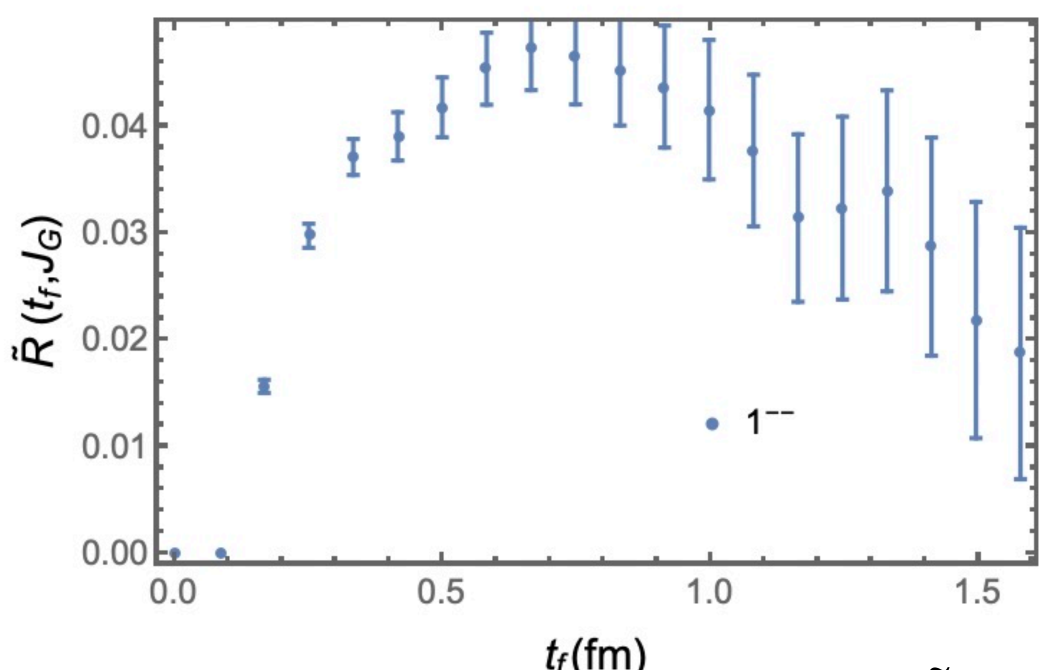
Matrix element at the ground state

The contamination of excited state

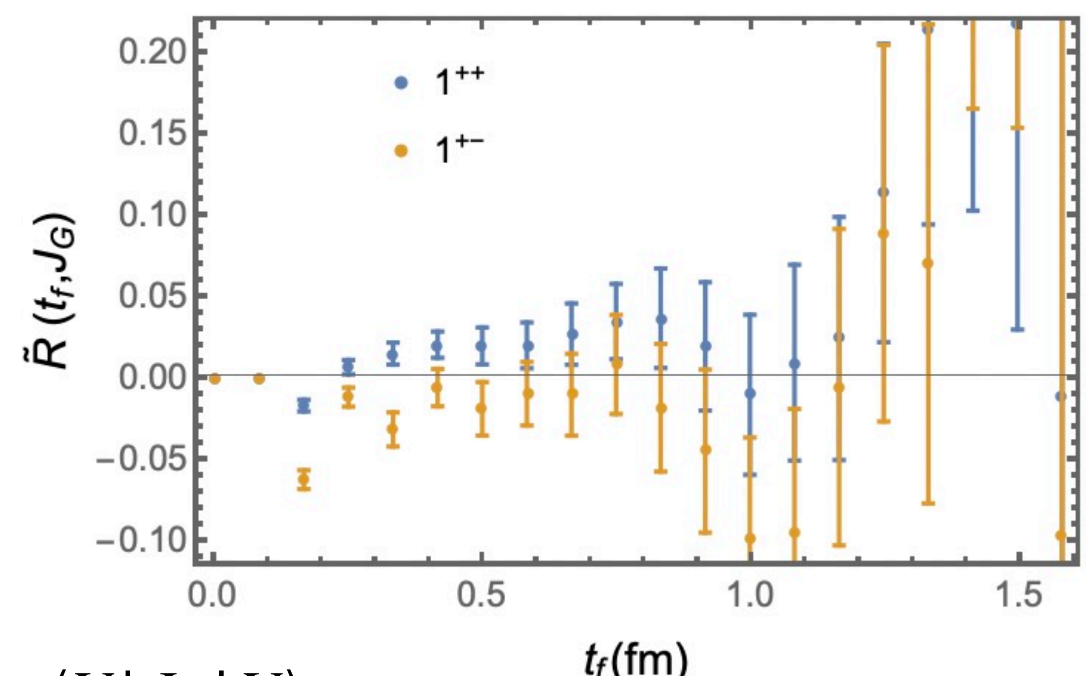
Numerical results of ratio of gluon angular momentum (Bare matrix)

- **The numerical results of gluon angular momentum**

Results of 1^{--} channel



Results of 1^{++} and 1^{+-} channel



$$\tilde{R}(t_f, J_G) \xrightarrow[\text{larger } t_f]{} \langle V | J_G | V \rangle$$

The bare matrix elements of gluon angular momentum operator in charmonium spin are very small.

Results

- **The contributions of quark spin and gluon angular momentum in different charmonium states**

$$S_q^R = Z_A S_q^B \qquad \begin{pmatrix} S_q + L_q \\ J_g \end{pmatrix}^R = \begin{pmatrix} Z_{qq} & Z_{qg} \\ Z_{gq} & Z_{gg} \end{pmatrix} \begin{pmatrix} S_q + L_q \\ J_g \end{pmatrix}^B \qquad L_q^R + S_q^R + J_g^R = 1$$

$Z_{qg} = 0.18(2)$

Renormalization constants are from

Y. Yang, et. al (χQCD Collaboration), PRL. 121 (2018)
F. He, et. al (χQCD Collaboration), 2204.09246

	S_q^R (Quark Spin)	L_q^R (Quark OAM)	J_g^R (Gluon AM)
$1^{--}(L=0, S=1)$	0.88(2)	-0.05(3)	0.17(3)
$1^{+-}(L=1, S=0)$	0.04(4)	0.81(6)	0.15(4)
$1^{++}(L=1, S=1)$	0.49(5)	0.40(6)	0.11(4)
$1^{-+}(L=?, S=?)$	0.46(11)	?	?
$2^{++}(L=1, S=1)$	0.46(8)	0.43(9)	0.11(4)

Renormalized results of gluon AM is larger than bare results due to the mixing with quark operator.

Summary

- **We studied spin decomposition in different charmonium state. The contribution of quark spin is compatible with the prediction of quark model.**

- **Though the bare matrix elements of gluon spin in charmonium is very small, the renormalized results are still large due to the mixing with quark operator.**

- **The quark spin contributes to half of spin of exotic 1^{-+} state.**

Thank you for your attention!